What is chiral susceptibility probing?

OSAKA UNIVERSITY
Live Locally, Grow Globally

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for JLQCD collaboration

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(and preliminary results in Nf=2+1simulations)

Acknowledgments

Resources:

- Fugaku (hp200130, hp210165, hp210231, hp220279)
- Oakforest-PACS [JCAHPC]
  HPCI projects : hp170061, hp180061, hp190090, hp200086, hp210104
  MCRP in CCS, U. Tsukuba : xg17i032 and xg18i023
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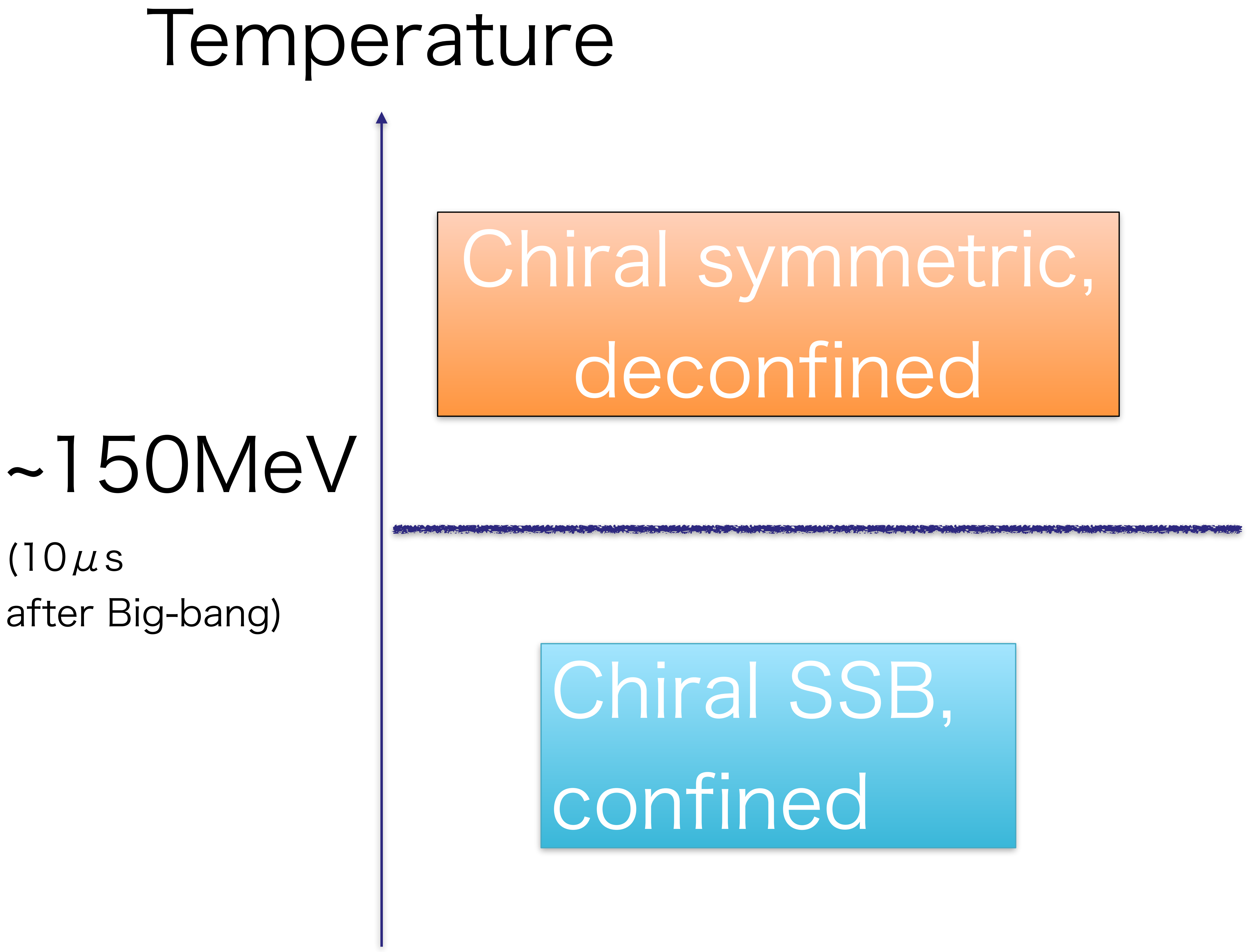
- Program for Promoting Researches on the Supercomputer Fugaku, Simulation for basic science: from fundamental laws of particles to creation of nuclei Joint
- Institute for Computational Fundamental Science (JICFuS)

QCD phase transition



Chiral condensate (at m=0) probes $SU(2)_L x SU(2)_R$ symmetry breaking/ restoration :

For T>Tc, $\langle \bar{q}q \rangle = 0$

For T<Tc, $\langle \bar{q}q \rangle \neq 0$

Chiral susceptibility

QCD partition function A : gluon fields

$$Z(m) = \int [dA] \det(D(A) + m)^{N_f} e^{-S_G(A)}$$

chiral condensate

$$-\langle \bar{q}q \rangle = \frac{1}{N_f V} \frac{\partial}{\partial m} \ln Z(m)$$

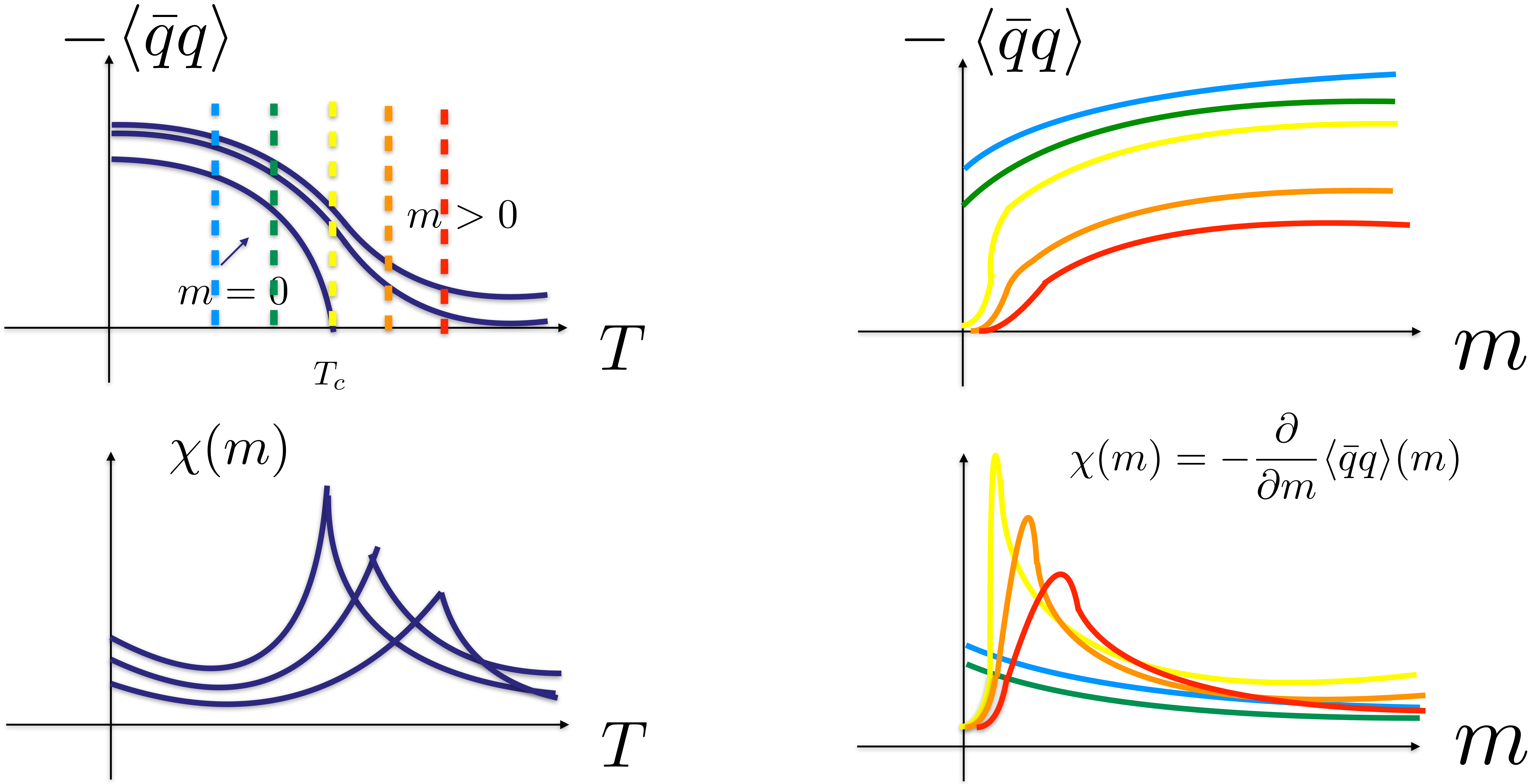
chiral susceptibility

$$\chi(m) = -\frac{\partial}{\partial m} \langle \bar{q}q \rangle (m)$$

In this talk, $N_f = 2 \quad (m_u = m_d = m)$

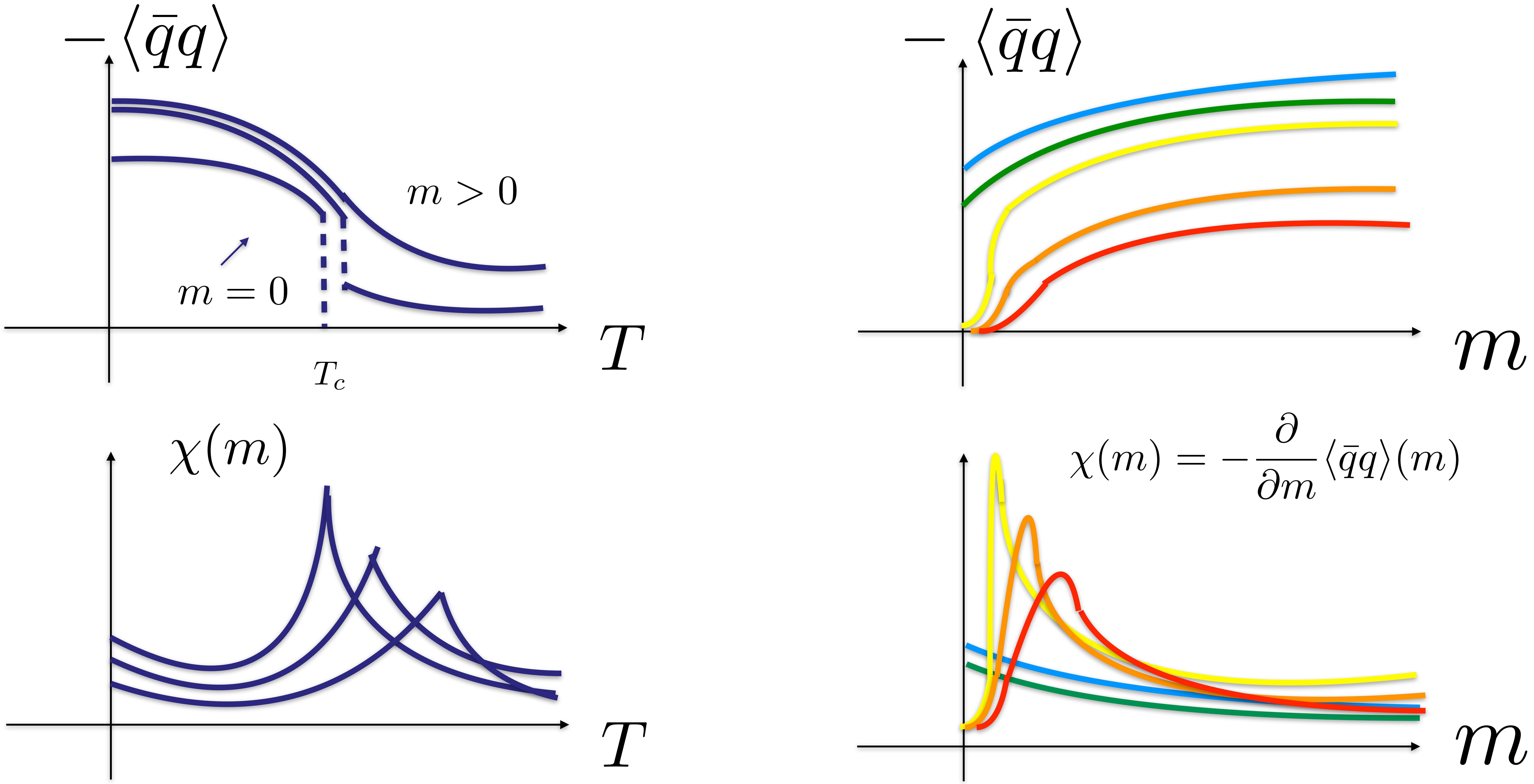
* strange quark is just a spectator.

Temperature(T) and mass(m) dependence



When the transition is 1st order

* But finite V effect makes the transition not sharp.



Chiral phase transition

Chiral condensate probes

$SU(2)_L xSU(2)_R$ symmetry breaking/restoration :

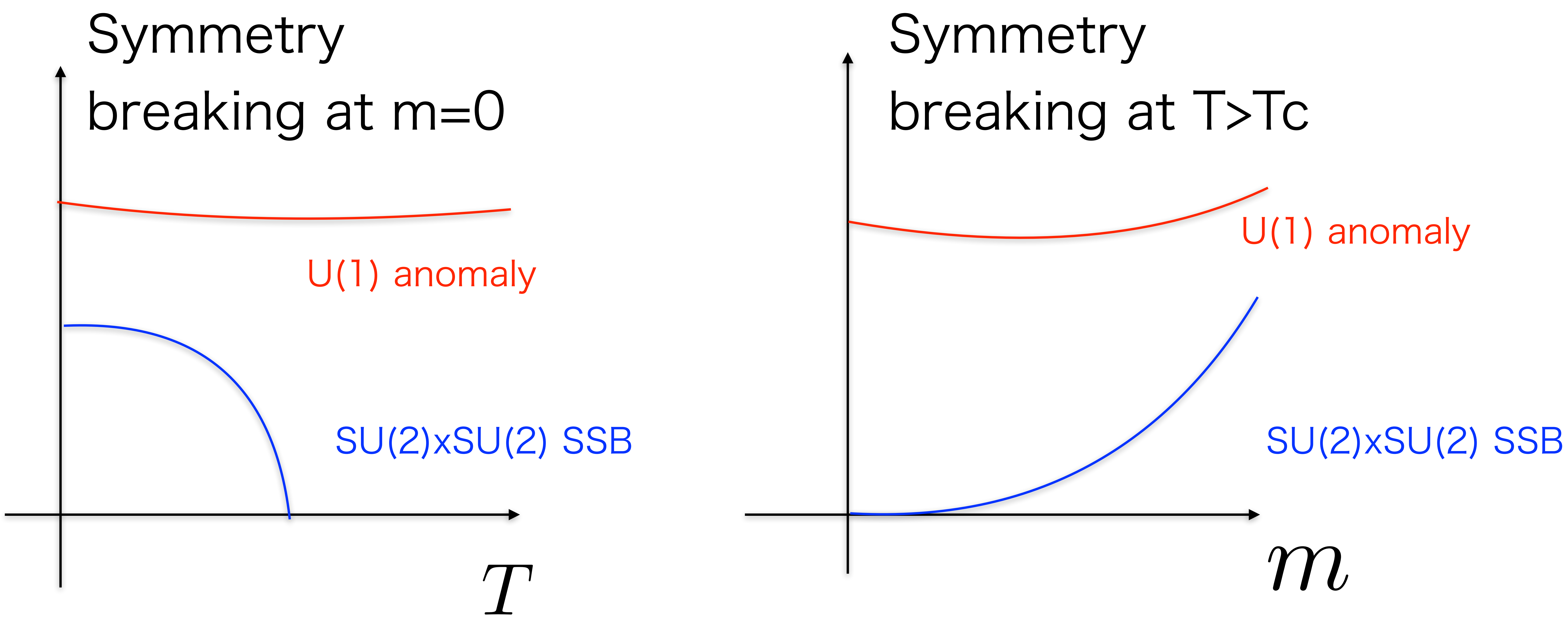
For T<Tc, $\langle \bar{q}q \rangle \neq 0$ For T>Tc, $\langle \bar{q}q \rangle = 0$

But $\langle \bar{q}q \rangle$ also breaks $U(1)_A$ symmetry.

Question:

How much does $U(1)_A$ (anomaly) contribute to the transition?

Naive expectation: U(1) anomaly exists at any energy scale (does not change much)



You may think that T and m dependences of chiral condensate should reflect $SU(2)_L x SU(2)_R$ breaking rather than U(1) anomaly.

But in early days of QCD

QCD founders in 70's and 80's thought

instanton → axial U(1) anomaly → SU(2)xSU(2) breaking.

Callan, Dashen & Gross 1978:

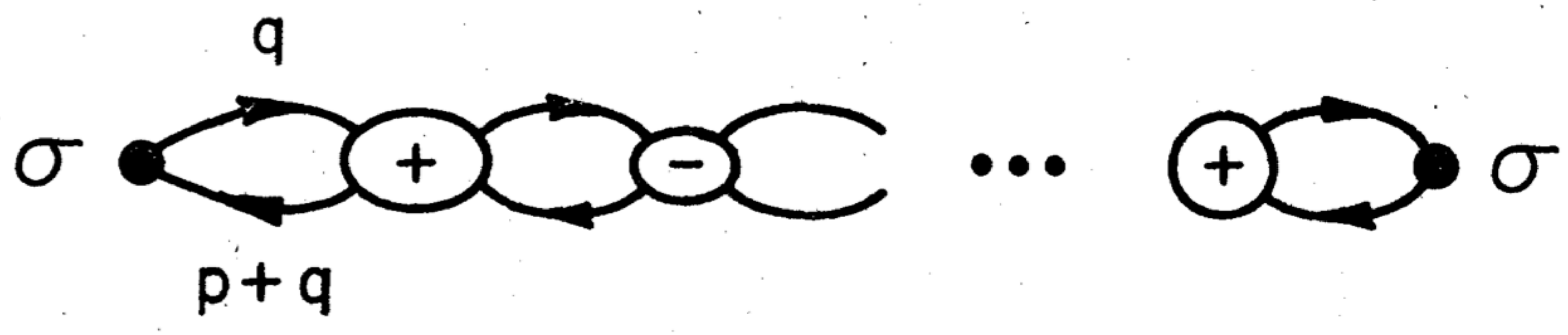


FIG. 9. The structure of the diagrams that produce a tachyon in the σ channel. The + (−) blobs refer to the effective determinantal four-fermion interaction induced by instantons (anti-instantons).

If this inverse is true, we should have

instanton disappears → anomaly disappears → SU(2)xSU(2) restored.

It has been difficult issue.

Analytic method:

Semi-classical QCD instantons are not enough to describe the low-energy dynamics of QCD.

Lattice simulations :

Staggered fermions explicitly breaks

$SU(2)_L \times SU(2)_R \times U(1)_A \rightarrow U(1)_{A'}$

Wilson fermion explicitly breaks

$SU(2)_L \times SU(2)_R \times U(1)_A \rightarrow SU(2)_V$

Moreover, we found that lattice artifacts are enhanced at high temperature (even for domain-wall fermions) [JLQCD 2015, 2016]

Our work

In this work we study chiral condensate and its susceptibility in 2- and 2+1-flavor QCD with exactly chiral symmetric Dirac operator. We separate the axial U(1) breaking (in particular topological) effect from others in a clean way.

Our result shows that signal of chiral susceptibility is dominated by axial U(1) breaking effect (at T>=Tc), rather than $SU(2)_L x SU(2)_R$.

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✔ 1. Introduction

We simulate the chiral phase transition of QCD with chiral fermions to investigate the role of axial U(1) anomaly.

2. $U(1)_A$ contribution to chiral susceptibility

3. Numerical results

4. Summary

Dirac eigenmode decomposition

$$Z(m) = \int [dA] \det(D(A)+m)^{N_f} e^{-S_G(A)} = \int [dA] \prod_\lambda (i\lambda(A)+m)^{N_f} e^{-S_G(A)}$$

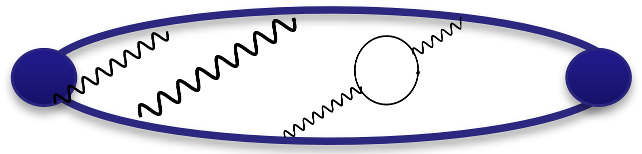
O(100) eigenvalues can be computed on the lattice.

$$-\langle \bar{q}q \rangle = \frac{1}{N_f V}\frac{\partial}{\partial m} \ln Z(m) = \frac{1}{V}\left\langle \sum_\lambda \frac{1}{i\lambda(A)+m} \right\rangle,$$

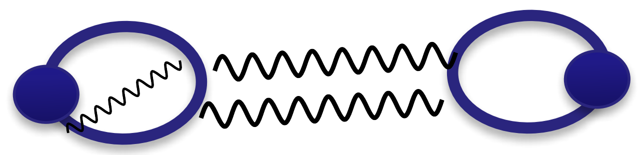
chiral susceptibility

$$\chi(m) = \frac{1}{N_f V}\frac{\partial^2}{\partial m^2} \ln Z(m) = \chi^{con.}(m) + \chi^{dis.}(m),$$

$$\chi^{con.}(m) = -\frac{\partial}{\partial m_{valence}} \langle \bar{q}q \rangle \Big|_{m_{valence}=m}$$



$$\chi^{dis.}(m) = -\frac{\partial}{\partial m_{sea}} \langle \bar{q}q \rangle \Big|_{m_{sea}=m}$$



Chiral rotations (with angle π)

$$S^0(x) = \bar{q}q(x) \quad \xleftrightarrow[\exp(i\pi\gamma_5\tau_a/2)]{SU(2)_L \times SU(2)_R} \quad P^a(x) = \bar{q}i\gamma_5\tau^a q(x)$$

$U(1)_A$ $\updownarrow$ $\exp(i\pi\gamma_5/2)$ $\qquad\qquad$ $\updownarrow$

$$P^0(x) = \bar{q}i\gamma_5 q(x) \quad \longleftrightarrow \quad S^a(x) = \bar{q}\tau^a q(x)$$

isospin singlet

isospin triplet

Relation to scalar susceptibility

$$L_{\rm QCD} = \left[\frac{1}{4}{\rm Tr}F_{\mu\nu}F^{\mu\nu} + \bar{q}\left(\gamma^\mu(\partial_\mu - igA_\mu) + m\right)q\right]$$

$$\chi(m) = \frac{1}{N_f V}\frac{\partial^2}{\partial m^2}\ln Z(m, \theta = 0)$$

$$= -\sum_x \langle S^0(x) S^0(0)\rangle - V\langle S^0\rangle^2 \qquad S^0(x) = \bar{q}q(x)$$

Relation to pseudoscalar susceptibility

$$Z(m,\theta) = \int [dA]\,\det(D(A)+m)^{N_f} e^{-S_G(A)+i\theta Q(A)}$$

$$= \int [dA]\,\det(D(A)+me^{i\gamma_5\theta/N_f})^{N_f} e^{-S_G(A)}$$

$\longleftarrow$ U(1)A rotation

$$\chi_{\text{top.}}(m) = -\frac{1}{N_f V}\frac{\partial^2}{\partial\theta^2}\ln Z(m,\theta)|_{\theta=0} = m\left[\frac{\partial}{\partial\theta}\langle\bar{q}i\gamma_5 e^{i\gamma_5\theta/N_f}q\rangle\right]|_{\theta=0}$$

$\Rightarrow$

$$\frac{N_f}{m^2}\chi_{\text{top.}}(m) = -\sum_x \langle P^0(x)P^0(0)\rangle - \frac{\langle\bar{q}q\rangle(m)}{m}.$$

$$P^0(x) = \bar{q}i\gamma_5 q(x)$$

$*N_f = 2$

Connected/disconnected pseudoscalar susceptibilities

From a Ward-Takahashi identity $0 = \langle \delta^a_{SU(2)} P^a(0) \rangle - \langle \delta^a_{SU(2)} S P^a(0) \rangle,$

we have

$$m \sum_x \langle P^a(x) P^a(0) \rangle + \langle S^0 \rangle = 0.$$

Therefore,

$$\frac{N_f}{m^2} \chi_{\text{top.}}(m) = -\sum_x \langle P^0(x) P^0(0) \rangle - \frac{\langle S(0) \rangle}{m}$$

$$= \sum_x \langle P^a(x) P^a(0) \rangle - \sum_x \langle P^0(x) P^0(0) \rangle$$

Symmetry structure of scalar/pseudoscalar susceptibilities

$\chi(m)$

$SU(2)_L \times SU(2)_R$

$$\sum_x \langle S^0(x) S^0(0) \rangle \longleftrightarrow \sum_x \langle P^a(x) P^a(0) \rangle$$

iso-singlet $\quad U(1)_A \quad\quad U(1)_A \quad$ iso-triplet

$$\sum_x \langle P^0(x) P^0(0) \rangle \longleftrightarrow \sum_x \langle S^a(x) S^a(0) \rangle$$

$SU(2)_L \times SU(2)_R$

$$-\frac{N_f}{m^2}\chi_{\mathrm{top.}}(m) - \frac{-\langle \bar{q}q \rangle(m)}{m}$$

See also LLNL/RBC Collaboration 2014, Nicola & Elvira 2018, Nicola 2020.

Connected/disconnected scalar susceptibilities

$$\chi(m) = \chi^{\text{con.}}(m) + \chi^{\text{dis.}}(m)$$

$$\chi^{\text{con.}}(m) = \sum_x \langle S^a(x)S^a(0)\rangle = \sum_x \langle S^a(x)S^a(0) - P^a(x)P^a(0)\rangle + \sum_x \langle P^a(x)P^a(0)\rangle$$

$$= -\Delta_{U(1)}(m) + \frac{-\langle\bar{q}q\rangle(m)}{m}$$

$$\chi^{\text{dis.}}(m) = \sum_x \langle S^0(x)S^0(0) - S^a(x)S^a(0)\rangle - V\langle S^0(0)\rangle^2$$

$$= \sum_x \langle \left[S^0(x)S^0(0) - \langle S^0(0)\rangle^2 - P^a(x)P^a(0)\right] + \left[P^a(x)P^a(0) - S^a(x)S^a(0)\right]\rangle$$

$$= \Delta^{(1)}_{SU(2)}(m) + \left[P^a(x)P^a(0) - P^0(x)P^0(0)\right] + \left[P^0(x)P^0(0) - S^a(x)S^a(0)\right]\rangle$$

$$= \Delta^{(1)}_{SU(2)}(m) + \frac{N_f\chi_{\text{top.}}(m)}{m^2} - \Delta^{(2)}_{SU(2)}(m)$$

Separating U(1)A breaking part

$$\chi(m) = \chi^{\mathrm{con.}}(m) + \chi^{\mathrm{dis.}}(m)$$

$$\chi^{\mathrm{con.}}(m) = \underbrace{-\Delta_{U(1)}(m) + \frac{\langle |Q(A)| \rangle}{m^2 V}}_{\text{U(1)A breaking contribution}} \underbrace{- \frac{-\langle \bar{q}q \rangle_{\mathrm{sub.}}(m)}{m}}_{\text{mixed}}$$

* quadratic divergence is subtracted using the data at reference quark mass mref=0.005.

$$\chi^{\mathrm{dis.}}(m) = \overbrace{\frac{N_f}{m^2}\chi_{\mathrm{top.}}(m)}^{\text{U(1)A breaking contribution}} \underbrace{+\Delta^{(1)}_{SU(2)}(m) - \Delta^{(2)}_{SU(2)}(m)}_{\text{SU(2)xSU(2) breaking}}$$

where $\Delta_{U(1)}(m) \equiv \sum_x \langle P^a(x)P^a(0) - S^a(x)S^a(0) \rangle$ axial U(1) susceptibility

$$\Delta^{(1)}_{SU(2)}(m) \equiv \sum_x \langle S^0(x)S^0(0) - P^a(x)P^a(0) \rangle \qquad \Delta^{(2)}_{SU(2)}(m) \equiv \sum_x \langle S^a(x)S^a(0) - P^0(x)P^0(0) \rangle$$

Lattice formulas

Using $\lambda_m = \text{eigenvalues of } H_m = \gamma_5[(1-m)D_{ov} + m]$

$$\Delta_{U(1)}(m) = \frac{1}{V(1-m^2)^2}\left\langle \sum_{\lambda_m} \frac{2m^2(1-\lambda_m^2)^2}{\lambda_m^4} \right\rangle,$$

$$-\langle\bar{q}q\rangle = \frac{1}{V(1-m^2)}\left\langle \sum_{\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2} \right\rangle.$$

$$\chi^{\text{dis.}}(m) = \frac{N_f}{V}\left[\frac{1}{(1-m^2)^2}\left\langle \left(\sum_{\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2}\right)^2 \right\rangle - |\langle\bar{q}q\rangle^{lat}|^2 V^2\right].$$

Remark.1 eigen functions do not matter.

Remark.2 chiral symmetry is essential for this decomposition.

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✔ can be separated using Ward-Takahashi identities etc.

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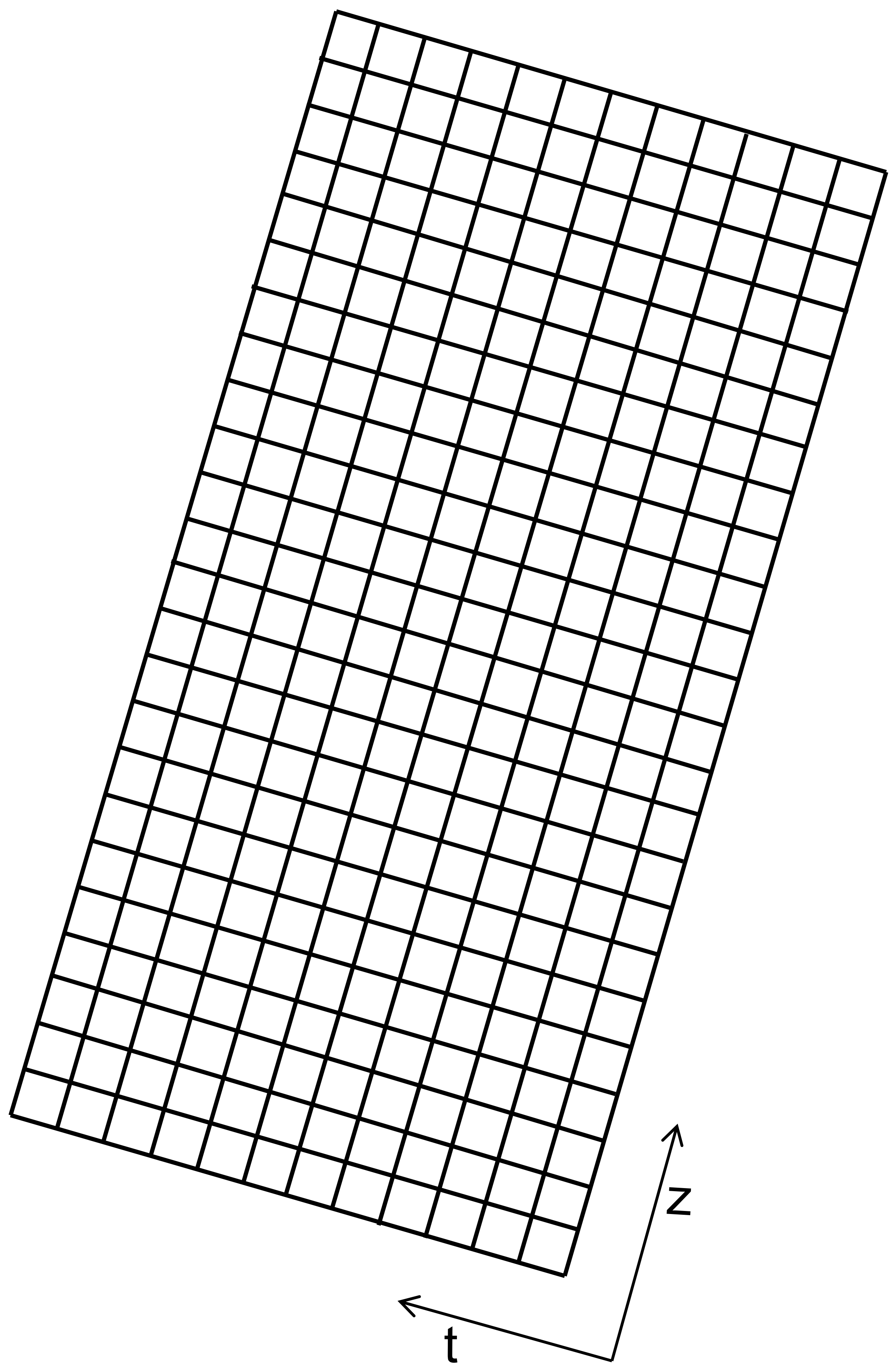
Simulation setup (Nf=2)



Nf=2 flavor QCD

1/a = 2.6 GeV (0.075fm)

Symanzik gauge action

L=24,32,40,48 [1.8-3.6fm] (at T=220MeV)

Mobius domain-wall fermions with m_{res}<1MeV

(and reweighted overlap fermion)

Quark mass from 3MeV (< phys. pt. ~4MeV) to 30MeV.

T=165 (~Tc), 195, 220, 260, 330 MeV (Lt=8,10,12,14,16)

Tc is estimated to be around 175MeV (from Polyakov loop)

Simulation codes : Irolro++ (https://github.com/coppolachan/Irolro)
Grid (https://github.com/paboyle/Grid)
Bridge++(https://bridge.kek.jp/Lattice-code/)

Simulation setup (Nf=2+1) [preliminary]

Nf=2+1 flavor QCD

1/a = 2.453GeV

L=32 (2.58,fm)

Mobius domain-wall fermion with m_{res}<1MeV

(and reweighted overlap fermion)

up-down quark mass from phys. pt. ~4MeV to 30MeV.

strange quark mass at phys.pt.

T=153(~Tc), 175, 220 MeV

Overlap vs. Mobius domain-wall

$$D_{\text{ov}}(m) = \left[\frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \,\text{sgn}(H_M)\right]$$

perfect chiral sym.

numerically m_{res} ~ 1keV

good chiral sym.

$$D^{4D}_{\text{DW}}(m) = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \frac{1-(T(H_M))^{L_s}}{1+(T(H_M))^{L_s}}$$

with L_5=16.

numerically m_{res} ~ 1MeV

$$H_M = \gamma_5 \frac{2D_W}{2+D_W}$$

OV is obtained by exactly computing the sgn function for low-modes of H_M.

Violation of chiral symmetry enhanced at finite T

Checking chiral sym. for EACH eigenmode

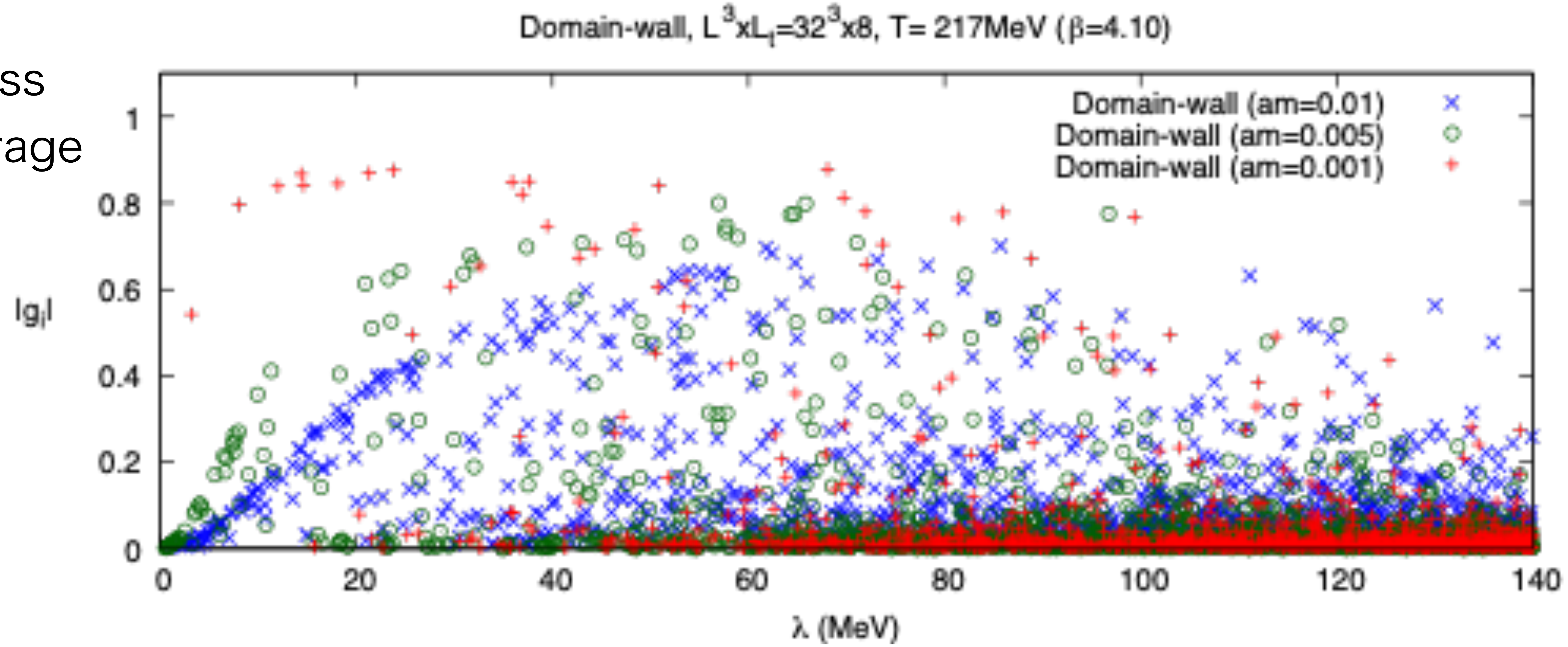
$$g_i = \left(v_i^\dagger, \frac{D\gamma_5 + \gamma_5 D - aRD\gamma_5 D}{\lambda_i} v_i \right)$$

Bad modes appear above Tc for a~0.1fm.

[JLQCD (Cossu et al.) 2015, JLQCD(Tomiya et al.) 2016]

Note: residual mass is (weighted) average of them.

For T=0, gi are consistent with residual mass.



Overlap/domain-wall reweighting

$$\langle O \rangle_{overlap} = \frac{\int dA O [\det D_{\rm ov}(m)]^2 e^{-S_G}}{\int dA [\det D_{\rm ov}(m)]^2 e^{-S_G}}$$

$$= \frac{\int dA O R [\det D_{\rm DW}^{\rm 4D}(m)]^2 e^{-S_G}}{\int dA R [\det D_{\rm DW}^{\rm 4D}(m)]^2 e^{-S_G}} \qquad R \equiv \frac{\det[D_{\rm ov}(m)]^2}{\det[D_{\rm DW}^{\rm 4D}(m)]^2}$$

$$= \frac{\langle O R \rangle_{domain-wall}}{\langle R \rangle_{domain-wall}}$$

* Use of overlap fermion in valence sector only is VERY DANGEROUS (anomaly is overestimated by1000% [JLQCD 2015]) !

Low-mode approximation

In the eigenvalue summations,

$$\Delta_{U(1)}(m) = \frac{1}{V(1-m^2)^2}\left\langle \sum_{\lambda_m} \frac{2m^2(1-\lambda_m^2)^2}{\lambda_m^4} \right\rangle,$$

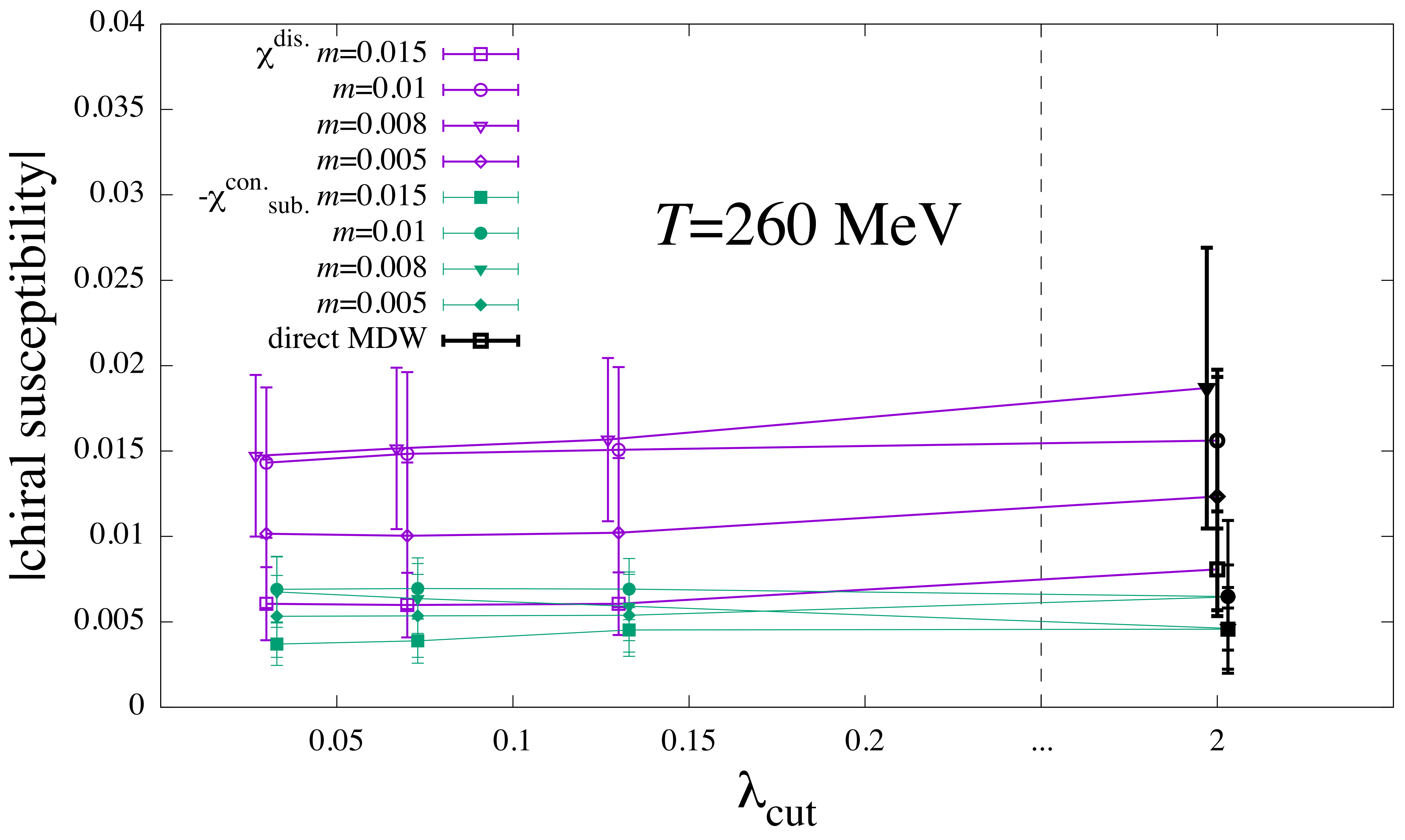
$$-\langle \bar{q}q \rangle = \frac{1}{V(1-m^2)}\left\langle \sum_{\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2} \right\rangle.$$

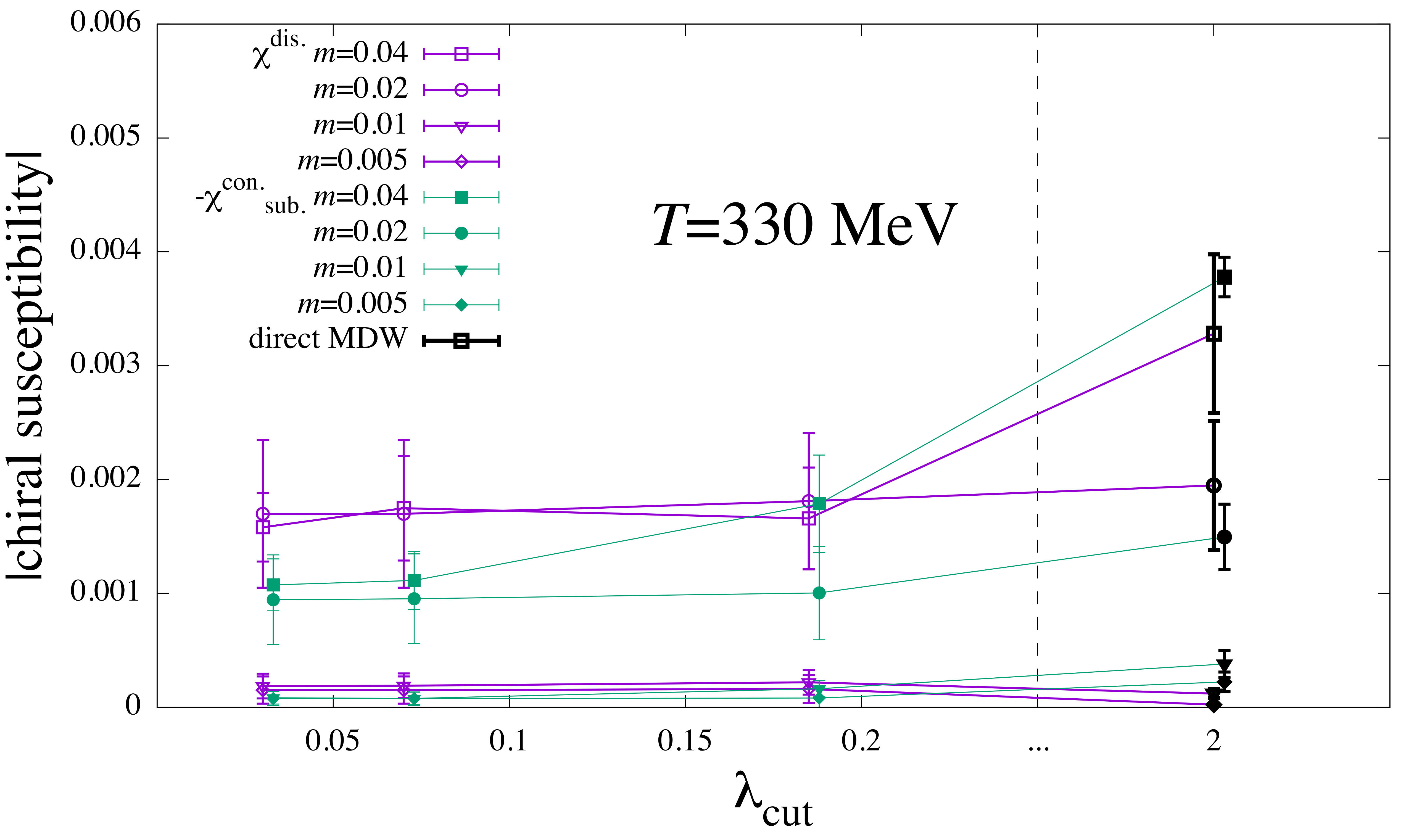
$$\chi^{\mathrm{dis.}}(m) = \frac{N_f}{V}\left[\frac{1}{(1-m^2)^2}\left\langle \left(\sum_{\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2} \right)^2 \right\rangle - |\langle \bar{q}q \rangle^{lat}|^2 V^2 \right].$$

where $\lambda_m = \text{eigenvalues of } H_m = \gamma_5[(1-m)D_{ov} + m]$

we truncate at 30-40th lowest mode ($\lambda_{\mathrm{threshold}} \sim 150\text{–}300\ \mathrm{MeV}$).

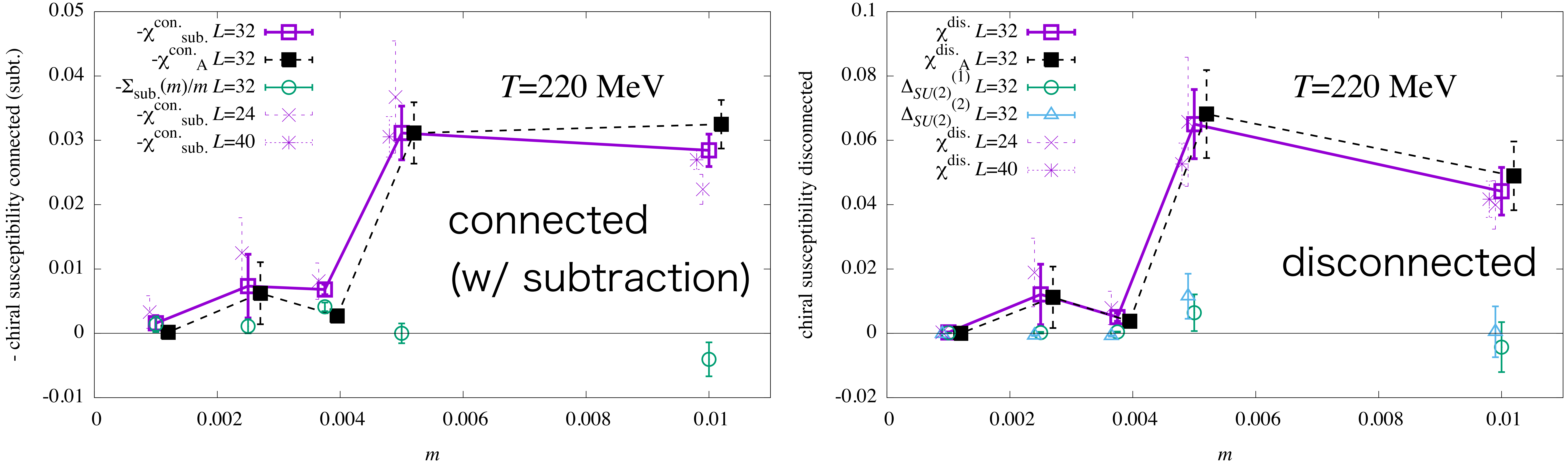
Low mode approximation





For T<= 260MeV, we find a good saturation and consistency with direct inversion of Mobius domain-wall Dirac operator (direct MDW) but T=330 MeV, it is not good; we use direct MDW.

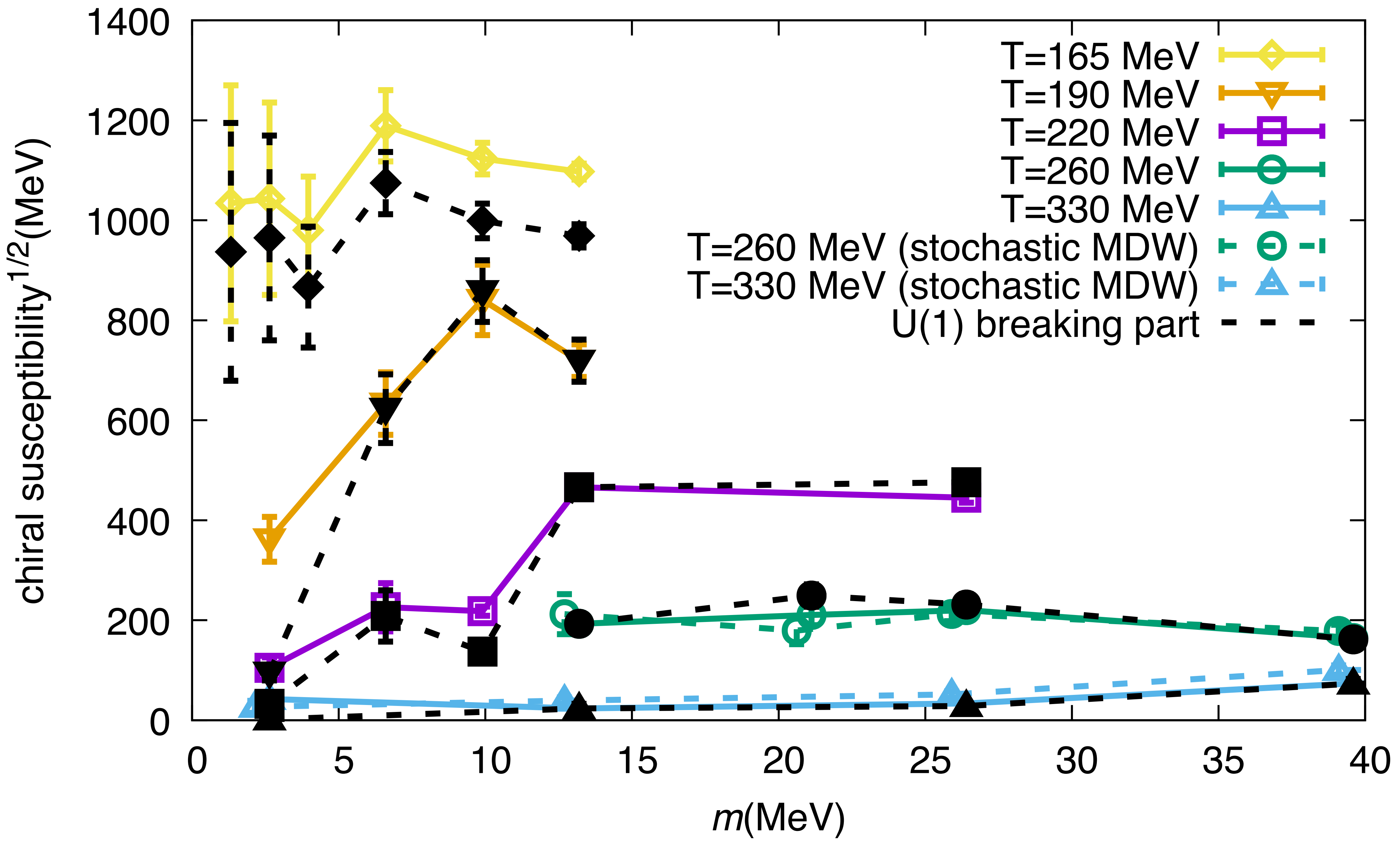
Nf=2 Result at T=220MeV

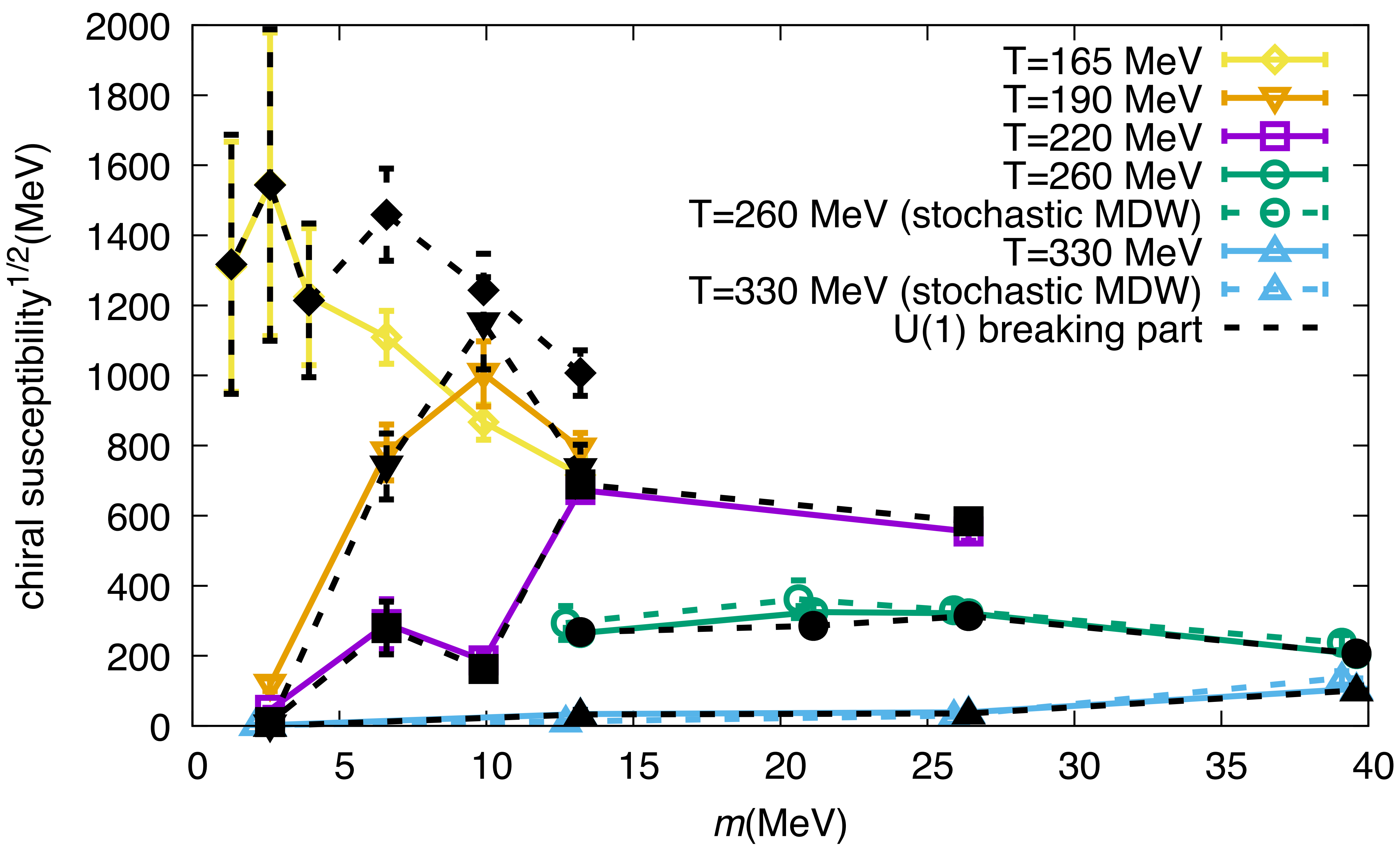


Axial U(1) anomaly dominates the signal:
connected part ~ U(1) susceptibility
disconnected ~ topological susceptibilityx2/m².
Finite V effects look under control.

Open squares : data for chiral susceptibility
filled squares : axial U(1) anomaly part
crosses and stars : data on different Vs

Nf=2 at different temperatures





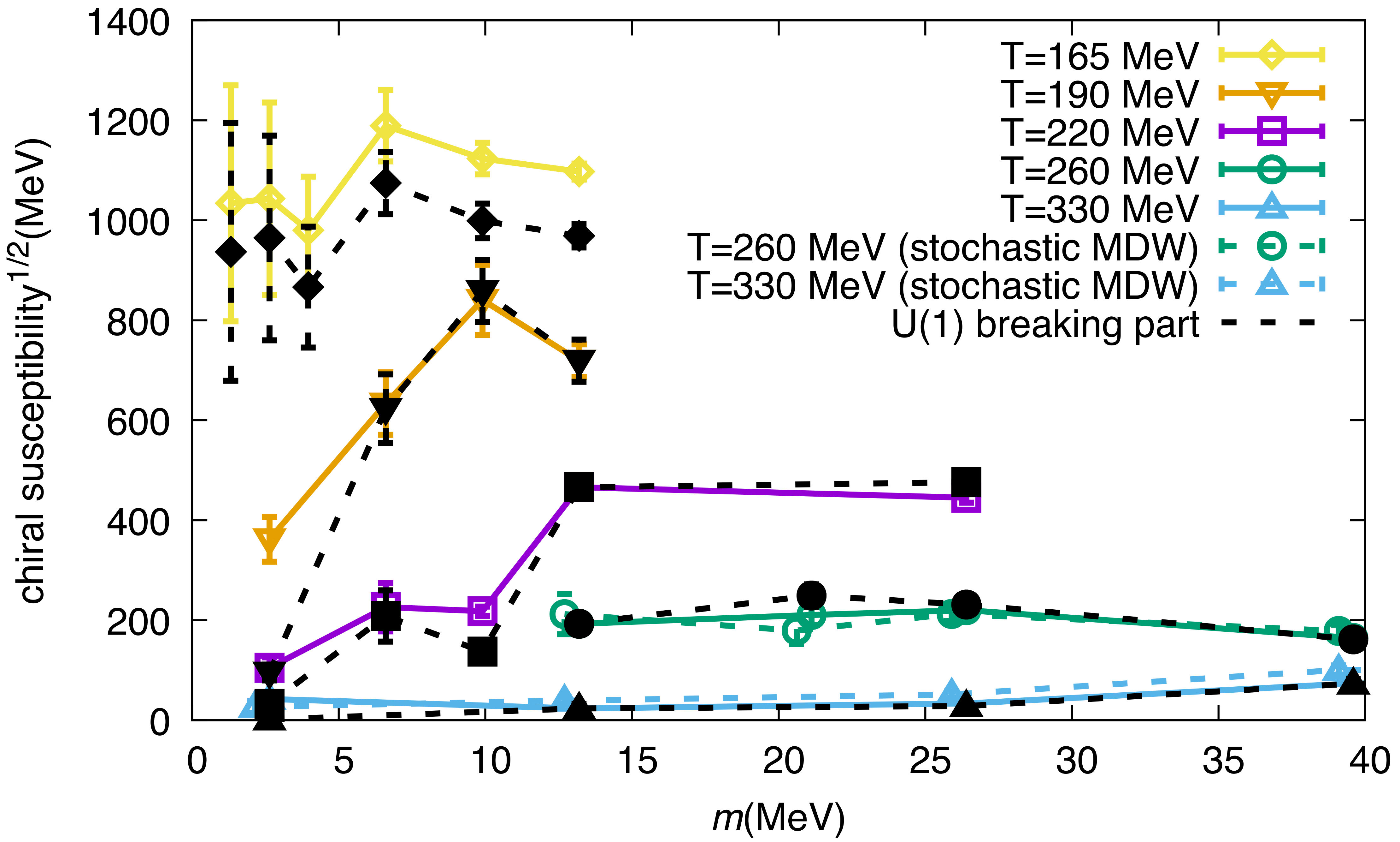
The dominance by axial U(1) anomaly is seen at 5 different Ts.

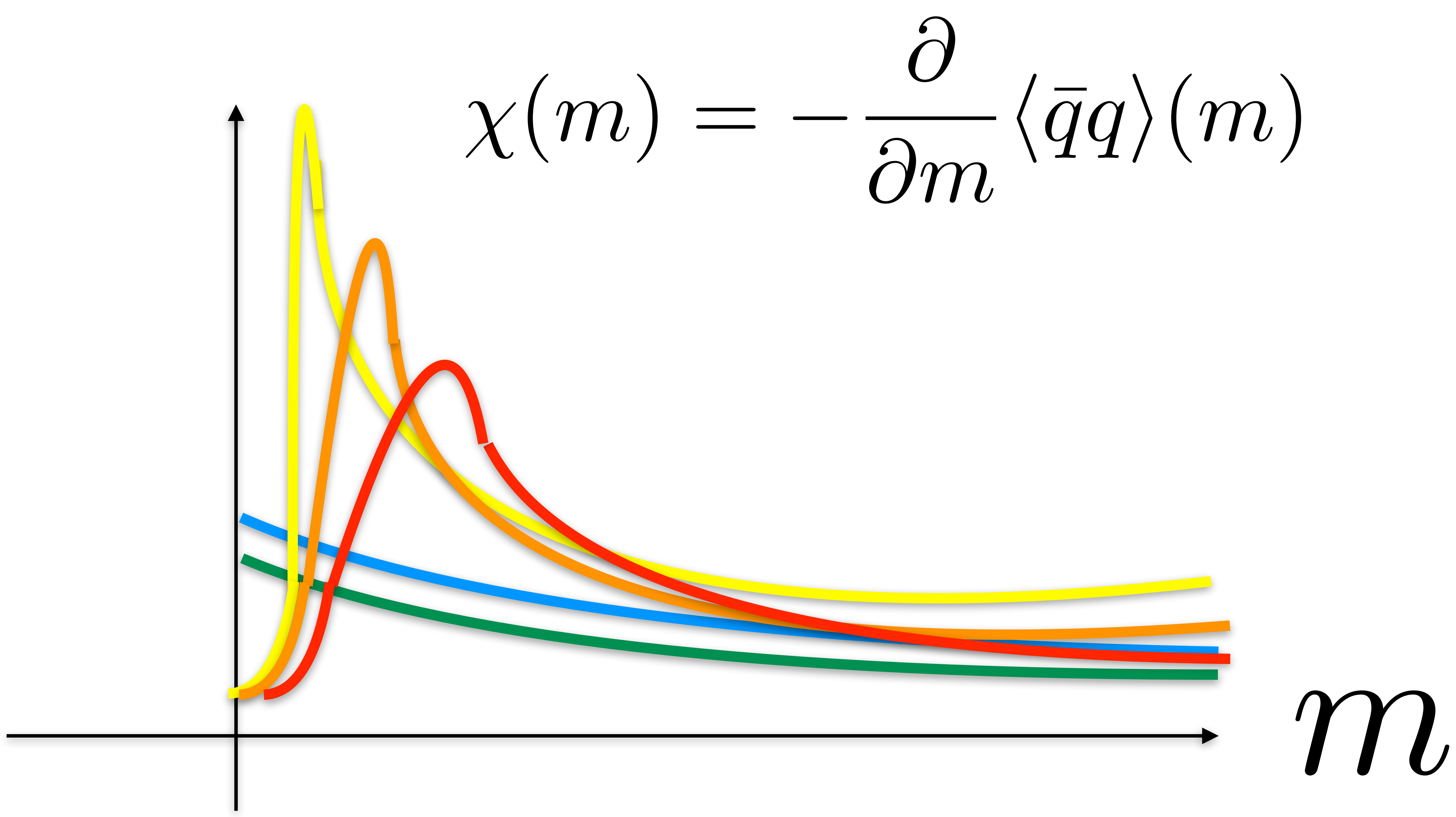
In fact, ~90% of the signal is from axial U(1) anomaly.

Also note that the chiral limit of anomaly part looks consistent with zero.

T=165 results are new.

Nf=2 at different temperatures





The dominance by axial U(1) anomaly is seen at 5 different Ts.

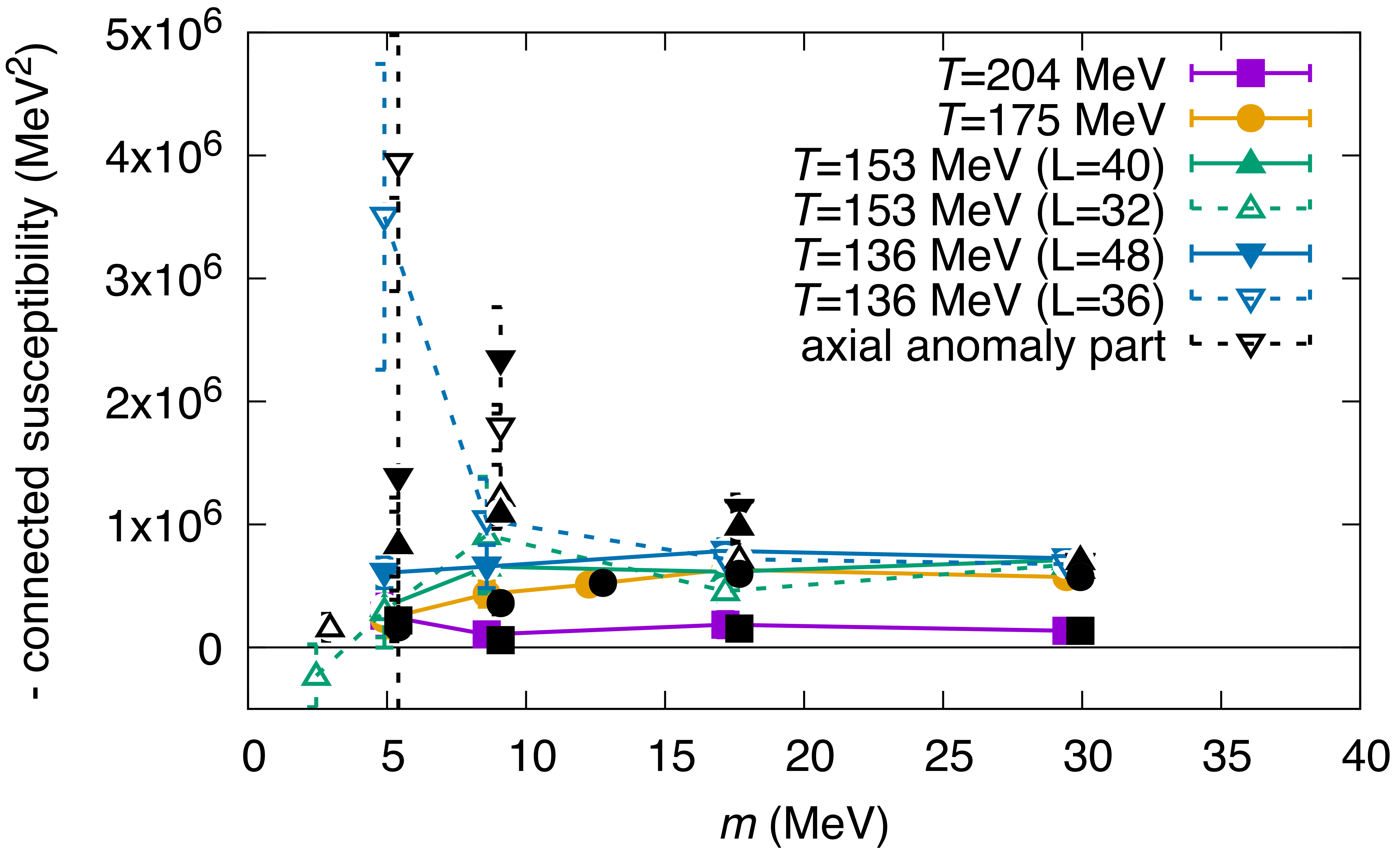
In fact, ~90% of the signal is from axial U(1) anomaly.

Also note that the chiral limit of anomaly part looks consistent with zero.

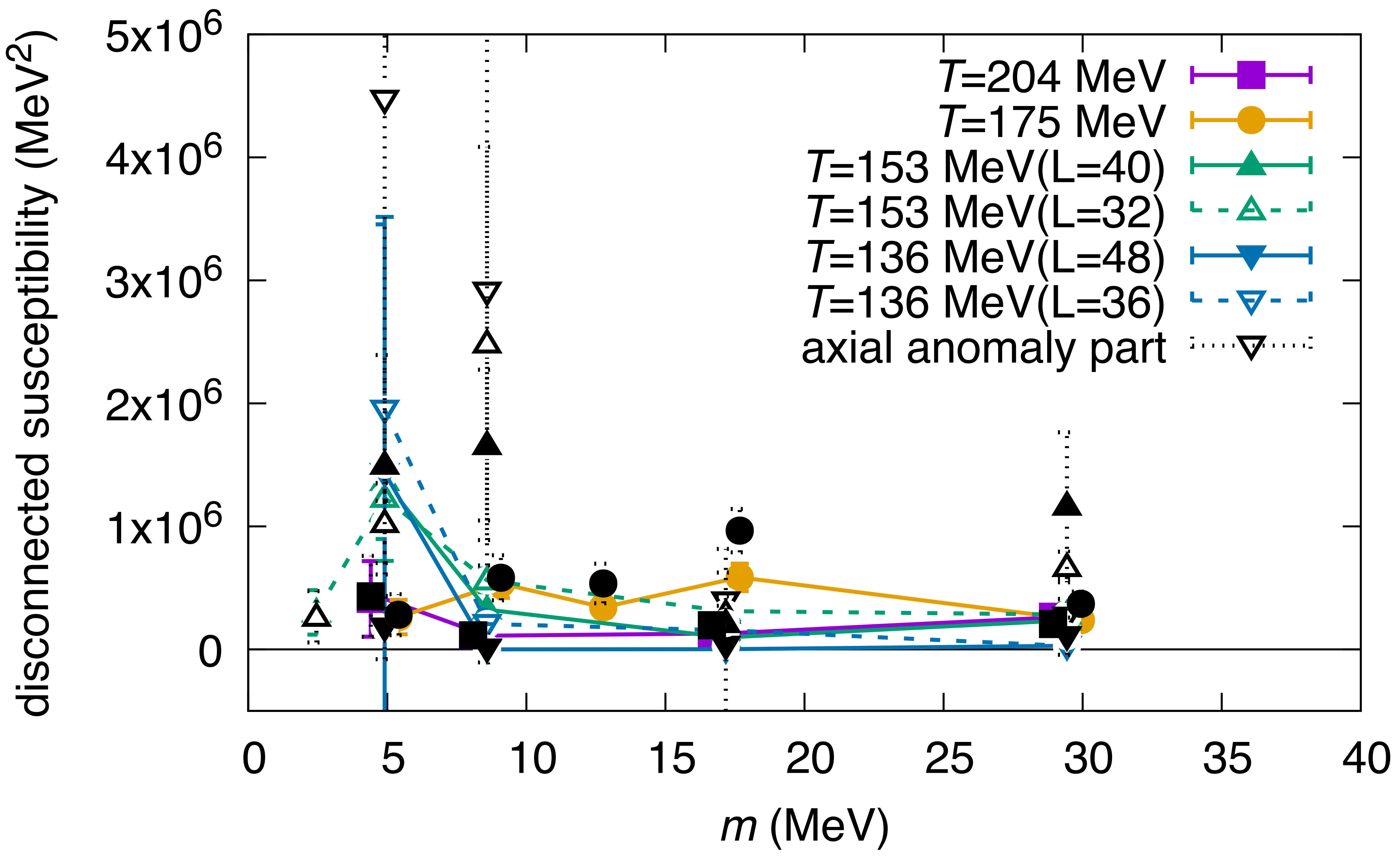
T=165MeV results are new.

Nf=2+1 preliminary results

connected



disconnected



The dominance by axial U(1) anomaly is seen at 3 different Ts.

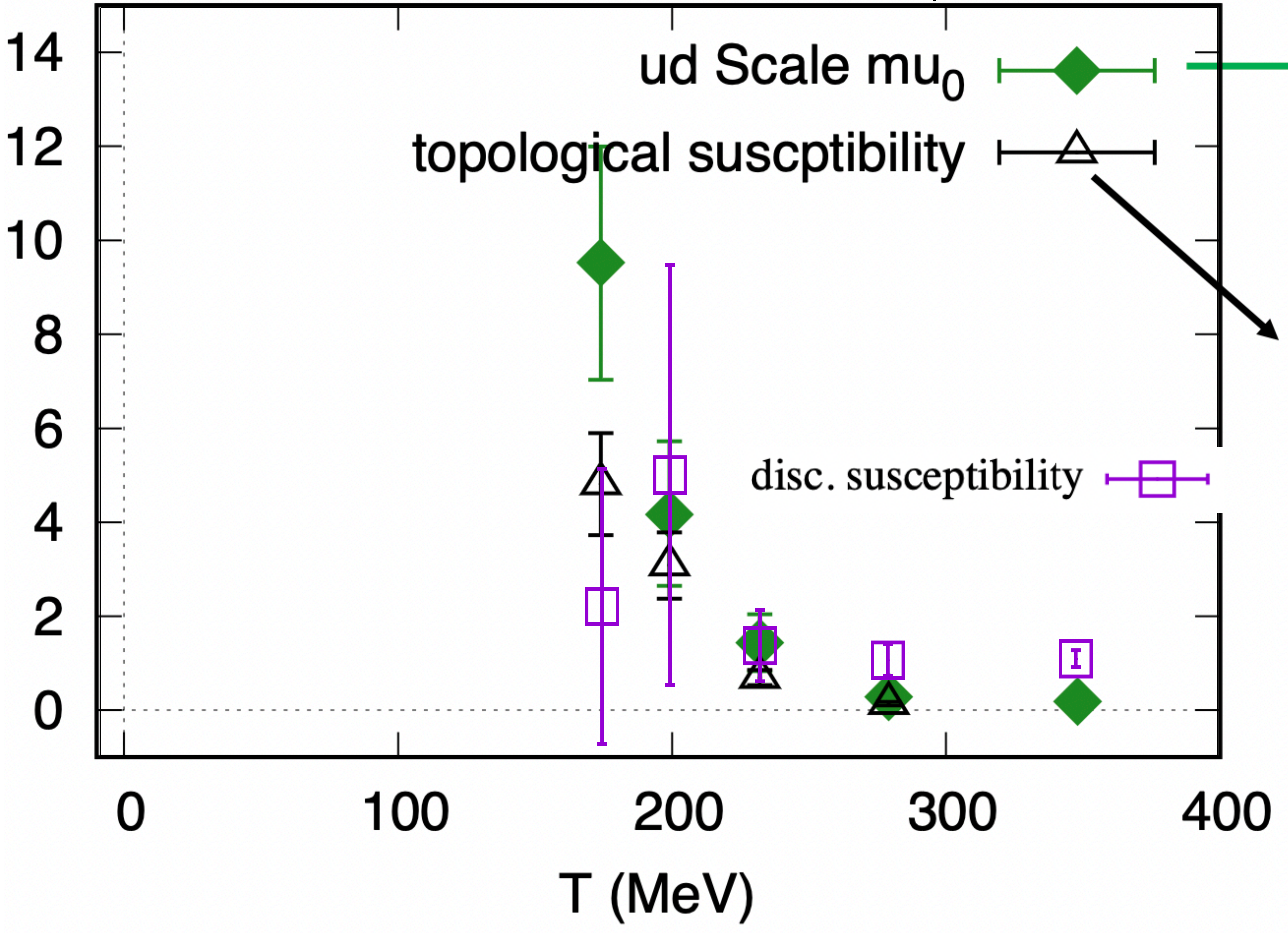
But the signals are still noisy to identify the peaks.

The same in WHOT-QCD collaboration (?)

We thank A. Baba, S. Ejiri and K.Kanaya for providing us the data.

WHOT-QCD Collaboration, Phys. Rev. D 95, no.5, 054502 (2017)

A. Baba et al. (WHOT-QCD Collaboration), talk at 76th JPS annual meeting 2021



$\chi^{\pi-\delta}/T^2$

$\chi_t/(T^2 m_{ud}^2)$

$\chi^{\text{dis.}}$ & $\chi_{\text{top.}}/m^2$

are consistent.

Subtlety in the total contribution

$$\chi(m) = \chi^{\text{con.}}(m) + \chi^{\text{dis.}}(m)$$

$$\chi^{\text{con.}}(m) = \underbrace{-\Delta_{U(1)}(m) + \frac{\langle |Q(A)| \rangle}{m^2 V}}_{\text{U(1)}_A \text{ breaking contribution}} \underbrace{- \frac{-\langle \bar{q}q \rangle_{\text{sub.}}(m)}{m}}_{\text{mixed}}$$

$$\chi^{\text{dis.}}(m) = \overbrace{\frac{N_f}{m^2}\chi_{\text{top.}}(m)}\underbrace{+\Delta^{(1)}_{SU(2)}(m) - \Delta^{(2)}_{SU(2)}(m)}_{\text{SU(2)xSU(2) breaking}}$$

a large cancellation

$O(1/V^{1/2})$ effect

It is difficult to see what survives in the total contribution.

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  We simulate the chiral phase transition of Nf=2 QCD with chiral fermions to investigate the role of axial U(1) anomaly.

- ✔ 2. $U(1)_A$ contribution to chiral susceptibility

  can be separated using Ward-Takahashi identities.

- ✔ 3. Numerical results

  The signal is dominated by axial U(1) anomaly.

- 4. Summary

Summary

1. We simulate Nf=2 and 2+1 lattice QCD.
2. Chiral condensate and susceptibility are related to both SU(2)xSU(2) and $U(1)_A$.
3. In the spectral decomposition of the Dirac operator with exact chiral symmetry, we can separate the purely U(1) anomaly effect.
4. Connected/disconnected susceptibilities are dominated by U(1) breaking at T>=Tc.

   Connected part ~ axial U(1) susceptibility.

   Disconnected part ~ top. susceptibility x $2/m^2$

Axial U(1) anomaly may play more important role in the QCD phase diagram than expected.

Take-home message

$$\frac{\partial}{\partial m}\langle\bar{\psi}\psi\rangle$$

is probing not only $SU(2)_L \times SU(2)_R$ but also $U(1)_A$ breaking/restoration.

At T>=165MeV in Nf=2 QCD (>=153MeV in Nf=2+1),

U(1)$_A$ anomaly dominates the signal of connected/disconnected susceptibilities.