

## IV. Phenomenology of nucleon spin decomposition

### Thomas' recent proposal toward the resolution of the nucleon spin puzzle

Recently, Thomas carried out an analysis of the proton spin contents in the context of the **refined cloudy bag (CB) model**, and concluded that the **modern spin discrepancy** can well be resolved in terms of the **standard features of the nonperturbative structure of the nucleon**, i.e.

- (1) **relativistic motion** of valence quarks
- (2) **pion cloud** required by chiral symmetry
- (3) **exchange current** contribution associated with the **OGE** hyperfine interactions

supplemented with **QCD scale evolution**.

(1) relativistic effect

$$\Delta\Sigma^Q \rightarrow 0.65 \times \Delta\Sigma^Q, \quad 2L^Q \rightarrow 0.35$$

- Lower  **$p$ -wave components** of relativistic wave functions -

## (2) pion cloud effects

physical nucleon = “bare nucleon” + pion cloud



3 valence quarks of nucleon core

bare nucleon probability  $Z \sim 1 - P_{N\pi} - P_{\Delta\pi} \sim 0.7$

$$P_{N\pi} \sim 0.20 - 0.25$$

$$P_{\Delta\pi} \sim 0.05 - 0.10$$

reduction of quark spin fraction  $\Delta\Sigma^Q$

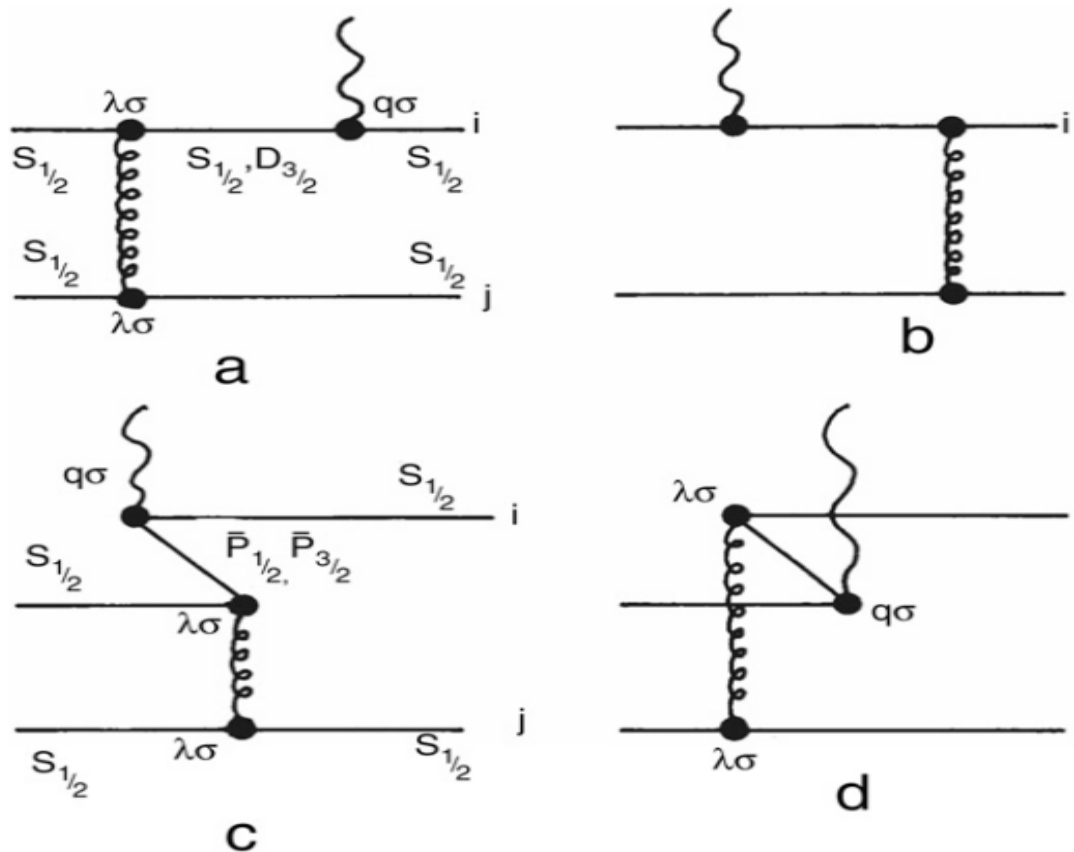
partial cancellation

$$\Delta\Sigma^Q \rightarrow \left( Z - \frac{1}{3} P_{N\pi} + \frac{5}{3} P_{\Delta\pi} \right) \Delta\Sigma^Q$$



main factor of reduction !

(3) one-gluon exchange correction (Myhrer and Thomas)



$$\Delta\Sigma^Q \rightarrow \Delta\Sigma^Q - 3G$$

$$G \propto \alpha_S \times (\text{certain bag model matrix elements})$$

## predictions of the refined CB model

- A. W. Thomas, Phys. Rev. Lett. 101 (2009) 102003.

	$2L^u$	$2L^d$	$\Delta\Sigma$
Non-relativistic	0	0	1.00
Relativistic	0.46	-0.11	0.65
+ OGE	0.52	-0.02	0.50
+ Pion Cloud	0.50	0.12	0.38

## characteristic features

$$\Delta\Sigma^Q \sim 0.4$$

$$L^u + L^d \sim 0.6, \quad L^u - L^d > 0$$

- CB model prediction corresponds to low energy model scale.  $\Leftrightarrow \sqrt{Q^2} \sim 0.4 \text{ GeV}$
- The quark OAM is a strongly scale-dependent quantity !

## Leading-order (LO) evolution equation for quark orbital angular momenta (OAM)

- X. Ji, J. Tang, and P. Hoodbhoy, Phys. Rev. Lett. 76 (1996) 740.

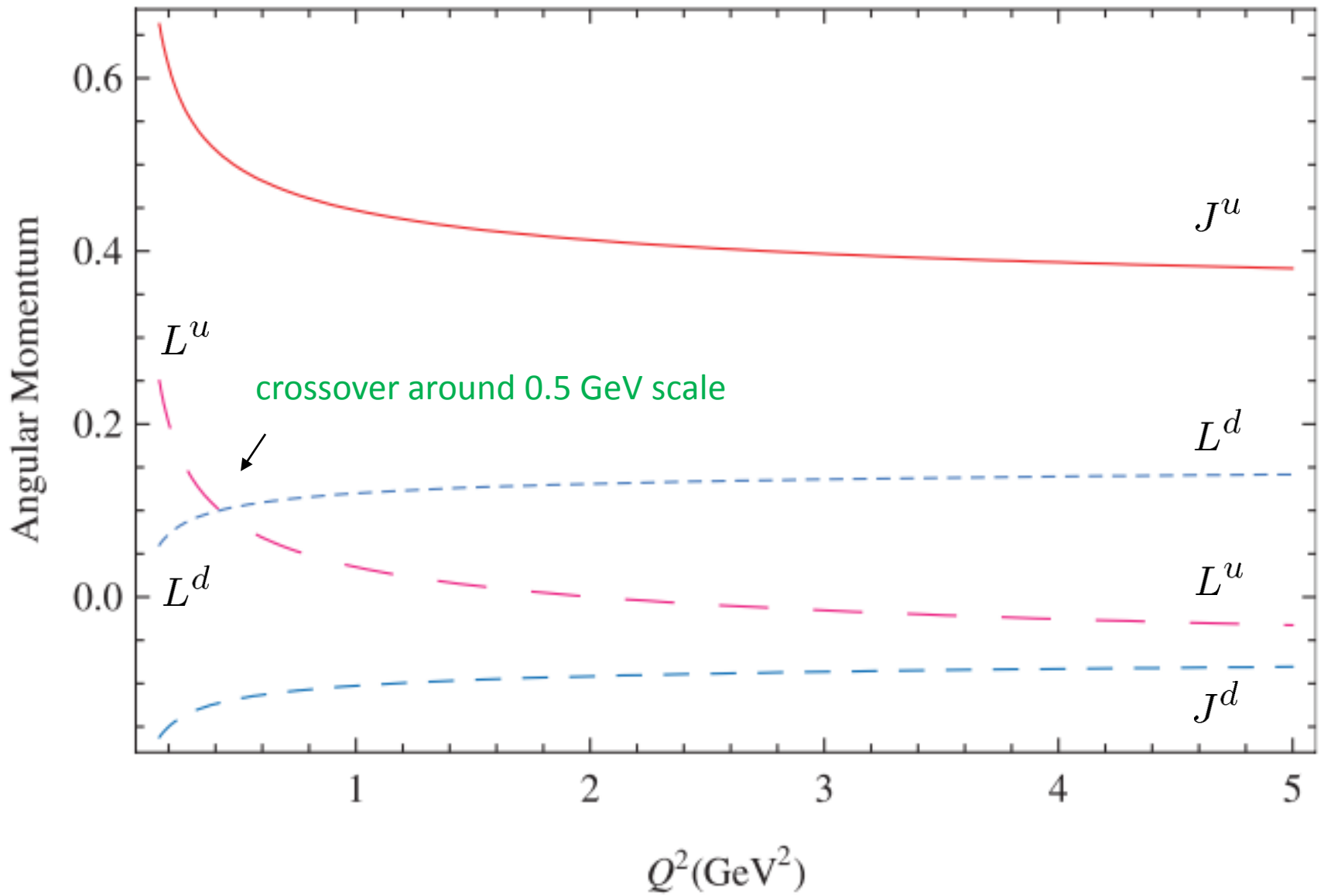
- flavor singlet channel

$$\begin{aligned} L^Q(t) &= -\frac{1}{2} \Delta \Sigma^Q + \frac{3n_f}{16 + 3n_f} \\ &+ \left( \frac{t}{t_0} \right)^{-2(16+3n_f)/9\beta_0} \left( L^Q(t_0) + \frac{1}{2} \Delta \Sigma^Q - \frac{1}{2} \frac{3n_f}{16 + 3n_f} \right) \\ L^g(t) &= -\Delta g(t) + \frac{16}{16 + 3n_f} \\ &+ \left( \frac{t}{t_0} \right)^{-2(16+3n_f)/9\beta_0} \left( L^g(t_0) + \Delta g(t_0) - \frac{1}{2} \frac{16}{16 + 3n_f} \right) \end{aligned}$$

- flavor non-singlet channel

$$L^{(NS)}(t) + \frac{1}{2} \Delta \Sigma^{(NS)} = \left( \frac{t}{t_0} \right)^{-32/9\beta_0} \left( L^{(NS)}(t_0) + \frac{1}{2} \Delta \Sigma^{(NS)} \right)$$

$$(NS) = u - d, \quad \text{or} \quad u + d - 2s$$



- A. W. Thomas, Phys. Rev. Lett. 101 (2008) 102003.

A remarkable feature is a **crossover** of  $L^u$  and  $L^d$  around 0.5 GeV scale !

This crossover is absolutely **necessary** for Thomas' scenario to hold, **because**

(1) Refined CBM prediction at low energy scale :

$$L^u - L^d > 0 \quad \text{at} \quad \sqrt{Q^2} \simeq 0.4 \text{ GeV}$$

(2) Asymptotic boundary condition dictated by the QCD evolution equation :

$$\lim_{Q^2 \rightarrow \infty} (L^u - L^d) = -g_A^{(3)}/2 < 0$$

See next page

Thomas then claims that, owing to this **crossover**, the predictions of the refined CB model **after taking account of QCD evolution** is qualitatively consistent with the recent lattice QCD data given at  $Q^2 = 4 \text{ GeV}^2$ , which gives

- $L^u(\text{LHPC}) = -0.196 \pm 0.044$ ,  $L^d(\text{LHPC}) = +0.200 \pm 0.044$
- $L^{u+d}(\text{LHPC}) \sim 0.06$  (**small**)  $\Leftrightarrow L^{u+d}(\text{CB model}) \sim 0.11$

**We shall show later that his statement is not necessarily justified !**

[Note] on the asymptotic boundary condition of  $L^u - L^d$

$$\lim_{Q^2 \rightarrow \infty} (L^u - L^d) = -g_A^{(3)} / 2$$

- M. W. and Y. Nakakoji, Phys. Rev. D77 (2008) 074011.

Leading-order evolution eq. for flavor nonsinglet channel

$$L^{u-d}(t) + \frac{1}{2} \Delta \Sigma^{u-d} = \left( \frac{t}{t_0} \right)^{-32/9\beta_0} \left( L^{u-d}(t_0) + \frac{1}{2} \Delta \Sigma^{u-d} \right)$$

with  $\beta_0 = 11 - 3n_f / 2$

Since right-hand-side becomes 0 as  $t \rightarrow \infty$ , we find that

$$\lim_{t \rightarrow \infty} L^{u-d}(t) = -\frac{1}{2} \Delta \Sigma^{u-d} = -\frac{1}{2} g_A^{(3)}$$

neutron beta-decay coupling constant !



## Our (nearly) model-independent analysis of proton spin

- M. W. , Eur. Phys. J. A44 (2010) 297 ; ibid. A46 (2010) 327.

based on

- M. W. and T. Kubota, Phys. Rev. D60 (1999) 034020.
- M. W., Phys. Rev. D67 (2003) 034005.
- M. W. and Y. Nakakoji, Phys. Rev. D74 (2006) 054006.
- M. W. and Y. Nakakoji, Phys. Rev. D77 (2008) 074011.

We try to carry out the analysis of the nucleon spin contents **as model-independently as possible** !

**[Starting point]** most general nucleon spin sum rule in QCD

$$J^Q + J^g = \frac{1}{2} \quad (Q \equiv u + d + s + \dots)$$

The point is that this **decomposition** can be made purely **experimentally** through the **GPD analyses** (X. Ji, 1997).

Ji's sum rule

$$J^Q = \frac{1}{2} \left[ \langle x \rangle^Q + B_{20}^Q(0) \right]$$

with

$$\langle x \rangle^Q = \int_0^1 x H^Q(x, \xi = 0, t = 0) dx \quad : \quad \text{quark momentum fraction}$$

$$B_{20}^Q(0) = \int_0^1 x E^Q(x, \xi = 0, t = 0) dx \quad : \quad \text{anomalous gravitomagnetic moment}$$

♣ Once  $J^Q$  is known,  $J^g$  is automatically known from  $J^g = 1/2 - J^Q$

For flavor decomposition, we also need non-singlet combination

$$J^{(NS)} = \frac{1}{2} \left[ \langle x \rangle^{(NS)} + B_{20}^{(NS)}(0) \right]$$

with

$$J^{(NS)} = J^{u-d}, \quad \text{or} \quad J^{u+d-2s}$$

## 1st key observation

♣ Ji showed that  $\langle x \rangle^q$  and  $J^q$  obey exactly the **same evolution equation** !

At the leading order (LO)

$$2 J^Q(Q^2) = \frac{3 n_f}{16 + 3 n_f} + \left( \frac{\alpha(Q^2)}{\alpha(Q_0^2)} \right)^{2(16+3 n_f)/9 \beta_0} \left( 2 J^Q(Q_0^2) - \frac{3 n_f}{16 + 3 n_f} \right)$$

with  $\beta_0 = 11 - \frac{2}{3} n_f$  and **similarly** for  $\langle x \rangle^Q$

$$2 J^{(NS)}(Q^2) = \left( \frac{\alpha(Q^2)}{\alpha(Q_0^2)} \right)^{32/9 \beta_0} 2 J^{(NS)}(Q_0^2)$$

and **similarly** for  $\langle x \rangle^{(NS)}$

♣ The **next key observation** now is that the quark and gluon **momentum fractions** are basically **known quantities** at least **above**  $Q^2 \simeq 1 \text{ GeV}^2$ , where the framework of **pQCD** can safely be applied !

♣ Neglecting small contribution of strange quarks, which is not essential for the present qualitative discussion, we are then left with **two unknowns** :

$$B_{20}^{u+d}(0) \quad \text{or} \quad B_{20}^{u-d}(0)$$

♣ Fortunately, the available predictions of lattice QCD for **these quantities** corresponds to the **renormalization scale**  $Q^2 = 4 \text{ GeV}^2$ , which is high enough for the framework of **pQCD** to work.

♣ An **interesting idea** is then to use the **QCD evolution equation** to estimate the nucleon spin contents **at lower energy scales** of nonperturbative QCD.

**inverse or downward evolution !**

- ♣ A natural question is how far down to the low energy scale we can trust the framework of **perturbative renormalization group equations**.
- ♣ Leaving this fundamental question aside, one may continue the downward evolution up to the scale  $\mu^2$ , where

$$\langle x \rangle^Q = 1, \quad \langle x \rangle^g = 0 \quad : \text{unitarity-violating limit}$$

- ♣ By starting with the MRST2004 values,  $\langle x \rangle^Q = 0.579, \langle x \rangle^g = 0.421$  at  $Q^2 = 4 \text{ GeV}^2$ , we find :

$$\begin{aligned} \mu^2 &\simeq 0.070 \text{ GeV}^2 && \text{with LO evolution eq.} \\ \mu^2 &\simeq 0.195 \text{ GeV}^2 && \text{with NLO evolution eq.} \end{aligned}$$

- ♣ As advocated by Mulders and Pollock, these scales may be regarded as a **matching scale** with **low energy effective quark models** without gluon degrees of freedom.
- ♣ Here, we take a little more **conservative viewpoint** that matching scale would be **somewhere** between  $\mu^2$  and  $1 \text{ GeV}^2$ .
- ♣ Nevertheless, one can say at the least that downward evolution **below** this **unitarity-violating limit**  $\mu^2$  is absolutely **meaningless**, since  $\langle x \rangle^g < 0$  there !

Now, we concentrate on getting reliable information on two unknowns :

$$B_{20}^{u+d}(0) \quad \text{and} \quad B_{20}^{u-d}(0)$$

(A) Isovector part  $B_{20}^{u-d}(0)$

- newest lattice QCD results given at  $Q^2 = 4 \text{ GeV}^2$

$$B_{20}^{u-d}(0) = 0.274 \pm 0.037 \quad \text{LHPC 2008}$$

$$B_{20}^{u-d}(0) = 0.269 \pm 0.020 \quad \text{QCDSF-UKQCD 2007}$$

close to each other !

- CQSM 2008 (M.W. and Y. Nakakoji)

$$B_{20}^{u-d}(0) \simeq 0.289$$

evolved from  $Q^2 = 0.30 \text{ GeV}^2$  to  $4.0 \text{ GeV}^2$  with NLO evolution eq.

To avoid starting energy dependence of CQSM estimate, here we simply use the central value of LHPC 2008 :

$$B_{20}^{u-d}(0) = 0.274 \quad \text{at} \quad Q^2 = 4 \text{ GeV}^2$$

(B) Isoscalar part  $B_{20}^{u+d}(0)$

Lattice QCD predictions are **sensitive** to the used method of  $\chi$ PT and **dispersed** !

$$B_{20}^{u+d}(0) = -0.094 \pm 0.050 \quad : \quad \text{LHPC2008 with covariant baryon } \chi\text{PT}$$

$$B_{20}^{u+d}(0) = +0.050 \pm 0.049 \quad : \quad \text{LHPC2008 with heavy baryon } \chi\text{PT}$$

$$B_{20}^{u+d}(0) = -0.120 \pm 0.023 \quad : \quad \text{QCDSF-UKQCD2007}$$

From the analysis of forward limit of unpolarized GPD  $E^{u+d}(x, \xi, t)$  within the CQSM, the 2nd moment of which gives  $B_{20}^{u+d}(0)$ , a reasonable theoretical bound for  $B_{20}^{u+d}(0)$  is obtained ( M.W. and Y. Nakakoji, 2008 )

$$0 \geq B_{20}^{u+d}(0) \geq -0.12 \quad ( = \kappa^{p+n} )$$

This works to **exclude** some range of lattice QCD predictions !

In the following analysis, we therefore regard  $B_{20}^{u+d}(0)$  as an **uncertain constant within the above bound**.

Once  $J^q$  ( $q = u, d, s$ ) is known, the (dynamical) quark OAM  $L^q$  is easily obtained by subtracting the known longitudinal quark polarization :

$$L^q(Q^2) = J^q(Q^2) - \frac{1}{2}\Delta\Sigma^q$$

$$\Delta\Sigma^q (q = u, d, s) \quad : \quad \text{scale indep. at LO}$$

$$\Delta\Sigma^{u-d} \equiv g_A^{(3)} = 1.269 \pm 0.003$$

$$\Delta\Sigma^{u+d-2s} \equiv g_A^{(8)} = 0.586 \pm 0.031$$

For  $\Delta\Sigma^{u+d+s}$ , we simply use here the central value of HERMES analysis :

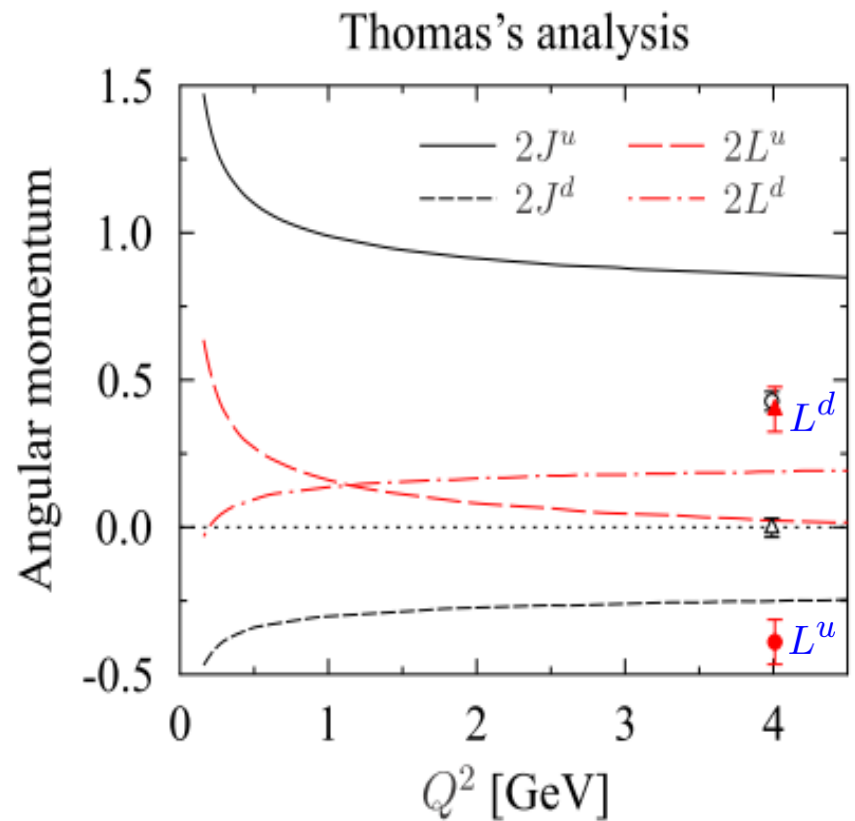
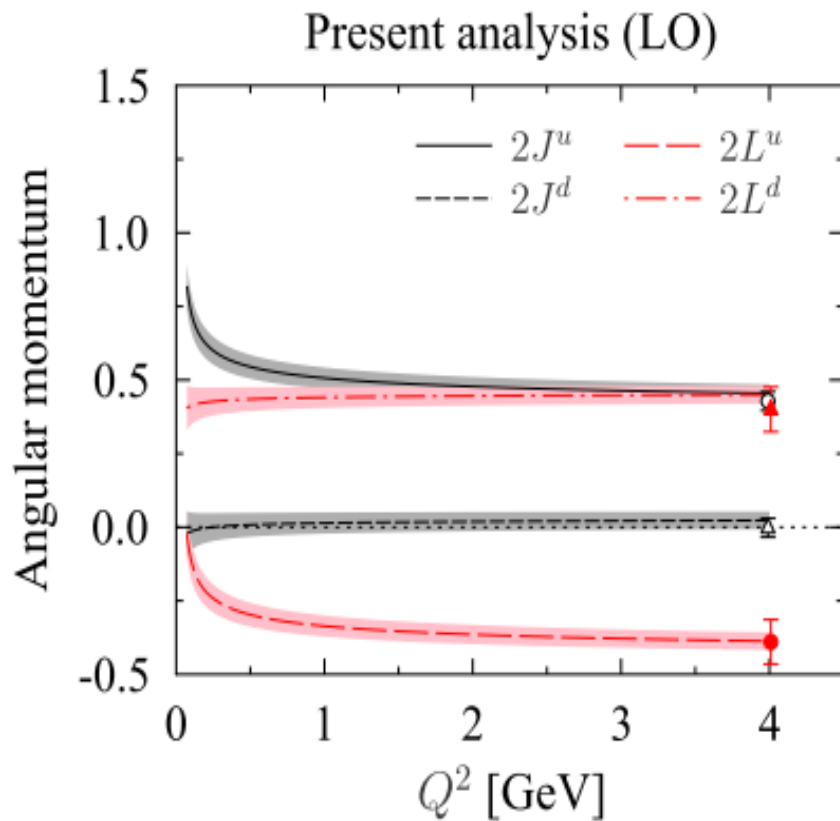
$$\Delta\Sigma^Q = 0.33$$

summary of complete initial conditions at  $Q^2 = 4 \text{ GeV}^2$

$$\begin{aligned} \langle x \rangle^Q &= 0.579, & \langle x \rangle^{u-d} &= 0.158, & \langle x \rangle^s &= 0.041 \\ B_{20}^{u-d}(0) &= 0.274, & -0.12 &\leq B_{20}^Q(0) = B_{20}^{u+d-2s}(0) \leq 0 \\ \Delta\Sigma^Q &= 0.33, & \Delta\Sigma^{u-d} &= 1.27, & \Delta\Sigma^{u+d-2s} &= 0.586 \end{aligned}$$



$J^u, J^d$ , and  $L^u, L^d$  as functions of  $Q^2$



Data at  $Q^2 = 4 \text{ GeV}^2$  are from LHPC2008.

- Ph. Haegler et al. (LHPC Collaboration), Phys. Rev. D77 (2008) 094502.

One sees that the **difference** between results of the two analyses is **quite large**, which also means that the **agreement** between Thomas' results and the lattice QCD predictions is not so good as he claimed .

♣ The most significant difference appears in the **quark OAM** !

♣ Thomas' analysis shows a **crossover** of  $L^u$  and  $L^d$  around 0.5 GeV scale.

It comes from the fact that

- refined CB model predicts  $L^u - L^d > 0$  at low energy model scale.
- QCD evolution dictates that  $\lim_{Q^2 \rightarrow \infty} (L^u - L^d) < 0$  .

♣ In contrast, **no crossover** of  $L^u$  and  $L^d$  is observed in our analysis.

$L^d$  remains to be larger than  $L^u$  even down to the **unitarity-violating limit**.

- ♣ One might suspect that the **uncertainties of the initial conditions** given at  $Q^2 = 4 \text{ GeV}^2$  might alter this remarkable conclusion.

It is clear by now, however, that the problem exists for the **isovector quark OAM**, for which the **uncertainties are fairly small**. In fact, in the r.h.s. of the relation

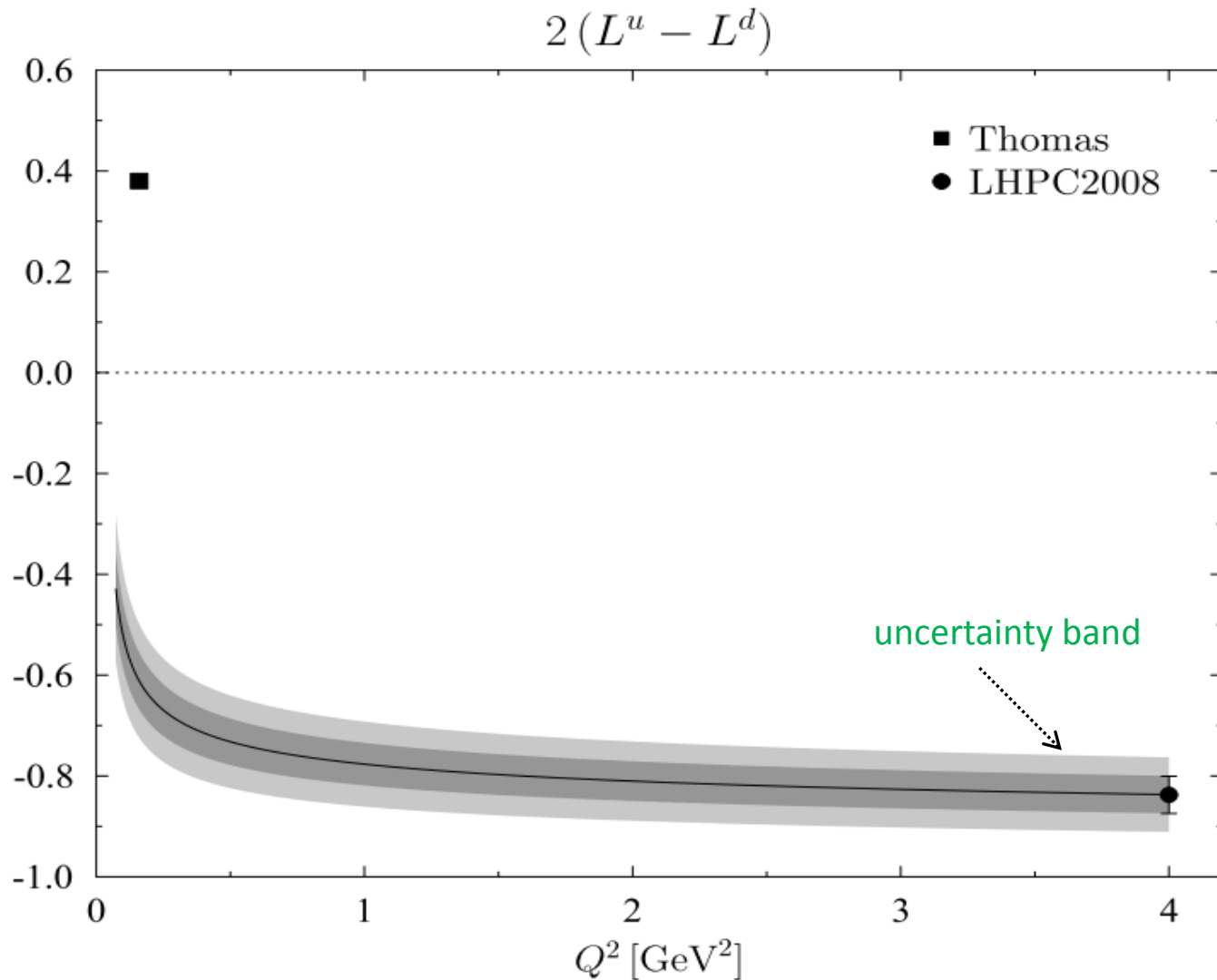
$$L^{u-d} = \frac{1}{2} \left[ \langle x \rangle^{u-d} + B_{20}^{u-d}(0) \right] - \frac{1}{2} \Delta \Sigma^{u-d}$$

uncertainties

$$\begin{aligned} \Delta \Sigma^{u-d} &= 1.2695 \pm 0.0029 && (\sim 0.29\%) \\ \langle x \rangle^{u-d} &= 0.158 && (< 1\%) \end{aligned}$$

Main uncertainty comes from **the isovector anomalous gravito-magnetic moment !**

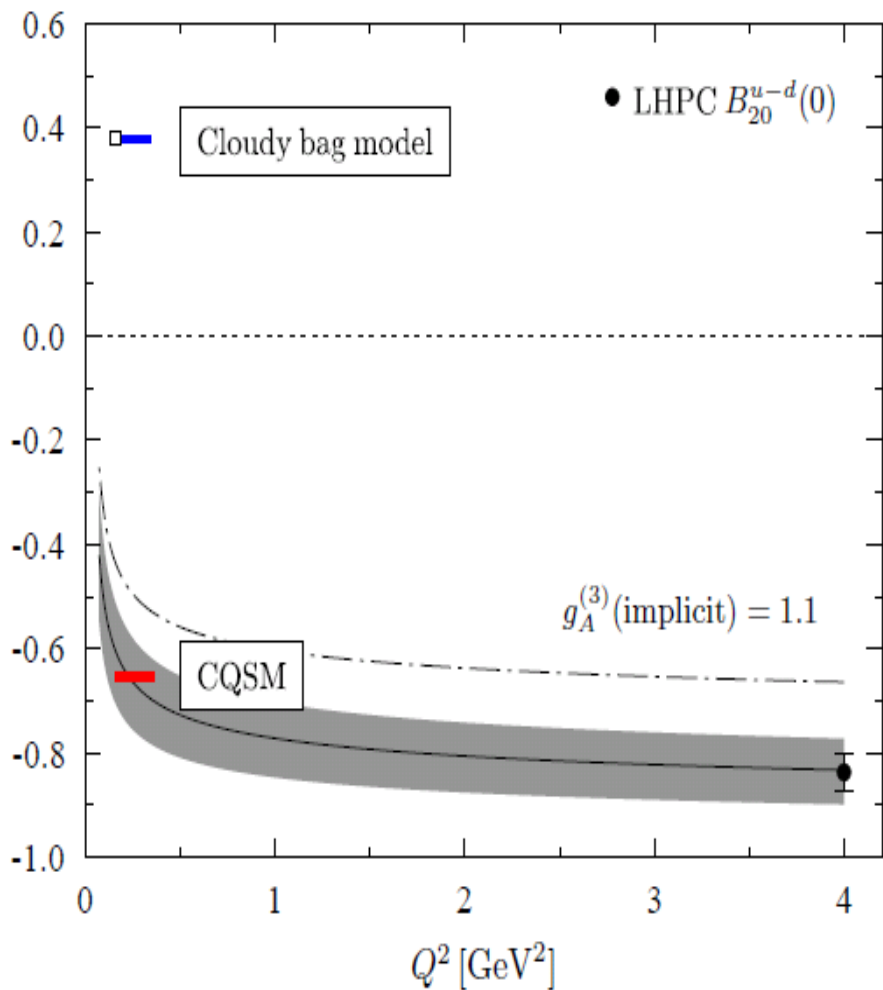
$$\begin{aligned} B_{20}^{u-d}(0) &= 0.274 \pm 0.037 \quad (13.5\%) && : \text{ LHPC2008} \\ &= 0.269 \pm 0.020 \quad (7.4\%) && : \text{ QCDSF-UKQCD2009} \\ &= 0.289 && : \text{ CQSM} \end{aligned}$$



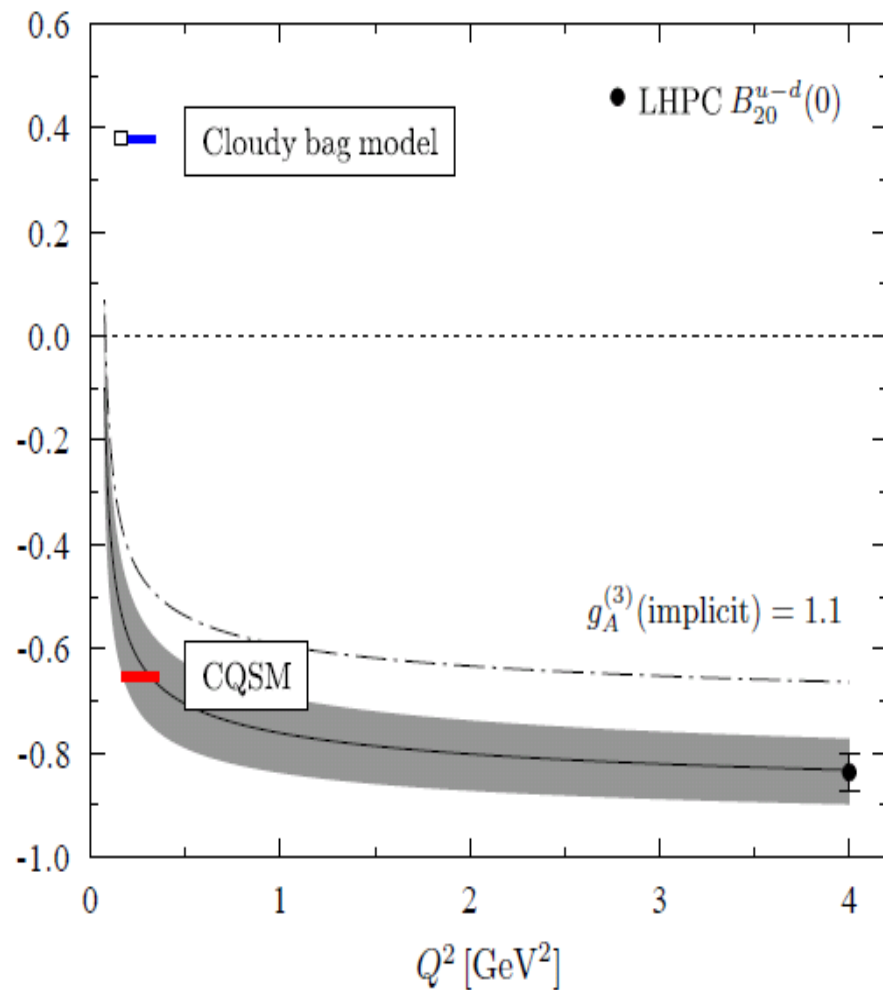
$L^u - L^d$  remains **negative** even down to the lower energy scale close to the unitarity-violating bound !

# sensitivity to the magnitude of QCD coupling constant

(a)  $2(L^u - L^d)$  at LO with  $\alpha_S(Q^2 = 4 \text{ GeV}^2) = 0.284$



(b)  $2(L^u - L^d)$  at LO with  $\alpha_S(Q^2 = 4 \text{ GeV}^2) = 0.320$



Also interesting would be a [direct comparison with the empirical information](#) on  $J^u$  and  $J^d$  extracted from the recent [GPD analyses](#).

See figure in the next page.

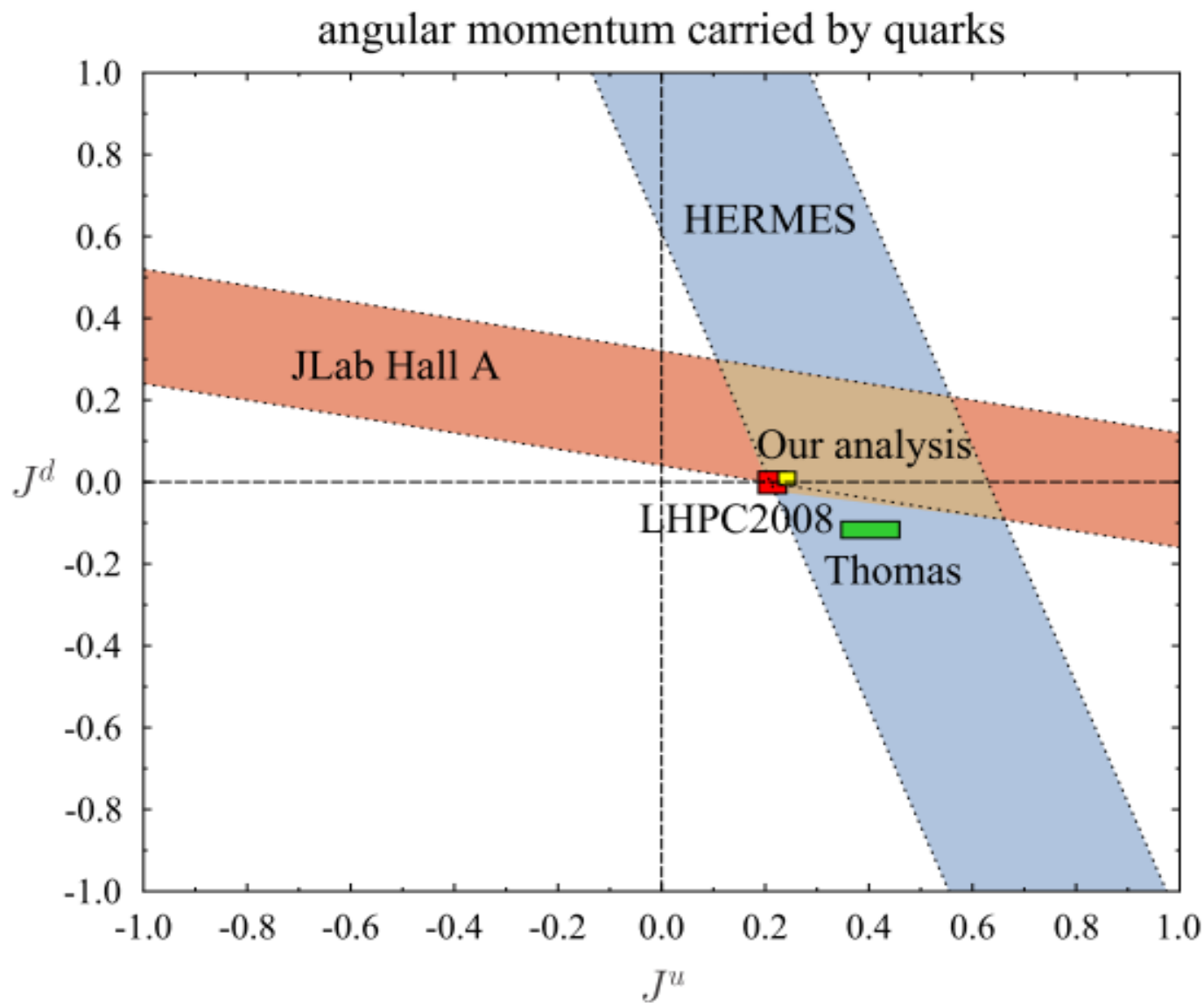
One sees that, by construction, the result of our semi-phenomenological analysis is fairly close to that of the lattice QCD simulations.

On the other hand, the result of Thomas' analysis significantly deviates from the other two, and outside the [error-band of JLab data](#).

Typical features of Thomas' predictions

- $J^u$  is [sizably larger](#) than the other two.
- $J^d$  is a little smaller than the other two.

Comparison with GPD extraction of  $J^u$ ,  $J^d$



Anyhow, our semi-phenomenological analysis, which is consistent with empirical information as well as the lattice QCD data at high energy scale **indicates** that

$L^u - L^d$  remains large and negative

even at low energy scale of nonperturbative QCD !

If this is really confirmed, it is a serious challenge to any low energy models of nucleon, because they must now explain

- small  $\Delta\Sigma^Q$
- large and negative  $L^{u-d}$  →

nonstandard !

simultaneously !

We might call it

**new or another proton spin puzzle**

in the sense that it is totally incompatible with the picture of the standard quark model, including the refined CB model of Thomas and Myhrer.



Is there **any low energy model** which can explain this **peculiar feature** ?

Interestingly or strangely , the CQSM can !

It has been long claimed that it can naturally explain very small quark spin fraction :

$$\Delta\Sigma^Q \simeq 0.35 \quad \text{at the model scale}$$

because of the **very nature of the model** ( i.e. the nucleon as a **rotating hedgehog** )

Very interestingly, its prediction for  $L^u - L^d$  given in

- M. W. and H. Tsujimoto, Phys. Rev. D71 (2005) 074001.

$$L^{u-d} \simeq -0.33 \quad \text{at the model scale}$$

**perfectly matches** the **scenario** emerged from the present **semi-empirical analysis** !

**But why ?**

The problem may have deep connection with the **definition of quark OAM** !

We have already pointed out that there are **2 kinds of quark OAMs**.

(1) **“dynamical” quark OAM** = Ji’s quark OAM

(2) **“canonical” quark OAM** = Jaffe-Manohar’s quark OAM  
(or its nontrivial gauge-invariant extension)

Remember the fact that the quark OAM defined through GPDs is the 1st one, i.e. the **“dynamical” quark OAM** .

It has been long recognized that the **quark OAM in the Ji decomposition** is manifestly **gauge invariant**, so that it contains **interaction term with the gluon**.

Since the **CQSM** is an **effective quark theory** that contains **no gauge field**, one might naively expect that there is no such ambiguity problem in the definition of the quark OAM.

However, it turns out that this is not necessarily the case. The point is that it is a **highly nontrivial interaction theory of quark fields**.

To explain it, we recall the past analyses of **GPD sum rules** within the **CQSM**.

## CQSM analyses of GPD sum rules :

- **Isoscalar channel** : J. Ossmann et al., Phys. Rev. D71,034001 (2005).
- **Isovector channel** : M. W. and H. Tsujimoto, Phys. Rev. D71,074001 (2005).

**Isoscalar case** : 2<sup>nd</sup> moment of  $E_M^{u+d}(x, 0, 0) \equiv H^{u+d}(x, 0, 0) + E^{u+d}(x, 0, 0)$

$$\frac{1}{2} \int_{-1}^1 x E_M^{u+d}(x, 0, 0) dx = L_f^{u+d} + \frac{1}{2} \Delta \Sigma^{u+d}$$

where

$$L_f^{u+d} \equiv \langle p \uparrow | \hat{L}_f^{u+d} | p \uparrow \rangle, \quad \Delta \Sigma^{u+d} \equiv \langle p \uparrow | \Delta \hat{\Sigma}^{u+d} | p \uparrow \rangle$$

with

$$\hat{L}_f^{u+d} = \int \psi^\dagger(x) [\mathbf{x} \times (-i \nabla)]_3 \psi(x) d^3x,$$

: free-field quark OAM operator

= canonical OAM op.

$$\Delta \hat{\Sigma}^{u+d} = \int \psi^\dagger(x) \Sigma_3 \psi(x) d^3x$$

Isovector case : 2<sup>nd</sup> moment of  $E_M^{u-d}(x, 0, 0) \equiv H^{u-d}(x, 0, 0) + E^{u-d}(x, 0, 0)$

$$\int_0^1 x E_M^{u-d}(x, 0, 0) dx = \frac{1}{2} \Delta \Sigma^{(I=1)} + \left( L_f^{(I=1)} + \delta L^{(I=1)} \right)$$

with

$$\Delta \Sigma^{(I=1)} = \langle p \uparrow | \int d^3x \psi^\dagger(\mathbf{x}) \tau_3 \Sigma_3 \psi(\mathbf{x}) | p \uparrow \rangle$$

$$L_f^{(I=1)} = \langle p \uparrow | \int d^3x \psi^\dagger(\mathbf{x}) \tau_3 (\mathbf{x} \times \mathbf{p})_3 \psi(\mathbf{x}) | p \uparrow \rangle$$

$$\delta L^{(I=1)} = -M \frac{N_c}{18} \sum_{n \in \text{occ}} \langle n | r \sin F(r) \gamma^0 [ \Sigma \cdot \hat{\mathbf{r}} \boldsymbol{\tau} \cdot \hat{\mathbf{r}} - \Sigma \cdot \boldsymbol{\tau} ] | n \rangle$$

	valence	sea	total
$L_f^{(I=1)}$	0.147	- 0.265	- 0.118
$\delta L^{(I=1)}$	- 0.289	0.077	- 0.212
$L_f^{(I=1)} + \delta L^{(I=1)}$	- 0.142	- 0.188	- 0.33

## Concluding remarks

- ♣ We have estimated the orbital angular momentum of up and down quarks in the proton as functions of the energy scale, by carrying out a downward evolution of available information at high energy, to find that  $L^u - L^d$  remains to be large and negative even at low energy scale of nonperturbative QCD ! We emphasized that, if it is really confirmed, it may be called

another nucleon spin puzzle ?

because it absolutely contradicts the picture of standard quark model !



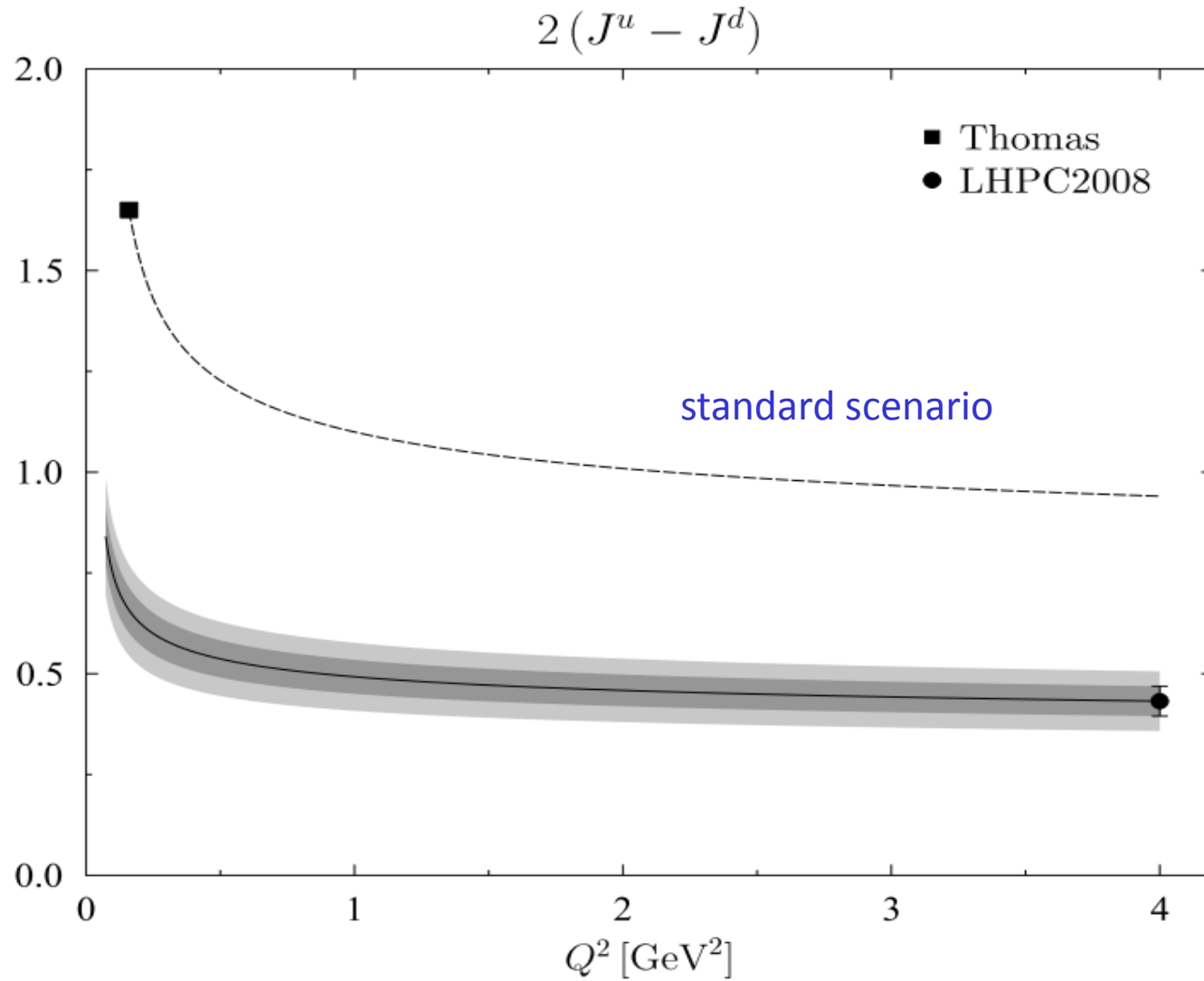
- ♣ Does the strong scale dependence of  $L^u - L^d$  rescue this puzzle, as Thomas claims ?

or

- ♣ Is it an indication of a big difference between

“dynamical” & “canonical” quark OAM ?

- ♣ A key is a precise measurement of  $2(J^u - J^d)$  at a few GeV scale.



[Appendix] On the **twist**  $t_O$  of an operator  $O$

$$t_O \equiv d_O - n$$

where

$d_O$  : canonical dimension of  $O$

$n$  : **spin** of  $O$

The lowest twist in operator tower in QCD is  $t_O = 2$ .

From dimensional analysis of hadron tensor  $W_{\mu\nu}$ , higher twist operators with  $t_O$  is **power suppressed** at large  $Q^2$  as

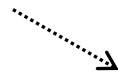
$$\left(M / \sqrt{Q^2}\right)^{t_O - 2}$$

[Some examples]

$$\bar{\psi} \gamma^\mu \psi : t_O = \left(\frac{3}{2} + \frac{3}{2}\right) - 1 = 2$$

$$\bar{\psi} \psi : t_O = \left(\frac{3}{2} + \frac{3}{2}\right) - 0 = 3$$

$$D^\mu : t_O = 1 - 1 = 0$$



derivative does not change twist !

[Appendix] On the **chiral symmetry** of **massless Dirac fields**

$$\mathcal{L}_{Dirac} = \bar{\psi} i \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi$$

Decomposition into **right- and left-handed components**

$$\psi = \psi_R + \psi_L \quad \text{with} \quad \psi_{R/L} \equiv \frac{1 \pm \gamma_5}{2} \psi$$

Under the **chiral transformation**

$$\begin{aligned} \psi_R &\rightarrow g_R \psi_R = e^{i\theta_R} \psi_R \\ \psi_L &\rightarrow g_L \psi_L = e^{i\theta_L} \psi_L \end{aligned}$$

the **kinetic term** is invariant, but the **mass term** breaks this invariance !

$$\begin{aligned} \bar{\psi} i \gamma^\mu \partial_\mu \psi &= (\bar{\psi}_R + \bar{\psi}_L) i \gamma^\mu \partial_\mu (\psi_R + \psi_L) \\ &= \bar{\psi}_R i \gamma^\mu \partial_\mu \psi_R + \bar{\psi}_L i \gamma^\mu \partial_\mu \psi_L \\ &\rightarrow \bar{\psi}_R e^{-i\theta_R} i \gamma^\mu \partial_\mu e^{i\theta_R} \psi_R + \bar{\psi}_L e^{-i\theta_L} i \gamma^\mu \partial_\mu e^{i\theta_L} \psi_L \\ &= \bar{\psi} i \gamma^\mu \partial_\mu \psi \quad : \text{invariant} \end{aligned}$$

$$\begin{aligned} \bar{\psi} \psi &= \bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R \\ &\rightarrow \bar{\psi}_R e^{-\theta_R} e^{i\theta_L} \psi_L + \bar{\psi}_L e^{-i\theta_L} e^{i\theta_R} \psi_R \\ &\neq \bar{\psi} \psi \quad : \text{not invariant} \end{aligned}$$