# Introduction to polarized high energy electron scattering

T.-A. Shibata

Nihon Univ. and Tokyo Tech

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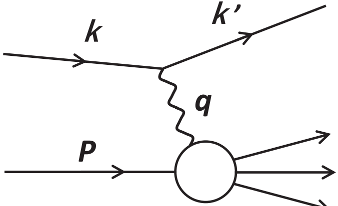
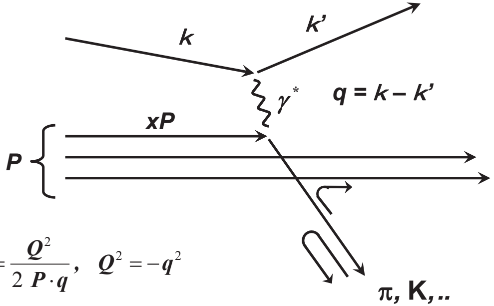
# 1. Kinematical variables of scattering, $Q^2, x, \ldots$




$$x = \frac{Q^2}{2\, P \cdot q}, \quad Q^2 = -q^2$$

The electron energy and the proton energy are asymmetric in collider experiments.
The electron energy is lower.

The laboratory frame $\neq$ the rest frame of the initial proton
$\neq$ the center of mass frame

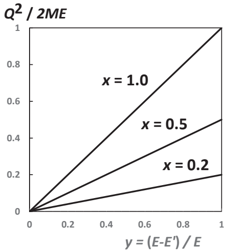
Important kinematical variables are Lorentz invariant,
so we make use of Lorentz invariance in experiments.
They can be evaluated in any frame.

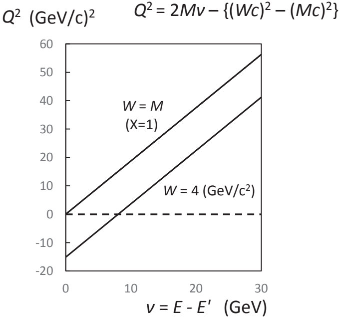
$Q^2$, $\nu$, $y$, $x$, $W$, $s$ are Lorentz invariant variables.

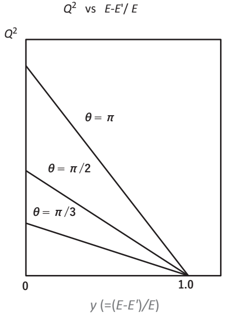
$$
\begin{aligned}
& k = (E/c, 0, 0, k^3) \approx (E/c, 0, 0, E/c) & \\
& k' = (E'/c, \vec{k}') & \\
& P = (P^0, 0, 0, P^3) & = (Mc, 0, 0, 0) \\
& q = k - k' = (\tfrac{E-E'}{c}, \vec{k} - \vec{k'}) & \\
& Q^2 = -q^2 \approx 4 \tfrac{E}{c} \tfrac{E'}{c} \sin^2 \tfrac{\theta}{2} & \\
& \nu = \tfrac{P \cdot q}{M} = (P^0 q^0 - P^3 q^3)/M & = E - E' \\
& y = \tfrac{P \cdot q}{P \cdot k} = (P^0 q^0 - P^3 q^3)/(P^0 k^0 - P^3 k^3) & = \tfrac{\nu}{E} = \tfrac{E-E'}{E} \\
& x = \tfrac{Q^2}{2\, P \cdot q} = \tfrac{Q^2}{2M\nu} = \tfrac{Q^2}{2\,(P^0 q^0 - P^3 q^3)} & \\
& (Wc)^2 = P'^2 = (P+q)^2 & \\
& \qquad = (Mc)^2 + 2P \cdot q - Q^2 & \\
& s = (P+k)^2 c^2 \approx (Mc)^2 + 2Pc \cdot kc & = (Mc)^2 + 2Mc^2 E \\
& \omega = \tfrac{1}{x} &
\end{aligned}
$$

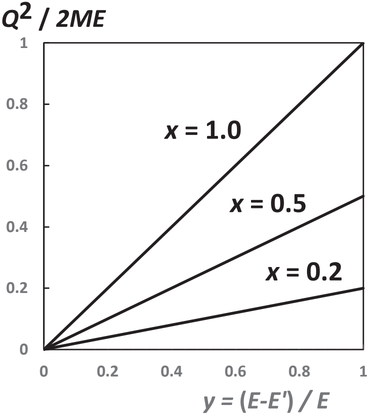
$M$ is the proton mass.
The right-hand side is for the frame where the initial proton is at rest.

The followings are the three basic plots of the kinematic variables.



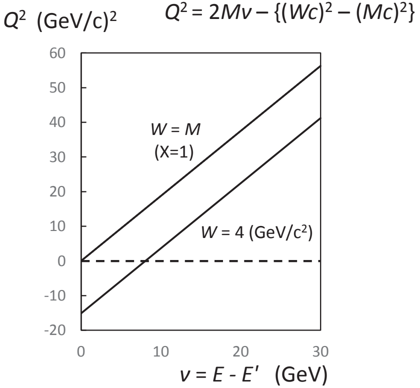






$W$ is the invariant mass of hadron system in the final state:
$W^2/c^2 = (P+q)^2 = P^2 + 2P \cdot q + q^2 = (Mc)^2 + 2P \cdot q - Q^2,$

$$W^2/c^2 - (Mc)^2 = 2P \cdot q - Q^2. \qquad (1)$$

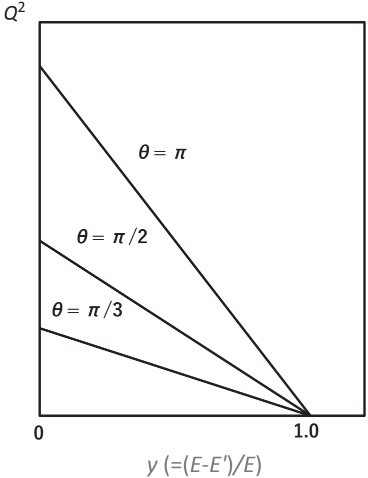
From this equation, one can see that $W^2/c^2 - (Mc)^2 = 0$ corresponds to $2P \cdot q - Q^2 = 0$ which means $x = 1$.




$$W^2/c^2 = (P+q)^2 = P^2 + 2P\cdot q + q^2 = (Mc)^2 + 2P\cdot q - Q^2,$$
$$Q^2 = 2M\nu - \{(Wc)^2 - (Mc)^2\}.$$

The deep inelastic region ($W > 4$ GeV) and the resonance region are identified in this plot.

$Q^2$ vs $E$-$E'$/ $E$




$$Q^2 = 4EE' \sin^2\frac{\theta}{2} = 4E(E-\nu)\sin^2\frac{\theta}{2} = 4E\sin^2\frac{\theta}{2}\cdot(E-\nu) \tag{2}$$

## 2. Structure functions

$$\frac{d^2\sigma}{dE'd\Omega} = \left(\frac{\alpha_0^2\,\cos^2\frac{\theta}{2}}{4E^2\sin^4\frac{\theta}{2}}\right)\left\{W_2 + 2\tan^2\frac{\theta}{2}\,W_1\right\} \tag{3}$$

In case of longitudinal polarization,

$$\frac{d^2\sigma^{\uparrow\downarrow}}{dE'd\Omega} - \frac{d^2\sigma^{\uparrow\uparrow}}{dE'd\Omega} \;\propto\; (E + E'\cos\theta)\,M\,G_1 - Q^2 G_2. \tag{4}$$

In the Bjorken limit ($x$ fixed, $Q^2, \nu \to \infty$),

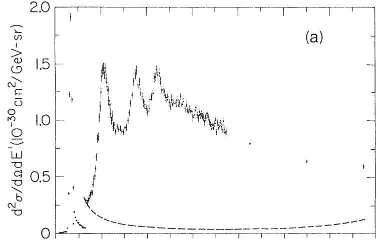
$$\nu W_2 \to F_2, \quad Mc^2 W_1 \to F_1, \quad M^2\,\nu\,G_1 \to g_1, \quad M\,\nu^2\,G_2 \to g_2. \tag{5}$$

Further structure functions when the proton is transversely polarized.

# 1. Scaling and deep inelastic scattering

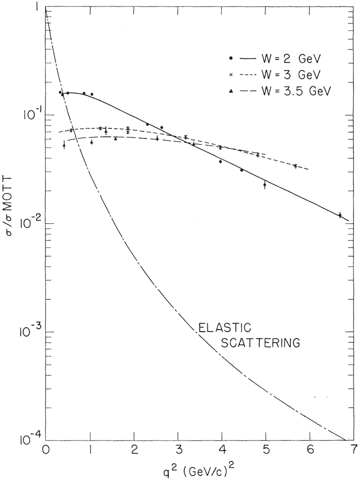
Nobel prize in physics, 1990,
J. I. Friedman, H. W. Kendall, R. E. Taylor
MIT-SLAC, Phys. Rev. Lett. 23 (1969) 930-934, 23 (1969) 935-939.

$E_e = 7 - 17.7$ GeV, $\theta = 6^\circ$, $10^\circ$.
$W$ dependence of the cross section




The horizontal axis is $W$ (GeV), ranging from 0.8 to 3.6 GeV. Resonances are seen between 0.938 to 2 GeV.

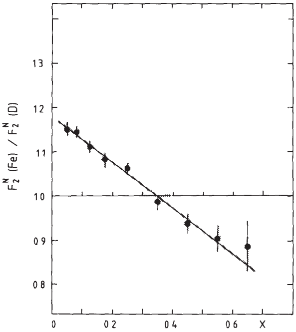
$Q^2$ dependence of the cross section at $W = 2,\ 3,\ 3{,}5$ GeV. Calculaton of elastic scattering (W = 0.938 GeV) is also shown.




$Q^2$ dependence is small at $W = 3$ and 3.5 GeV

# 2. EMC-effect Modulation of $F_2$ in nuclei

$F_2$(Fe) / $F_2$(D)




$x$

EMC, J.J. Aubert et al., Phys. Lett. 123B 275-278 (1983)

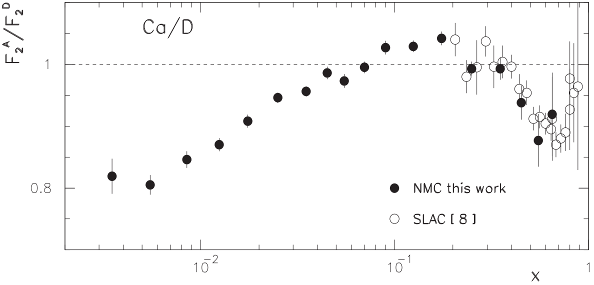
muon beam energy:
100, 200, 250, 280 GeV,
covered $x$ range: $0.03 - 0.65$

$+15\%$ enhancement,
$-10\%$ depletion.

$F_2$(Fe) / $F_2$(D) was fitted with $a + b\,x$

NMC, P. Amaudruz et al., Nucl. Phys. B441, 3-11 (1995)

$F_2(\mathrm{Ca})\ /\ F_2(\mathrm{D})$




muon beam energy: 200 GeV,
covered $x$ range: $0.0035 < x < 0.65$

Electron scattering at large $x$, $x \geq 1$, JLab
$x = 1$ corresponds to eN elastic scattering,
scattering from correlated nucleons

# 3. Proton spin problem

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_z^q + L_z^G, \tag{6}$$

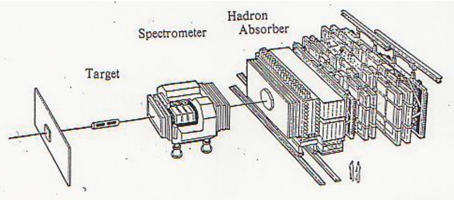
$$\frac{1}{2}\Delta\Sigma = \frac{1}{2}\int_0^1 (u\uparrow(x) - u\downarrow(x))dx + \frac{1}{2}\int_0^1 (d\uparrow(x) - d\downarrow(x))dx \tag{7}$$

$$+ \frac{1}{2}\int_0^1 (s\uparrow(x) - s\downarrow(x))dx = \frac{1}{2}(\Delta u + \Delta d + \Delta s). \tag{8}$$

Contribution of spins of quarks and anti-quarks to the proton spin.

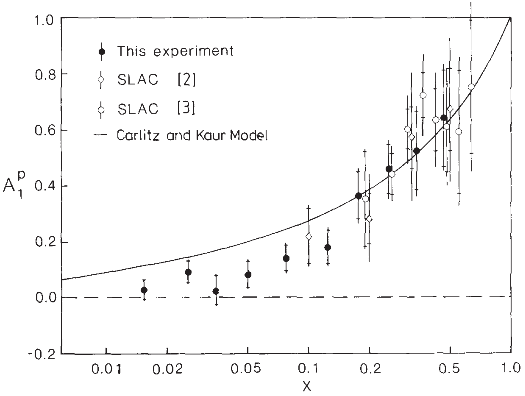
EMC J. Ashman et al., Phys. Lett. B 206 (1988) 364,
EMC J. Ashman et al., Nucl. Phys. B 328 (1989) 1.

Longitudinally polarized muon and longitudinally polarized proton,
Muon beam energy 100, 120, 200 GeV at CERN.



$$A = \frac{\sigma_{\uparrow\downarrow} - \sigma_{\uparrow\uparrow}}{\sigma_{\uparrow\downarrow} + \sigma_{\uparrow\uparrow}} \quad \longrightarrow \quad \frac{g_1(x)}{F_2(x)} \tag{9}$$

$$\begin{aligned} \int_0^1 g_1^{\mathrm{p}}(x)\, dx &= \frac{1}{2}\left(\frac{4}{9}\Delta\mathrm{u} + \frac{1}{9}\Delta\mathrm{d} + \frac{1}{9}\Delta\mathrm{s}\right) \qquad (10) \\ &= \frac{1}{12}\Big[(\Delta\mathrm{u} - \Delta\mathrm{d}) + \frac{1}{3}(\Delta\mathrm{u} + \Delta\mathrm{d} - 2\Delta\mathrm{s}) + \frac{4}{3}(\Delta\mathrm{u} + \Delta\mathrm{d} + \Delta\mathrm{s})\Big] \end{aligned}$$



EMC's result: $\Delta\Sigma = 0.12 \pm 0.09 \pm 0.14$,
Contribution of the spins of quarks and anti-quarks to the proton spin is $(12 \pm 9 \pm 14)\%$.

Further studies by SMC, HERMES, COMPASS, PHENIX, STAR, JLab experiments, SpinQuest,..

## 4. Flavor asymmetry of sea quarks in the proton

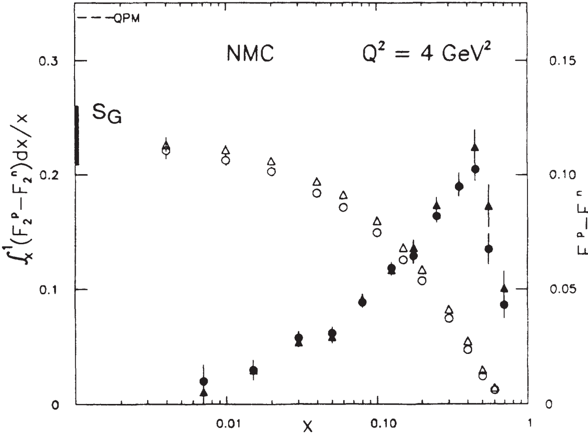
Gottfried sum is defined as

$$S_{\mathrm{G}} = \int_0^1 dx \, \frac{F_2^{\mathrm{p}}(x) - F_2^{\mathrm{n}}(x)}{x} \tag{11}$$

$$F_2^{\mathrm{p}}(x)/x = \frac{4}{9}(u_v^p + u_s^p + \bar{u}^p) + \frac{1}{9}(d_v^p + d_s^p + \bar{d}^p) + \frac{1}{9}(s^p + \bar{s}^p)$$
$$F_2^{\mathrm{n}}(x)/x = \frac{1}{9}(d_v^n + d_s^n + \bar{d}^n) + \frac{4}{9}(u_v^n + u_s^n + \bar{u}^n) + \frac{1}{9}(s^n + \bar{s}^n)$$

$$S_{\mathrm{G}} = \frac{1}{3} + \frac{2}{3}\left(\int_0^1 \bar{u}^p \, dx - \int_0^1 \bar{d}^p \, dx\right). \tag{12}$$

NMC, P. Amaudruz et al., Phys. Rev. Lett. 66 (1991) 2712-2715,
NMC, M. Arneodo et al., Phys. Rev. 50D (1994) R1-R3.
muon beam energy: 90 and 280 GeV,
covered x range: 0.004 – 0.8.



$$S_{\mathrm{G}} = \frac{1}{3} + \frac{2}{3}\left(\int_0^1 \bar{u}^p\, dx - \int_0^1 \bar{d}^p\, dx\right) = 0.235 \pm 0.026,$$

$$\int_0^1 \bar{d}^p\, dx - \int_0^1 \bar{u}^p\, dx = 0.147 \pm 0.039. \tag{13}$$

$\bar{d}$ is more abundant than $\bar{u}$ in the proton.

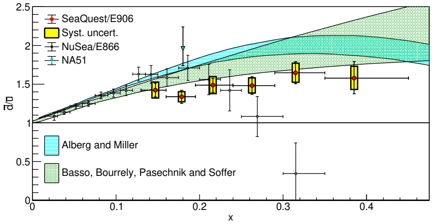
Flavor asymmetry of sea quarks in the proton.

Drell-Yan process in pp and pd reactions.
NA51, NuSea/E866, SeaQuest

$$q + \bar{q} \to \gamma^* \to \mu^+\mu^-$$

SeaQuest, J. Dove et al., SeaQuest, Nature 590 561-565 (2021)  FNAL, 120 GeV proton beam

$\bar{d}/\bar{u}$




$x$

The ratio is about 1.5 at $x = 0.13 - 0.45$.
Data from lepton deep inelastic scattering and hadron reactions are complementary.

## Summary

- In the first half of the talk, I showed the relation between the kinematical variables.

In designing and building the detector, $k'$ or $(E', \theta, \phi)$ in the laboratory frame are the important variables as the detector sits in the laboratory frame.

Events are detected only when $E', \theta, \phi$ are inside the range of the detector.
Once the events are detected, the cross sections can be converted using Jacobian determinant between $(E', \theta, \phi)$ and $(Q^2, x, \phi)$.

• In the second half of the talk, I showed four examples of the experiments.

After the initial finding of scaling and deep-inelastic scattering at the electron accelerator of SLAC, high energy muon beams at the proton accelerators benefitted from its capability to access the small $x$ region.

Experiments were extended to the e-p collider.

Hadron physics will be further extended in future with the new facility of polarized e-p and e-A collider.