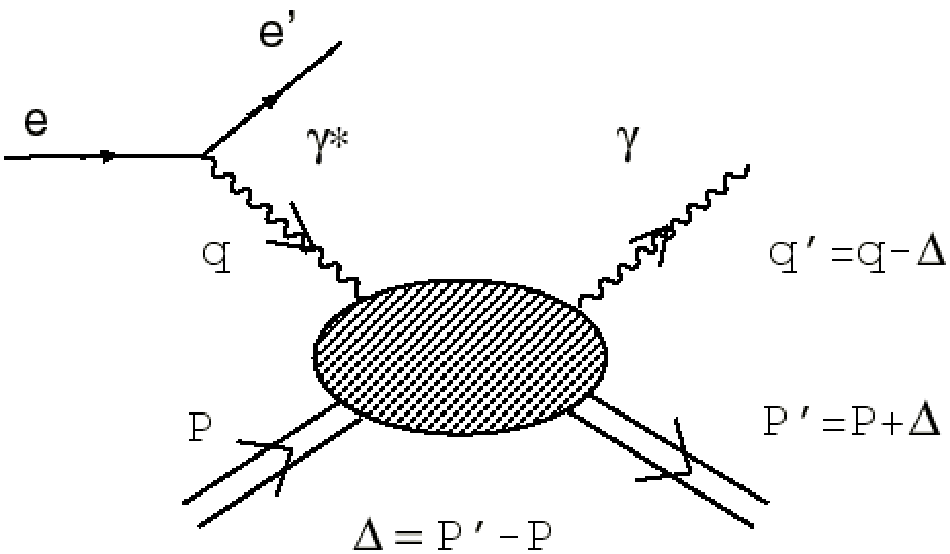
III. **Generalized parton distributions and nucleon spin decomposition**

deeply-virtual Compton scatterings (DVCS)

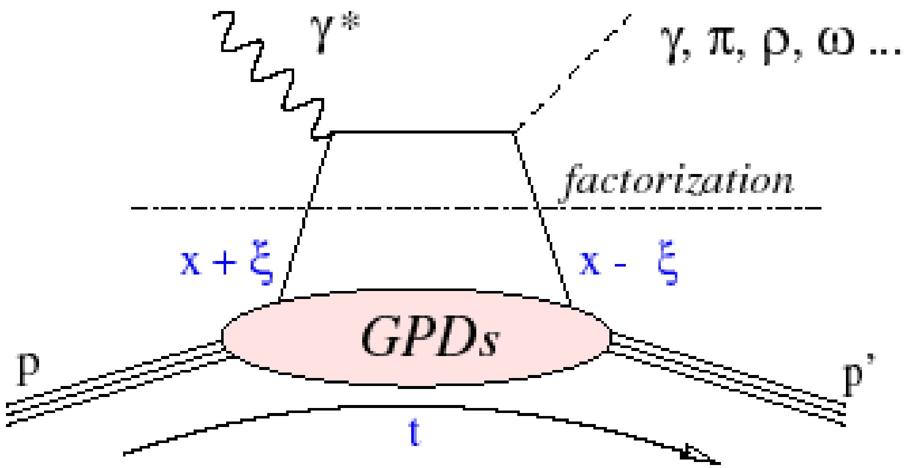


kinematics of deep-inelastic scatterings

$$Q^2 \equiv -q^2 \longrightarrow \infty, \quad P \cdot q \longrightarrow \infty, \quad \frac{Q^2}{P \cdot q} = \text{finite}$$

DIS processes ⇒ **tractable within the framework of pQCD**

DVCS amplitude dominant in the Bjorken limit



Handbag Diagram

soft part is parametrized by 4 generalized parton distributions (GPDs)

$$H(x,\xi,t),\ \ E(x,\xi,t)\quad \&\quad \tilde{H}(x,\xi,t),\ \ \tilde{E}(x,\xi,t)$$

depending on 3 kinematical variables

$t = (P' - P)^2$: nucleon 4-momentum transfer square

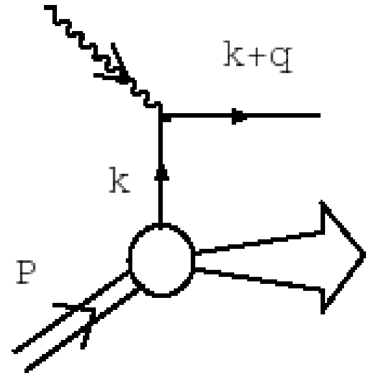
x : Broken variable

ξ : Skewdness parameter

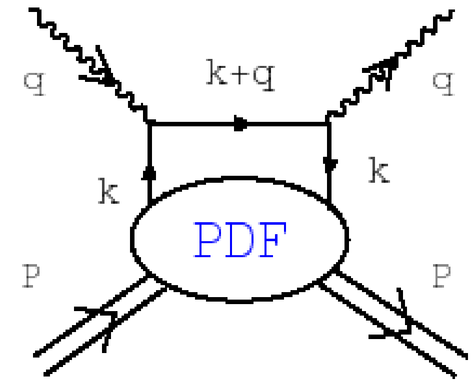
[Reminder] relation between inclusive scattering & DVCS

forward Compton amplitude

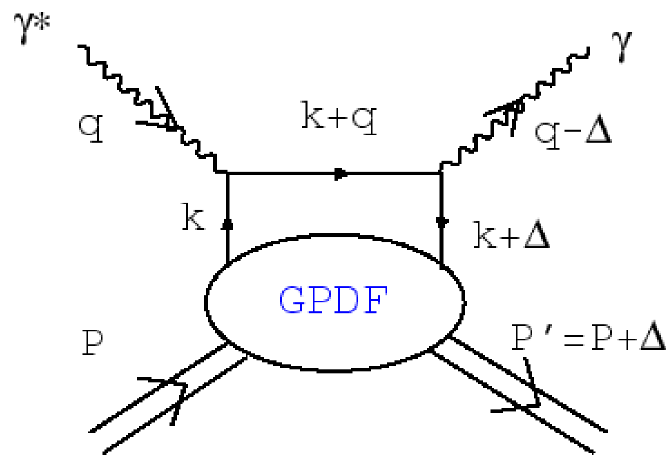
$$d\sigma_{inclusive} \propto \sum_f \left| \right.$$



$$\left. \right|^2 \propto \mathrm{Im}$$



$$d\sigma_{DVCS} \propto \left| \right.$$



$$\left. \right|^2$$

compare !

relation between PDF & GPDF (from field theoretical viewpoint)

usual PDF

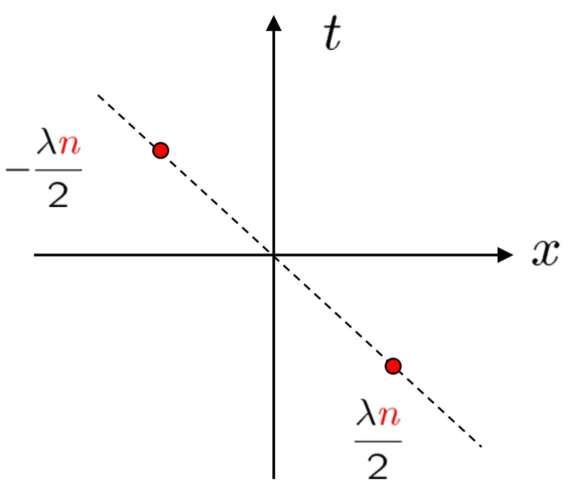
$$q(x) = \langle P|O(x)|P\rangle, \qquad |P\rangle : \text{ nucleon state with momentum } P$$

light-cone operator

$$O(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x}\, \bar{\psi}\left(-\frac{\lambda n}{2}\right) \not{n}\, \mathcal{L}_g\, \psi\left(\frac{\lambda n}{2}\right)$$

$\mathcal{L}_g$: gauge link

$n \propto (1,0,0,-1)$: light-like vector



PDF

⇕

quark-quark light-cone correlation in the nucleon

light-cone operator $O(x)$ is equivalent to the following tower of twist-2 local operators

$$O^{\mu_1} = \bar{\psi}\,\gamma^{\mu}\,\psi$$

$$O^{\{\mu_1,\mu_2\}} = \bar{\psi}\,\gamma^{\{\mu_1}\,iD^{\mu_2\}}\psi$$

$$O^{\{\mu_1,\mu_2,\mu_3\}} = \bar{\psi}\,\gamma^{\{\mu_1}\,iD^{\mu_2} iD^{\mu_3\}}\psi$$

$$\cdots\cdots\cdots$$

→ e.m. current carried by quarks

→ quark part of QCD energy-momentum tensor

→ higher rank tensors

nucleon forward matrix element

$$\langle P | O^{\{\mu_1,\cdots,\mu_n\}} | P \rangle = \boxed{a_n} \cdot 2\,P^{\mu_1}\cdots P^{\mu_n} \qquad (n = 1, 2, \cdots)$$

not calculable in pQCD !

define **quark distribution function** $q(x)$

$$\int_{-1}^{1} dx\, x^{n-1}\, q(x) \equiv a_n, \quad \text{or} \quad q(x) = \frac{-1}{2\,\pi\,i} \int_{c-i\,\infty}^{c+i\,\infty} x^{-n}\, a_n\, dn$$

n-th moment of PDF

nucleon non-forward matrix element of the same twist-2 operators defines GPDs

(Here, we limit to the case $\xi = 0$, for simplicity.)

$$\begin{aligned}
\langle P' | O^{\mu_1} | P \rangle &= A_{10}(t) \cdot \bar{U}(P') \gamma^{\mu_1} U(P) \\
&+ B_{10}(t) \cdot \frac{i}{2M} \bar{U}(P') \sigma^{\mu_1 \alpha} \Delta_\alpha U(P)
\end{aligned}$$

$$\begin{aligned}
\langle P' | O^{\{\mu_1, \mu_2\}} | P \rangle &= A_{20}(t) \cdot \bar{P}^{\{\mu_1} \bar{U}(P') \gamma^{\mu_2\}} U(P) \\
&+ B_{20}(t) \cdot \frac{i}{2M} \bar{P}^{\{\mu_1} \bar{U}(P') \sigma^{\mu_2\}\alpha} \Delta_\alpha U(P) \\
&+ C_{20}(t) \cdot \frac{1}{M} \Delta^{\{\mu_1} \Delta^{\mu_2\}}
\end{aligned}$$

....

with

$$\Delta = P' - P, \qquad \bar{P} = \frac{1}{2}(P' + P)$$

In particular

$$A_{10}(t) = F_1(t) \quad : \quad \text{Dirac F.F.}$$
$$B_{10}(t) = F_2(t) \quad : \quad \text{Pauli F.F.}$$

For this reason

$$A_{n0}(t),\; B_{n0}(t),\; C_{n0}(t) \qquad (n = 2, 3, \cdots)$$

are called the **generalized form factors**.

GPDs (with $\xi = 0$) are defined by the following equation.

$$\int x^{n-1}\, H(x, 0, t)\, dx = A_{n0}(t)$$
$$\int x^{n-1}\, E(x, 0, t)\, dx = B_{n0}(t)$$

Clearly

$$A_{n0}(t = 0) = a_n$$

The forward limit of $H(x, \xi, t)$ is then reduced to the usual unpolarized PDF $q(x)$.

$$H(x, 0, 0) = q(x)$$

Ji's nucleon spin sum rule

define angular momentum operator in terms of QCD energy-momentum tensor

$$J^i = \frac{1}{2}\varepsilon^{ijk}\int d^3x\, M^{0jk}$$

where

$$M^{\alpha\mu\nu} = T^{\alpha\nu}\, x^\mu - T^{\alpha\mu}\, x^\nu$$

key observation

$$T_q^{\mu\nu} = \bar{\psi}\,\gamma^{(\mu}\, i\, D^{\mu)}\,\psi \;=\; O^{\{\mu,\nu\}}$$

The quark part of QCD energy-momentum tensor $T_q^{\mu\nu}$ is nothing but the twist-2 operator $O^{\{\mu,\nu\}}$ appearing in the definitions of PDFs & GPDFs.

Based on this fact, Ji showed that

$$J_q \;=\; \langle P\uparrow|\int d^3x\,(\boldsymbol{x}\times\boldsymbol{T}_q)^z\,|P\uparrow\rangle \;=\; \frac{1}{2}[A_{20}(0)+B_{20}(0)]$$

$$=\; \frac{1}{2}\int_0^1 x\,[\,H^q(x,0,0)\;+\;E^q(x,0,0)\,]\,dx$$

and similarly for the gluon part.

$$J_g \;=\; \frac{1}{2}\int_0^1 x\,[\,H^g(x,0,0)\;+\;E^g(x,0,0)\,]\,dx$$

Ji's nucleon spin sum rule

$$J^Q + J^G = \frac{1}{2} \qquad (Q = u + d + s + \cdots)$$

with

$$J^Q = \frac{1}{2} \int x \left\{ H^Q(x,0,0) + E^Q(x,0,0) \right\} dx$$
$$J^G = \frac{1}{2} \int x \left\{ H^G(x,0,0) + E^G(x,0,0) \right\} dx$$

A natural next question is whether we can further decompose the **total angular momenta** of quarks and gluons into their intrinsic spin and orbital angular momenta ?

$$J^Q \stackrel{?}{=} \frac{1}{2} \Delta\Sigma^Q + L^Q$$
$$J^G \stackrel{?}{=} \Delta G + L^G$$

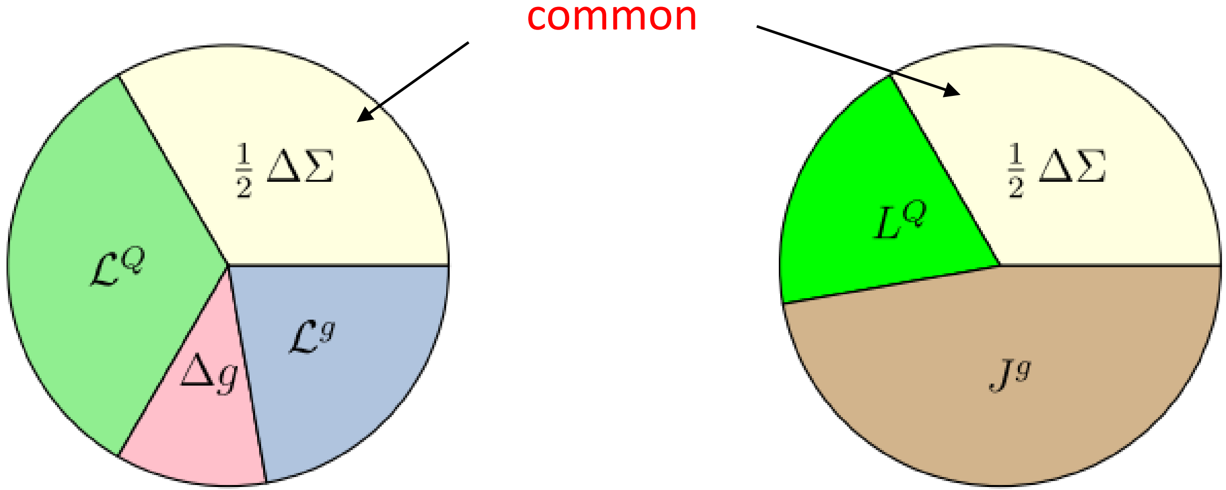
⇓

This is a highly nontrivial question, which causes a lot of controversies !

nucleon spin decomposition problem

Two popular decompositions of the nucleon spin in the market

Jaffe-Manohar | Ji



Jaffe-Manohar:

$$\begin{aligned}\boldsymbol{J}_{QCD} &= \int \psi^\dagger \frac{1}{2}\boldsymbol{\Sigma}\,\psi\, d^3x \\ &+ \int \psi^\dagger\, \boldsymbol{x} \times \frac{1}{i}\nabla\, \psi\, d^3x \\ &+ \int \boldsymbol{E}^a \times \boldsymbol{A}^a\, d^3x \\ &+ \int E^{ai}\, \boldsymbol{x} \times \nabla\, A^{ai}\, d^3x\end{aligned}$$

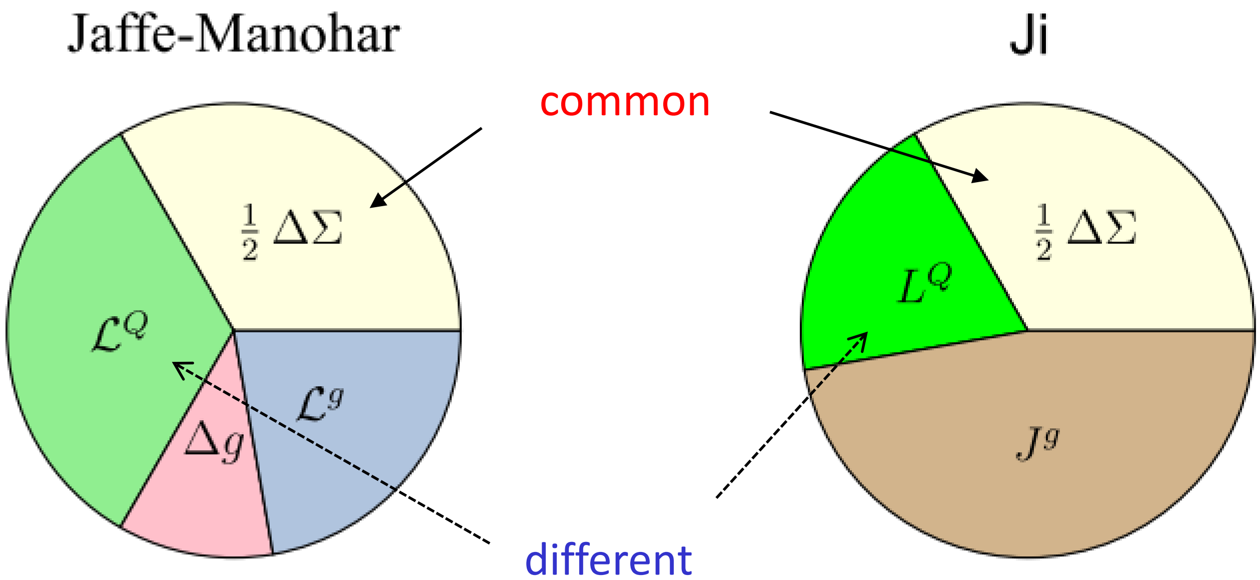
Ji:

$$\begin{aligned}\boldsymbol{J}_{QCD} &= \int \psi^\dagger \frac{1}{2}\boldsymbol{\Sigma}\,\psi\, d^3x \\ &+ \int \psi^\dagger\, \boldsymbol{x} \times \frac{1}{i}\boldsymbol{D}\, \psi\, d^3x \\ &+ \int \boldsymbol{x} \times (\boldsymbol{E}^a \times \boldsymbol{B}^a)\, d^3x\end{aligned}$$

Each term is not separately gauge-invariant !

No further decomposition of J^g !

Two popular decompositions of the nucleon spin (continued)



An especially important observation is that, since

$$\mathcal{L}^Q \neq L^Q$$

one must conclude that

$$\Delta g + \mathcal{L}^g \neq J^g$$

New gauge-invariant decomposition by Chen et al.

X.-S. Chen et al., Phys. Rev. Lett. 103, 062001 (2009) ; 100, 232002 (2008).

The basic idea

$$A^\mu \;=\; A^\mu_{phys} \;+\; A^\mu_{pure}$$

with

$$F^{\mu\nu}_{pure} \;\equiv\; \partial^\mu A^\nu_{pure} - \partial^\nu A^\mu_{pure} - i\,g\,[A^\mu_{pure}, A^\nu_{pure}] \;=\; 0$$

and

$$A^\mu_{phys}(x) \;\rightarrow\; U(x)\,A^\mu_{phys}(x)\,U^{-1}(x)$$
$$A^\mu_{pure}(x) \;\rightarrow\; U(x)\left(A^\mu_{pure}(x) - \frac{i}{g}\,\partial^\mu\right)U^{-1}(x)$$

Answer

$$\begin{aligned} \boldsymbol{J}_{QCD} \;&=\; \int \psi^\dagger \frac{1}{2}\boldsymbol{\Sigma}\,\psi\,d^3x \;+\; \int \psi^\dagger\,\boldsymbol{x}\times(\boldsymbol{p} - g\,\boldsymbol{A}_{pure})\,\psi\,d^3x \\ &+\; \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{phys}\,d^3x \;+\; \int E^{aj}\,(\boldsymbol{x}\times\nabla)\,\boldsymbol{A}^{aj}_{phys}\,d^3x \\ &=\; \boldsymbol{S}'^q \;+\; \boldsymbol{L}'^q \;+\; \boldsymbol{S}'^g \;+\; \boldsymbol{L}'^g \end{aligned}$$

- Each term is separately gauge-invariant !
- It reduces to the gauge-variant **Jaffe-Manohar decomposition** in a special gauge !

$$\boldsymbol{A}_{pure} \;=\; 0, \qquad \boldsymbol{A} \;=\; \boldsymbol{A}_{phys}$$

Chen et al. also advocated the following decomposition of linear momentum

$$\boldsymbol{P}_{QCD} \;=\; \int \psi^\dagger \frac{1}{i} \boldsymbol{D}_{pure} \psi \, d^3x \;+\; \int E^i \, \mathcal{D}_{pure} \, A^i_{phys} \, d^3x$$

where

$$\boldsymbol{D}_{pure} \;=\; \nabla - i\, g\, \boldsymbol{A}_{pure}, \qquad \mathcal{D}_{pure} \;=\; \nabla - i\, g\, [\, \boldsymbol{A}_{pure}, \cdot\,]$$

This decomposition is different from the standardly-accepted decomposition

$$\boldsymbol{P}_{QCD} \;=\; \int \psi^\dagger \frac{1}{i} \boldsymbol{D} \, \psi \, d^3x \;+\; \int \boldsymbol{E} \times \boldsymbol{B} \, d^3x$$

and they claim that it leads to the following nonstandard prediction for the asymptotic values of quark and gluon momentum fractions :

$$\lim_{Q^2 \to \infty} \langle x \rangle^Q \;=\; \frac{3\, n_f}{\frac{1}{2} n_g + 3\, n_f} \;\overset{n_f = 6}{\simeq}\; 0.82$$

$$\lim_{Q^2 \to \infty} \langle x \rangle^g \;=\; \frac{\frac{1}{2} n_g}{\frac{1}{2} n_g + 3\, n_f} \;\overset{n_f = 6}{\simeq}\; 0.18$$

However, this claim is probably **wrong**, as we shall discuss later !

In a recent paper (M.W., Phys. Rev. D81 (2010) 114010), we have shown that the way of gauge-invariant decomposition of nucleon spin is not necessarily unique, and proposed another gauge-invariant decomposition :

$$\boldsymbol{J}_{QCD} \;=\; \boldsymbol{S}^q \;+\; \boldsymbol{L}^q \;+\; \boldsymbol{S}^g \;+\; \boldsymbol{L}^g$$

where

$$\boldsymbol{S}^q \;=\; \int \psi^\dagger \frac{1}{2} \boldsymbol{\Sigma} \, \psi \, d^3x$$

$$\boldsymbol{L}^q \;=\; \int \psi \, \boldsymbol{x} \times (\boldsymbol{p} - g\,\boldsymbol{A}) \, \psi \, d^3x$$

$$\boldsymbol{S}^g \;=\; \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{phys} \, d^3x$$

$$\boldsymbol{L}^g \;=\; \int E^{aj} \, (\boldsymbol{x} \times \nabla) \, A^{aj}_{phys} \, d^3x \;+\; \boxed{\int \rho^a \, (\boldsymbol{x} \times \boldsymbol{A}^a_{phys}) \, d^3x}$$

"potential angular momentum"

- The quark part of this decomposition is common with the **Ji decomposition**.
- The quark and gluon intrinsic spin parts are common with the **Chen decomp**.
- A crucial difference with the Chen decomp. appears in the orbital parts

$$\boldsymbol{L}^q \;+\; \boldsymbol{L}^g \;=\; \boldsymbol{L}'^q \;+\; \boldsymbol{L}'^g$$

$$\boldsymbol{L}^g - \boldsymbol{L}'^g \;=\; -\,(\,\boldsymbol{L}^q - \boldsymbol{L}'^q) \;=\; \int \rho^a \, (\boldsymbol{x} \times \boldsymbol{A}^a_{phys}) \, d^3x$$

The QED correspondent of this term is the orbital angular momentum carried by electromagnetic field, appearing in the famous Feynman paradox in his textbook.

An arbitrariness of the spin decomposition arises, since this potential angular momentum term is solely gauge-invariant !

$$\int \rho^a \, \boldsymbol{x} \times \boldsymbol{A}^a_{phys} \, d^3x \;=\; g \int \psi^\dagger(x) \, \boldsymbol{x} \times \boldsymbol{A}_{phys}(x) \, \psi(x) \, d^3x$$

$$\rightarrow \; \text{gauge invariant}$$

since

$$\boldsymbol{A}_{phys}(x) \;\rightarrow\; U^\dagger(x) \, \boldsymbol{A}_{phys}(x) \, U(x)$$

$$\psi^\dagger(x) \;\rightarrow\; \psi^\dagger(x), \qquad \psi(x) \;\rightarrow\; U(x) \, \psi(x)$$

This means that one has a freedom to include this potential OAM term into the quark OAM part in **our decomposition**, which leads to the **Chen decomposition**.

$$\boldsymbol{L}^q \,(\text{Ours}) \;+\; \text{potential angular momentum}$$

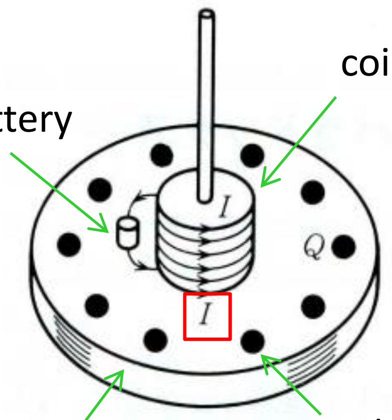
$$= \int \psi^\dagger \, \boldsymbol{x} \times (\boldsymbol{p} - g \, \boldsymbol{A}) \, \psi \, d^3x \;+\; g \int \psi^\dagger \, \boldsymbol{x} \times \boldsymbol{A}_{phys} \, \psi \, d^3x$$

$$= \int \psi^\dagger \, \boldsymbol{x} \times (\boldsymbol{p} - g \, \boldsymbol{A}_{pure}) \, \psi \, d^3x \;=\; \boldsymbol{L}'^q \,(\text{Chen})$$

A short review of the **Feynman paradox**

1. Initially, the disk is at rest.

2. Shut off the electric current at some moment.



Question

Does the disk begin to rotate, or does it continue to be at rest ?

Answer (A)

♣ Since an electric current is flowing through the coil, there is a magnetic flux along the axis.

♣ When the current is stopped, due to the electromagnetic induction, an electric field along the circumference of a circle is induced.

♣ Since the charged metal ball receives forces by this electric field, the disk begins to rotate !

Answer (B)

- Since the disk is initially at rest, its angular momentum is zero.

- Because of the conservation of angular momentum, the disk continue to be at rest !

⇓

2 totally conflicting answers !

Feynman's paradox

The paradox is resolved, if one takes account of the angular momentum carried by the electromagnetic field or potential generated by an electric current !

$$\boldsymbol{L}_{e.m.} = \int \boldsymbol{r} \times \rho \, \boldsymbol{A} \, d^3 r$$

The answer (A) is correct !

Covariant extension of gauge-invariant decomposition of nucleon spin

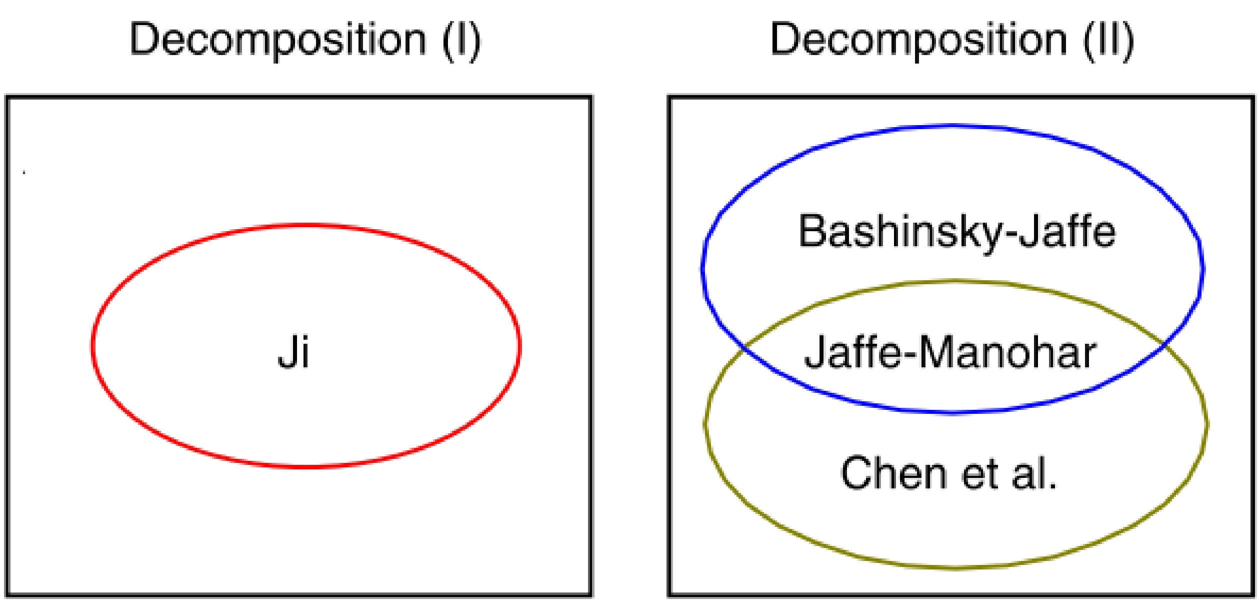
- M. W., Phys. Rev. D83 (2011) 014012.

covariant generalization of the decomposition has twofold advantages.

(1) It is essential to prove Lorentz frame-independence of the decomposition.

(2) It generalizes and unifies the nucleon spin decompositions in the market.

Basically, we find two essentially different decompositions (I) and (II) .



The starting point is again the decomposition of gluon field, similar to Chen et al.

$$A^\mu = A^\mu_{phys} + A^\mu_{pure}$$

Here, we impose only the following quite general conditions.

$$F^{\mu\nu}_{pure} \equiv \partial^\mu A^\nu_{pure} - \partial^\nu A^\mu_{pure} - i\, g\, [A^\mu_{pure}, A^\nu_{pure}] = 0$$

and

$$A^\mu_{phys}(x) \rightarrow U(x)\, A^\mu_{phys}(x)\, U^{-1}(x)$$

$$A^\mu_{pure}(x) \rightarrow U(x) \left(A^\mu_{pure}(x) - \frac{i}{g}\, \partial^\mu \right) U^{-1}(x)$$

- As already mentioned, these conditions are not enough to fix gauge uniquely !
- However, the **point of our argument** is that we can postpone a concrete gauge-fixing until later stage, while accomplishing a gauge-invariant decomposition of $M^{\mu\nu\lambda}$ based on the **above general conditions** only.

Again, we find the way of gauge-invariant decomposition is not unique.

decomposition (I) & decomposition (II)

Gauge-invariant decomposition (II) : covariant generalization of Chen et al's

$$M^{\mu\nu\lambda}_{QCD} = M'^{\mu\nu\lambda}_{q-spin} + M'^{\mu\nu\lambda}_{q-OAM} + M'^{\mu\nu\lambda}_{g-spin} + M'^{\mu\nu\lambda}_{g-OAM} + \text{boost} + \text{total divergence}$$

with

$$M'^{\mu\nu\lambda}_{q-spin} = \frac{1}{2}\,\epsilon^{\mu\nu\lambda\sigma}\,\bar{\psi}\,\gamma_\sigma\,\gamma_5\,\psi$$

$$M'^{\mu\nu\lambda}_{q-OAM} = \bar{\psi}\,\gamma^\mu\,(\,x^\nu\,i\,D^\lambda_{pure} - x^\lambda\,i\,D^\nu_{pure}\,)\,\psi$$

$$M'^{\mu\nu\lambda}_{g-spin} = 2\,\mathrm{Tr}\,\{\,F^{\mu\lambda}\,A^\nu_{phys} - F^{\mu\nu}\,A^\lambda_{phys}\,\}$$

$$M'^{\mu\nu\lambda}_{q-OAM} = 2\,\mathrm{Tr}\,\{\,F^{\mu\alpha}\,(\,x^\nu\,D^\lambda_{pure} - x^\lambda\,D^\nu_{pure}\,)\,A^{phys}_\alpha\,\}$$

This decomposition reduces to any ones of Bashinsky-Jaffe, of Chen et al., and of Jaffe-Manohar, after an appropriate gauge-fixing in a suitable Lorentz frame, which means that **these 3 decompositions are all gauge-equivalent** !

They are not recommendable decompositions, however, because the quark and gluon OAMs in those do not correspond to **known** experimental observables !

Gauge-invariant decomposition (I) : our recommendable decomposition

$$M^{\mu\nu\lambda} = M^{\mu\nu\lambda}_{q-spin} + M^{\mu\nu\lambda}_{q-OAM} + M^{\mu\nu\lambda}_{g-spin} + M^{\mu\nu\lambda}_{g-OAM}$$
$$+ \text{ boost } + \text{ total divergence}$$

with

full covariant derivative

$$M^{\mu\nu\lambda}_{q-spin} = M'^{\mu\nu\lambda}$$
$$\boxed{M^{\mu\nu\lambda}_{q-OAM}} = \bar{\psi}\,\gamma^\mu\,(\,x^\nu\, i\, D^\lambda - x^\lambda\, i\, D^\nu\,)\,\psi \neq \boxed{M'^{\mu\nu\lambda}_{q-OAM}}$$

$$M^{\mu\nu\lambda}_{g-spin} = M'^{\mu\nu\lambda}_{g-spin}$$
$$\boxed{M^{\mu\nu\lambda}_{g-OAM}} = \boxed{M'^{\mu\nu\lambda}_{g-OAM}} + 2\,\mathrm{Tr}\,[\,(\,D_\alpha\,F^{\alpha\mu}\,)\,(\,x^\nu\,A^\lambda_{phys} - x^\lambda\,A^\nu_{phys}\,)\,]$$

⇧

generalized potential OAM term !

The superiority of this decomposition is that the **quark and gluon OAMs** in this decomposition can be related to known **experimental observables** !

[Digression] **decomposition of linear momentum fraction**

$T^{\mu\nu}_{QCD}$	$T^{\mu\nu}_q$	$T^{\mu\nu}_g$
(1) standard	$\frac{1}{2}\bar{\psi}\,(\gamma^\mu\, i\, D^\nu + \gamma^\nu\, i\, D^\mu)\,\psi$	$2\,\mathrm{Tr}\,[F^{\mu\alpha}\, F^\nu_\alpha]$ $+\frac{1}{2}\,g^{\mu\nu}\,\mathrm{Tr}\,F^2$
(2) Jaffe-Manohar	$\frac{1}{2}\bar{\psi}\,(\gamma^\mu\, i\,\partial^\nu + \gamma^\nu\, i\,\partial^\mu)\,\psi$	$-\,\mathrm{Tr}\,[\,F^{\mu\alpha}\,\partial^\nu\, A_\alpha + F^{\nu\alpha}\partial^\mu\, A_\alpha\,]$ $+\frac{1}{2}\,g^{\mu\nu}\,\mathrm{Tr}\,F^2$
(3) Chen et al.	$\frac{1}{2}\bar{\psi}\,(\gamma^\mu\, i\, D^\nu_{pure} + \gamma^\nu\, i\, D^\mu_{pure})\,\psi$	$-\,\mathrm{Tr}\,[F^{\mu\alpha}\, D^\nu_{pure}\, A_{\alpha,phys} + F^{\nu\alpha}\, D^\mu_{pure}\, A_{\alpha,phys}\,]$ $+\frac{1}{2}\,g^{\mu\nu}\,\mathrm{Tr}\,F^2$
(4) Ours	$\frac{1}{2}\bar{\psi}\,(\gamma^\mu\, i\, D^\nu + \gamma^\nu\, i\, D^\mu)\,\psi$	$-\,\mathrm{Tr}\,[\,F^{\mu\alpha}\, D^\nu_{pure}\, A_{\alpha,phys} + F^{\nu\alpha}\, D^\mu_{pure}\, A_{\alpha,phys}\,]$ $-\,\mathrm{Tr}\,[\,D_\alpha\, F^{\mu\alpha}\, A^\nu_{phys} + D_\alpha\, F^{\nu\alpha}\, A^\mu_{phys}\,]$ $+\frac{1}{2}\,g^{\mu\nu}\,\mathrm{Tr}\,F^2$

generalized potential momentum term !

What do these decompositions mean for the **momentum sum rule** of QCD ?

Take light-cone (LC) gauge $(A^+ = 0)$

$$A^+_{phys} \rightarrow 0, \quad A^+_{pure} \rightarrow 0$$

$$D^+ \equiv \partial^+ - i\,g\,A^+ \rightarrow \partial^+, \quad D^+_{pure} \equiv \partial^+ - i\,g\,A^+_{pure} \rightarrow \partial^+$$

$$F^{+\alpha} = \partial^+ A^\alpha - \partial^\alpha A^+ + g\,[A^+, A^\alpha] \rightarrow \partial^+ A^\alpha$$

T^{++} component in **any of the 4 decompositions** then reduce to

$$T^{++} = i\,\psi^\dagger_+\,\partial^+\,\psi_+ + \mathrm{Tr}\,(\partial^+ \boldsymbol{A}_\perp)^2$$

Interaction-dependent part drops in the LC gauge and infinite-momentum frame !

Thus, from

- Jaffe -

$$\langle P_\infty\,|\,T^{++}\,|\,P_\infty\rangle\,/\,2\,(P^+_\infty)^2 = 1$$

we obtain the **standard momentum sum rule** of QCD : $\langle x \rangle^q + \langle x \rangle^g = 1$

Even Chen decomposition gives the standard sum rule, contrary to their claim !

The point is that the difference between

$$T'^{++}_q = \frac{1}{2}\bar{\psi}(\gamma^+ i\partial^+ + \gamma^+ i\partial^+)\psi \quad : \quad \text{canonical momentum}$$

$$T^{++}_q = \frac{1}{2}\bar{\psi}(\gamma^+ i D^+ + \gamma^+ i D^+)\psi \quad : \quad \text{dynamical momentum}$$

does not appear in the **longitudinal momentum sum rule**, since $A^+ = 0$!

However, this is not the case for the **angular momentum sum rule**.

In fact, the difference between

$$M'^{\mu\nu\lambda}_{q-OAM} = \frac{1}{2}\bar{\psi}\gamma^\mu(x^\nu i\partial^\lambda + x^\lambda i\partial^\nu)\psi \quad : \quad \text{canonical OAM}$$

$$M^{\mu\nu\lambda}_{q-OAM} = \frac{1}{2}\bar{\psi}\gamma^\mu(x^\nu i D^\lambda + x^\lambda i D^\nu)\psi \quad : \quad \text{dynamical OAM}$$

does not vanish even in LC gauge and IMF, since

$$M^{+12}_{q-OAM} - M'^{+12}_{q-OAM} = g\bar{\psi}\gamma^+(x^1 A^2_\perp - x^2 A^1_\perp)\psi$$

physical components, which cannot be transformed away by any gauge transformation !

This is also clear from a **"toy model" analysis** of

- M. Burkardt and Hikmat BC, Phys. Rev. D79, 071501 (2009).

Using

scalar diquark model & QED and QCD to order α

they compared the **fermion OAMs** obtained from Jaffe-Manohar decomposition and Ji decomposition.

In our terminology, these two fermion OAMs are nothing but

canonical OAM & dynamical OAM

[**Their findings**]

- 2 decompositions give the same fermion OAMs in scalar diquark model, but they do not in QED and QCD (gauge theories).
- x- distribution of fermion OAMs are different even in scalar diquark model.
- in QED and QCD at order α

$$L^e(\mathrm{Ji}) - \mathcal{L}^e(\text{Jaffe-Manohar}) = -\frac{\alpha}{4\pi} < 0 \quad : \quad (\mathrm{QED})$$

$$L^q(\mathrm{Ji}) - \mathcal{L}^q(\text{Jaffe-Manohar}) = -\frac{\alpha_S}{3\pi} < 0 \quad : \quad (\mathrm{QCD})$$

Unfortunately, these conclusions are heavily **model-dependent** !

An **important lesson** is that one should clearly distinguish two kinds of OAMs :

canonical OAM (or its nontrivial gauge-invariant extension) & dynamical OAM

the difference of which is **nothing spurious**, i.e., **physical** !

The following shows a **power balance** of supporters of two kinds of OAMs :

canonical OAM party

- Jaffe-Manohar
- Bashinsky-Jaffe
- Chen et al.
- Cho et al.
- Leader

dynamical OAM party

- Ji
- Wakamatsu

Neutral party

- Burkardt-BC

- **Superiority of the decomposition (I)**

The **key relations** are the following identities, which hold in our decomposition (I) :

quark :
$$x^\nu T_q^{\mu\lambda} - x^\lambda T_q^{\mu\nu} = M_{q-spin}^{\mu\nu\lambda} + M_{q-OAM}^{\mu\nu\lambda} + \text{total divergence}$$

and

gluon :
$$x^\nu T_g^{\mu\lambda} - x^\lambda T_g^{\mu\nu} - \text{boost} = M_{g-spin}^{\mu\nu\lambda} + M_{g-OAM}^{\mu\nu\lambda} + \text{total divergence}$$

with

$$T_{QCD}^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu} \quad : \quad \text{Belinfante tensor}$$

Evaluating the nucleon forward M.E. of the $(\mu\nu\lambda) = (012)$ component (in rest frame) or $(\mu\nu\lambda) = (+12)$ component (in IMF) of the above equalities, we can prove the following crucial relations :

For the quark part

$$\begin{aligned} L_q &= \frac{1}{2}\int_{-1}^{1} x\,[\,H^q(x,0,0) + E^q(x,0,0)\,]\,dx - \frac{1}{2}\int_{-1}^{1} \Delta q(x)\,dx \\ &= J_q - \frac{1}{2}\Delta q \\ &= \langle p\uparrow \,|\, M^{012}_{q-OAM} \,|\, p\uparrow\rangle \end{aligned}$$

with

$$M^{012}_{q-OAM} = \bar{\psi}\left(\boldsymbol{x}\times\frac{1}{i}\boldsymbol{D}\right)^3\psi \neq \begin{cases} \bar{\psi}\left(\boldsymbol{x}\times\frac{1}{i}\nabla\right)^3\psi \\ \bar{\psi}\left(\boldsymbol{x}\times\frac{1}{i}\boldsymbol{D}_{pure}\right)^3\psi \end{cases}$$

$$\Downarrow$$

In other words

the quark OAM extracted from the combined analysis of GPD and polarized PDF is "dynamical OAM" (or "mechanical OAM") not "canonical OAM" !

This conclusion is nothing different from Ji's claim !

For the gluon part (this is totally new)

$$
\begin{aligned}
L_g &= \frac{1}{2}\int_{-1}^{1} x\,[\,H^g(x,0,0) \;+\; E^g(x,0,0)\,]\,dx \;-\; \int_{-1}^{1} \Delta g(x)\,dx \\
&= J_g \;-\; \Delta g \\
&= \langle p\uparrow \,|\, M^{012}_{g-OAM} \,|\, p\uparrow\rangle
\end{aligned}
$$

with

$$
\begin{aligned}
M^{012}_{g-OAM} \;&=\; 2\,\mathrm{Tr}\,[\,E^j\,(\boldsymbol{x}\times\boldsymbol{D}_{pure})^3\,A^{phys}_j\,] \qquad : \quad \text{canonical OAM} \\
&+\; 2\,\mathrm{Tr}\,[\,\rho\,(\boldsymbol{x}\times\boldsymbol{A}_{phys})^3\,] \qquad : \quad \text{potential OAM term}
\end{aligned}
$$

The gluon OAM extracted from the combined analysis of GPD and polarized PDF contains "potential OAM" term, in addition to "canonical OAM" !

It is natural to call the whole part the gluon "dynamical OAM" .

A natural next question is why the dynamical OAM can be observed ?

- motion of a charged particle in static electric and magnetic fields

(See the textbook of J.J. Sakurai, for instance.)

$$\boldsymbol{E} = -\nabla\phi, \quad \boldsymbol{B} = \nabla \times \boldsymbol{A}$$

Hamiltonian

$$H = \frac{1}{2\,m}\,(\boldsymbol{p} - e\,\boldsymbol{A})^2 + e\,\phi$$

Heisenberg equation

$$\frac{d x_i}{dt} = \frac{1}{i\,\hbar}\,[\,x_i, H\,] = \frac{p_i - e\,A_i}{m}$$

One finds

$$\boldsymbol{\Pi} \overset{def}{\equiv} m\,\frac{d\boldsymbol{x}}{dt} = \boldsymbol{p} - e\,\boldsymbol{A} \neq \boldsymbol{p}$$

$\boldsymbol{\Pi}$: mechanical (or dynamical) momentum

$\boldsymbol{p}$: canoninal momentum

Equation of motion

$$m\frac{d^2\boldsymbol{x}}{dt^2} = \frac{d\boldsymbol{\Pi}}{dt} = e\left[\boldsymbol{E} + \frac{1}{2}\left(\frac{d\boldsymbol{x}}{dt}\times\boldsymbol{B} - \boldsymbol{B}\times\frac{d\boldsymbol{x}}{dt}\right)\right]$$

- What appears in Newton's equation of motion is dynamical momentum $\boldsymbol{\Pi}$ not canonical momentum $\boldsymbol{p}$.

- "Equivalence principle" of Einstein then dictates that the "flow of mass" can in principle be detected by using gravitational force as a probe.

- As a matter of course, the gravitational force is too weak to be used as a probe of mass flow in microscopic system.

- However, remember that the 2nd moments of unpolarized GPDs are also called the gravito-electric and gravito-magnetic form factors.

- The fact that the dynamical OAM as well as dynamical linear momentum can be extracted from GPD analysis is therefore not a mere accident !

♣ A final comment concerning **quantum-loop effects**

general reasoning deduced from the widely-accepted decomposition :

$$\frac{1}{2} = J_q + J_G$$

both gauge-invariant and measurable !

quark part (transparent)

$\Delta\Sigma$: gauge-invariant and measurable !

$$\Rightarrow L_q \equiv J_q - \frac{1}{2}\Delta\Sigma$$: gauge-invariant and measurable !

gluon part (delicate)

logical conclusion

If ΔG is really gauge-invariant and measurable !

$$\Rightarrow L_G \equiv J_G - \Delta G$$: gauge-invariant and measurable !

key question

Is ΔG really gauge-invariant ?

In fact, it was sometimes claimed that ΔG has its meaning only in the LC gauge and in the infinite-momentum frame (IMF).

More specifically, in

- P. Hoodbhoy, X. Ji, and W. Lu, Phys. Rev. D59 (1999) 074010.

they claim that ΔG evolves differently in the LC gauge and the Feynman gauge.

However, the gluon spin operator used in their Feynman gauge calculation is

$$M^{+12}_{g-spin} = 2\,\mathrm{Tr}\,[\,F^{+1}\,A^2 \; - \; F^{+2}\,A^1\,]$$

which is delicately different from our gauge-invariant gluon spin operator

$$M^{+12}_{g-spin} = 2\,\mathrm{Tr}\,[\,F^{+1}\,A^2_{phys} \; - \; F^{+2}\,A^1_{phys}\,]$$

The problem is how to take account of this difference in the Feynman rule of evaluating 1-loop anomalous dimension of the quark and gluon spin operator.

This problem was attacked and solved in our latest paper

- M. W., arXiv : 1104.1465 [hep-ph].

♣ We find that the calculation in the Feynman gauge (as well as in any covariant gauge including the Landau gauge) reproduces the answer obtained in the LC gauge, which is also the answer obtained by the celebrated Altarelli-Parisi method.

Our finding is important also from another context.

♣ So far, a direct check of the answer of **Altarelli-Pasiri method** for the evolution equation of ΔG within the Operator-Produce-Expansion (OPE) framework was limited to the LC gauge calculation, because it was believed that there is no gauge-invariant definition of gluon spin in the OPE framework.

♣ This is the reason why the question of gauge-invariance of ΔG has been left in unclear status for a long time !

♣ Now we can definitely say that the gauge-invariant gluon spin operator appearing in our nucleon spin decomposition (although nonlocal) certainly provides us with a satisfactory operator definition of gluon spin operator (with gauge invariance), which has been searched for nearly 40 years.

Summary of gauge-invariant decomposition of nucleon spin

- ♣ We have discussed the OAM in composite particles, with particular emphasis upon the existence of two kinds of OAM, i.e.

  canonical OAM & dynamical OAM

  and also

  canonical momentum & dynamical momentum

- ♣ The canonical momentum is certainly a fundamental ingredient in theoretical framework of quantum mechanics and quantum field theory, but whether it corresponds to an observable is a different thing !

- ♣ In fact, we have shown that the dynamical OAM of quarks and gluons in the nucleon can in principle be extracted model-independently from combined analysis of GPD measurements and polarized DIS measurements.

- ♣ This means that we now have a satisfactory theoretical basis toward a complete decomposition of the nucleon spin, which is a strongly-coupled relativistic bound state of quarks and gluons.

- One must recognize that this is an exceptionally fortunate situation, which has never been observed for other composite system like atomic nuclei.

- Undoubtedly, we must thank Buddha (and also Xiangdong Ji) for this boon !

On the observability of "canonical" orbital angular momentum (OAM)

We have argued that the "dynamical" OAM can be observed through the combined analyses of unpolarized GPDs and longitudinally polarized PDFs.

⇓

Is there any possibility to extract "canonical" OAM by means of direct measurements ?

⇓

We are a little pessimistic about this possibility by the reason explained below.

?

"canonical" OAM $\neq$ "observable"

for strongly coupled bound system

Model-dependent insight into the OAM inside composite particle

(A) some examples from nuclear physics

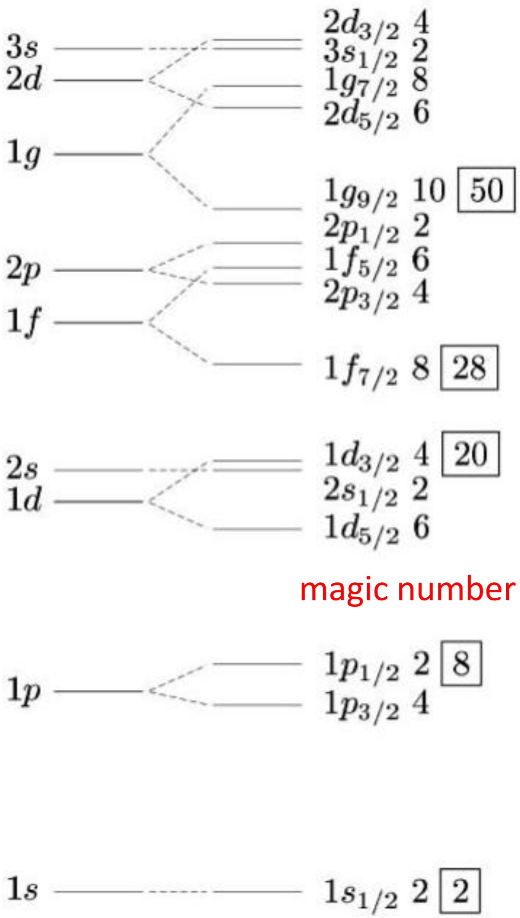
- **magnetic moments of closed shell ± 1 nuclei**

$$\mu_{Schmidt} = \begin{cases} l\, g^{(l)} + \frac{1}{2} g^{(s)} & (j = l + \frac{1}{2}) \\ \frac{j}{j+1} [(l+1)\, g^{(l)} - \frac{1}{2} g^{(s)}] & (j = l - \frac{1}{2}) \end{cases}$$

$g^{(l)}$: orbital g-factor

$g^{(s)}$: spin g-factor

$l \Leftrightarrow$ orbital angular momentum



OAM plays important role, but the concept is critically model-dependent, since it holds only within "Shell Model"

Shell Model s.p. orbits

- **magnetic moment of deuteron** (in the simplest approximation)

$$\mu_d = \mu_p + \mu_n - \frac{3}{2} P_D \left(\mu_p + \mu_n - \frac{1}{2} \right), \qquad P_D \ : \ \text{D-state probability}$$

S- and D-state probabilities

$$P_S = \int_0^\infty u^2(r)\, r^2\, dr, \quad P_D = \int_0^\infty w^2(r)\, r^2\, dr$$

deuteron w.f. and Schrödinger eq.

$$\psi_d(\boldsymbol{r}) = \left[u(r) + \frac{S_{12}(\hat{\boldsymbol{r}})}{\sqrt{8}} w(r) \right] \frac{\chi}{\sqrt{4\pi}}$$

$$\left[-\frac{\hbar^2}{2\mu} \Delta + V_{central}(\boldsymbol{r}) + V_{tensor}(\boldsymbol{r}) \right] \psi_d(\boldsymbol{r}) = E_d\, \psi_d(\boldsymbol{r})$$

angular momentum decomposition of deuteron spin

$$\begin{aligned} \langle J_3 \rangle &= \langle L_3 \rangle + \langle S_3 \rangle \\ &= \frac{3}{2} P_D + \left(P_S - \frac{1}{2} P_D \right) = P_S + P_D = 1 \quad ! \end{aligned}$$

Several obstacles of this simple thought are

relativistic corrections, meson exchange currents, ...

Most serious would be the fact that the D-state probability is not direct observable !

- R.D. Amado, Phys. Rev. C20 (1979) 1473.
- J.L. Friar, Phys. Rev. C20 (1979) 325.

♣ The “interior” of a bound state w.f. cannot be determined empirically.

♣ 2-body unitary transformation arising in the theory of meson-exchange currents can change the D-state probability, while keeping the deuteron observables intact.

♣ The D-state probability, for instance, depends on the cutoff Λ of short range physics in an effective theory of 2-nucleon system.

- S.K. Bogner et al., Nucl. Phys. A784 (2007) 79.

⇩

See the figure in the next page !

Deuteron **D-state probability** in an effective theory

Bogner et al, 2007

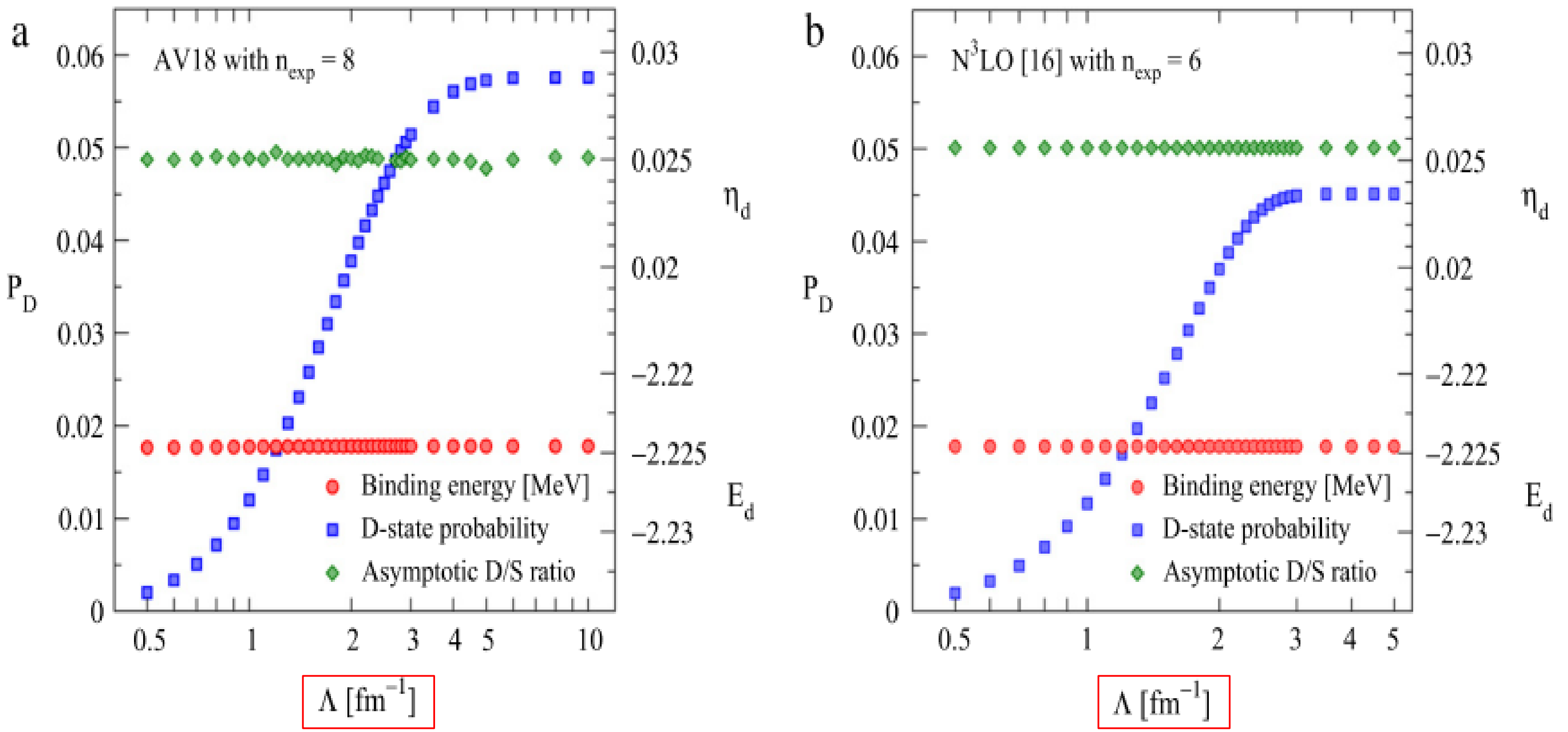


Fig. 57. D-state probability P_D (left axis), binding energy E_d (lower right axis), and asymptotic D/S-state ratio η_d (upper right axis) of the deuteron as a function of the cutoff [6], starting from (a) the Argonne v_{18} [18] and (b) the N^{3}LO NN potential of Ref. [20] using different smooth $V_{\mathrm{low}\,k}$ regulators. Similar results are found with SRG evolution.

(B) examples from nucleon structure

TMD distribution predicted by the **Chiral Quark Soliton Model** (CQSM)

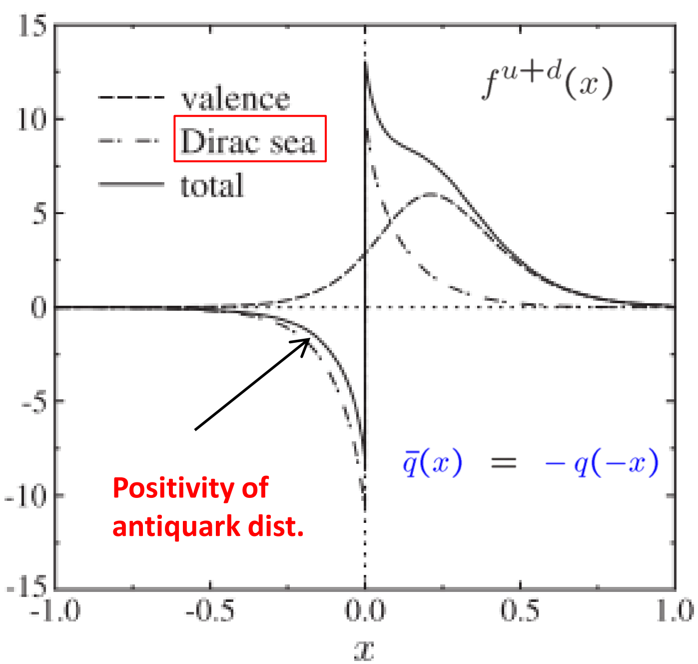
So far, only the iso-singlet combination of unpolarized TMD was calculated.

$$f^{u+d}(x, \boldsymbol{k}_\perp) \qquad : \text{ M. W., Phys. Rev. D79 (2009) 094028.}$$

A prominent feature of the CQSM prediction is self-evident from the shape of x-distribution obtained after integrating over the transverse momentum $\boldsymbol{k}_\perp$.

$$f^{u+d}(x) = \int \frac{d^2 \boldsymbol{k}_\perp}{(2\pi)^2} f^{u+d}(x, \boldsymbol{k}_\perp)$$

a dominant role of vacuum-polarized Dirac-sea quarks in the small x region !



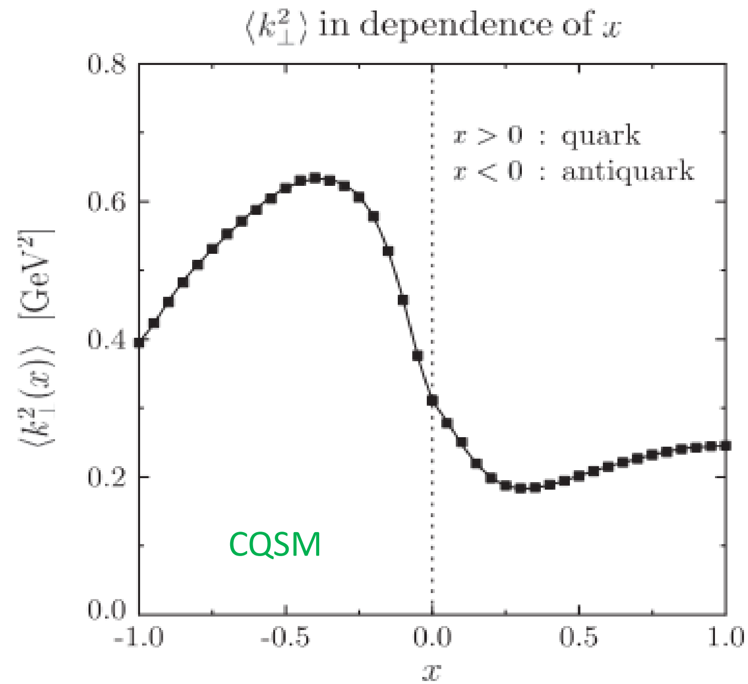
Test of **factorized ansatz**

$$f^q(x, \boldsymbol{k}_\perp) \overset{?}{\simeq} f^q(x) \times e^{-\boldsymbol{k}_\perp^2 / \langle \boldsymbol{k}_\perp^2 \rangle} / \pi \langle k_\perp^2 \rangle$$

$$\langle k_\perp^2(x) \rangle = \frac{\int d^2\boldsymbol{k}_\perp k_\perp^2 f^{u+d}(x, \boldsymbol{k})}{\int d^2\boldsymbol{k}_\perp f^{u+d}(x, \boldsymbol{k})}$$

$$\langle k_\perp^2(x) \rangle \neq \text{constant}$$

drastically broken !



average transverse momentum (square) for quarks and antiquarks

$$\langle k_\perp^2 \rangle^Q = 0.224 \text{ GeV}^2,$$

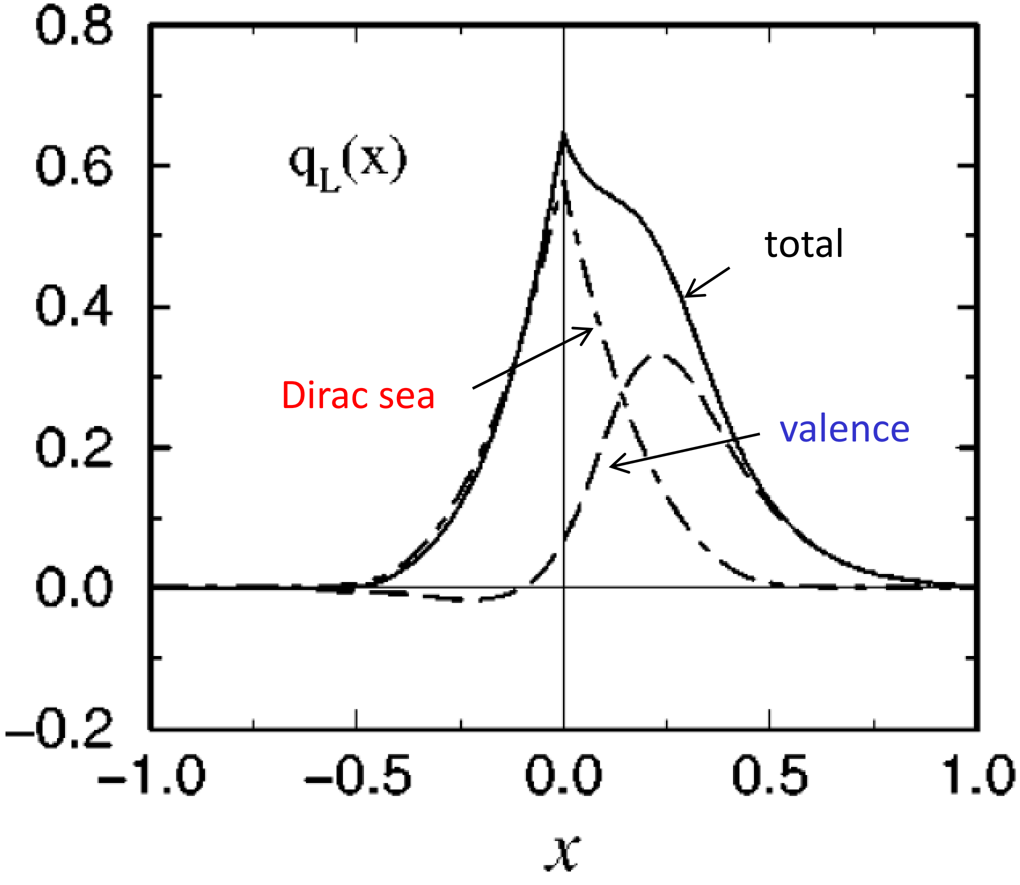
$$\langle k_\perp^2 \rangle^{\bar{Q}} = 0.445 \text{ GeV}^2,$$

antiquarks have larger extension in $\boldsymbol{k}_\perp$ -distribution !

large contribution to the z-component of OAM L_z ?

quark and antiquark OAM distribution in CQSM

- M.W. and T. Watabe, Phys. Rev. D62 (2000) 054009.



- quarks and antiquarks with small Bjorken *x* carry sizable amount of OAM !

Unfortunately, **highly model-dependent statement** !

More on the relation between TMD distributions and OAM

- Strong correlation between **Sivers function** and **GPD** $E(x, \xi, t)$

$$f_{1T}^{\perp q}(x, \boldsymbol{k}^2) \;\Leftrightarrow\; \varepsilon(x, \boldsymbol{b}_\perp^2)$$: M. Burkardt (2002)

caution !

♣ naïve **T-odd** Sivers function vanishes without **FSI** !

♣ on the other hand, GPD $E(x, \xi, t)$ exists irrespectively of **FSI** !

average transverse momentum of an unpol. quark in a transversally pol. target

$$\langle k_T^{q,i}(x) \rangle_{UT} = -\int d^2 k_\perp \, k_\perp^i \, \frac{\epsilon_\perp^{jk} \, k_\perp^j \, S_\perp^k}{M} \, f_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2)$$

$$\simeq + \int d^2 b_\perp \, \mathcal{I}^{q,i}(x, \boldsymbol{b}_\perp) \, \frac{\epsilon_\perp^{jk} \, b_\perp^j \, S_\perp^k}{M} \, \left(\mathcal{E}^q(x, \boldsymbol{b}_\perp^2) \right)'$$

$\mathcal{E}^q(x, \boldsymbol{b}_\perp^2)' \equiv \dfrac{\partial}{\partial b_\perp^2} \mathcal{E}^q(x, \boldsymbol{b}_\perp^2)$: impact parameter rep. of $E^q(x, \xi, t)$

$\mathcal{I}^{q,i}(x, \boldsymbol{b}_\perp)$: lensing function (effect of FSI due to gluon)

Final state interactions mix into the relation in a **model-dependent way** !

A quantity, which has **more direct connection with OAM** in the nucleon

♣ **pretzolosity distribution** (**T-even, chiral-odd** TMD distribution)

$$\Phi(x, \boldsymbol{k}_\perp, \boldsymbol{S}) \propto f_1(x, \boldsymbol{k}_\perp^2) \not{n} + \cdots + \frac{(\boldsymbol{k}_\perp \cdot \boldsymbol{S})}{M} h_{1T}^\perp(x, \boldsymbol{k}_\perp^2) \frac{[\not{k}_\perp, \not{n}]}{2M} + \cdots$$

in MIT bag model (later, also in scalar diquark model)

- H. Avakian et al., Phys. Rev. D78, 114024 (2008).

$$h_{1T}^{(1)\perp q}(x) \equiv \int \frac{\boldsymbol{k}_\perp^2}{2M} h_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2) \, d^2\boldsymbol{k}_\perp = g_1^q(x) - h_1^q(x)$$

$$\int h_{1T}^{(1)\perp q}(x) \, dx = \Delta q - \Delta_T q = (\text{axial charge}) - (\text{tensor charge})$$

- pretzolosity gives a measure of relativistic effects or quark OAM !
- it also gives a measure of the deviation from spherical shape of the nucleon !

- G. A. Miller, Phys. Rev. C68, 022201 (2003).

More direct statement is possible in MIT bag model.

- H. Avakian et al., arXiv : Phys. Rev. D81 (2010) 074035.

$$- L_3^q = \int dx\, h_{1T}^{(1)\perp q}(x) = \int dx \int d^2\boldsymbol{k}_\perp \frac{\boldsymbol{k}_\perp^2}{2M} h_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2)$$

by measuring **pretzolocity** : $A_{UT}^{\sin(3\phi - \phi_S)}$ ⇨ quark OAM ?

The above relation can easily be deduced from the previous relation

$$\int h_{1T}^{(1)\perp q}(x)\, dx = \Delta q - \Delta_T q$$

In fact, from the ground state w.f. of MIT bag model

$$\psi_{g.s.} = \begin{pmatrix} f(r)\,\chi_s \\ i\,\boldsymbol{\sigma}\cdot\hat{\boldsymbol{r}}\, g(r)\,\chi_s \end{pmatrix}$$

we have

$$\Delta q = \int \left\{ [f(r)]^2 - \frac{1}{3}[g(r)]^2 \right\} r^2\, dr \quad : \quad \text{axial charge}$$

$$\Delta_T q = \int \left\{ [f(r)]^2 + \frac{1}{3}[g(r)]^2 \right\} r^2\, dr \quad : \quad \text{tensor charge}$$

From these

$$\Delta q - \Delta_T q \;=\; -\frac{2}{3}\int [g(r)]^2\, r^2 dr$$

On the other hand

$$\begin{aligned} L_3^Q &= \frac{1}{2} - \frac{1}{2}\Delta q \\ &= \frac{1}{2}\int \left\{ [f(r)]^2 + [g(r)]^2 \right\} r^2\, dr \;-\; \frac{1}{2}\int \left\{ [f(r)]^2 - \frac{1}{3}[g(r)]^2 \right\} r^2\, dr \\ &= \frac{2}{3}\int [g(r)]^2\, r^2\, dr \end{aligned}$$

Angular momentum decomposition of the nucleon spin in MIT bag model

$$\begin{aligned} \langle J_3 \rangle &= \langle L_3 \rangle + \frac{1}{2}\langle \Sigma_3 \rangle \\ &= \frac{2}{3} P_P + \frac{1}{2}\left(P_S - \frac{1}{3} P_P \right) \\ &= \frac{1}{2}\left(P_S + P_P \right) = \frac{1}{2} \;\; ! \end{aligned}$$

♣ MIT bag model is not a good model of bound state of nearly zero-mass quarks !

- importance of chiral symmetry
- clouds of Goldstone pions
- breakdown of SU(6)-like picture

More serious would be the neglect of gluon degrees of freedom, which are widely believed to carry sizable amount of nucleon momentum fraction.

♣ In any case, one should clearly recognize the fact that, even in much simpler bound system like the deuteron, the D-state probability or the OAM content is not direct observable !

♣ We point out that the OAM, which we were talking about here, is an expectation value of "canonical OAM operator" between some Fock-state eigenvectors !

♣ The canonical momentum and canonical OAM are fundamental ingredients of quantum mechanics and quantum field theory. However, whether they correspond to direct observables is a totally different story !

[Appendix] Non-spherical shape of the nucleon

the nucleon magnetic moment in MIT bag model is given by

$$\begin{aligned}\mu &= \frac{1}{2}\int \psi^{\dagger}_{g.s.}(\boldsymbol{r})\, \boldsymbol{r}\times\boldsymbol{\alpha}\, \psi_{g.s.}(\boldsymbol{r})\, d^3r \\ &= \frac{1}{2}\int \psi^{\dagger}_{g.s.}(\boldsymbol{r}) \begin{pmatrix} 0 & \boldsymbol{r}\times\boldsymbol{\sigma} \\ \boldsymbol{\alpha}\times\boldsymbol{r} & \end{pmatrix} \psi_{g.s.}(\boldsymbol{r})\, d^3r \\ &\propto \int f(r)\, g(r)\, r^3\, dr \\ &= 0 \quad \text{without OAM} \quad \text{(lower p-wave component)}\end{aligned}$$

This means, within the framework of MIT bag model, nonzero magnetic moment of the nucleon already dictates the existence of nonzero OAM in a nucleon !

but ⇨ So what ?