

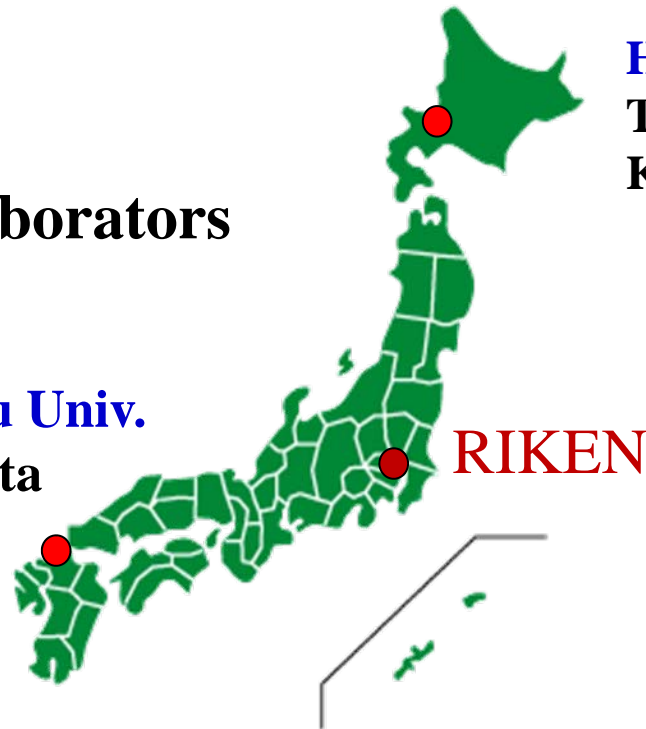
Recent development of CDCC

M. Yahiro
Kyushu University

Collaborators

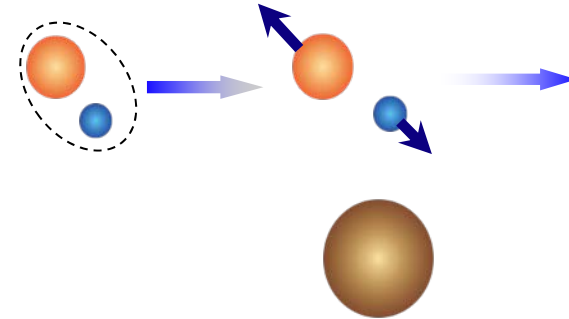
Kyushu Univ.
K. Ogata

Hokkaido Univ.
T. Matsumoto
K. Kato



CDCC

(The method of Continuum-Discretized Coupled Channels)



Review Papers

Kamimura, Yahiro, Iseri, Sakuragi, Kameyama and Kawai, PTP Suppl.89,1(1986)

Austern, Iseri, Kamimura, Kawai, Rawitscher and Yahiro, Phys. Rep. 154(1987),126.

Theoretical foundation

Austern, Yahiro and Kawai, PRL 63, 2649(1989)

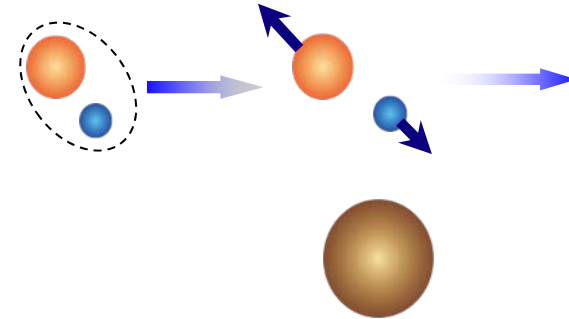
Austern, Kawai and Yahiro, PRC 53, 394(1996)

Numerical comparison between CDCC and Faddeev solutions

A. Deltuva, A. M. Moro, E. Cravo, F. M. Nunes, and A. C. Fonseca, Phys. Rev. C 76 (2007), 064602.

CDCC

(The method of Continuum-Discretized Coupled Channels)



Review Papers

Kamimura, Yahiro, Iseri, Sakuragi, Kameyama and Kawai, PTP Suppl.89,1(1986)

Austern, Iseri, Kamimura, Kawai, Rawitscher and Yahiro, Phys. Rep. 154(1987),126.

Theoretical foundation

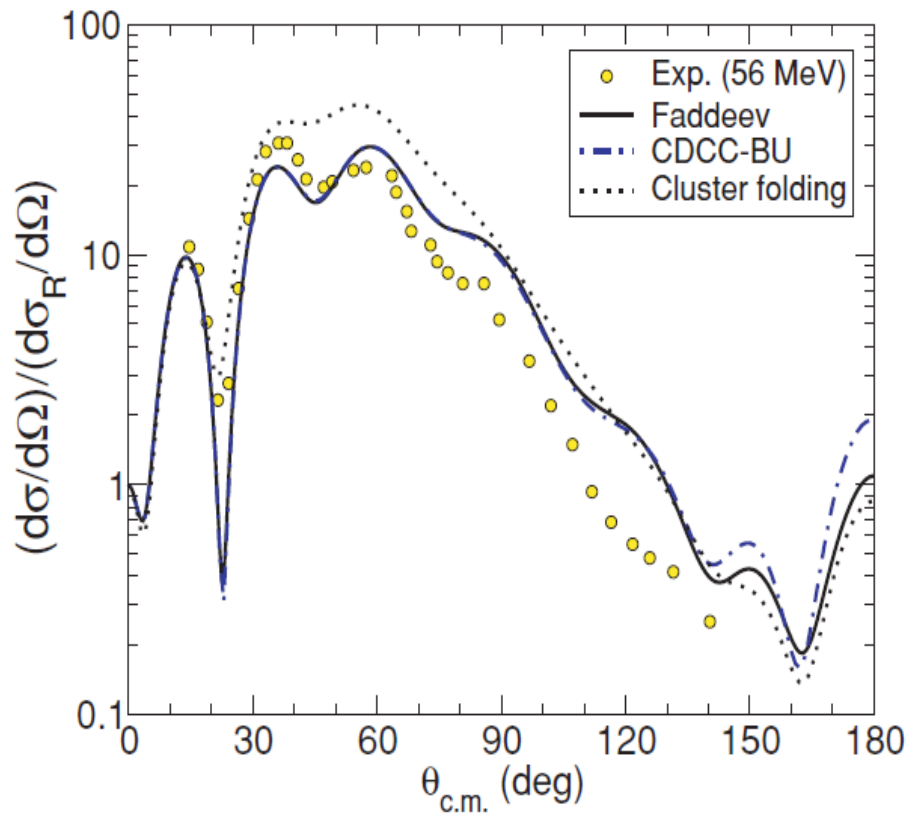
Austern, Yahiro and Kawai, PRL 63, 2649(1989)

Austern, Kawai and Yahiro, PRC 53, 394(1996)

Numerical comparison between CDCC and Faddeev solutions

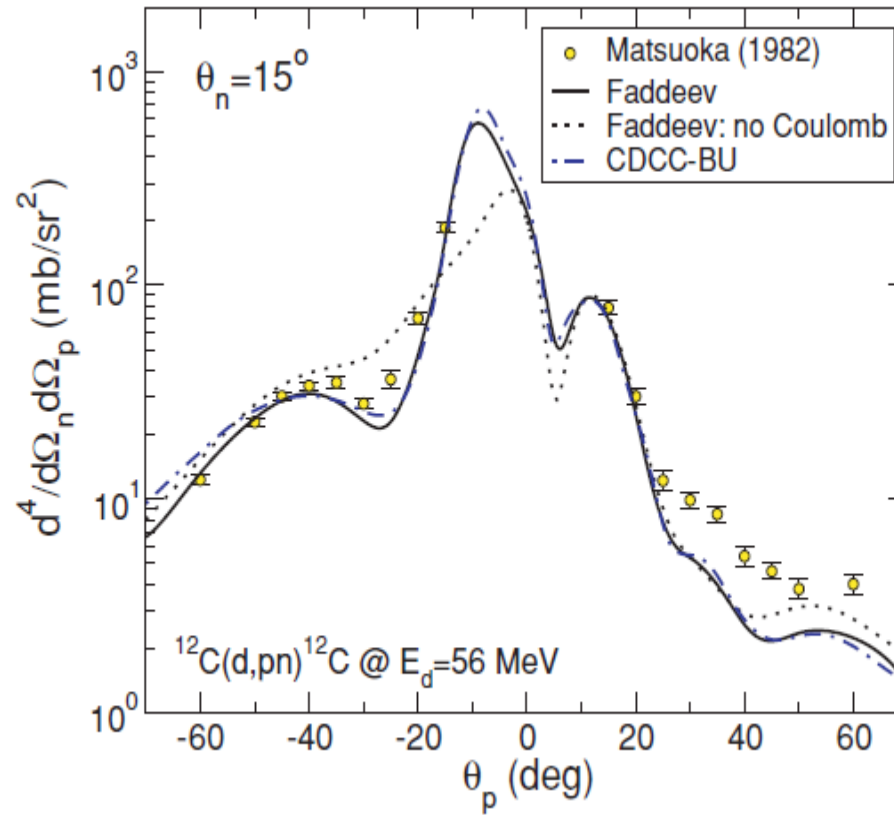
A. Deltuva, A. M. Moro, E. Cravo, F. M. Nunes, and A. C. Fonseca, Phys. Rev. C 76 (2007), 064602.

Comparison between CDCC and Faddeev solutions



$d+^{12}\text{C}$ at 56 MeV
Elastic scattering

$^{12}\text{C}(d,pn)$ at 56 MeV



Recent development of CDCC

1. Eikonal CDCC:  Ogata

The method for treating Coulomb breakup reactions.

2. Eikonal reaction theory (ERT):  Minomo

An extension of CDCC to inclusive reactions such as one-nucleon removal reaction.

3. Four-body CDCC:  Matsumoto

The method for treating four-body breakup reactions

3. Four-body CDCC

T. Matsumoto

T. Matsumoto, K. Kato and M. Yahiro, Phys. Rev. C82, 051602(R) (2010).

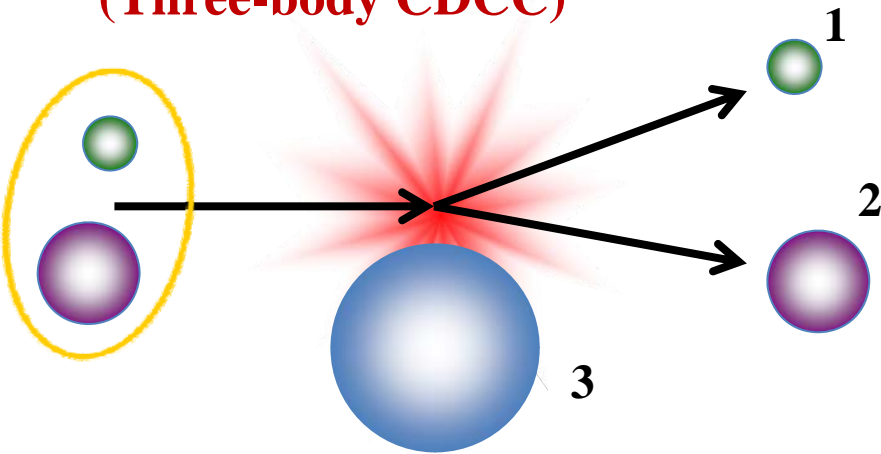
T. Egami, T. Matsumoto, K. Ogata and M. Yahiro, Prog. Theor. Phys. 121(2009), 789-807.

T. Matsumoto, E. Hiyama, K. Ogata, Y. Iseri, M. Kamimura, S. Chiba and M. Yahiro, Phys. Rev. C70(2004), 061601.

T. Matsumoto, T. Egami, K. Ogata, Y. Iseri, M. Kamimura and M. Yahiro. Phys. Rev. C73(2006),051602.

Breakup Reaction

Three-body Breakup Reaction: (Three-body CDCC)



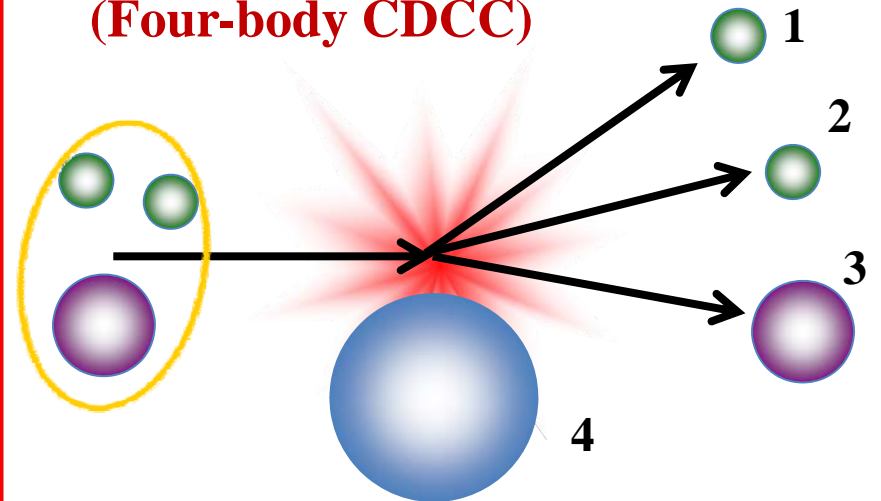
The projectile breaks up into **2 particles**.

Projectile (**2-body**) + target (**1-body**)
→ **3-body** breakup reaction

Ex.) d , ${}^6\text{Li}$, ${}^{11}\text{Be}$, ${}^8\text{B}$, ${}^{15}\text{C}$, etc..

One-neutron halo

Four-body Breakup Reaction (Four-body CDCC)



The projectile breaks up into **3 particles**.

Projectile (**3-body**) + target (**1-body**)
→ **4-body** breakup reaction

Ex.) ${}^6\text{He}$, ${}^{11}\text{Li}$, ${}^{14}\text{Be}$, etc..

Two-neutron halo

Breakup Reaction

Three-body Breakup Reaction: (Three-body CDCC)



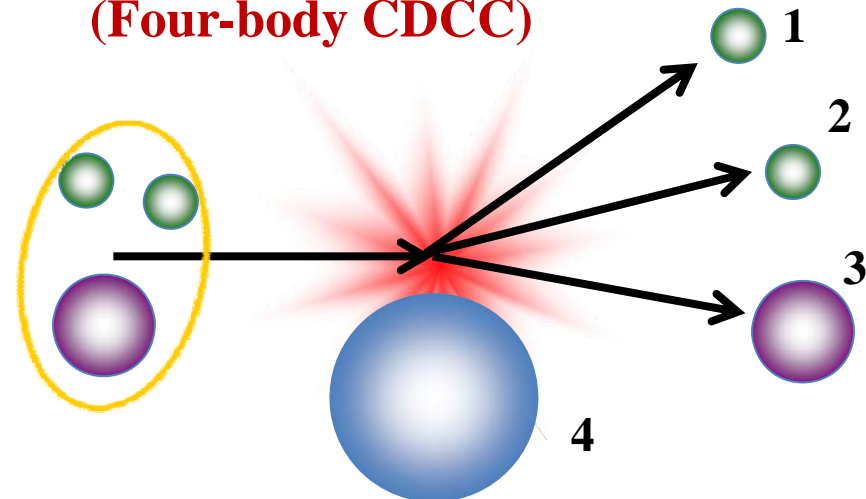
The projectile breaks up into **2 particles**.

Projectile (**2-body**) + target (**1-body**)
→ **3-body** breakup reaction

Ex.) d , ${}^6\text{Li}$, ${}^{11}\text{Be}$, ${}^8\text{B}$, ${}^{15}\text{C}$, etc..

One-neutron halo

Four-body Breakup Reaction (Four-body CDCC)



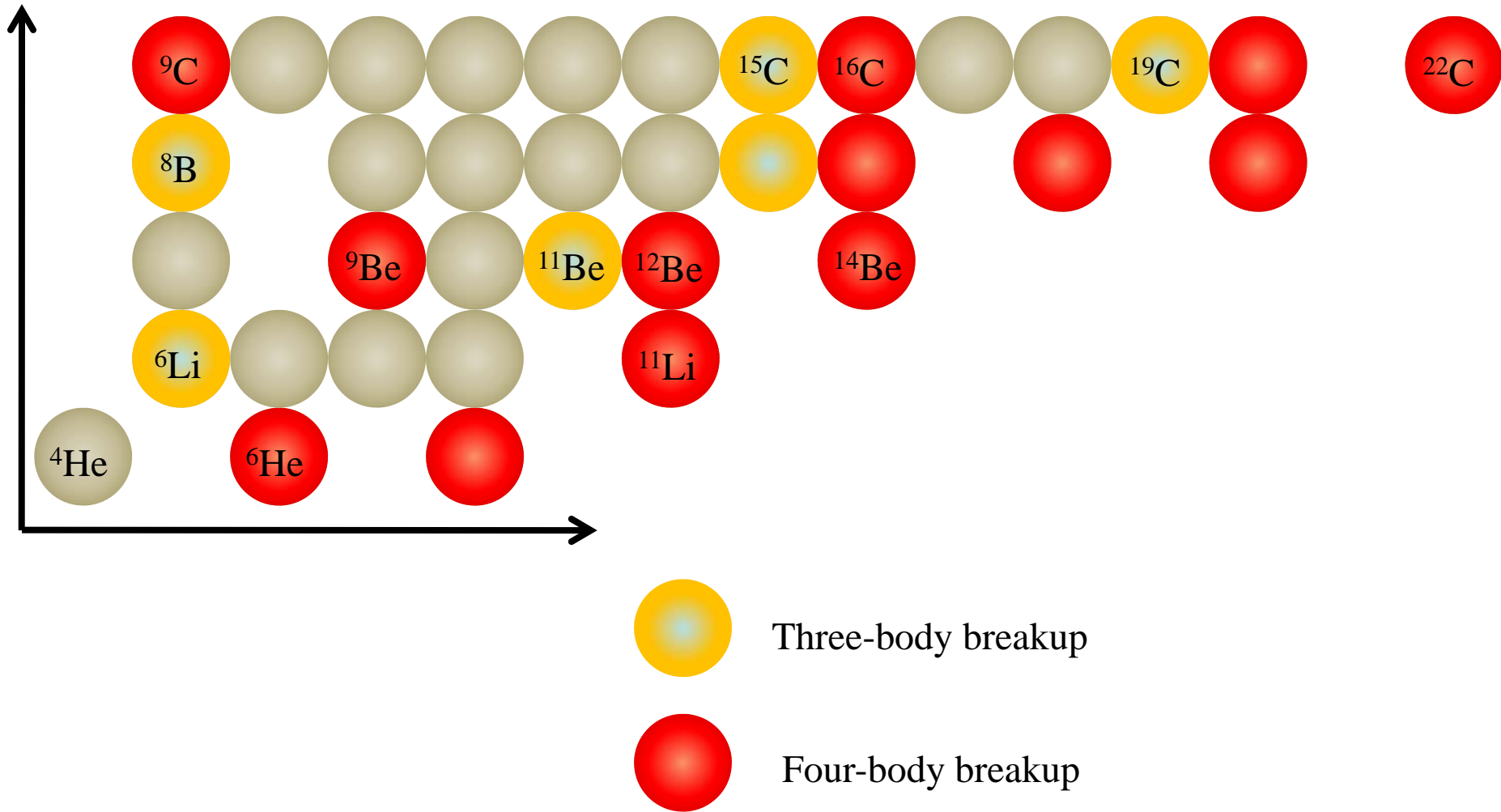
The projectile breaks up into **3 particles**.

Projectile (**3-body**) + target (**1-body**)
→ **4-body** breakup reaction

Ex.) ${}^6\text{He}$, ${}^{11}\text{Li}$, ${}^{14}\text{Be}$, etc..

Two-neutron halo

Nuclear Chart



Scattering of ${}^6\text{He}$

Four-body Schrodinger equation

$$(E - K - U - H_6) |\Psi\rangle = 0$$

$$U = U_{nT} + U_{nT} + U_{HeT}$$

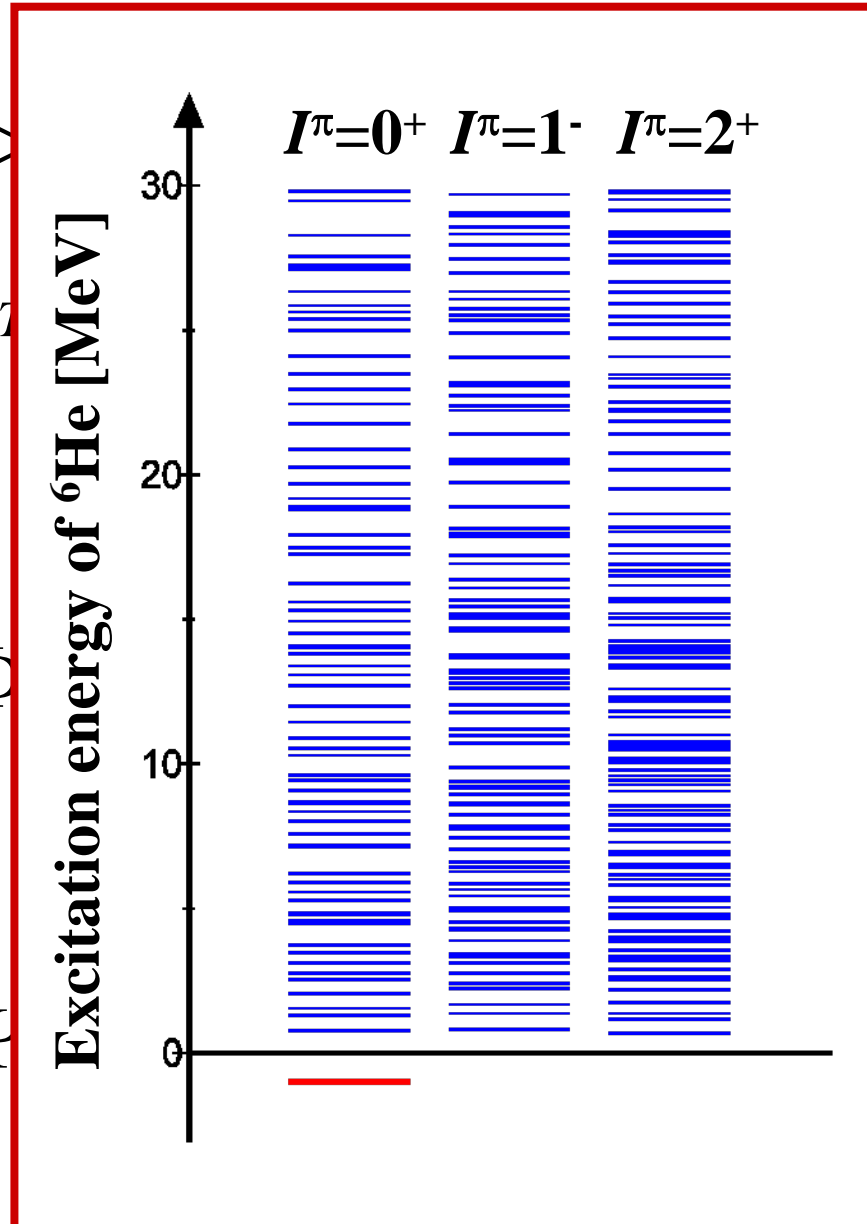
Gaussian basis functions

$$\langle \Phi_i | H_6 | \Phi_j \rangle = e_i \delta_{ij}$$

Model space $P = \sum_i |\Phi_i\rangle\langle\Phi_i|$

CDCC equation

$$P(E - K - U - H_6)P |\Psi\rangle = 0$$



Scattering of ${}^6\text{He}$

Four-body Schrodinger equation

$$(E - K - U - H_6) |\Psi\rangle = 0$$

$$U = U_{nT} + U_{nT} + U_{HeT}$$

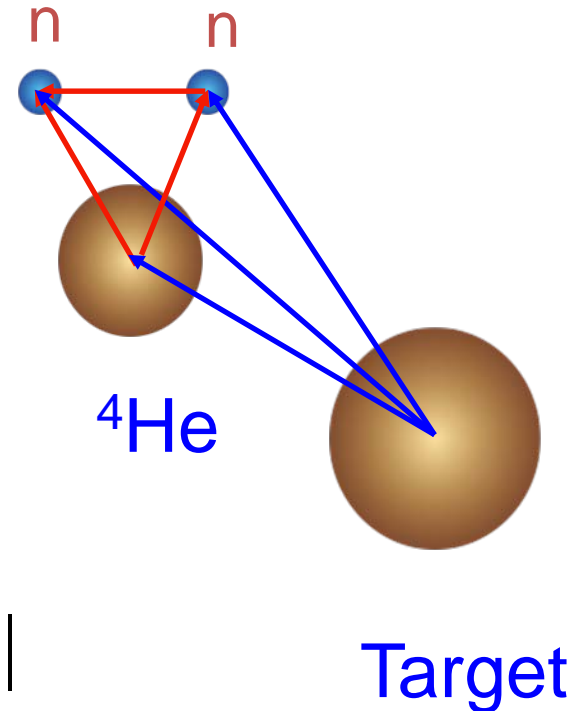
Gaussian basis functions

$$\langle \Phi_i | H_6 | \Phi_j \rangle = e_i \delta_{ij}$$

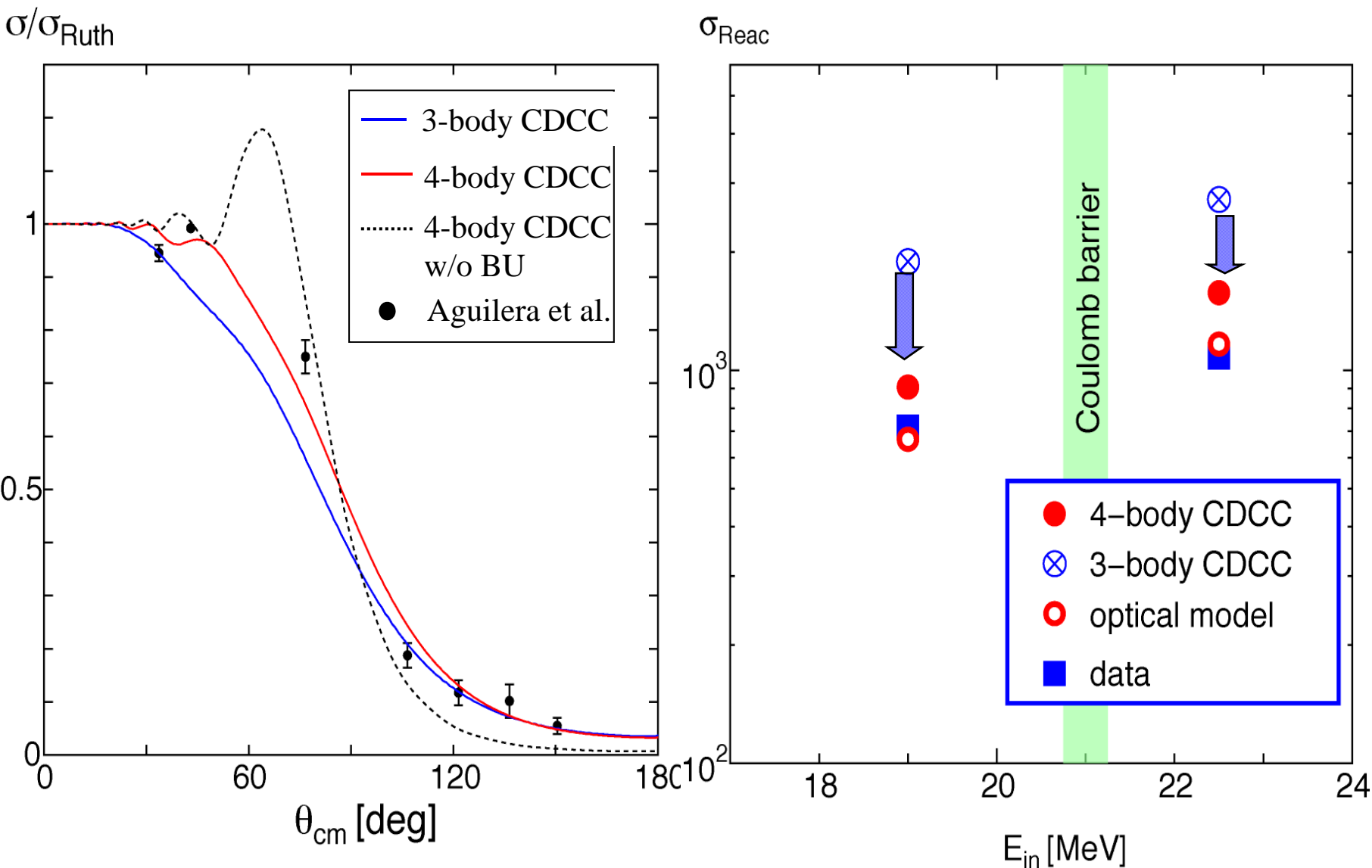
Model space $P = \sum_i |\Phi_i\rangle \langle \Phi_i|$

CDCC equation

$$P(E - K - U - H_6)P |\Psi\rangle = 0$$



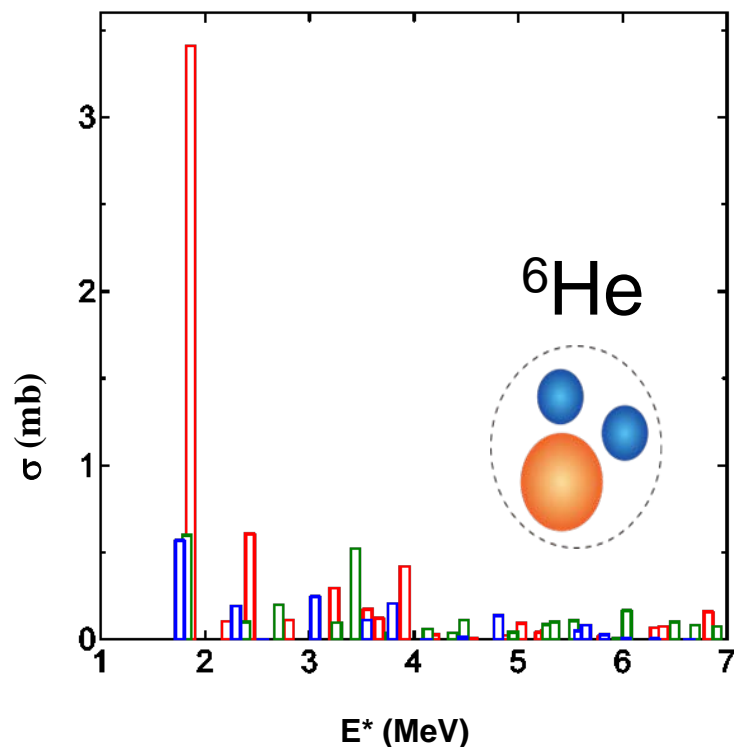
${}^6\text{He}-{}^{209}\text{Bi}$ scattering at 22.5 MeV



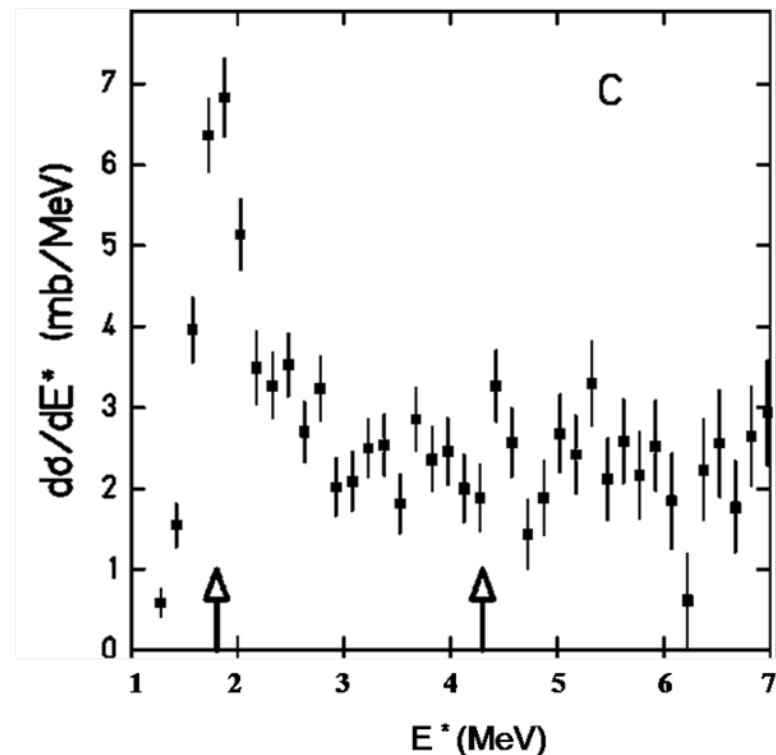
Breakup Cross Section of ${}^6\text{He}$ scattering

${}^6\text{He}+{}^{12}\text{C}$ scattering at 240 MeV/nucleon.

4-body CDCC calc.



PRC59, 1252(1999), T. Aumann *et al.*



How to smooth the discrete spectrum

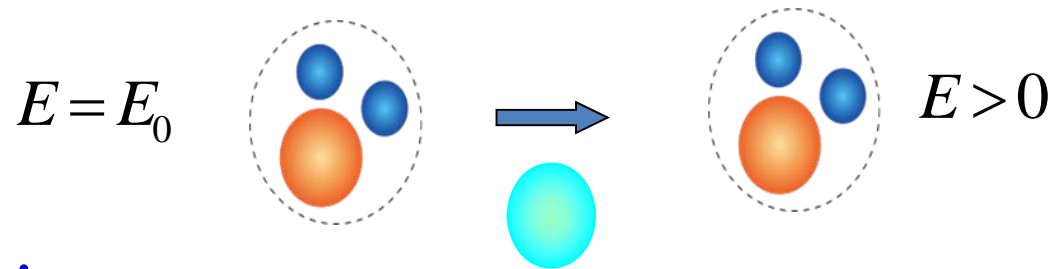
New Smoothing Procedure with *Complex Scaling Method*

T. Matsumoto, K. Kato and M. Yahiro, Phys. Rev. C82, 051602(R) (2010).

$$\frac{d\sigma}{dE} = \int d\vec{k}' d\vec{p}' d\vec{P}' \delta(E - E') |T(E)|^2 = \frac{1}{\pi} \text{Im}R(E)$$

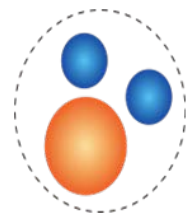
$$T(E) = \langle \psi^{(-)}(E, \xi) \chi_C^{(-)}(\mathbf{R}) | V | \Psi^{(+)}(\xi, \mathbf{R}) \rangle$$

Continuum state of ${}^6\text{He}$ with positive E



Response function

$$\mathcal{R}(E) = \int d\xi d\xi' \langle \Psi^{(+)}(\xi, \mathbf{R}) | V^* | \chi_C^{(-)}(\mathbf{R}) \rangle_{\mathbf{R}} \mathcal{G}^{(-)}(E, \xi, \xi') \langle \chi_C^{(-)}(\mathbf{R}) | V | \Psi^{(+)}(\xi, \mathbf{R}) \rangle_{\mathbf{R}}$$



$E > 0$

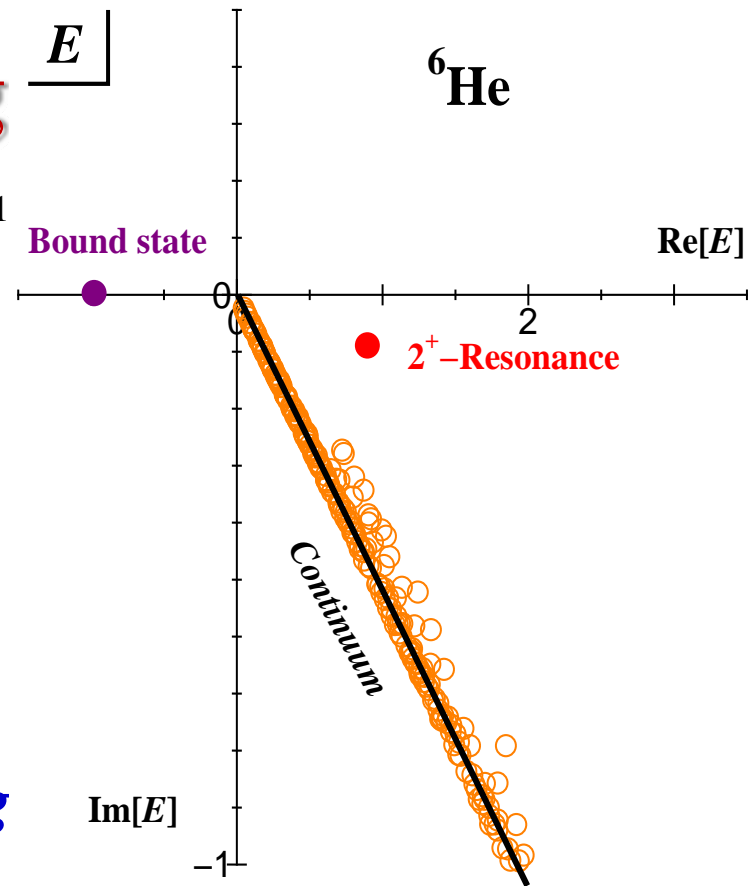
$$G^{(-)}(E) = \frac{1}{E - H_6 - i\varepsilon}$$

Complex scaling

T. Matsumoto, K. Kato and M. Yahiro, Phys. Rev. C82, 051602(R) (2010)

T. Matsumoto, T. Egami, K. Ogata and M. Yahiro, Prog. Theor. Phys. 121(2009), 885-894.

Kikuchi, Myo, Takashina, Kato, Ikeda, PTP122 (2009) 499



Scaling operator $U(\theta)$

$$\langle \vec{r} | U(\theta) | f \rangle = e^{i3\theta/2} f(\vec{r}e^{i\theta})$$

Green's function with Complex-Scaling

$$G^{(-)}(E) = \frac{1}{E - H_6 - i\varepsilon} = U^{-\theta} \frac{1}{E - H^\theta - i\varepsilon} U^\theta \approx \sum_{\nu} U^{-\theta} \frac{|\Phi_{\nu}^{\theta}\rangle \langle \tilde{\Phi}_{\nu}^{\theta}|}{E - E_{\nu}^{\theta}} U^{\theta}$$

$$1 = U^{-1}(\theta)U(\theta)$$

$$H^{\theta} = U^{-1}H_6U$$

$$1 = \sum_i |\Phi_i\rangle \langle \Phi_i|$$

$$\langle \tilde{\Phi}_{\mu}^{\theta} | H^{\theta} | \Phi_{\nu}^{\theta} \rangle = E_{\nu}^{\theta} \delta_{\mu\nu} \quad \langle \Phi_i | H_6 | \Phi_j \rangle = E_i \delta_{ij}$$

New Smoothing Procedure with Complex Scaling Method

Green's function with Complex-Scaling Method

$$\mathcal{G}^{(-)}(E, \xi, \xi') \approx \sum_{\nu} \sum_{i,j} |\Phi_i\rangle \frac{\langle \Phi_i | U^{-\theta} | \Phi_{\nu}^{\theta} \rangle \langle \tilde{\Phi}_{\nu}^{\theta} | U^{\theta} | \Phi_j \rangle}{E - E_{\nu}^{\theta}} \langle \Phi_j |$$

Response function

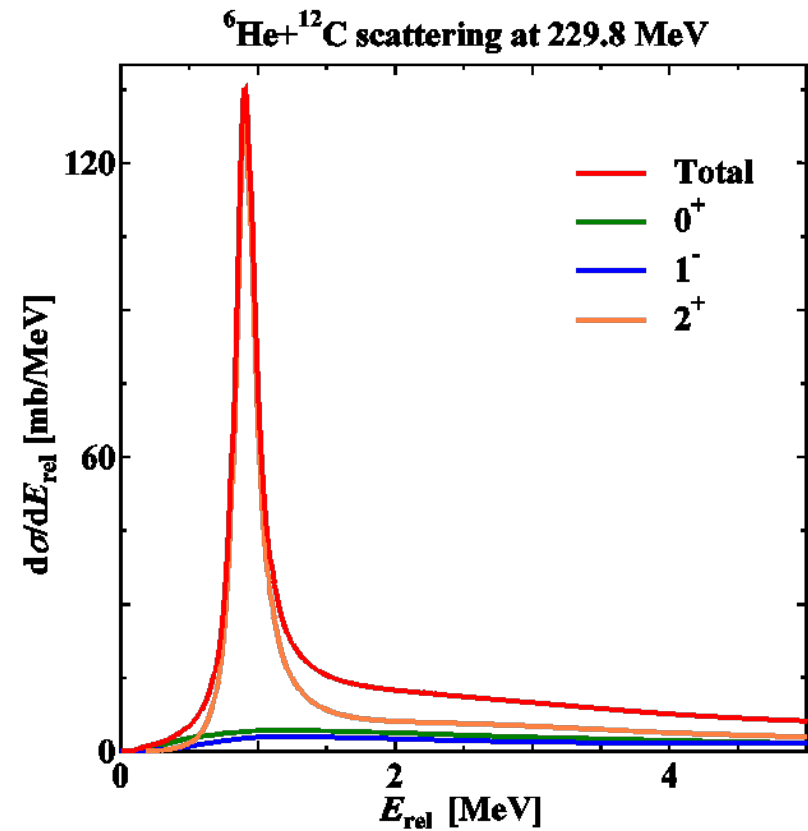
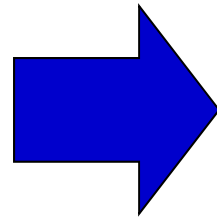
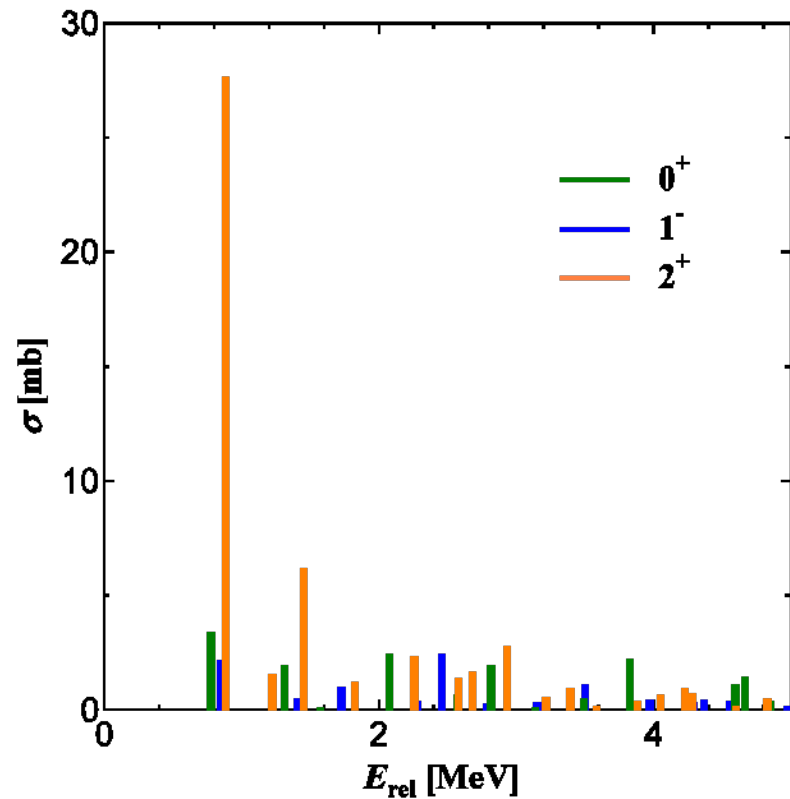
$$\mathcal{R}(E) = \int d\xi d\xi' \langle \Psi^{(+)}(\xi, \mathbf{R}) | V^* | \chi_C^{(-)}(\mathbf{R}) \rangle_{\mathbf{R}} \mathcal{G}^{(-)}(E, \xi, \xi') \langle \chi_C^{(-)}(\mathbf{R}) | V | \Psi^{(+)}(\xi, \mathbf{R}) \rangle_{\mathbf{R}}$$

$$\mathcal{R}(E) = \sum_{\nu} \sum_{i,j} \langle \Psi^{(+)} | V^* | \chi_C^{(-)} \Phi_i \rangle \frac{\langle \Phi_i | U^{-\theta} | \Phi_{\nu}^{\theta} \rangle \langle \tilde{\Phi}_{\nu}^{\theta} | U^{\theta} | \Phi_j \rangle}{E - E_{\nu}^{\theta}} \langle \Phi_j \chi_C^{(-)} | V | \Psi^{(+)} \rangle$$

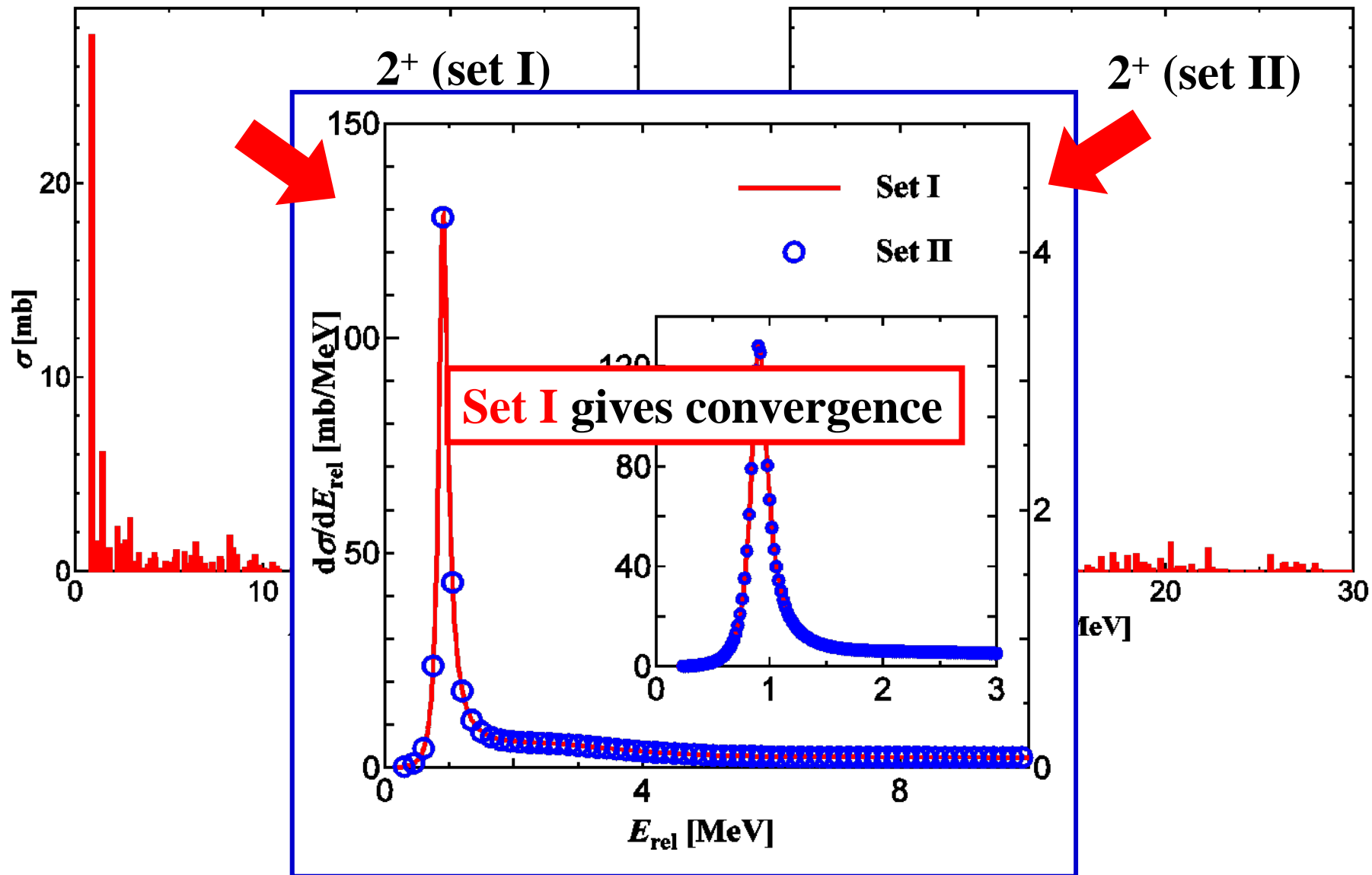
T-matrix of CDCC

Breakup Spectrum

$$\frac{d\sigma}{dE} = \frac{1}{\pi} \text{Im} \sum_{\nu} \sum_{i,j} T_i^{\text{CDCC}\dagger} \frac{\langle \Phi_i | U^{-\theta} | \Phi_{\nu}^{\theta} \rangle \langle \tilde{\Phi}_{\nu}^{\theta} | U^{\theta} | \Phi_j \rangle}{E - E_{\nu}^{\theta}} T_j^{\text{CDCC}}$$



Convergence of Breakup-spectrum (2^+)



Application of the new method

1. ${}^6\text{He}+{}^{12}\text{C}@240\text{ MeV/nucl.}$

2. ${}^6\text{He}+{}^{208}\text{Pb}@240\text{ MeV/nucl.}$

measured @ GSI

PRC59, 1252 (1999), T. Aumann *et al.*

Proton target

Conclusion

1. Four-body CDCC is applied to ${}^6\text{He}$ breakup reactions from p, ${}^{12}\text{C}$ and ${}^{208}\text{Pb}$ targets.
2. Four-body CDCC is successful in describing the breakup reactions.