A computationally universal phase of quantum matter

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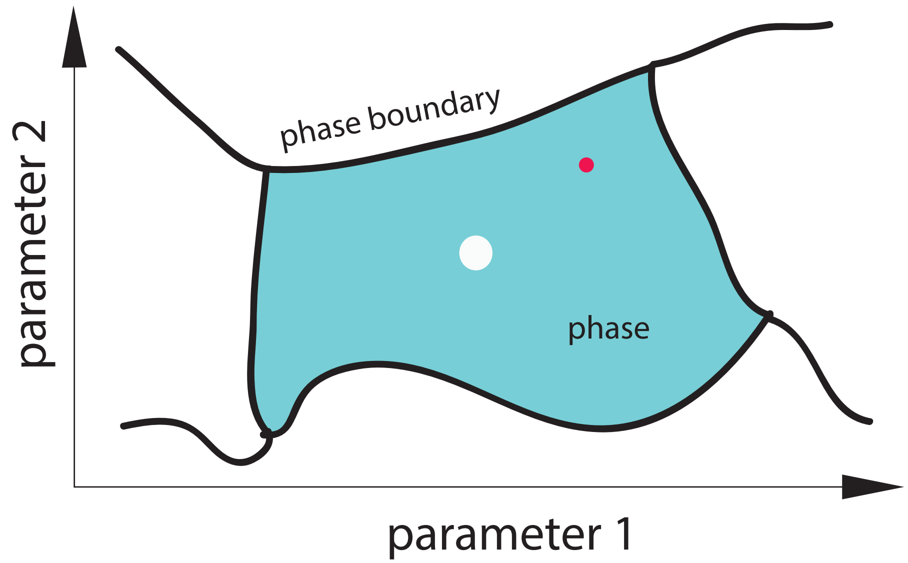
joint work with D.-S. Wang, D.T. Stephen, C. Okay, and H.P. Nautrup

The liquid phase of water

A quantum phase of spins in 2D

... which supports universal quantum computation

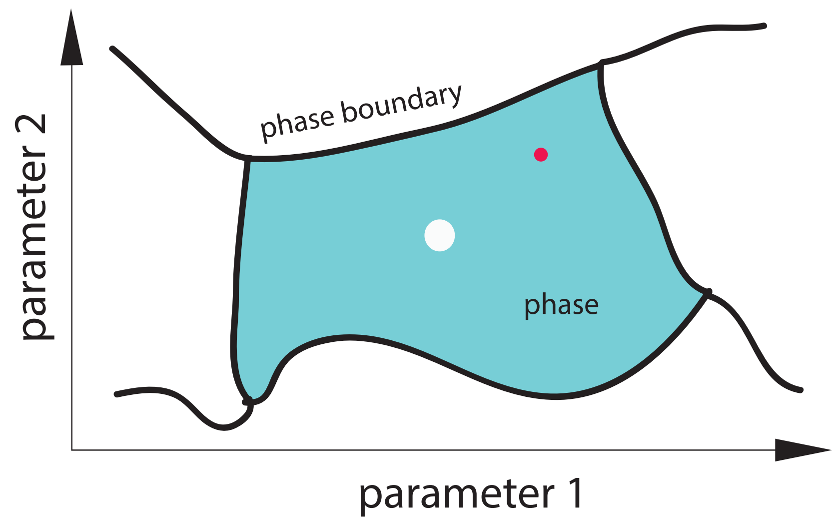


We consider:

- Phases of unique ground states of spin Hamiltonians, at $T = 0$,
- In the presence of symmetry,
- In spatial dimension 2 (a lattice of spin 1/2 particles)

A quantum phase of spins in 2D

... which supports universal quantum computation



We show: for measurement-based quantum computation,

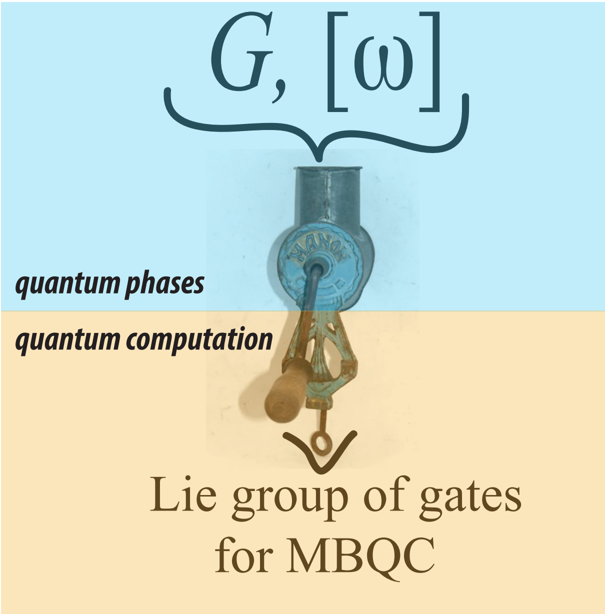
- There exists a quantum phase of matter which is universal for quantum computation
- The computational power is *uniform* across the phase.

Outline

1. "Computational phases of quantum matter":
   - Our motivation
   - Background: SPT & MBQC
   - A short history of the question

2. A computationally universal phase of matter in 2D

Motivation #1: MBQC and symmetry



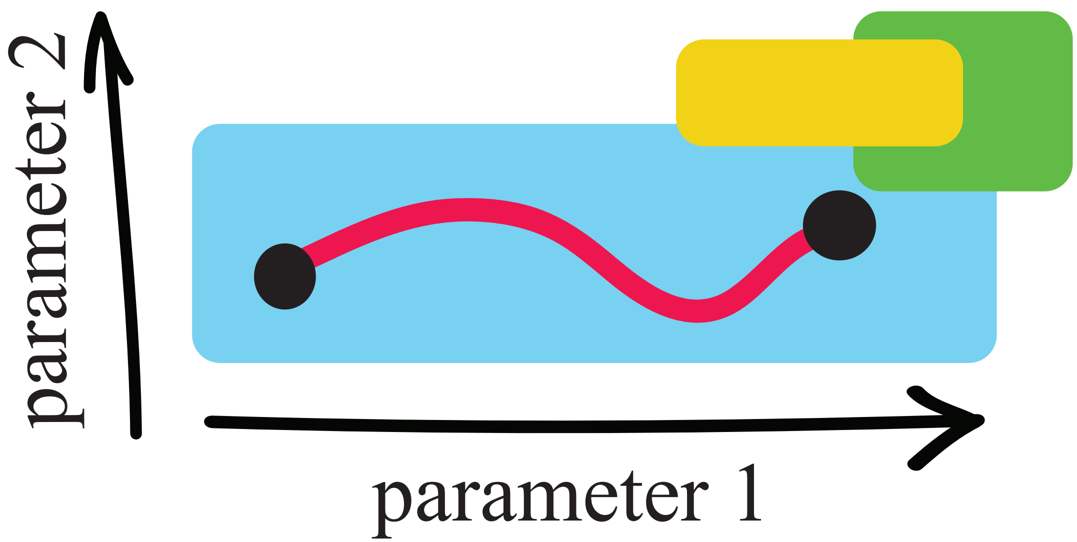
Can MBQC schemes be classified by symmetry, in a similar way as, say, elementary particles can?

If so, does this have a bearing on quantum algorithms?

Part I:

Background on SPT & MBQC

Symmetry-protected topological order

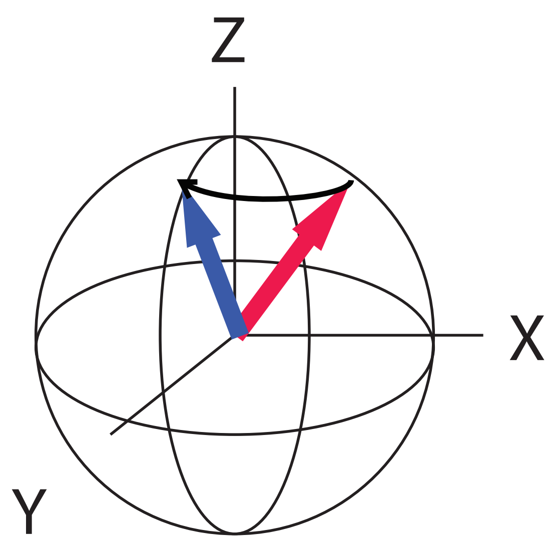


Two points in parameter space lie in the same SPT phase iff they can be connected by a path of Hamiltonians such that

1. At every point on the path, the corresponding Hamiltonian is invariant under G.
2. Along the path the energy gap never closes.

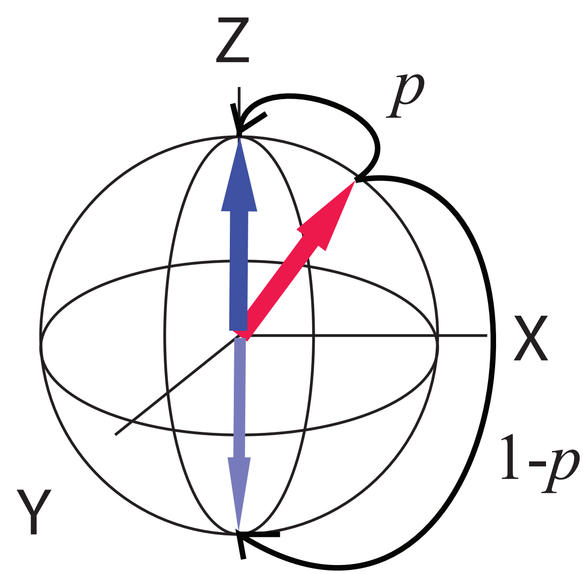
Measurement-based quantum computation

Unitary transformation



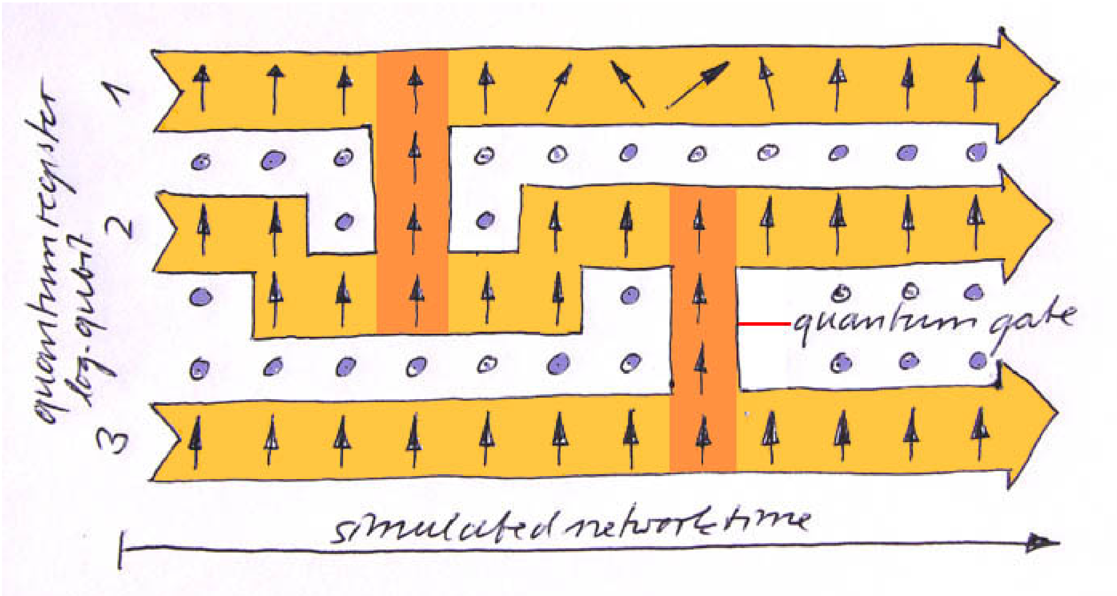
deterministic,
reversible

Projective measurement



probabilistic,
irreversible

Measurement-based quantum computation



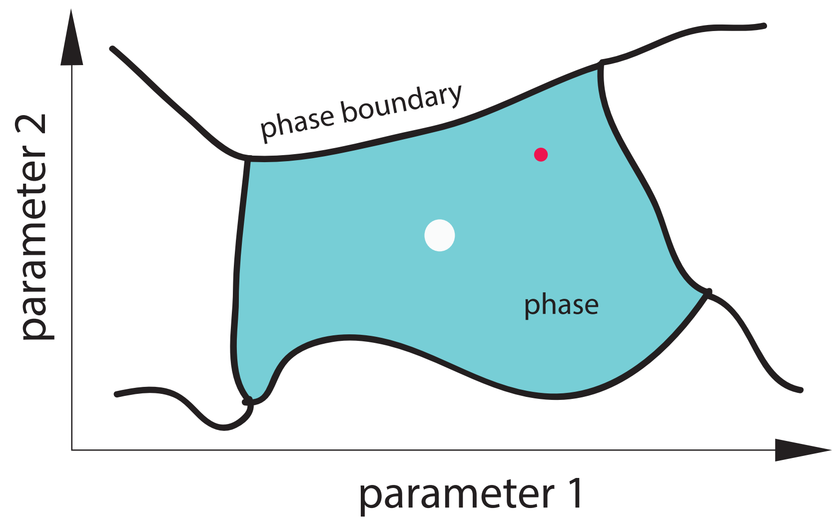
measurement of Z ($\odot$), X ($\uparrow$), $\cos\alpha\, X + \sin\alpha\, Y$ ($\nearrow$)

- Information written onto the resource state, processed and read out by one-qubit measurements only.
- Universal computational resources exist: cluster state, AKLT state.

R. Raussendorf, H.-J. Briegel, Physical Review Letters 86, 5188 (2001).

A quantum phase of spins in 2D

... which supports universal quantum computation



We show: for measurement-based quantum computation,

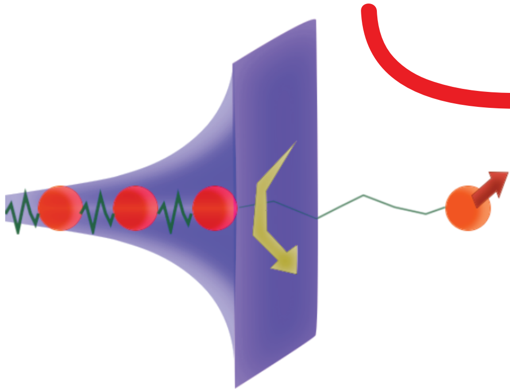
- There exists a quantum phase of matter which is universal for quantum computation
- The computational power is *uniform* across the phase.

A short history of

“computational phases of quantum matter”

1. Symmetry protects computation

we observe low-maintenance features of the ground-code MQC in that this computation is doable without an exact (classical) description of the resource ground state as well as without an initialization to a pure state. It

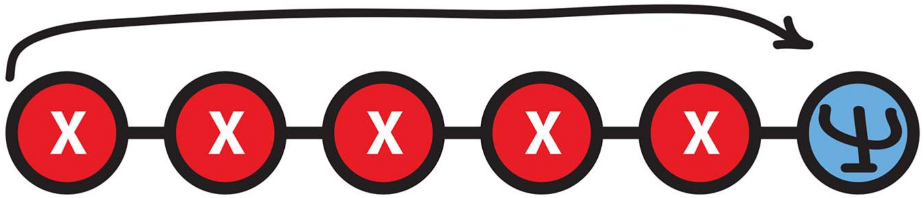


It turns out these features are deeply intertwined with the physics of the 1D Haldane phase (cf. Fig. 1), that is well characterized as the symmmetry-protected topological order in a modern perspective [6, 7]. We believe our approach must bring the study of MQC, conventionally based on the analysis of the model entangled states (e.g., [1, 8, 9]), much closer to the condensed matter physics, which is aimed to describe characteristic physics based on the Hamiltonian.

A. Miyake, Phys. Rev. Lett. 105, 040501 (2010).

2. Symmetry-protected wire in MBQC

Else Schwartz Doherty Bartlett

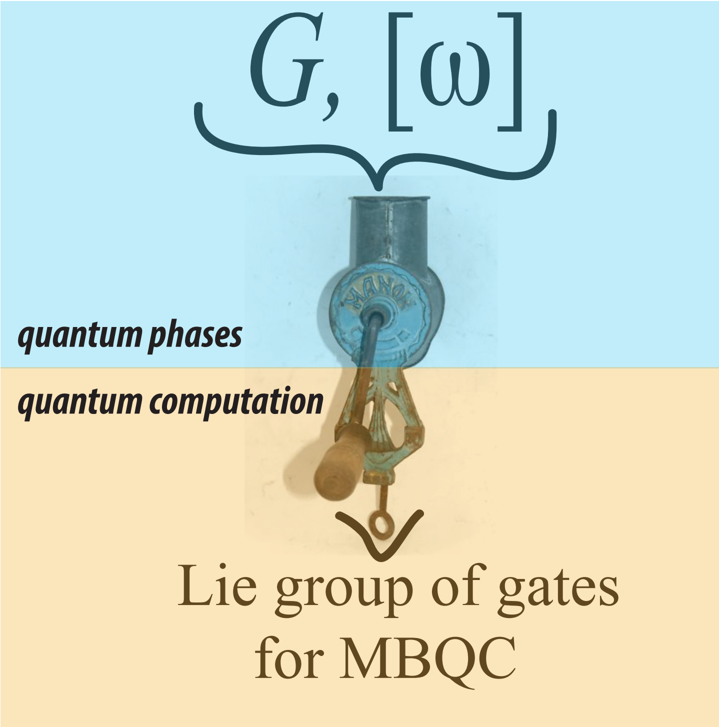


- Computational wire persists throughout symmetry-protected phases in 1D.
- Imports <u>group cohomology</u> from the classification of SPT phases.

D.V. Else, I. Schwartz, S.D. Bartlett and A.C. Doherty, PRL 108 (2012).

F. Pollmann *et al.*, PRB B 81, 064439 (2010); N. Schuch, D. Perez-Garcia, and I. Cirac, PRB 84, 165139 (2011); X. Chen, Z.-C. Gu, and X.-G. Wen, PRB 83, 035107 (2011).

3. The SPT⇒MBQC meat grinder



Hints at the classification of MBQC schemes by symmetry.

J. Miller and A. Miyake, Phys. Rev. Lett. 114, 120506 (2015) [first 1D comp. phase].

A. Prakash and T.-C. Wei, Phys. Rev. A (2016).

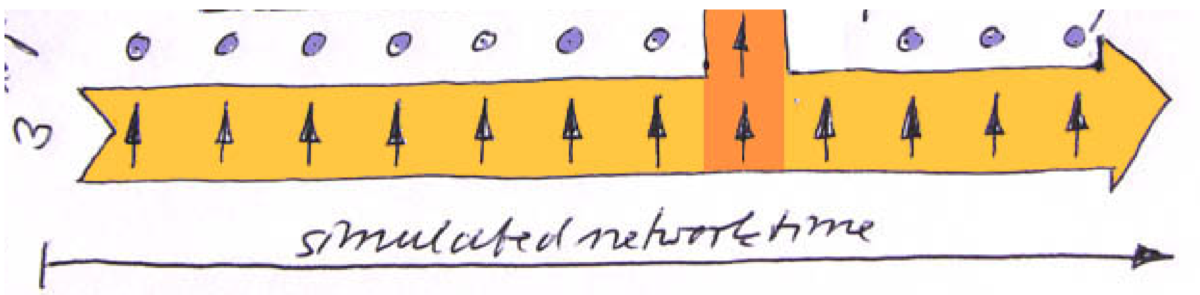
RR, A.Prakash, D.-S. Wang, T.-C.Wei, D.T. Stephen, Phys. Rev. A (2017).

Inspection

The above waypoints are about 1D systems.

1D is not sufficient for universal MBQC

here is why:



- MBQC in spatial dimension D maps to the circuit model in dimension $D-1$

$\Rightarrow$ Require $D \geq 2$ for universality.

Are there computationally universal quantum phases in two dimensions?

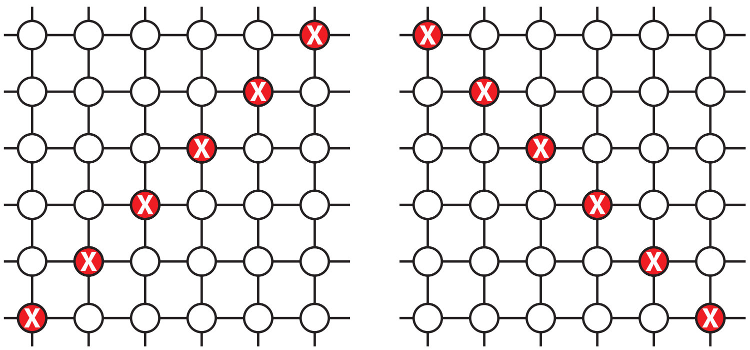
This talk describes one.

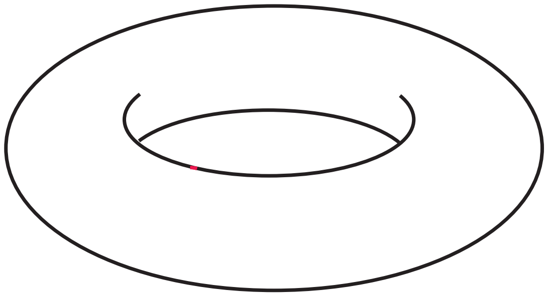
Part II:

A computationally universal SPT phase in 2D

Description of the 2D phase & result

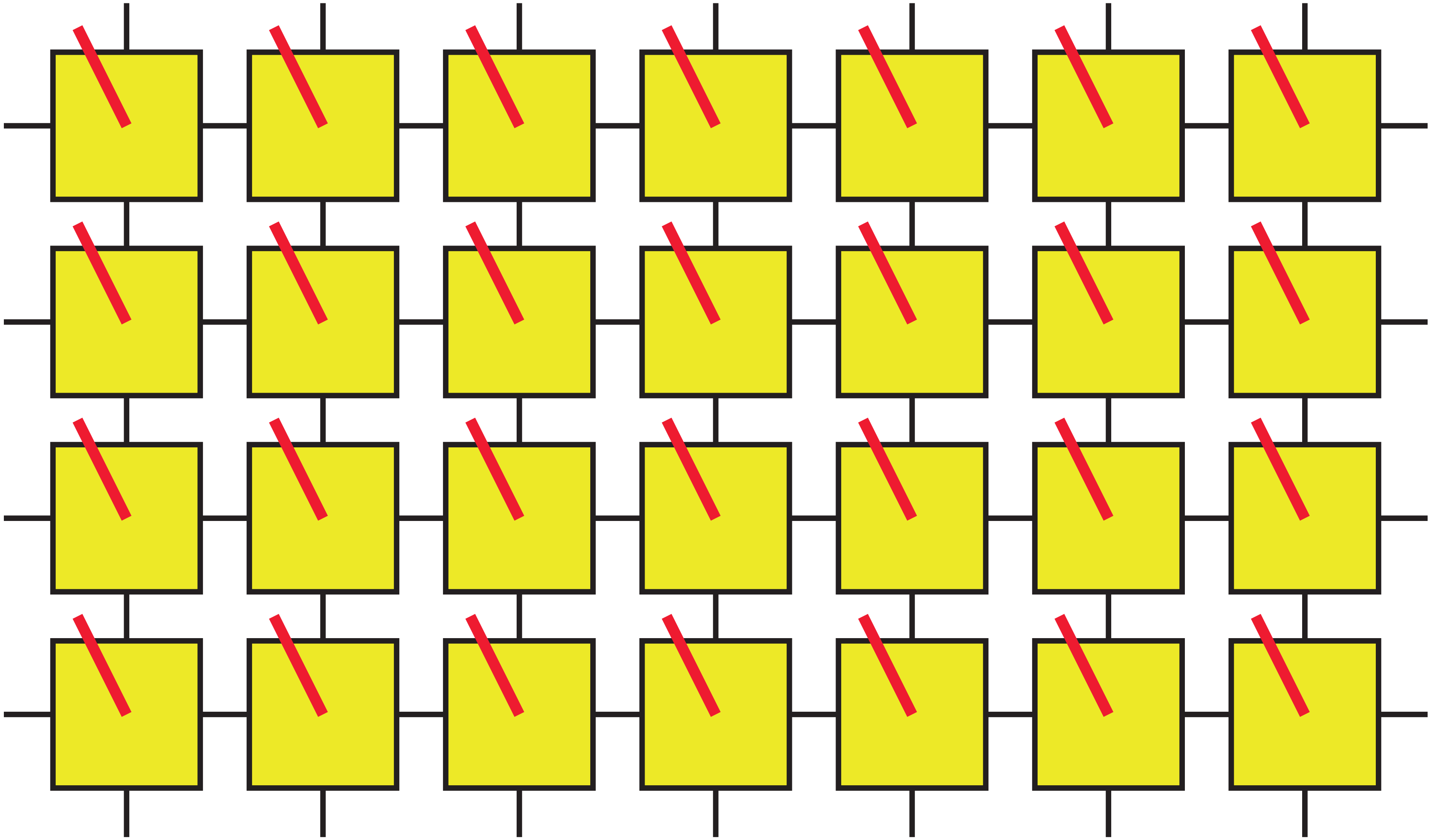
- The symmetries of the phase are





- The 2D cluster state is inside the phase

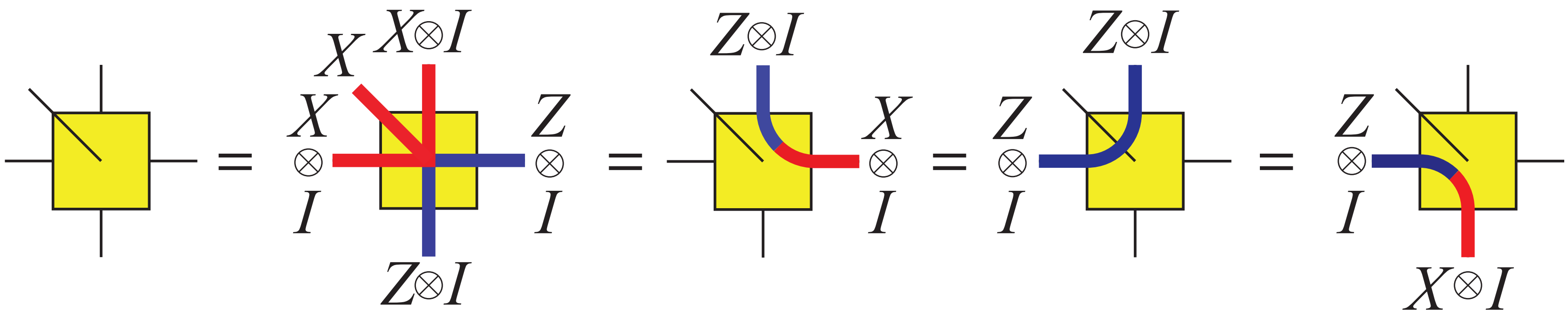
Result. For a spin-1/2 lattice on a torus with circumferences n and Nn, with n even, all ground states in the 2D cluster phase, except a possible set of measure zero, are universal resources for measurement-based quantum computation on $n/2$ logical qubits.



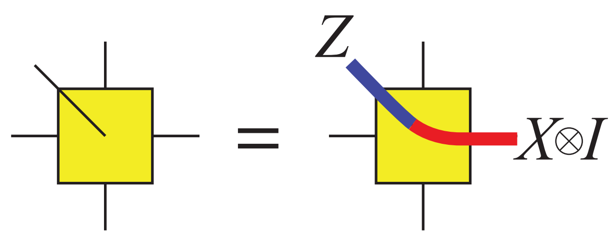
Consider MBQC resource states as tensor networks

Cluster-like states

... have PEPS tensors with the following symmetries



The cluster states have the additional symmetry



(We do not require the latter symmetry for cluster-like states)

Splitting the problem into halves

Part A:

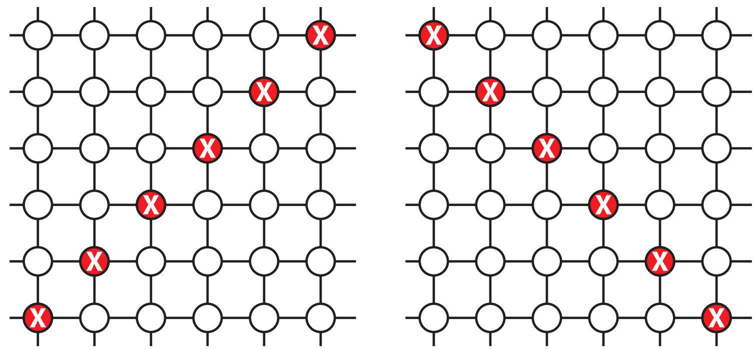
Lemma 1. All states in the 2D cluster phase are cluster-like.

Part B:

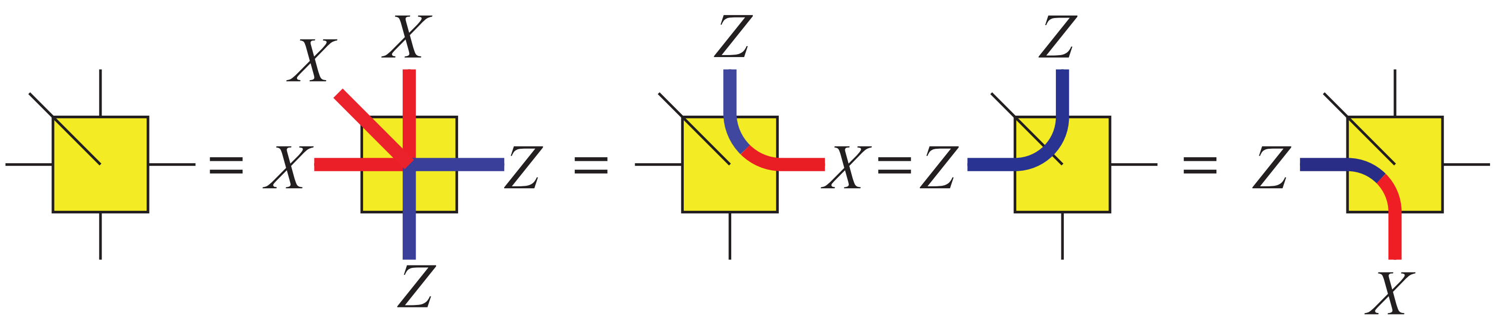
Lemma 2. All cluster-like states, except a set of measure zero, are universal for MBQC.

Part A: PEPS tensor symmetries

The physical symmetries

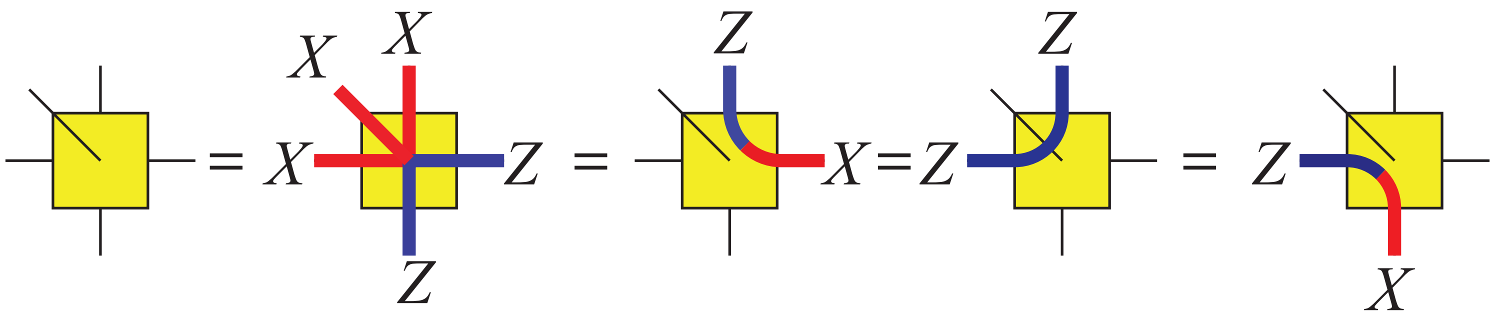


in the 2D cluster phase imply the local PEPS tensor symmetries,



Part B: Symmetry Lego

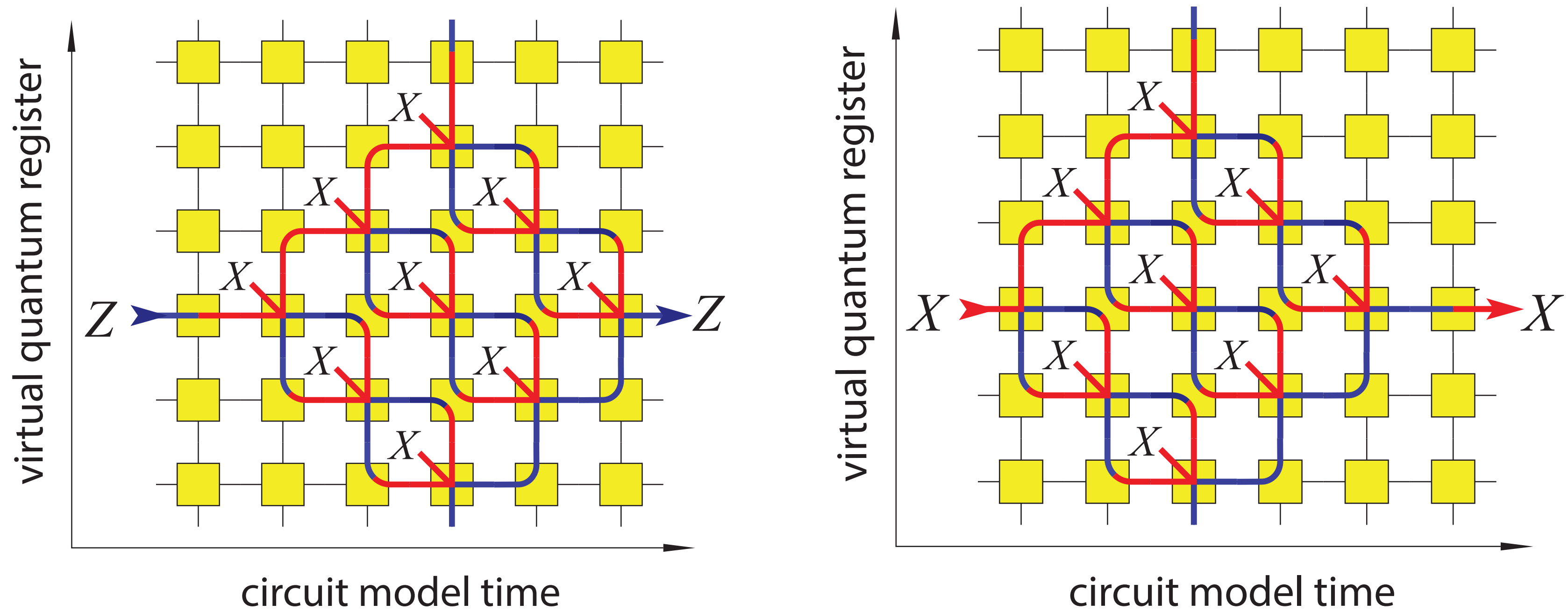
Now weave the PEPS tensor symmetries



into larger patterns.

B: Cluster-like $\Rightarrow$ universal

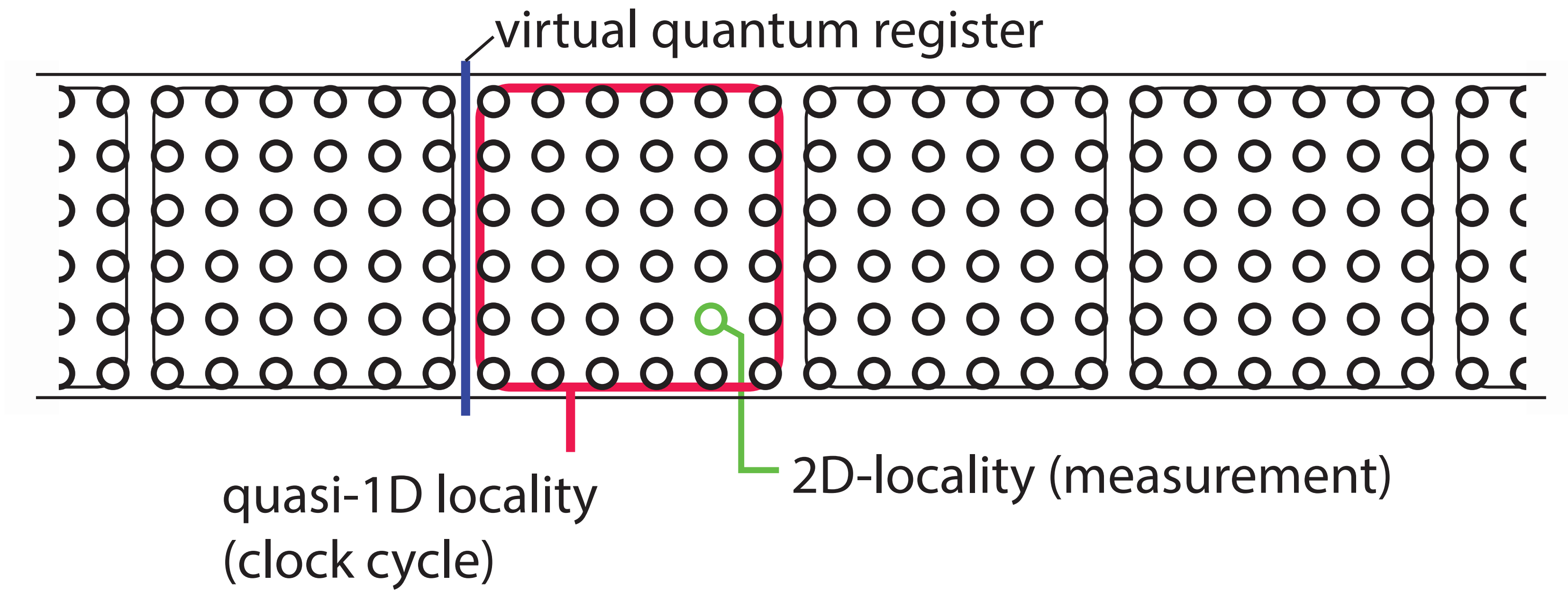
The clock cycle:



- Every byproduct operator is mapped back to itself after n columns (n = circumference).

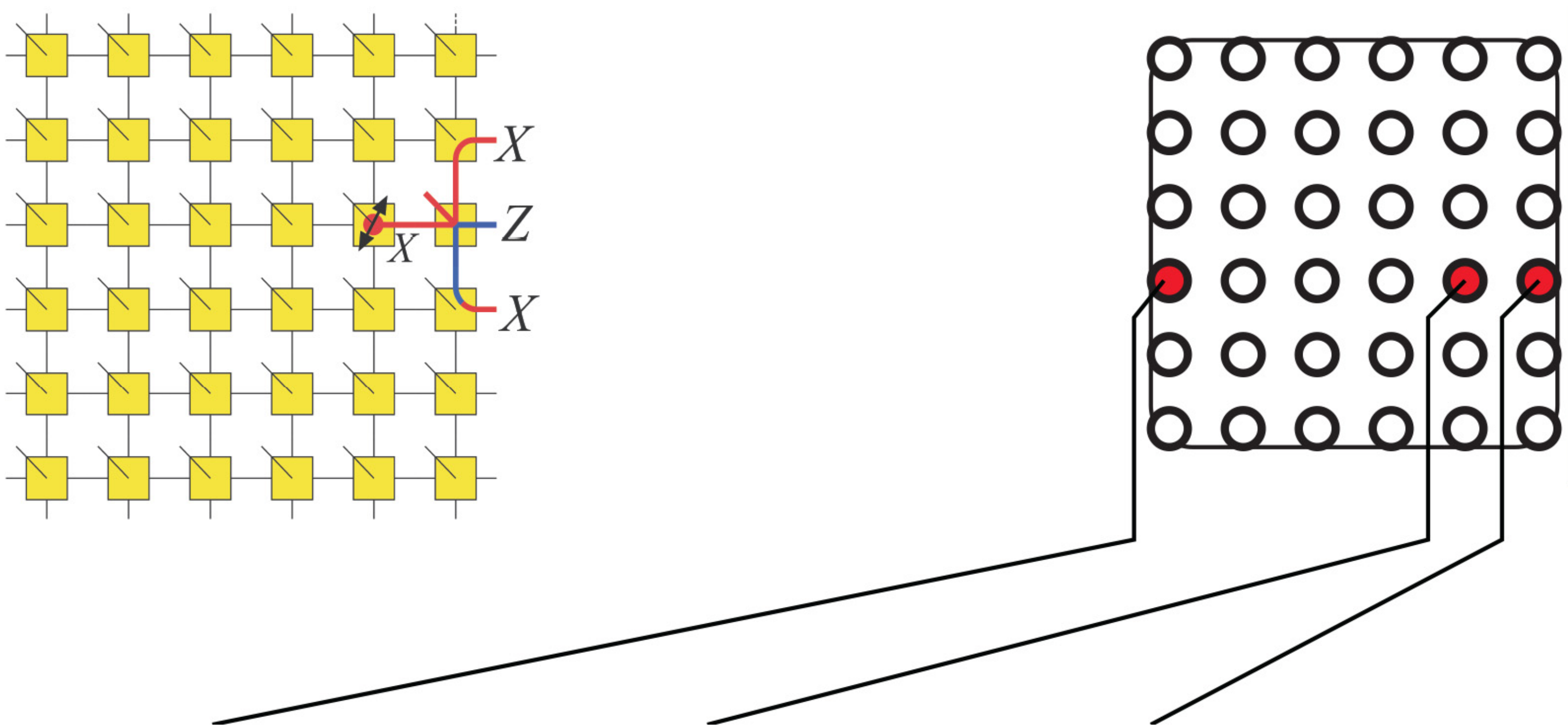
$\Rightarrow$ If a gate can be done once, it can be done many times.

B: Cluster-like $\Rightarrow$ universal



- Map 2D system to effective 1D system

B: Cluster-like $\Rightarrow$ universal

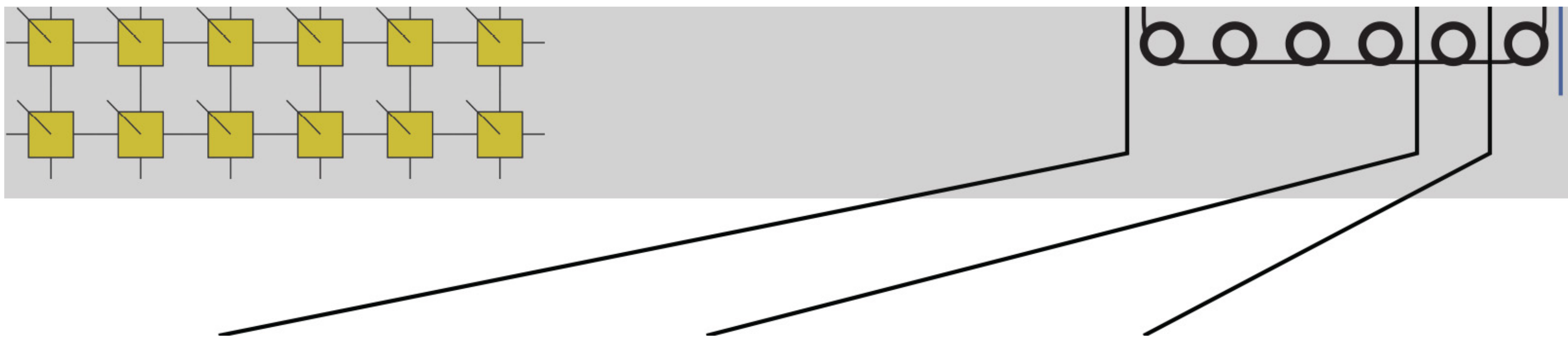


$e^{id\alpha\, Z_k}$ $\qquad$ $e^{id\alpha\, X_{k-1} Z_k X_{k+1}}$ $\qquad$ $e^{id\alpha\, X_k}$

Universal gate set on $n/2$ qubits

B: Cluster-like $\Rightarrow$ universal

2D cluster state:



$e^{id\alpha\, Z_k}$ $\qquad$ $e^{id\alpha\, X_{k-1}Z_kX_{k+1}}$ $\qquad$ $e^{id\alpha\, X_k}$

Throughout the phase:

$e^{i|\nu|d\alpha\, Z_k}$ $\qquad$ $e^{i|\nu|d\alpha\, X_{k-1}Z_kX_{k+1}}$ $\qquad$ $e^{i|\nu|d\alpha\, X_k}$

$$|\nu| \leq 1$$

(ν depends on the location in the phase)

About ν: RR, A.Prakash, D.-S. Wang, T.-C.Wei, D.T. Stephen, Phys. Rev. A (2017).

Summary and outlook

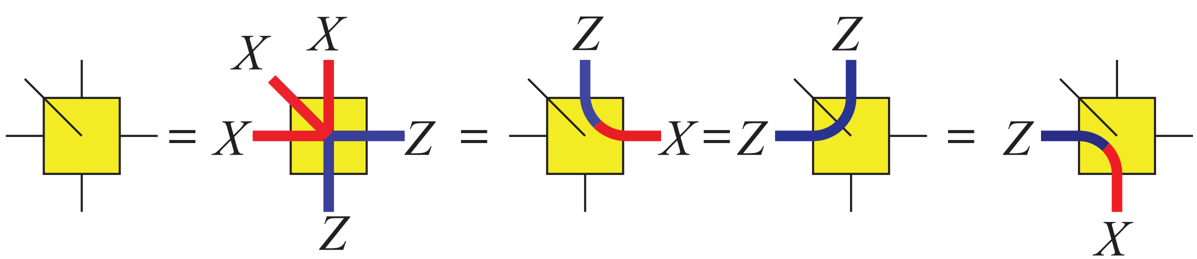
- There exists a symmetry-protected phase in 2D with uniform universal computational power for MBQC.

- *Can we have a classification of MBQC schemes in 2D, based on symmetry?*

- Symmetry Lego is fun—Try it!

PRL 122, 090501 (2019)

Related: **Quantum 3, 162 (2019)**



A: In cluster phase $\Rightarrow$ cluster-like

Lemma 3. [*] Symmetric gapped ground states in the same SPT phase are connected by symmetric local quantum circuits of constant depth.

For any state $|\Phi\rangle$ in the phase,

$$|\Phi\rangle = U_k U_{k-1} .. U_1 \, |\text{2D cluster}\rangle.$$

Look at an individual symmetry-respecting gate in the circuit,

$$U = \sum_j c_j T_j, \text{ with } T_j \in \mathcal{P}.$$

Which Pauli observables T_j can be admitted in the expansion?

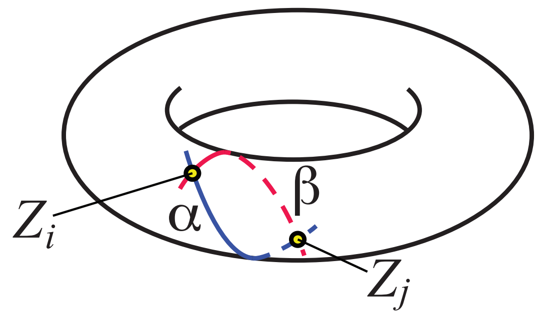
[*] X. Chen, Z.C. Gu, and X.G. Wen, Phys. Rev. B **82**, 155138 (2010).

A: In cluster phase $\Rightarrow$ cluster-like

Which Paulis T_j can be admitted in the expansion $U = \sum_j c_j T_j$?

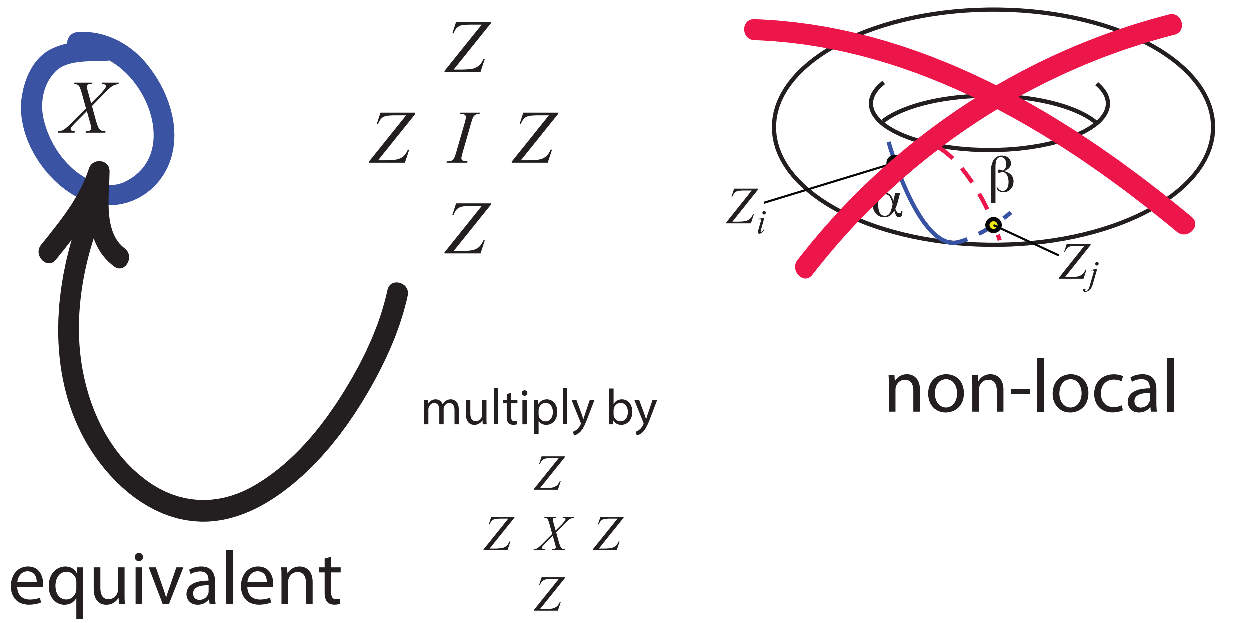
X

$$\begin{matrix} & Z & \\ Z & I & Z \\ & Z & \end{matrix}$$



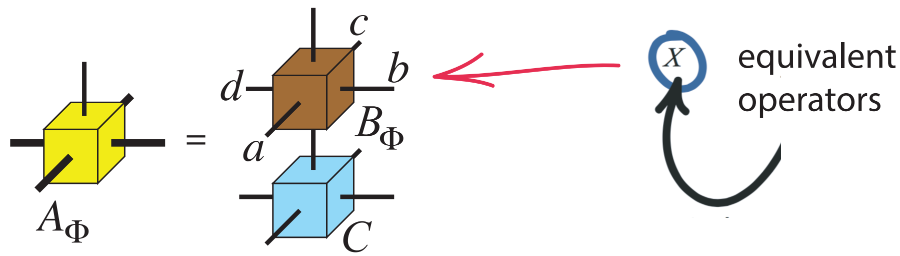
A: In cluster phase $\Rightarrow$ cluster-like

Which Paulis T_j *can be admitted in the expansion* $U = \sum_j c_j T_j$*?*

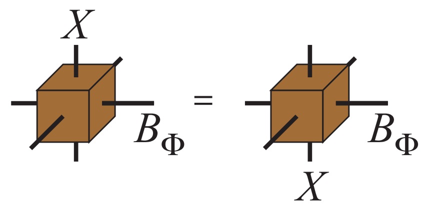


Only X*-type Pauli operators survive in the expansion.*

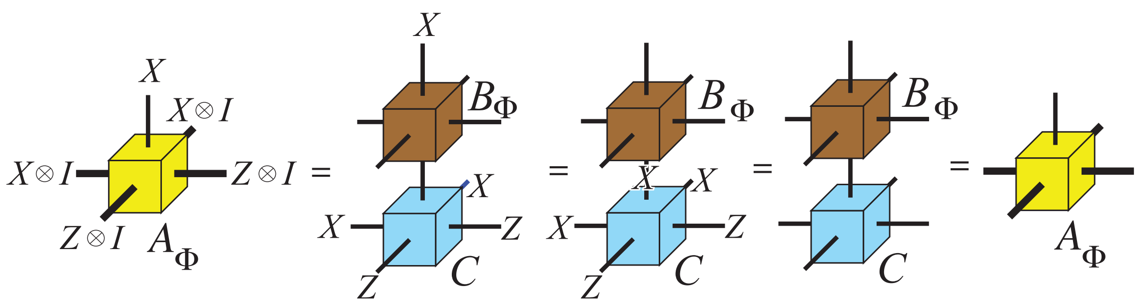
Description of the local tensors:



With



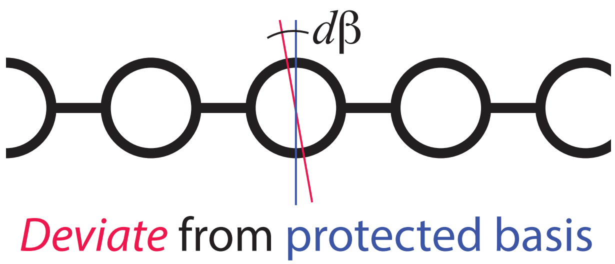
Hence



- Local tensors A_Φ describing $|\Phi\rangle$ are invariant under the cluster-like symmetries.

The parameter ν

There is a complex-valued parameter ν, $|\nu| \leq 1$, that needs to be known about the location of the resource state within the phase.



For infinitesimal angles $d\beta$, this results in a logical rotation [*]

$$e^{id\beta|\nu|\,T},$$

for some Pauli operator T. (E.g., $T = Z_k, X_k, X_{k-1}Z_kX_{k+1}$).

We require that $\nu \neq 0$.

[*] RR, D.-S. Wang, A. Prakash, T.-C. Wei, D.T. Stephen, PRA 96 (2017).