生成座標法のバリア透過過程への適用

「理論と実験で拓く中性子過剰核の核分裂」@理研RIBF　2023/2/17

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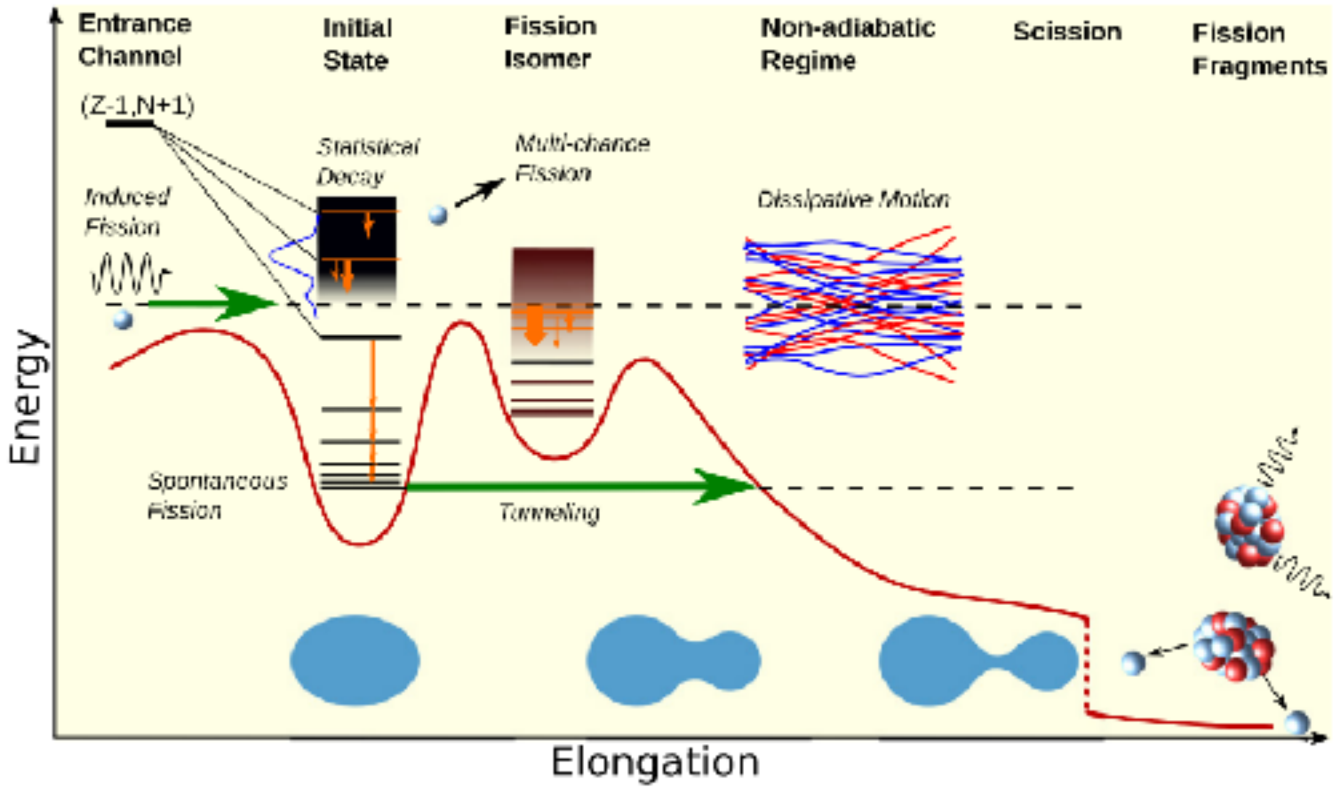
Introduction: Induced Fission

Phenomenological model: The statistical model, Transport theoretical approach …

Microscopic approach: Density Functional Theory, Generator Coordinate Method …



M. Bender et al., J. Phys. G: Nucl. Part. Phys. 47 113002 (2020) .

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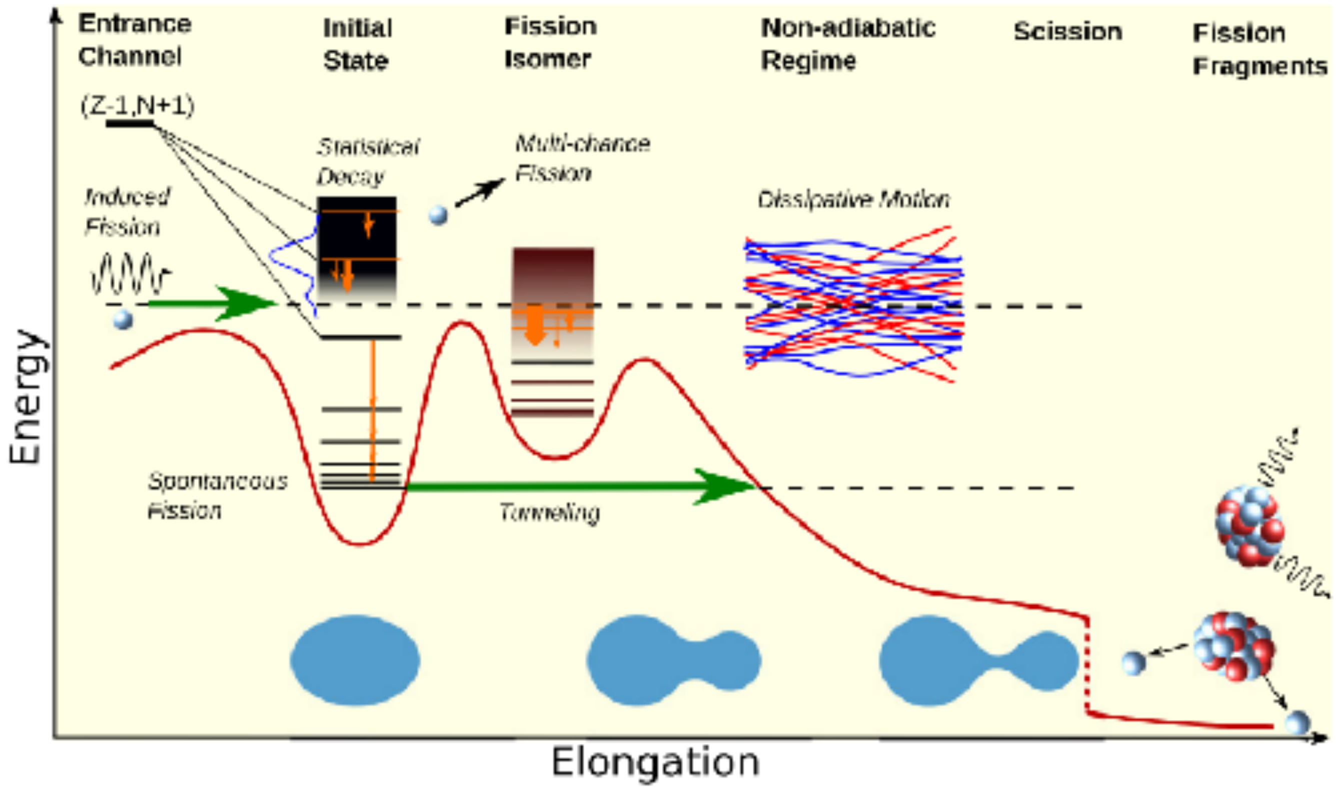
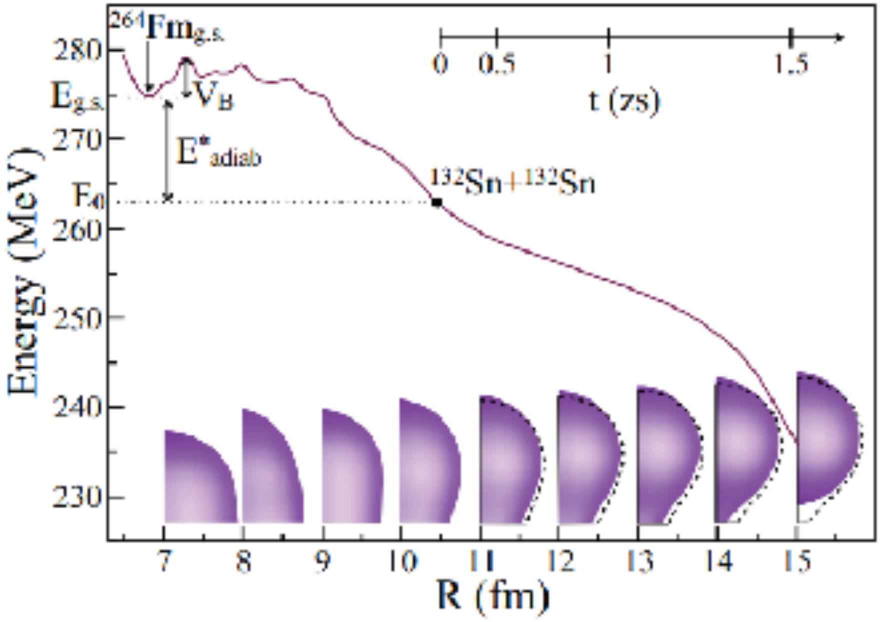
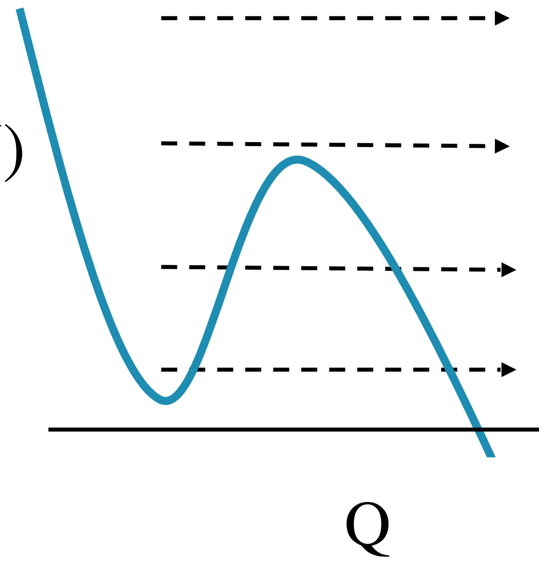




C. Simenel and A. S. Umar Phys. Rev. C 89, 031601(R) (2014).

Motivation

- fast neutron ($E_n > 1$ MeV)
- barrier top fission (E^*~6 MeV)
- neutron-rich nuclei
- spontaneous fission ($E^* = 0$)



Unified theory for the barrier transmission problem
&
Microscopic understanding of nuclear fission

⬇

Configuration Interaction (CI) approach based on GCM

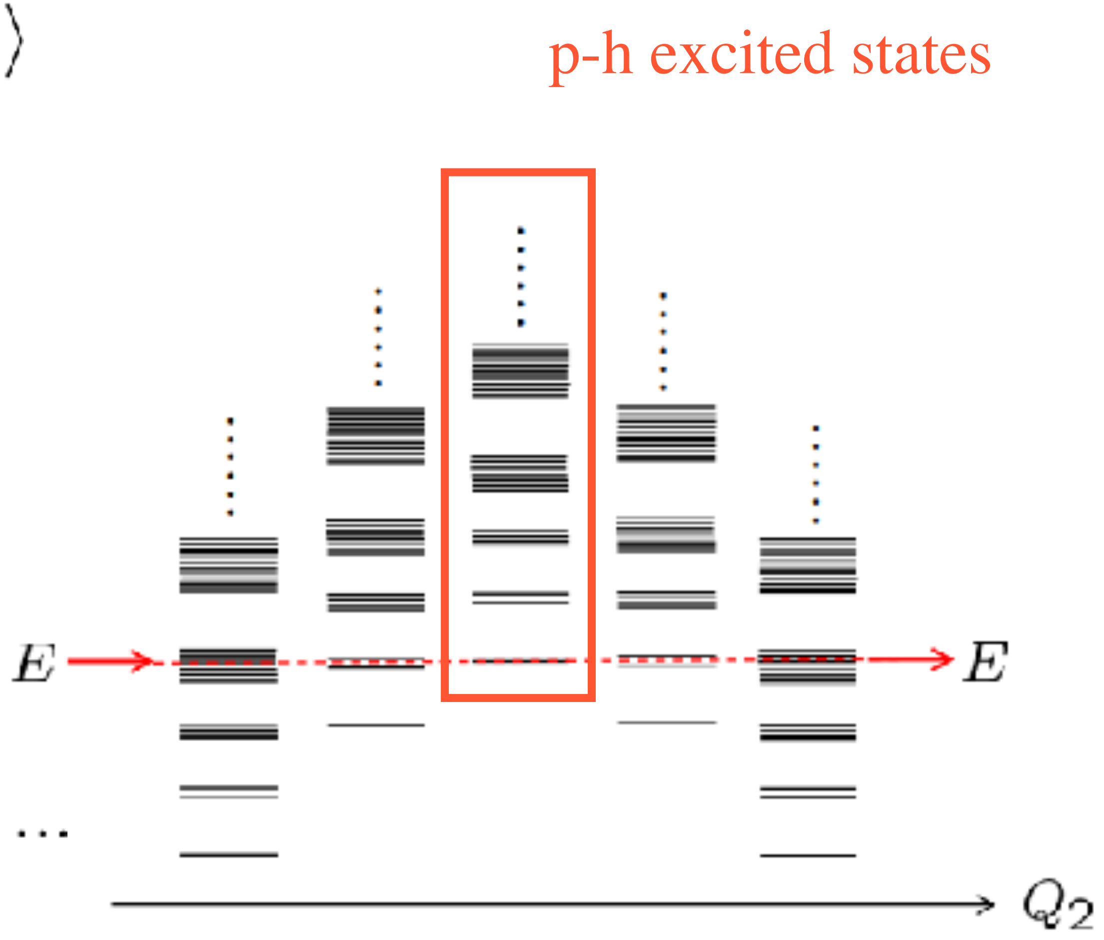
Extension of the GCM ansatz

Superpose mean-field wavefunction $|\Phi(Q)\rangle$
in Generator coordinate method (GCM)

$$|\Psi\rangle = \int dQ f(Q)|\Phi(Q)\rangle$$

Extend usual GCM ansatz
and superpose p-h excited states

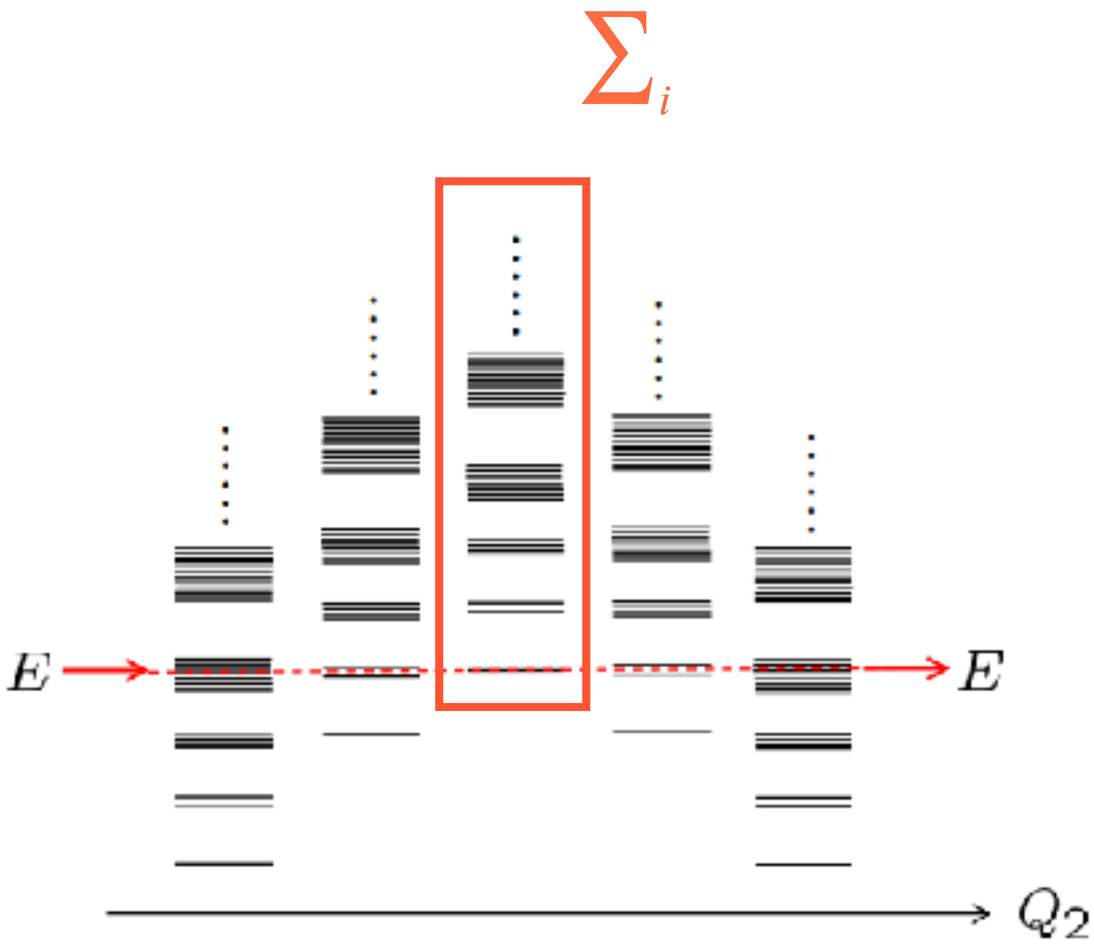
$$|\Phi_i(Q)\rangle = a_p^\dagger a_h|\Phi(Q)\rangle,\ a_{p'}^\dagger a_p^\dagger a_{h'} a_h|\Phi(Q)\rangle \ldots$$



1. Superpose mean field wave function (GCM ansatz)

$$|\Psi\rangle = \sum_i \int dQ\, f_i(Q) |\Phi_i(Q)\rangle$$

p-h excited states



1. Superpose mean field wave function (GCM ansatz)

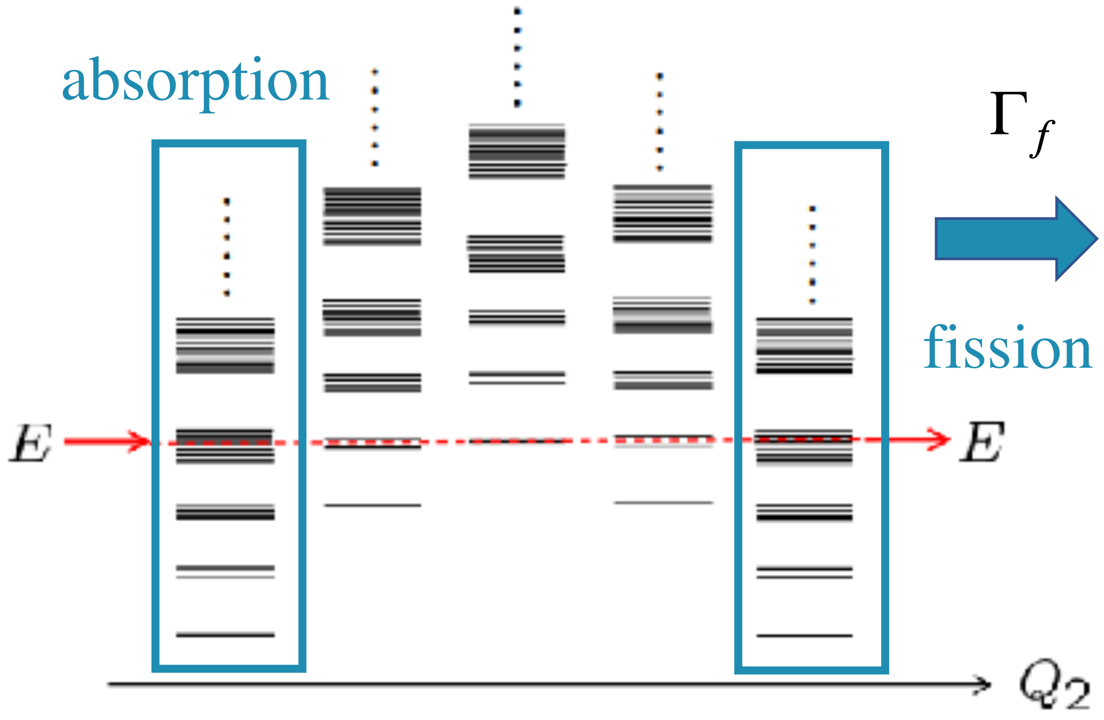
$$|\Psi\rangle = \sum_i \int dQ f_i(Q)|\Phi_i(Q)\rangle$$

2. Calculate GCM kernel and decay width Γ

$$N(Q,Q')_{i,j} = \langle\Phi_i(Q)|\Phi_j(Q')\rangle$$

$$H(Q,Q')_{i,j} = \langle\Phi_i(Q)|\hat{H}|\Phi_j(Q')\rangle$$

$$(\Gamma_j)_{kk'} = \gamma_j(N^{\frac{1}{2}})_{jk}(N^{\frac{1}{2}})_{jk'}$$



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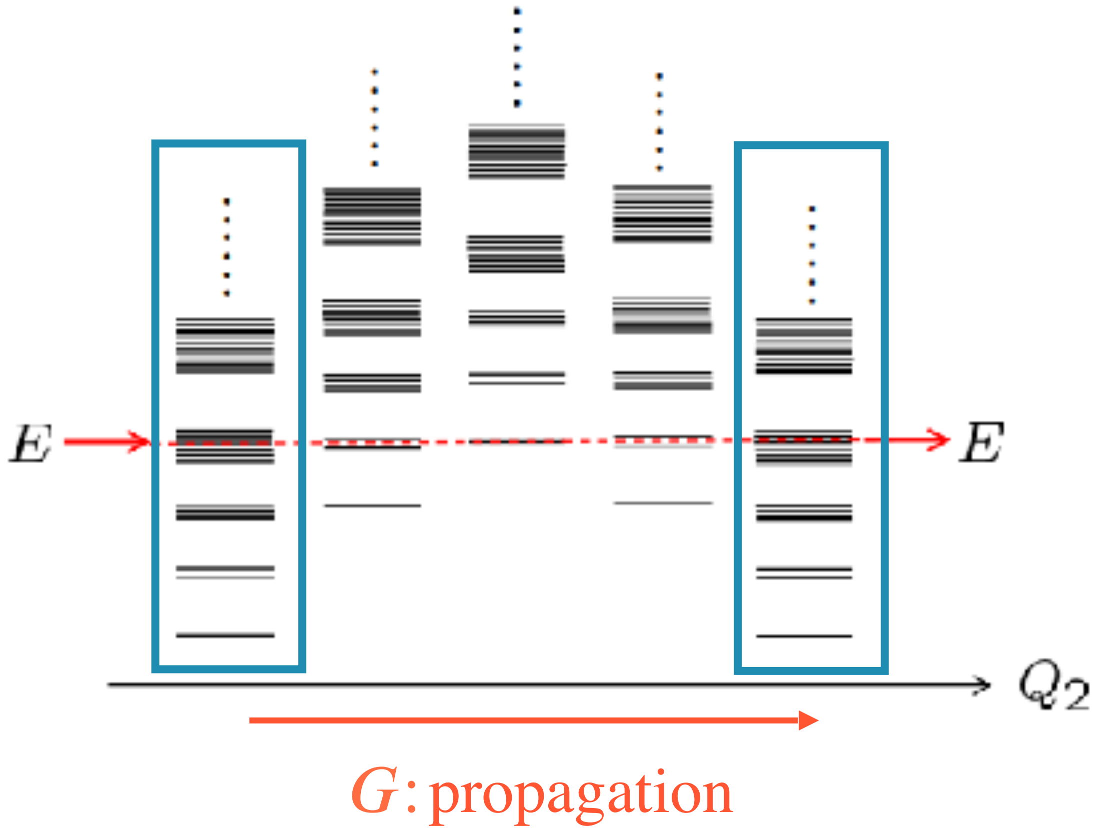
3. Calculate Green's function

$$G(E) = (H - i\Gamma/2 - EN)^{-1}$$



4. Transmission coefficient $T_{ab}(E)$ obtained by the Datta formula

$$|S_{a,b}(E)|^2 \equiv T_{a,b}(E) = \mathrm{Tr}(\Gamma_a G(E) \Gamma_b G^\dagger(E))$$

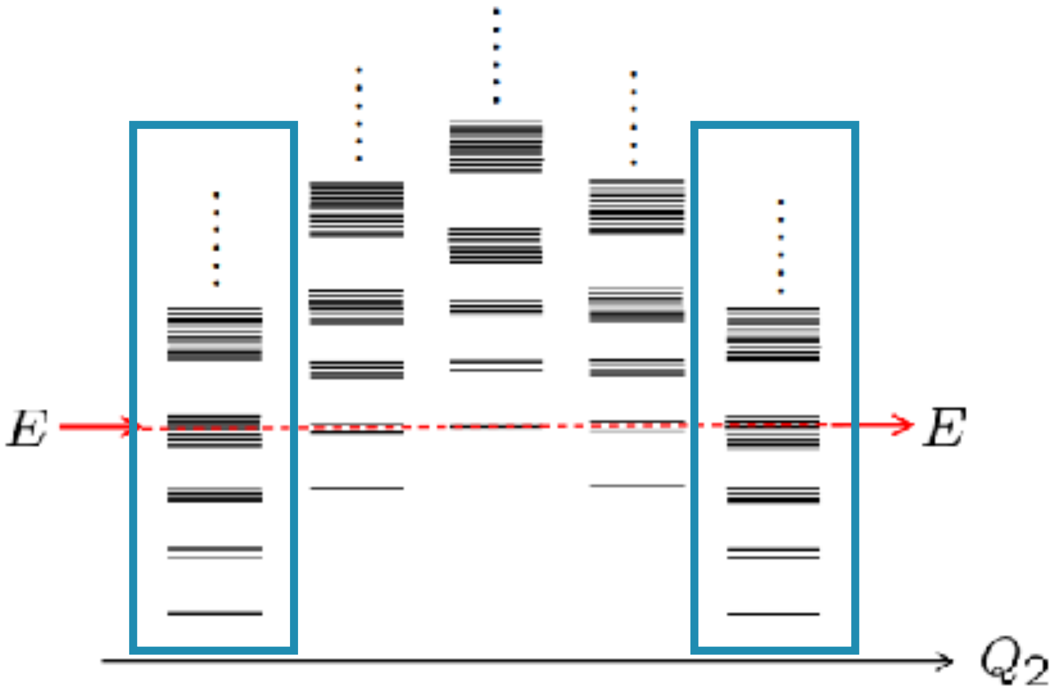
Neutron absorption (channel a)

↓

Formation of the compound nucleus

↓

Fission or Capture (channel b)



Γ_a:input channel Γ_b:output channel

G: propagation

Hamiltonian and model space

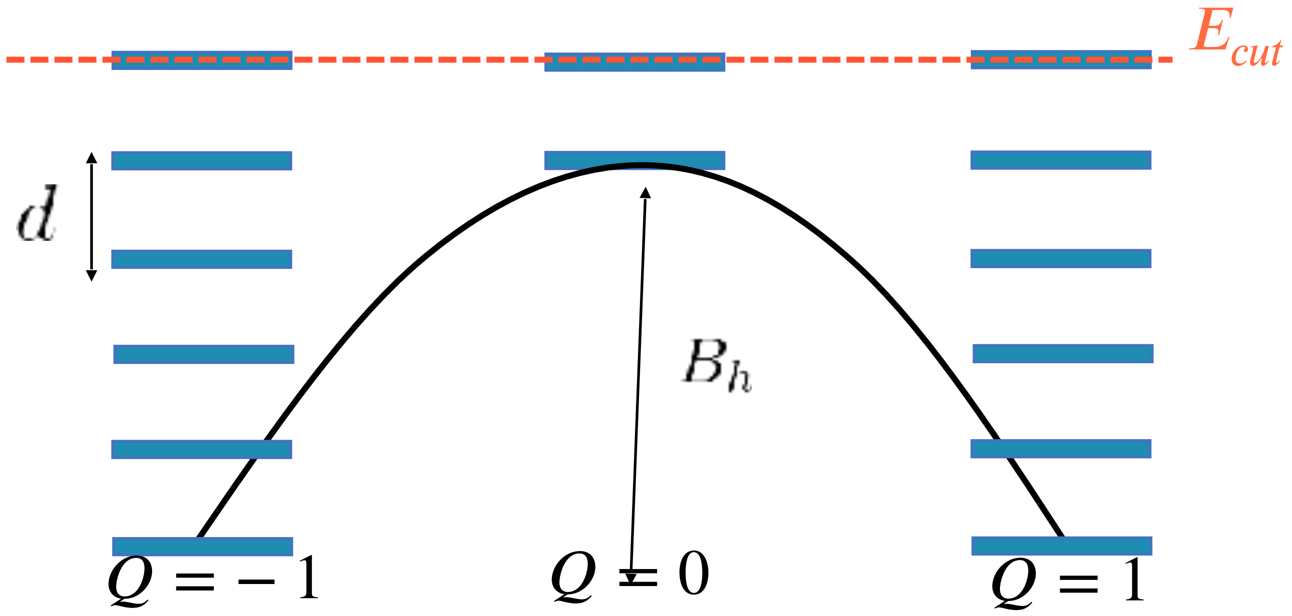
$$H = V(Q) + H_{ph} + H_{ran} + H_{pair}$$

✓Deformation Energy

$$V(Q) = \begin{cases} 0 & (Q = -1, 1) \\ B_h (> 0) & (Q = 0) \end{cases}$$

✓ Particle-Hole excitation

$$H_{ph} = d \sum_{\alpha : n_\alpha > 0} a_\alpha^\dagger a_\alpha + d \sum_{\alpha : n_\alpha \leq 0} a_\alpha a_\alpha^\dagger$$



Hamiltonian and model space

$$H = V(Q) + H_{ph} + H_{ran} + H_{pair}$$

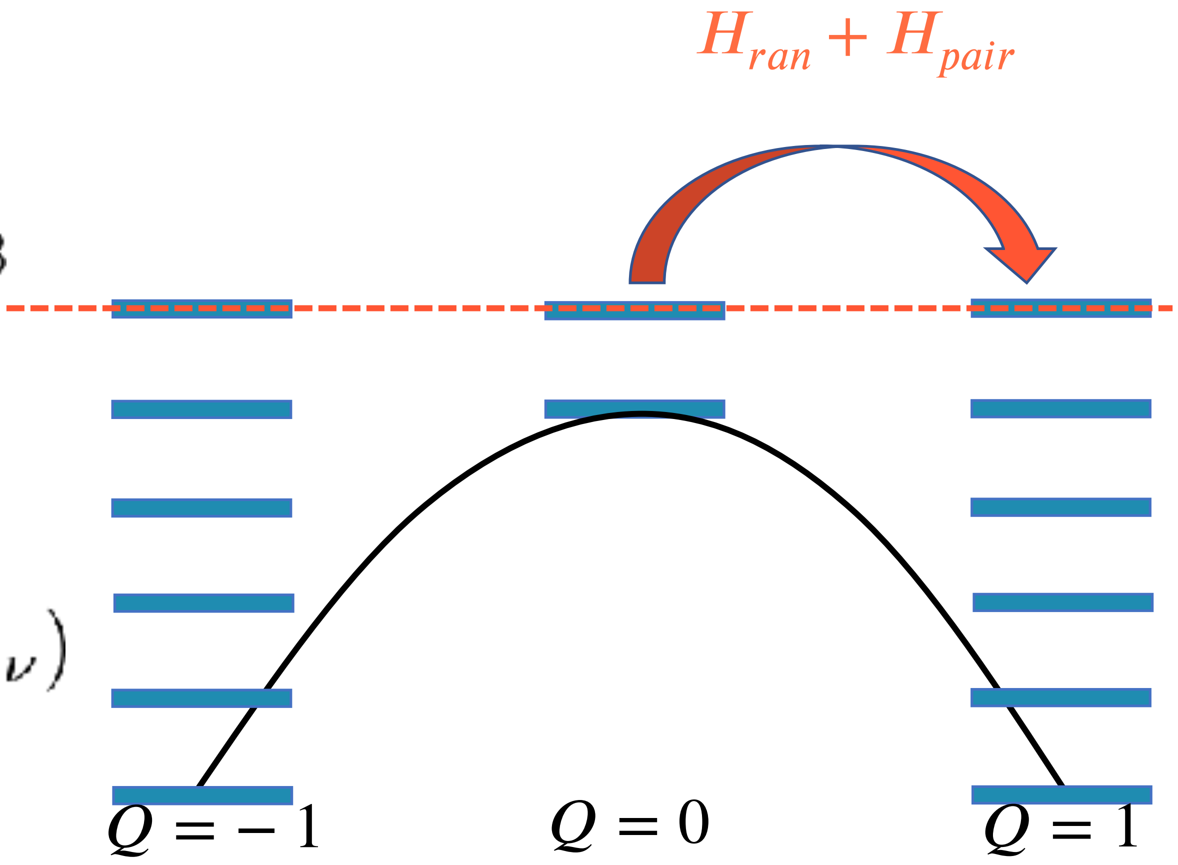
✓ Proton-Neutron random interaction

$$H_{ran} = v_{np} \sum r\, a^{\dagger}_{\alpha 1} a^{\dagger}_{\alpha 2} a_{\alpha 4} a_{\alpha 3}$$

random number

✓ Pairing interaction

$$H_{pair} = -G\mathrm{P}^{\dagger}\mathrm{P} \;\; (\mathrm{P} = \textstyle\sum_{\nu} a_{\bar{\nu}} a_{\nu})$$



Overlap kernel

Mean-field w.f. $|\Phi_i(Q)\rangle$ is not orthogonal in general

Overlap kernel $N_{i,j}(Q, Q')$ express size of non-orthogonality

$$N_{i,j}(Q,Q') = \langle\Phi_i(Q)|\Phi_j(Q')\rangle \simeq \exp(-\lambda(Q-Q')^2)\delta_{i,j}$$

We assume the form of Gaussian Overlap Approximation ($\lambda = 1$)
and $a_\mu(Q) \simeq a_\mu(Q')$

Neutron width :

$$\left(\Gamma_n\right)_{kk'} = \gamma_n \Sigma_{i:Q=-1} \left(N^{1/2}\right)_{ik} \left(N^{1/2}\right)_{ik'}$$

Capture width:

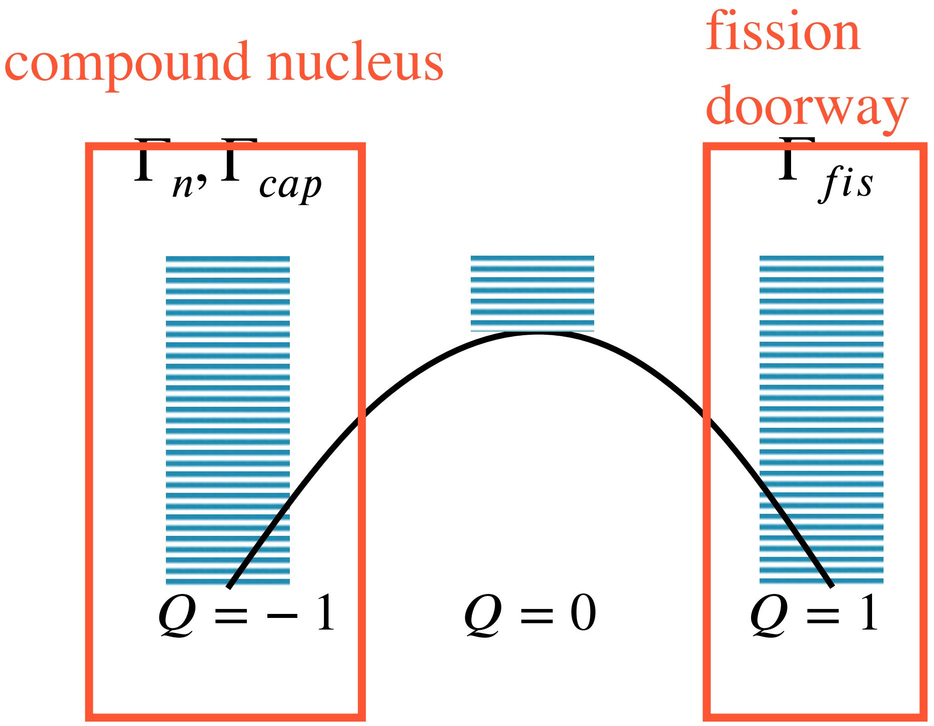
$$\left(\Gamma_{cap}\right)_{kk'} = \gamma_{cap} \Sigma_{i:Q=-1} \left(N^{1/2}\right)_{ik} \left(N^{1/2}\right)_{ik'}$$

Fission width:

$$\left(\Gamma_{fis}\right)_{kk'} = \gamma_{fis} \Sigma_{i:Q=1} \left(N^{1/2}\right)_{ik} \left(N^{1/2}\right)_{ik'}$$

Datta's formula

$$T_{ab}(E) = \mathrm{Tr}(\Gamma_a G(E) \Gamma_b G^{\dagger}(E))$$



$Q = -1$: compound nucleus $\Rightarrow \Gamma_n, \ \Gamma_{cap}$

$Q = 1$: fission doorway states $\Rightarrow \Gamma_{fis}$

Diabatic interaction

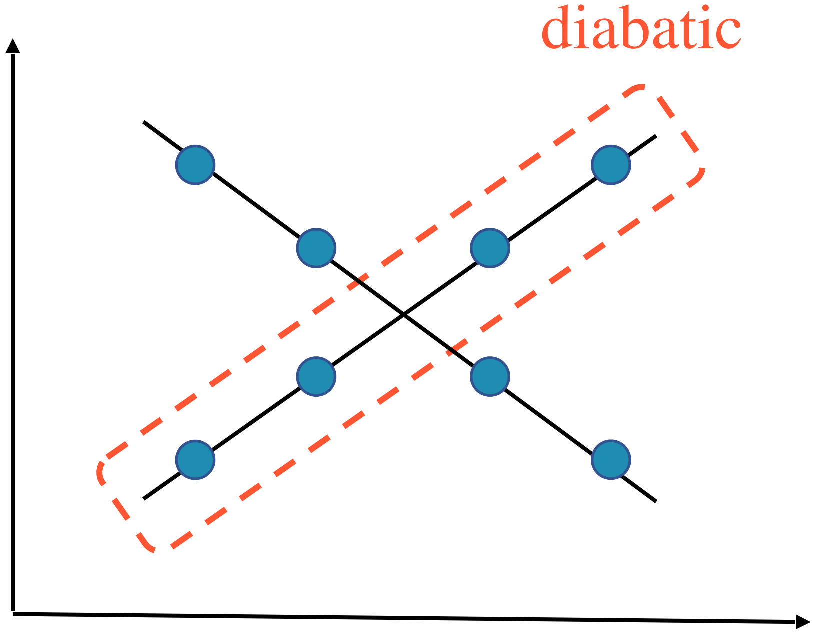
$$\langle\Phi_i(Q)|\sum\epsilon_\mu a^\dagger_\mu a_\mu|\Phi_j(Q')\rangle = -h_2\langle\Phi_i(Q)|\Phi_j(Q')\rangle(Q-Q')^2$$

size of diabatic int.

Diabatic interaction connects diabatic configurations (as the name suggests)

↕

Adiabatic configurations are connected by the pairing

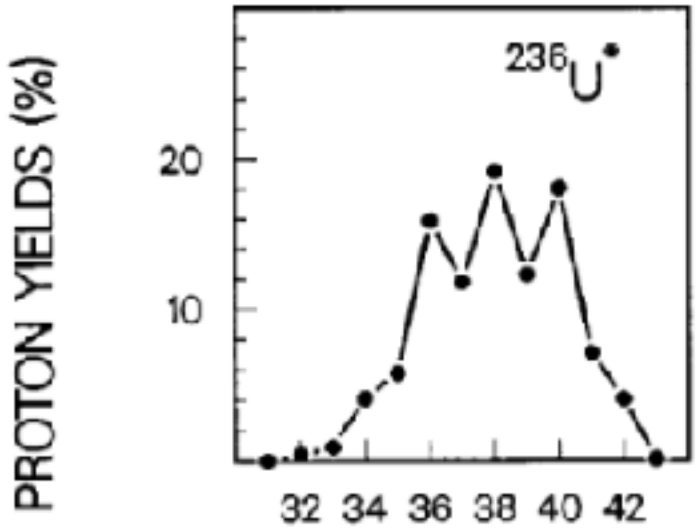


Result

Result 1. Effects of the pairing on the induced fission

Result 2. Role of the diabatic interaction with non-orthogonality

Result 3. Comparison with the Bohr-Wheeler theory



J. P. Bocquet and R. Brissot,

Result 1: Effects of the pairing

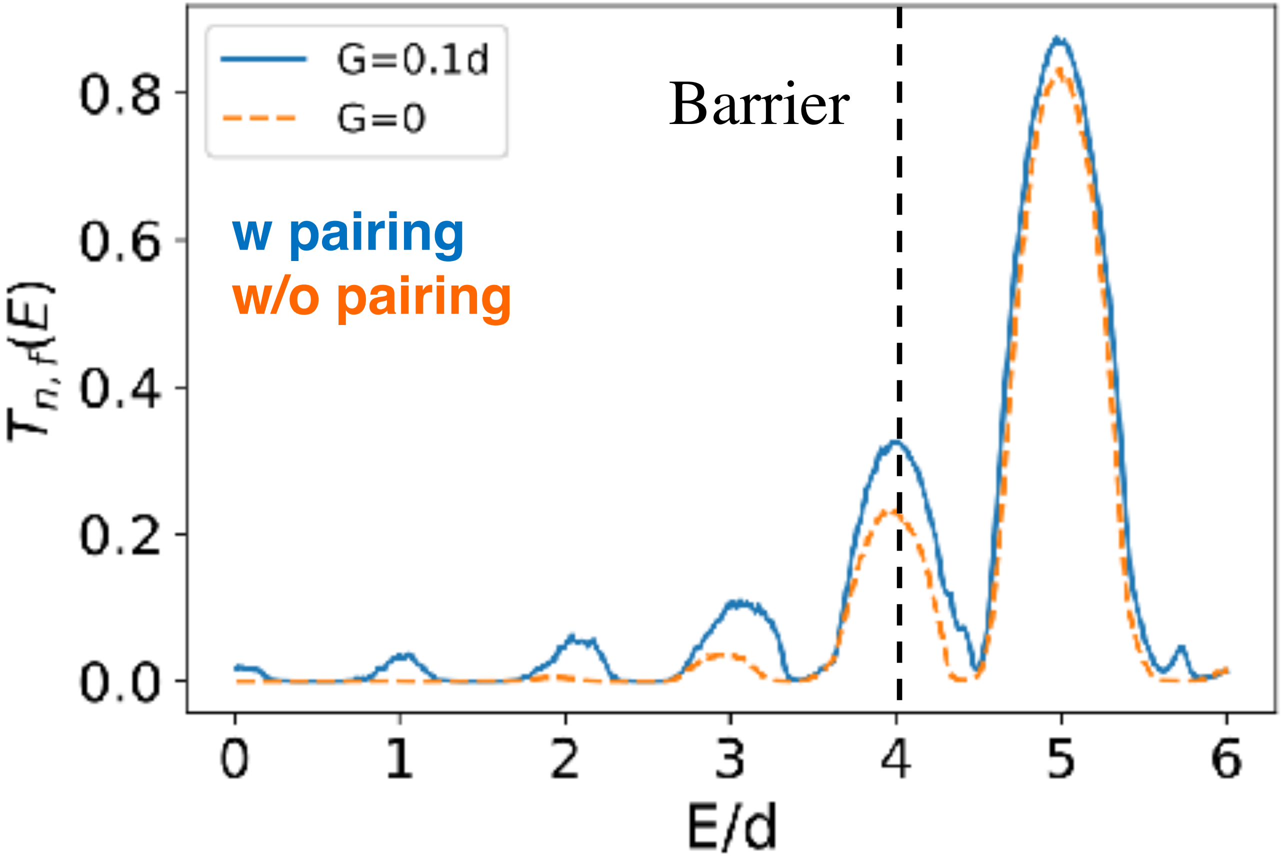
Calculate $T_{n,fis}(E)$ with different G

$$H_{pair} = -G\mathrm{P}^{\dagger}\mathrm{P}$$

Apply orthogonal basis for simplicity

Pairing plays a role mainly in the sub-barrier region

$T_{n,fis}(E)$



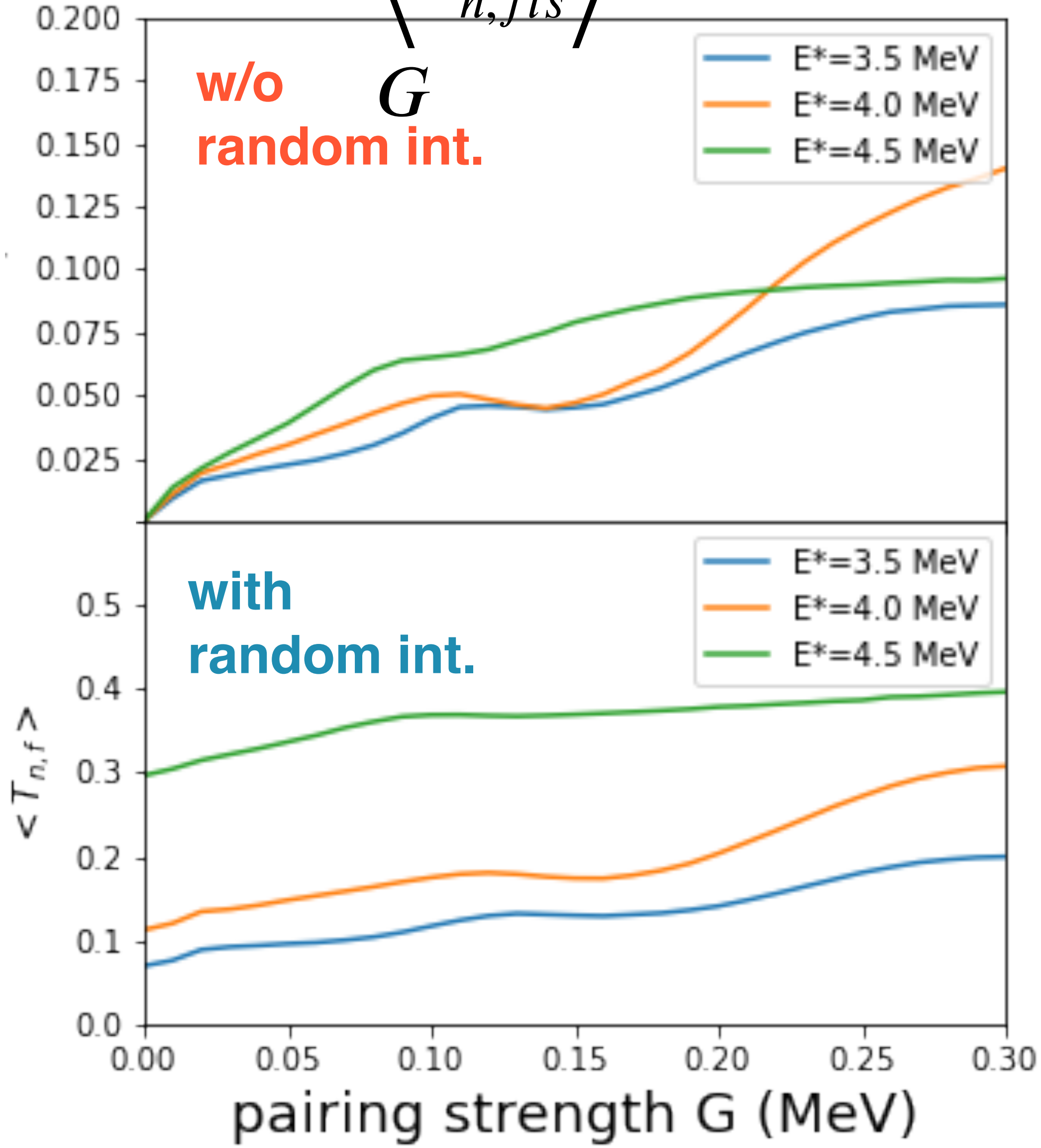
parameters:($v_{np} = 0.03d$, $B_h = 4d$, $\gamma_n = 0.001\mathrm{d}$, $\gamma_f = 0.1d$)

Energy averaged transmission coefficient $\left\langle T_{n,fis} \right\rangle$

$$\langle T_{n,fis}(E)\rangle = \frac{1}{\Delta E}\int_{E-\Delta E/2}^{E+\Delta E/2} dE'\, T_{n,fis}(E')$$

- ✓ Pairing enhances transmission probabilities mainly in the sub-barrier region
- ✓ Random interaction smears out coherent pairing interaction

$\left\langle T_{n,fis} \right\rangle$ vs G



Result 2: Role of the diabatic interaction

Non-orthogonality leads to diabatic interaction

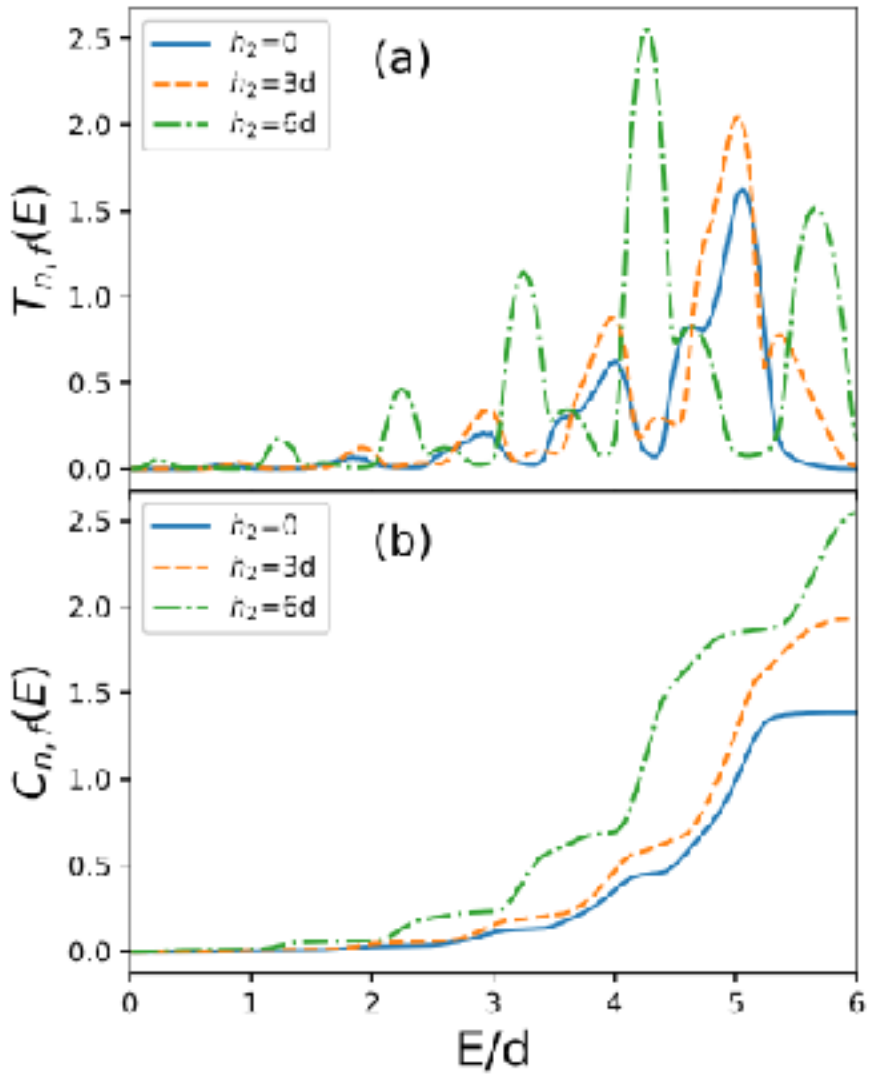
$$\frac{\langle nQ|v_{db}|nQ'\rangle}{\langle nQ|nQ'\rangle} = \frac{E(nQ) + E(nQ')}{2} - h_2(Q - Q')^2$$

Define cumulative sum of $T_{n,fis}(E)$

$$C_{n,fis}(E) = \int_0^E T_{n,fis}(E')dE' \ .$$

Diabatic transition increases barrier transmission

$T_{n,fis}(E)$ and $C_{n,fis}(E)$



$(v_{np} = 0.03d \ , B_h = 4d, \ G = 0, \ \lambda = 1)$

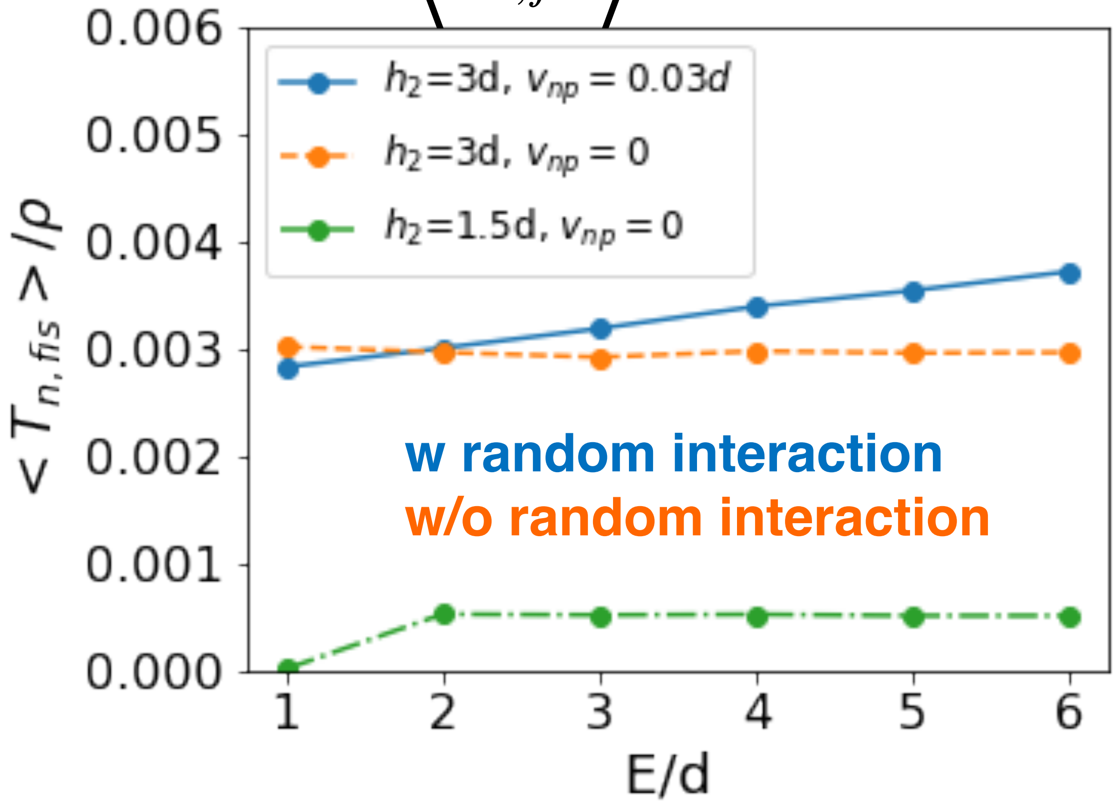
Relation between $\langle T_{n,fis} \rangle (E)$ and excitation energy E

- $\langle T_{n,fis} \rangle$ increases as E with finite v_{np}
- $\langle T_{n,fis} \rangle$ is insensitive to E if $v_{np} = 0$

✓Transmission by h_2 is insensitive to E

✓Diabatic transition becomes important relatively in low-energy induced fission

$\langle T_{n,fis} \rangle (E) / \rho(E)$



$(v_{np} = 0.03d\ , B_h = 4d,\ G = 0,\ \lambda = 1)$

Results 3: Transition state theory

$$\Gamma_{\rm BW} = \frac{1}{2\pi\rho}\sum_c T_c$$

Γ_{BW}:fission decay width

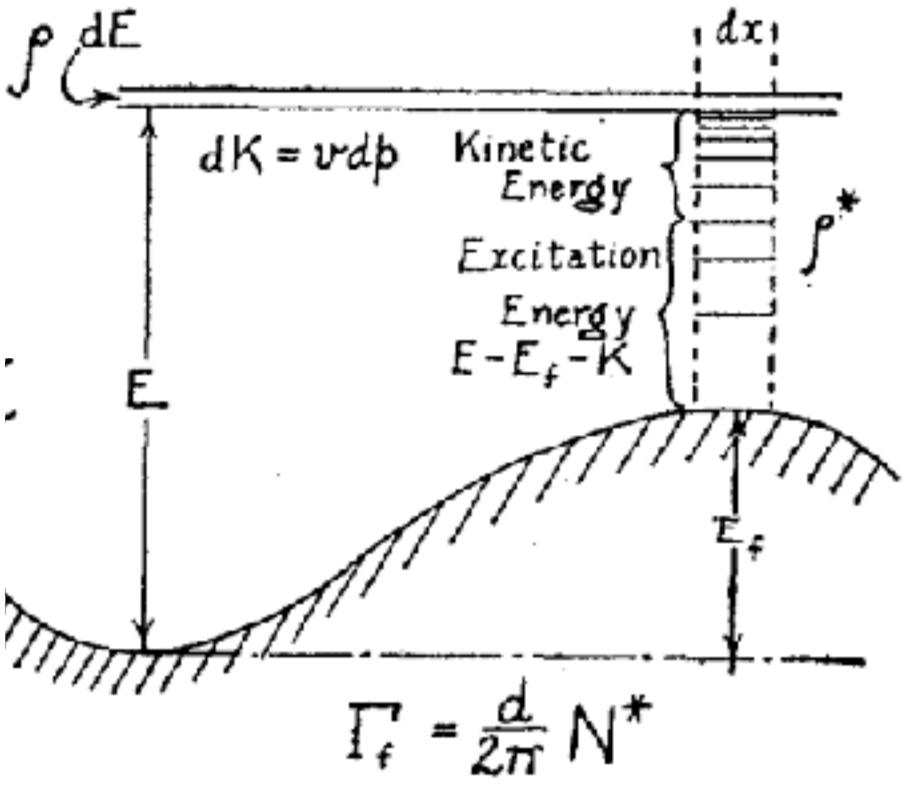
ρ: level density of the compound nuclei

T_c: transmission probability via transition state

B-W formula assumes

- ➢ mother nucleus is in statistical equilibrium
- ➢ transition states determine the fission dynamics

Justify the assumption microscopic viewpoint

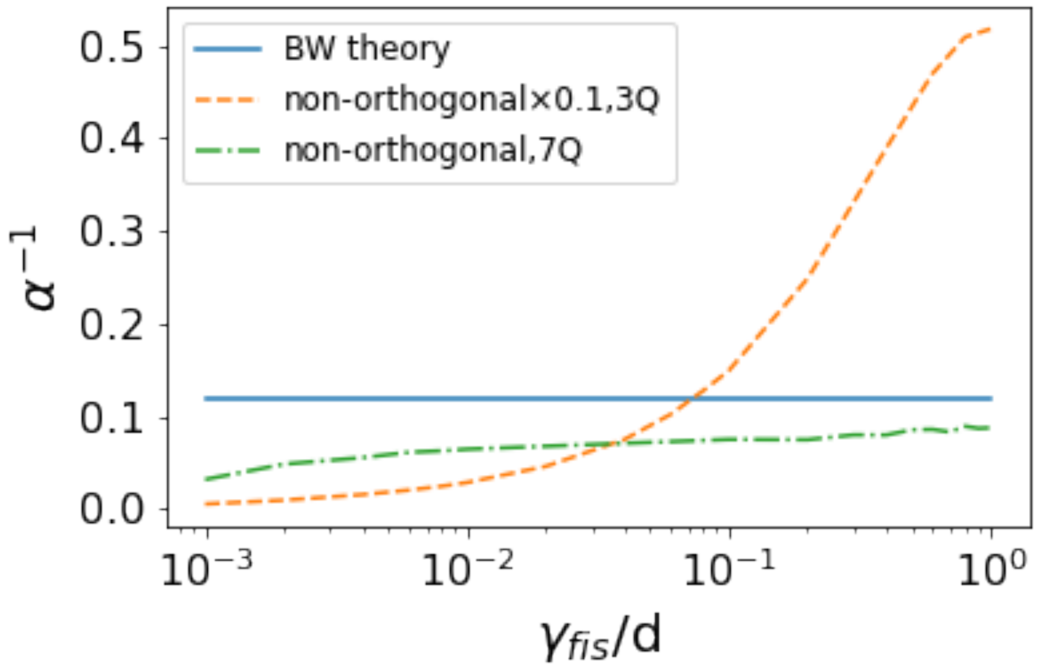


N. Bohr and A. Wheeler, Phys. Rev. 56 426 (1939).

Focus on the capture-fission branching ratio α^{-1},

$$\alpha^{-1} = \frac{\int dE' \, T_{n,fis}(E')}{\int dE' \, T_{n,cap}(E')}$$

α^{-1} vs γ_{fis}(fission width)



✓ α^{-1} is constant in the BW theory

✓ α^{-1} increases as γ_f in Q=(-1, 0, 1) case

(not consistent with BW theory)

✓ 7-Q block model reproduces BW theory's results

⇒justify the transition state hypothesis microscopically

$(v_{np} = 0.03d \, , B_h = 4d, \; G = 0, \; \lambda = 1)$

Summary

Conclusion

- Apply GCM+CI approach to the barrier transmission problem
- Pairing increases fission probability and competes random interaction
- Diabatic transition enhances fission probability and show insensitivity to E
- Transition states hypothesis is justified microscopically within our approach

Future perspectives

- Realistic calculation with DFT and development of efficient numerical methods