# Velocity of Sound beyond the High-Density Relativistic Limit from Lattice Simulation of Dense Two-Color QCD

Etsuko Itou (RIKEN/ Keio U. / Osaka U.)

Based on K.Iida and EI, PTEP 2022 (2022) 11, 111B01

Challenges and opportunities in Lattice QCD simulations and related fields, 2023/02/17, R-CCS, RIKEN (Kobe)

# $c_s^2/c^2 > 1/3$ is found by Lattice Simulation in Dense Two-Color QCD

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# Why 2-color QCD at finite density?

# 2color QCD ≈ 3color QCD at $\mu = 0$

- 2color QCD
  reduced model with color d.o.f. in real QCD

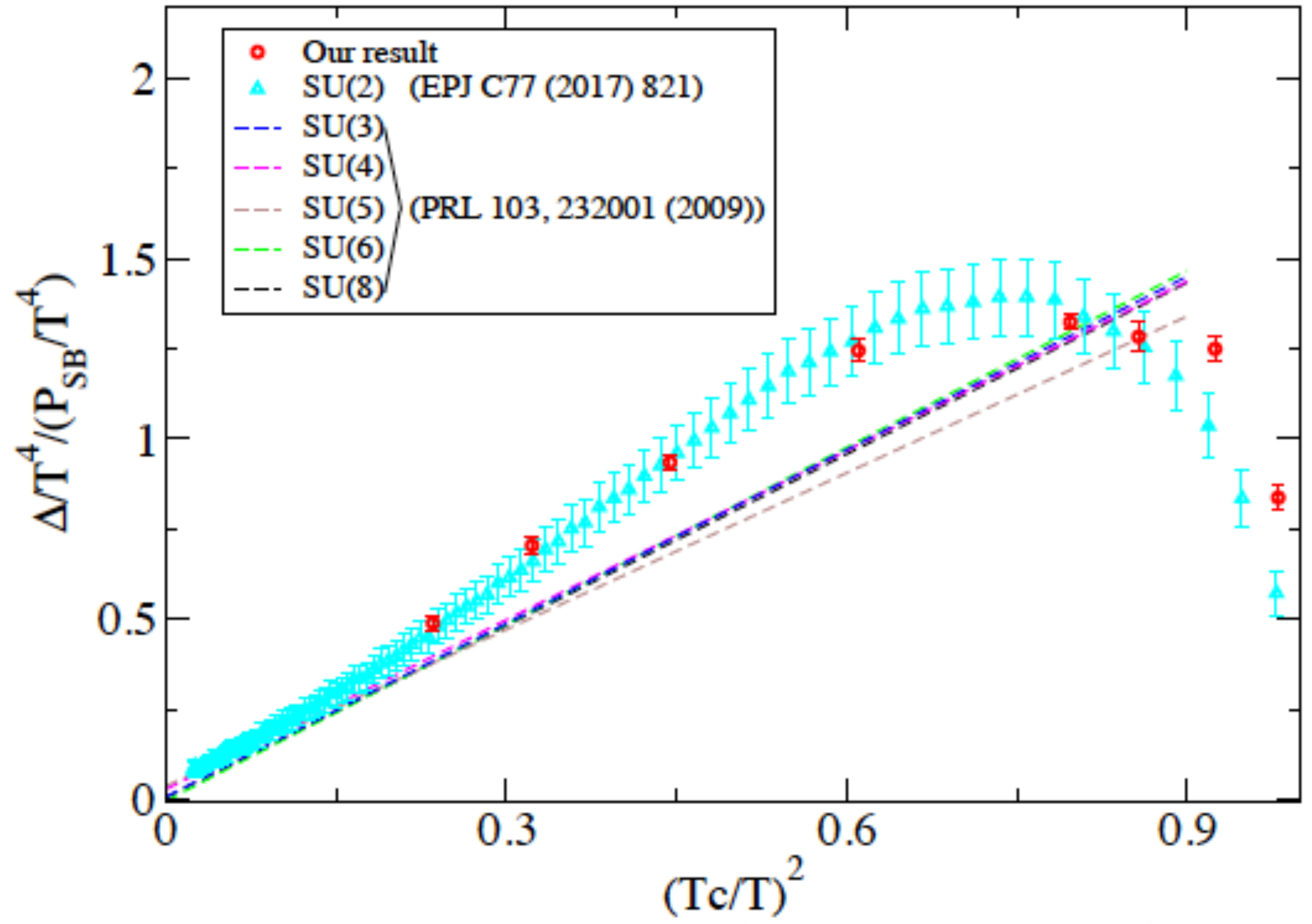
- Properties of 3color QCD at $\mu = 0$
  - asymptotic freedom
  - finite T transition (chiral/confinement)
  - pseudo-scalar meson is lightest(pion)
    cf.) QCD inequality
  - EoS(energy, pressure)

- Qualitatively, 2color QCD has the same ones

- Quantitatively, EoS shows very similar
  at least quenched QCD case

Trace anomaly ($\Delta = (\epsilon - 3p)$) of pure SU(Nc)

gauge theories with several Nc



**T. Hirakida, EI, H. Kouno**
**PTEP 2019 (2019) 033B01**

In 2color QCD at $\mu \neq 0$,

the sign problem is absent.

Find qualitative property of real dense 3color QCD

# Our projects

- K.Iida, EI, T.-G. Lee: JHEP2001(2020)181

  Phase diagram by Lattice simulation
- T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

  Phase diagram by 't Hooft anomaly matching
- K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0

  Scale setting of Lattice simulation
- K.Iida, K.Ishiguro, EI, arXiv: 2111.13067 (PoS, Lattice 2021)

  Flux tube and quark confinement by Lattice simulation
- K.Iida, EI, PTEP 2022 (2022) 11, 111B01

  Velocity of sound by Lattice simulation
- D. Suenaga, K.Murakami, EI, K.Iida, arXiv:2211.01789 (to appear in PRD)

  Mass spectrum using effective model
- K.Murakami, D.Suenaga, K.Iida, EI, arXiv:2211.13472 (PoS, Lattice 2022)

  Mass spectrum by Lattice simulation

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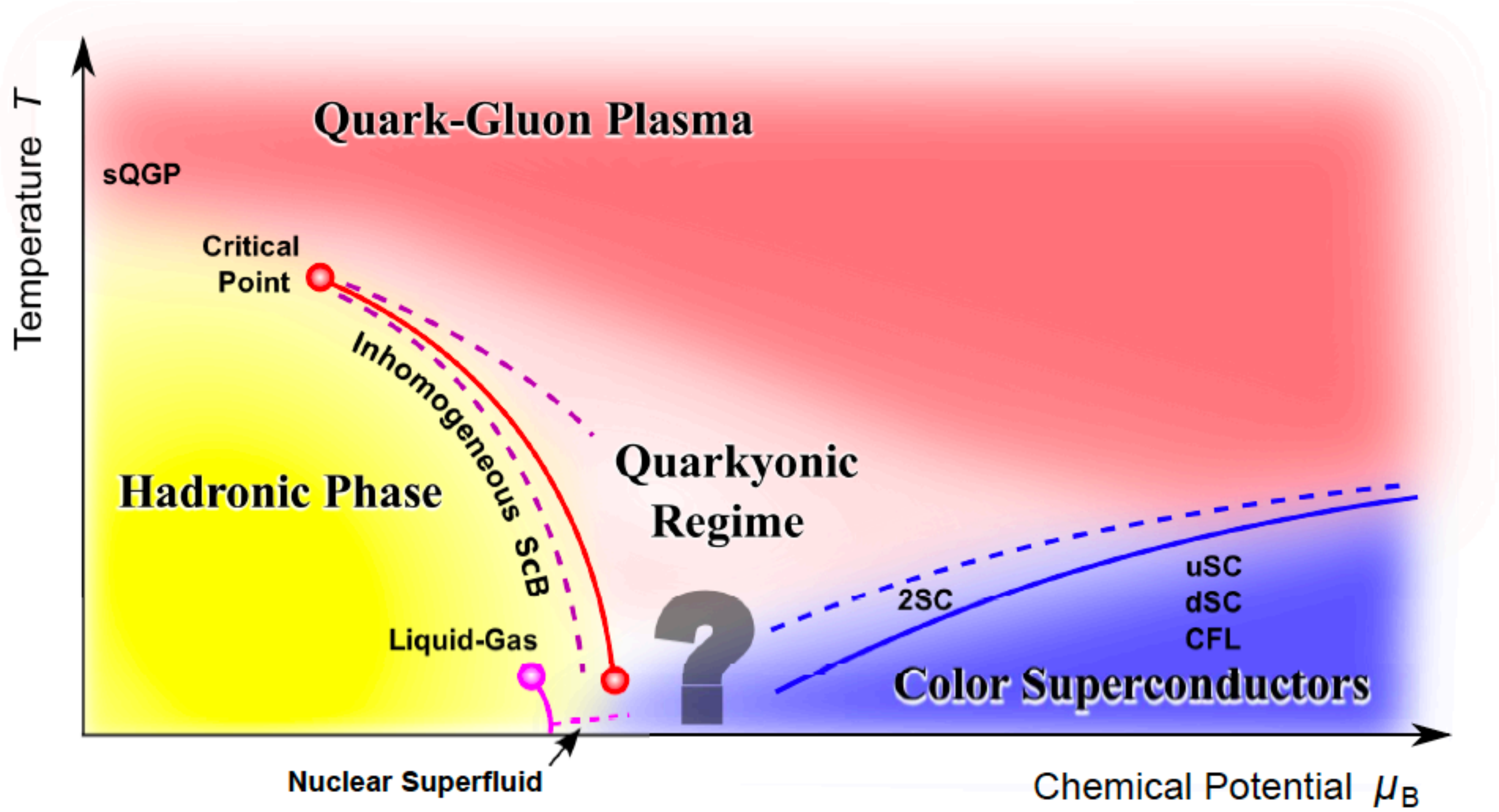
# Introduction: finite-T transition

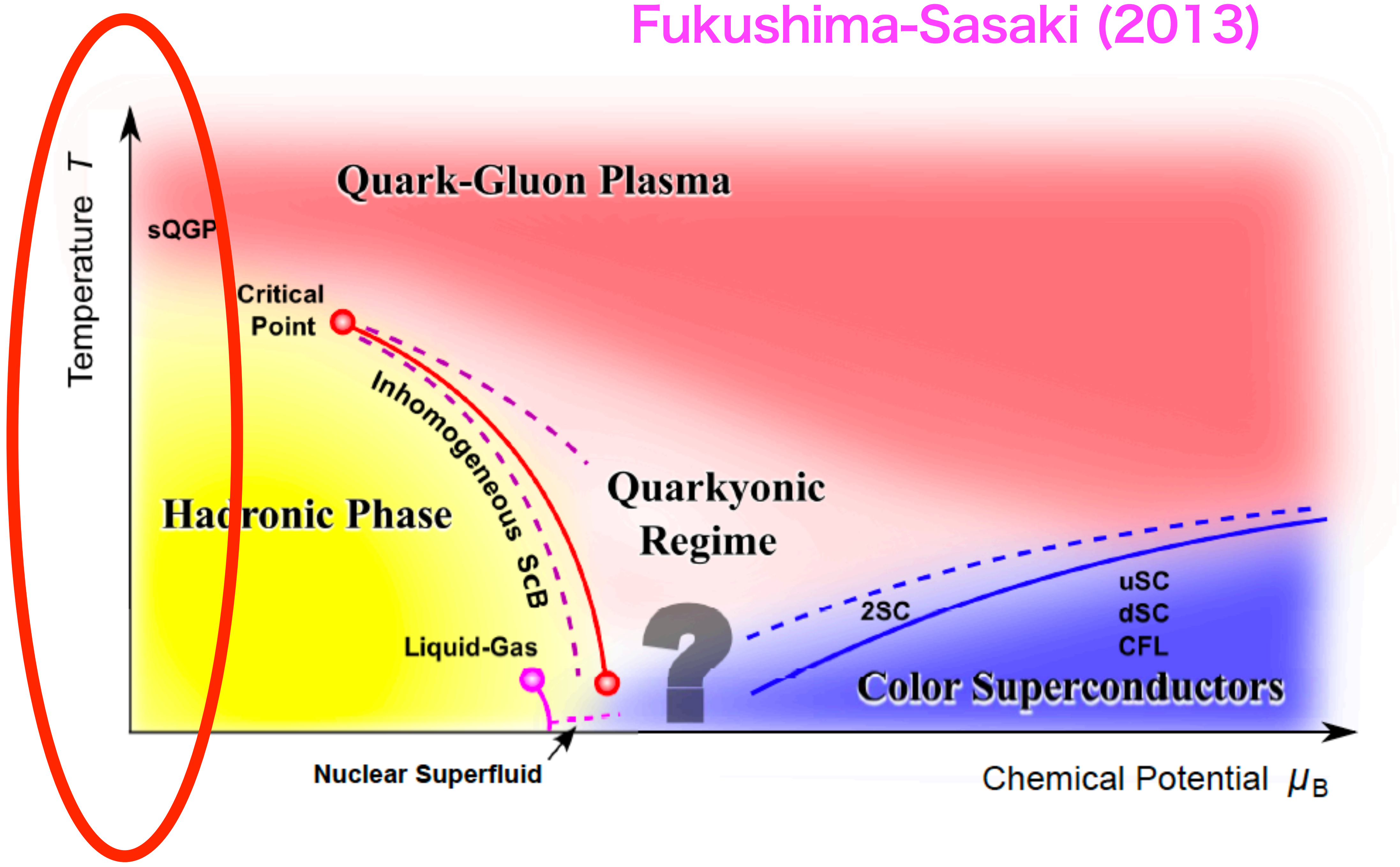
Fukushima-Hatsuda (2010)
Fukushima-Sasaki (2013)



# Introduction: finite-T transition

## EoS and sound velocity at zero-$\mu$

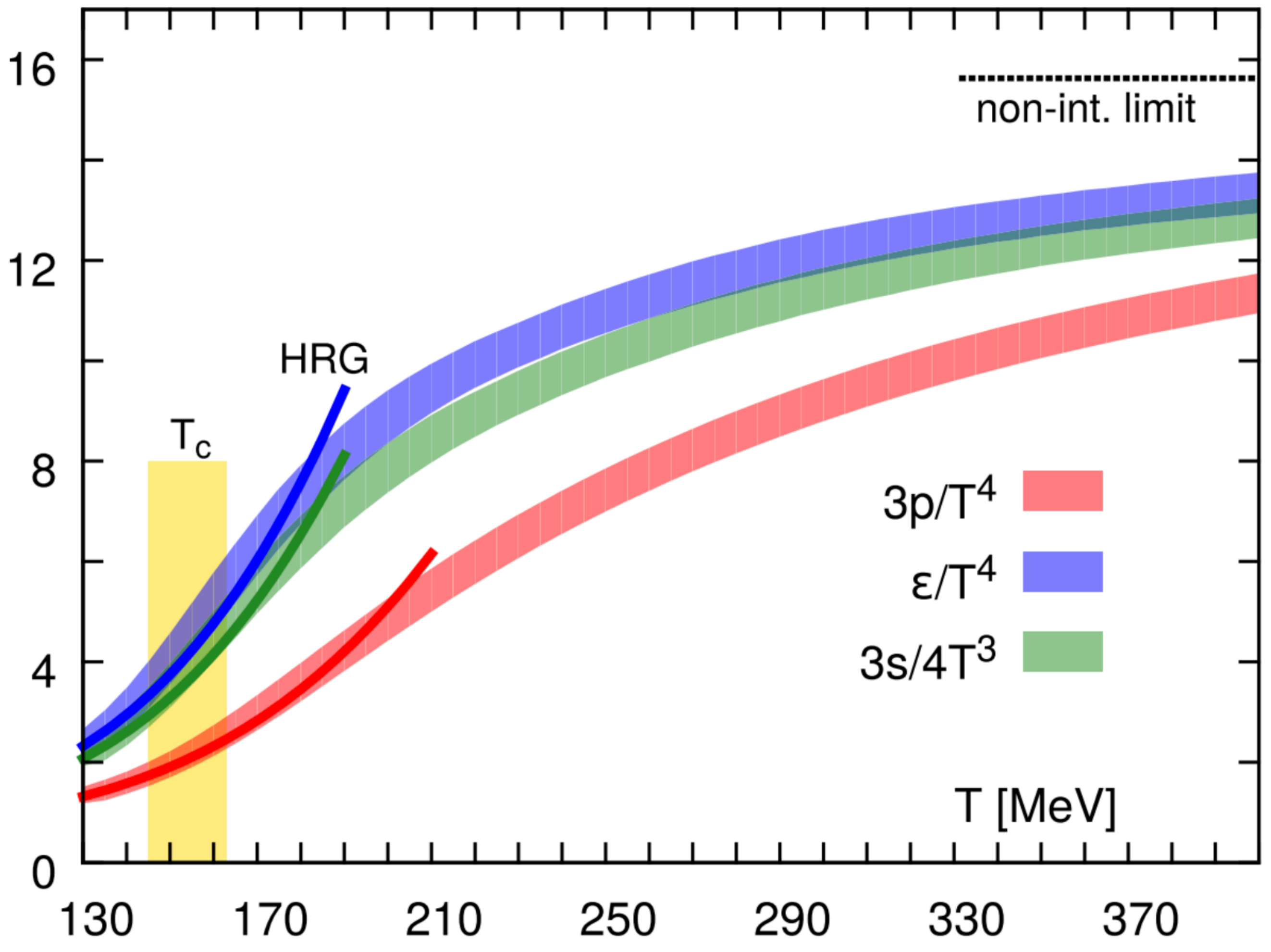
Fukushima-Hatsuda (2010)
Fukushima-Sasaki (2013)



### Finite Temperature transition (Nf=2+1 QCD)

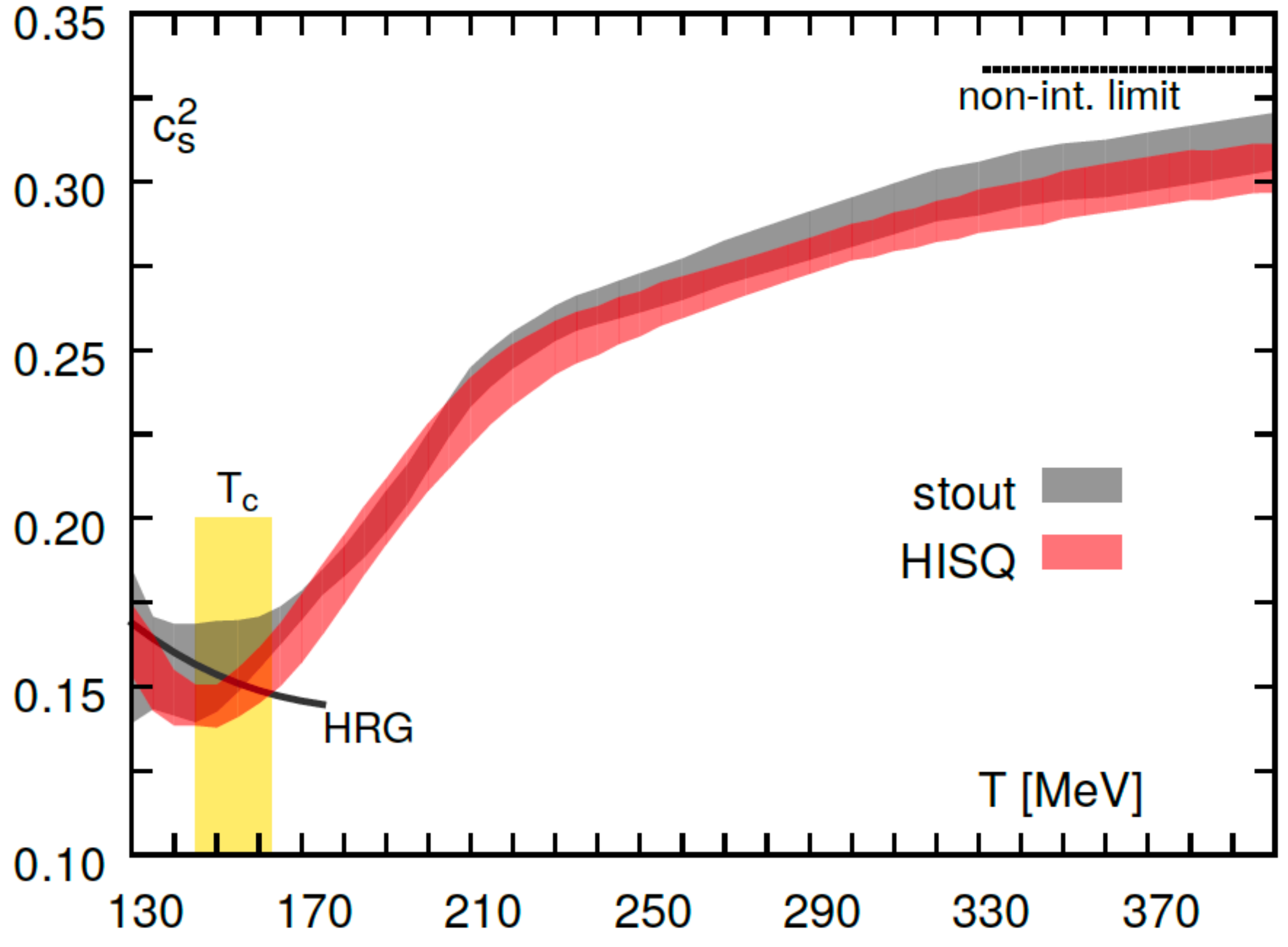
EoS

($p$ and $\varepsilon$)



Sound velocity

$c_s^2 = \partial p/\partial\epsilon$

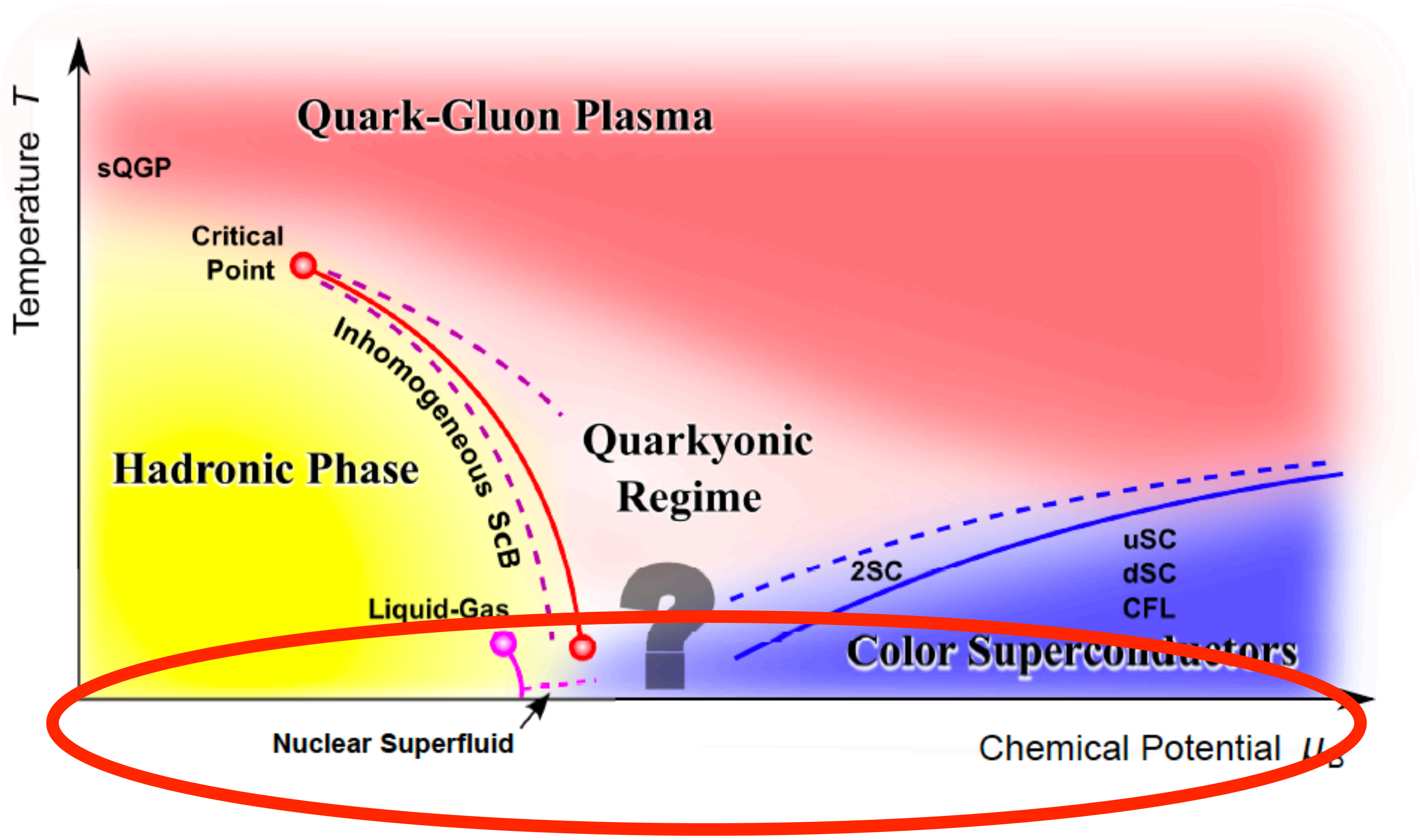


HotQCD (2014)

# Introduction: Today's talk

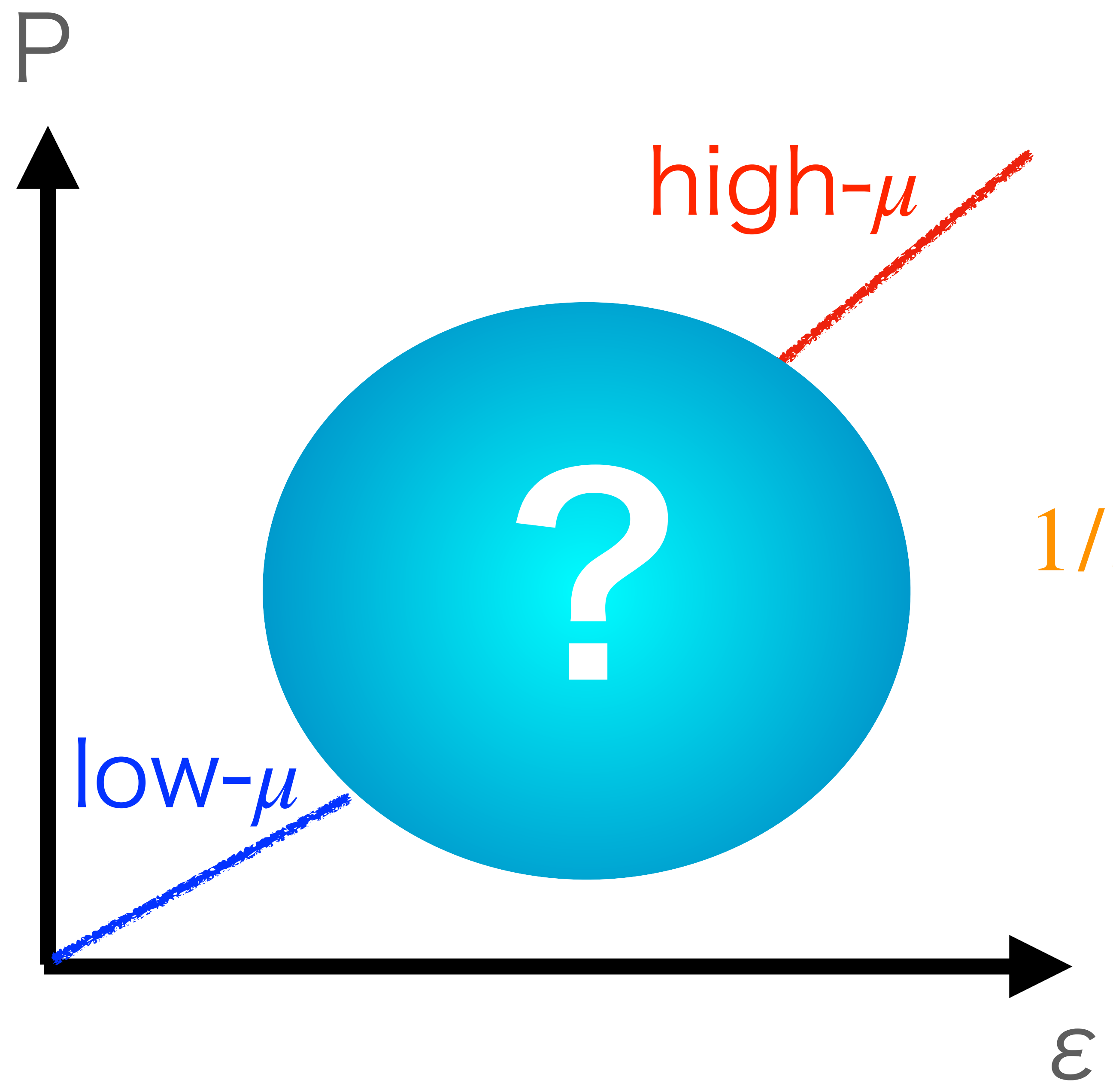
## EoS and sound velocity at low-T and high-$\mu$

Fukushima-Hatsuda (2010)
Fukushima-Sasaki (2013)



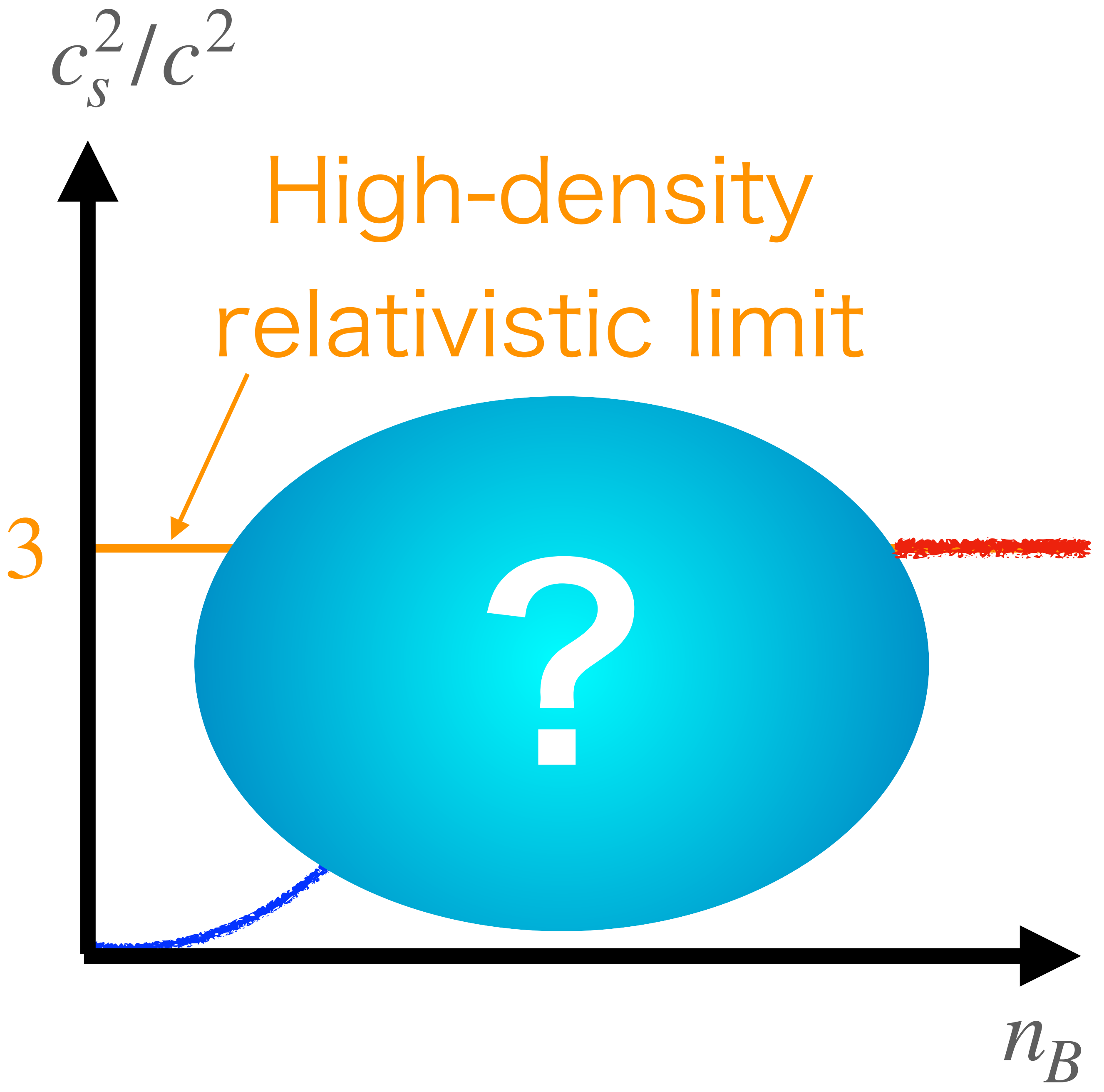
EoS

$p(\mu)$ vs $\varepsilon(\mu)$



Sound velocity

$c_s^2 = \partial p / \partial \epsilon$



low -$\mu$ ($n_B \lesssim 2n_0$): Hadronic matter

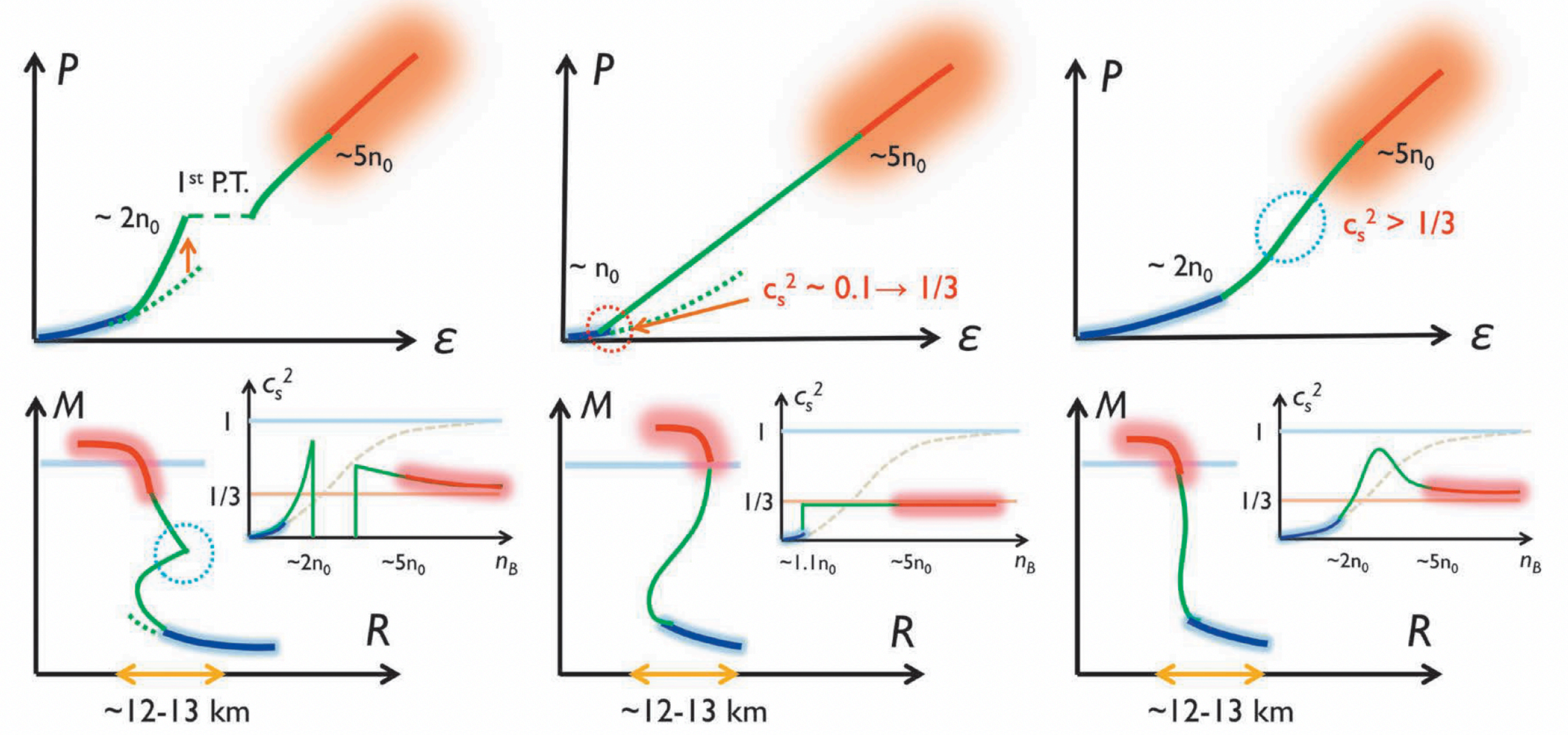
high-$\mu$ ($5n_0 < n_B$): Quark matter

-> pQCD ($50n_0 < n_B$)

# EoS, $c_s$ and neutron star

## Mass and radius of neutron star

T. Kojo, arXiv:2011.10940



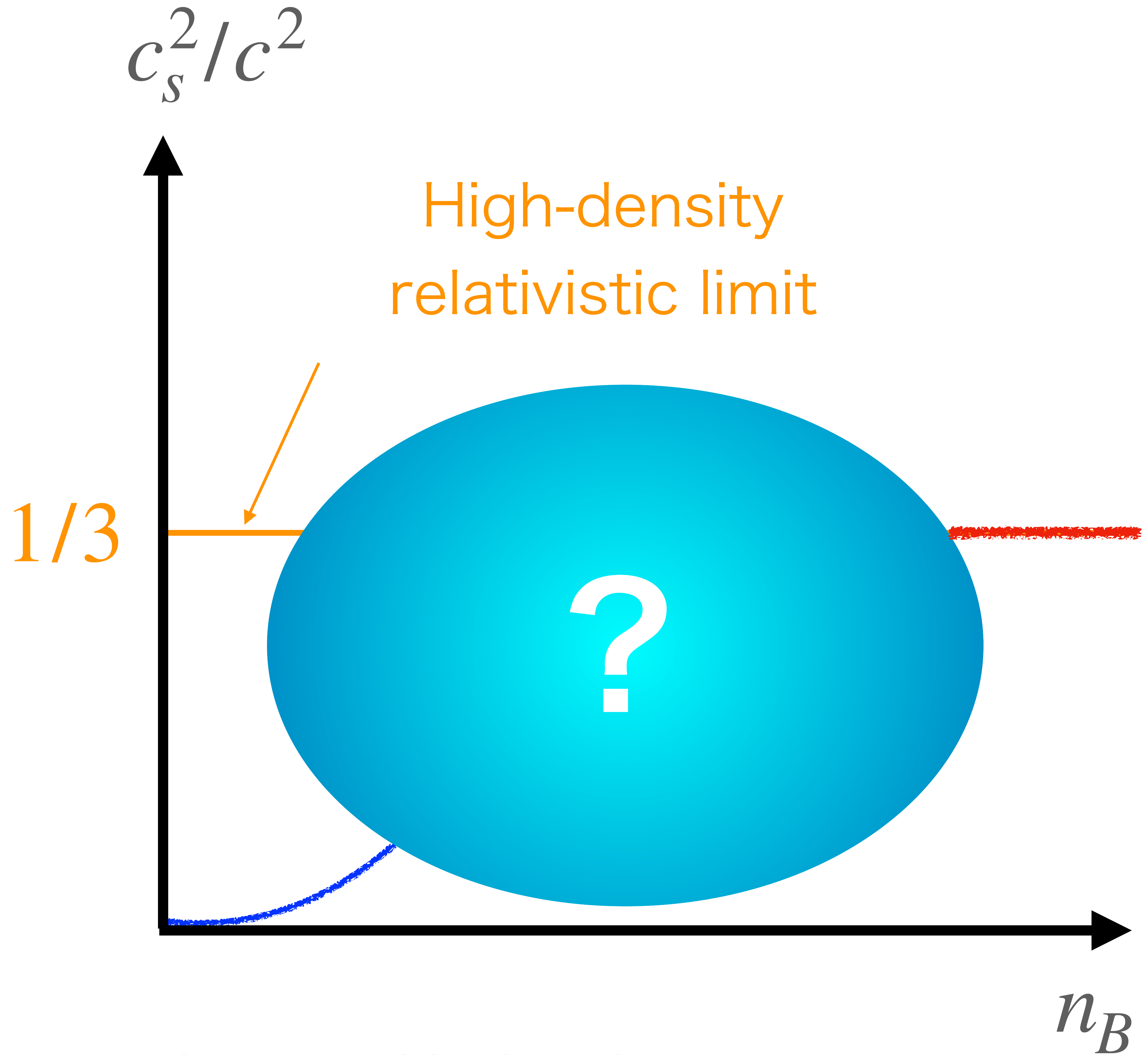
Sound velocity

$$c_s^2 = \partial p/\partial \epsilon$$

Mass-Radius of neutron star <=> EoS in dense QCD

# Prediction by phenomenology and effective models

## Sound velocity has a peak?



low -$\mu$: Hadronic matter

high-$\mu$: Quark matter ~ pQCD

- Quark-hadron crossover picture consistent with observed neutron stars (M-R) suggests $c_s^2$ peaks at $n_B = 1 - 10n_0$

Masuda,Hatsuda,Takatsuka (2013)
Baym, Hatsuda, Kojo(2018)

- Quarkyonic matter model $c_s^2$ peaks at $n_B = 1 - 5n_0$

McLerran and Reddy (2019)

- Microscopic interpretation on the origin of the peak = quark saturation (work for any # of color)

Kojo (2021), Kojo and Suenaga (2022)

➡ Lattice study on 2color dense QCD
the sign problem is absent!!

# Two problems at low-T high-$\mu$ QCD

- Sign problem (at $\mu \neq 0$ $S_E[U]$ takes complex value)

Reduce the color dof, 2color QCD
quarks becomes pseudo-real reps.
The sign problem is absent from 2color QCD with even Nf

- Onset problem in low-T, high-$\mu$ (e.g. $\mu_q > m_\pi/2, \ \ m_N/3$),

It comes from the phase transition to superfluid phase(SSB of baryon sym.)

**Kogut et al.** NPB642 (2002)18

Add an explicit breaking term of the sym., then take $j \rightarrow 0$ limit

$$S_F^{cont.} = \underbrace{\int d^4x \bar{\psi}(x)(\gamma_\mu D_\mu + m)\psi(x)}_{\text{QCD}} + \underbrace{\mu \hat{N}}_{\text{Number op.}} - \underbrace{\frac{j}{2}(\bar{\psi}_1 K \bar{\psi}_2^T - \psi_2^T K \psi_1)}_{\text{diquark source}}$$

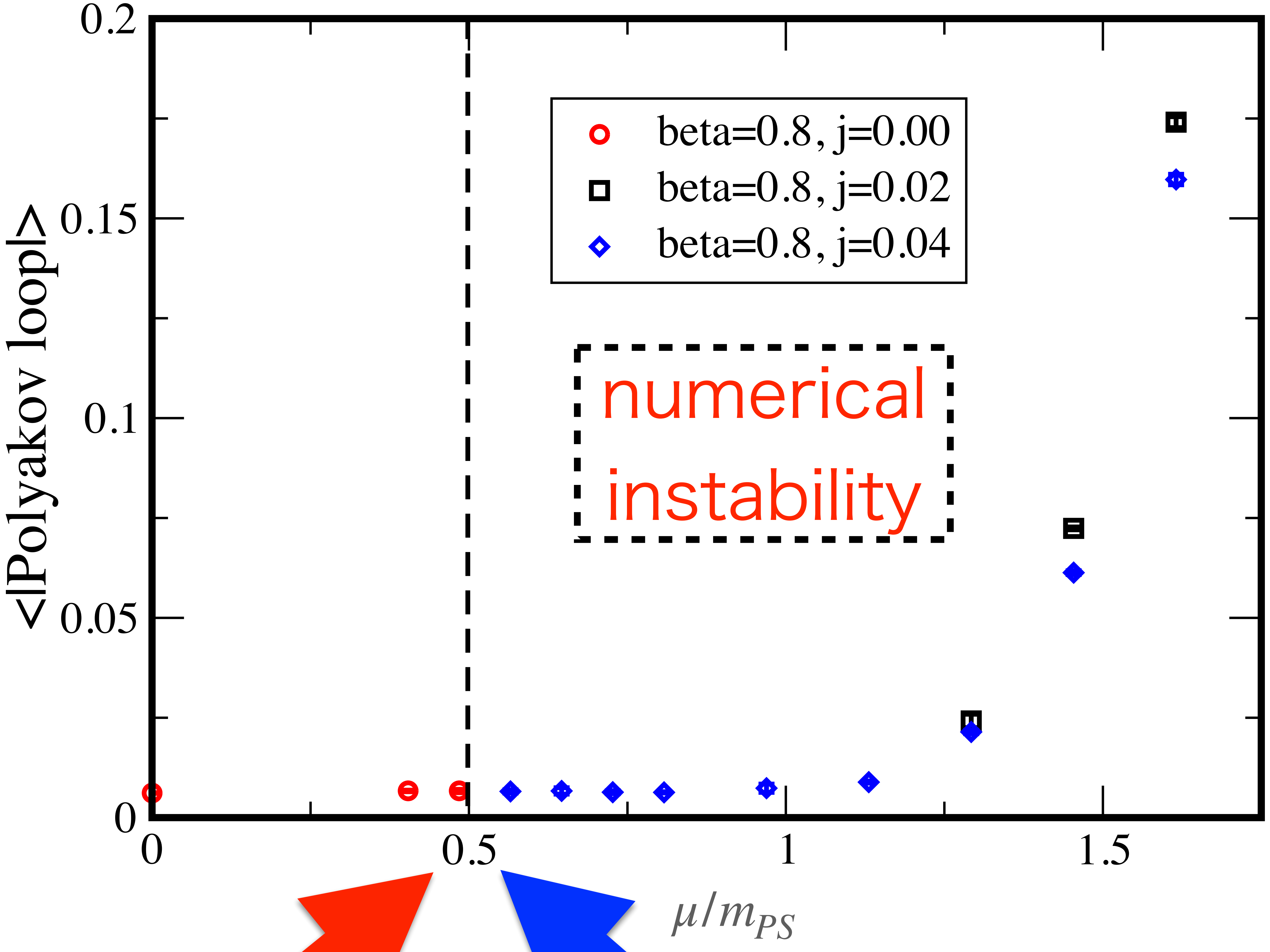
HMC simulations for whole T-$\mu$ regime are doable!

(j->0 extrapolation is taken in all plots today)

# HMC calculation w or w/o diquark source term

According to chiral perturbation theory,

the hadronic-superfluid phase transition occurs at $\mu/m_{PS} \sim 0.5$



numerical instability

HMC without j is doable
(minimum MC step ~ 1/800)

HMC without j cannot run even with
a tiny MC step(~1/1000)

# Implementation QC2D with diquark source term

$$S_F^{cont.} = \underbrace{\int d^4x \bar{\psi}(x)(\gamma_\mu D_\mu + m)\psi(x)}_{\text{QCD}} + \underbrace{\mu \hat{N}}_{\text{Number op.}} \underbrace{- \frac{j}{2}(\bar{\psi}_1 K \bar{\psi}_2^T - \psi_2^T K \psi_1)}_{\text{diquark source}}$$

construct a single bilinear form of fermion fields

$$S_F = (\bar{\psi}_1 \;\; \bar{\varphi}) \begin{pmatrix} \Delta(\mu) & J\gamma_5 \\ -J\gamma_5 & \Delta(-\mu) \end{pmatrix} \begin{pmatrix} \psi_1 \\ \varphi \end{pmatrix} \equiv \bar{\Psi}\mathcal{M}\Psi$$

Here, $\Psi = \begin{pmatrix} \psi_1 \\ \varphi \end{pmatrix}$

$$\bar{\varphi} = -\bar{\psi}_2^T C \tau_2, \quad \varphi = C^{-1}\tau_2 \bar{\psi}_2^T$$

$\mathscr{M}$ has non-diagonal components, calculations of det[M] and inverse of M are hard…

$$\mathcal{M}^\dagger \mathcal{M} = \begin{pmatrix} \Delta^\dagger(\mu)\Delta(\mu) + |\bar{J}|^2 & 0 \\ 0 & \Delta^\dagger(-\mu)\Delta(-\mu) + |J|^2 \end{pmatrix}$$

J (=$j\kappa$) term lifts the eigenvalue of Dirac op.

Note that $\Psi$ denotes 2-flavor, $\det \mathcal{M}$ gives Nf=2 action

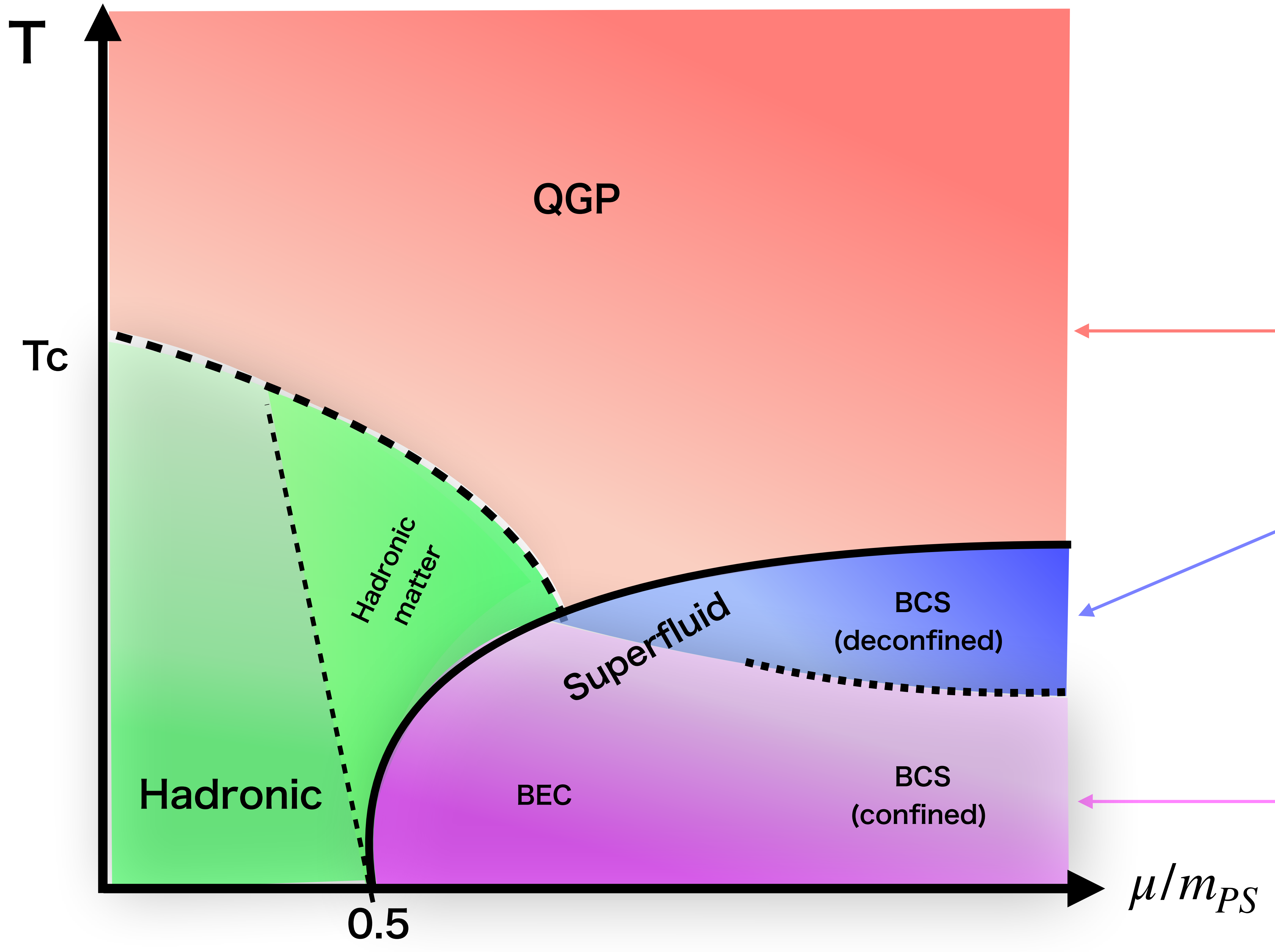
$\det \mathcal{M}^\dagger \mathcal{M}$ is 4-flavor theory

RHMC algorithm

# 2color QCD phase diagram

(1) K.Iida, K.Ishiguro , EI, arXiv: 2111.13067
(2) K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0
(3) K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
(4) T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

# Current status on 2color QCD phase diagram



**At least three independent group studying the phase diagram**

**(1) S. Hands group : Wilson-Plaquette gauge + Wilson fermion**
**(2) Russian group : tree level improved Symanzik gauge + rooted staggered fermion**
**(3) Our group : Iwasaki gauge + Wilson fermion, Tc=200 MeV to fix the scale**

T=158 MeV (**deconfined**, hadron -> QGP phase transition occurs)
T=130 MeV (**deconfined? QGP phase?** , 2019)

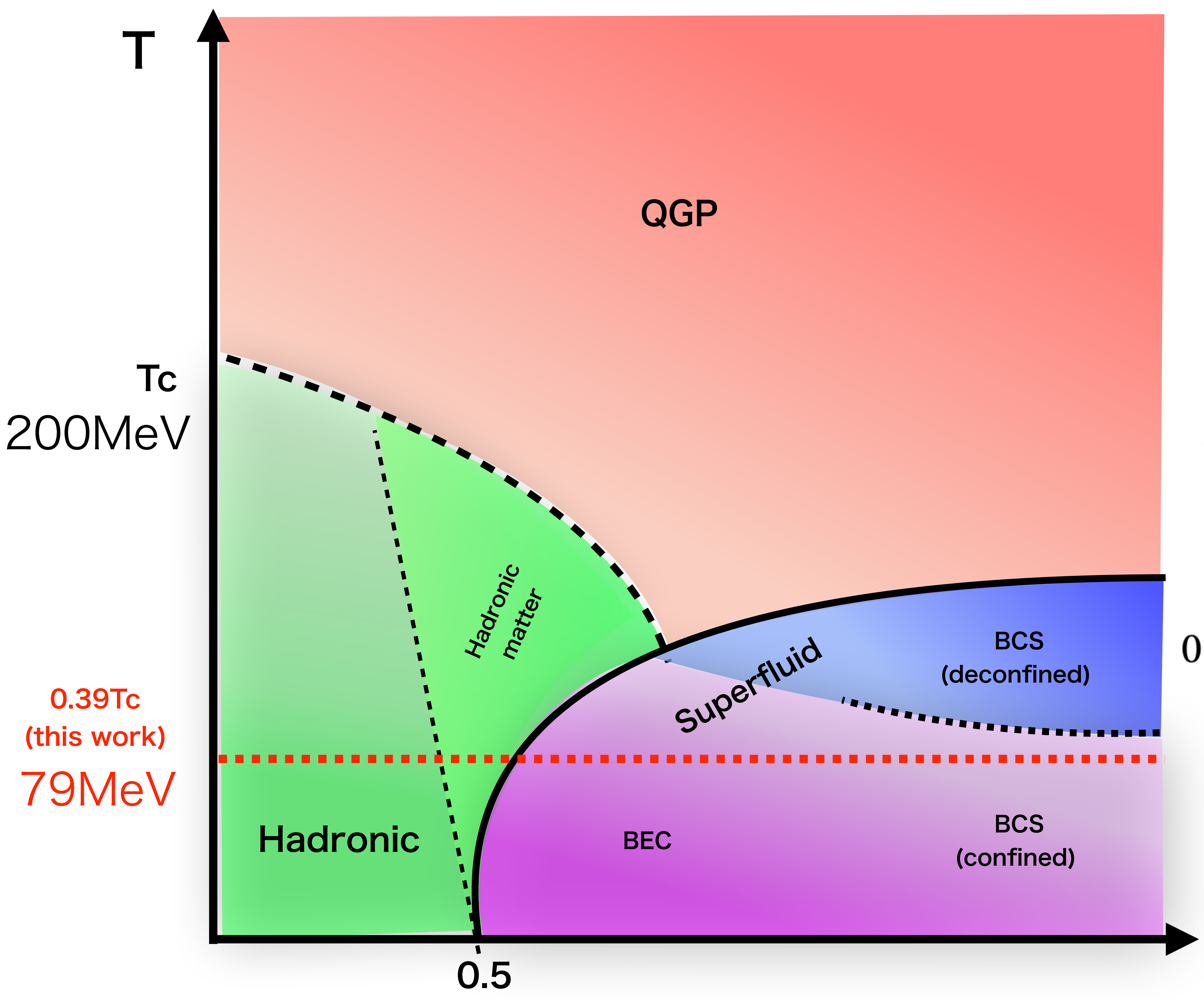
T=140 MeV (**deconfined** in high mu, <qq> is not zero, 2017, 2018, 2020)
T= 93 MeV (**deconfined** in high mu ?, also <qq> is not zero?, 2017)

T=87 MeV (**confined** in 2019)
T=79 MeV (**confined** even in high mu)
T=55 MeV (**confined** in high mu, 2016)
T=47 MeV (**deconfined** coarse lattice in 2012, but **confined** in 2019)
T=45 MeV (**confined** in 2019)

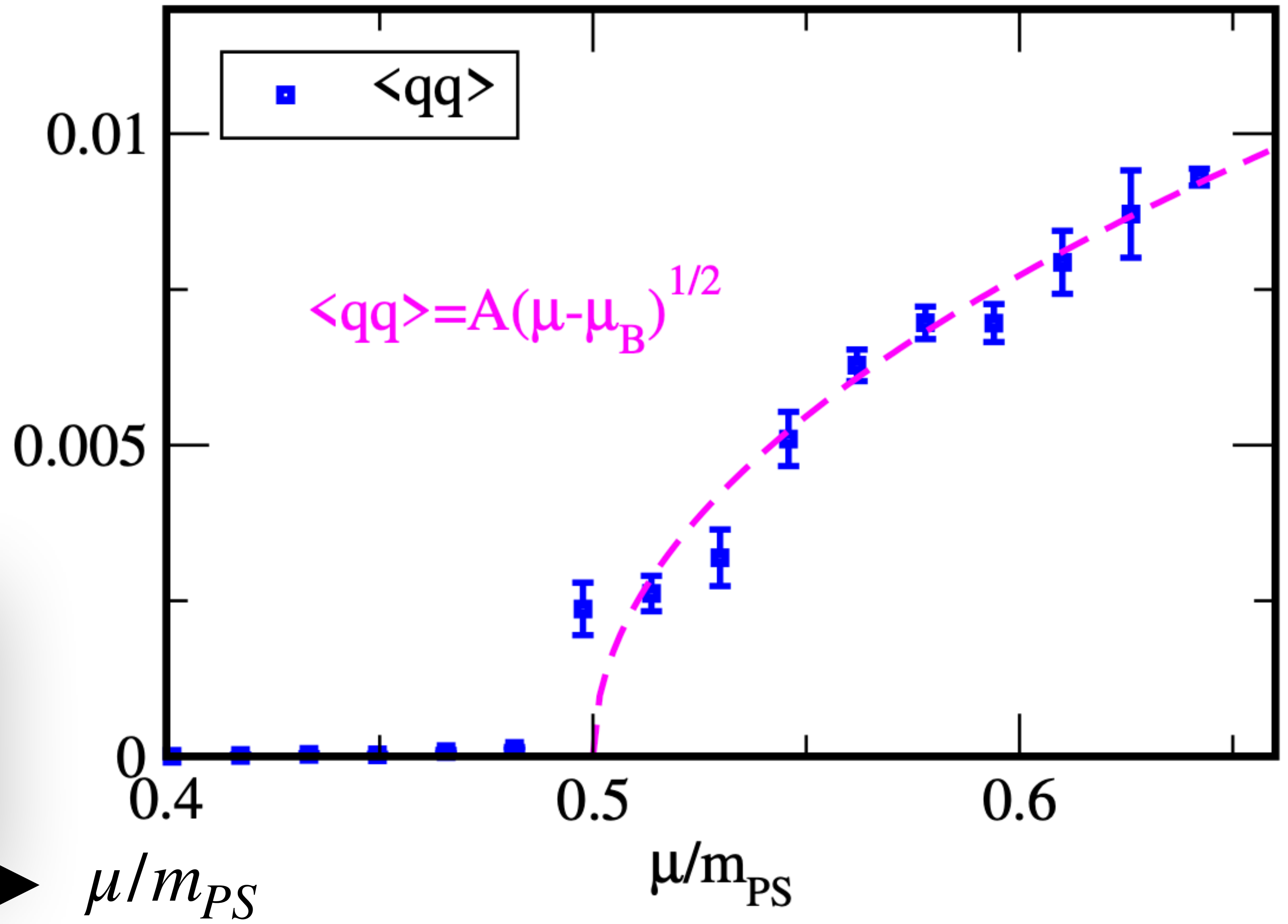
- Even $T \approx 100\text{MeV}$ and $\mu/m_{PS} = 0.5$, superfluid phase emerges
- 2color QCD phase diagram has been determined by independent works!

# Phase diagram of 2color QCD

K.Iida, EI, T.-G. Lee: JHEP2001(2020)181



| | Hadronic | Hadronic-matter | QGP | Superfluid | |
|---|---|---|---|---|---|
| | | | | BEC | BCS |
| $\langle \lvert L \rvert \rangle$ | zero | zero | non-zero | | |
| $\langle qq \rangle$ | zero | zero | zero | non-zero | $\propto \Delta(\mu)\mu^2$ |
| $\langle n_q \rangle$ | | non-zero | | non-zero | $n_q/n_q^{\rm tree} \approx 1$ |



Scaling law of order param.
is consistent with ChPT.

Kogut et al., NPB 582 (2000) 477

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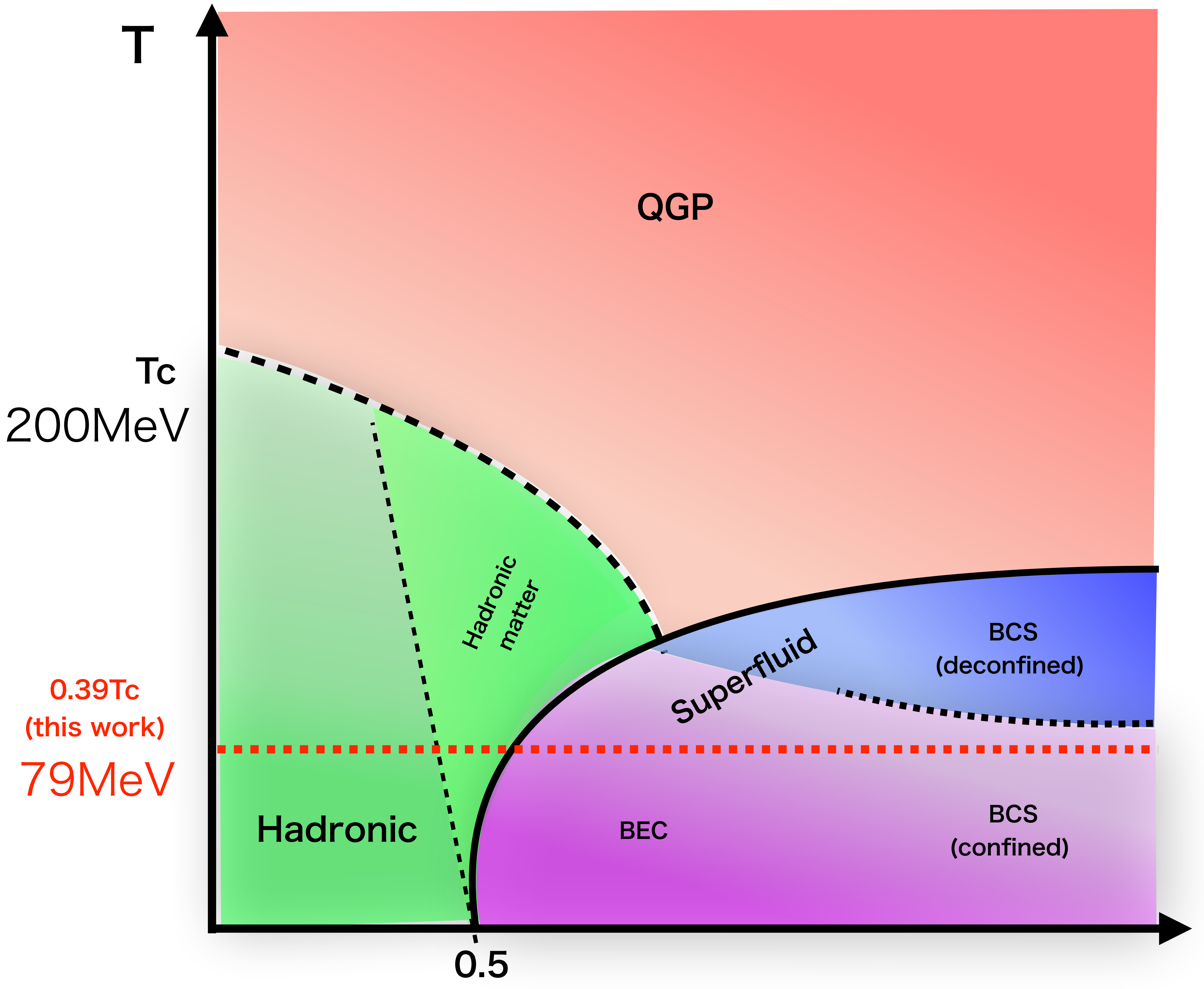
K.Iida, EI, T.-G. Lee: JHEP2001(2020)181



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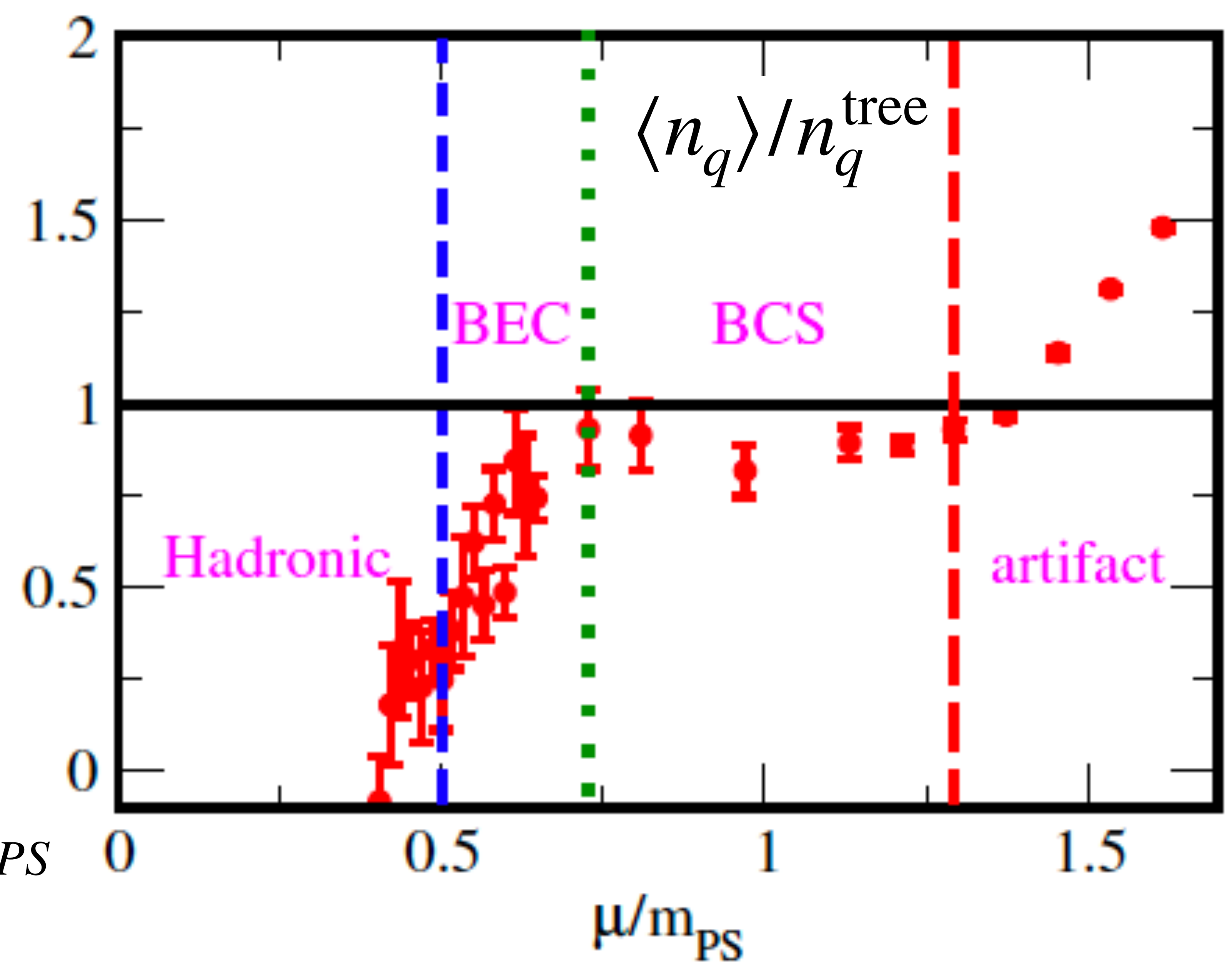
| $\langle qq \rangle$ | zero | zero | zero | non-zero | $\propto \Delta(\mu)\mu^2$ |
| $\langle n_q \rangle$ | | non-zero | | non-zero | $n_q/n_q^{\mathrm{tree}} \approx 1$ |



In high-$\mu$, $\langle n_q \rangle \approx n_q^{\mathrm{tree}}$

number density of free particle

BEC-BCS crossover

# Equation of state

K.Iida and EI, PTEP 2022 (2022) 11, 111B01

# Equation of state

- Fixed scale approach ($\mu \neq 0$ version)

  beta=0.80 (Iwasaki gauge)

  lattice size = $16^4$

  T=79MeV, j->0 extrapolation is taken

EoS in dense 2color QCD
Hands et al. (2006)
Hands et al. (2012), T~47MeV (coarse lattice)
Astrakhantsev et al. (2020), T~140MeV

- trace anomaly: $\epsilon - 3p = \frac{1}{N_s^3}\left( a\frac{d\beta}{da}|_{LCP}\underline{\langle \frac{\partial S}{\partial \beta} \rangle}_{sub.} + a\frac{d\kappa}{da}|_{LCP}\langle \frac{\partial S}{\partial \kappa} \rangle_{sub.} + a\underline{\cancel{\frac{\partial j}{\partial a}\langle \frac{\partial S}{\partial j} \rangle}} \right)$

  No renormalization for $\mu$

  $\langle \cdot \rangle_{sub.} = \langle \cdot \rangle_{\mu} - \langle \cdot \rangle_{\mu=0}$

  Zero at $j \to 0$

- pressure: $p(\mu) = \int_{\mu_o}^{\mu} n_q(\mu')d\mu'$

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  Zero at $j \to 0$

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**Technical steps**

(1) Measure the gauge action and chiral cond.

(2) Calculate the beta fn. at $\mu = 0$

(3) Numerical integration of $n_q$

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  Zero at $j \to 0$

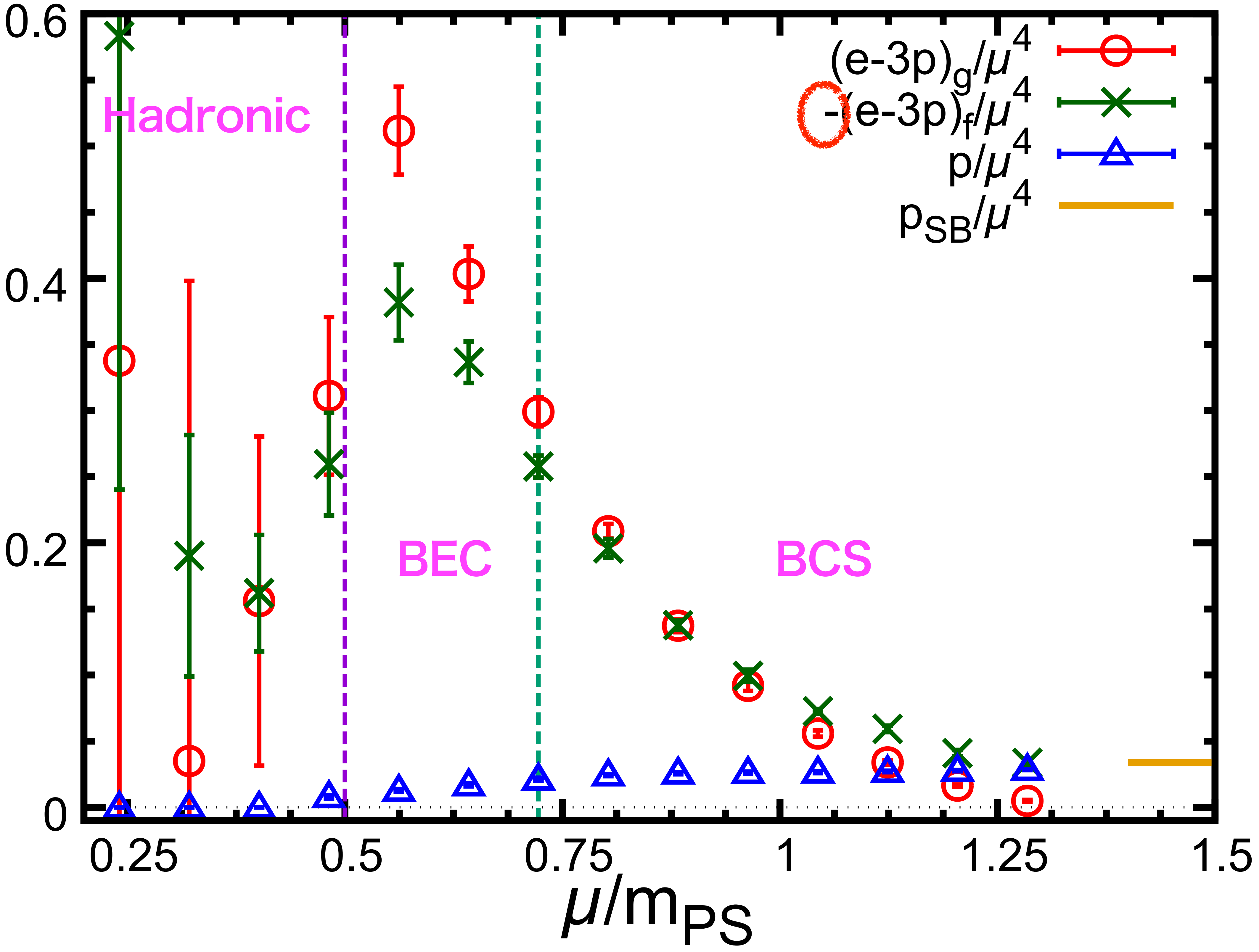
- pressure: $p(\mu) = \int_{\mu_o}^{\mu} n_q(\mu')d\mu'$

**Nonperturbative beta-fn.**

$a\frac{d\beta}{da} = -0.3521, \quad a\frac{d\kappa}{da} = 0.02817$

K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0

# Trace anomaly and pressure



- Sum of trace anomaly, $(e-3p)_g + (e-3p)_f$

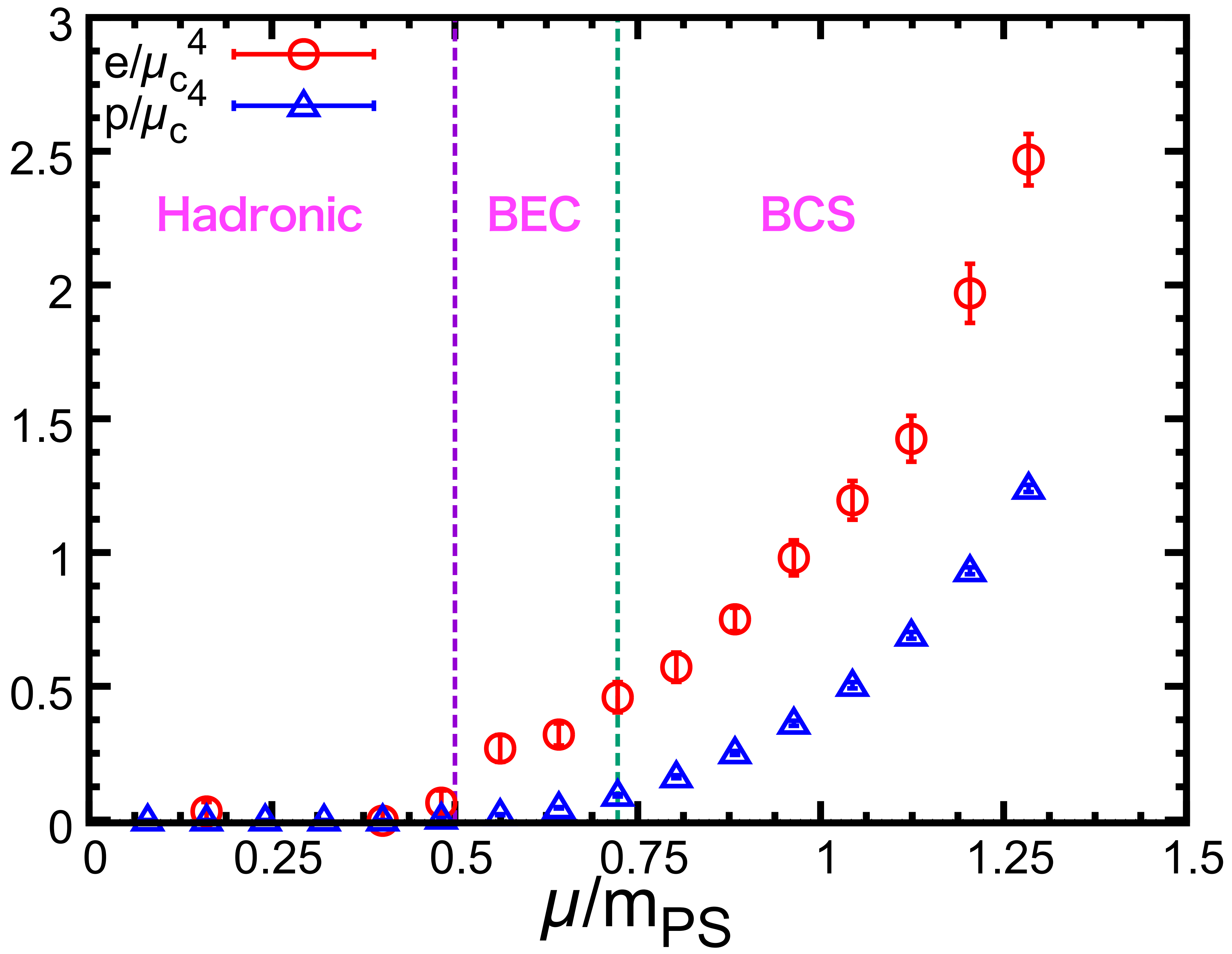
  zero in Hadronic phase
  positive in BEC phase
  positive -> negative in BCS phase
  Finally, fermions give the larger contribution

- Pressure increase monotonically

  In high density, it approaches
  $p_{SB}/\mu^4 = N_c N_f/(12\pi^2) \approx 0.03$

# P and e as a function of $\mu$

## (Normalized by $1/\mu_c^4$ to be dim-less)

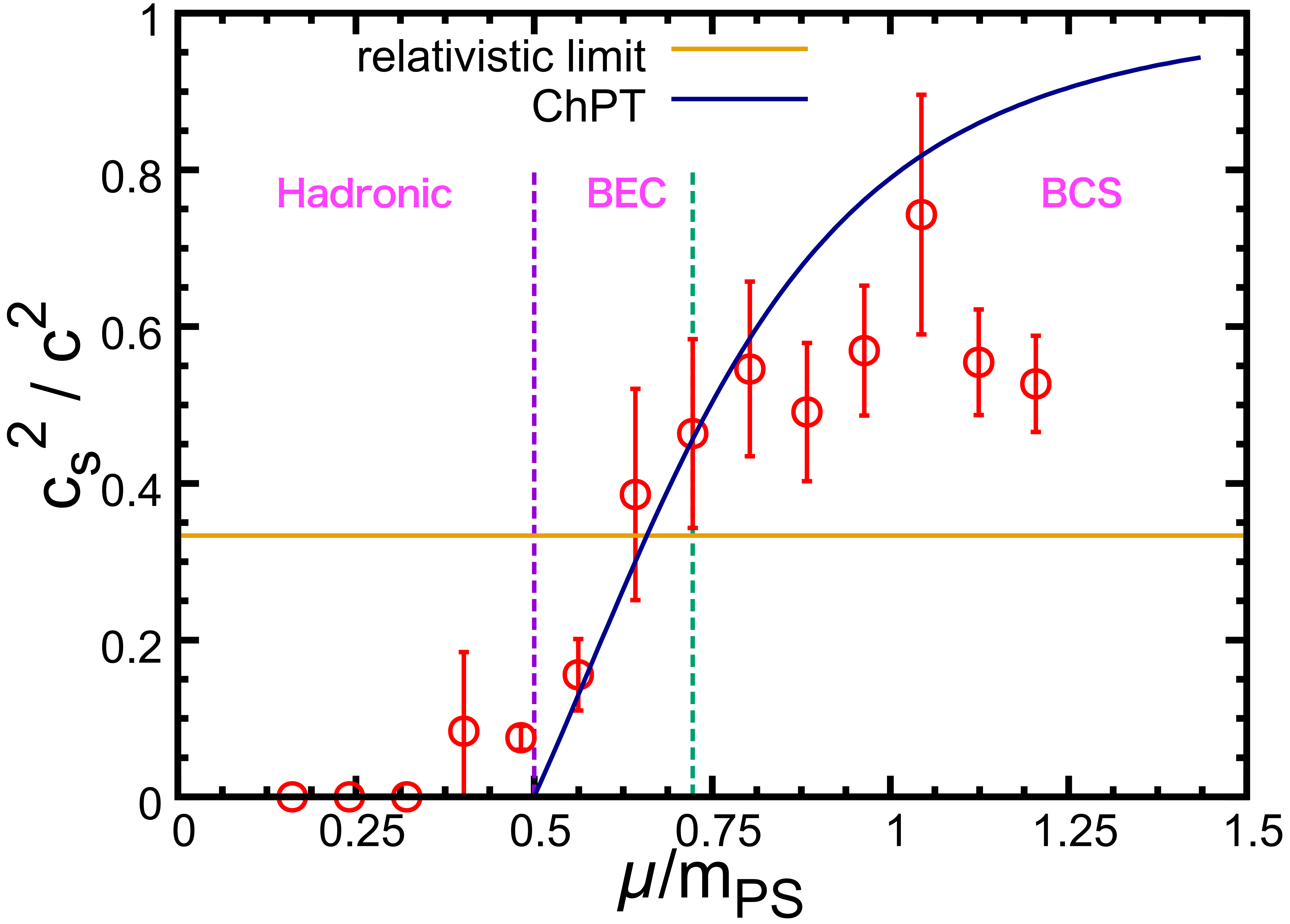


- P is zero in Hadronic phase since $n_q = 0$
- e is also zero in Hadronic phase by the cancelation between $(e-3p)_g$ and $(e-3p)_f$

From these data, the sound velocity is obtained

$$c_s^2/c^2 = \frac{\Delta p}{\Delta e} = \frac{p(\mu+\Delta\mu) - p(\mu-\Delta\mu)}{e(\mu+\Delta\mu) - e(\mu-\Delta\mu)}$$

# Sound velocity ( $c_s^2/c^2 = \Delta p/\Delta e$)



Chiral Perturbation Theory (ChPT)

$$c_s^2/c^2 = \frac{1-\mu_c^4/\mu^4}{1+3\mu_c^4/\mu^4}$$ : no free parameter!!

Son and Stephanov (2001) : 3color QCD with isospin $\mu$

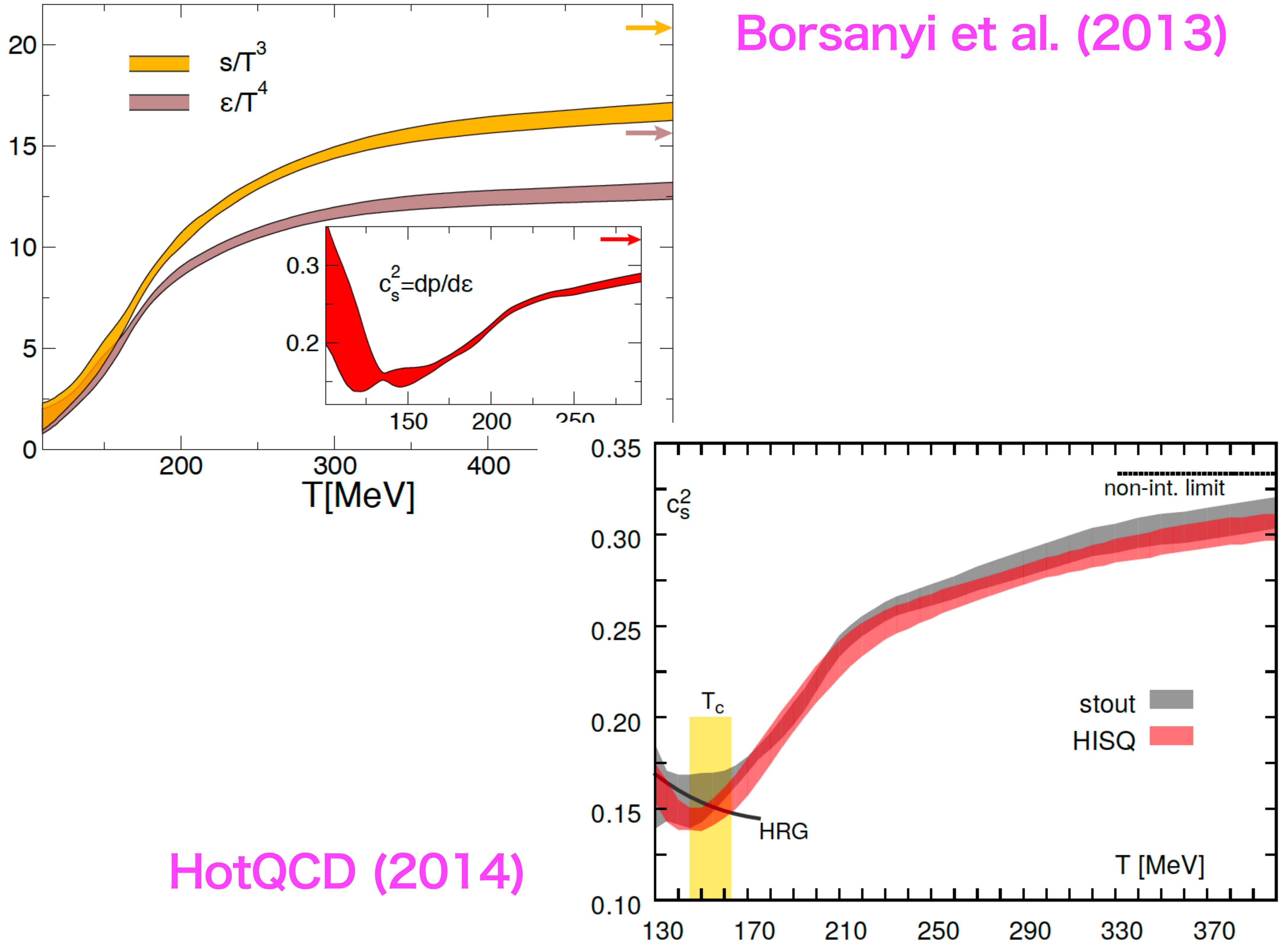
Hands, Kim, Slullerud (2006) : 2color QCD with real $\mu$

- In BEC phase, our result is consistent with ChPT.
- $c_s^2/c^2$ exceeds the relativistic limit
- In high-density, it peaks around $\mu \approx m_{PS}$.

"Stiffen" and then "soften" picture as density increases

# Sound velocity and phase transition

## Finite Temperature transition (Nf=2+1 QCD)



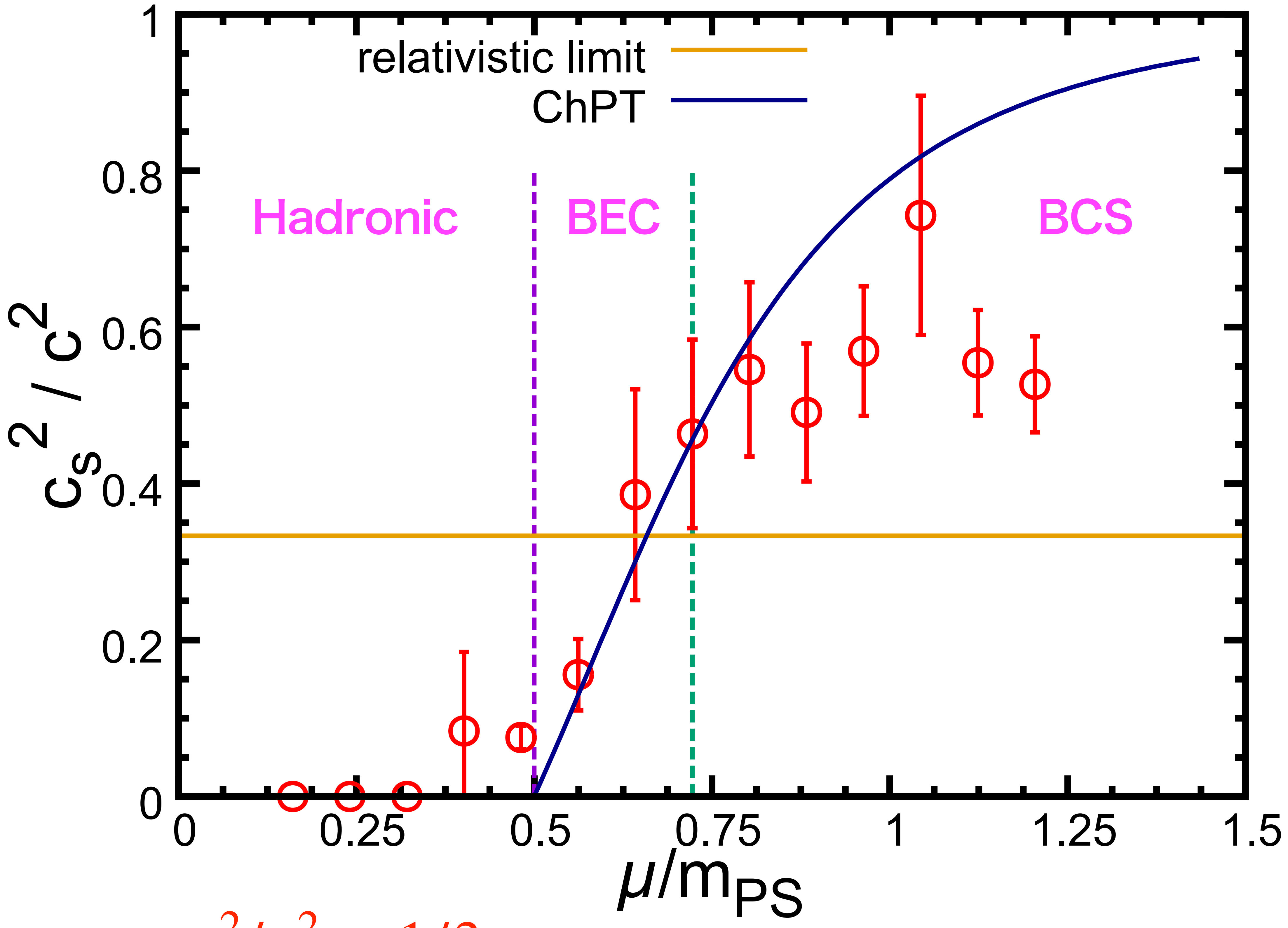
Borsanyi et al. (2013)

HotQCD (2014)

- Minimum around Tc
- Monotonically increases to $c_s^2/c^2 = 1/3$

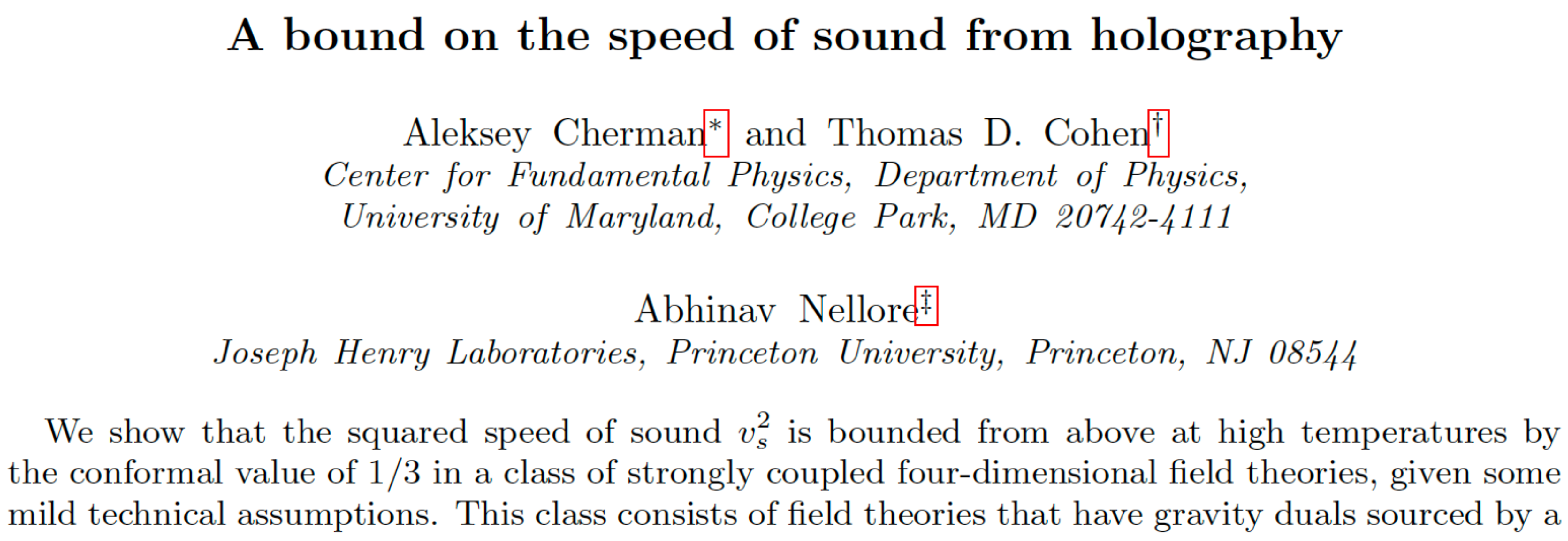
## Finite Density transition (Nf=2 2color QCD)

Iida and Itou arXiv: 2207.01253



- $c_s^2/c^2 > 1/3$
- previously unknown from any lattice calculations for QCD-like theories.

# Holography bound?



### A bound on the speed of sound from holography

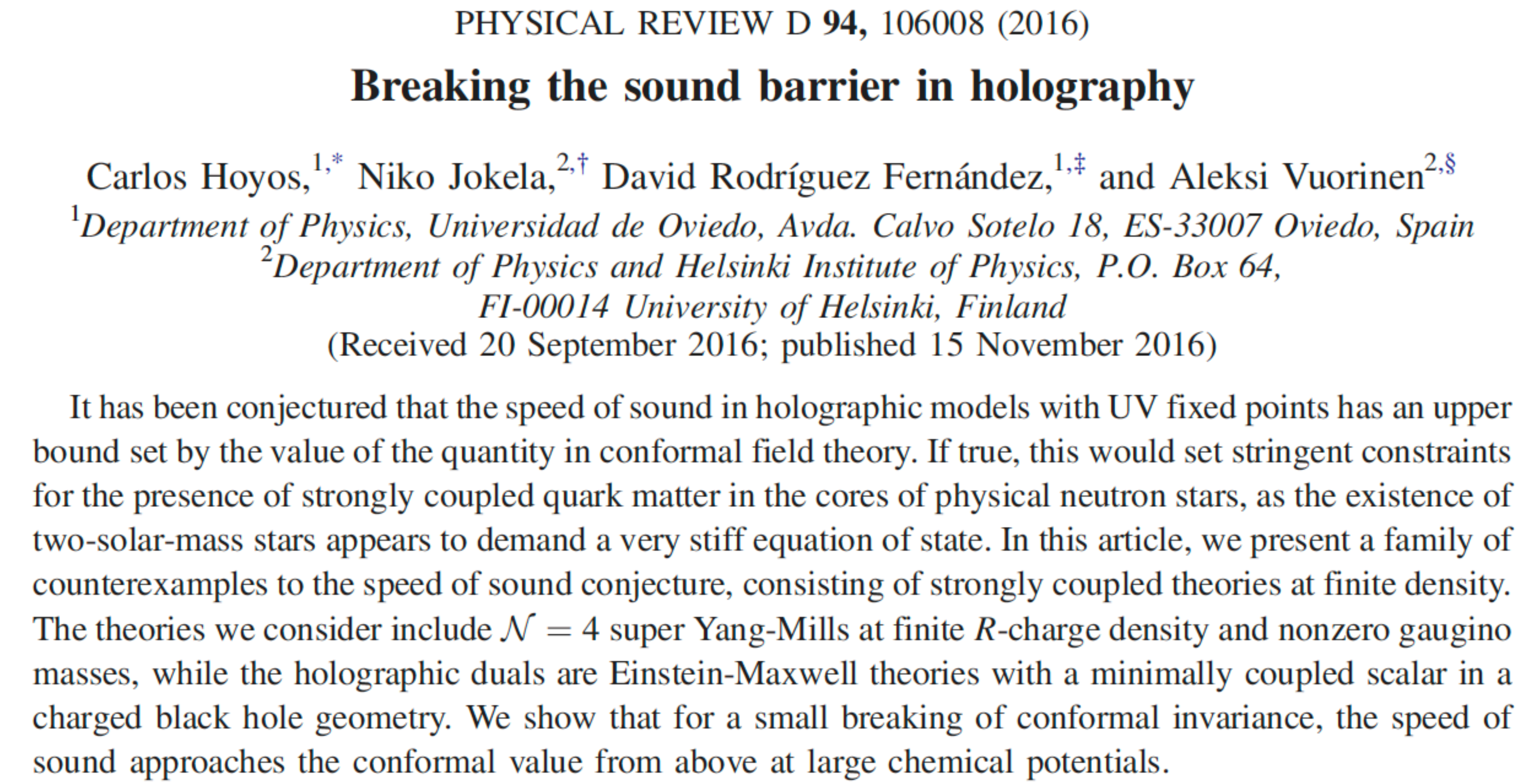
Aleksey Cherman[*] and Thomas D. Cohen[†]
*Center for Fundamental Physics, Department of Physics,*
*University of Maryland, College Park, MD 20742-4111*

Abhinav Nellore[‡]
*Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544*

We show that the squared speed of sound $v_s^2$ is bounded from above at high temperatures by the conformal value of 1/3 in a class of strongly coupled four-dimensional field theories, given some mild technical assumptions. This class consists of field theories that have gravity duals sourced by a single scalar field. There are no known examples to date of field theories with gravity duals for which $v_s^2$ exceeds 1/3 in energetically favored configurations. We conjecture that $v_s^2 = 1$ upper bound for a broad class of four-dimensional theories.

$c_s^2/c^2 \leq 1/3$ at high T

conjecture it is valid for a broad class of 4-dim. theories



PHYSICAL REVIEW D **94,** 106008 (2016)

### Breaking the sound barrier in holography

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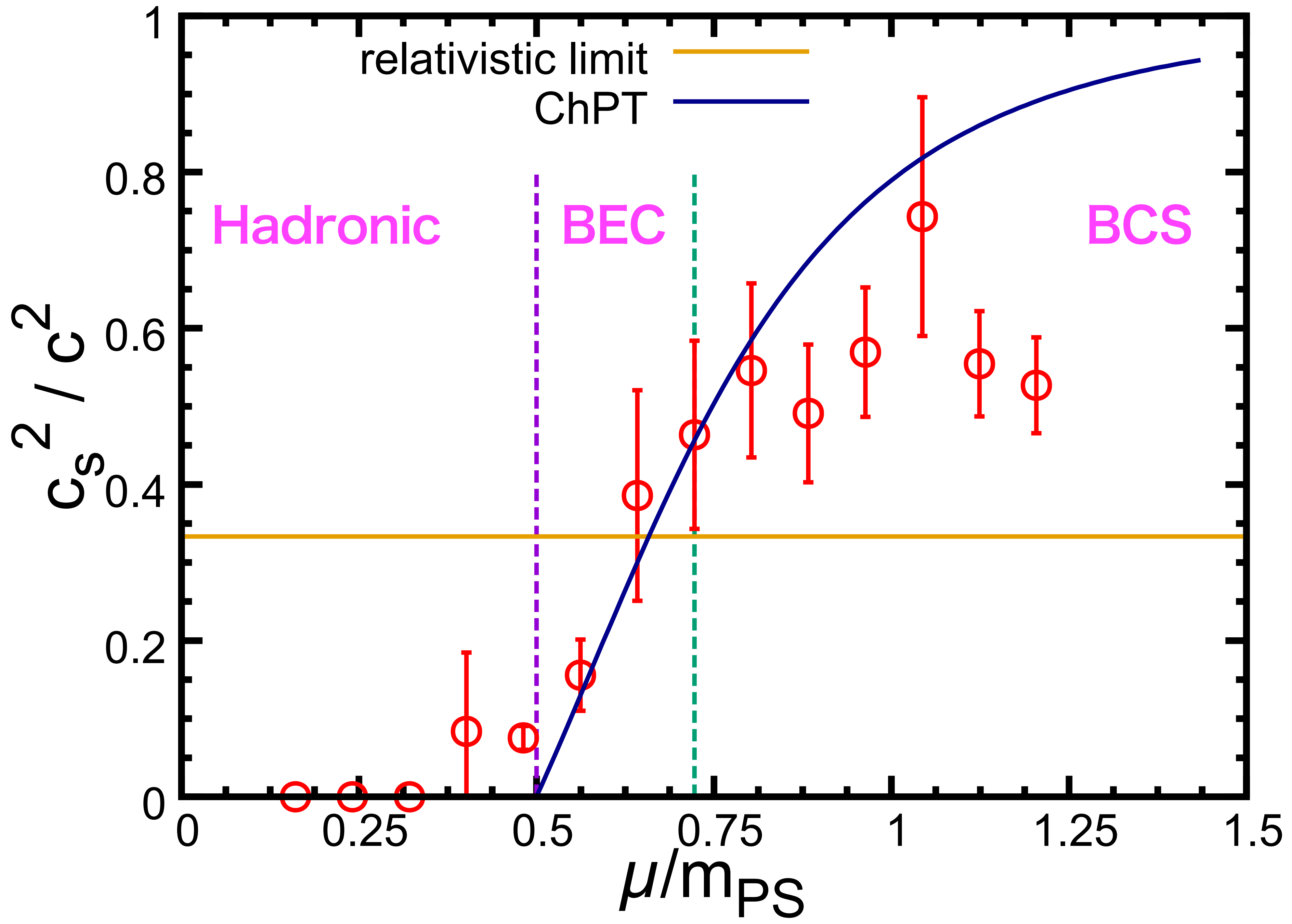
[2]*Department of Physics and Helsinki Institute of Physics, P.O. Box 64,*
*FI-00014 University of Helsinki, Finland*

(Received 20 September 2016; published 15 November 2016)

It has been conjectured that the speed of sound in holographic models with UV fixed points has an upper bound set by the value of the quantity in conformal field theory. If true, this would set stringent constraints for the presence of strongly coupled quark matter in the cores of physical neutron stars, as the existence of two-solar-mass stars appears to demand a very stiff equation of state. In this article, we present a family of counterexamples to the speed of sound conjecture, consisting of strongly coupled theories at finite density. The theories we consider include $\mathcal{N} = 4$ super Yang-Mills at finite $R$-charge density and nonzero gaugino masses, while the holographic duals are Einstein-Maxwell theories with a minimally coupled scalar in a charged black hole geometry. We show that for a small breaking of conformal invariance, the speed of sound approaches the conformal value from above at large chemical potentials.

Counterexample
for N=4 SYM at finite density

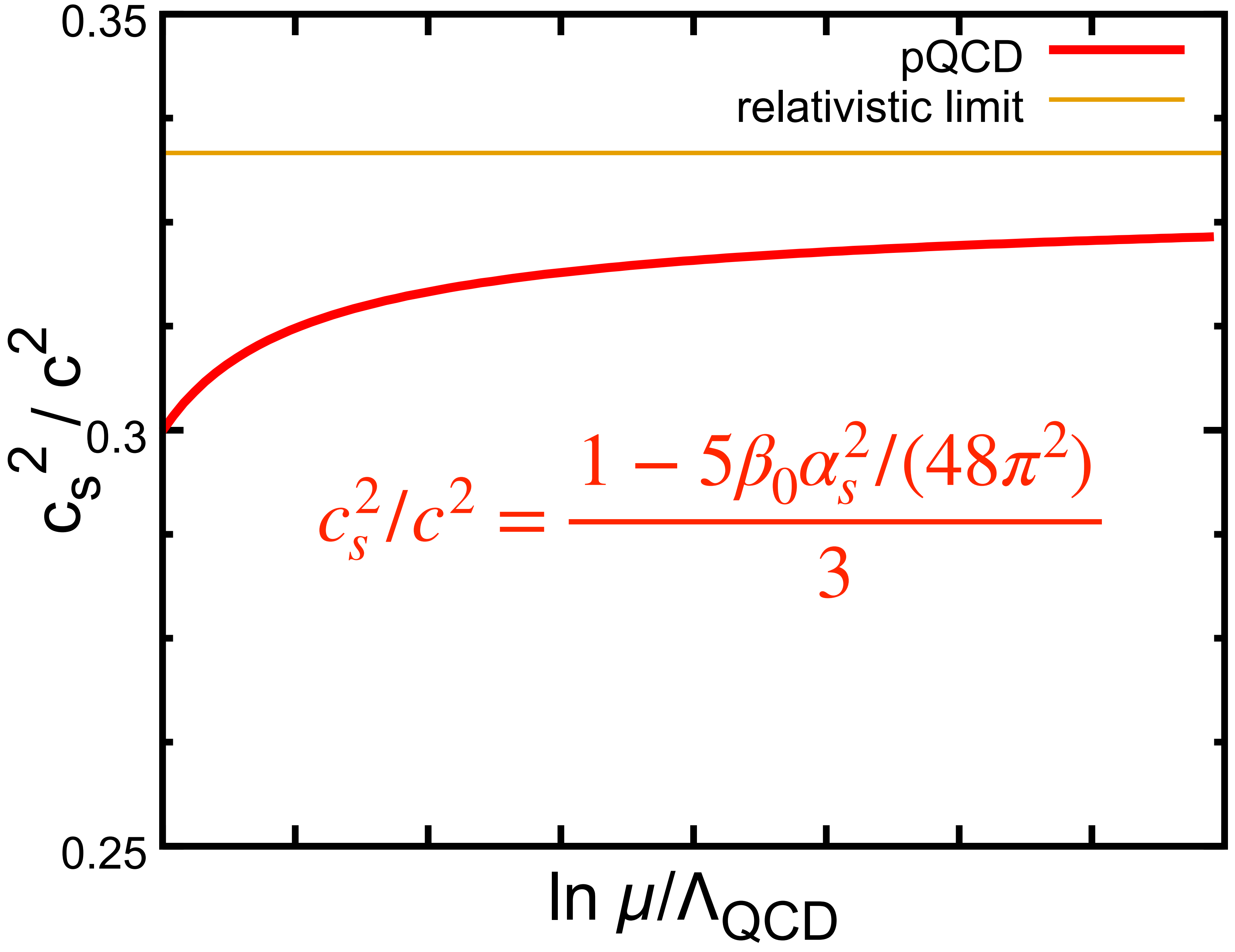
# Further high density?



Kojo, Baym, Hatsuda (2021)

pQCD prediction

(Ultra high-density regime)



$$c_s^2/c^2 = \frac{1 - 5\beta_0\alpha_s^2/(48\pi^2)}{3}$$

- Upper bound of chemical potential in lattice simulation comes from $a\mu \ll 1$

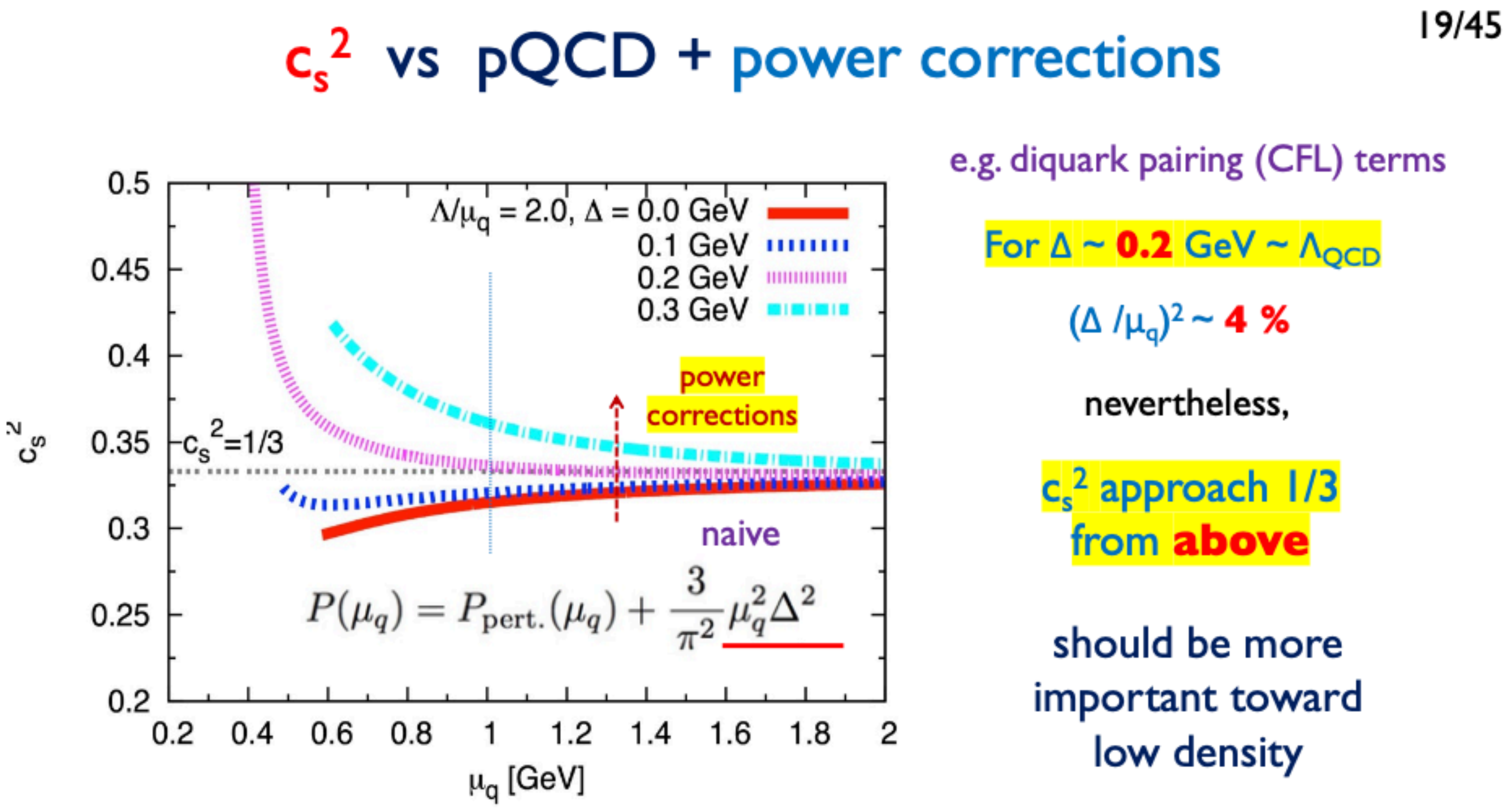
  (Here, we take $a\mu \leq 0.8$)

- To study high-density, the lighter mass / finer lattice spacing are needed

# Further high density?

pQCD + power correction due to diquark gap

Slide by Kojo (2019)



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## $c_s^2$ vs pQCD + power corrections

$$P(\mu_q) = P_{\mathrm{pert.}}(\mu_q) + \frac{3}{\pi^2}\mu_q^2\Delta^2$$

e.g. diquark pairing (CFL) terms

For $\Delta \sim$ **0.2** GeV ~ $\Lambda_{QCD}$

$(\Delta/\mu_q)^2 \sim$ **4 %**

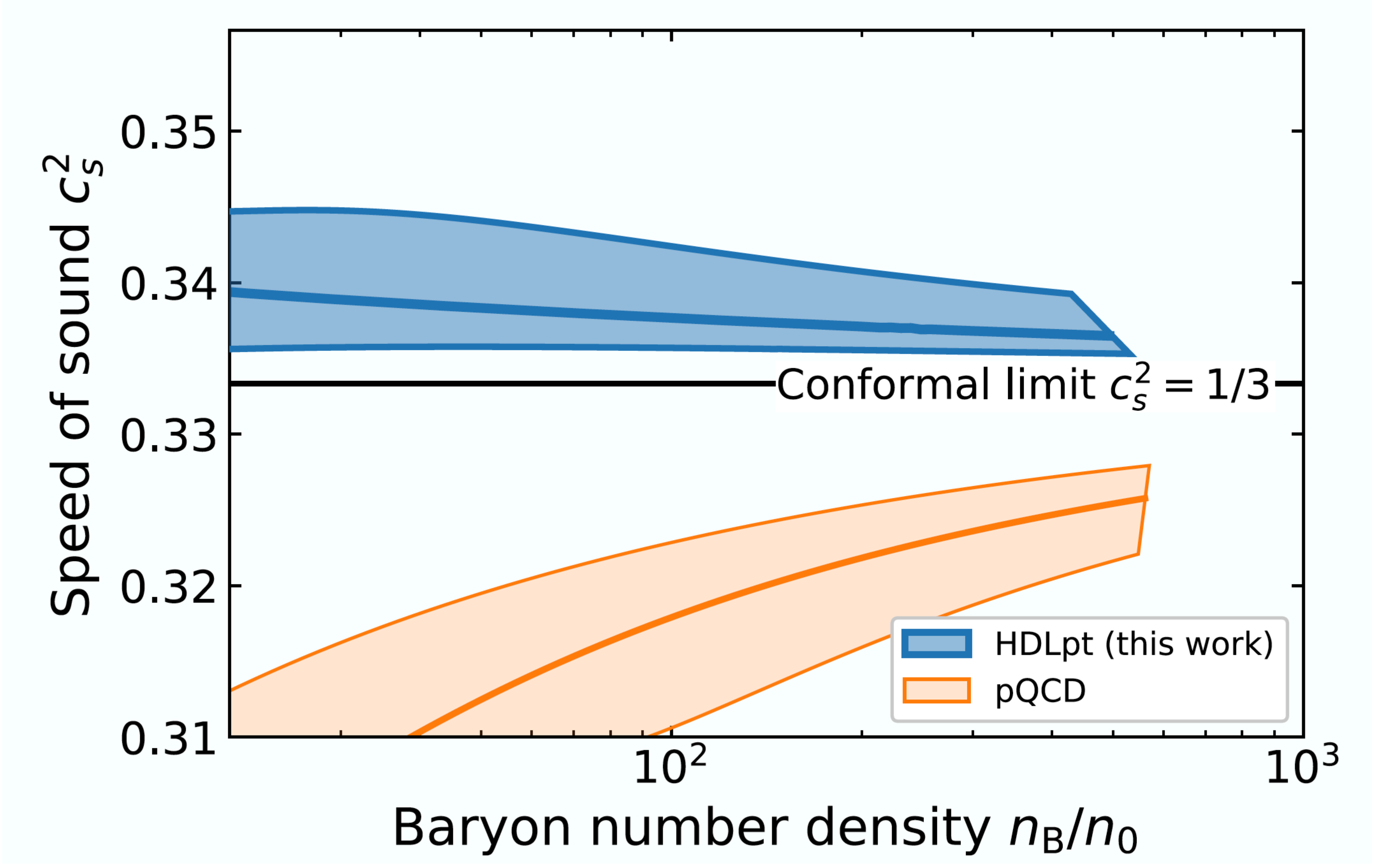
nevertheless,

$c_s^2$ approach 1/3
from **above**

should be more
important toward
low density

Hard thermal loop resummation

Fujimoto and Fukushima(2021)



- Open question: How $c_s^2/c^2$ approaches 1/3; from below or from above?

# Summary and future work

- In BEC phase, our result is consistent with ChPT.
  Sound velocity exceeds the relativistic limit and has a peak after BEC-BCS crossover
  cf.) cond-mat model study also find it   Tajima and Liang (2022)

- Find a mechanism of a peak structure
  - quark saturation?(Kojo,Suenaga), strong coupling with trace anomaly?(McLerran,Fukushima et al.), others?
  - attractive or repulsive force between hadrons?
    => extended HAL QCD method in finite density
    => mass spectrum in superfluid phase
  - independent of the color dof?

work in progress with
K.Murakami (YITP, Kyoto U.)

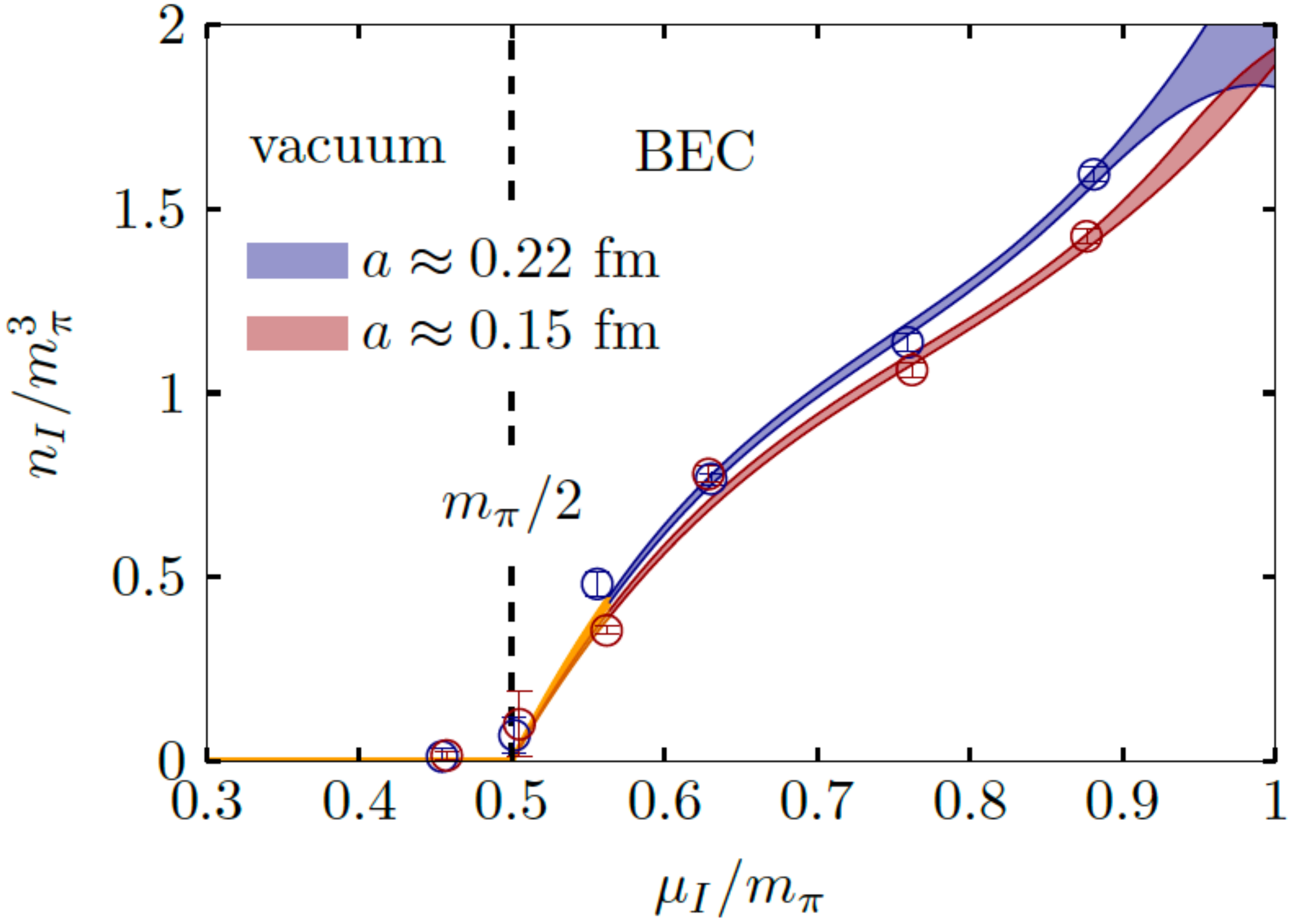
Suenaga, Murakami, EI, Iida (appear in PRD)
K.Murakami's Lattice proceedings

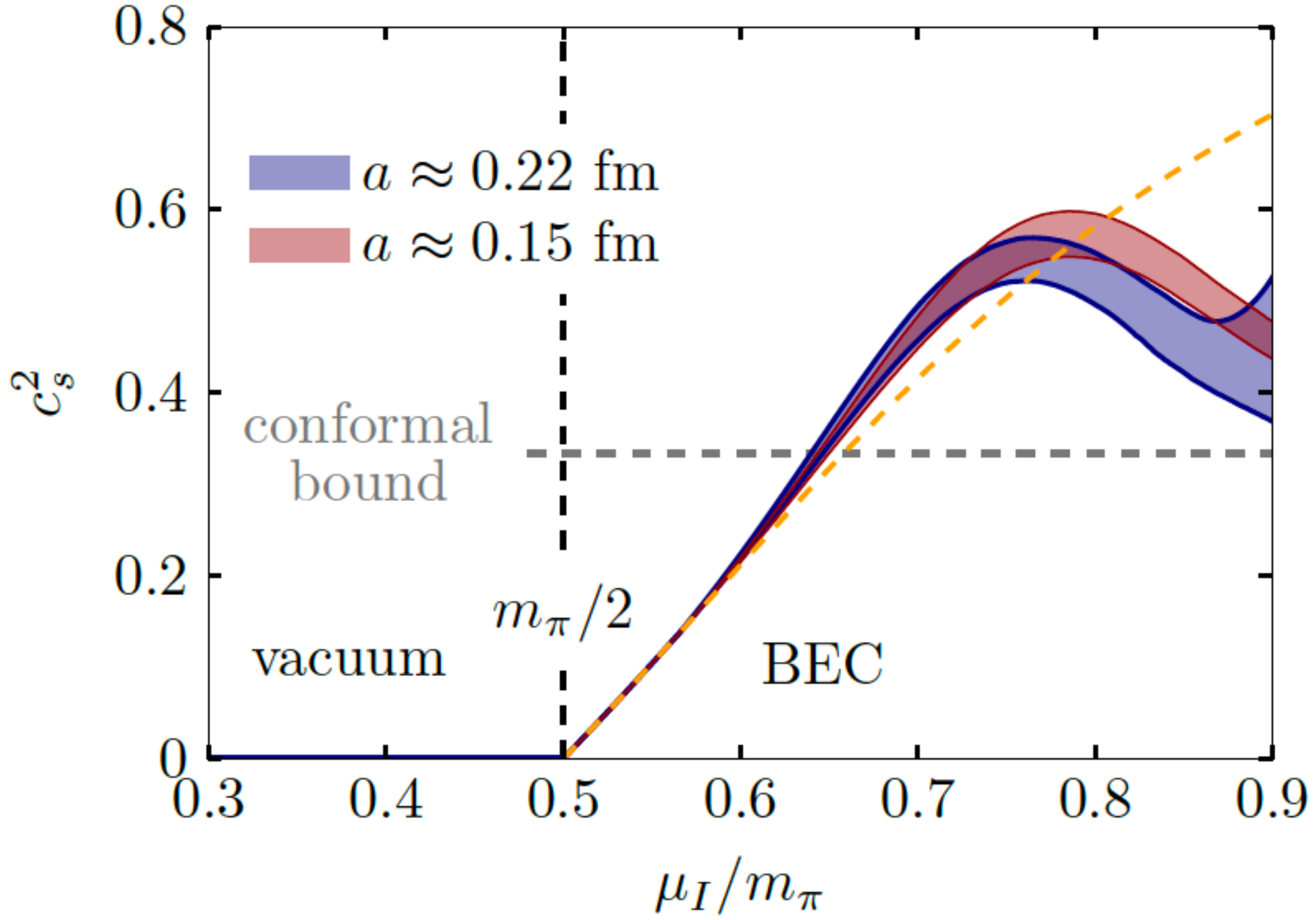
# 3 color QCD with isospin chemical potential

## Isospin-$\mu_I$ QCD $\approx$ real $\mu$ 2color QCD

B. B. Brandt, F. Cuteri , G. Endrodi, arXiv: 2212.14016

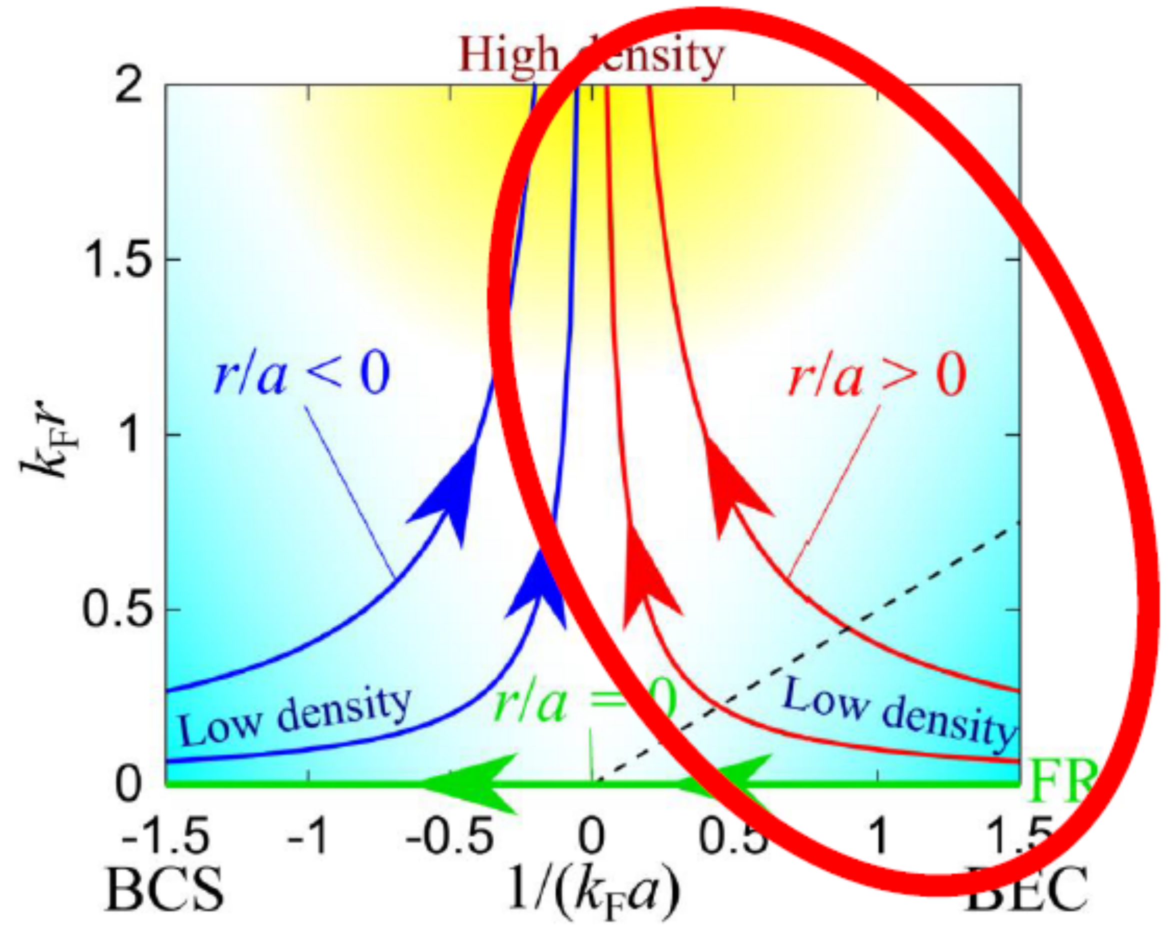
result with spline interpolation

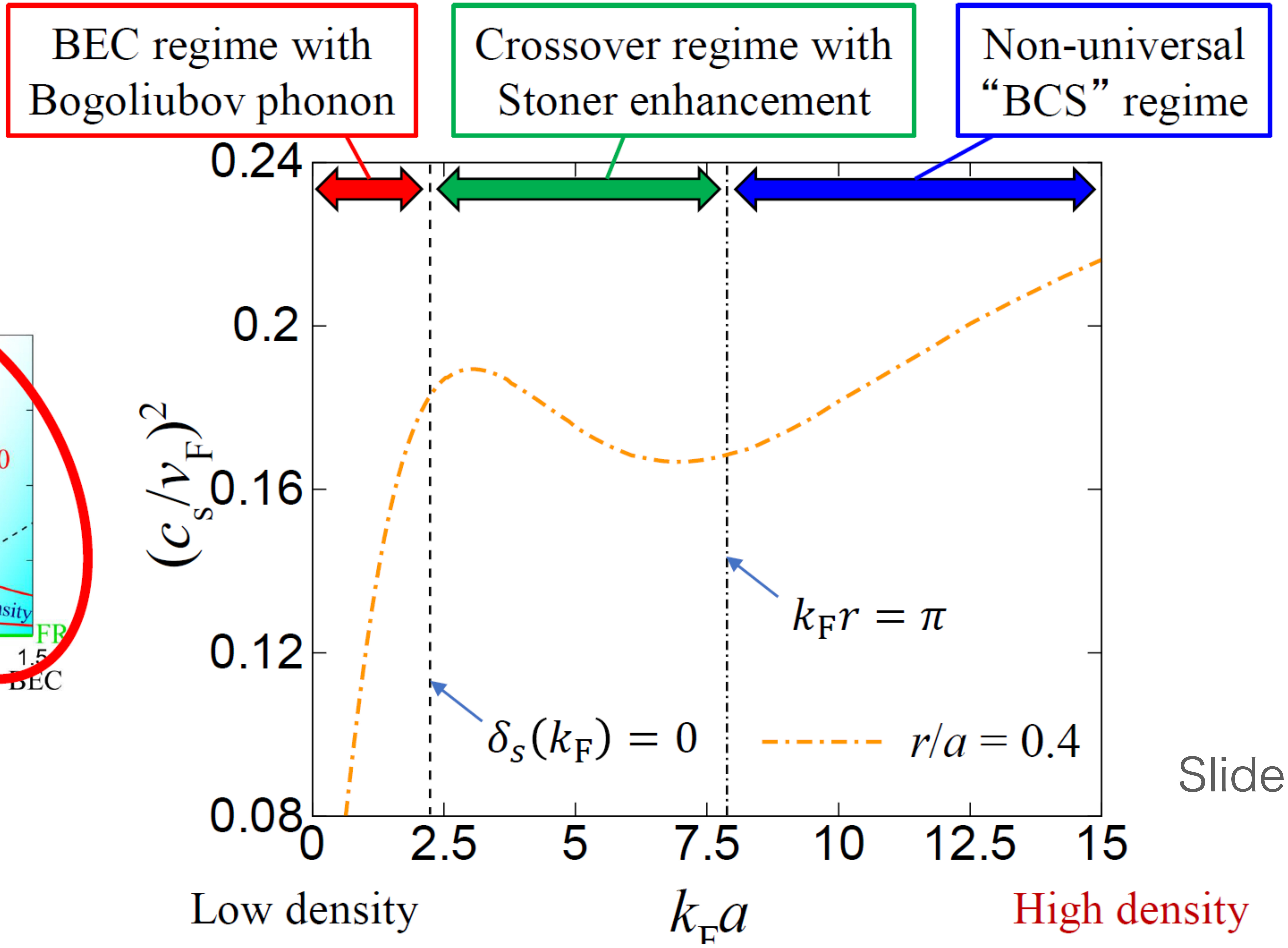




# backup

# Example of cond.mat. model





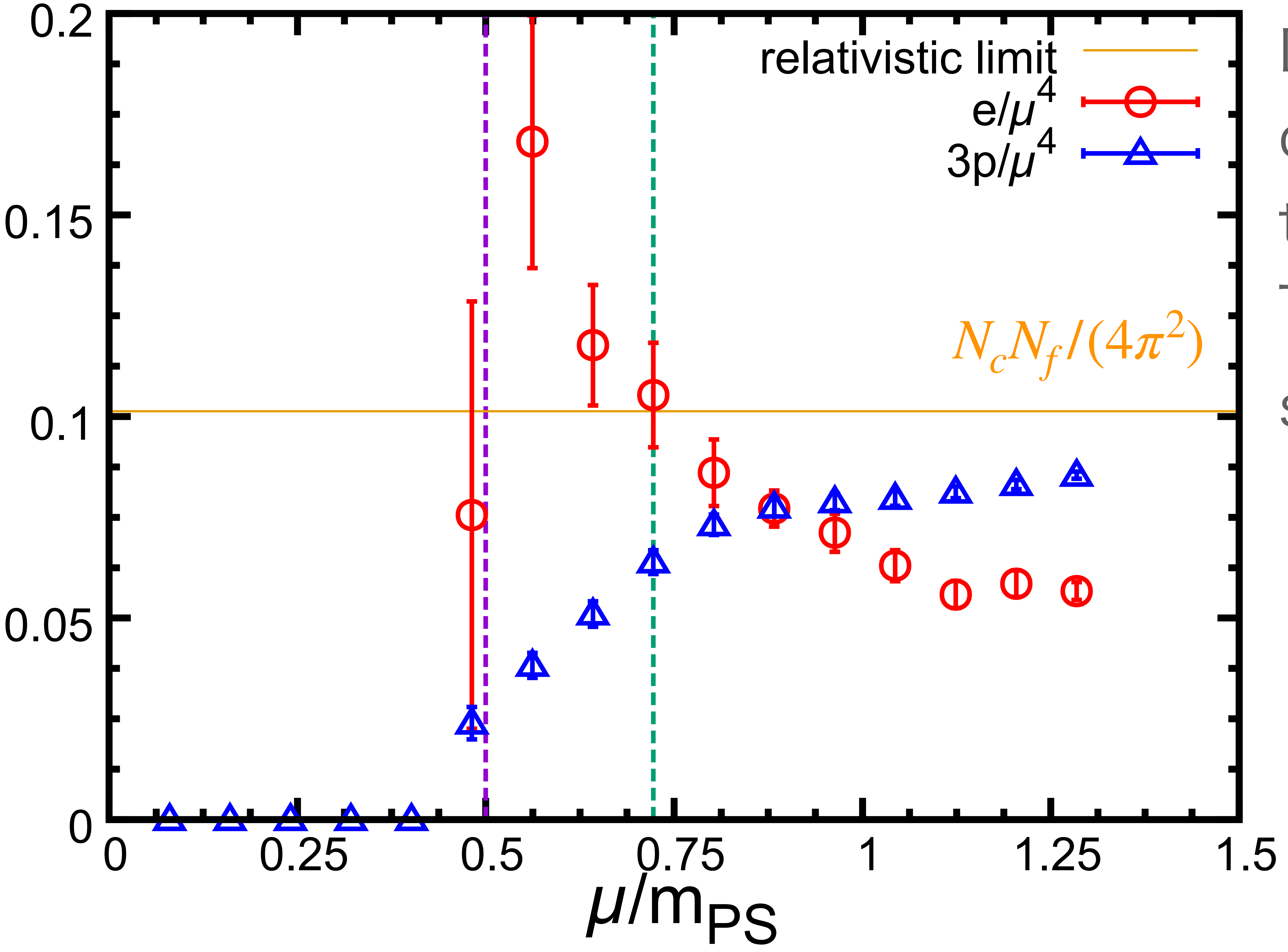
BEC regime with Bogoliubov phonon

Crossover regime with Stoner enhancement

Non-universal "BCS" regime

Slide by H.Tajima

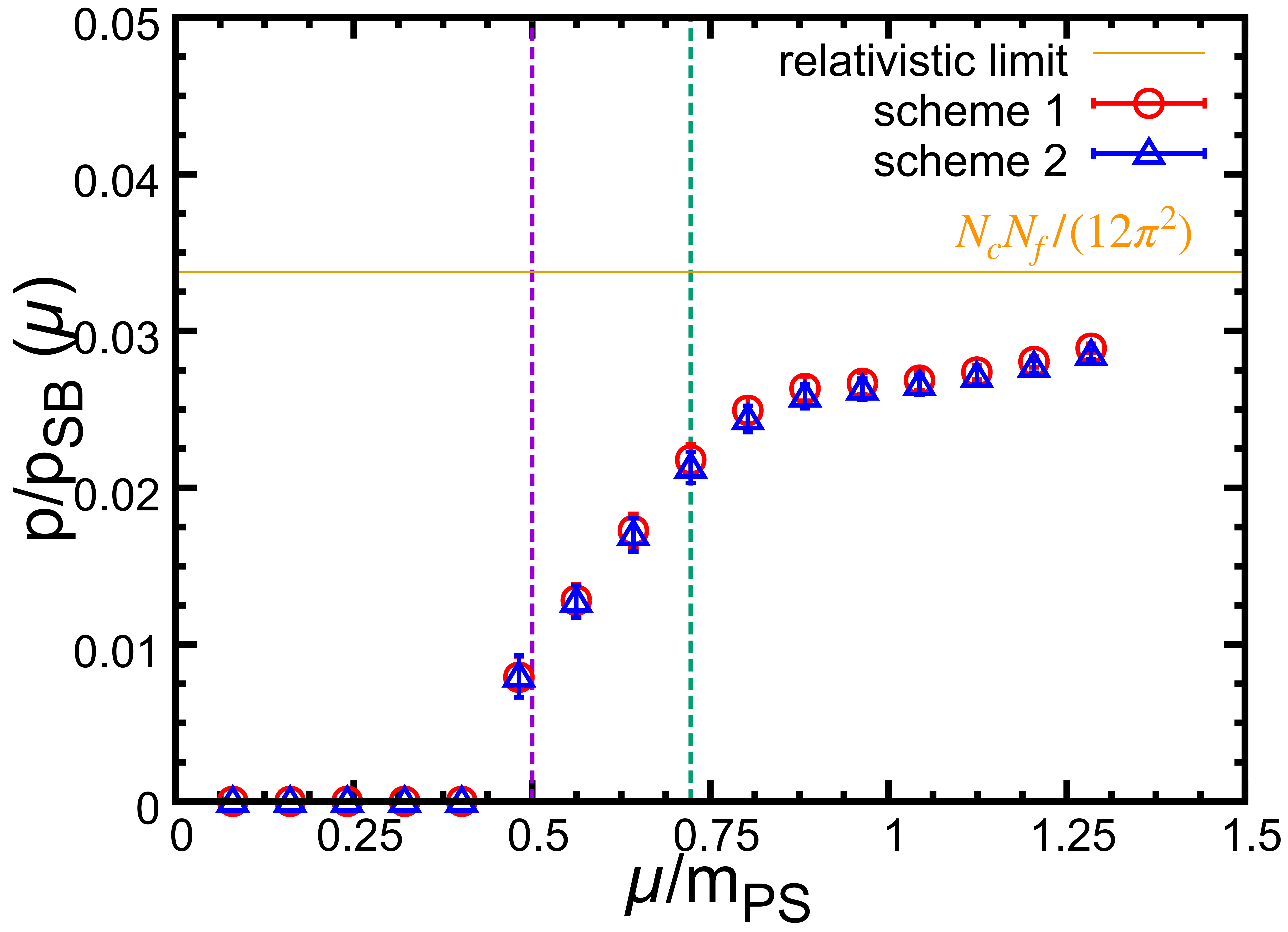
# scaling of p and e in high density



In massive fermion theory, the trace anomaly does not vanish because the mass term breaks the scale invariance.

The mass term will give a negative contribution, so that we expect $e/\mu^4 < e_{SB}/\mu^4 = N_cN_f/(4\pi^2)$

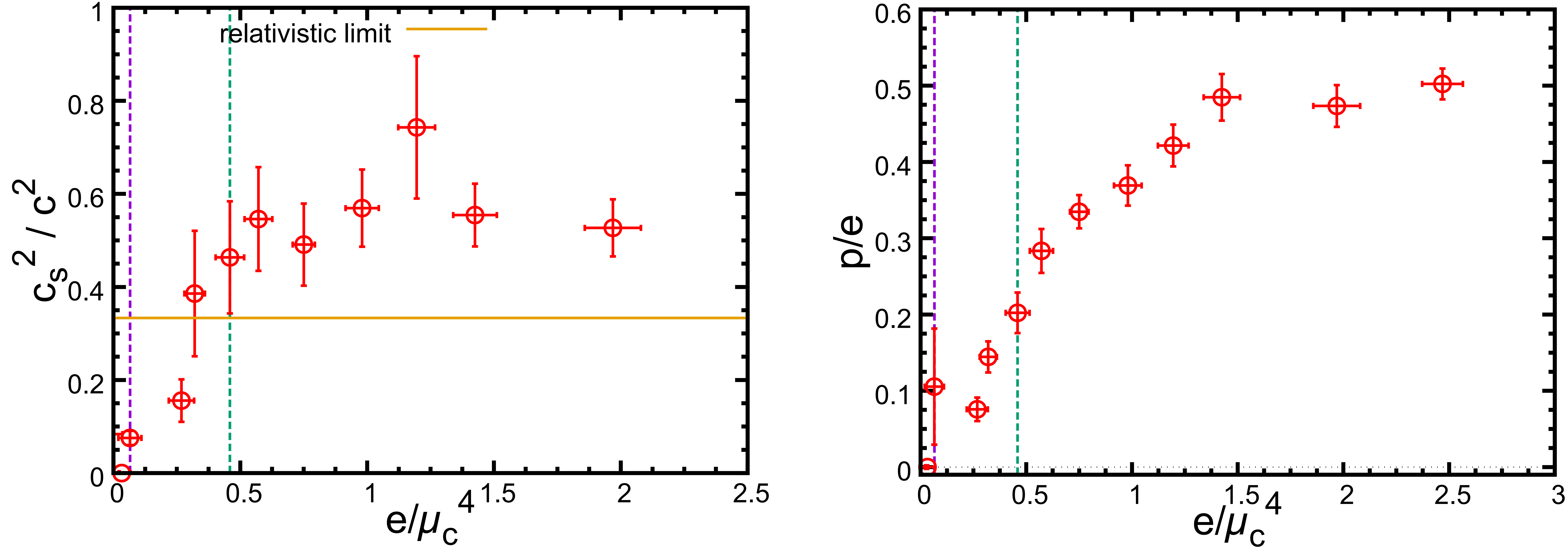
# Scheme dependence of pressure



$$\mathrm{I:}\quad \frac{p}{p_{SB}}(\mu) = \frac{\int_{\mu_o}^{\mu} n_q(\mu')d\mu'}{\int_{\mu_o}^{\mu} n_{SB}^{\mathrm{lat}}(\mu')d\mu'}\,; \tag{28}$$

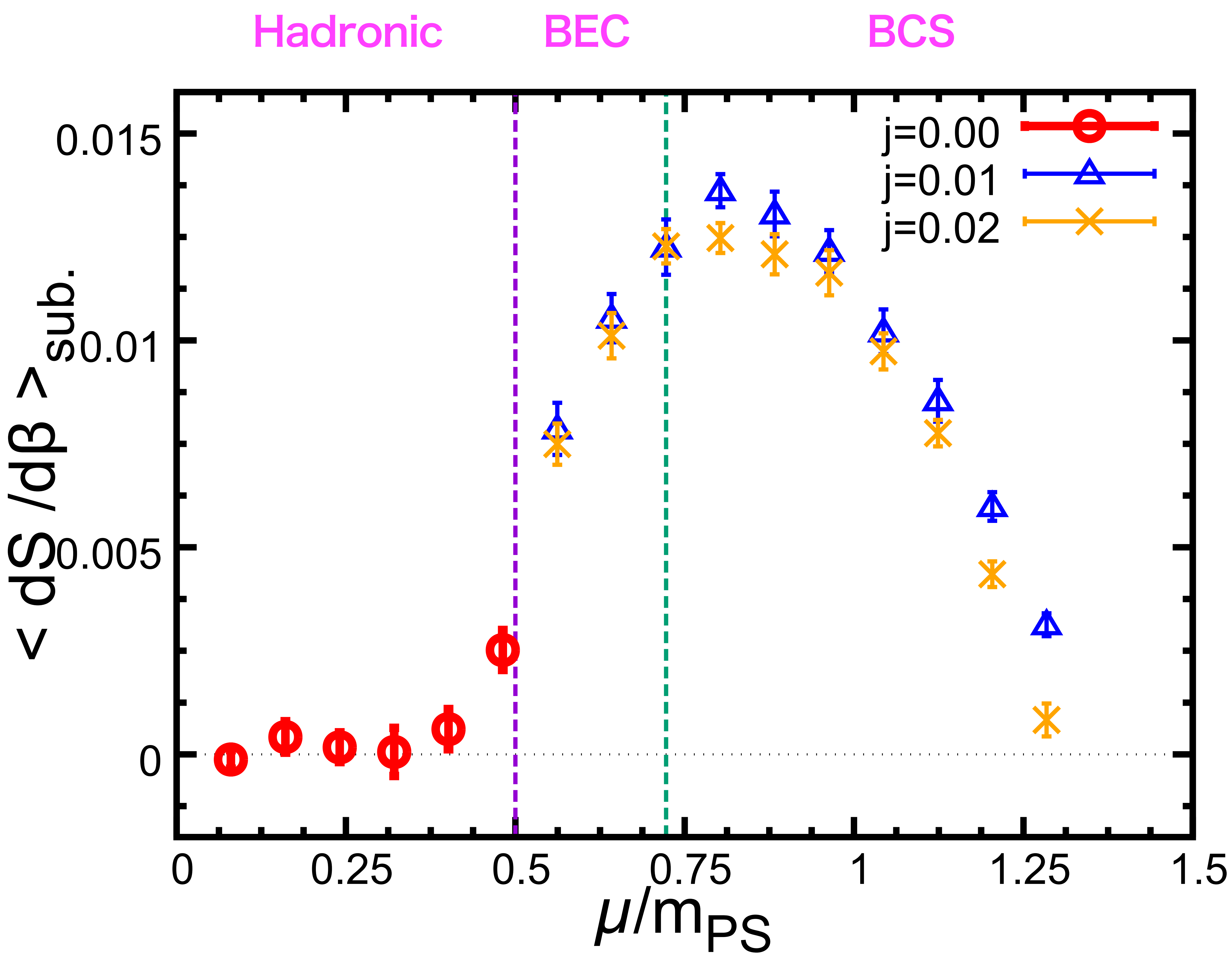
$$\mathrm{II:}\quad \frac{p}{p_{SB}}(\mu) = \frac{\int_{\mu_o}^{\mu} \frac{n_{SB}^{\mathrm{cont}}}{n_{SB}^{\mathrm{lat}}}(\mu')n_q(\mu')d\mu'}{\int_{\mu_o}^{\mu} n_{SB}^{\mathrm{cont}}(\mu')d\mu'}\,, \tag{29}$$

# Sound velocity (ratio $\Delta p/\Delta e$) vs energy



# $\mu$-dependence of gauge action

## value of Iwasaki gauge action knows the phase structure!



Our definition of each phase

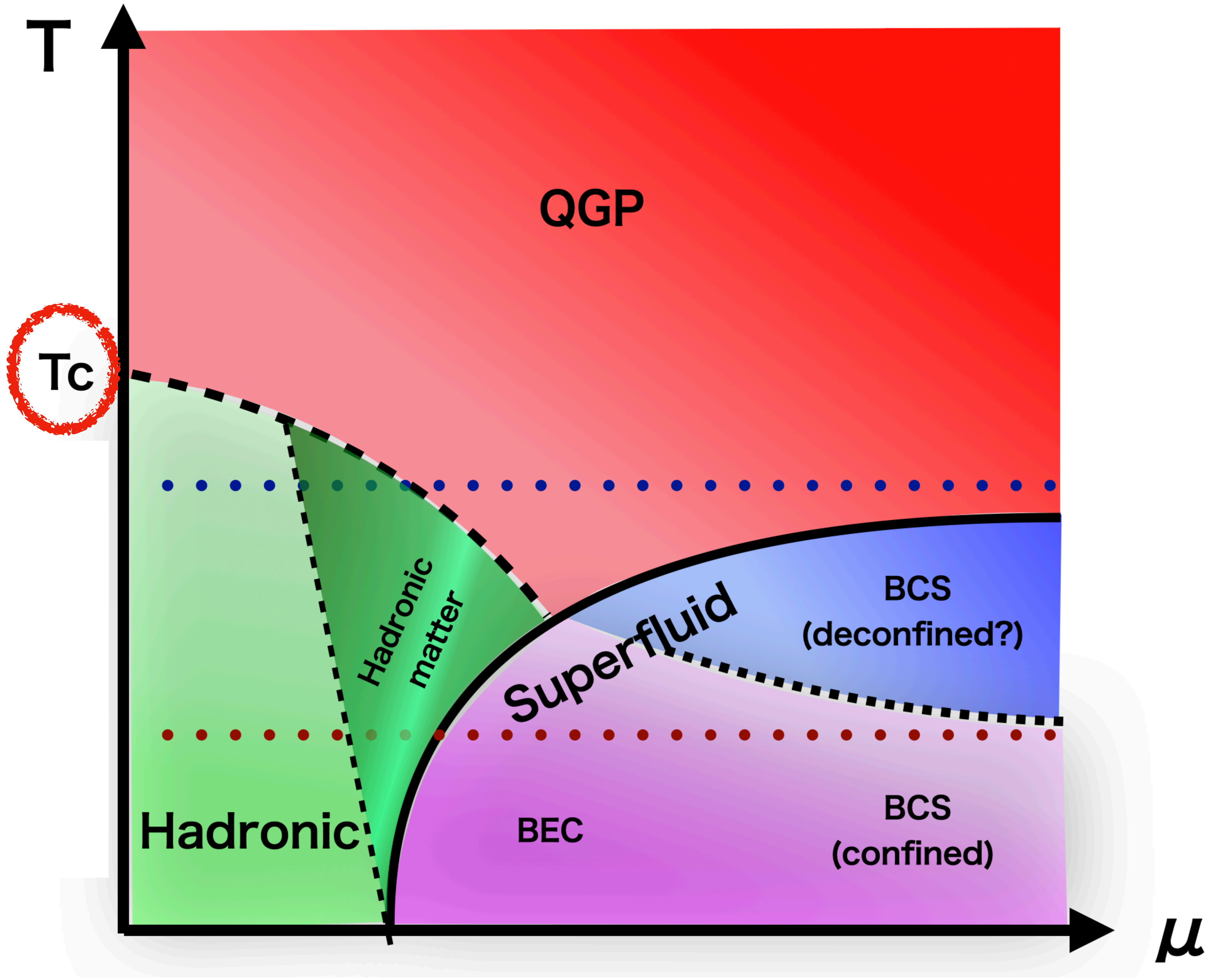
| | Hadronic | Hadronic-matter | QGP | Superfluid | |
|---|---|---|---|---|---|
| | | | | **BEC** | **BCS** |
| $\langle \lvert L \rvert \rangle$ | zero | zero | non-zero | | |
| $\langle qq \rangle$ | zero | zero | zero | non-zero | $\propto \Delta(\mu)\mu^2$ |
| $\langle n_q \rangle$ | | non-zero | | non-zero | $n_q/n_q^{\mathrm{tree}} \approx 1$ |

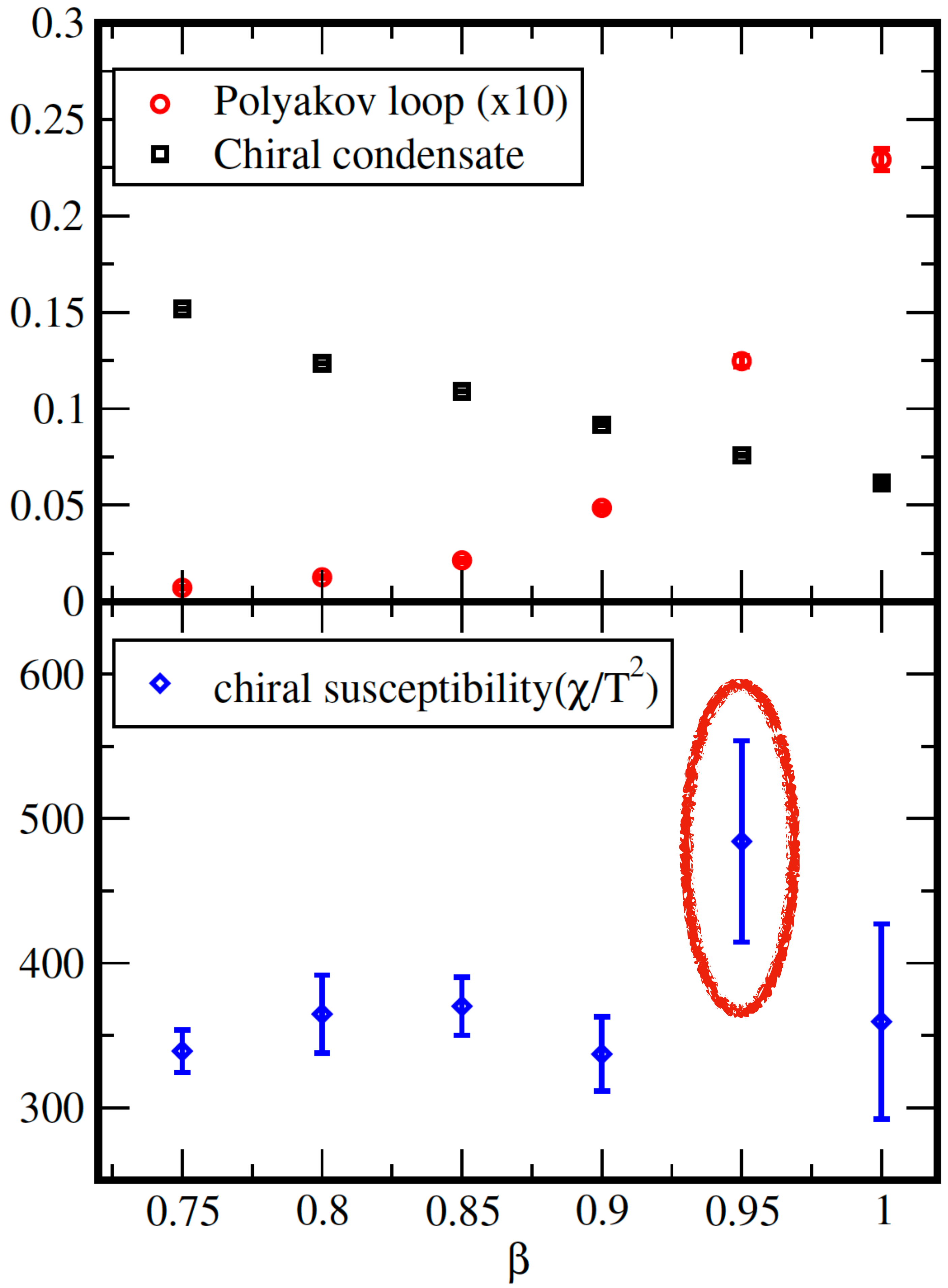
# Phase diagram

# Scale setting at $\mu = 0$

K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0

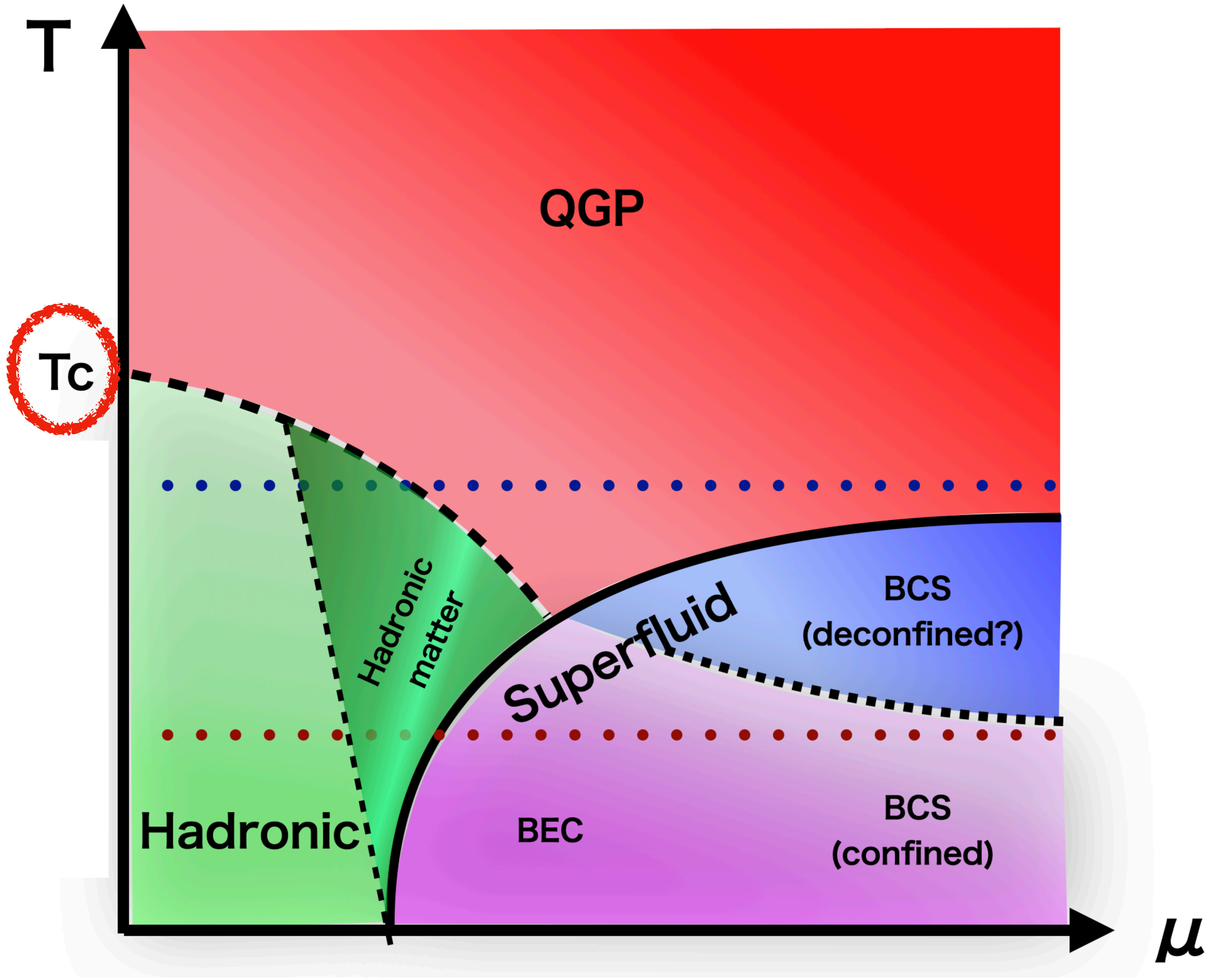
- Tc at $\mu = 0$ from chiral susceptibility





# Scale setting at $\mu = 0$

K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0

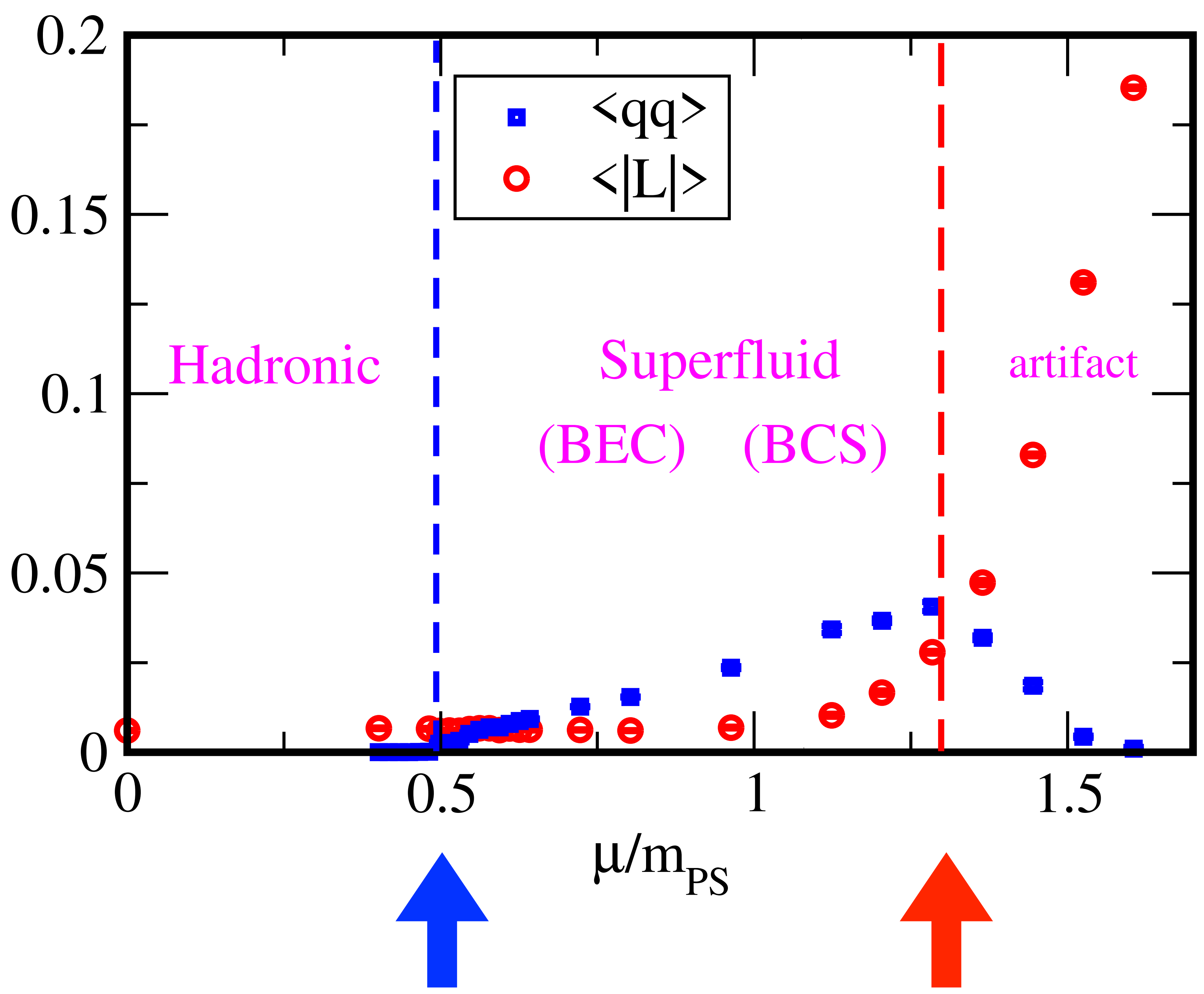


- Tc at $\mu = 0$ from chiral susceptibility
- Assume Tc=200MeV

  Tc is realize Nt=10, $\beta = 0.95$ (a=0.1[fm])
- Find relationship between $\beta$ (lattice bare coupling) and $a$ (lattice spacing)

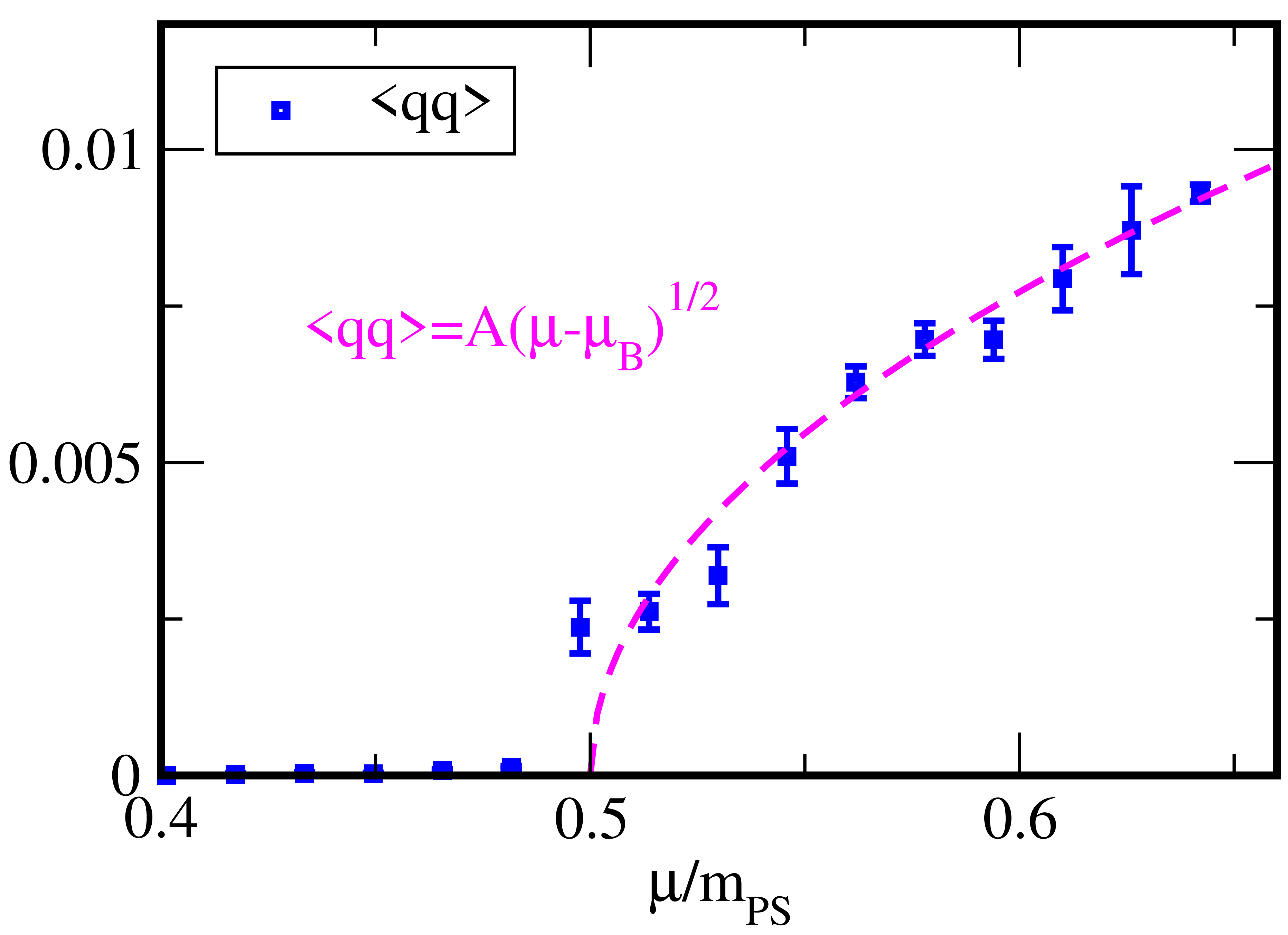
  In finite density simulation, a=0.1658[fm]

# Order parameters in j=0 limit



$\mu_B/m_{\rm PS} \simeq 0.50$

$\mu/m_{\rm PS} \simeq 1.28$
$(\mu_D/m_{\rm PS} \simeq 1.44)$

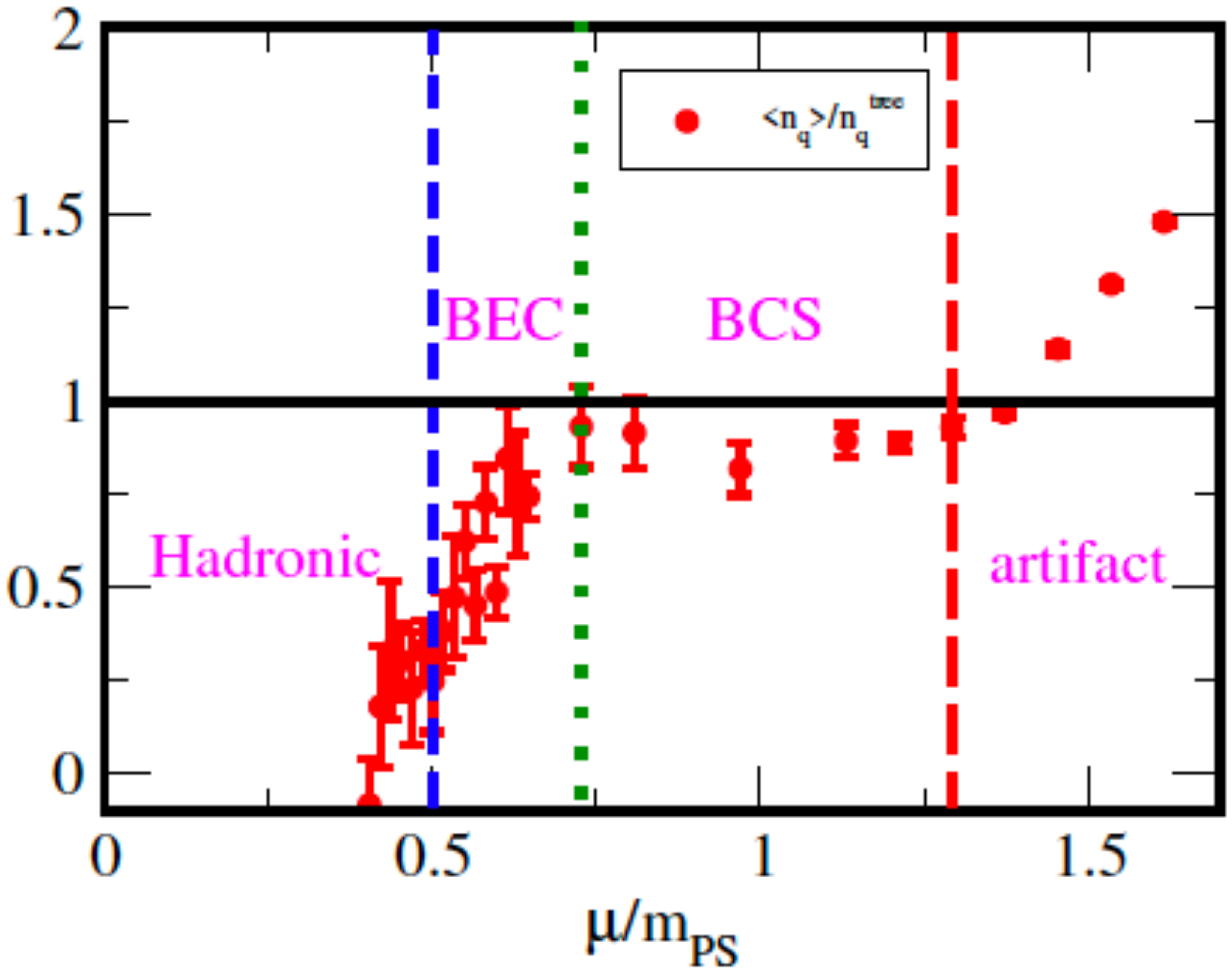


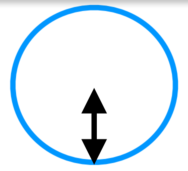
Scaling law of order param.
is consistent with ChPT.

Ref.) Kogut, Stephanov, Toublan, Verbaarshot, Zhitnitsly
NPB 582 (2000) 477

At T=0.39Tc, we find the BCS with confined phase until $\mu \lesssim 1152 MeV$.

# BEC/BCS crossover

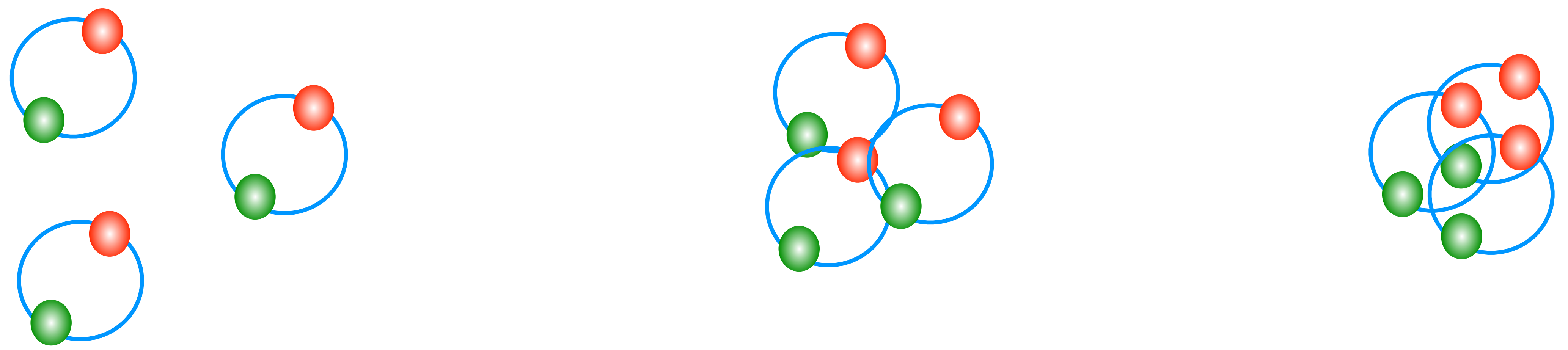




Δ: gap energy

BEC phase

BCS phase



Distance between quarks $\gg \Delta^{-1}$

Distance between quarks $\ll \Delta^{-1}$

Quarks behave free particles

density

Number density of free particle

$$n_q^{\text{tree}}(\mu) = \frac{4N_cN_f}{N_s^3N_\tau}\sum_k \frac{i\sin\tilde{k}_0[\sum_i \cos k_i - \frac{1}{2\kappa}]}{[\frac{1}{2\kappa} - \sum_\nu \cos\tilde{k}_\nu]^2 + \sum_\nu \sin^2\tilde{k}_\nu}$$

# J->0 extrapolation

## Diquark condensate has a strong j dependence

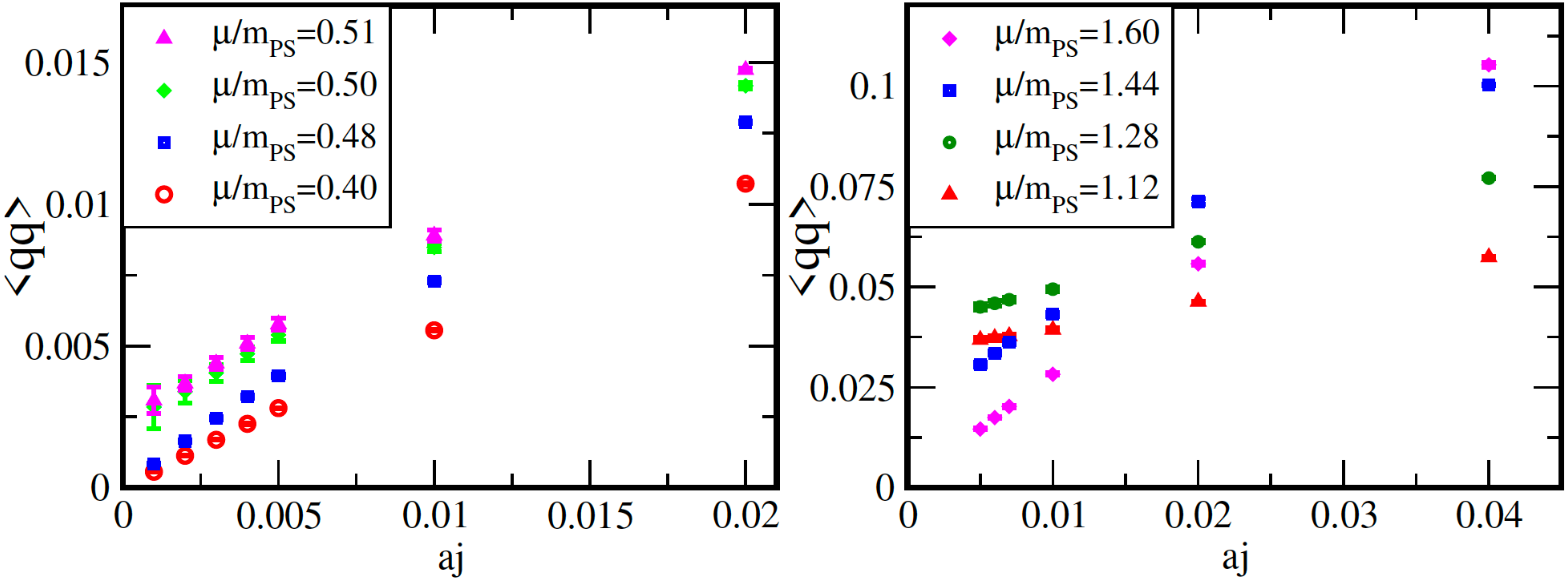
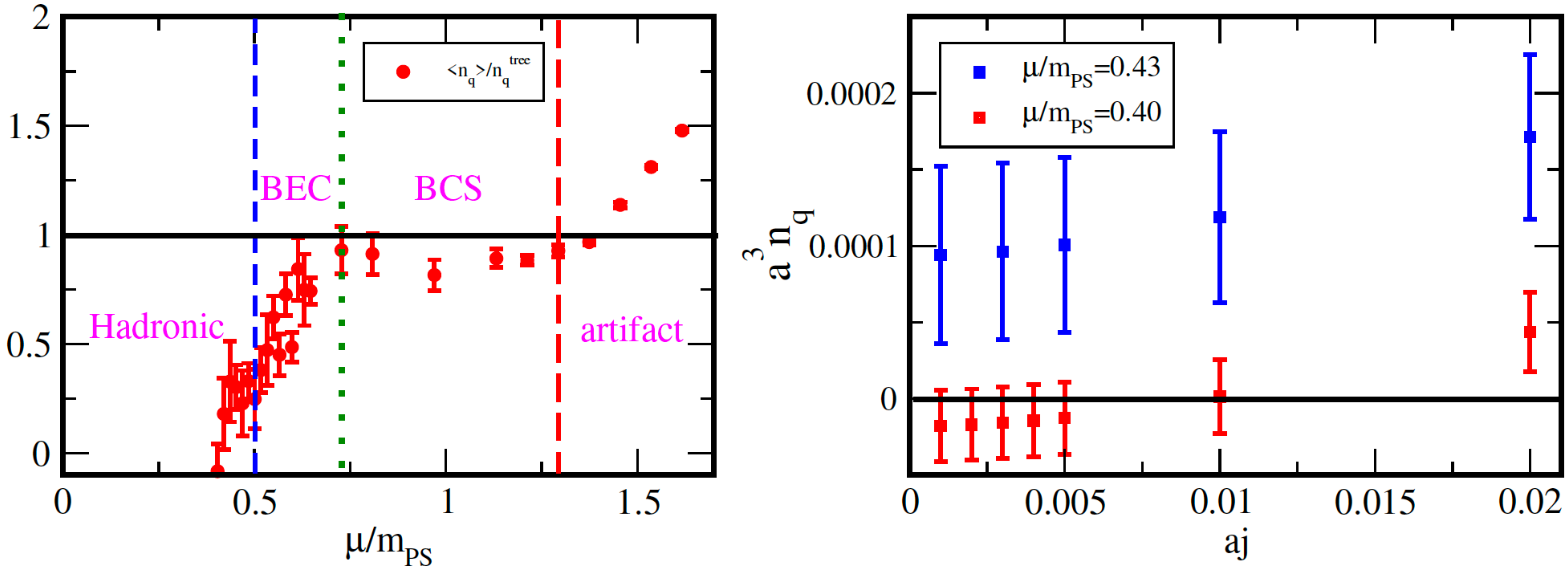


**Figure 5.** The $j$-dependence of the diquark condensate for several $\mu/m_{\rm PS}$.

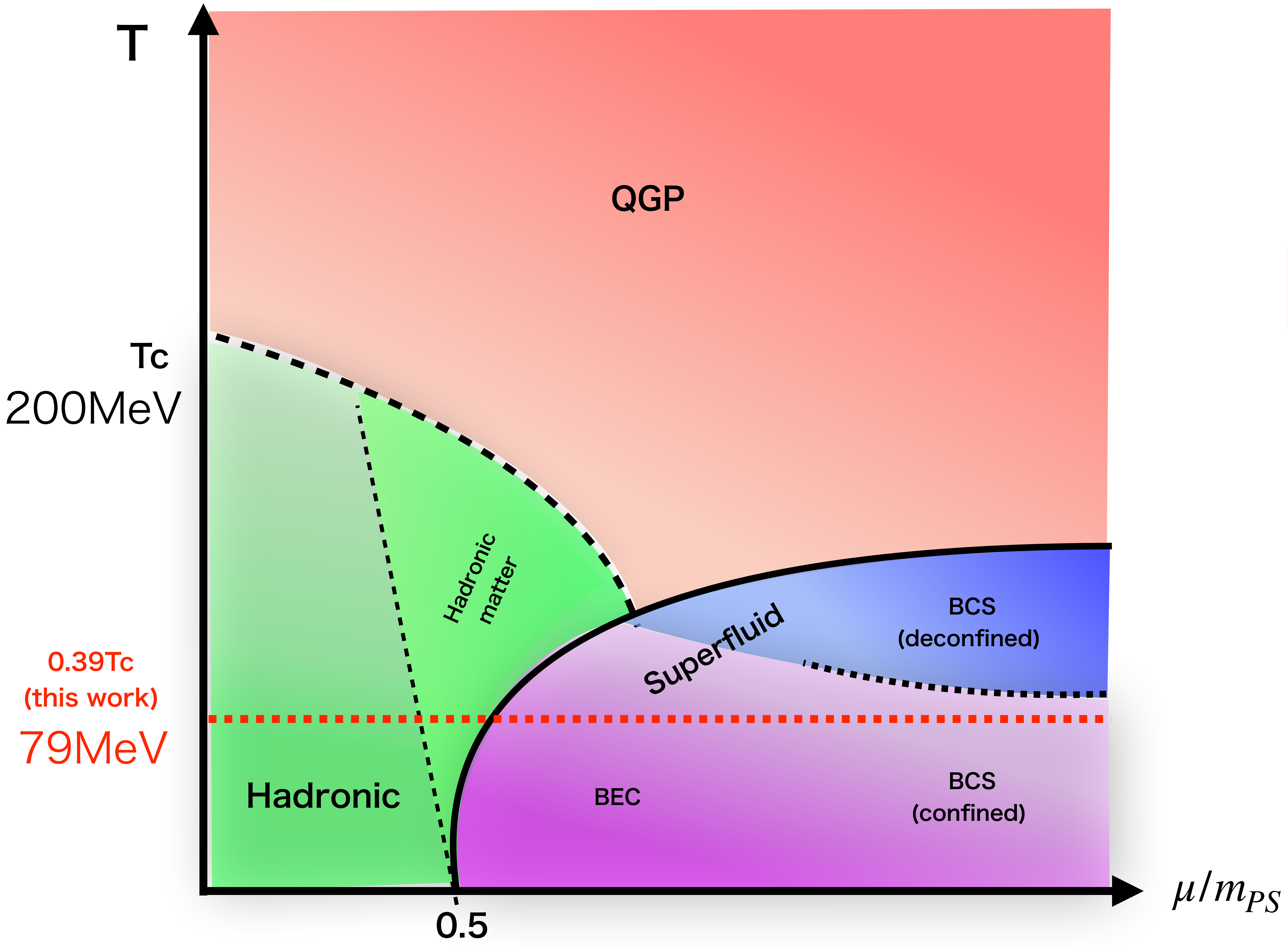
# J->0 extrapolation

Chiral condensate and $n_q$ have a mild j-dependence



# Phase diagram of 2color QCD

## Comparison with 3color QCD



Fukushima-Hatsuda (2010)