II. Model predictions and phenomenology of PDFs (- continued -)

Comparative analysis of transversities & longitudinally polarized distributions

We are interested in the difference between

$$\Delta q(x) \quad \text{and} \quad \Delta_T q(x)$$

The most important quantities characterizing these are their 1st moments, called

$$\text{axial charge } g_A \quad \& \quad \text{tensor charge } g_T$$

$$g_A^{(I=0)} = \int_0^1 \left\{ [\,\Delta u(x) + \Delta d(x)\,] + [\,\Delta \bar{u}(x) + \Delta \bar{d}(x)\,] \right\} dx$$

$$g_A^{(I=1)} = \int_0^1 \left\{ [\,\Delta u(x) - \Delta d(x)\,] + [\,\Delta \bar{u}(x) - \Delta \bar{d}(x)\,] \right\} dx$$

$$g_T^{(I=0)} = \int_0^1 \left\{ [\,\Delta_T u(x) + \Delta_T d(x)\,] - [\,\Delta_T \bar{u}(x) + \Delta_T \bar{d}(x)\,] \right\} dx$$

$$g_T^{(I=1)} = \int_0^1 \left\{ [\,\Delta_T u(x) - \Delta_T d(x)\,] - [\,\Delta_T \bar{u}(x) - \Delta_T \bar{d}(x)\,] \right\} dx$$

understanding of isospin dependencies is crucially important for disentangling non-perturbative chiral dynamics hidden in the PDFs

known basic facts

(A) Naïve quark model

$$g_A^{(I=1)} = g_T^{(I=1)} = \frac{5}{3}, \qquad g_A^{(I=0)} = g_T^{(I=0)} = 1$$

(B) MIT bag model

$$g_A^{(I=1)} = \frac{5}{3} \cdot \int \left(f^2 - \frac{1}{3} g^2 \right) r^2 \, dr, \qquad g_A^{(I=0)} = 1 \cdot \int \left(f^2 - \frac{1}{3} g^2 \right) r^2 \, dr$$

$$g_T^{(I=1)} = \frac{5}{3} \cdot \int \left(f^2 + \frac{1}{3} g^2 \right) r^2 \, dr \qquad g_T^{(I=0)} = 1 \cdot \int \left(f^2 + \frac{1}{3} g^2 \right) r^2 \, dr$$

$f(r), g(r)$: upper & lower components of g.s. w.f.

Important observation

$$\frac{g_A^{(I=0)}}{g_A^{(I=1)}} = \frac{g_T^{(I=0)}}{g_T^{(I=1)}} = \frac{3}{5}$$

in both of NQM & MIT bag model

Comparison between the predictions of MIT bag and CQSM

	MIT bag		CQSM	Experiment
$g_A^{(I=1)}$	1.06		1.31	1.267
$g_A^{(I=0)}$	0.64	$\leftarrow\text{---}\rightarrow$	0.35	$0.330 \pm 0.040 \quad (Q^2 = 5\text{GeV}^2)$
$g_T^{(I=1)}$	1.34		1.21	
$g_T^{(I=0)}$	0.88	$\leftarrow\text{---}\rightarrow$	0.68	
$g_A^{(I=0)} / g_A^{(I=1)}$	0.60	$\leftarrow\text{---}\rightarrow$	0.27	~ 0.26
$g_T^{(I=0)} / g_T^{(I=1)}$	0.60	$\leftarrow\text{---}\rightarrow$	0.56	

CQSM predicts for tensor and axial charges

$$g_T^{(I=1)} \simeq g_A^{(I=1)}, \quad \text{while} \quad g_T^{(I=0)} \gg g_A^{(I=0)}$$

⬇

expected features for transversities and longitudinally polarized PDF

$$\Delta_T q^{(I=1)}(x) \simeq \Delta q(x)^{(I=1)}, \quad \Delta_T q^{(I=0)}(x) \gg \Delta q^{(I=0)}(x)$$

Then, from

$$\Delta_T u(x) = \frac{1}{2}\left[\Delta_T q^{(I=0)}(x) + \Delta_T q^{(I=1)}(x)\right]$$
$$\Delta_T d(x) = \frac{1}{2}\left[\Delta_T q^{(I=0)}(x) - \Delta_T q^{(I=1)}(x)\right]$$

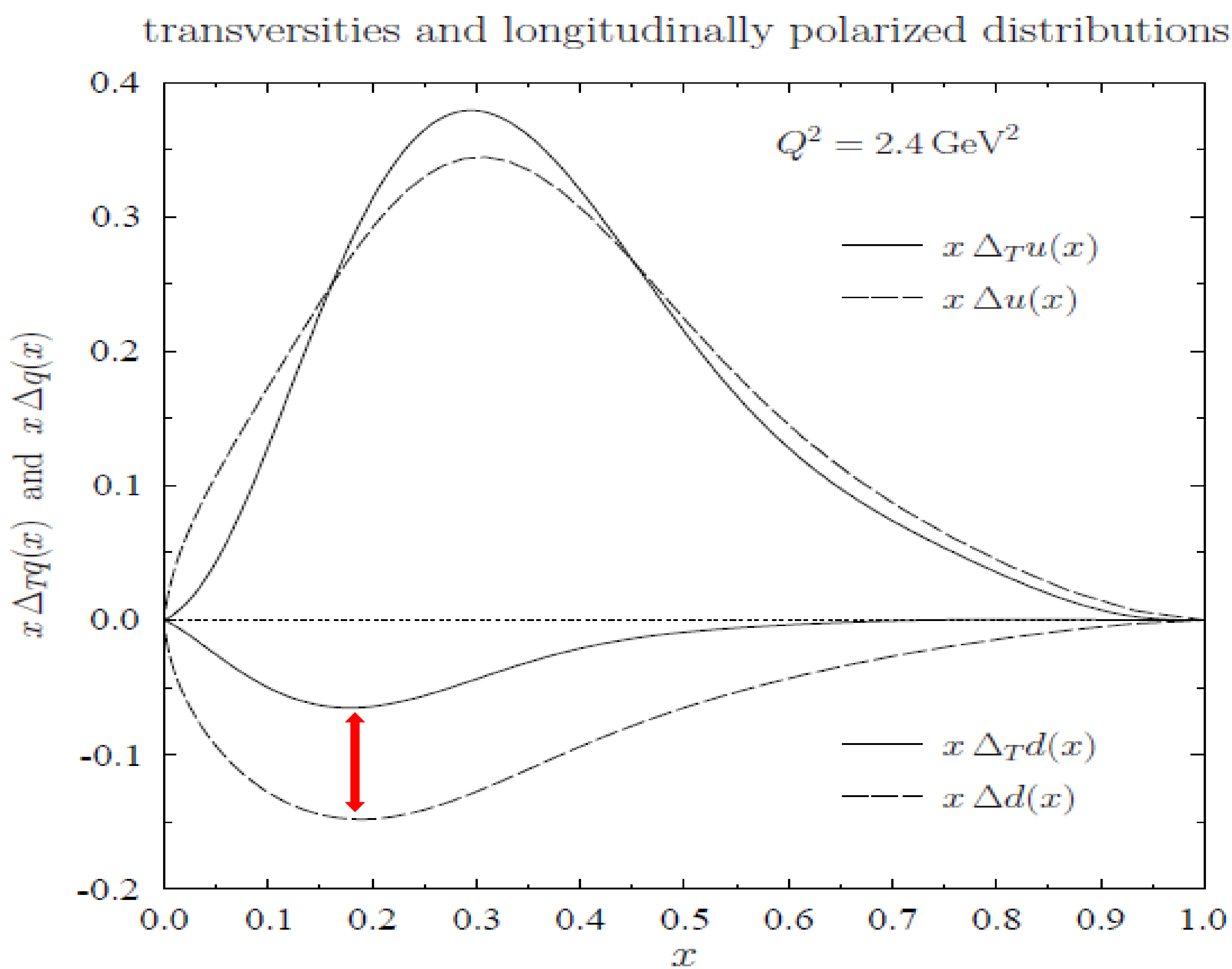
together with the analogous relation for the longitudinally polarized PDFs ,

We anticipate that

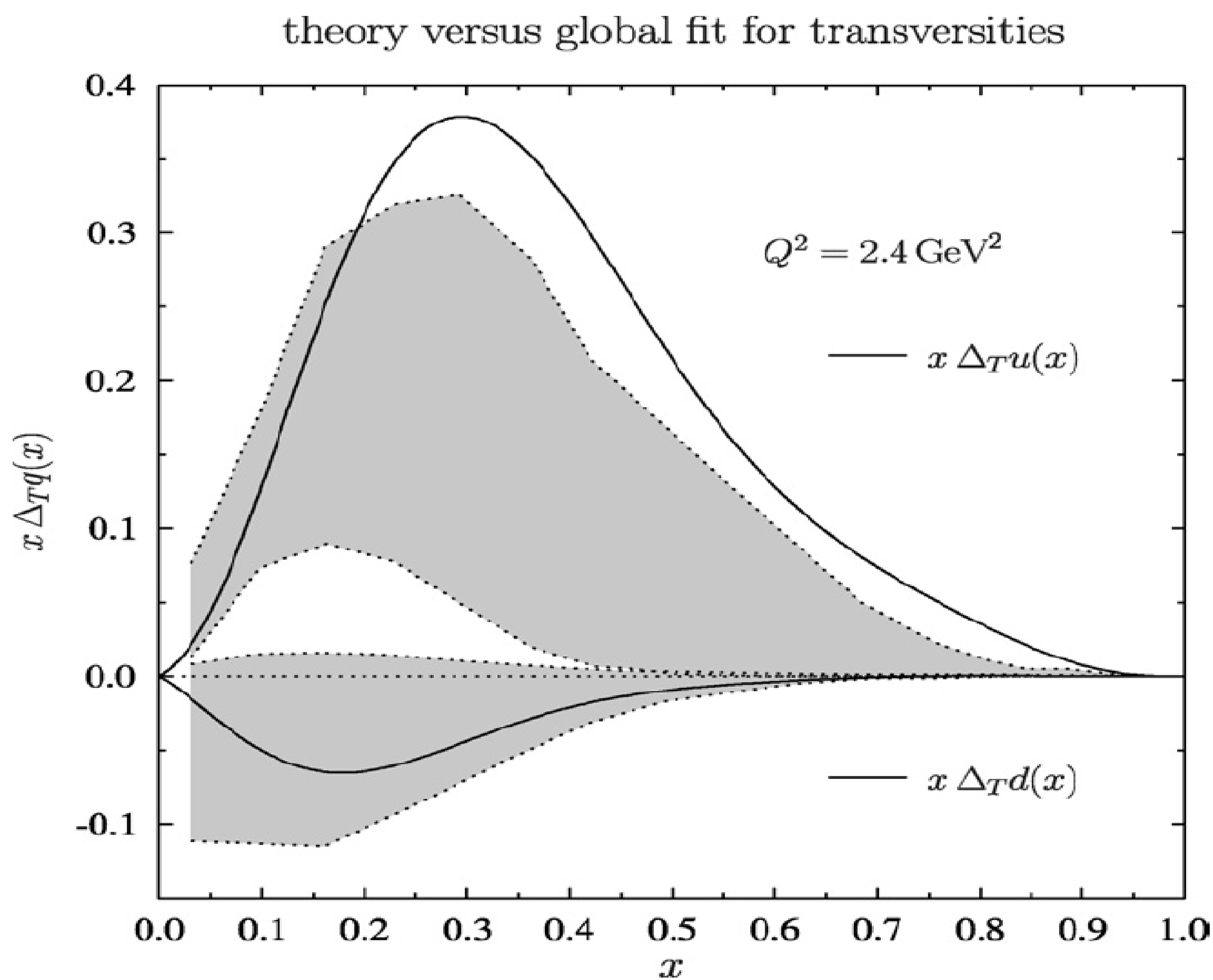
$$|\Delta_T d(x)| \ll |\Delta d(x)|$$
$$|\Delta_T u(x)| \simeq |\Delta u(x)|$$

CQSM predictions evolved to $Q^2 = 2.4\,\mathrm{GeV}^2$

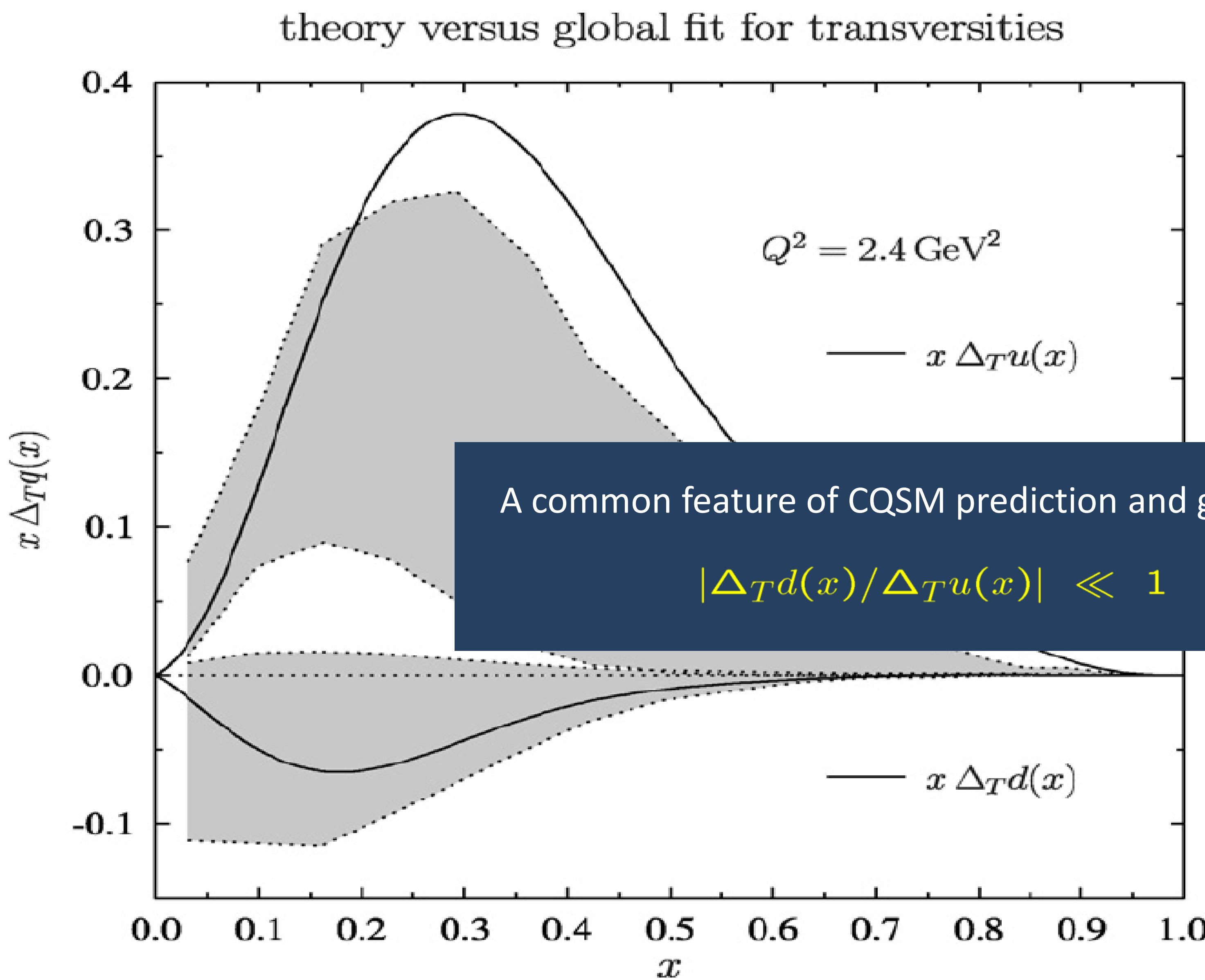
energy scale corresponding to Anselmino et al's phenomenological fits



Comparison with global fit by Anselmino et al.



Comparison with global fit by Anselmino et al.

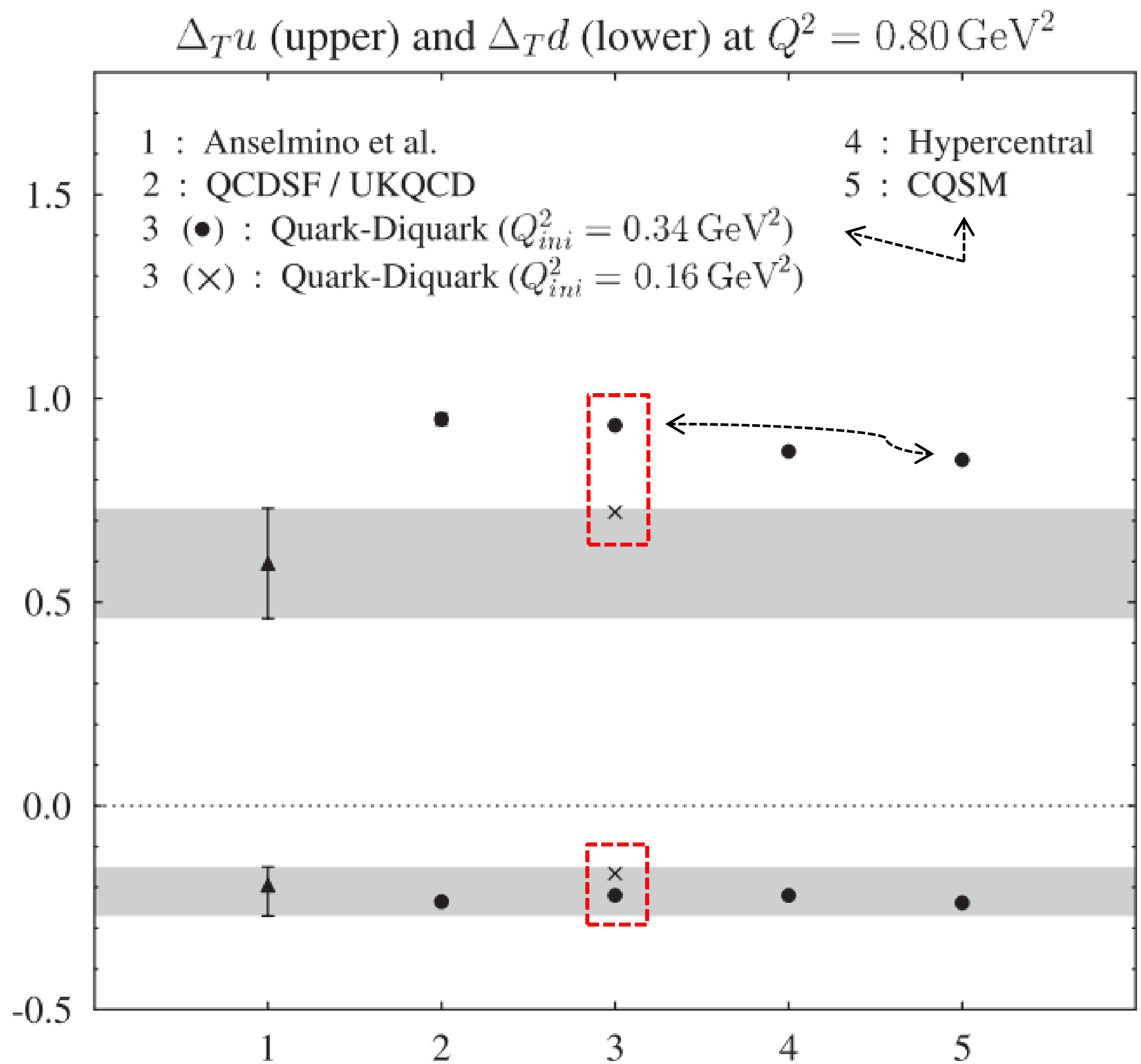


A common feature of CQSM prediction and global fit

$$|\Delta_T d(x)/\Delta_T u(x)| \ll 1$$

Caution about strong scale dependence of transversity around model energy scales

• M. W., Phys. Rev. D79 (2009) 014033.

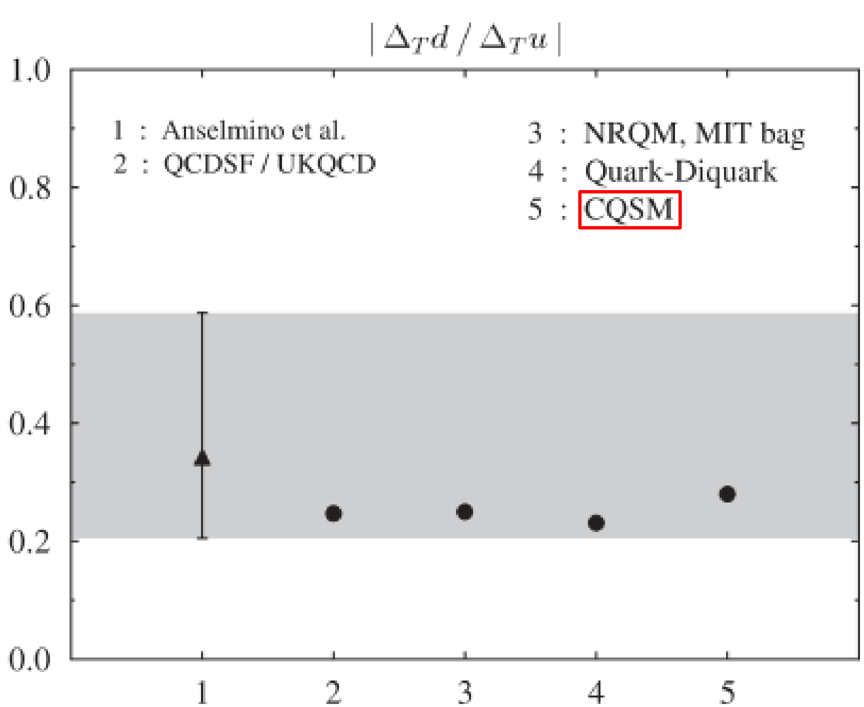


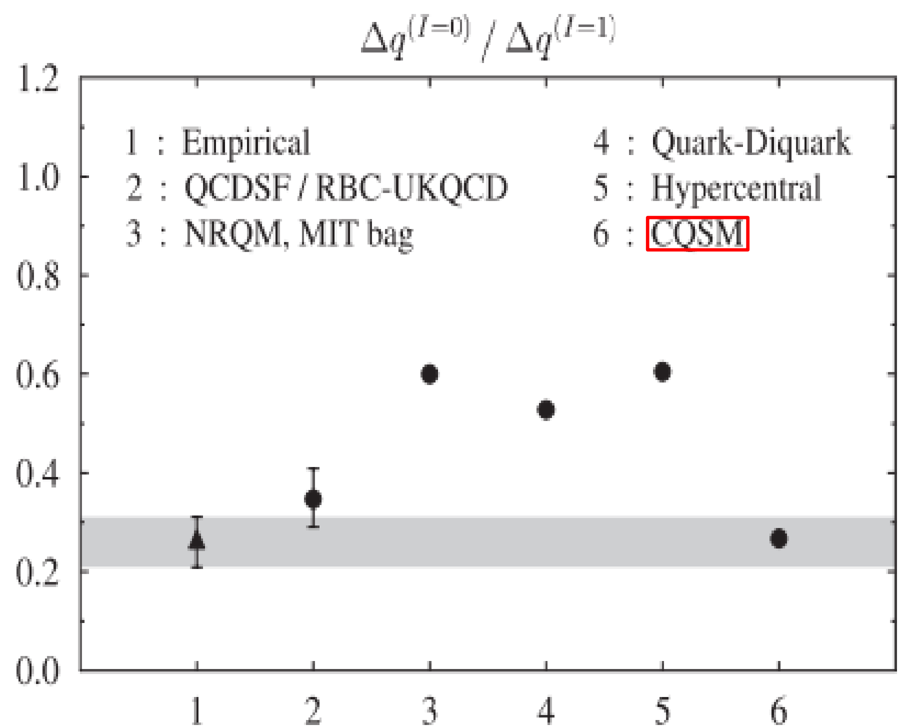
Comparison of various model predictions for scale independent ratios

• M. W., Phys. Rev. D79 (2009) 014033.

tensor-charge ratio

axial-charge ratio





scale independent !

nearly scale independent !

Short summary on the transversity distributions

♣ When one compares the model predictions of transversities with the empirical ones extracted from high-energy measurements, one must be very careful about the strong scale-dependence of transversities in the nonperturbative low energy domain.

♣ Model predictions are very sensitive to the starting energy scale of evolution !

⇓

♣ A safer comparison would therefore be made for the ratios like

$$\Delta_T u \,/\, \Delta_T d \quad \text{and/or} \quad \Delta q_T^{(I=0)} \,/\, \Delta q_T^{(I=1)}$$

which are scale-independent, because of the flavor-independent nature of evolution equations for chiral-odd transversities, which does not couple to gluons !

Chiral-odd twist-3 distribution function $e(x)$

- M. W. and Y. Ohnishi, Phys. Rev. D67 (2003) 114011.
- Y. Ohnishi and M. W., Phys. Rev. D69 (2004) 114002 .

chiral-odd

twist-2	twist-3	twist-4
$f_1(x) = q(x)$	$e(x)$	$f_4(x)$
$g_1(x) = \Delta q(x)$	$h_2(x)$	$g_3(x)$
$h_1(x) = \Delta_T q(x)$	$g_T(x)$	$h_3(x)$

Why is it interesting ?

- $\int_{-1}^{1} e(x)\, dx \quad \propto \quad \pi N$ Sigma term
- existence of $\delta(x)$-type sigularity ?

M.Burkardt and Y.Koike (2002)

↓

What is the **physical origin** of this peculiar delta-function singularity ?

Origin of delta-function singularity in $e(x)$

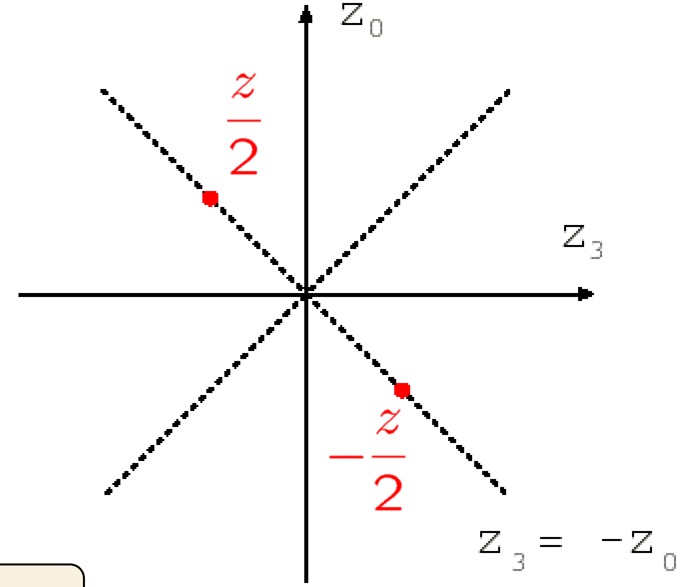
theoretical definition of $e(x)$

$$e(x) = M_N \int_{-\infty}^{\infty} \frac{dz^0}{2\pi} e^{-ixM_N z^0} E(z^0)$$

with

$$E(z_0) = \langle N | \bar{\psi}\left(-\frac{z}{2}\right) \psi\left(\frac{z}{2}\right) | N \rangle |_{z_3=-z_0, z_\perp=0}$$

measures light-cone correlation of scalar type



existence of delta-function singularity in $e(x)$ indicates

$$E(z_0) \xrightarrow{z_0 \to \infty} \text{constant} \quad \text{(does not damp)}$$

long-range (infinite range) correlation

Within the CQSM, we could analytically confirm this behavior

M.W. and Y.Ohnishi, Phys. Rev. D67 (2003) 114011

We found the existence of this infinite-range correlation is inseparably connected with

nontrivial vacuum structure of QCD

spontaneous χSB and nonvanishing vacuum quark condensate

⇓

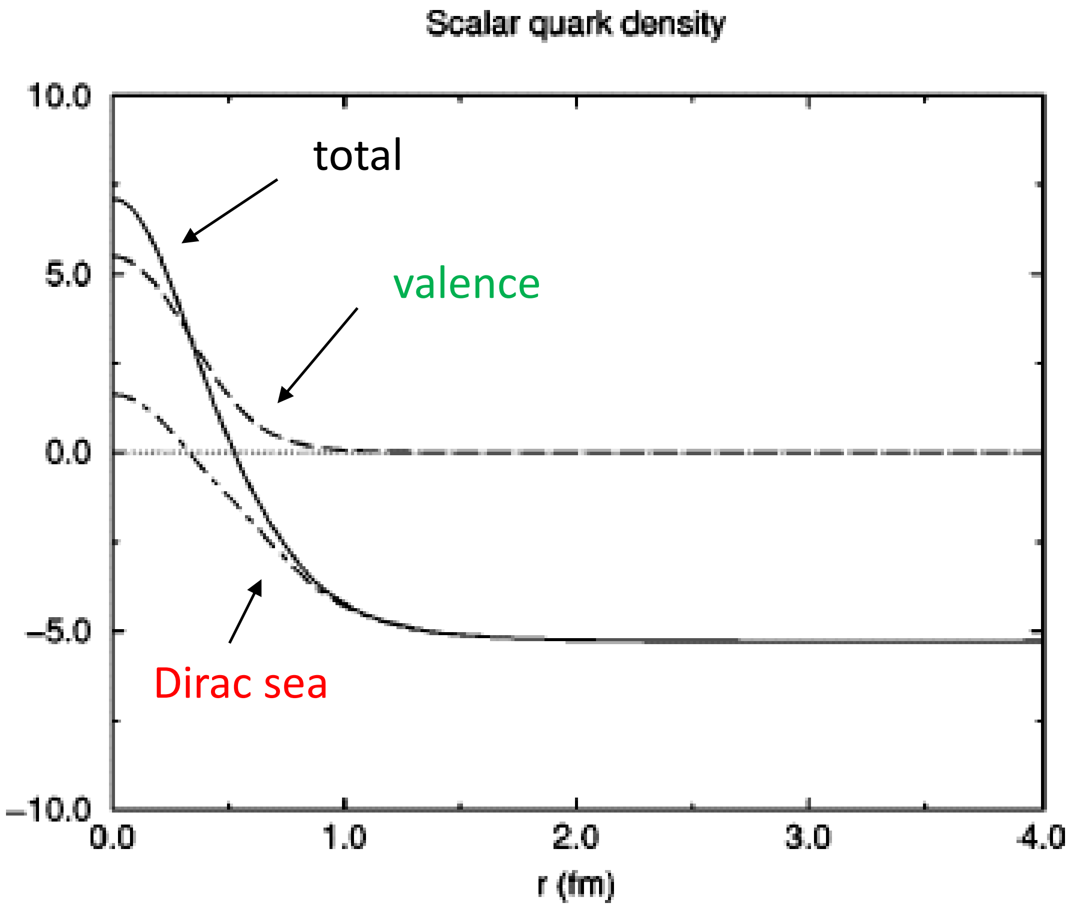
Why does vacuum property come into nucleon (baryon) observable ?

⇓

due to extraordinary nature of the scalar quark density in the nucleon

$$S(r) \equiv \langle N | \bar{\psi} \psi (\boldsymbol{r}) | N \rangle$$

nucleon scalar quark density in the CQSM



$\langle 0 \,|\, \bar{\psi}\,\psi \,|\, 0 \rangle \;<\; 0$

vacuum quark condensate

as $r \longrightarrow \infty$

valence contribution $\longrightarrow$ 0

Dirac-sea contribution $\longrightarrow$ nonzero vacuum quark condensate

This in turn indicates that

$$E(z_0) \;=\; \langle N |\, \bar{\psi}\left(-\frac{z}{2}\right)\, \psi\left(\frac{z}{2}\right)\, | N \rangle \,|_{z_3 = -z_0,\, z_\perp = 0} \;\longrightarrow\; \text{nonzero}$$

(as $z_0 \longrightarrow \infty$)

⇓

We thus conclude that

Nonvanishing quark condensate as a signal of the spontaneous χSB of the QCD vacuum is the physical origin of $\delta(x)$-type singularity in $e(x)$

Sophisticated numerical method to treat $e^{(I=0)}(x)$ containing $\delta(x)$

Y. Ohnishi and M. W., Phys. Rev D69 (2004) 114002

We find that

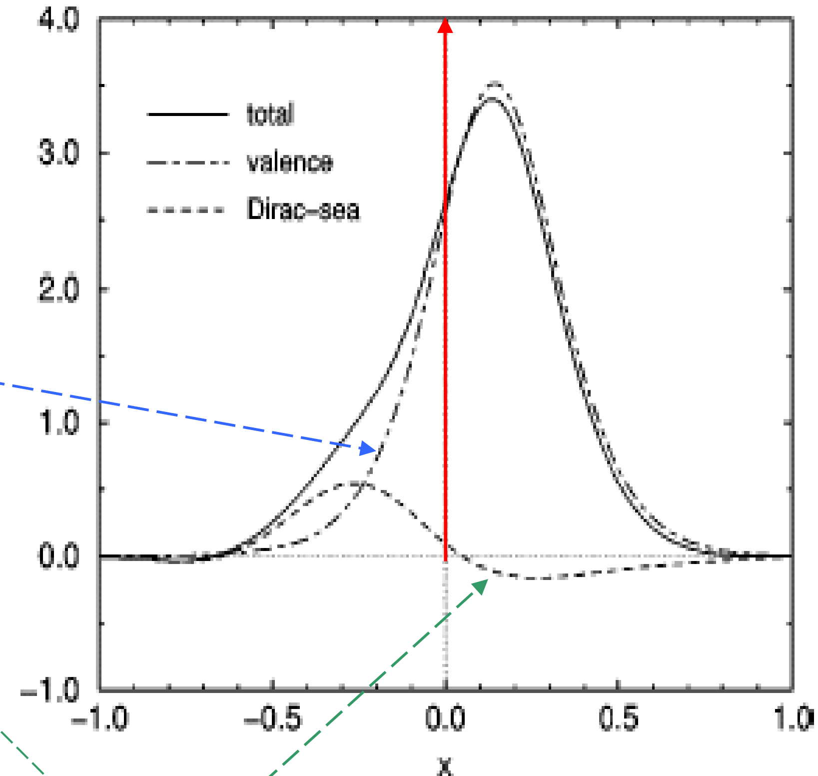
$$e^{(I=0)}(x) = e^{(I=0)}_{valence}(x) + e^{(I=0)}_{sea}(x)$$

where

$$e^{(I=0)}_{sea}(x) = e^{(I=0)}_{sing}(x) + e^{(I=0)}_{regl}(x)$$

with

$$e^{(I=0)}_{sing}(x) = C\,\delta(x), \quad C \simeq 9.9$$



1st moment sum rule for isoscalar $e(x)$

$$\int_{-1}^{1} e^{(I=0)}(x)\, dx = \bar{\sigma} \quad : \quad \text{nucleon scalar charge}$$

numerically

$$\bar{\sigma} = \bar{\sigma}_{valence} + \bar{\sigma}_{sea}{}^{regl} + \bar{\sigma}_{sea}{}^{sing}$$

$$\simeq \quad 1.7 \quad + \quad 0.2 \quad + \quad \boxed{9.9} \quad \simeq \quad 11.8$$

dominant

with $m_q \simeq (4 \sim 7)\,\text{MeV}$, this gives

$$\Sigma_{\pi N} \equiv m_q \bar{\sigma} \simeq (47 \sim 83)\ \text{MeV}$$

reproduce fairly large πN sigma term

[Cf.] Nucleon scalar charge in the MIT bag model

$$\bar{\sigma} \;=\; \langle N \,|\, \int \bar{\psi}(\boldsymbol{r})\,\psi(\boldsymbol{r})\,d^3 r \,|\, N \rangle \;=\; N_c \int \left\{ f(r)^2 \;-\; g(r)^2 \right\} r^2\,dr$$

However, since

$$\int \left\{ f(r)^2 \;+\; g(r)^2 \right\} r^2\,dr \;=\; 1$$

we find the **inequality**

$$\int \left\{ f(r)^2 \;-\; g(r)^2 \right\} r^2\,dr \;<\; 1$$

⇓

compare

then $\bar{\sigma} \;<\; N_c \;=\; 3$ ⟺ $\bar{\sigma}_{CQSM} \;\simeq\; 11.8$

quark models with **only 3 valence quark degrees freedom** cannot reproduce large πN sigma term !

Short summary on $e(x)$

(1) delta-function singularity in chiral-odd twist-3 distribution $e(x)$ is

a rare manifestation of **nontrivial vacuum structure** of QCD in baryon observable

(2) existence of this singularity will be observed as

violation of πN sigma-term sum rule of $e^{(I=0)}(x)$

⇓

need very precise experimental information on this quantity in wider range of x.

especially in small x region

- A CLASS collaboration proposal, H. Avakian et al.

flavor SU(3) CQSM and strange sea distribution in the nucleon

model lagrangian

$$\mathcal{L} = \bar{\psi}(x)\,(i\,\not\partial - M\,U^{\gamma_5}(x) - \Delta m_s\,P_s\,)\,\psi(x)$$

with

$$U^{\gamma_5}(x) = e^{i\gamma_5\,\pi(x)/f_\pi}, \quad \pi(x) = \pi_a(x)\,\lambda_a \quad (a = 1,\cdots,8)$$

$$\Delta m_s\,P_s = \begin{pmatrix} 0 & & \\ & 0 & \\ & & \Delta m_s \end{pmatrix} \quad : \quad \textbf{SU(3) breaking term}$$

basic dynamical assumptions

(1) lowest energy classical solution is obtained by **embedding** of SU(2) hedgehog sol.

$$U_0^{\gamma_5}(x) = \begin{pmatrix} e^{i\gamma_5\,\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\,F(r)} & 0 \\ 0 & 1 \end{pmatrix} \quad \in \quad \textbf{SU(3)}$$

(2) quantization of soliton **rotational motion** in SU(3) collective coordinate space.

(3) **perturbative treatment of SU(3) breaking term**.

$$\Delta\tilde{H} = \Delta m_s \cdot \gamma^0\,A^\dagger(t)\left(\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8\right)A(t), \qquad \Delta m_s = 100 \pm 20\,\text{MeV}$$

Comparison with high energy data

- only 1 parameter of the SU(3) CQSM is fixed to be

$$\Delta m_s = 100 \pm 20\,\text{MeV}$$

⬇

no other free parameter

- use predictions of CQSM as initial-scale distributions given at

$$Q^2_{ini} = 0.30\,\text{GeV}^2 \simeq (550\,\text{MeV})^2$$

$$u(x),\ d(x),\ s(x), \qquad \Delta u(x),\ \Delta d(x),\ \Delta s(x)$$

$$\bar{u}(x),\ \bar{d}(x),\ \bar{s}(x), \qquad \Delta \bar{u}(x),\ \Delta \bar{d}(x),\ \Delta \bar{s}(x)$$

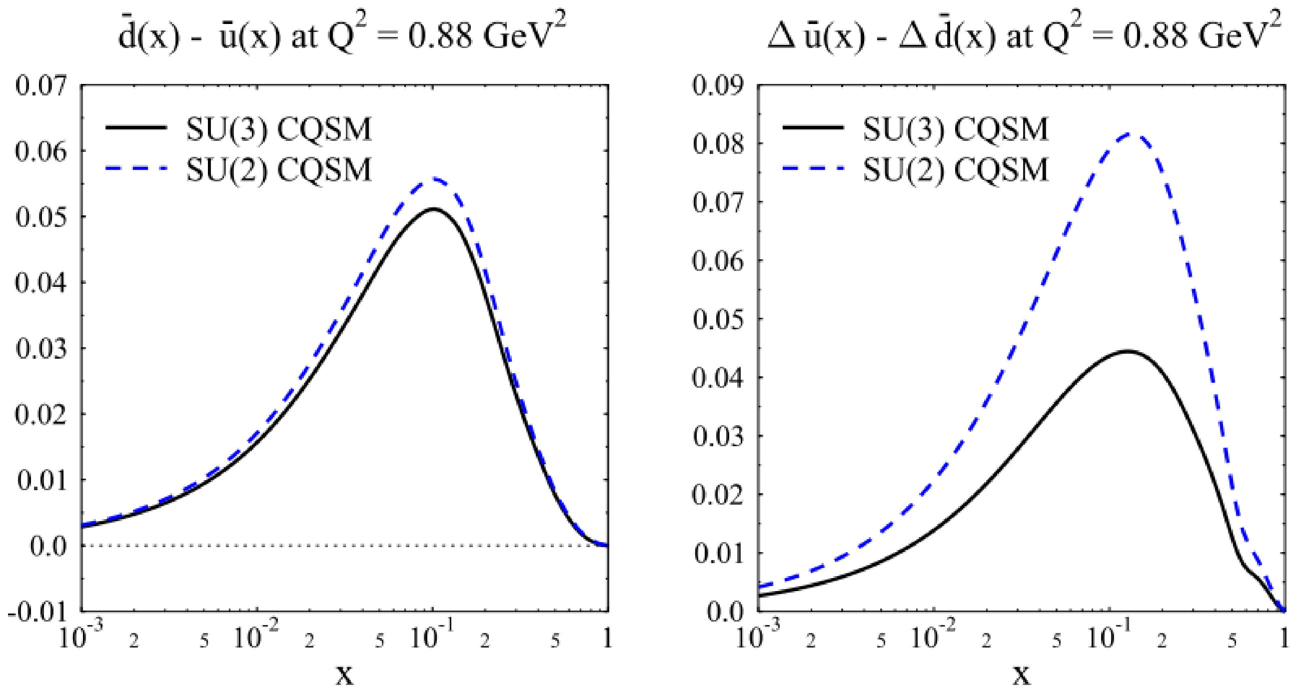
$$g(x) = 0, \qquad \Delta g(x) = 0$$

relevant 1st moments

	SU(2) CQSM	SU(3) CQSM	Experiment
$g_A^{(3)}$	1.41	1.20	1.257 ± 0.016
$g_A^{(8)}$	—	0.59	0.579 ± 0.031
$g_A^{(0)}$	**0.35**	**0.36**	**0.33 ± 0.04**
Δu	0.88	0.82	0.842 ± 0.010
Δd	-0.53	-0.38	-0.427 ± 0.010
Δs	**0**	**-0.08**	**-0.08 ± 0.02**
F	—	0.45	0.459 ± 0.008
D	—	0.76	0.798 ± 0.008
F/D	—	**0.59**	0.575 ± 0.016

problem of isospin asymmetry of sea quark distributions

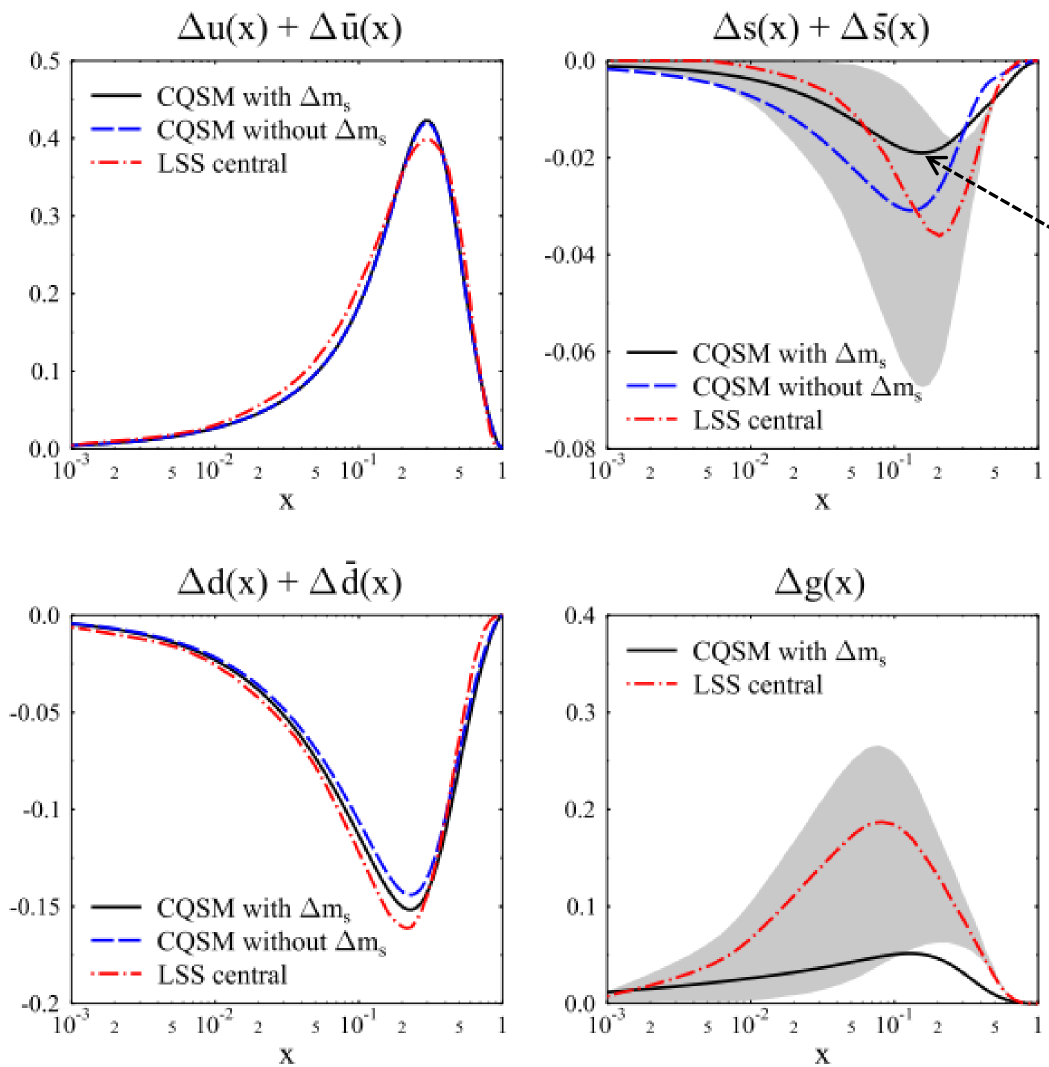
SU(2) CQSM predicts $\begin{cases} \bar{u}(x) - \bar{d}(x) < 0 \\ \Delta\bar{u}(x) - \Delta\bar{d}(x) > 0 \end{cases}$ ⇒ SU(3) CQSM ?



- $[\bar{d}(x) - \bar{u}(x)]^{SU(3)} \simeq [\bar{d}(x) - \bar{u}(x)]^{SU(2)}$
- $[\Delta\bar{d}(x) - \Delta\bar{u}(x)]^{SU(3)} < [\Delta\bar{d}(x) - \Delta\bar{u}(x)]^{SU(2)}$

several typical predictions of the SU(3) CQSM (continued)

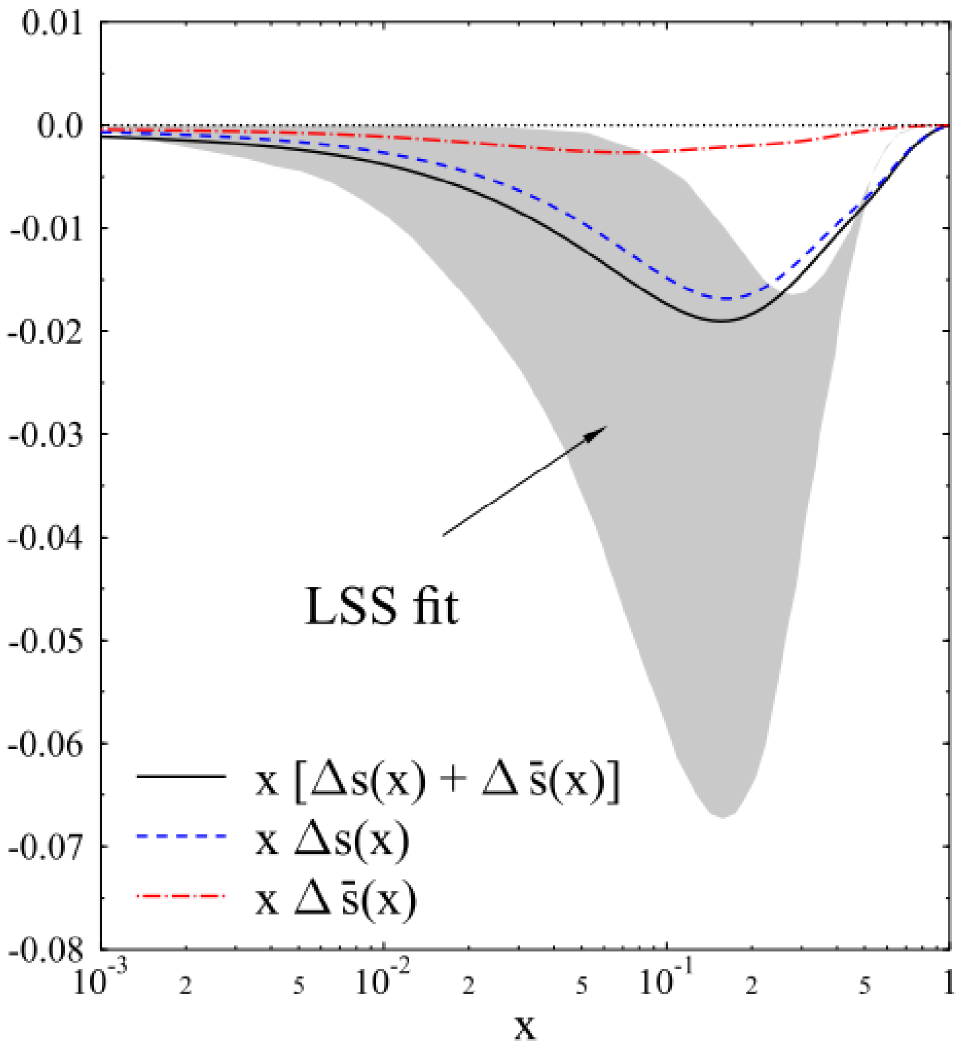
(A) longitudinally polarized strange quark distributions



$\Delta s(x) + \Delta \bar{s}(x) < 0$

LSS NLO fits
at $Q^2 = 1\ \mathrm{GeV}^2$

separate contributions of $\Delta s(x)$ & $\Delta\bar{s}(x)$



We find that

$$|\Delta\bar{s}(x)| \ll |\Delta s(x)|$$

⇩

consistent with the physical picture of meson-baryon fluctuation model

(Brodsky-Ma, 1996)

$$p \to \Lambda + K^+$$

$$(\Lambda \sim uds, \ K^+ \sim u\bar{s})$$

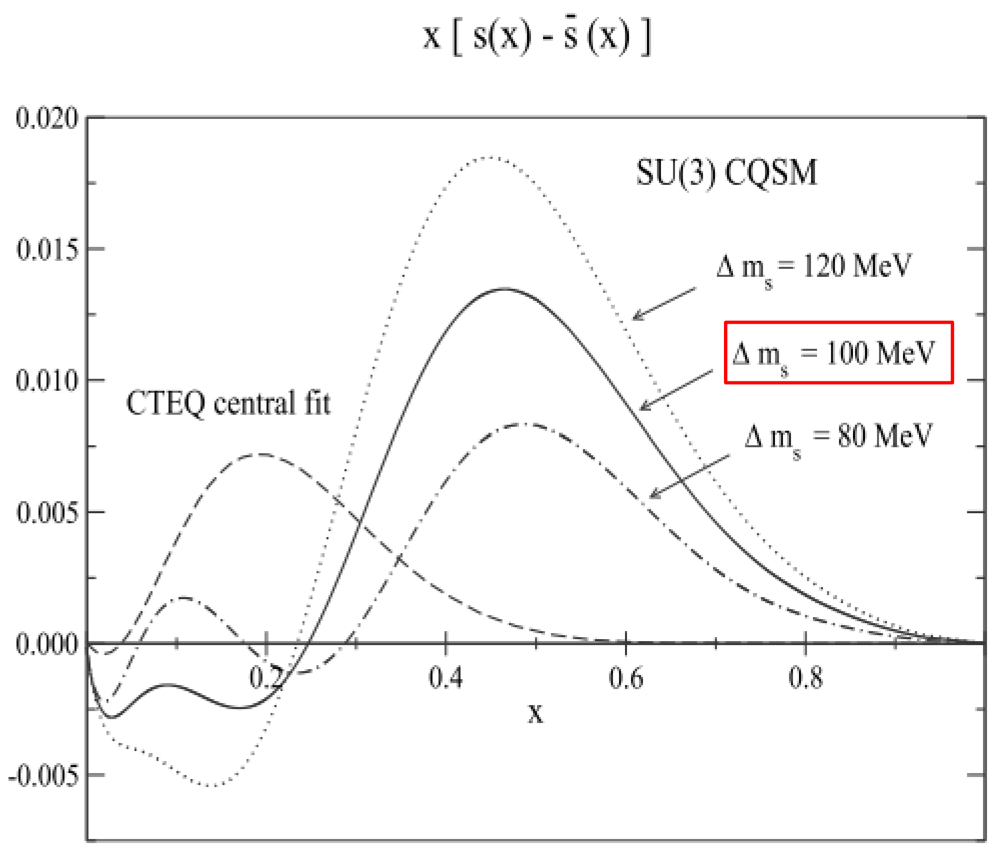
Note the asymmetry

$s \in$ spin 1/2 baryon

$\bar{s} \in$ spin 0 meson

asymmetry of unpolarized strange sea

$$\int_0^1 [\, s(x) - \bar{s}(x) \,] \, dx \;=\; 0 \quad : \quad \text{net strange-quark number}$$



This is also consistent with the physical picture of meson-baryon fluctuation model

$$p \to \Lambda + K^+$$

$$(\Lambda \sim uds, \;\; K^+ \sim u\bar{s})$$

Note the asymmetry

$s \in$ baryon

$\bar{s} \in$ meson

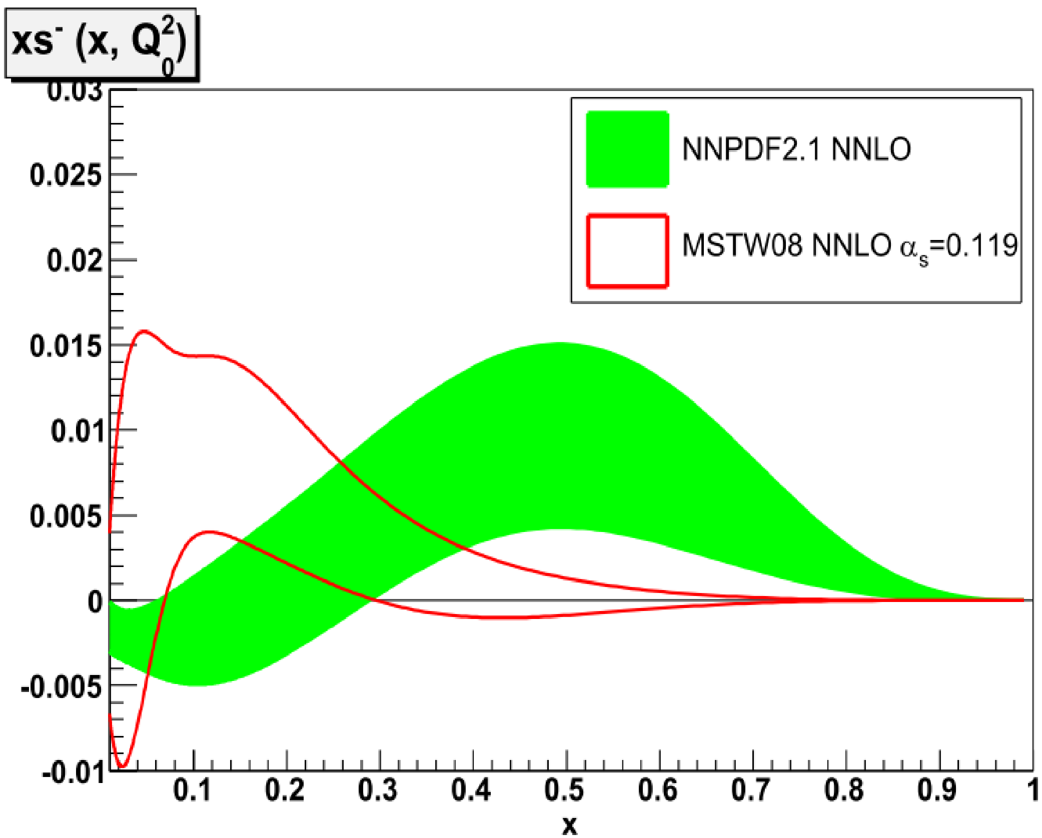
⬇

s-quark has valence-like harder component ?

Unbiased global determination of parton distributions and their uncertainties at NNLO and at LO

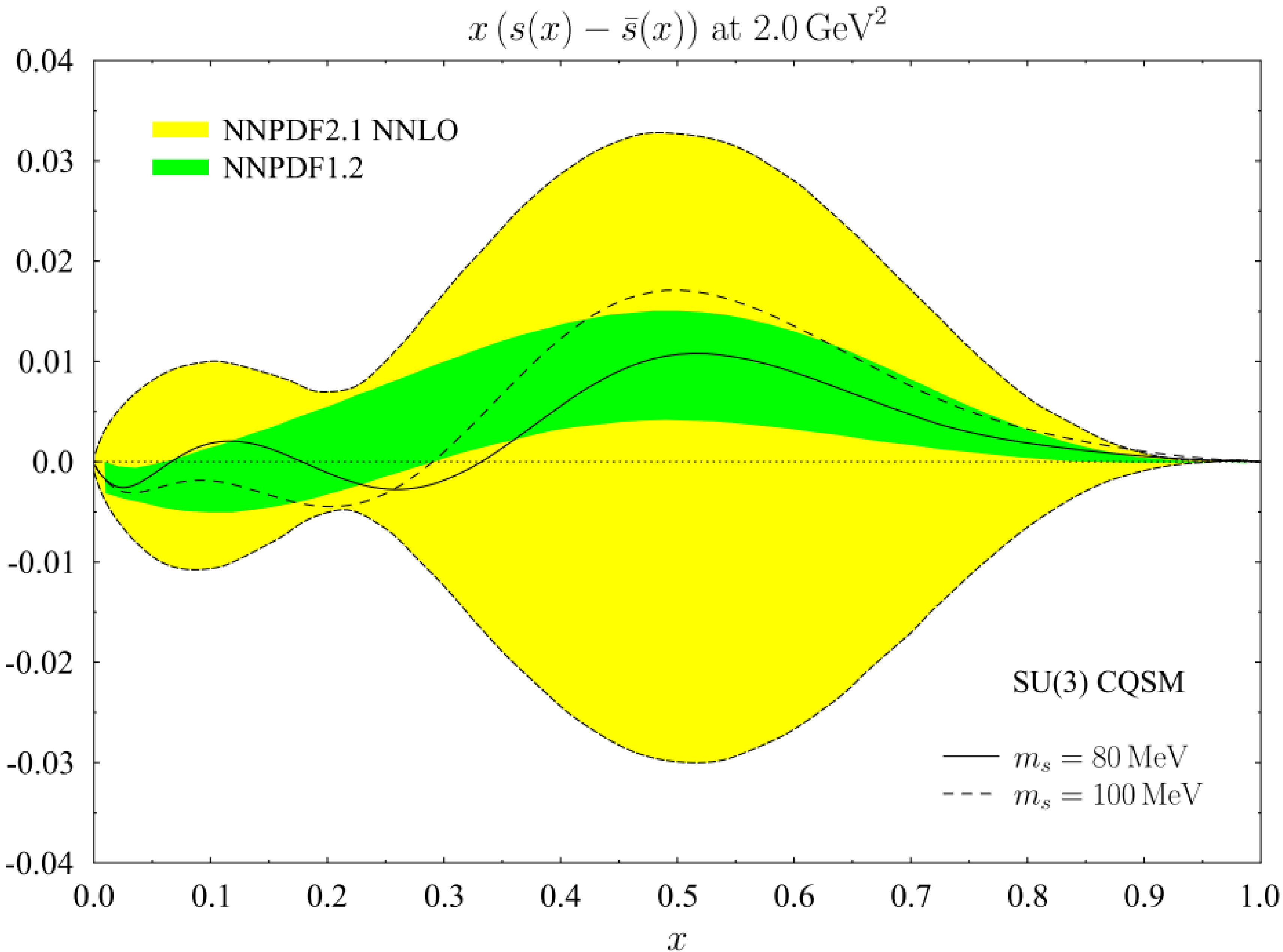
arXiv : 1107.2652 [hep-ph] , 13 July 2011.

The NNPDF Collaboration:
Richard D. Ball[1,5], Valerio Bertone[2], Francesco Cerutti[3], Luigi Del Debbio[1], Stefano Forte[4], Alberto Guffanti[2,5], José I. Latorre[3], Juan Rojo[4] and Maria Ubiali[6].



strange sea asymmetry

$$s^{-}(x) \equiv s(x) - \bar{s}(x)$$



$x\,(s(x) - \bar{s}(x))$ at 2.0 GeV^2
0.04
0.03
0.02
0.01
0.0
-0.01
-0.02
-0.03
-0.04
NNPDF2.1 NNLO
NNPDF1.2
SU(3) CQSM
$m_s = 80$ MeV
$m_s = 100$ MeV
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
x