QCD Theory and Models

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Plan of Lecture

I. Introduction to QCD, models and the physics of deep-inelastic scatterings

II. Model predictions and phenomenology of parton distribution functions

III. Generalized parton distributions and nucleon spin decomposition

IV. Phenomenology of nucleon spin decomposition

I. Introduction to QCD, models and the physics of deep-inelastic scatterings

QCD = nonabelian gauge theory

basic lagrangian

$$\mathcal{L}_{QCD} = \mathcal{L}_{chiral-sym} + \mathcal{L}_{mass}$$

chiral symmetric part

$$\mathcal{L}_{chiral-sym} = \bar{\psi}\, i\, \gamma^{\mu}\, D_{\mu}\, \psi - \frac{1}{2} \mathrm{Tr}\, (F_{\mu\nu}\, F^{\mu\nu})$$

covariant derivative

$$D_{\mu} \equiv \partial_{\mu} - i\, g\, A_{\mu} \quad : \quad A_{\mu} \equiv A^{a}_{\mu}\, T_{a} = A^{a}_{\mu} \frac{\lambda_a}{2} \quad (a\, :\, \text{summed})$$

$$a = 1, 2, \cdots, 8 \quad : \quad \text{8 colored gluons}$$

gluon field strength tensor

nonlinear term

$$F_{\mu\nu} \equiv \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} - i\, g\, [A_{\mu}, A_{\nu}]$$

chiral symmetry breaking part (quark mass term)

$$\mathcal{L}_{mass} = -\sum_{a=1}^{3}\sum_{f=1}^{6} m_f\, \bar{q}^{fa} q^{fa}$$

$$a = 1,2,3 \quad : \quad \text{3 colored quarks}$$

$$f = 1,2,\cdots,6 \quad : \quad \text{6 flavored quarks}$$

each flavored quark has 3 colors !

Although the quarks masses cannot directly be measured due to confinement, the following masses called the current quark masses have been extracted from the analyses of meson masses, etc. based on current algebra (see, PDG).

$$m_u = (1.5-3.3)\,\text{MeV}, \; m_d = (3.5-6.0)\,\text{MeV}, \; m_s \simeq 105\,\text{MeV}$$

$$m_c \simeq 1.27\,\text{GeV}, \quad m_b \simeq 4.20\,\text{GeV}, \quad m_t \simeq 171.2\,\text{GeV}$$

flavor symmetry of QCD

$$m_u,\ m_d,\ m_s \ll \Lambda_{QCD} \Rightarrow \text{SU(3) symmetry is relatively good !}$$

$$m_u \simeq m_d \ll m_s \ll \Lambda_{QCD} \Rightarrow \text{SU(2) symmetry is very good !}$$

$\mathcal{L}_{QCD}$ is invariant under the **nonabelian gauge transformation** :

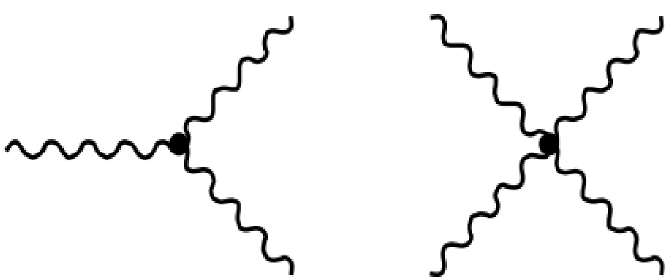
$$\psi(x) \;\rightarrow\; U(x)\,\psi(x)$$
$$\bar{\psi}(x) \;\rightarrow\; \bar{\psi}(x)\,U^{\dagger}(x)$$
$$A_{\mu}(x) \;\rightarrow\; U(x)\left(A_{\mu}(x) \;+\; \frac{i}{g}\,\partial_{\mu}\right)U^{\dagger}(x)$$

with

Space-time dep. functions

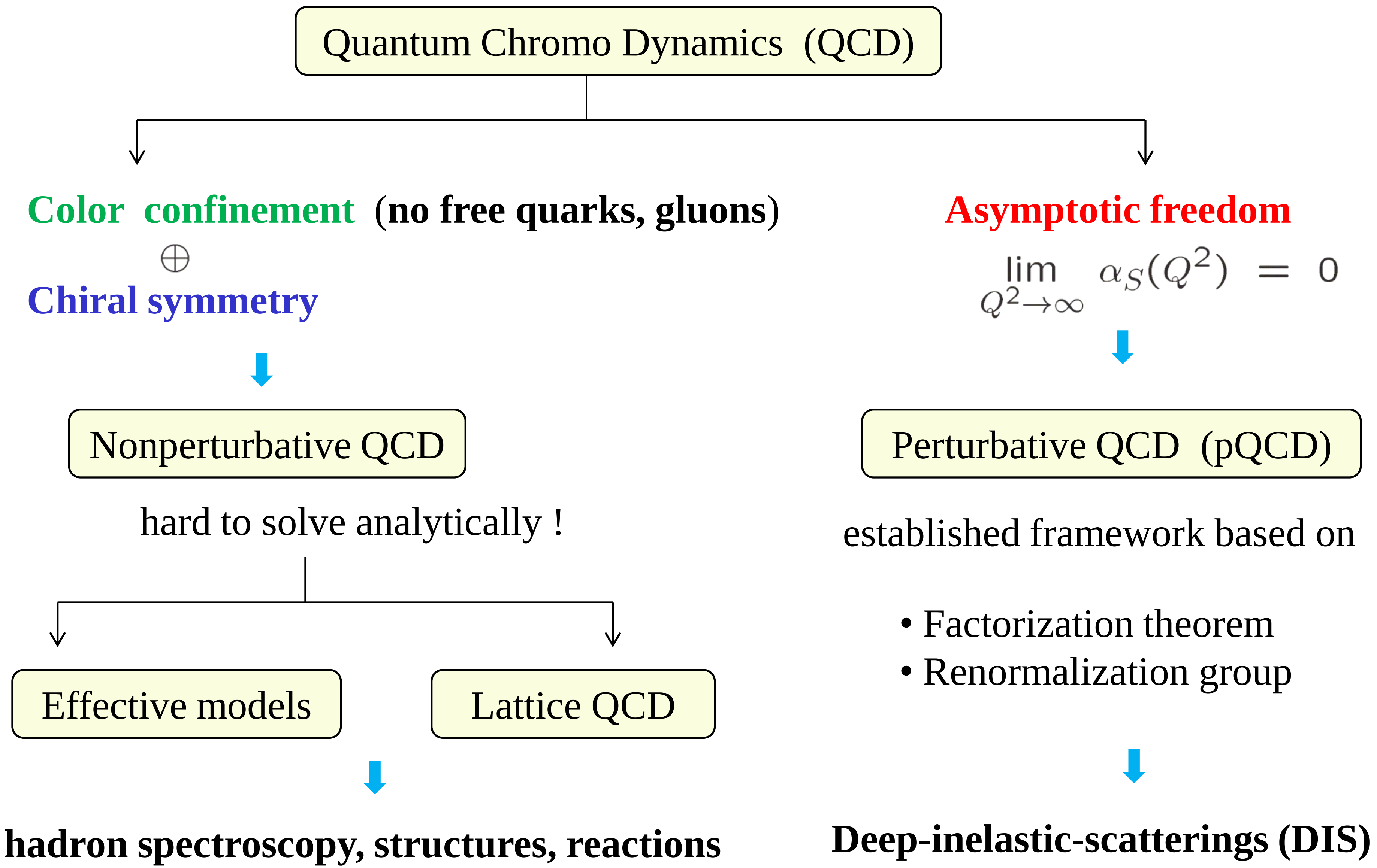
$$U(x) \;\equiv\; e^{\,i\,\omega_a(x)\,T_a}$$

Due to the nonabelian nature of QCD lagrangian, not only the quark-gluon coupling, which is an analogue of electron-photon coupling in the QED case, the gluon self-interactions (3- and 4-point gluon vertices) appear !



gluon self-interactions

The most important properties of QCD lagrangian



Importance of **chiral symmetry** in QCD

QCD with massless quark has chiral symmetry.

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chiral symmetry is however spontaneously broken !

Consequences

- Nontrivial vacuum with nonzero quark condensate

$$\langle 0 \,|\, \bar{q}\, q \,|\, 0 \rangle \;\neq\; 0$$

- Appearance of massless Nambu-Goldstone (N.-G.) modes

N.-G. modes = collective excitations in QCD vacuum

Yukawa's pions and light mesons

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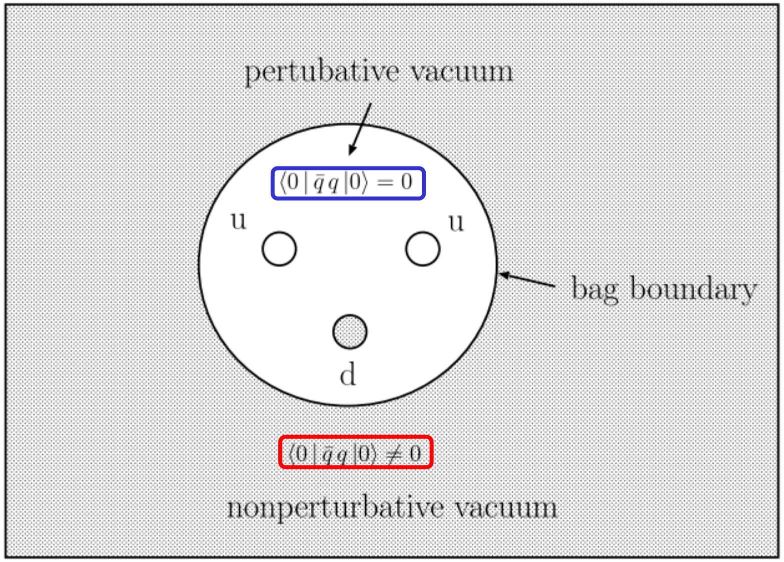
What are nucleons (baryons) like in the nontrivial QCD vacuum, then ?

A proton in the QCD vacuum

MIT bag model picture of a proton in the QCD vacuum



Some important physics neglected in the MIT bag model.

- chiral symmetry of QCD !

  - clouds of N.-G. pions surrounding core of three valence quarks -

- gluon degrees of freedom !

$$| N \rangle \sim | q^3 \rangle + | q^4 \bar{q} \rangle + | q^3 g \rangle + \cdots$$

How can we probe internal structure of the nucleon ?

(I) elastic electron-nucleon scatterings : simplest experimental probe

$$e(k) \ + \ N(P) \ \rightarrow \ e(k') \ + \ N(P')$$

sensitive only to charge and magnetization distributions in the nucleon !

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Any probe, which is more sensitive to the internal structure of the nucleon ?

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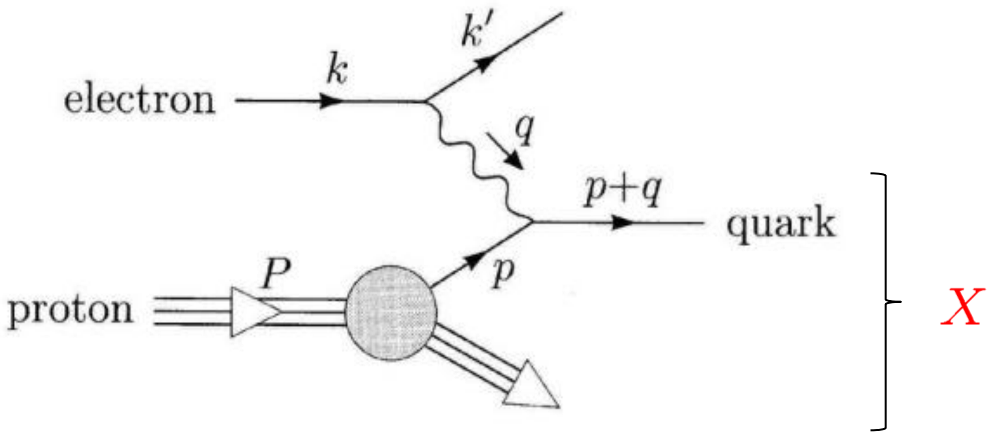
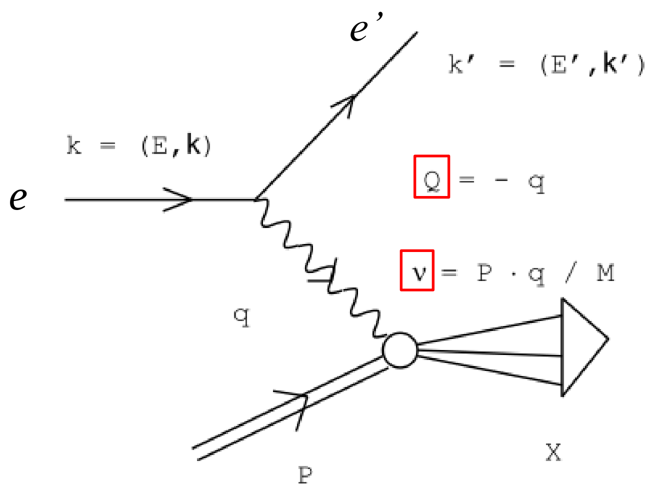
(II) deep-inelastic electron-nucleon scatterings : **inclusive** processes

$$e(k) \ + \ N(P) \ \rightarrow \ e(k') \ + \ X \text{ (anything)}$$



inclusive deep-inelastic-scattering cross section



$$\frac{d^2\sigma}{dE'\,d\Omega'} = \frac{\alpha^2}{M\,Q^4}\,\frac{E'}{E}\,l_{\mu\nu}\,W^{\mu\nu}$$

$l_{\mu\nu}$: lepton tensor

$W_{\mu\nu}$: hadron tensor

hadron tensor (unpolarized target)

electromagnetic current

$$W^{\mu\nu} \equiv \frac{1}{4\,\pi}\int d^4\xi\; e^{iq\cdot\xi}\,\langle P|[J^\mu(\xi), J^\nu(0)]|P\rangle$$
$$= W_1(Q^2,\nu)\left(-\,g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)$$
$$+\; W_2(Q^2,\nu)\,\frac{1}{M^2}\left(P^\mu - \frac{(P\cdot q)q^\mu}{q^2}\right)\left(P^\nu - \frac{(P\cdot q)q^\nu}{q^2}\right)$$

$W_1(Q^2,\nu),\; W_2(Q^2,\nu)$: structure functions of nucleon

In the Bjorken limit $(Q^2,\ \nu\ \to\ \infty)$

$$W_{1,2}(Q^2,\nu)\ \to\ W_{1,2}(Q^2/2M\nu\ \equiv\ x)$$: Bjorken scaling

Forward virtual Compton amplitude

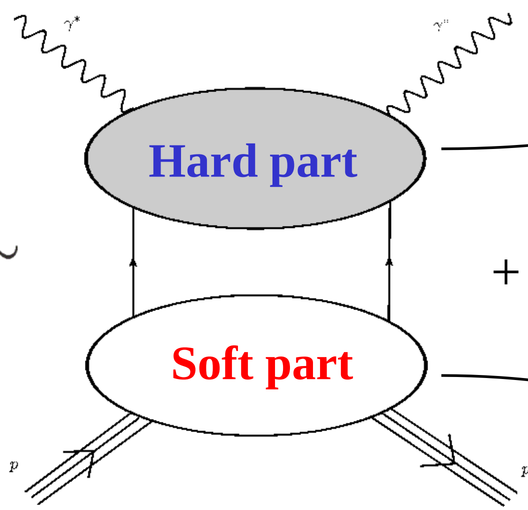
$$T_{\mu\nu}\ \equiv\ i\int d^4x\, e^{i\,q\cdot x}\,\langle p\,|\,T\,[J_\mu(x)\,J_\nu(0)]\,|\,p\rangle$$

Optical theorem

$$W_{\mu\nu}\ =\ \frac{1}{\pi}\,\mathrm{Im}\ T_{\mu\nu}$$

Factorization theorem of pQCD as a foundation of DIS physics

$T_{\mu\nu}\ \sim$



+ crossed

tractable in **pQCD**

information on **nonperturbative quark-gluon structure** of nucleon

- Soft part can be parametrized by parton distribution functions (PDFs) .

$$u(x, Q^2) \; ; \; \text{unpolarized } u\text{-quark distribution}$$
$$\bar{u}(x, Q^2) \; : \; \text{unpolarized } \bar{u}\text{-quark distribution}$$
$$g(x, Q^2) \; : \; \text{unpolarized gluon distrubtion} \quad \text{etc.}$$

- They are not calculable within the framework of **pQCD**. They are rather determined empirically to fit DIS cross sections at a certain Q^2.

- The pQCD however can predict Q^2-dependence (scale-dependence) of PDFs through **evolution equation** (**DGLAP equation**) !

- Theoretical prediction of PFDs needs to solve nonperturbative bound state problem, which is hard to carry out. At the moment, we must resort to

Effective models (or theories) of QCD and/or **lattice QCD**

Remark on the **antiquark distributions** (for unpolarized distribution)

$$q(x) \;=\; \int_{-\infty}^{\infty} dz_0 \; e^{\,i\,x M_N\,z_0} \, \langle\, N \,|\, \bar{\psi}(0)\,(1 + \gamma^0\,\gamma^3)\,\psi(z)\,|\, N\,\rangle$$

$$\bar{q}(x) \;=\; \int_{-\infty}^{\infty} dz_0 \; e^{\,i\,x M_N\,z_0} \, \langle\, N \,|\, \bar{\psi}^c(0)\,(1 + \gamma^0\,\gamma^3)\,\psi^c(z)\,|\, N\,\rangle$$

where

$$\psi^c \;=\; C\,\bar{\psi}^T, \qquad C \;:\; \text{charge conjugation matrix}$$

one can prove

$$\boxed{\bar{q}(x) \;=\; -\,q(-x), \qquad (0 < x < 1)}$$

for longitudinally polarized distribution

$$\Delta q(x) \;=\; \int_{-\infty}^{\infty} dz_0 \; e^{\,i\,x M_N\,z_0} \, \langle\, N \,|\, \bar{\psi}(0)\,(1 + \gamma^0\,\gamma^3)\,\gamma_5\,\psi(z)\,|\, N\,\rangle$$

we have

$$\boxed{\Delta\bar{q}(x) \;=\; +\,\Delta q(-x), \qquad (0 < x < 1)}$$

How to harmonize nonperturbative and perturbative domains of QCD ?

model predictions of PDFs given at low Q^2 → empirically extracted PDFs given at high Q^2

related through **DGLAP equation**

matching problem

- difficult to specify the exact initial energy scale of evolution !

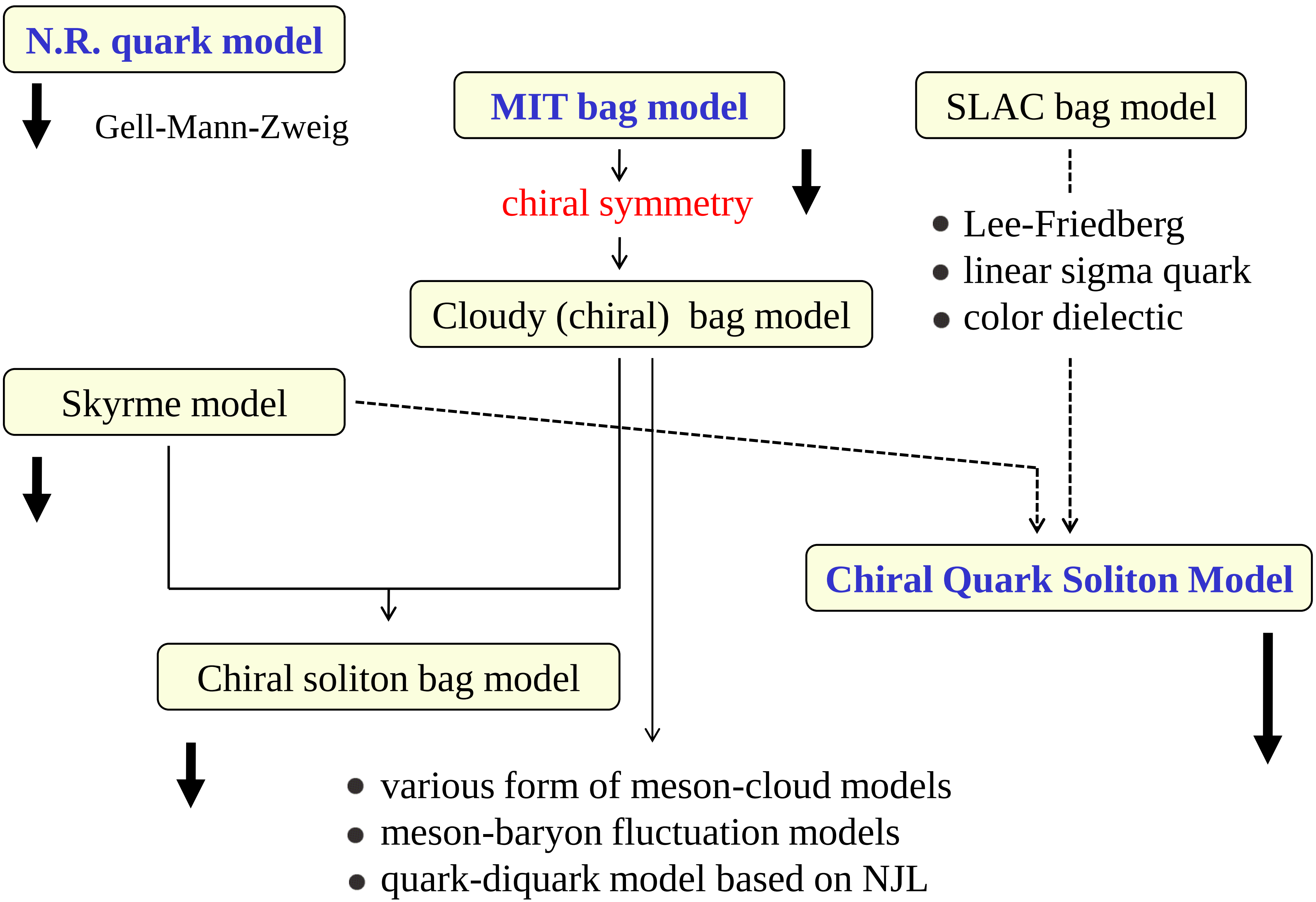
most effective models like MIT bag model : $Q^2_{ini} \simeq (400\,\mathrm{MeV})^2$

Chiral Quark Soliton Model (CQSM) : $Q^2_{ini} \simeq (600\,\mathrm{MeV})^2$

- validity of using perturbative RG eq. (DGLAP eq.) at low energy scale ?

diverging behavior of QCD running coupling constant $\alpha_S(Q^2)$!

genealogy of models of baryons



Non-relativistic quark model

$$\Psi_B(1,2,3) = \phi_{space}(1,2,3)\,\chi_{spin-flavor}(1,2,3)\,X_{color}(1,2,3)$$

color part

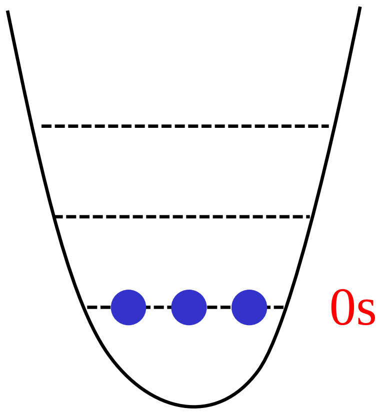
$$X_{color}(1,2,3) = \frac{1}{\sqrt{3!}}\sum_P (-1)^P P\, r(1)\, b(2)\, g(3)$$

: totally antisymmeric

spatial wave function

For octet and decuplet baryons, all the 3 quarks are in the lowest-energy (0s)-orbit.

$$\phi_{space}(1,2,3) = \phi_{0s}(1)\,\phi_{0s}(2)\,\phi_{0s}(3)$$



spin-flavor wave function (example) : SU(6) w.f.

$$\chi_{\Delta^{++},+3/2} = |\,u_\uparrow(1)\,u_\uparrow(2)\,u_\uparrow(3)\rangle$$

$$\chi_{p,+1/2}(1,2,3) = \frac{1}{3\sqrt{2}}\Big\{2\,|\,u_\uparrow u_\uparrow d_\downarrow\rangle + 2\,|\,u_\uparrow d_\downarrow u_\uparrow + 2\,|\,d_\downarrow u_\uparrow u_\uparrow\rangle$$
$$- |\,u_\uparrow d_\uparrow u_\downarrow\rangle - |\,d_\uparrow u_\uparrow u_\downarrow\rangle - |\,u_\uparrow u_\downarrow d_\uparrow\rangle$$
$$- |\,u_\downarrow d_\uparrow u_\uparrow\rangle - |\,u_\downarrow u_\uparrow d_\uparrow\rangle - |\,d_\uparrow u_\downarrow u_\uparrow\rangle\Big\}$$

MIT bag model

$$ {}_{pert}\langle 0 \,|\, \bar{q}\, q \,|\, 0 \rangle_{pert} \;=\; 0 \quad : \quad \text{inside bag} $$

$$ {}_{QCD}\langle 0 \,|\, \bar{q}\, q \,|\, 0 \rangle_{QCD} \;<\; 0 \quad : \quad \text{outside bag} $$

lagrangian density

$$ \mathcal{L}_{bag} \;=\; \left(\mathcal{L}_{QCD} \;-\; B \right) \theta(\bar{q}\, q), \qquad B \;:\; \text{bag constant} $$

difference in energy density between the QCD and perturbative vacuum

static spherical cavity approximation with bag radius *R*

$$ (\, i \;\not{\partial} \;-\; m \,)\; q \;=\; 0 \qquad r < R $$

with boundary condition

$$ i\, n^{\mu}\, \gamma_{\mu}\, q \;=\; q, \qquad n_{\mu}\, \partial\, (\bar{q}\, q) \;=\; 2\, B $$

ground state (single-quark) wave function

$$ \psi_{g.s.} \;=\; \frac{1}{\sqrt{4\,\pi}} \begin{pmatrix} j_0(k_0\, r/R)\, \chi_s \\ j_1(k_0\, r/R)\, i\, \boldsymbol{\sigma} \cdot \hat{\boldsymbol{r}}\, \chi_s \end{pmatrix} \;\equiv\; \begin{pmatrix} f(r)\, \chi_s \\ g(r)\, i\, \boldsymbol{\sigma} \cdot \hat{\boldsymbol{r}}\, \chi_s \end{pmatrix} $$

ϕ_{0s}

lower p-wave component

Chiral Quark Soliton Model

basic lagrangian

$$\mathcal{L}_{CQSM} = \bar{\psi}(x)\,(\,i\,\not\partial \;-\; M\,e^{\,i\,\gamma_5\,\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)\,/\,f_\pi}\,)\,\psi(x)$$

no kineic term for $\boldsymbol{\pi}(x)$

effective meson action $S_{eff}[\boldsymbol{\pi}]$

$$Z = \int \mathcal{D}\boldsymbol{\pi}\,\mathcal{D}\psi\,\mathcal{D}\,\psi^\dagger\,e^{\,i\int d^4x\,\mathcal{L}_{CQSM}} = \int \mathcal{D}\boldsymbol{\pi}\,e^{\,i\,S_{eff}[\boldsymbol{\pi}]}$$

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derivative expansion

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$S_{eff}[\boldsymbol{\pi}] =$ Skyrmion action with Wess-Zumino term

$+$ destabilizing 4-th derivative term

$+ \cdots\cdots\cdots$

Soliton construction without using derivative expansion

$$\boldsymbol{\pi}(\boldsymbol{x}) = \hat{\boldsymbol{r}}\, F(r) \qquad F(0) - F(\infty) = n\,\pi$$

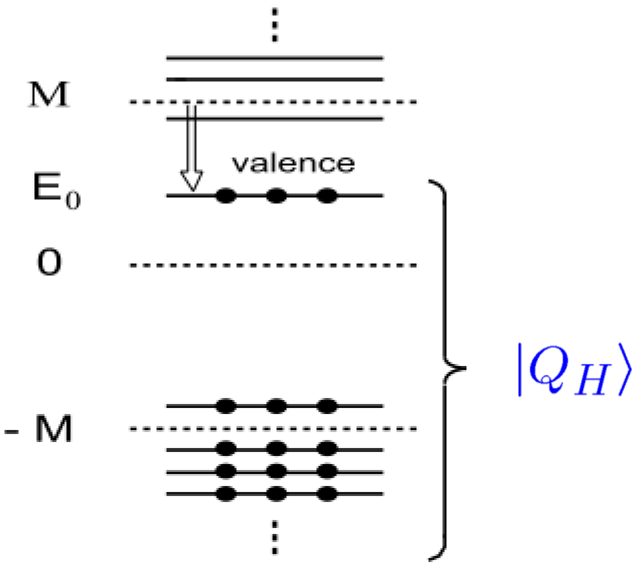
Mean field for quarks $\qquad$ n : winding number

M.F. Dirac equation

$$H\,|\,m\rangle = E_m\,|\,m\rangle$$

$$H = \frac{\boldsymbol{\alpha}\cdot\nabla}{i} + M\beta\left(\cos F(r) + i\gamma_5\, \boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\, \sin F(r)\right)$$

breaks rotational symmetry



Energy of $|Q_H\rangle$

$$E_{static} = N_c E_0 + E_{v.p.}$$

$$E_{v.p.} \sim N_c \left(\sum_{m<0} E_m - \sum_{k<0} \epsilon_k \right) \Rightarrow \text{regularize with } \Lambda$$

Hartree condidion

$$\frac{\delta}{\delta\, F(r)} E_{static}[F(r)] = 0 \quad \Rightarrow \quad F(r)$$

model needs **regularization**

$$S_{eff}[U] \;=\; -i\,N_c \,\mathrm{Sp}\,\log\,[\,i\,\partial\!\!\!/ - MU^{\gamma_5}] \;=\; \frac{4N_c}{f_\pi^2}\,I_2(M)\cdot\frac{1}{2}(\partial_\mu \boldsymbol{\pi})^2 \;+\;\cdots$$

where

$$I_2(M) \;\equiv\; i\int\frac{d^4k}{(2\pi)^4}\,\frac{M^2}{(k^2-M^2)^2} \qquad : \quad \text{log divergence}$$

Pauli-Villars regularization scheme

$$\boxed{S_{eff}^{reg} \;\equiv\; S_{eff}^{M} \;-\; \left(\frac{M}{M_{PV}}\right)^2 S_{eff}^{M_{PV}}}$$

then

$$I_2^{reg} \;\equiv\; I_2(M) - \left(\frac{M}{M_{PV}}\right)^2 I_2(M_{PV}) \;=\; \frac{M^2}{16\pi^2}\,\log\left(\frac{M_{PV}}{M}\right)^2$$

$$\frac{N_c}{4\pi^2}\,M^2\,\log\left(\frac{M_{PV}}{M}\right)^2 = f_\pi^2 \quad\Rightarrow\quad \boldsymbol{M_{PV}} \qquad : \ \text{uniquely fixed !}$$

other observables

$$\boxed{\langle O\rangle^{reg} \;\equiv\; \langle O\rangle^{M} \;-\; \left(\frac{M}{M_{PV}}\right)^2 \langle O\rangle^{M_{PV}}}$$

Noteworthy achievements of CQSM for low energy baryon observables :

(1) reproduce small quark spin fraction of N consistent with EMC observation !

$$\Delta\Sigma \sim 0.35$$

(2) reproduce large πN sigma term !

$$\Sigma_{\pi N} \simeq 60\,\mathrm{MeV}$$

(3) resolve $g_A^{(I=1)}$ problem of the Skyrme model !

$$g_A^{(Skyrme)} = g_A(\Omega^0) + g_A(\Omega^1) \simeq 0.8 + 0 = 0.8$$
$$g_A^{(CQSM)} = g_A(\Omega^0) + \boxed{g_A(\Omega^1)} \simeq 0.8 + 0.4 = 1.2$$

- Still, most low energy baryon observables are insensitive to model difference !

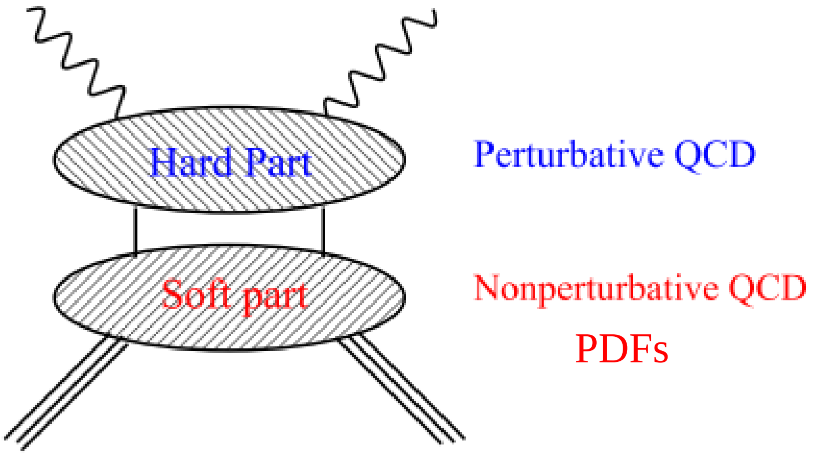
- We demonstrate that the potential ability of CQSM manifests most clearly in its predictions of internal partonic structure of the nucleon (or baryons) !

II. Model predictions and phenomenology of parton distribution functions

some remarkable achievements of CQSM in deep-inelastic-scattering physics

- Standard approach to DIS physics

factorization theorem



Soft part is treated as a **black box**, which should be determined via experiments !

reasonable strategy !

We however believe that, even if this part is completely fixed by experiments, one still wants to know why those PDFs take the form so determined !

- Nonstandard but complementary approach to DIS physics is necessary here to understand hidden chiral dynamics of soft part, based on models or lattice QCD

Merits of CQSM over many other effective models of baryons :

- it is a relativistic mean-field theory of quarks, consistent with

  large N_c QCD and $1/N_c$ expansion

- field theoretical nature of the model (nonperturbative inclusion of polarized Dirac-sea quarks) enables reasonable estimation of antiquark distributions.

- only 1 parameter of the model (dynamical quark mass M) was already fixed from low energy phenomenology

  [$M = (375 - 400)\,\mathrm{MeV}$]

parameter-free predictions for PDFs

Default

lack of explicit gluon degrees of freedom

How should we use predictions of CQSM ?

Follow the spirit of empirical PDF fit by Glueck-Reya-Vogt (GRV)

- They start the QCD evolution at fairly low energy scales like

$$Q^2 = 0.23\,\mathrm{GeV}^2 \quad \text{at LO case}$$
$$= 0.35\,\mathrm{GeV}^2 \quad \text{at NLO case}$$

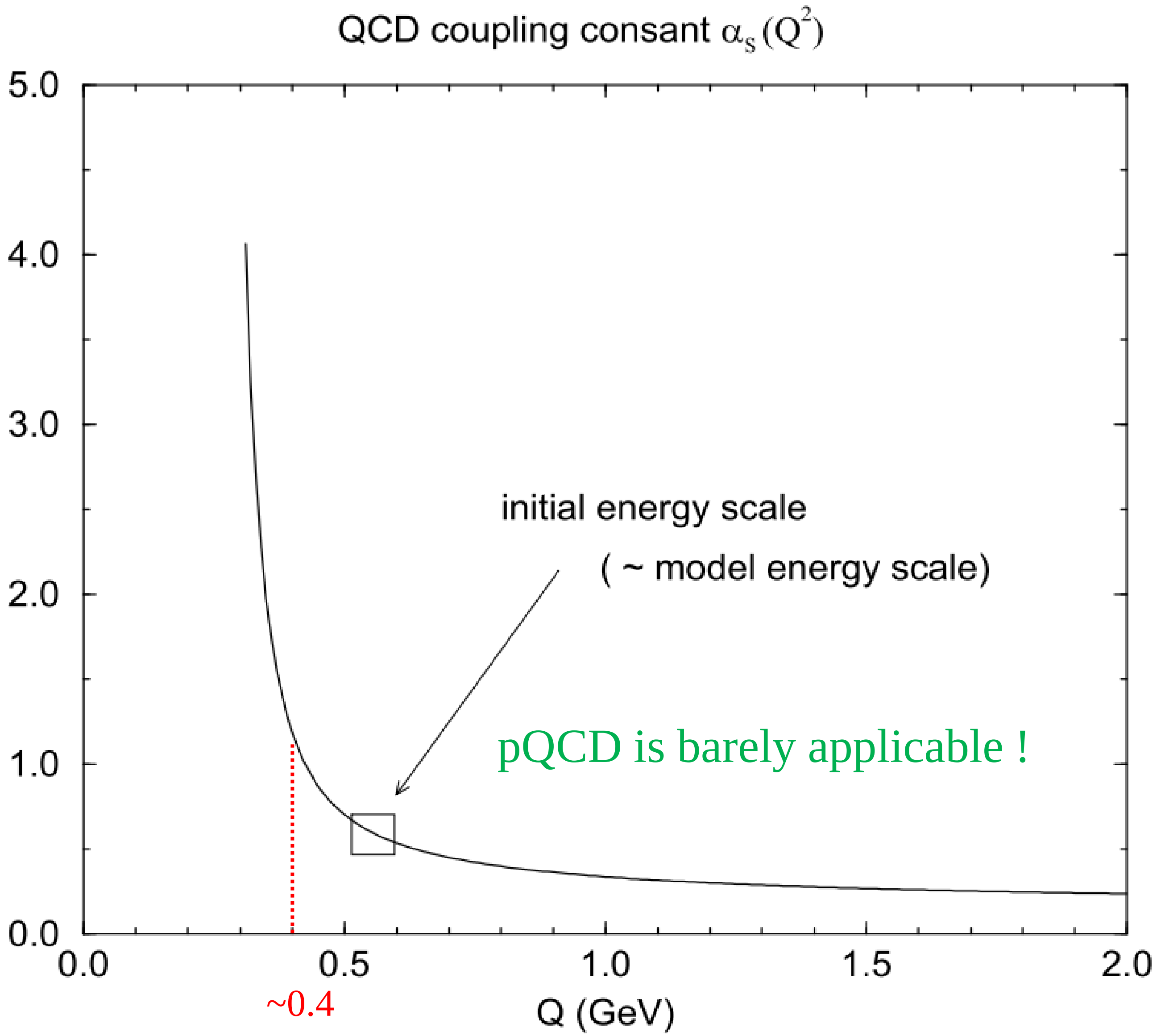
- They found that, even at such low energy scales, one needs nonperturbatively generated sea-quarks, which may be identified with effects of meson clouds.

Our general strategy

- use predictions of CQSM as initial-scale distributions of **DGLAP equation**
- initial energy scale is fixed to be (similarly to the GRV PDF fitting program)

$$Q^2_{ini} = 0.30\,\mathrm{GeV}^2 \simeq (550\,\mathrm{MeV})^2$$

QCD running coupling constant at the next-to-leading order (NLO)



parameter free predictions of CQSM for **3 twist-2 PDFs**

- unpolarized PDFs

$$q(x)$$

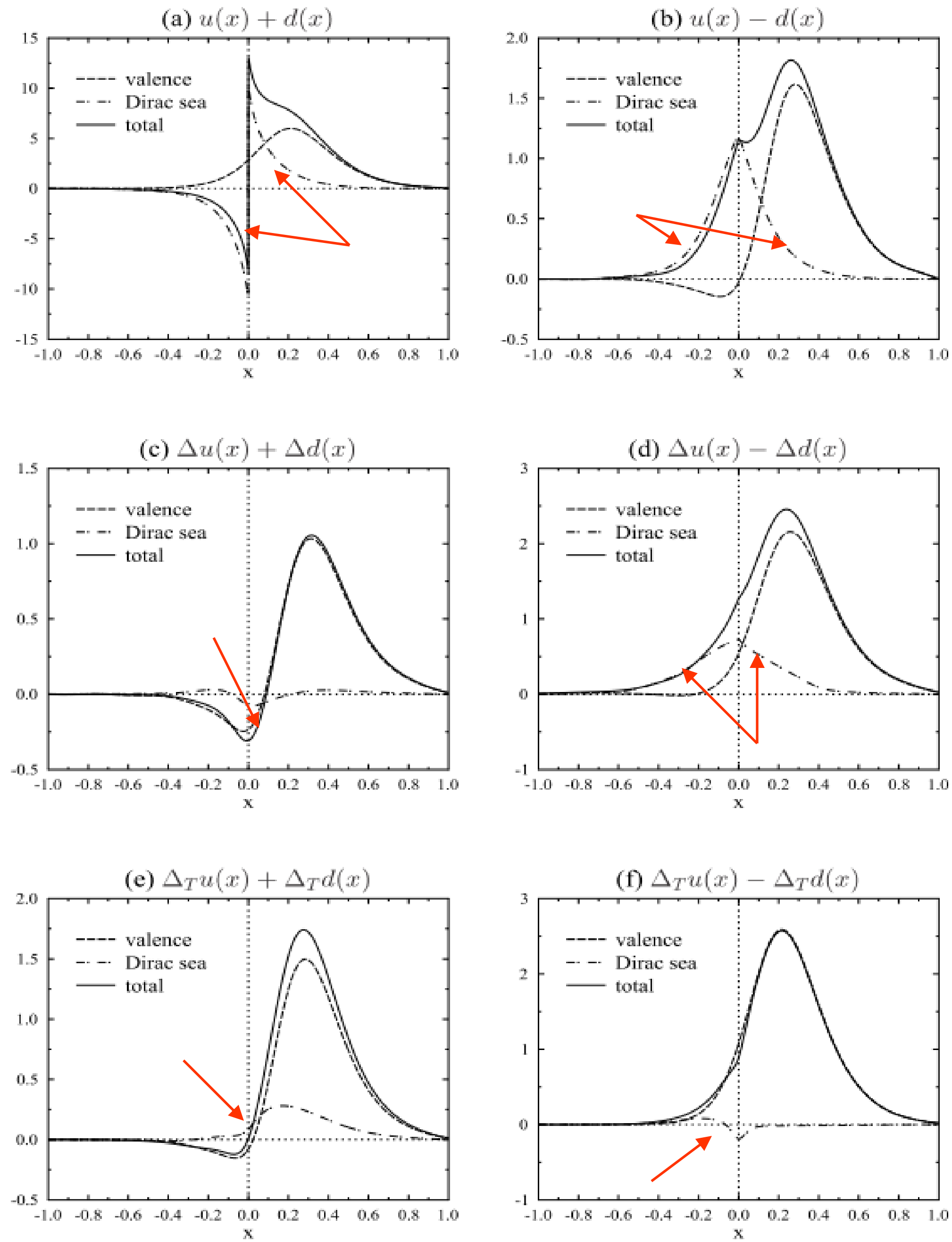
- longitudinally polarized PDFs

$$\Delta q(x)$$

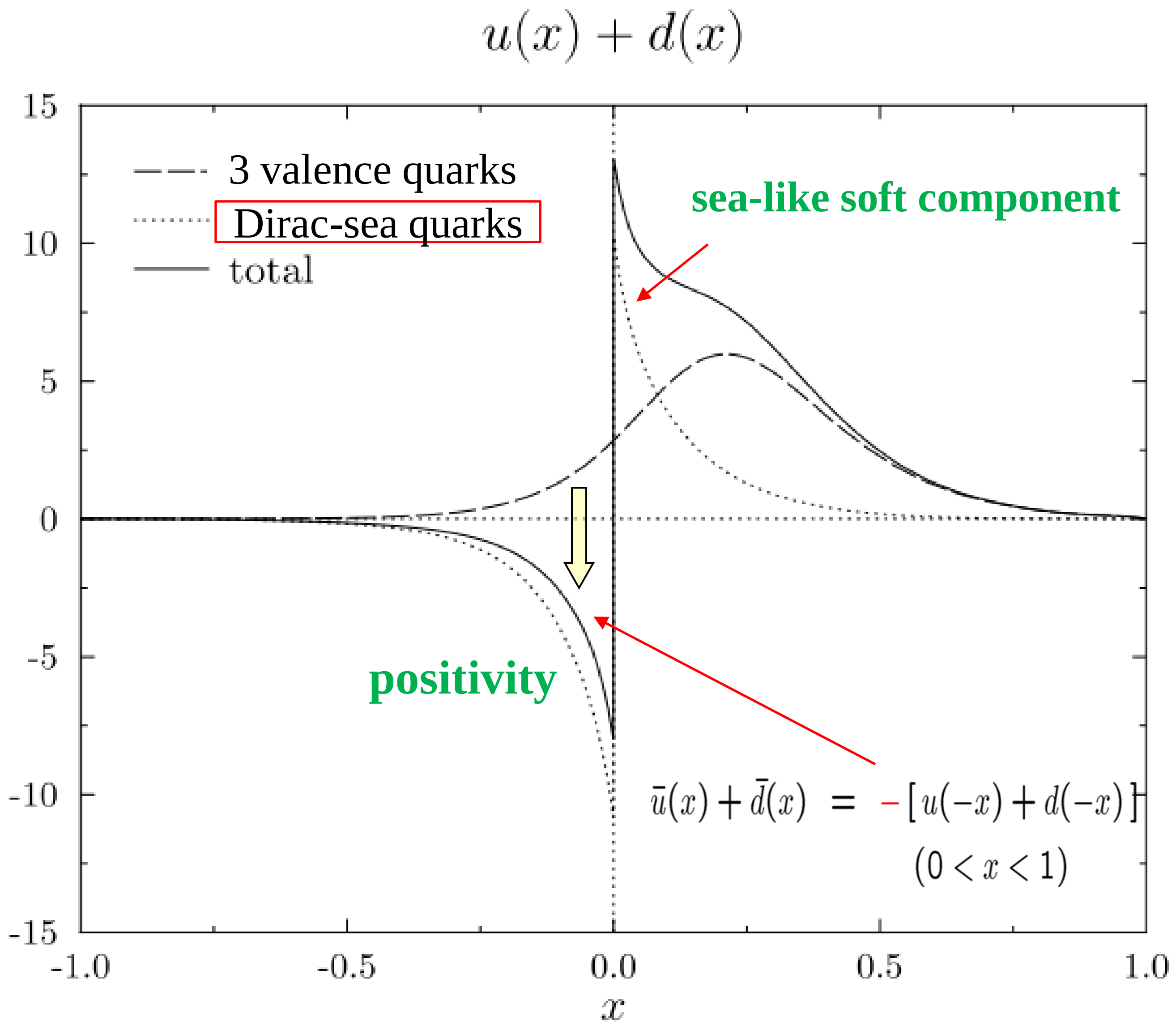
- transversities (chiral-odd)

$$\Delta_T q(x)$$

totally different behavior of the Dirac-sea contributions in different PDFs !

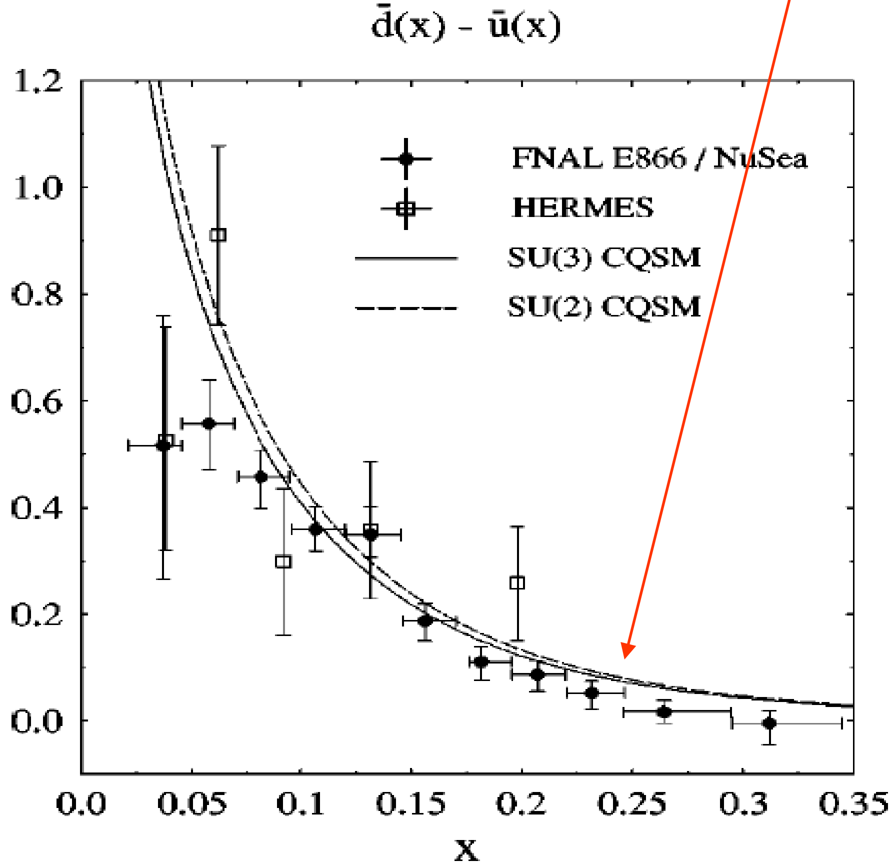


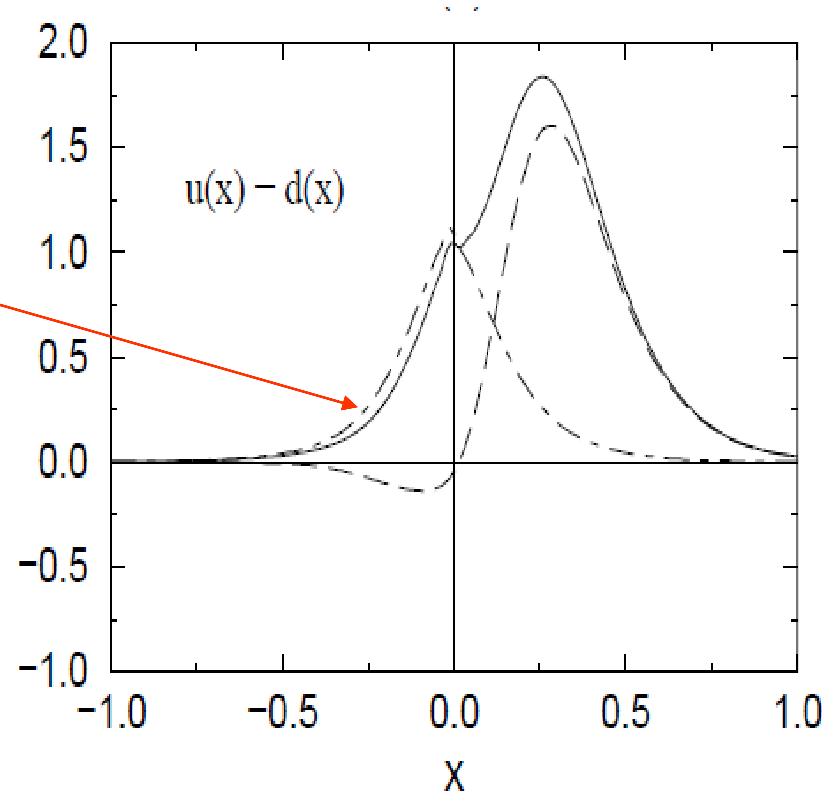
Isoscalar unpolarized PDF



Isovector unpolarized PDF

- NMC observation -



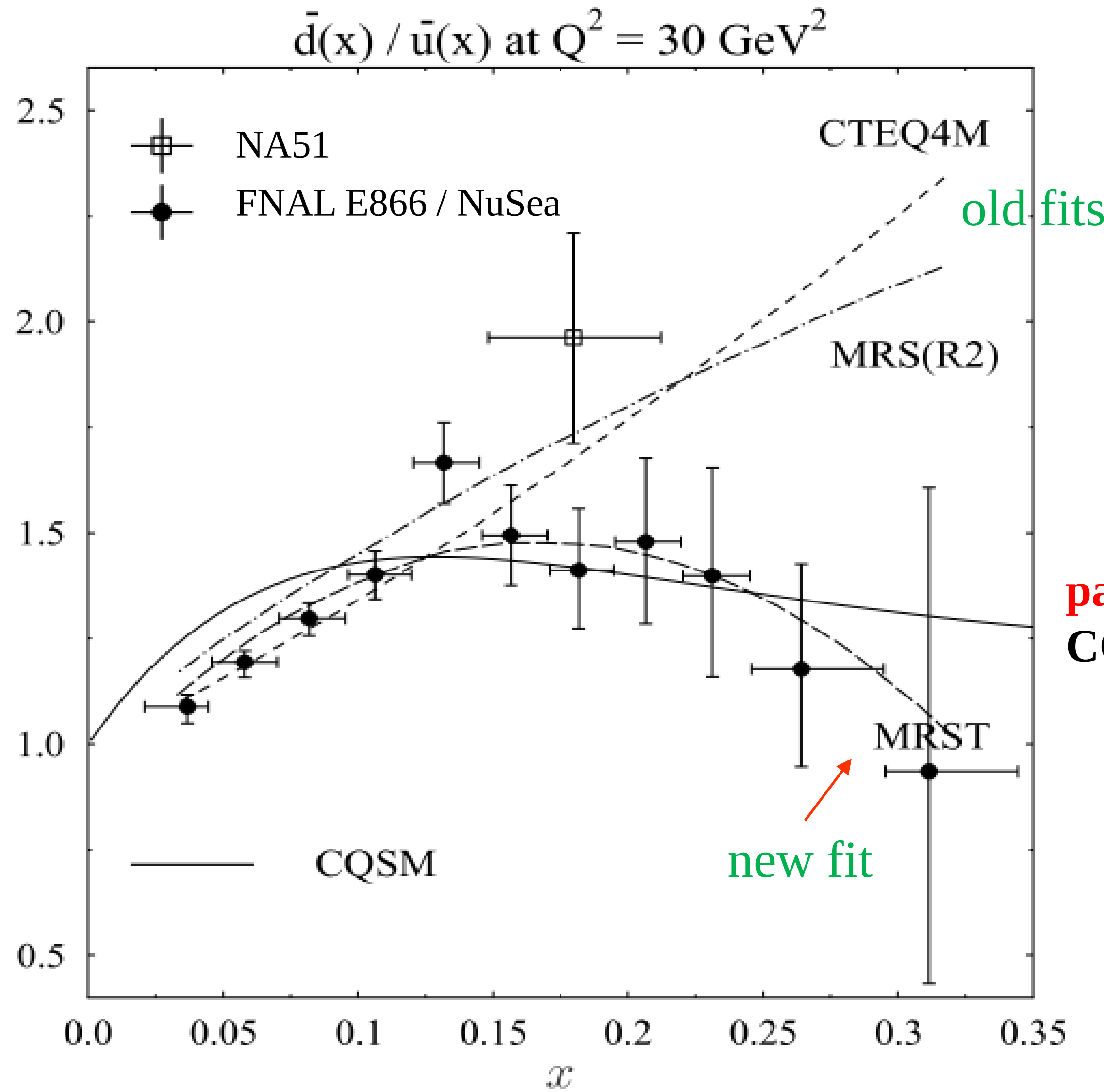


$$\bar{u}(x) - \bar{d}(x) = -[u(-x) - d(-x)] \quad (0 < x < 1)$$

$$\Downarrow$$

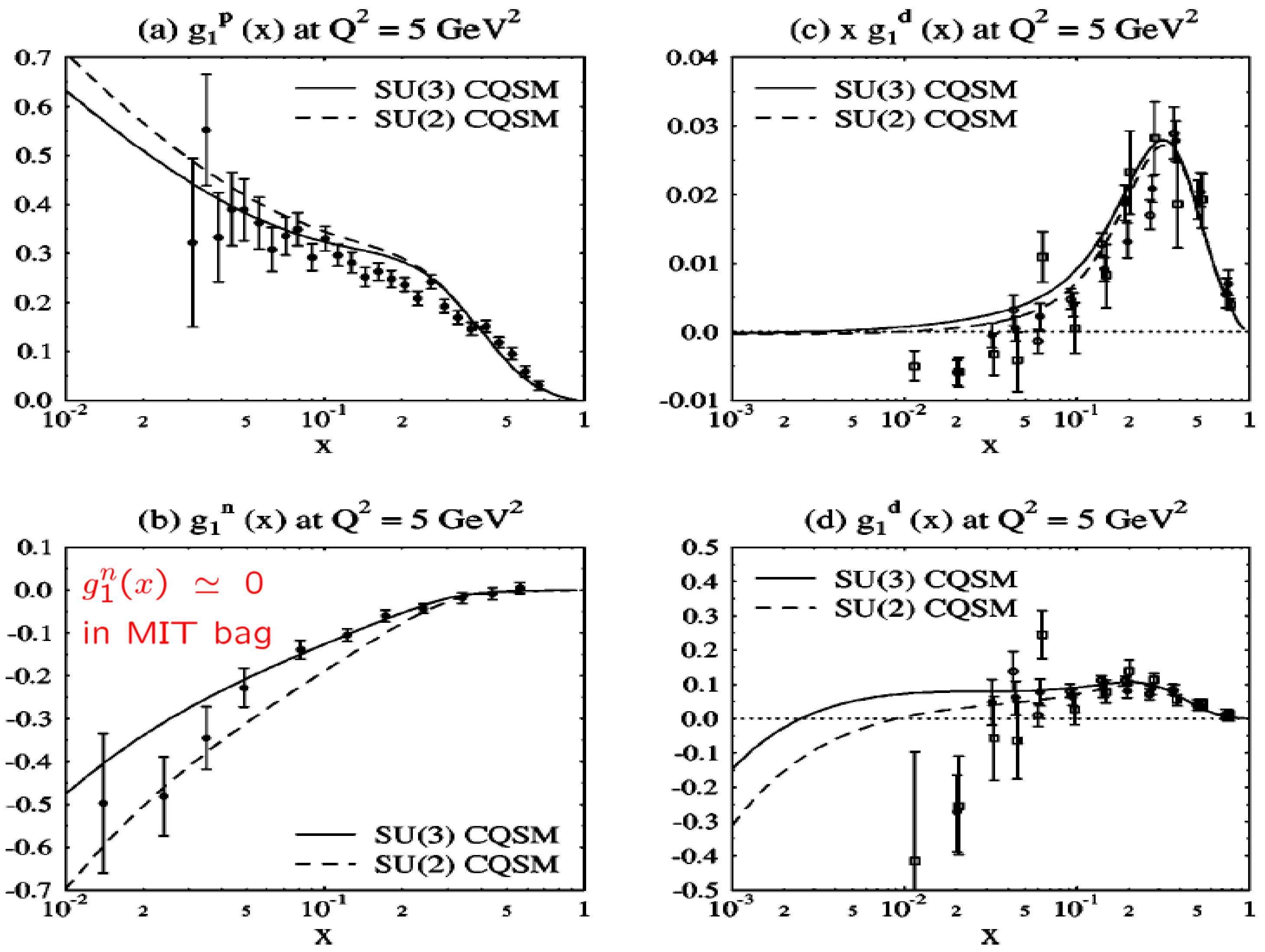
$$\bar{u}(x) - \bar{d}(x) < 0 \; !$$

$\bar{d}(x)/\bar{u}(x)$ ratio in comparison with Fermi-Lab. Drell-Yan data



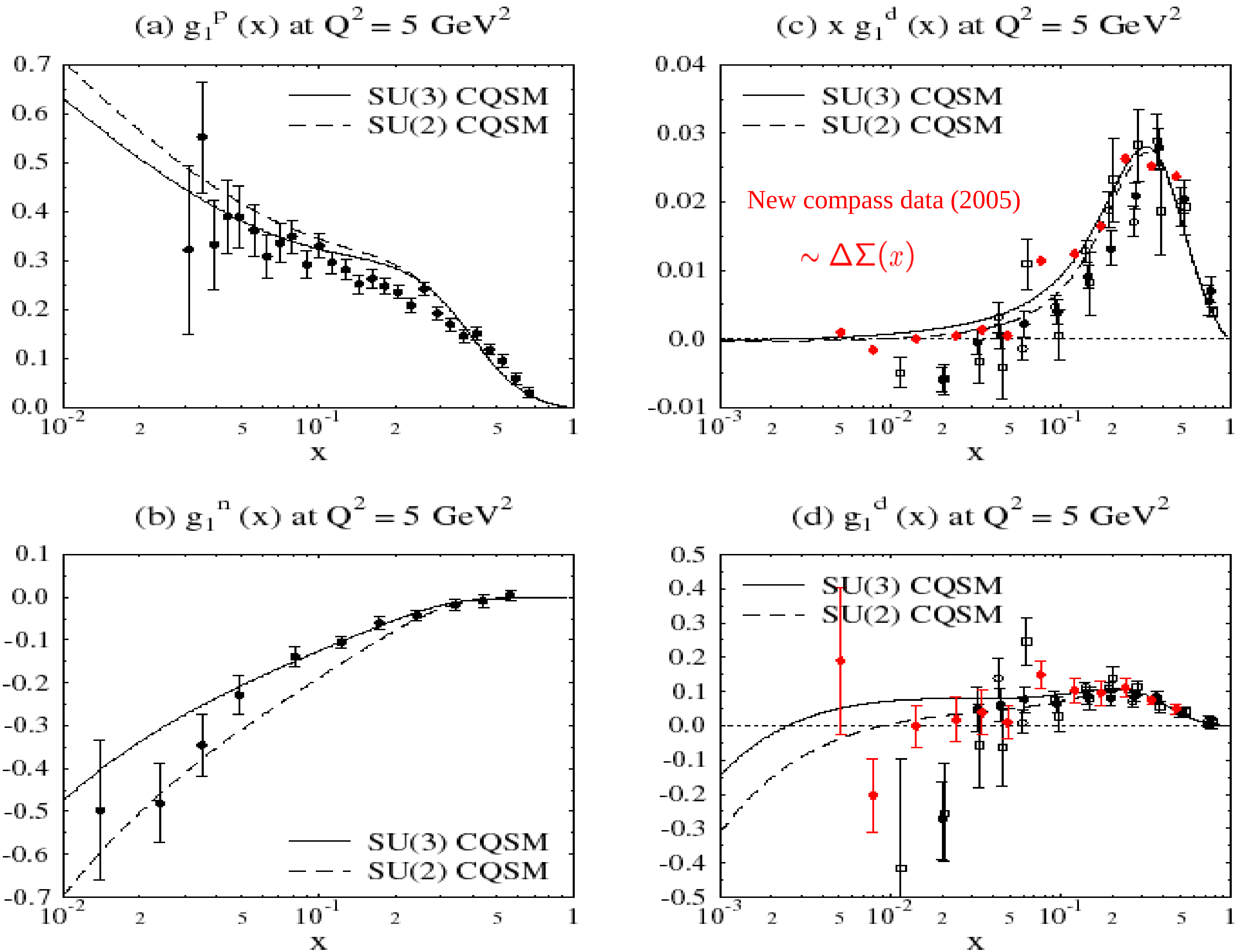
parameter free
CQSM prediction

longitudinally polarized structure functions for p, n, D : (data before 2003)



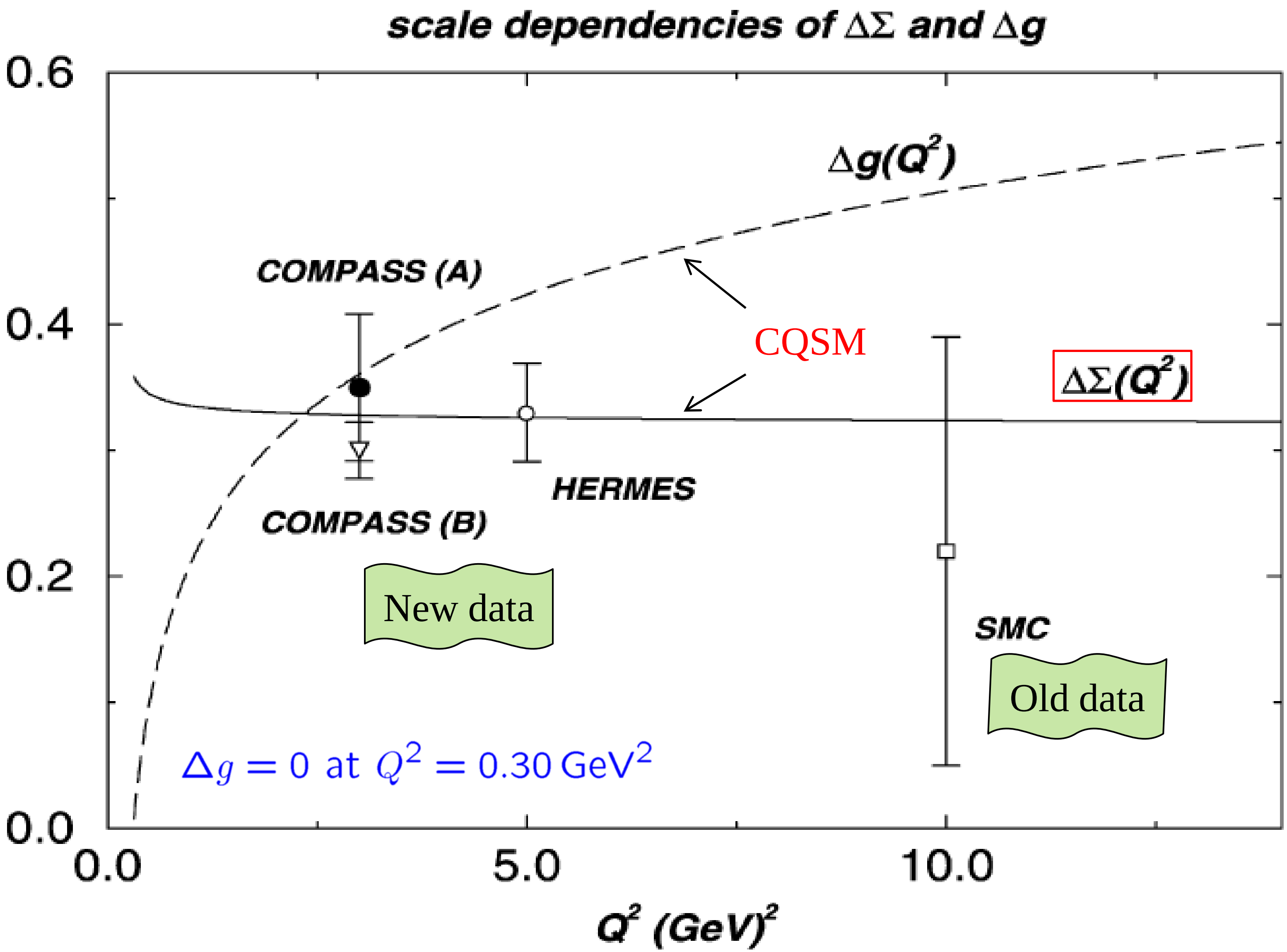
SU(2) : M. W. and T. Kubota, Phys. Rev. D60 (1999) 034022

SU(3) : M. Wakamatsu, Phys. Rev. D67 (2003) 034005



(a) g1^p (x) at Q^2 = 5 GeV^2
SU(3) CQSM
SU(2) CQSM
(c) x g1^d (x) at Q^2 = 5 GeV^2
SU(3) CQSM
SU(2) CQSM
New compass data (2005)
~ ΔΣ(x)
(b) g1^n (x) at Q^2 = 5 GeV^2
SU(3) CQSM
SU(2) CQSM
(d) g1^d (x) at Q^2 = 5 GeV^2
SU(3) CQSM
SU(2) CQSM
x

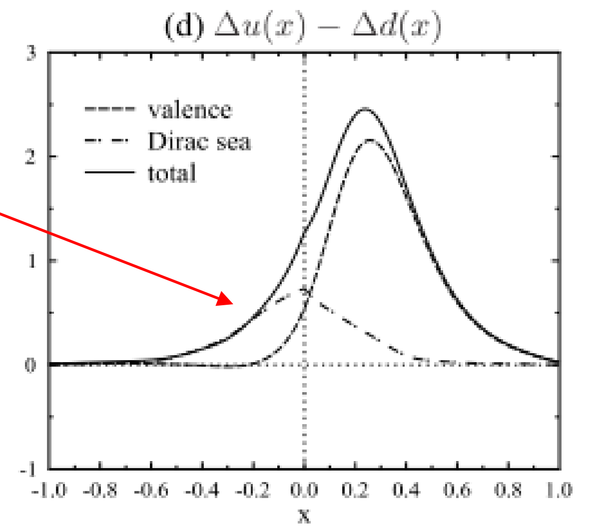
New COMPASS and HERMES fits for $\Delta\Sigma$ together with CQSM prediction



Isovector longitudinally polarized PDF

CQSM predicts $\Delta\bar{u}(x) - \Delta\bar{d}(x) > 0$

This means that antiquarks gives sizable positive contribution to Bjorken S.R.



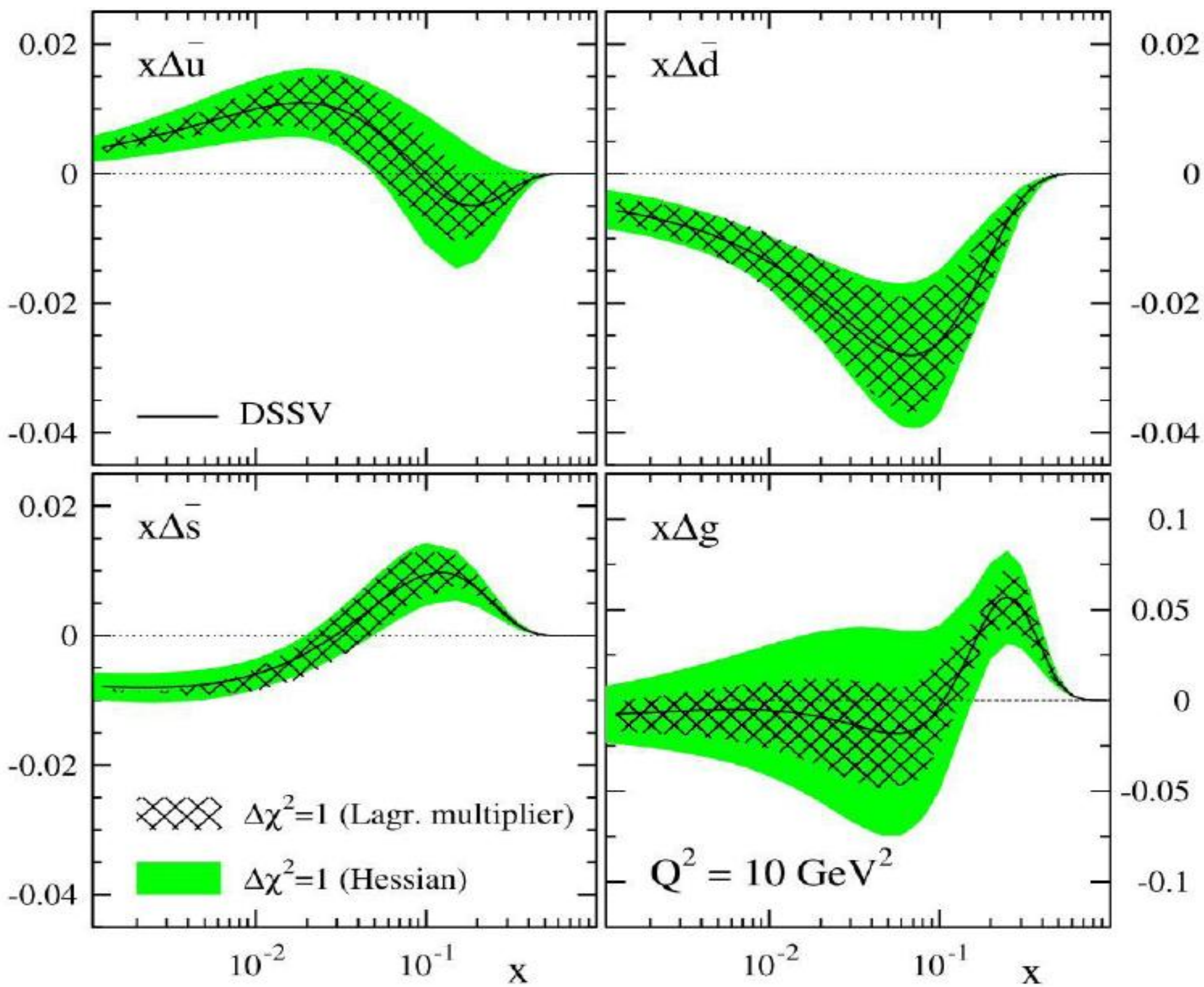
1st moment or Bjorken sum rule in CQSM

CQSM	$-1 < x < 0$	$0 < x < 1$	$-1 < x < 1$
$\Delta u + \Delta d$	-0.0472	0.399	0.352
$\Delta u - \Delta d$	0.2315	1.092	1.323
Δu	0.092	0.745	0.838
Δd	-0.139	-0.346	-0.485

$$\Delta\bar{u} \simeq 0.092, \quad \Delta\bar{d} \simeq -0.139, \quad |\Delta\bar{u}| < |\Delta\bar{d}|$$

A recent global fit including polarized pp data at RHIC

- D. Florian, R. Sassot, M. Strattmann, W. Vogelsang, Phys. Rev. D80, 034030 (2009).



$\Delta\bar{u} > 0, \quad \Delta\bar{d} < 0$

$|\Delta\bar{u}| < |\Delta\bar{d}|$

⇕

consistent with CQSM ?

But

$\Delta\bar{s} > 0$?

⇕

inconsistent with
inclusive DIS data !

⇓

Need more complete
understanding of
spin-dependent
SISIS mechanism

A recent global fit including polarized pp data at RHIC

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