Operational, gauge-free quantum tomography

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Quantum state tomography

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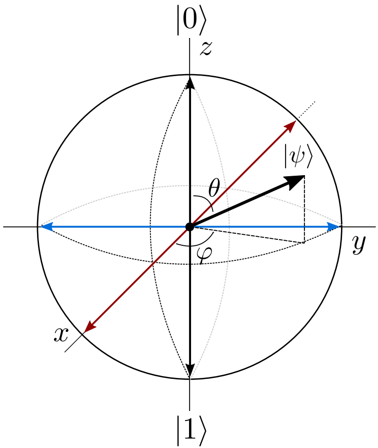
Reconstruct it by taking an informationally complete set of measurements.



Measure:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Quantum process tomography

How can we learn what an unknown quantum process is doing?

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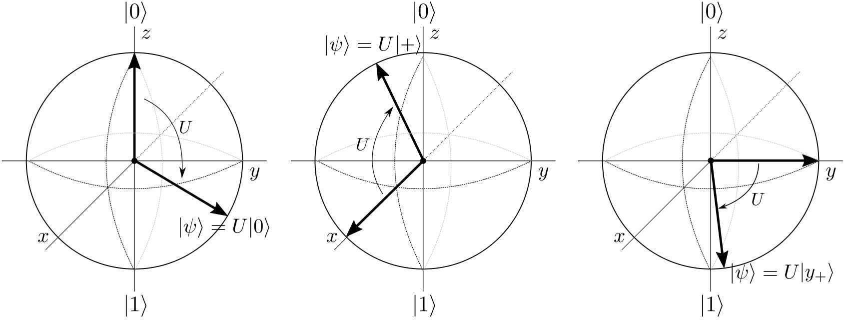
Reconstruct an operation based on how it acts on known states.



(Example: a unitary operation on a single qubit.)

QCVV: quantum characterization, verification, and validation

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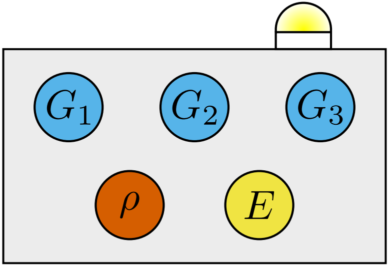
But in real physical systems, **S**tate **P**reparation **A**nd **M**easurement (SPAM) are also noisy processes!

Gate set tomography (GST)

Treat everything we can do to our quantum system equally. State preparation, operations, and measurement are ‘buttons’ that we can push to act on our quantum system.



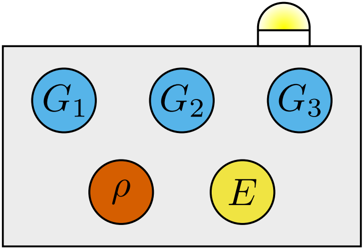
We want to learn what all of the buttons do.

Merkel, S. T., et al. (2013). *Self-consistent quantum process tomography*. Physical Review A, 87(6).

Blume-Kohout, R., Gamble, J. K., Nielsen, E., Mizrahi, J., Sterk, J. D., & Maunz, P. (2013). *Robust, self-consistent, closed-form tomography of quantum logic gates on a trapped ion qubit.* `http://arxiv.org/abs/1310.4492`

Gate set tomography (GST)

Mathematically, we represent every button as a *superoperator* - our initial task will be to learn their contents.



$$
\begin{aligned}
|\rho\rangle\rangle &= \begin{pmatrix} * & * & * & * \end{pmatrix}^T \\
|E\rangle\rangle &= \begin{pmatrix} * & * & * & * \end{pmatrix}^T \\
G_1 &= \begin{pmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{pmatrix} \\
G_2 &= \ldots
\end{aligned}
$$

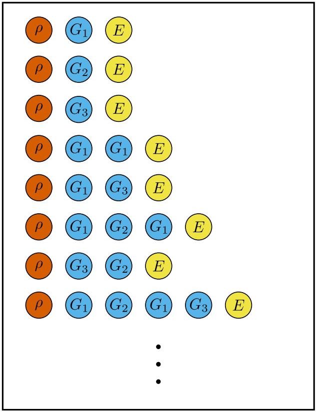
Assumption: buttons have the same action any time they are pressed.

Learning the superoperators

How?

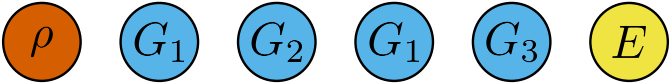
By pushing a series of buttons chosen in a clever way.

We can reconstruct the superoperators by using the outcome frequencies from a variety of experiments.



Learning the superoperators

Given an experiment, e.g.,



The probability that the light turns on is

$$\langle\langle E|G_3 G_1 G_2 G_1|\rho\rangle\rangle = \mathrm{Tr}\left(|\rho\rangle\rangle\langle\langle E|G_3 G_1 G_2 G_1\right)$$

where G_i, $|\rho\rangle\rangle$, and $|E\rangle\rangle$ are superoperators of the buttons.

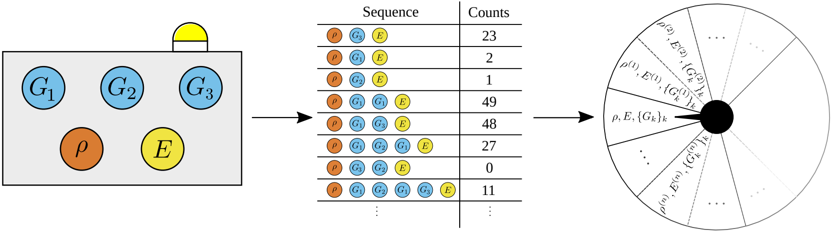
If you know the superoperators, you can predict the outcome probabilities for *any* experiment on the system.

Learning the superoperators

Transforming all superoperators by the same linear transformation B (a *gauge transformation*) results in the same probabilities:

$$\mathrm{Tr}\left(|\rho\rangle\rangle\langle\langle E|\, G_3\, G_1\, G_2\, G_1\right) =$$
$$\mathrm{Tr}\left(B^{-1}|\rho\rangle\rangle\langle\langle E|\, B B^{-1} G_3\, B B^{-1} G_1\, B B^{-1} G_2\, B B^{-1} G_1\, B\right)$$

The transformation B is not accessible experimentally! We learn one possible set from the *gauge orbit* of potential superoperators.



Dealing with gauge freedom in gate set tomography

Option 1: Gauge-fixing.

Run a computational procedure to find a B that makes your superoperators close to what you think they should be.

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Issues:

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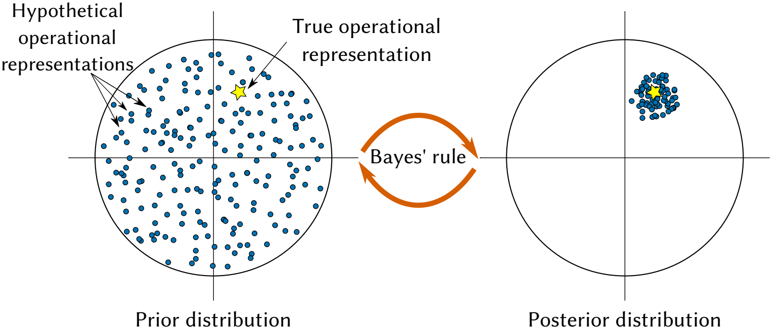
Option 2: Work with gauge-*independent* quantities instead.

Build a model that characterizes the system based on experimental outcome probabilities directly: *operational quantum tomography*.

Operational Quantum Tomography

Use Bayesian inference to learn a finite set of gauge-independent model parameters called the *operational representation.*



These model parameters are related to the outcome probabilities of a small set of *fiducial* experiments.

Fiducial experiments

Fiducial experiments, or *fiducial sequences*, are short sequences of only one or two button presses that gives us a point of reference.

The set of fiducial sequences must be informationally complete (more on this in a minute!)

$$F_0 =$$

$$F_1 = G_1$$

$$F_2 = G_2$$

$$F_3 = G_1 \quad G_2$$

An operational representation

Let F_i represent fiducial experiments. Use them to construct:

$$\begin{aligned}
\tilde{E}_i &= \langle\langle E|F_i|\rho\rangle\rangle \\
\tilde{F}_{ij} &= \langle\langle E|F_i F_j|\rho\rangle\rangle \\
\tilde{G}_{ij}^{(k)} &= \langle\langle E|F_i G_k F_j|\rho\rangle\rangle
\end{aligned}$$

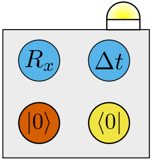
These quantities are gauge-*independent*, and can be used to compute the same probabilities as the superoperators. For a sequence of button presses **s**,

$$\begin{aligned}
&\mathrm{Tr}\left(|\rho\rangle\rangle\langle\langle E|G_{s_k}\cdots G_{s_1}\right) \\
&= \mathrm{Tr}\left(\tilde{F}^{-1}\tilde{E}\cdot\tilde{E}^T\cdot\tilde{F}^{-1}\tilde{G}^{(s_k)}\cdot\tilde{F}^{-1}\tilde{G}^{(s_{k-1})}\ldots\tilde{F}^{-1}\tilde{G}^{(s_1)}\right)
\end{aligned}$$

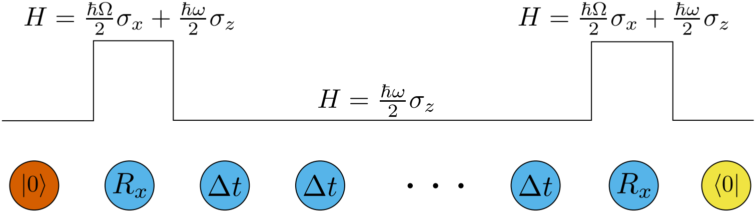
If we can learn $\tilde{E}$, $\tilde{F}$, and $\tilde{G}^{(k)}$, we can predict the outcome of any future experiment – we call them the operational representation.

Example: OQT for Ramsey interferometry

We can use OQT to perform Ramsey interferometry and learn the Rabi oscillation frequency of a qubit.



In this framework Ramsey interferometry experiments look like:



OQT for Ramsey interferometry

Steps for performing Ramsey interferometry using OQT:

1. Choose a prior distribution for what we think the operational representation should look like.

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Steps for performing Ramsey interferometry using OQT:

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   It's not obvious what properties arbitrary operational representations should have, except all values are $\in [0, 1]$.

   Instead, we choose a prior over the *superoperators*, e.g.

   $$R_x\left(\frac{\pi}{2}\right) \to R_x\left(\frac{\pi}{2} + \epsilon\right), \quad \epsilon \in \mathcal{N}(0, \sigma^2)$$

   and use these to later convert to the gauge-independent form.

OQT for Ramsey interferometry

2. Choose a set of fiducial experiments.[1]

   We chose:

   $$F_0 = \qquad\qquad F_2 = R_x \; R_x$$

   $$F_1 = R_x \qquad\qquad F_3 = R_x \; \Delta t \; R_x$$

   An ‘empty’ fiducial indicates an experiment where we perform only SPAM.

[1] Choice of fiducials must yield invertible $\tilde{F}$, where $\tilde{F}_{ij} = \langle\langle E | F_i F_j | \rho \rangle\rangle$.

3. Initialize a particle cloud with many hypothetical operational representations.

   Sample superoperators from their priors. In this example, we initialize a cloud of 10000 particles with prior assumptions:

   - State preparation creates a depolarized[2] $|0\rangle$ with $p \in \mathcal{U}(0, 0.1)$

   Use samples to analytically compute hypothetical $(\tilde{E}, \tilde{F}, \tilde{G}^{(k)})$.

[2] The depolarizing channel sends $\rho \to (1-p)\rho + \frac{p}{3}(X\rho X + Y\rho Y + Z\rho Z)$

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   - Δt pulled from $R_z(\omega \cdot dt),\ \ \omega \in \mathcal{U}(0, 1),\ \ dt = 1$

   Use samples to analytically compute hypothetical $(\tilde{E}, \tilde{F}, \tilde{G}^{(k)})$.

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OQT for Ramsey interferometry

4. Perform Bayesian inference

   Using either true experimental data, or simulated data, perform a series of experiments of the following form:

   $$|0\rangle \quad R_x \quad \Delta t \quad \cdots \quad \Delta t \quad R_x \quad \langle 0|$$

   We performed simulated experiments with a ‘true’ gateset sampled from our prior.

   Perform Ramsey experiments consisting of n presses of Δt, n from 2 to 50, and update the particle cloud of hypothetical operational representations according to Bayes’ rule[3].

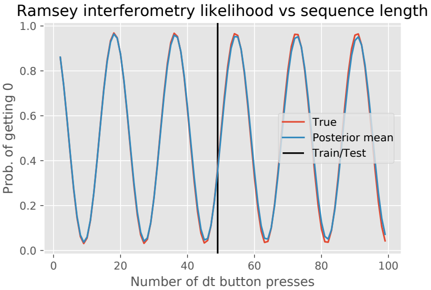
[3]We used Sequential Monte Carlo techniques to do this.

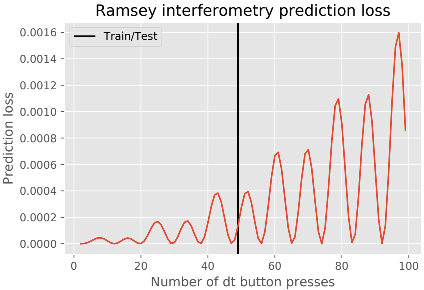
OQT for Ramsey interferometry

5. Assess the quality of our reconstruction.

   We use a prediction loss: for true probability p_s and mean posterior probability $\hat{p}_s$, the loss is given by

$$\mathrm{Loss}(p_s, \hat{p}_s) = (\hat{p}_s - p_s)^2$$





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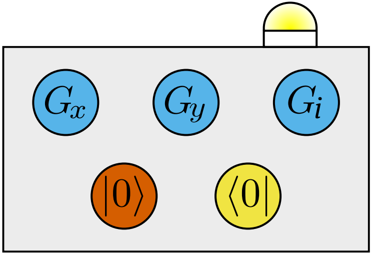
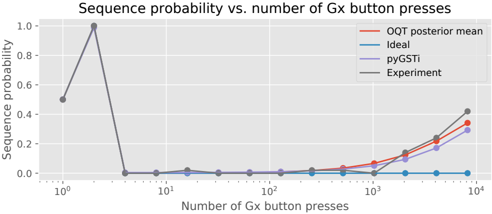
So far we have also successfully performed:

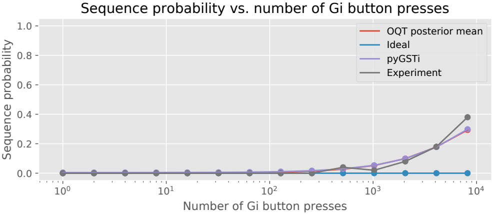
- Quantum state tomography
- Quantum process tomography (with simulated and real data)
- Randomized benchmarking

OQT for experimental trapped-ion qubit data

Use data from long-sequence GST experiments on a trapped-ion qubit.





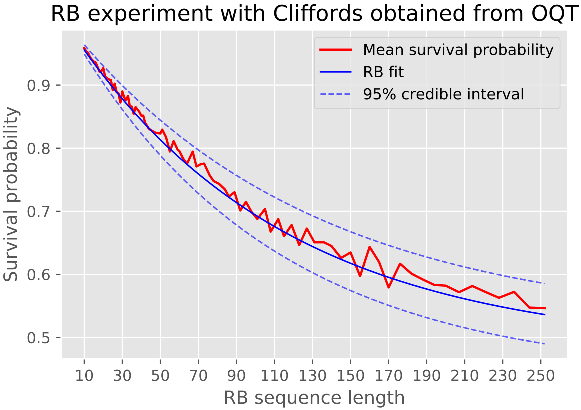


OQT is competitive with existing techniques!

Experimental data: Blume-Kohout, R., Gamble, J. K., Nielsen, E., Rudinger, K., Mizrahi, J., Fortier, K., & Maunz, P. (2017). *Demonstration of qubit operations below a rigorous fault tolerance threshold with gate set tomography.* Nature Communications, 8, 14485.

Randomized benchmarking

Use OQT to learn H and S (and SPAM) and perform randomized benchmarking.



OQT provides posterior distribution, enabling deeper analysis.

Conclusions and future work

Operational quantum tomography enables us to characterize and learn about a wide variety of quantum systems.

Learning the operational representation allows us to predict the outcome of future experiments in a way that overcomes the gauge-related challenges in other procedures.

Next steps for OQT:

- Scaling up to multi-qubit systems, and multi-state and multi-measurement cases
- More sophisticated noise models (e.g., how to consider non-Markovianity?)
- Take it to the lab! Characterize in-house hardware at UBC.

Thank you for your attention!

Thanks also to:

- My co-authors John, Chris, Kenny, and Nathan
- QuArC group at Microsoft Research, for hosting me as an intern and frequent visitor

Software and all examples are available open source with the arXiv version of the work: `https://arxiv.org/abs/2007.01470`