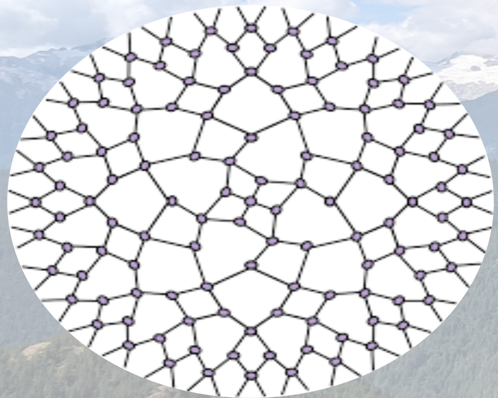
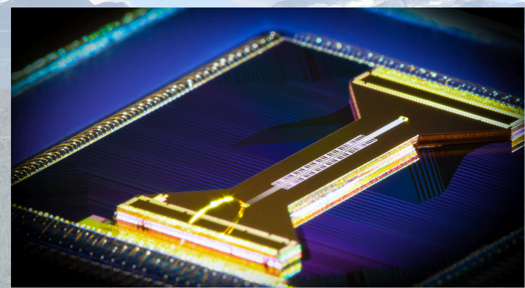
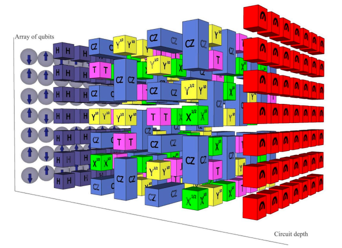
# Simulating highly-entangled matter with quantum tensor networks

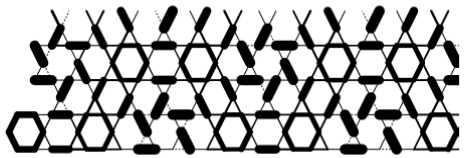
Andrew C. Potter (UT Austin -> UBC)





# Practical quantum computation?





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~50 qubits

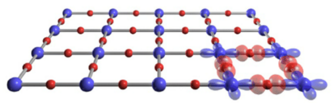
100's - 1000's of qubits

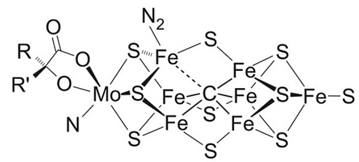
\# logical qubits

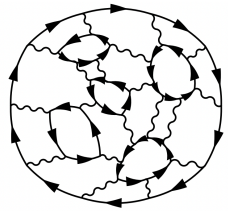
**Quantum "Supremacy"**

*Useful Quantum Advantage??*

Simulating quantum matter



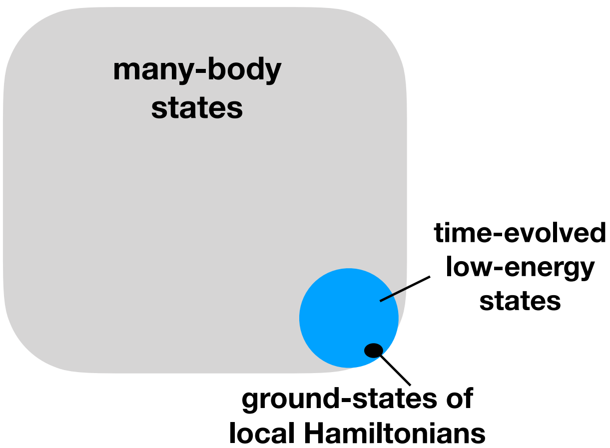


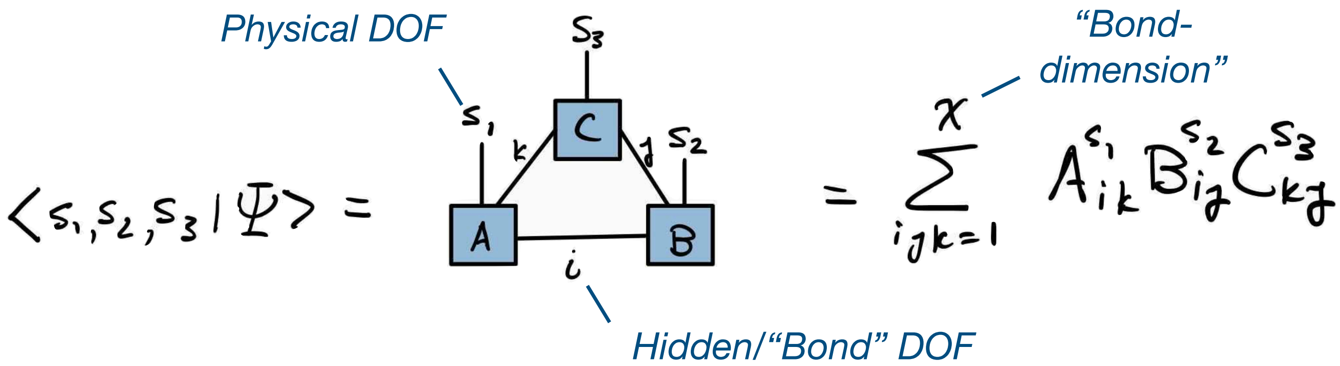


**Goal:** Port successful classical strategies **(tensor-network representations)** to the quantum realm

**Strategy:** Focus quantum resources on classically-hard aspects of computation

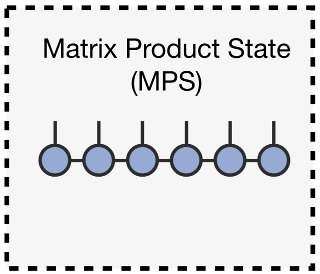
# Tensor Network States (TNS)

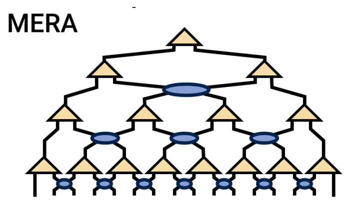


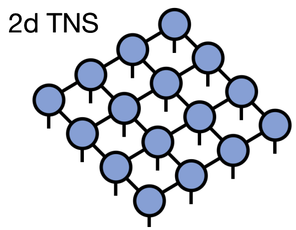



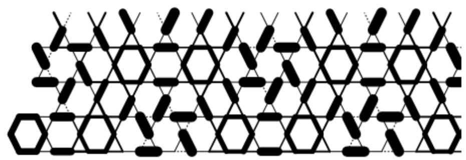

**Higher-d MPS:** $\chi \sim e^{L^{d-1}}$ $L^p$ (gapless)

**Can simulate 100's-1000's of spins!**



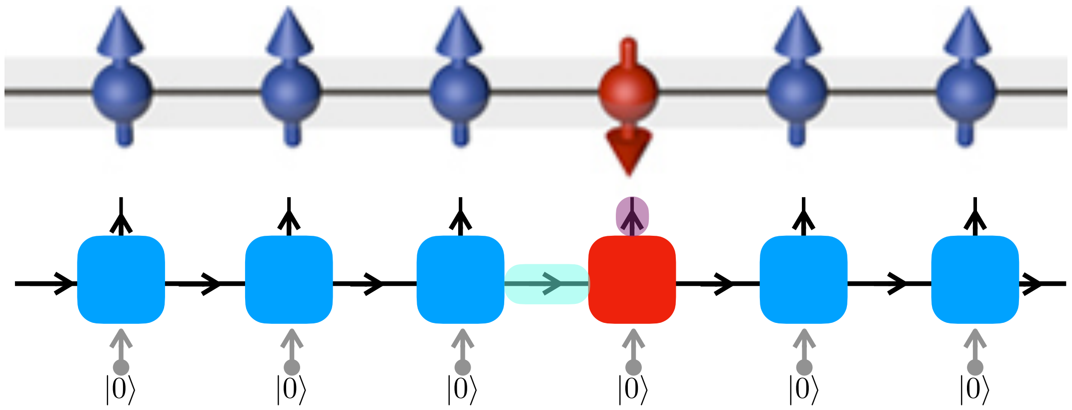



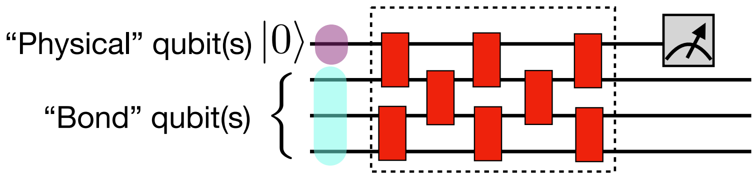
*e.g. Yan, Huse, White, Science '10*

# Quantum Matrix Product States (qMPS)




$$\chi = 2^{\#\text{bond-qubits}}$$

$$\#\ \text{hardware qubits} = \log_2 \chi + L^{d-1}$$

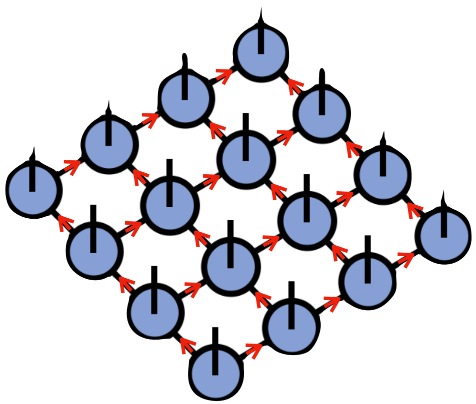



**“Holographic” simulation:**
Simulate d-dimensions w/ (d-1) dimensions’ worth of qubits

*Schoen, Hammerer, Wolf, Cirac PRA ‘07*
*Foss-Feig, Hayes, … ACP PRR ’21*
*Barratt, Dborin, Bal, Stojevic, Green, Pollmann NPJQI ’21*

# Representational Power of qTNS?

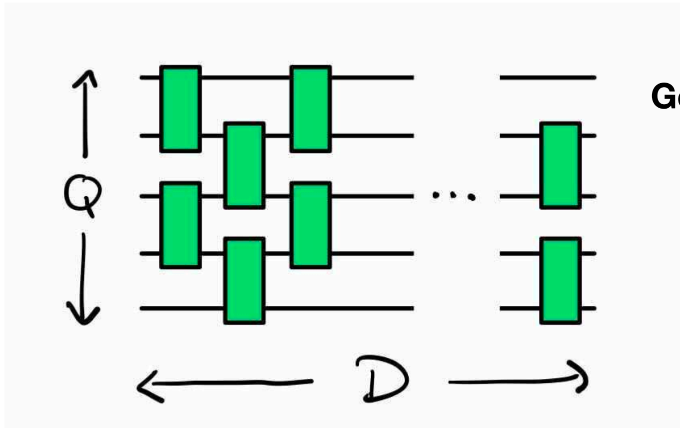
**Restriction 1: Tensors = Isometries**



- **1d:** No Restriction (“Canonical Form” )
- **2d, 3d: ??** [Many interesting states have low-bond dimension isoTNS representations]

*Soejima, Siva, Bultinck, Chatterjee, Pollmann, Zaletel PRB ‘20*

**Restriction 2: Circuit Resources**




**Generic “Dense” Unitary:**

$$D \sim e^{Q}$$

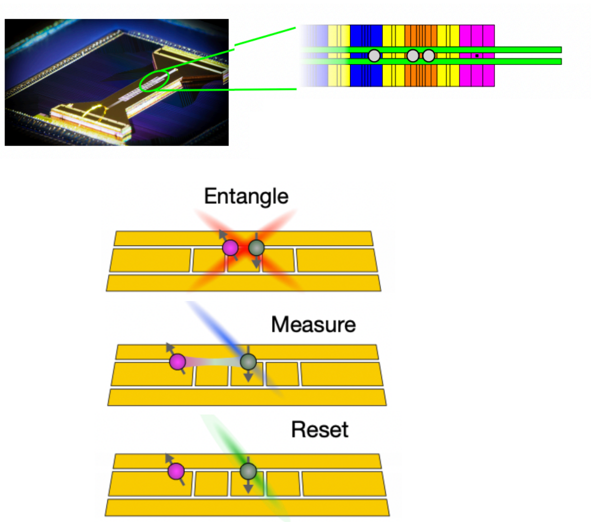
*(e.g. Solovay, Kitaev)*

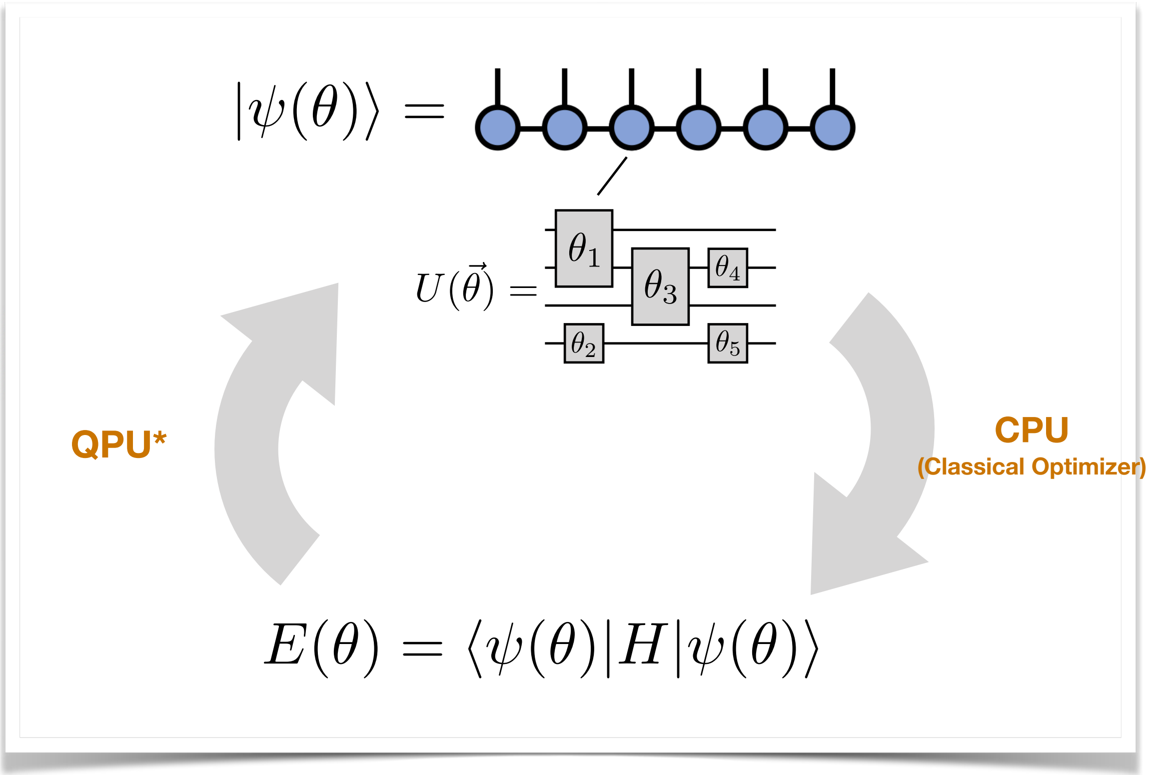
- **Key Question:** What class of states can be captured w/ D~poly(Q,L)?
- **(Partial) Answer:**
  **-** Anything continuously connected to non-interacting electrons
  - (Non-chiral) topological orders

*Niu, Zhang, Haghenas, Chan, ACP [to appear]*

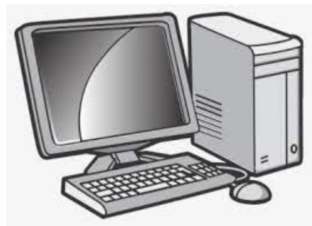
# qMPS Ground-State Preparation: “Holographic VQE”

**Honeywell H1**
**Trapped-Ion Processor**



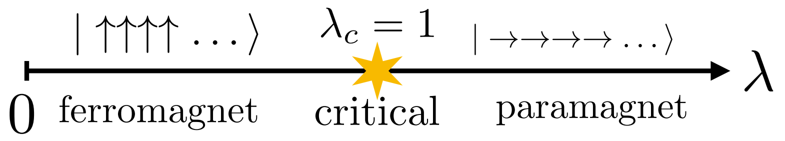

**CPU**
**(Classical Optimizer)**



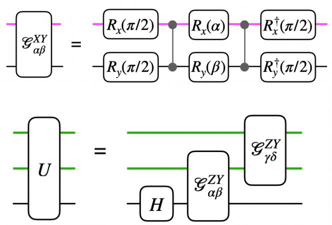
**Data shown will be classically pre-optimized, then implemented on QPU*

# Variational qMPS: Experimental Demo

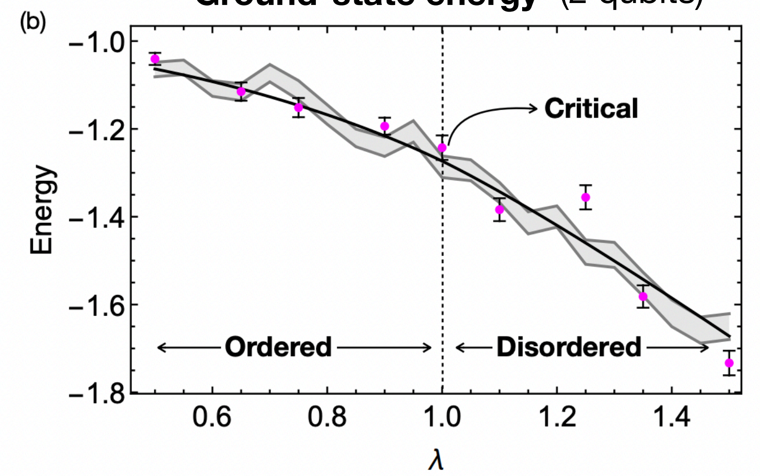
$$H_{\mathrm{TFIM}} = -\sum_j \left(\sigma^z_j \sigma^z_{j+1} + \lambda \sigma^x_i\right)$$



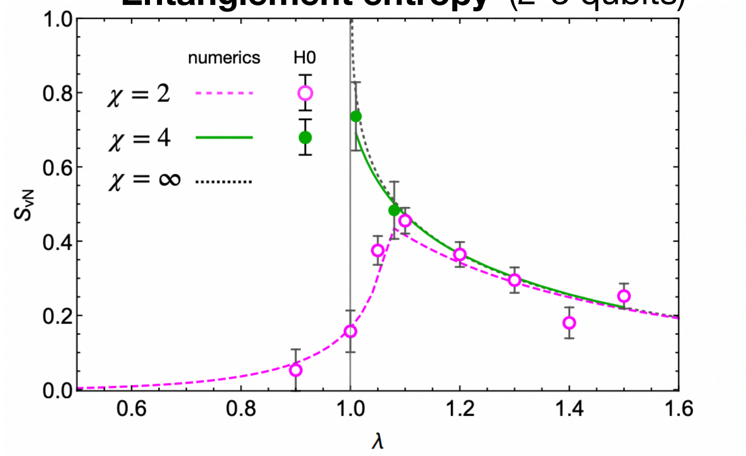

$\langle \sigma_r \sigma_0 \rangle \sim 1/r^{2\Delta}$ $\quad S \sim \frac{1}{12} \log L$ *Cardy, Calabrese*




**Ground-state energy** (2 qubits)




**Entanglement entropy** (2-3 qubits)

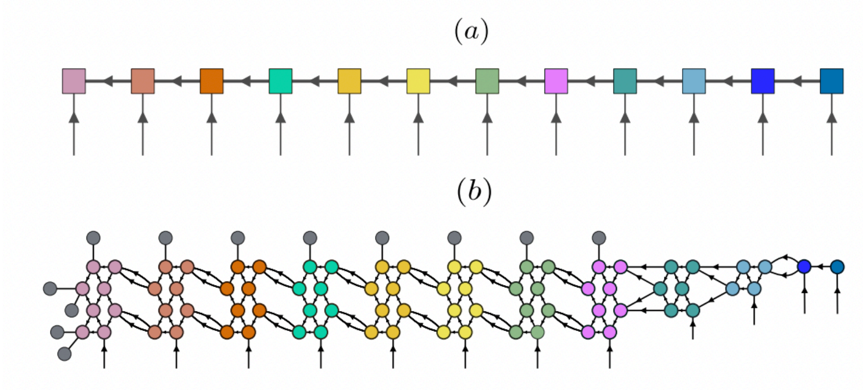



**# Qubits used: 2-3**

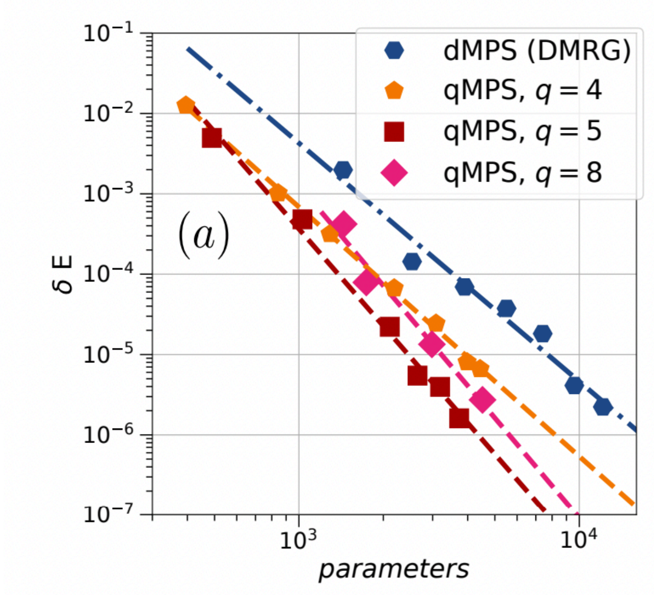
**Vs. >~ 40 qubits!!**

*Foss-Feig, Hayes, et al. [Honeywell + ACP] arXiv:2104.11235*

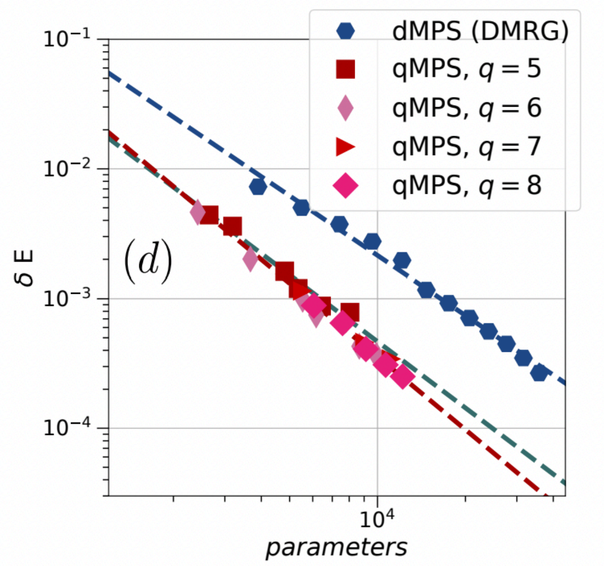
# Variational qMPS: Systematics




**Heisenberg spin-chain**




**Fermi-Hubbard Chain**
(partial filling = metal)




**Takeaways:**

- Local circuits can achieve similar performance to DMRG
- Extrapolation: qMPS can be more expressive than dense-MPS

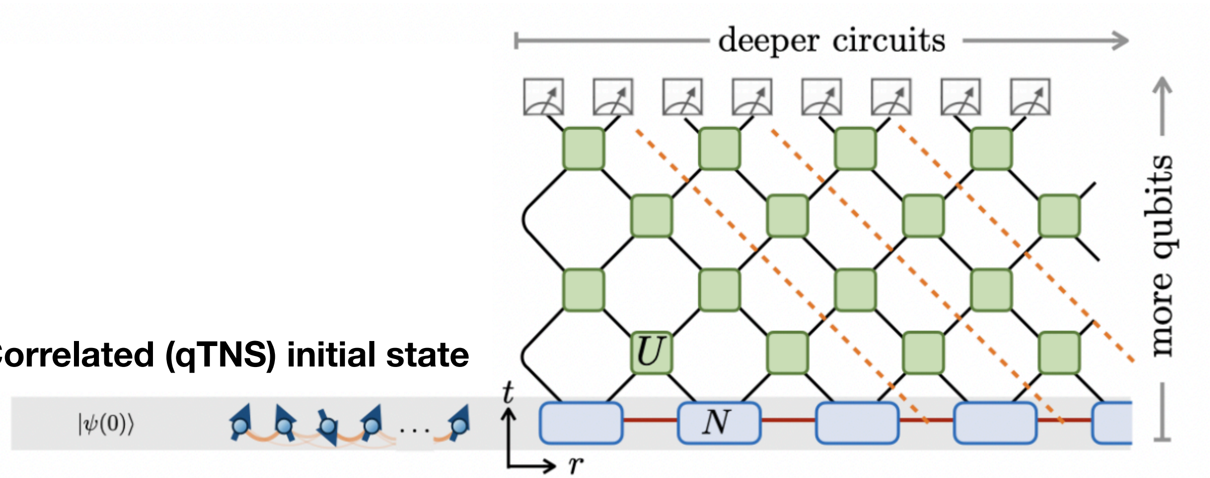
*Haghenas, Gray, ACP, G Chan arXiv:2107.01307*

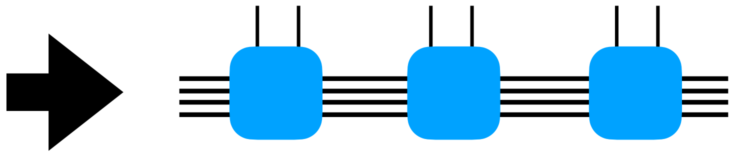
# Non-equilibrium dynamics

**Transport, Scattering, Thermalization, Localization, Quantum-Chaos, Scrambling, Dynamical critical phenomena & phases…**

Classical simulations (TEBD): $\chi \sim \exp(\text{time})$ **Hard even in 1d!!**

**Correlated (qTNS) initial state**



**Space-time tradeoff:**

$$Q \sim t$$

$$\chi \sim \exp(t)$$

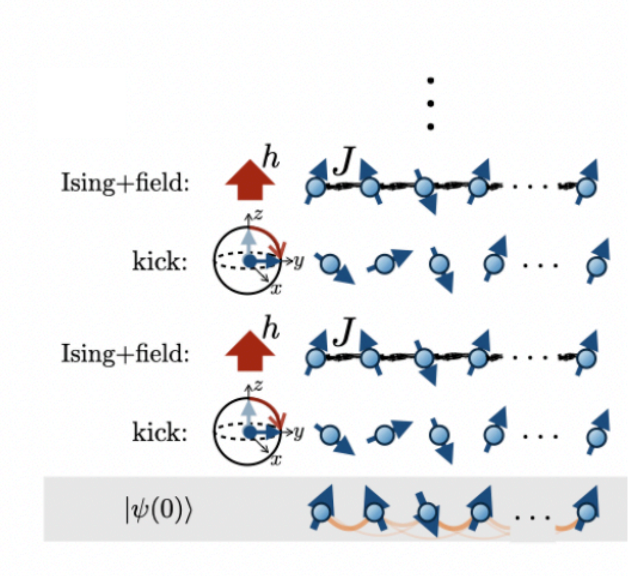
*Foss-Feig, Hayes, … ACP PRR ’21*

# Benchmark: Chaotic Floquet Dynamics

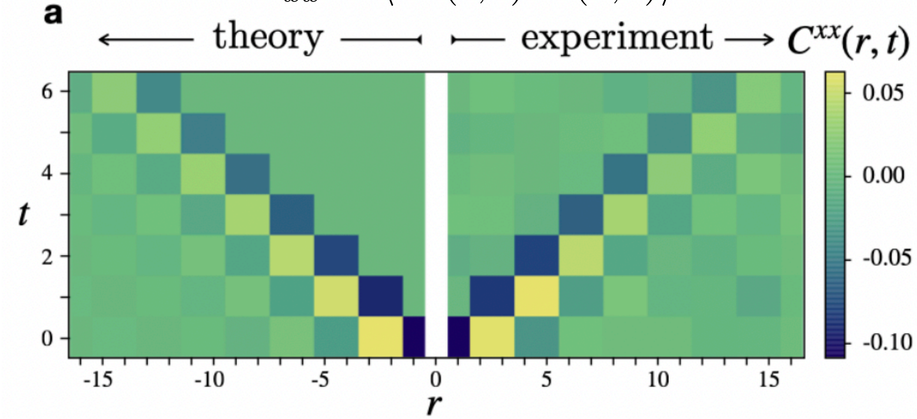
**Self-Dual Kicked Ising Model:** Non-integrable, chaotic, but solvable w/ certain MPS initial states
*(quantum algorithm does not exploit solvability)*

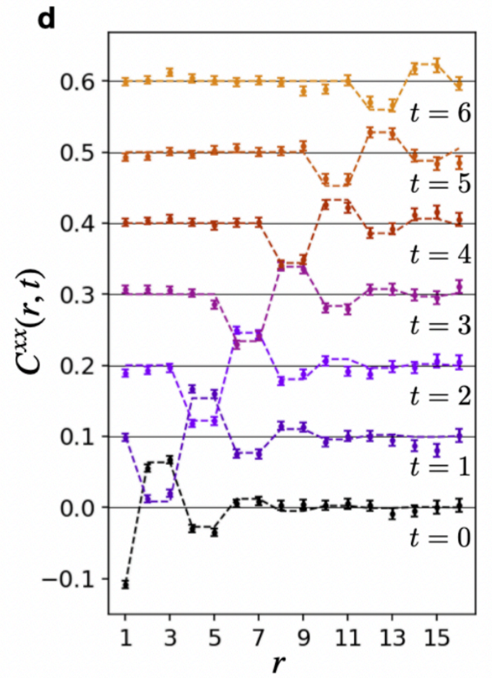
$$H(t) = \sum_i \left( \frac{\pi}{4} \sigma_i^z \sigma_{i+1}^z + h\sigma_i^z + \frac{\pi}{4} \sum_{n\in\mathbb{Z}} \delta(t-n)\sigma_i^x \right)$$

*Bertini, Kos, Prosen*




$$C_{xx} = \langle \sigma^x(r,t)\sigma^x(0,t)\rangle$$




- Quantitatively Accurate Simulations of L=32 spins w/ ≤ 8 qubits
- Essentially no post-processing or error-mitigation

*Chertkov, … Foss-Feig (Honeywell + ACP) arXiv: 2105.09324*

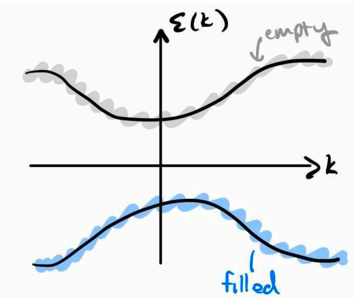
# Simulating Electrons

### Gaussian (Mean-field) States

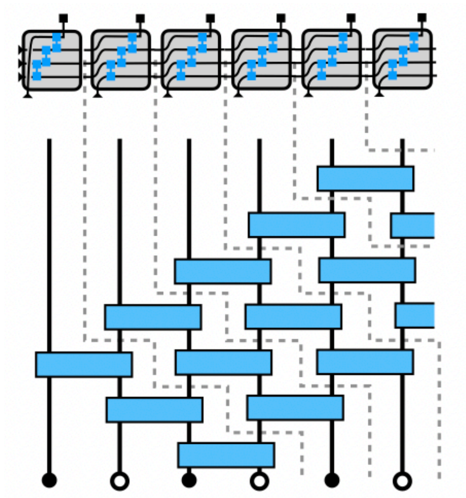
occupied

$$G_{ij} = \langle c_i^\dagger c_j \rangle = W \begin{pmatrix} \mathbb{1} & 0 \\ 0 & 0 \end{pmatrix} W^\dagger$$

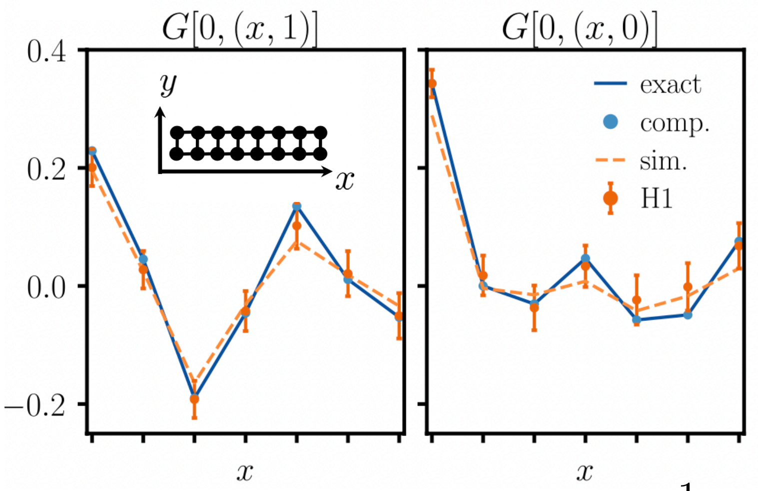
empty




### GMPS Compression: *Fishman, White*



### Experimental demo




$$\#\text{qubits} \sim L^{d-1} \cdot \log L \cdot \log \frac{1}{\epsilon}$$

- Approximate localized basis for filled states
- Construct circuit that rotates into that basis
- Convert to qMPS

- Compare to Google's "Hartree-Fock demo" #Q ~ $L^{2d}$ *Arute et al. Science '20*
- **+ adiabatic time evolution =>**
  **efficient qMPS for large-class of states**
  (metals, insulators, superconductors, magnets, some spin-liquids, etc…)

*Niu, Heghanas, Zhang, Chan, ACP (to appear)*

# Outlook

- **Future directions:**
  - Other tensor network architectures (2d, 3d, qMERA, etc…)
  - Mixed/Thermal states, open-system dynamics (qMPDO)
  - Alternative hardware platforms (circuit QED)

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Sajant Anand (UC Berkeley)

Mike Foss-Feig (Honeywell)

Dave Hayes (Honeywell)

Mike Zaletel (Berkeley)

Garnet Chan (Caltech)

Shyam Shankar (UT Austin)

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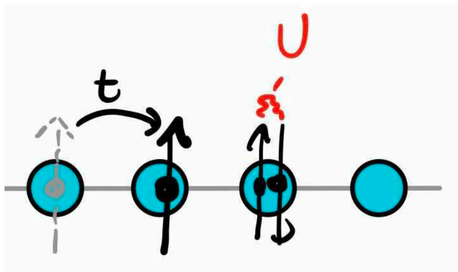
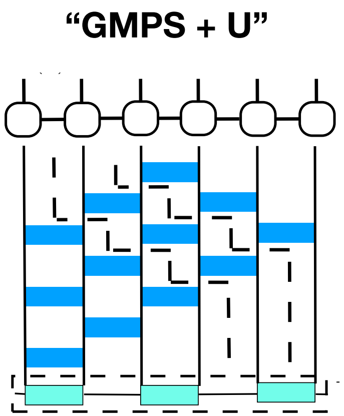
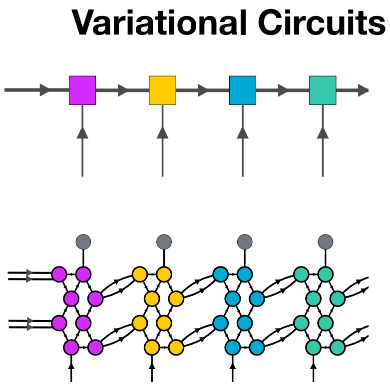
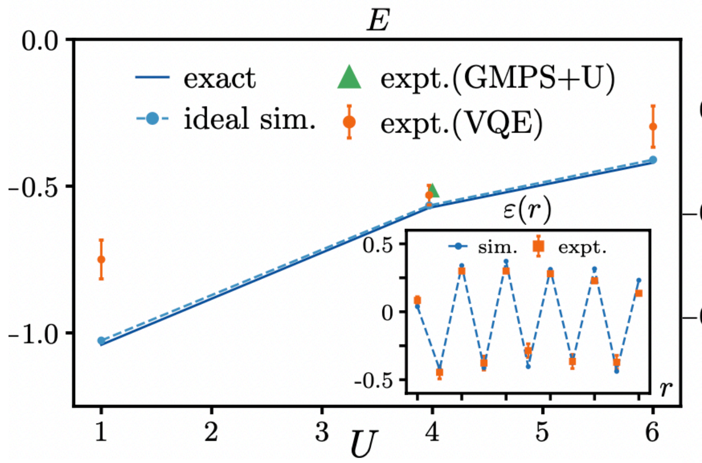
# Simulating Correlated Electrons

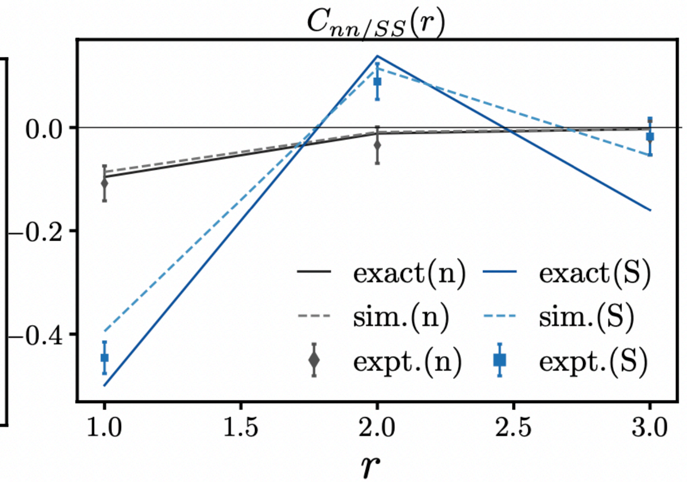



$$H_{\mathrm{FH}} = -\sum_{x,\sigma}\left(c^{\dagger}_{x+1,\sigma}c_{x,\sigma} + \mathrm{h.c.} + \mu n_x\right) + \sum_x \frac{U}{2} n_x(n_x - 1)$$




$$C_{SS}(r) = \langle S^z_0 S^z_r\rangle - \langle S^z_0\rangle\langle S^z_r\rangle$$
$$C_{nn}(\boldsymbol{r},\boldsymbol{r}') = \langle n_{\boldsymbol{r}} n_{\boldsymbol{r}'}\rangle - \langle n_{\boldsymbol{r}}\rangle\langle n_{\boldsymbol{r}'}\rangle$$

*Niu, Heghanas, Zhang, Chan, ACP (to appear)*