Erosion of N = 28 shell gap: shape coexistence and monopole transitions in the vicinity of ^{44}S

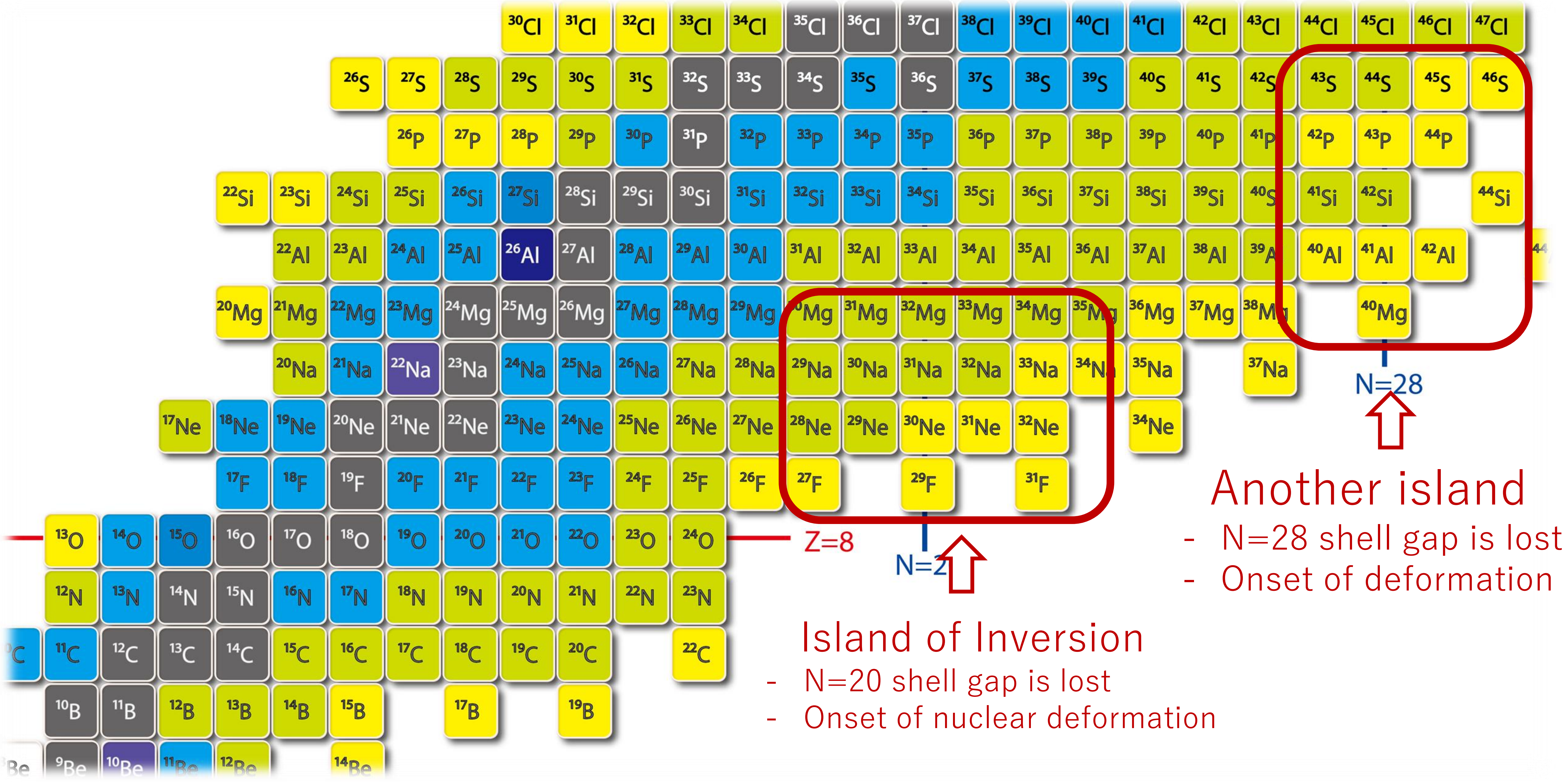
M. Kimura, Y. Suzuki and W. Horiuchi

Y. Suzuki and MK, PRC104, 024327 (2021)

Y. Suzuki W. Horiuchi and MK, to be appear on arXiv tomorrow (19th Jan)

1. Introduction
2. Numerical method
3. Results & Discussion
   - Shape coexistence in N=28 isotones
   - Nuclear shape and erosion of N=28 shell closure
   - Monopole transition as a probe for shape coexistence
4. Summary & Perspective

Quenching of N=28 shell gap and shape coexistence in neutron-rich N=28 isotones

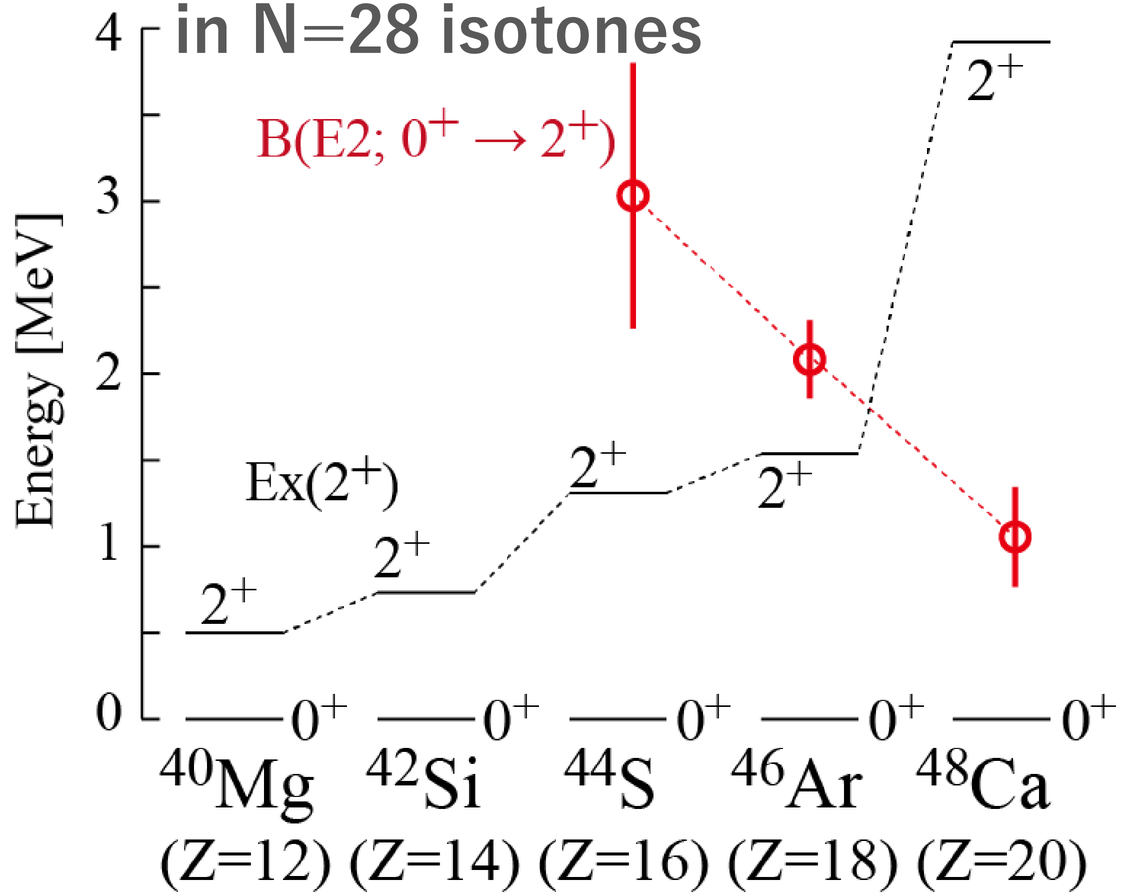


Another island
- N=28 shell gap is lost
- Onset of deformation

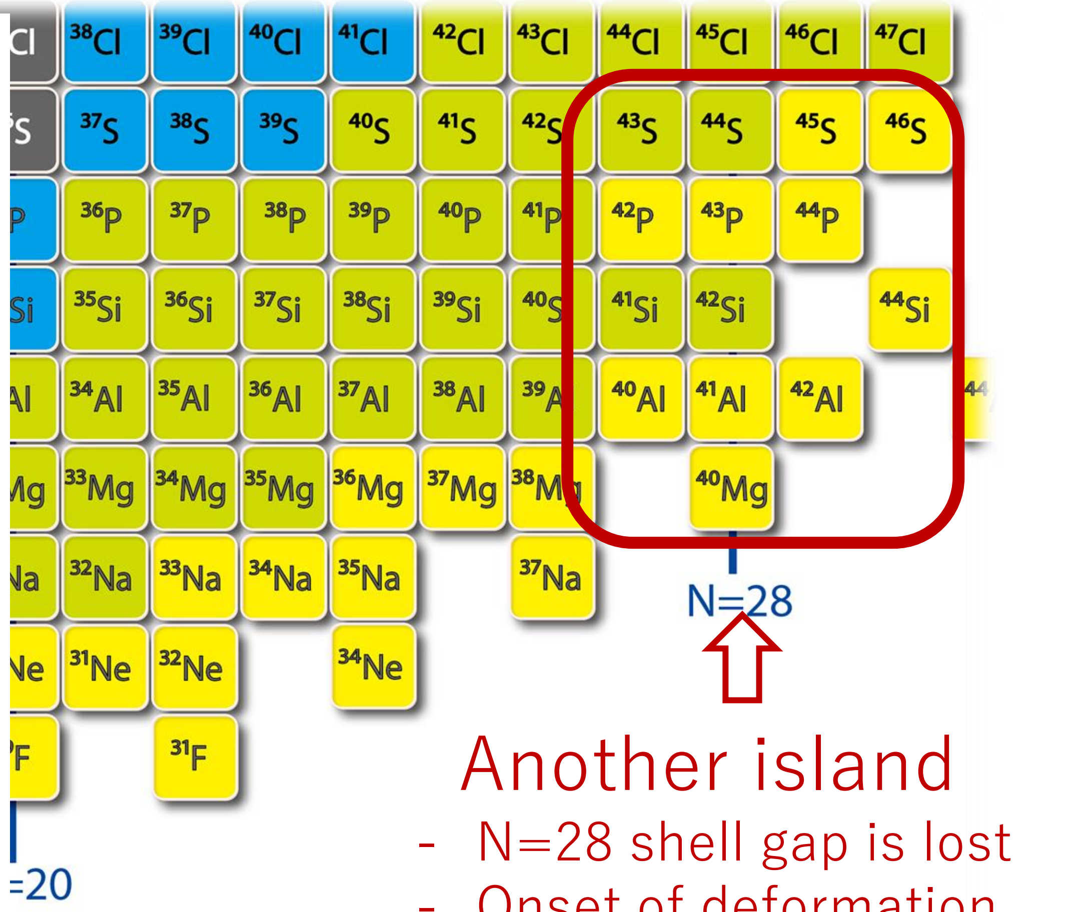
Island of Inversion
- N=20 shell gap is lost
- Onset of nuclear deformation

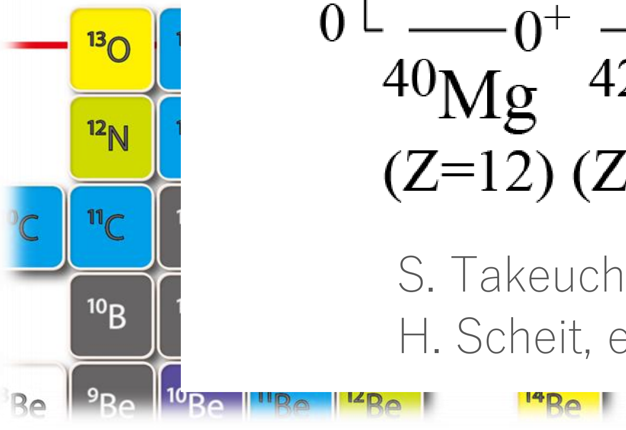
Quenching of N=28 shell gap and shape coexistence in neutron-rich N=28 isotones

Onset of the deformation in N=28 isotones



S. Takeuchi, et al., PRL109, 1 (2012).
H. Scheit, et al., 77, 3967 (1996).





Another island
- N=28 shell gap is lost
- Onset of deformation

What is new?
What is interesting?
⇨ “Shape coexistence”

Quenching of N=28 shell gap and shape coexistence in neutron-rich N=28 isotones

○ **N=28 is the smallest magic number created by the spin-orbit splitting**

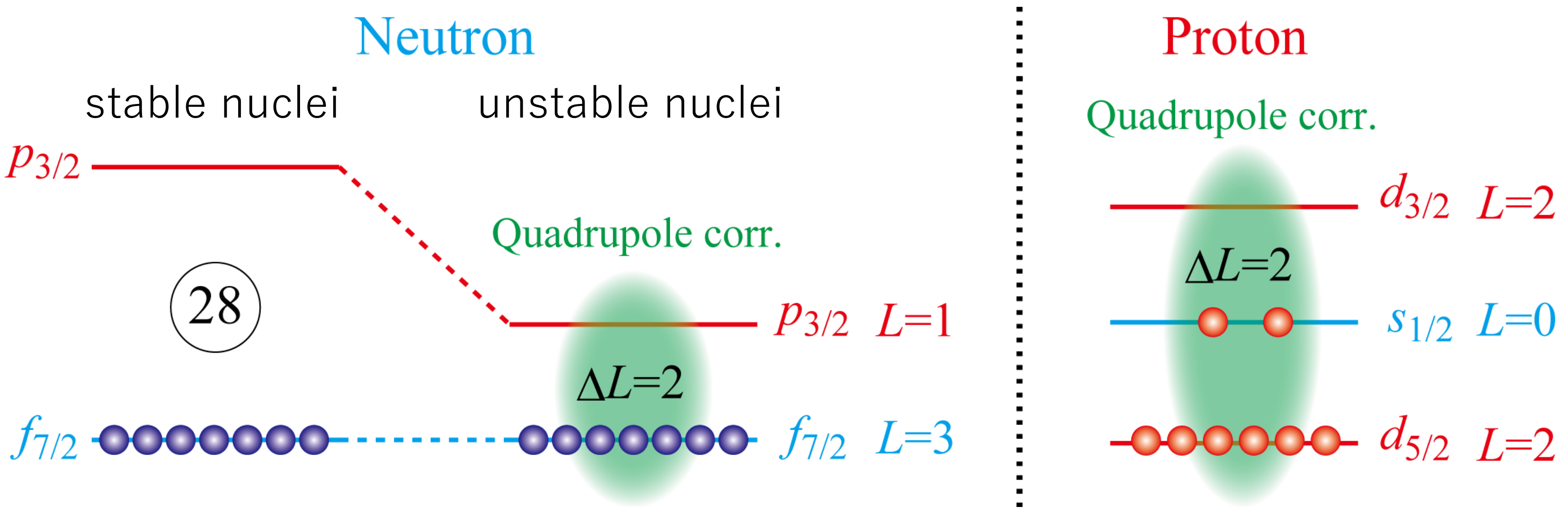
⇨ Quenching of N=28 shell gap leads to the degeneracy of p- and f-wave

⇨ Quadrupole correlations between valence neutrons

○ **Mg, Si and S (Z=12,14 and 16) have proton half-filling of sd-shell**

⇨ Quadrupole correlations between protons and neutrons

⇨ **Shape coexistence:** various deformed shapes coexist at small energies

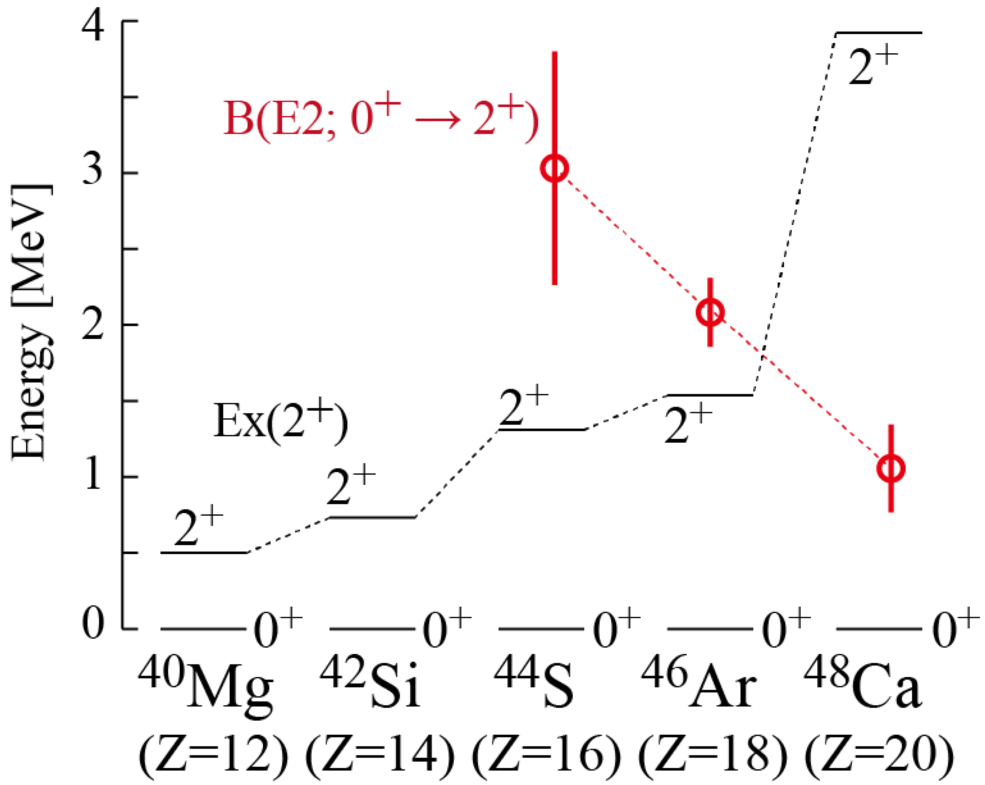


Quenching of N=28 shell gap and shape coexistence in neutron-rich N=28 isotones

Onset of the ground state deformation is well known

- Reduction of 2^+ state energy
- Increase of B(E2)

e.g., S. Takeuchi, et al., PRL109, 1 (2012).
H. Scheit, et al., 77, 3967 (1996)



However, the deformation of non-yrast states (shape coexistence) is not known well

We focus on

1. Shapes coexistence in N=28 isotones; ^{40}Mg, ^{42}Si and ^{44}S
2. Relationship between nuclear shape and the erosion of N=28 shell closure
3. Monopole transition as a probe for shape coexistence

Numerical Method: Antisymmetrized Molecular Dynamics (AMD)

To describe various deformed states, we have employed AMD framework

○ Hamiltonian Gogny D1S density functional J. F. Berger et al., CPC 63, 365 (1991)

$$\hat{H} = \sum_{i}^{A} \hat{t}_i - \hat{t}_{c.m.} + \sum_{i<j}^{A} \hat{v}_{\mathrm{GognyD1S}}(r_{ij}) + \sum_{i<j}^{Z} \hat{v}_{\mathrm{Coulomb}}(r_{ij})$$

○ Model wave function Antisymmetrized product of nucleon wave packets

$$\Psi^{\pi} = \frac{1+\pi\hat{P}_r}{2}\mathcal{A}\{\varphi_1, \varphi_2, ..., \varphi_A\}, \quad \varphi_i(\boldsymbol{r}) = \exp\left\{ -\nu(\boldsymbol{r} - \boldsymbol{Z}_i)^2 \right\} \cdot (a_i \left|\uparrow\right\rangle + b_i \left|\uparrow\right\rangle)$$

○ Energy variation with constraint

Model parameters (nucleon position and momenta, spins) are determined to minimize the energy with the constraint on the nuclear shape

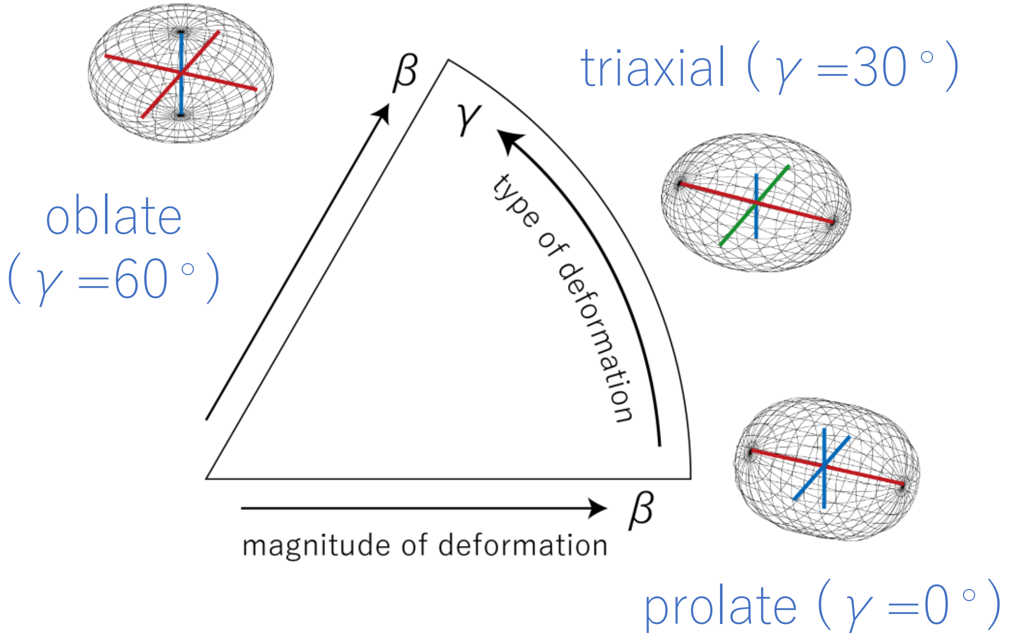
Numerical Method: Antisymmetrized Molecular Dynamics (AMD)

We calculate the energy surface as function of the quadrupole deformation parameters of nuclear shape

$$E(\beta,\gamma) = \frac{\langle P^J_{MK}\Phi^\pi(\beta,\gamma)|H|P^J_{MK}\Phi^\pi(\beta,\gamma)\rangle}{\langle P^J_{MK}\Phi^\pi(\beta,\gamma)|P^J_{MK}\Phi^\pi(\beta,\gamma)\rangle}.$$

β : magnitude of the deformation

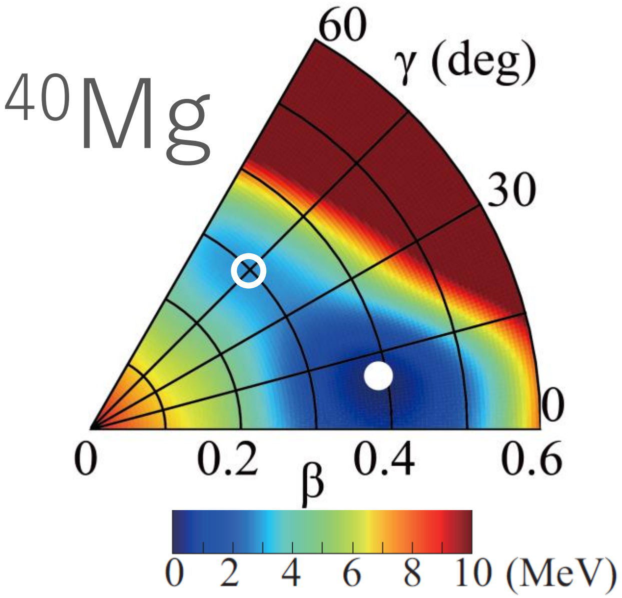
γ : type of deformation (prolate, oblate, triaxial)



The energy surface tells us which shape is favored

e.g., ^{40}Mg has the energy minimum at prolate shape

it also has local energy minimum at oblate shape



Numerical Method: Antisymmetrized Molecular Dynamics (AMD)

Mixing of different shapes (generator coordinate method; GCM)

$$\Psi^{J\pi}_{M\alpha} = \sum_{iK} \underbrace{g_{iK\alpha}}_{\text{amplitude}} P^{J}_{MK} \underbrace{\Phi^{\pi}(\beta_i, \gamma_i)}_{\text{wave function with definite shape}},$$

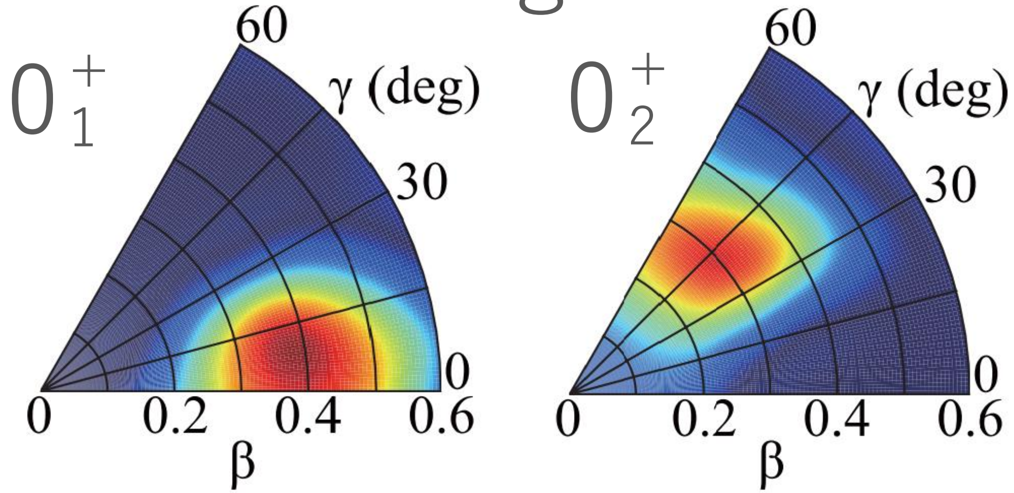
- The wave functions with various shapes are superposed ⇒ Shape fluctuation
- The amplitude and eigen-energy are obtained by diagonalizing Hamiltonian

The GCM amplitude tells us equilibrium nuclear shape and fluctuation around it

$$f(\beta,\gamma) = \frac{\langle \Psi^{J\pi}_{M\alpha} | P^{J}_{MK} \Phi^{\pi}(\beta,\gamma)\rangle}{\sqrt{\langle P^{J}_{MK}\Phi^{\pi}(\beta,\gamma) | P^{J}_{MK}\Phi^{\pi}(\beta,\gamma)\rangle}}.$$

e.g., ^{40}Mg has prolately-deformed ground state, and oblately-deformed 2nd 0^+ state.

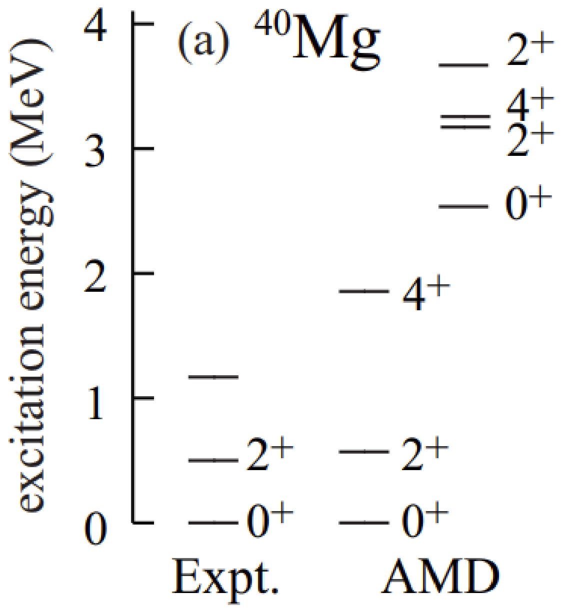
^{40}Mg



Results & Discussion : Shape coexistence in N=28 isotones

^{40}Mg exhibits the coexistence of

- **Prolate deformed ground band**
- **Oblate deformed excited band**

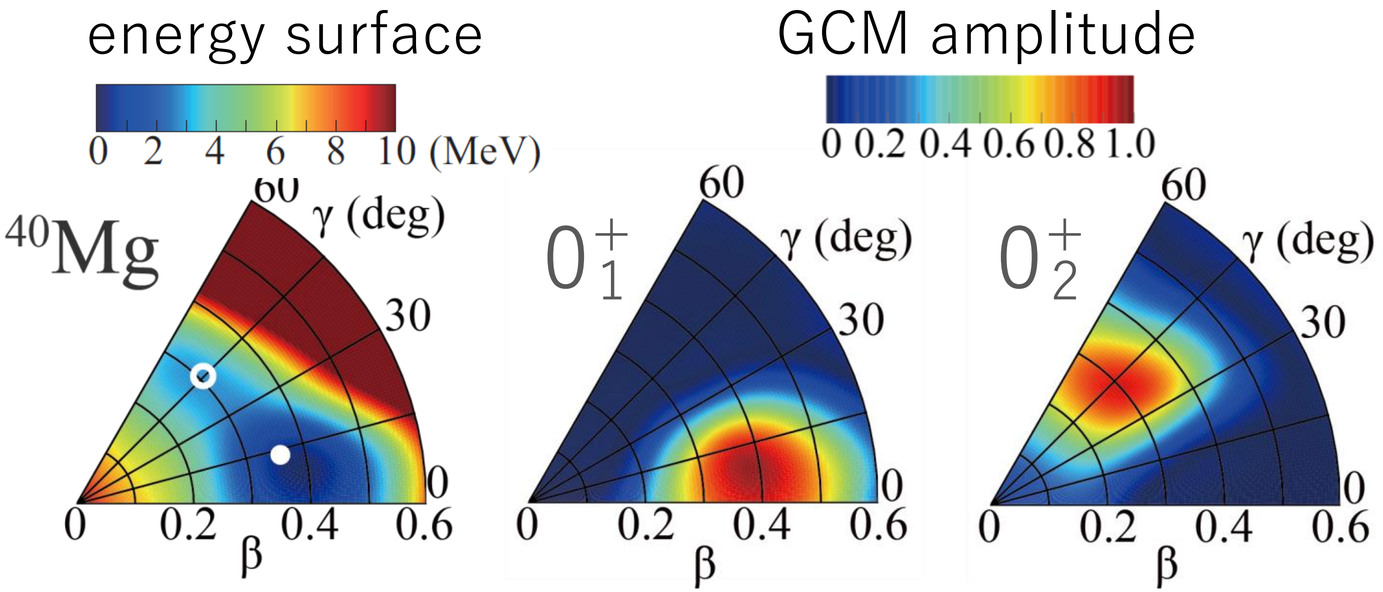


○ ^{40}Mg has the energy minimum at <u>prolate shape</u> and a local minimum at <u>oblate shape</u>

○ GCM amplitude is localized in the prolate and oblate deformed regions

○ Small fluctuation of GCM amp.

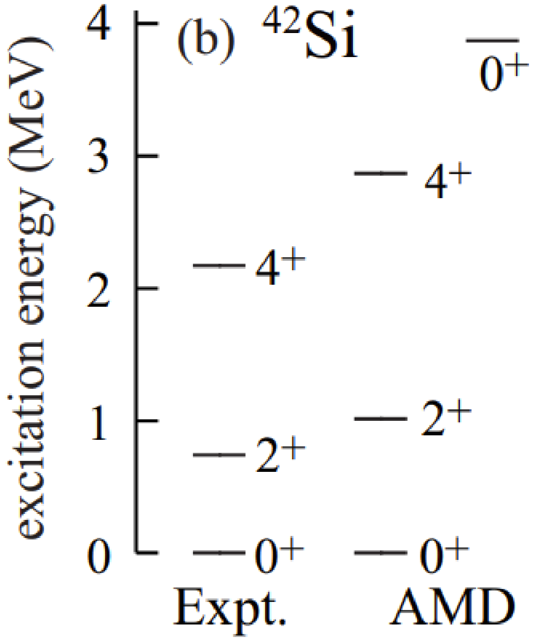
Coexistence of rigid rotors with different shapes



Results & Discussion : Shape coexistence in N=28 isotones

^{42}Si exhibits the coexistence of

- **Oblate deformed ground band**
- **A spherical state**

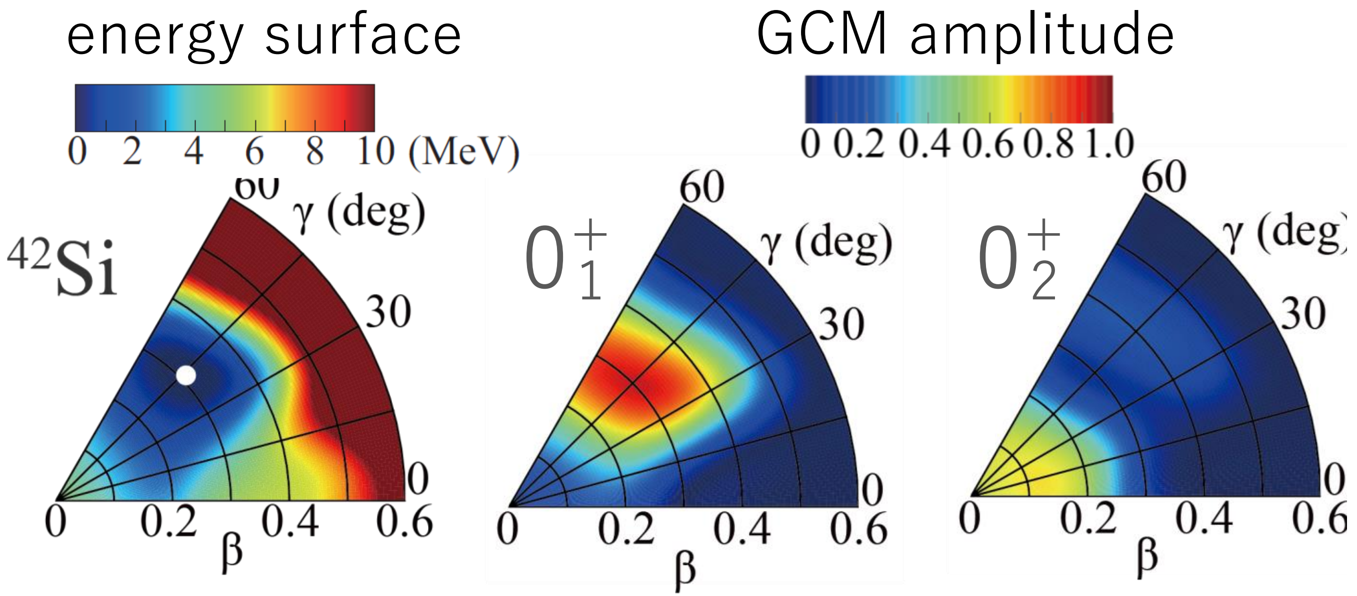


○ ^{42}Si has the energy minimum at oblate shape

○ GCM amplitude is localized in the oblate deformed and spherical regions

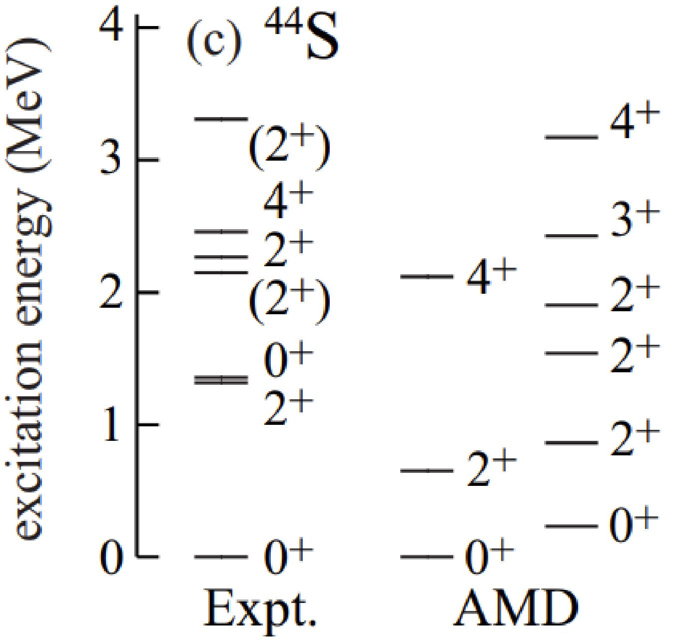
○ Small fluctuation of GCM amp.

Coexistence of a rigid rotor and a spherical state



Results & Discussion : Shape coexistence in N=28 isotones

^{44}S exhibits "large amplitude collective motion"



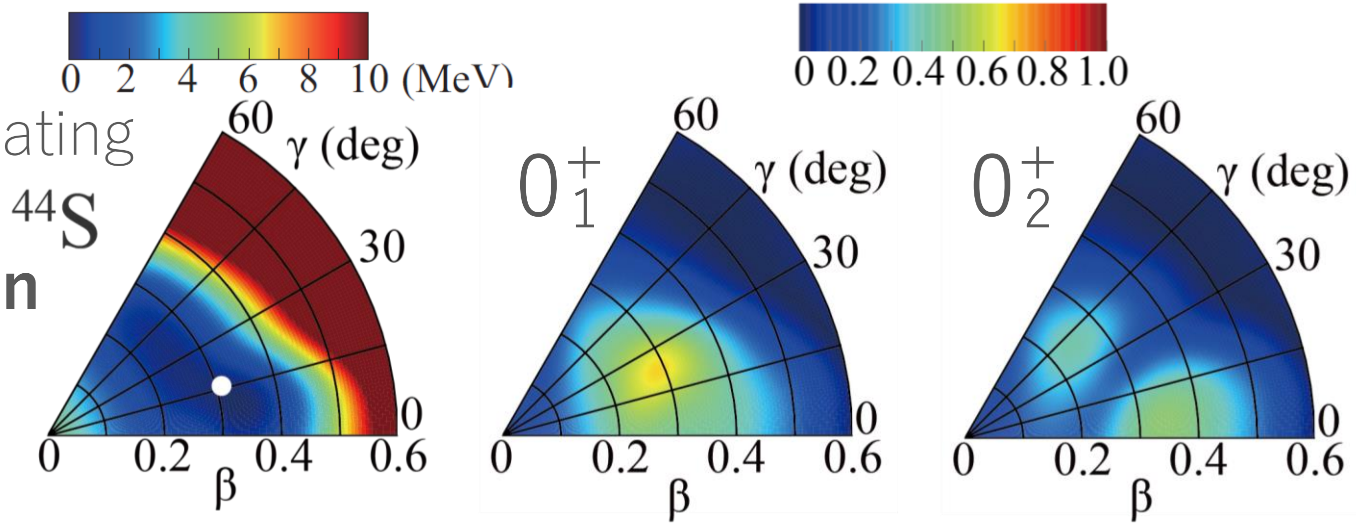
○ ^{44}Si has flat energy surface against γ deformation

○ GCM amplitude has broad and non-localized distribution ⇒ Large shape fluctuation

○ Both the ground and 2nd 0^+ have no shape. Their shapes are fluctuating

energy surface

GCM amplitude



Large amplitude collective motion

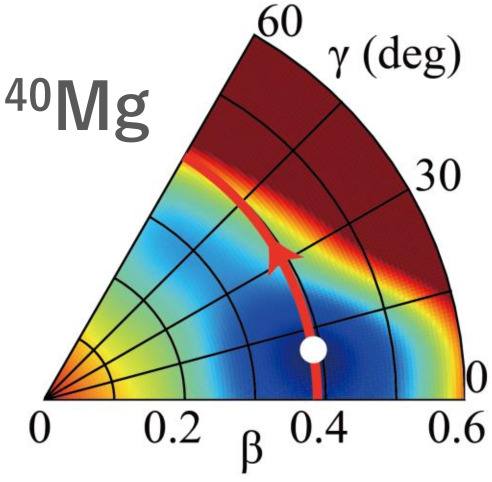
Collective motion beyond small amplitude approximation (RPA)

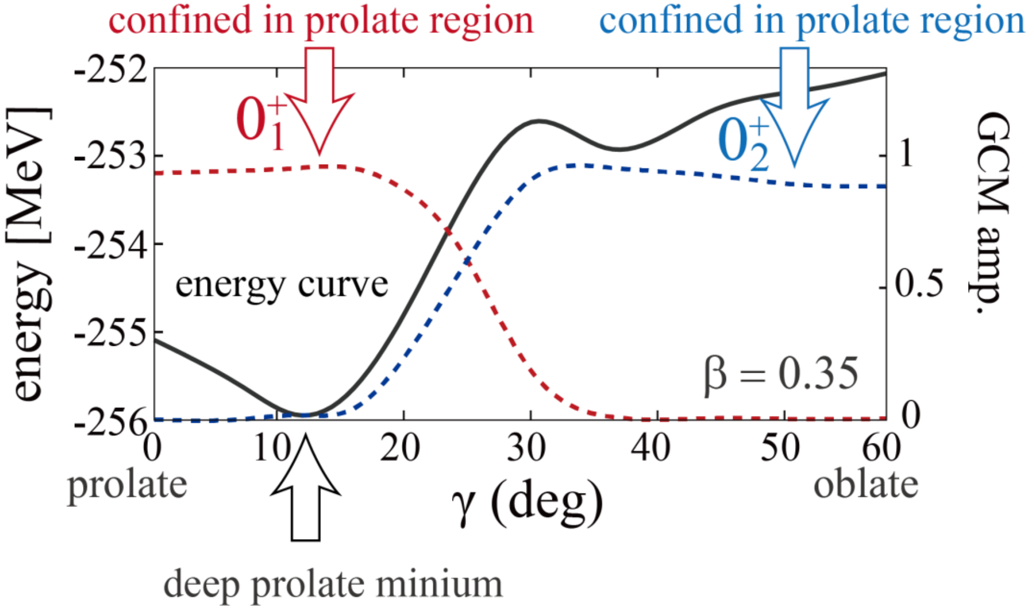
In comparison

^{40}Mg (^{42}Si): Rigid rotors

- Deep energy minimum
- GCM amp are localized

⇨ Definite shape with small fluctuation

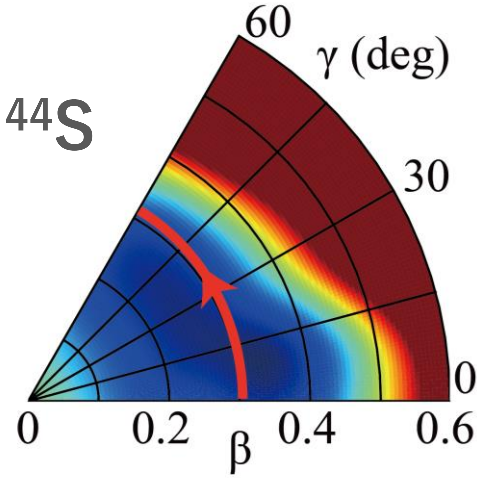


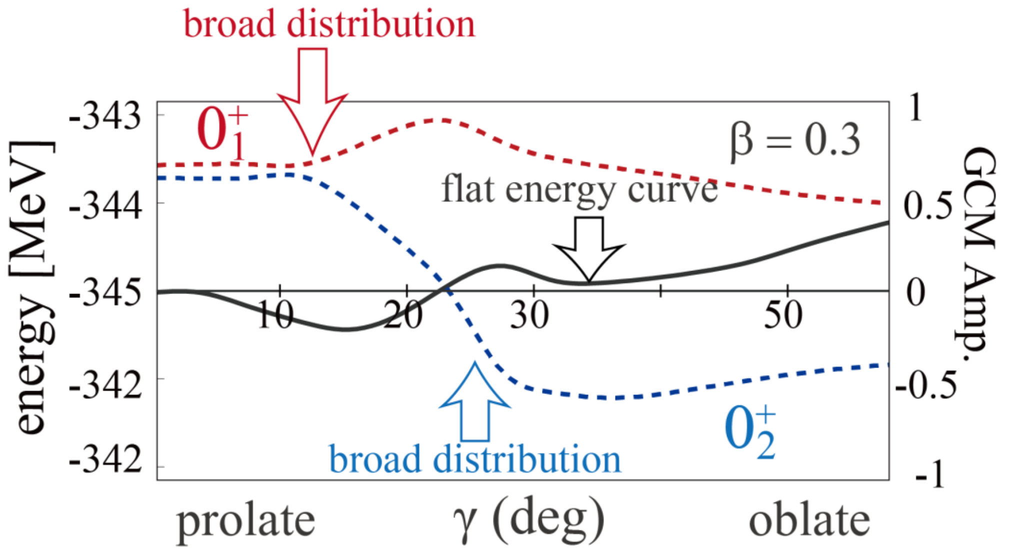


^{44}S: Large amplitude collective motion

- Flat energy surface
- Non-localized GCM

⇨ Large Amp. Coll. Motion





Results & Discussion : Shape coexistence in N=28 isotones

Summary for “shape coexistence in N=28 isotones”

^{40}Mg: Rigid shape

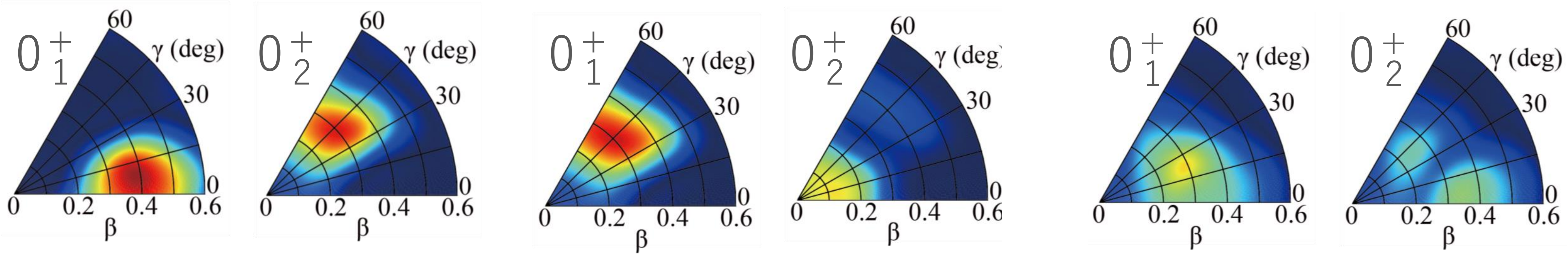
$|0_1^+\rangle$ Prolate rigid rotor
$|0_2^+\rangle$ Oblate rigid rotor

^{42}Si: Rigid shape

$|0_1^+\rangle$ Oblate rigid rotor
$|0_2^+\rangle$ Spherical state

^{44}S: No shape

$|0_1^+\rangle$ Large amplitude
$|0_2^+\rangle$ collective motion



N=28 isotones manifest different pattern of shape coexistence

In the next part I will discuss

- Relationship between nuclear shape and single-particle orbits
- Relationship between nuclear shape and erosion of N=28 shell closure

Results & Discussion : Nuclear shape and erosion of N=28 shell closure

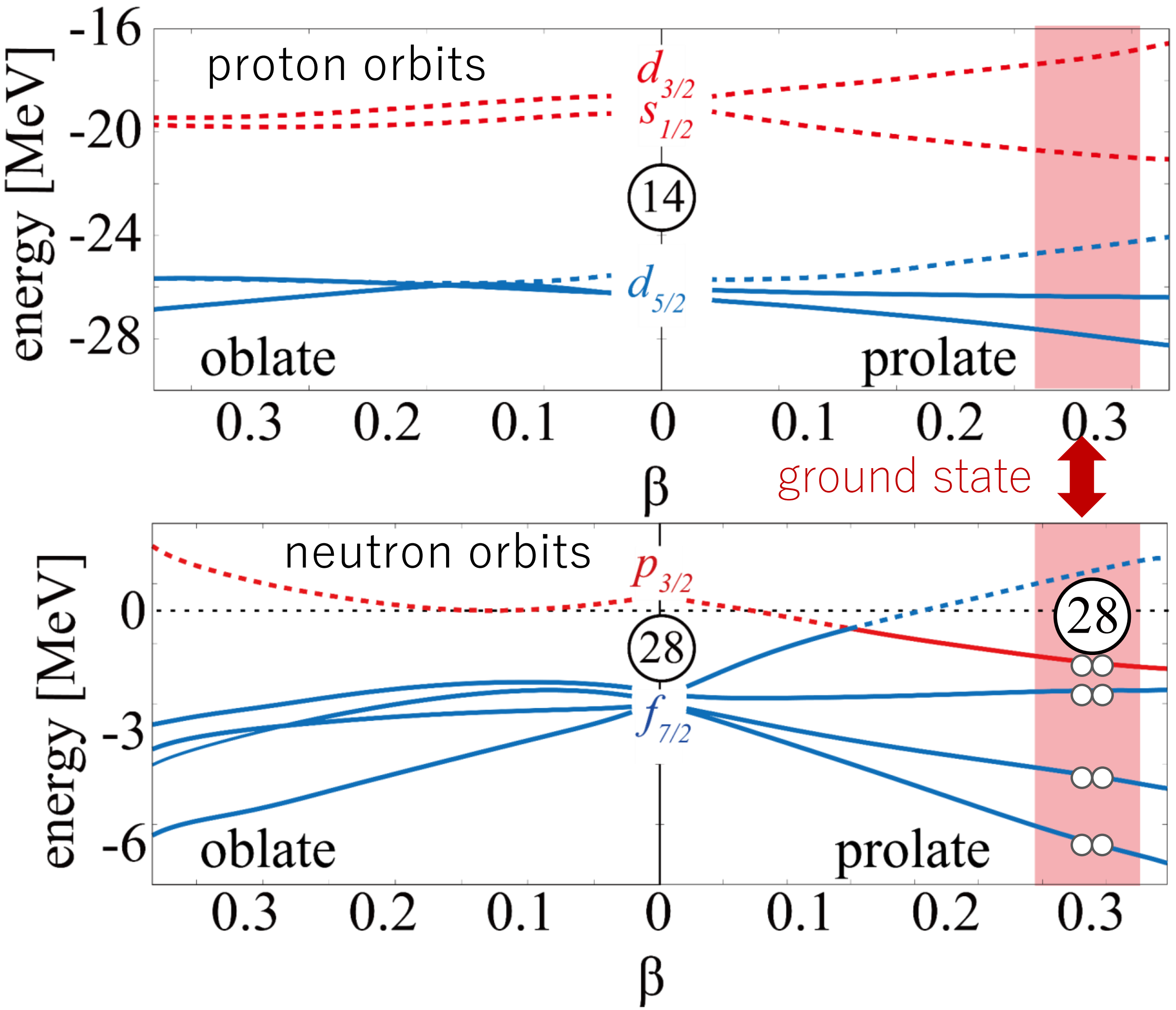
Nuclear shape and structure of Fermi surface are closely related

^{40}Mg

○ N=28 shell gap is considerably reduced (less than 2 MeV at spherical shape)
⇨ An intruder orbit from $p_{3/2}$ quickly comes down
⇨ A new shell gap N=28 is created in prolately-deformed region

⇨ Prolately-deformed ground state

Note that N=28 shell closure is explicitly broken by the inversion of neutron orbits



Results & Discussion : Nuclear shape and erosion of N=28 shell closure

Nuclear shape and structure of Fermi surface are closely related

^{40}Mg

○ N=28 shell gap is considerably reduced (less than 2 MeV at spherical shape)

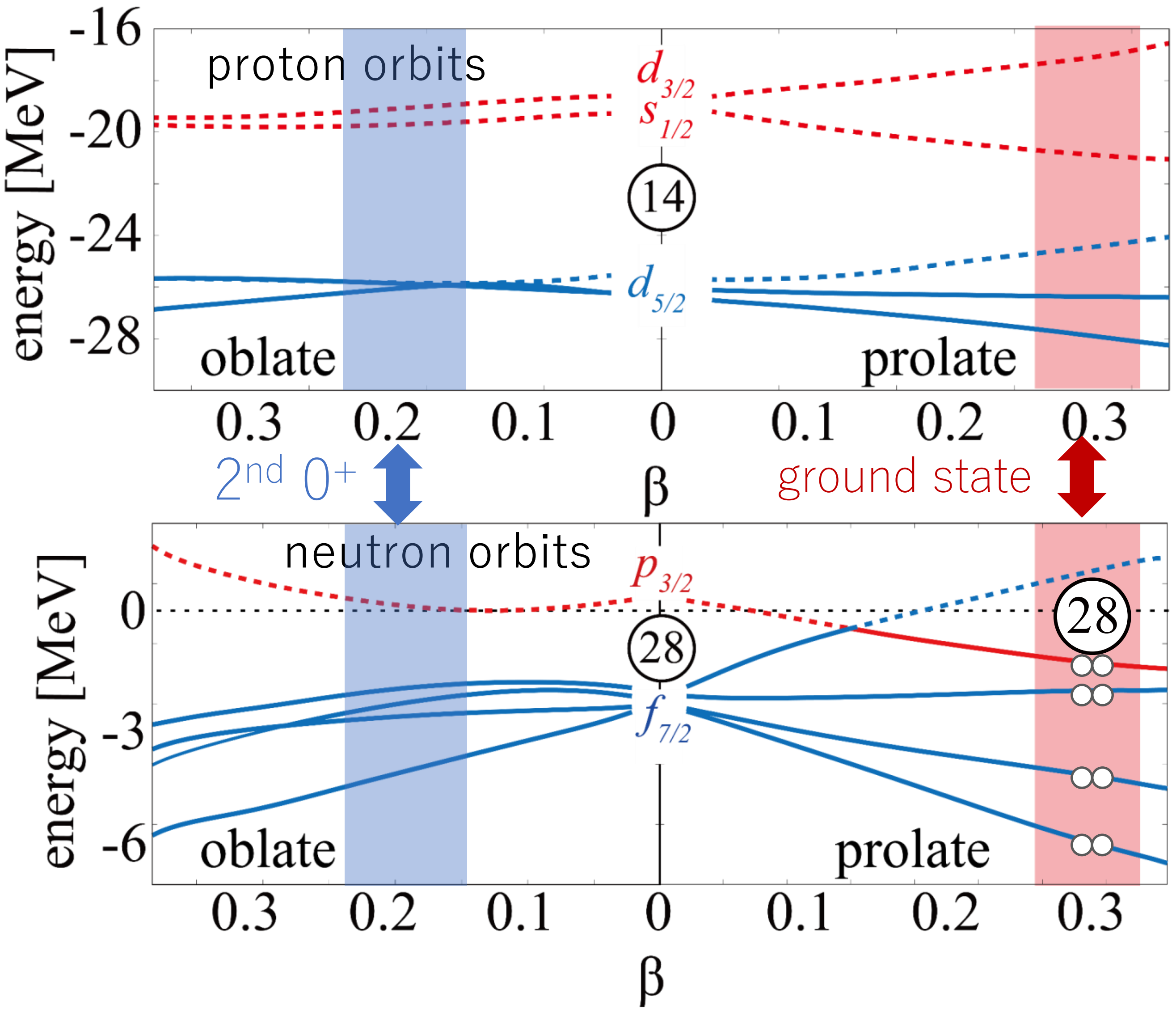
⇨ An intruder orbit from $p_{3/2}$ quickly comes down

⇨ A new shell gap N=28 is created in prolately-deformed region

⇨ Prolately-deformed ground state

○ Large shell gap (Z=14, N=28) is also maintained in oblate region

⇨ Oblately-deformed state appears as the 2^{nd} 0^+ state



Results & Discussion : Nuclear shape and erosion of N=28 shell closure

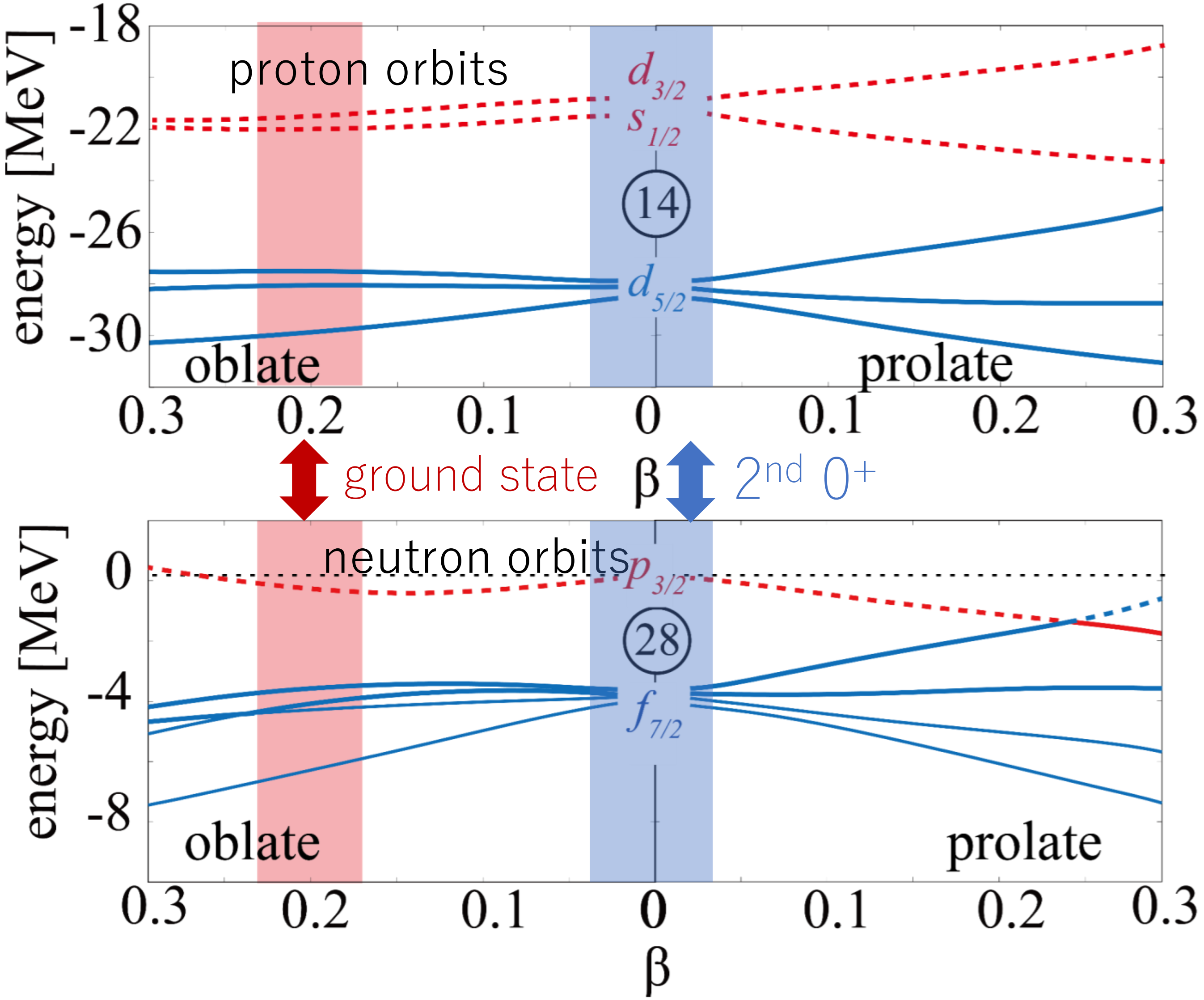
Nuclear shape and structure of Fermi surface are closely related

^{42}Si

- ○ Protons disfavors prolate deformation
- ○ N=28 shell gap is larger than ^{40}Mg
  ⇨ Prolately-deformed N=28 shell gap is not large enough
- ○ Large shell gap (Z=14, N=28) in oblately-deformed region

  ⇨ Oblately-deformed ground state

  ⇨ Spherical 2^{nd} 0^{+} state



Results & Discussion : Nuclear shape and erosion of N=28 shell closure

Nuclear shape and structure of Fermi surface are closely related

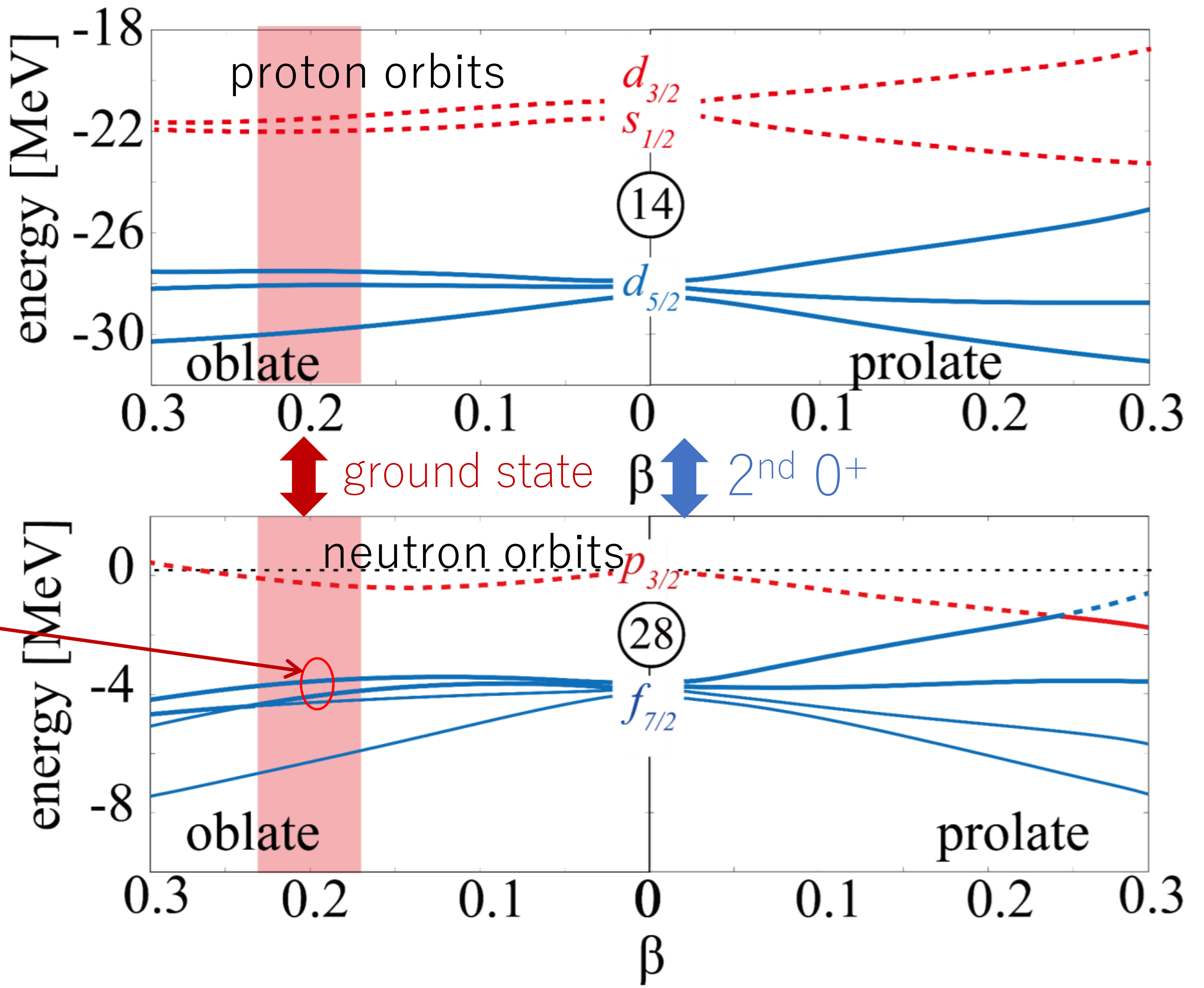
^{42}Si

Note that N=28 shell closure is also lost in the oblate deformed ground state of ^{42}Si, although there is no inversion of the neutron orbits.

Due to the deformation and weak-binding, the valence orbits with $\Omega = 1/2$ and $3/2$ are the mixture of the f- and p-waves.

e.g., I. Hamamoto, PRC 93, 054328 (2016).

Because of this mixing, valence neutrons partially occupy p-wave, which breaks the N=28 shell closure without level inversion.



Results & Discussion : Nuclear shape and erosion of N=28 shell closure

Summary for "nuclear shape and erosion of N=28 shell closure"

○ Shape of each nucleus can be roughly explained from
(shell gap, size of shell gap, …)

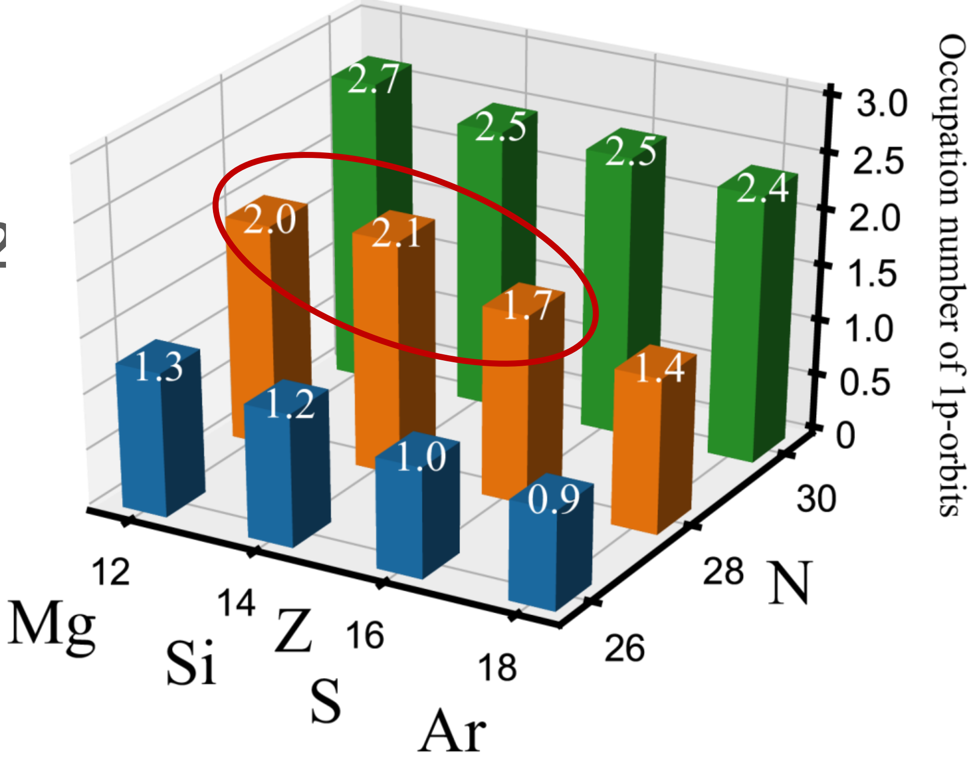
○ **Nuclear shape is closely related to how the N=2**

^{40}Mg has prolate-deformed ground state

⇨ The N=28 shell closure is explicitly lost with

^{42}Si has oblate-deformed ground state

⇨ The N=28 shell closure is implicitly lost with



○ **However, these differences do not affect the occupation number of p-wave**

Number of neutrons in p-wave : ^{40}Mg 2.0, ^{42}Si 2.1 ^{44}S 1.7
(these should be zero for N=28 shell closure)

⇨ occupation number is not a good probe

⇨ **Monopole transition strengths as an alternative probe for nuclear shape**

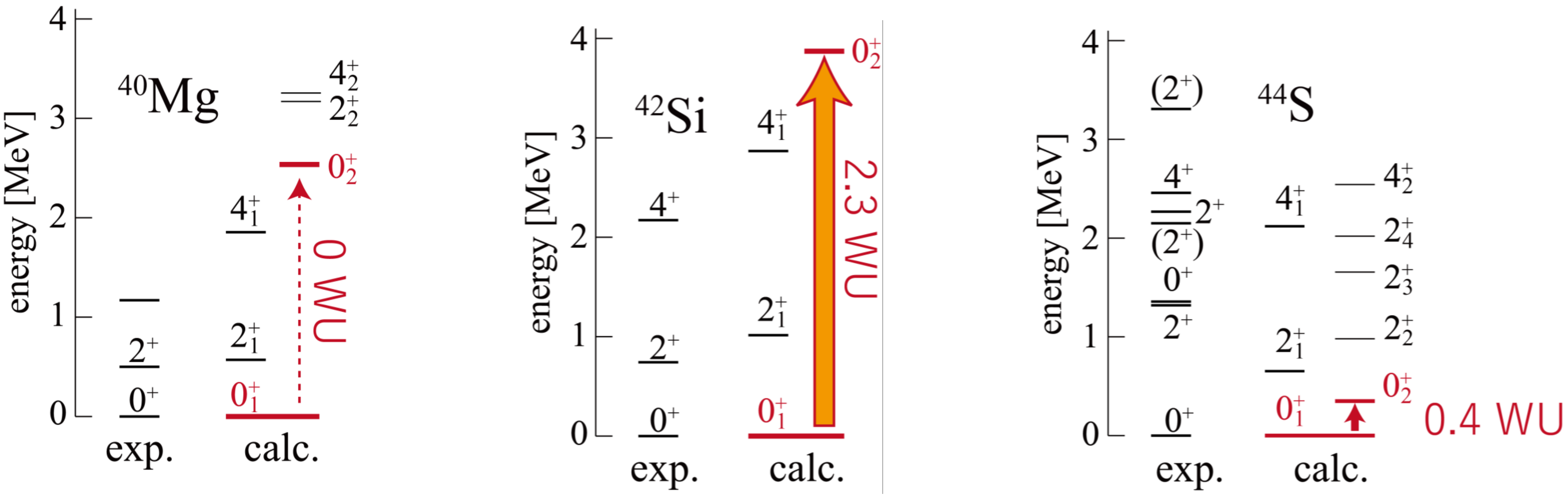
Results & Discussion : Monopole transition as a probe for shape coexistence

We propose the monopole transition as a probe for shape coexistence

Monopole transition strengths are different in order of magnitudes
⇨ This reflects the shape and structure of individual nuclei

monopole operators

$$\mathcal{M}_{E0} = \sum_{i=1}^{A} r_i^2 \frac{1+\tau_z}{2}, \quad \mathcal{M}_{IS0} = \sum_{i=1}^{A} r_i^2,$$



Results & Discussion : Monopole transition as a probe for shape coexistence

Relationship between shape coexistence and monopole transition

K. Heyde and J. L. Wood, Rev. Mod. Phys. 83, 1467 (2011).

○ Let $|A\rangle$ and $|B\rangle$ are the state vectors with different nuclear shape.
And the ground and 2nd 0^+ states are described by their linear combinations

○ The monopole transition matrix between the ground and 2nd 0^+ states is given as

wave functions

$$|0_1^+\rangle = a\,|A\rangle + b\,|B\rangle\,,$$

$$|0_2^+\rangle = -b\,|A\rangle + a\,|B\rangle\,,$$

monopole transition matrix

$$\langle 0_2^+|\mathcal{M}|0_1^+\rangle = ab\,\{\langle B|\mathcal{M}|B\rangle - \langle A|\mathcal{M}|A\rangle\} + (a^2 - b^2)\,\langle B|\mathcal{M}|A\rangle$$

$$\mathcal{M}_{E0} = \sum_{i=1}^{A} r_i^2 \frac{1+\tau_z}{2}, \qquad \mathcal{M}_{IS0} = \sum_{i=1}^{A} r_i^2,$$

○ The first term becomes large when the mixing of two states is large ($a \approx b \approx 1/\sqrt{2}$)
and the radii of $|A\rangle$ and $|B\rangle$ are different

○ The second term vanishes when the particle-hole configurations of $|A\rangle$ and $|B\rangle$ differ
by more than 2p2h, because $\mathcal{M}$ is a one-body operator

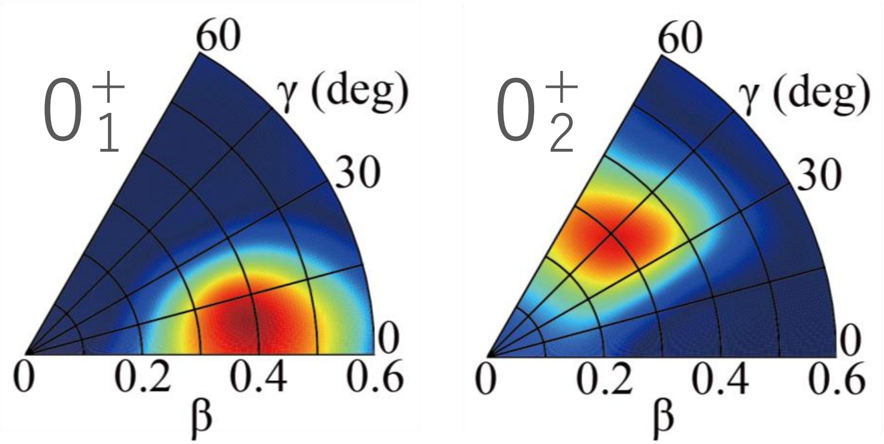
Results & Discussion : Monopole transition as a probe for shape coexistence

Monopole transition is strongly hindered in ^{40}Mg (B(IS0) ∼ 0 Wu)

Coexistence of prolate and oblate shapes

$$|A\rangle = |\text{prolate}\rangle,\ |B\rangle = |\text{oblate}\rangle,\ a = 1,\ b = 0$$

$$|0_1^+\rangle = a\,|A\rangle + b\,|B\rangle = |\text{prolate}\rangle$$
$$|0_2^+\rangle = -b\,|A\rangle + a\,|B\rangle = |\text{oblate}\rangle$$



Monopole matrix element

$$\langle 0_2^+|\mathcal{M}|0_1^+\rangle = ab\,\{\langle B|\mathcal{M}|B\rangle - \langle A|\mathcal{M}|A\rangle\} + (a^2 - b^2)\,\langle B|\mathcal{M}|A\rangle \;= \langle \text{oblate}|\mathcal{M}|\text{prolate}\rangle\,.$$

This matrix element vanishes due to the following reasons

Results & Discussion : Monopole transition as a probe for shape coexistence

Monopole transition is strongly hindered in ^{40}Mg (B(IS0) ~ 0 Wu)

Monopole matrix element for ^{40}Mg $\langle 0_2^+|\mathcal{M}|0_1^+\rangle = \langle \text{oblate}|\mathcal{M}|\text{prolate}\rangle$.

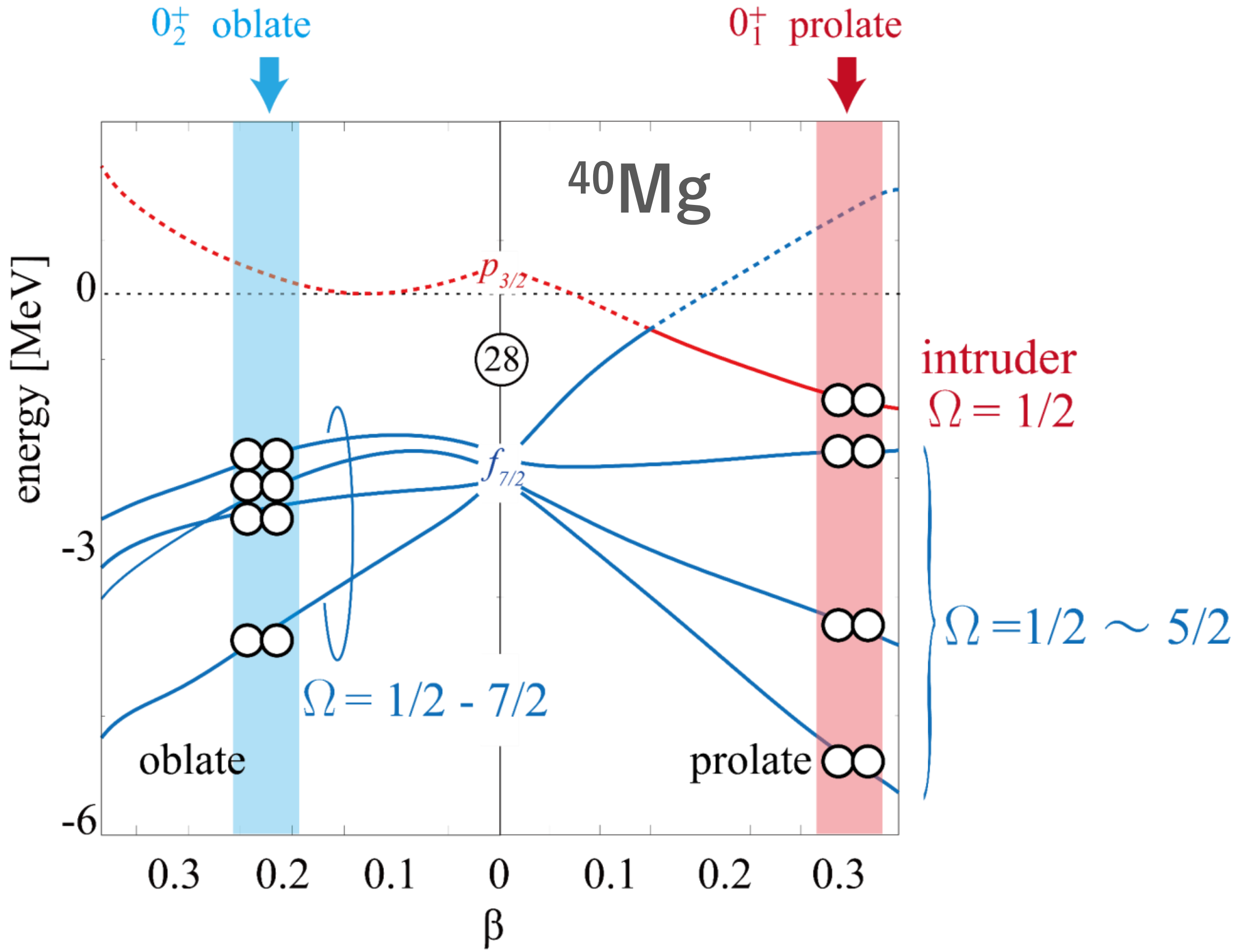
Single-particle configurations
of eight valence neutrons
[Here Ω denotes j_z of neutron orbit]

$$|\text{prolate}\rangle = |\Omega = 1/2\rangle^2 |3/2\rangle^2 |5/2\rangle^2 \underbrace{|1/2\rangle^2}_{\text{intruder}}$$

$$|\text{oblate}\rangle = |\Omega = 1/2\rangle^2 |3/2\rangle^2 |5/2\rangle^2 |7/2\rangle^2$$

$|\text{prolate}\rangle$ and $|\text{oblate}\rangle$ are different by 2p2h

⇒ forbidden transition

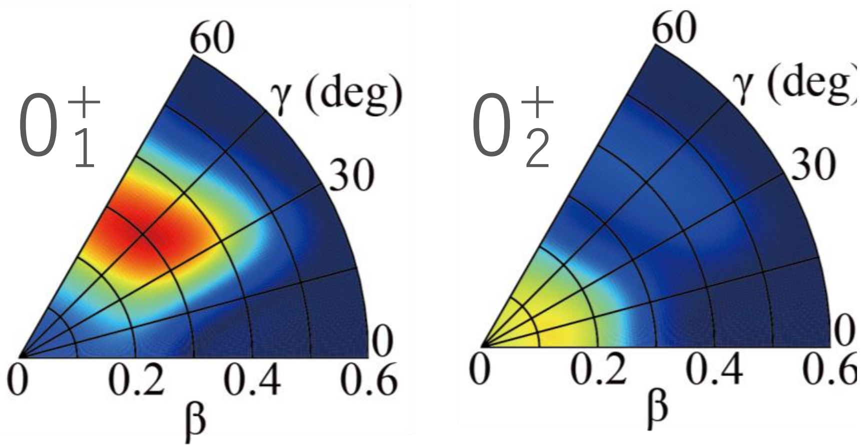


Monopole transition is strongly enhanced in ^{42}Si (B(IS0) ~ 2 Wu)

Coexistence of oblate and spherical shapes

$$|A\rangle = |\text{oblate}\rangle,\ |B\rangle = |\text{spherical}\rangle,\ a = 1,\ b = 0$$

$$|0_1^+\rangle = a\,|A\rangle + b\,|B\rangle = |\text{oblate}\rangle$$
$$|0_2^+\rangle = -b\,|A\rangle + a\,|B\rangle = |\text{spherical}\rangle$$



Monopole matrix element

$$\langle 0_2^+|\mathcal{M}|0_1^+\rangle = ab\,\{\langle B|\mathcal{M}|B\rangle - \langle A|\mathcal{M}|A\rangle\} + (a^2 - b^2)\,\langle B|\mathcal{M}|A\rangle \;= \langle\text{spherical}|\mathcal{M}|\text{oblate}\rangle$$

This matrix element becomes large because …

Results & Discussion : Monopole transition as a probe for shape coexistence

Monopole transition is strongly enhanced in ^{42}Si (B(IS0) ∼ 2 Wu)

Monopole matrix element for ^{42}Si $\langle 0_2^+|\mathcal{M}|0_1^+\rangle = \langle \mathrm{spherical}|\mathcal{M}|\mathrm{oblate}\rangle$

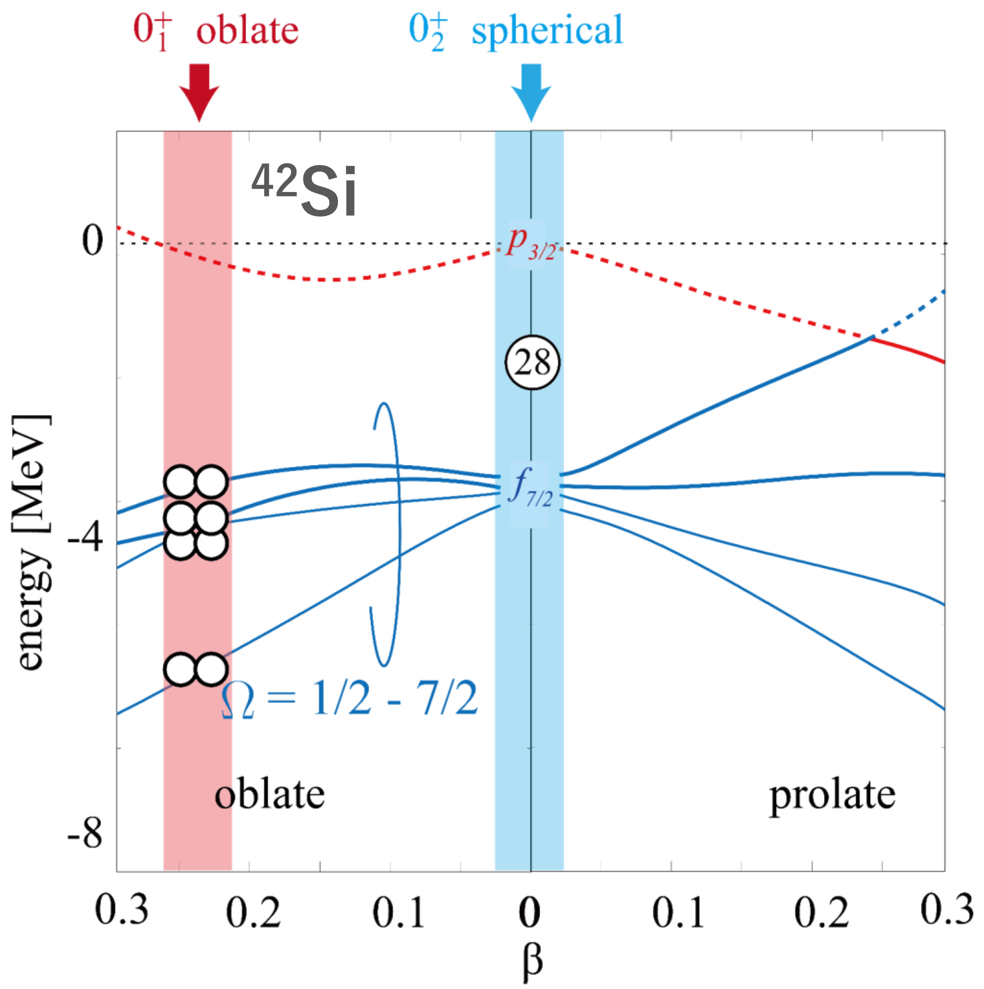
Single-particle configurations of eight valence neutrons
[Here Ω denotes j_z of neutron orbit]

$$|\mathrm{oblate}\rangle = |\Omega=1/2\rangle^2\,|3/2\rangle^2\,|5/2\rangle^2\,|7/2\rangle^2$$

$$|\mathrm{spherical}\rangle = |\Omega=1/2\rangle^2\,|3/2\rangle^2\,|5/2\rangle^2\,|7/2\rangle^2$$

$|\mathrm{oblate}\rangle$ and $|\mathrm{spherical}\rangle$ belong the same set of the single-particle orbits and smoothly transform as function of deformation

⇒ allowed transition, enhanced



Results & Discussion : Monopole transition as a probe for shape coexistence

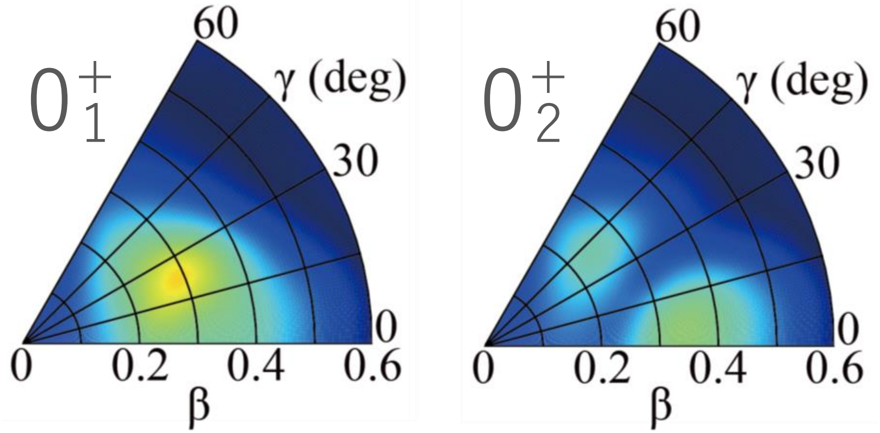
Monopole transition is enhanced in ^{44}S (B(IS0) ~ 0.4 Wu)

Large amplitude collective motion

$$|A\rangle = |\mathrm{prolate}\rangle,\ |B\rangle = |\mathrm{oblate}\rangle,\ a = b = 1/\sqrt{2}$$

$$|0_1^+\rangle = a\,|A\rangle + b\,|B\rangle = 1/\sqrt{2}\,\big(|\mathrm{prolate}\rangle + |\mathrm{oblate}\rangle\big)$$

$$|0_2^+\rangle = -b\,|A\rangle + a\,|B\rangle = 1/\sqrt{2}\,\big(|\mathrm{prolate}\rangle - |\mathrm{oblate}\rangle\big)$$



Monopole matrix element

$$\langle 0_2^+|\mathcal{M}|0_1^+\rangle = ab\,\{\langle B|\mathcal{M}|B\rangle - \langle A|\mathcal{M}|A\rangle\} + (a^2 - b^2)\,\langle B|\mathcal{M}|A\rangle$$

$$= \frac{1}{2}\,\underline{\{\langle \mathrm{oblate}|\mathcal{M}|\mathrm{oblate}\rangle - \langle \mathrm{prolate}|\mathcal{M}|\mathrm{prolate}\rangle\}} \sim 0.4\ \mathrm{Wu}$$

Proportional to the size difference of prolate and oblate shapes

This matrix can be enhanced due to the different radii of prolate and oblate shapes

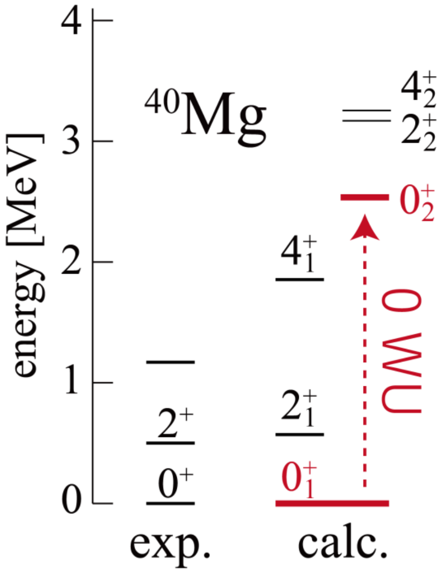
Summary: Erosion of N=28 shell closure; shape coexistence & monopole transition

^{40}Mg: Rigid shape

$|0_1^+\rangle$ Prolate rigid rotor
$|0_2^+\rangle$ Oblate rigid rotor

Erosion of N=28 closure with the inversion of neutron single-particle orbit

⇒ Forbidden monopole transition

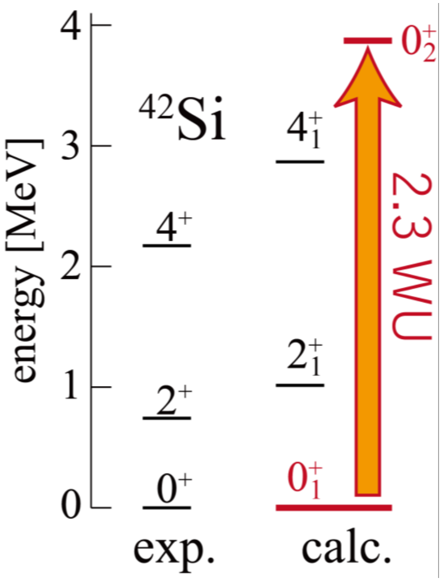


^{42}Si: Rigid shape

$|0_1^+\rangle$ Oblate rigid rotor
$|0_2^+\rangle$ Spherical state

Erosion of N=28 closure without the inversion of neutron single-particle orbit

⇒ Enhanced monopole transition

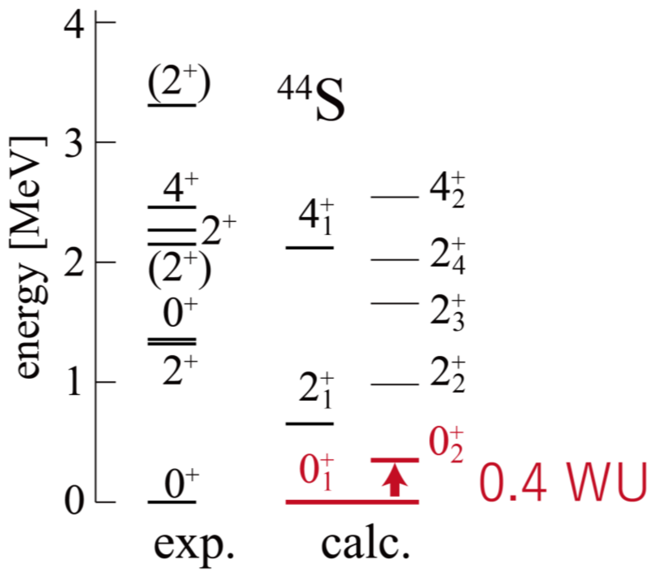


^{44}S: No shape

$|0_1^+\rangle$ Large amplitude
$|0_2^+\rangle$ collective motion

Superposition of shapes with different radii

⇒ Enhanced monopole strength



Summary & Perspectives

Summary

Quenching of N=28 shell gap induces the interesting features of N=28 isotones

- Different pattern in shape coexistence depending on proton number
- Different pattern in the erosion of N=28 shell closure
- There differences are clearly reflected to the monopole transition strengths
  ⇒ Candidate of a promising probe for shape coexistence & erosion of shell closure

Especially, ^{44}S is a very interesting research target as it exhibits "large amplitude collective motion", which is beyond the ordinarily collective motion described by RPA

The measurement of the monopole transition will provide us deeper understanding of this unique collective motion

Perspective

○ Model and interaction dependence of ^{44}S properties must be investigated

○ Search for other experimental probes for the large amplitude collective motion,
e.g., neutron removal and transfer reactions, electric transitions and moments, ⋯