Pion light-cone distribution amplitude from a Heavy-quark OPE

THE HOPE COLLABORATION

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Challenges and Opportunities in Lattice QCD Simulations and Related Fields
RIKEN RCCS, Kobe
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References

W. Detmold and CJDL, Phys. Rev. **D 73** (2006) 014501

HOPE Collaboration, W. Detmold *et al.*, Phys. Rev. **D 104** (2021) 7, 074511

HOPE Collaboration, W. Detmold *et al.*, Phys. Rev. **D 105** (2022) 3, 034506

HOPE Collaboration, W. Detmold *et al.*, arXiv: 2211.17009
(based upon talk presented by R. Perry at **Lattice 2022)**

THE HOPE COLLABORATION

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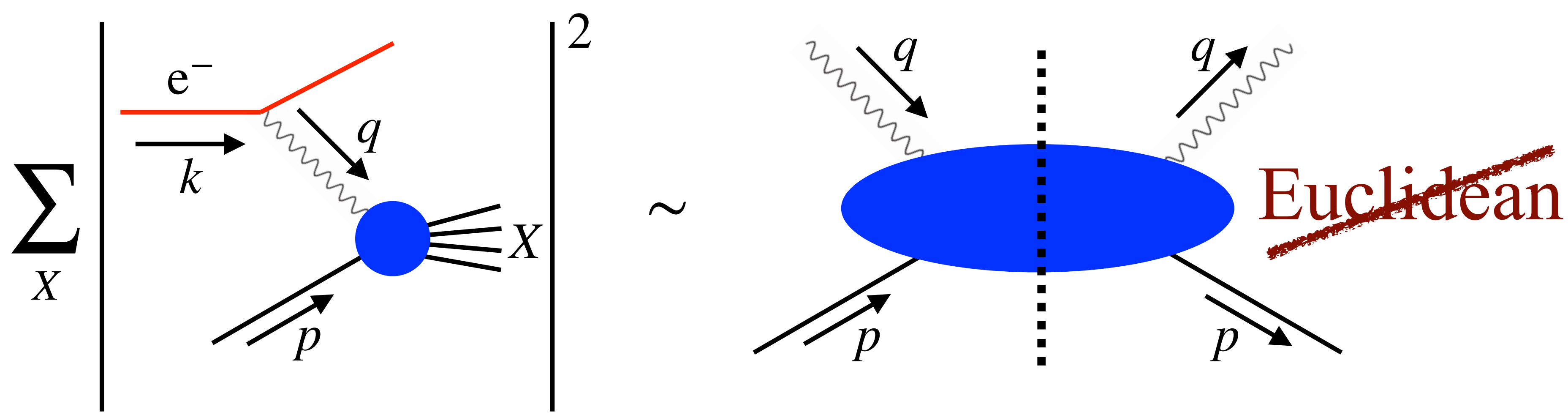
Outline

- ★ General issues: parton physics from Euclidean lattice QCD
- ★ The HOPE method
- ★ Pion light-cone distribution amplitude from HOPE
  - → Numerical results of the 2nd moment
  - → Exploratory numerical study of the 4th moment
- ★ Conclusion and outlook

General issues and introducing the HOPE method

Challenges in parton physics from lattice QCD



$$\text{Im of } T_S^{\mu\nu}(p,q) = \int d^4z\, \mathrm{e}^{iq\cdot z}\, \langle p,S\,|\,T[J^\mu(z)J^\nu(0)\,|\,p,S\rangle$$

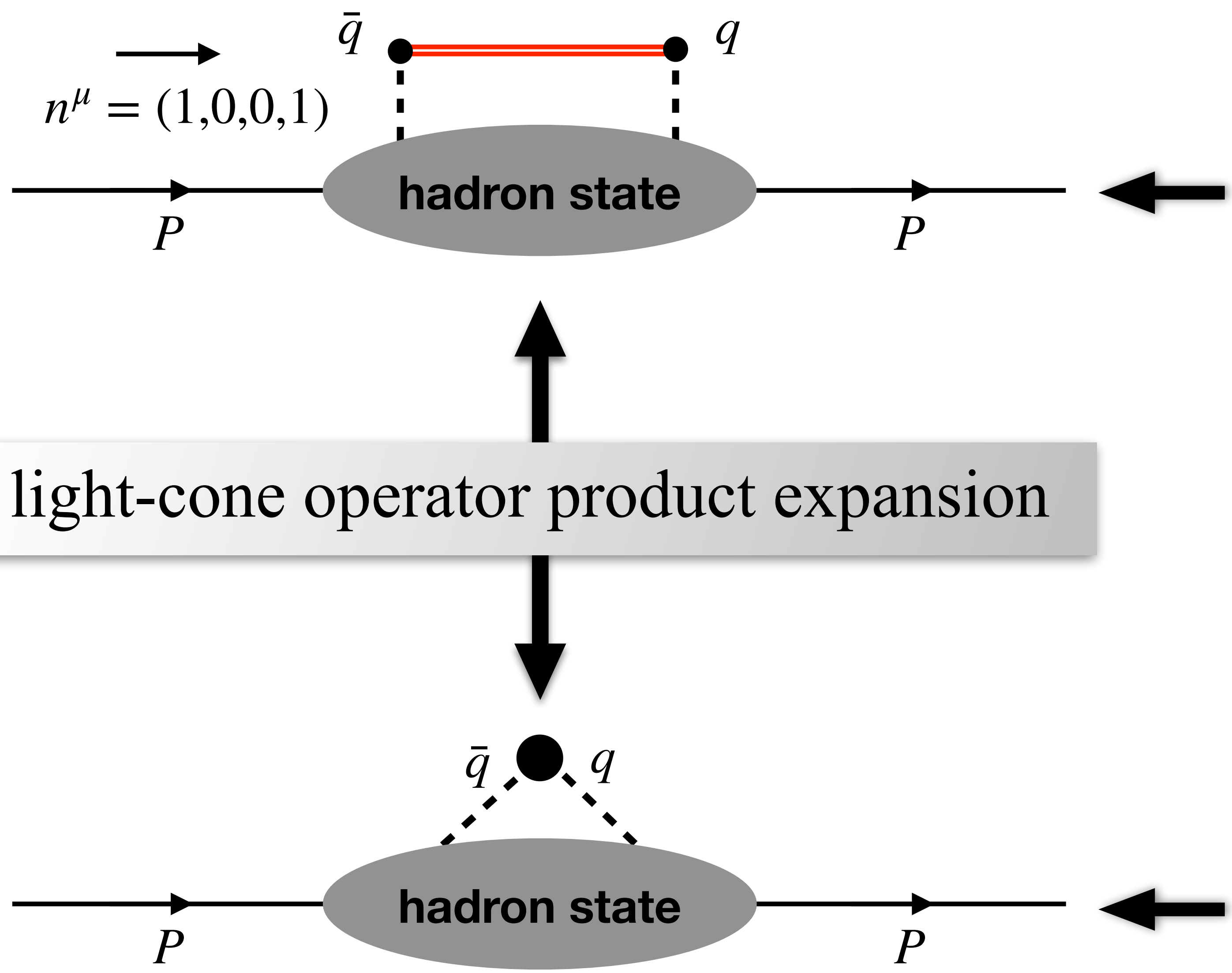
factorisation → twist-2

$$\sum_a \int dx\, H_a(xp, Q^2, \mu)\, f_{a/H}(x,\mu)$$

$$f_{q/H}(x,\mu) = \int \frac{dz^-}{4\pi}\, \mathrm{e}^{-ixp^+z^-}\, \langle p,S\,|\,\bar{q}(z^-)\Gamma W[z^-,0]q(0)\,|\,p,S\rangle$$

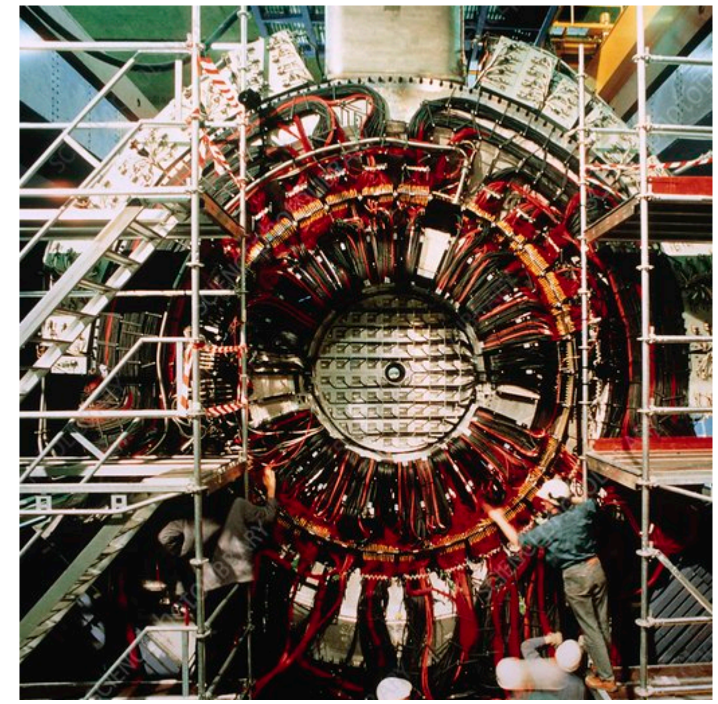
★ Non-perturbative QCD dynamics on the light cone → ~~Euclidean~~

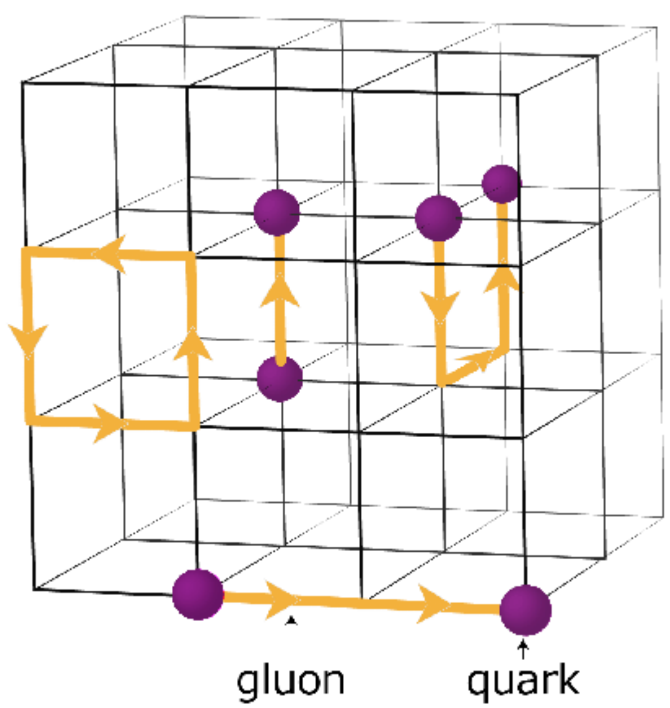
Conventional LQCD approach



twist-2 Mellin moments

twist = dim - spin





Conventional LQCD approach

★ Light-cone OPE

$$T[J^\mu(x)J^\nu(0)] = \sum_{i,n} C_i(x^2,\mu^2)\, x_{\mu_1}\ldots x_{\mu_n}\, \mathcal{O}_i^{\mu\nu\mu_1\ldots\mu_n}(\mu) + \text{higher twists}$$

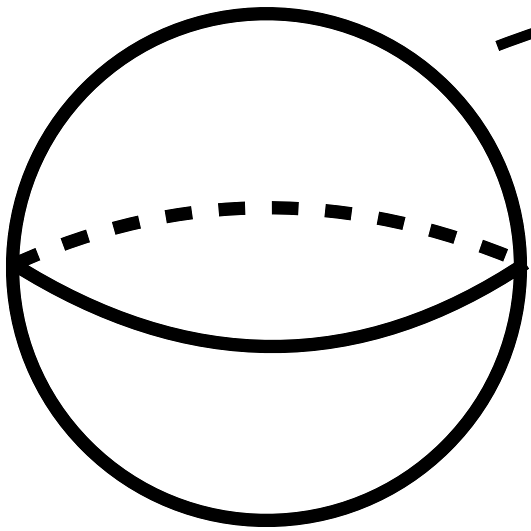
Twist-2 Mellin moments ⇒ parton distribution functions

★ The twist-2 operators

$$\mathcal{O}_i^{\nu\mu\mu_1\ldots\mu_n} = \bar{\psi}\Gamma_{i,\nu} D^\mu D^{\mu_1}\ldots D^{\mu_n}\psi - \text{ traces}$$

Issue with computing the Mellin moments

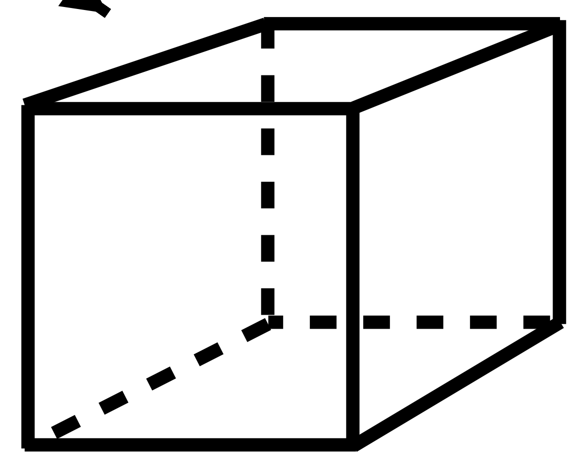
Continuum



SU(2) symmetry

Twist = Dim - Spin

Lattice



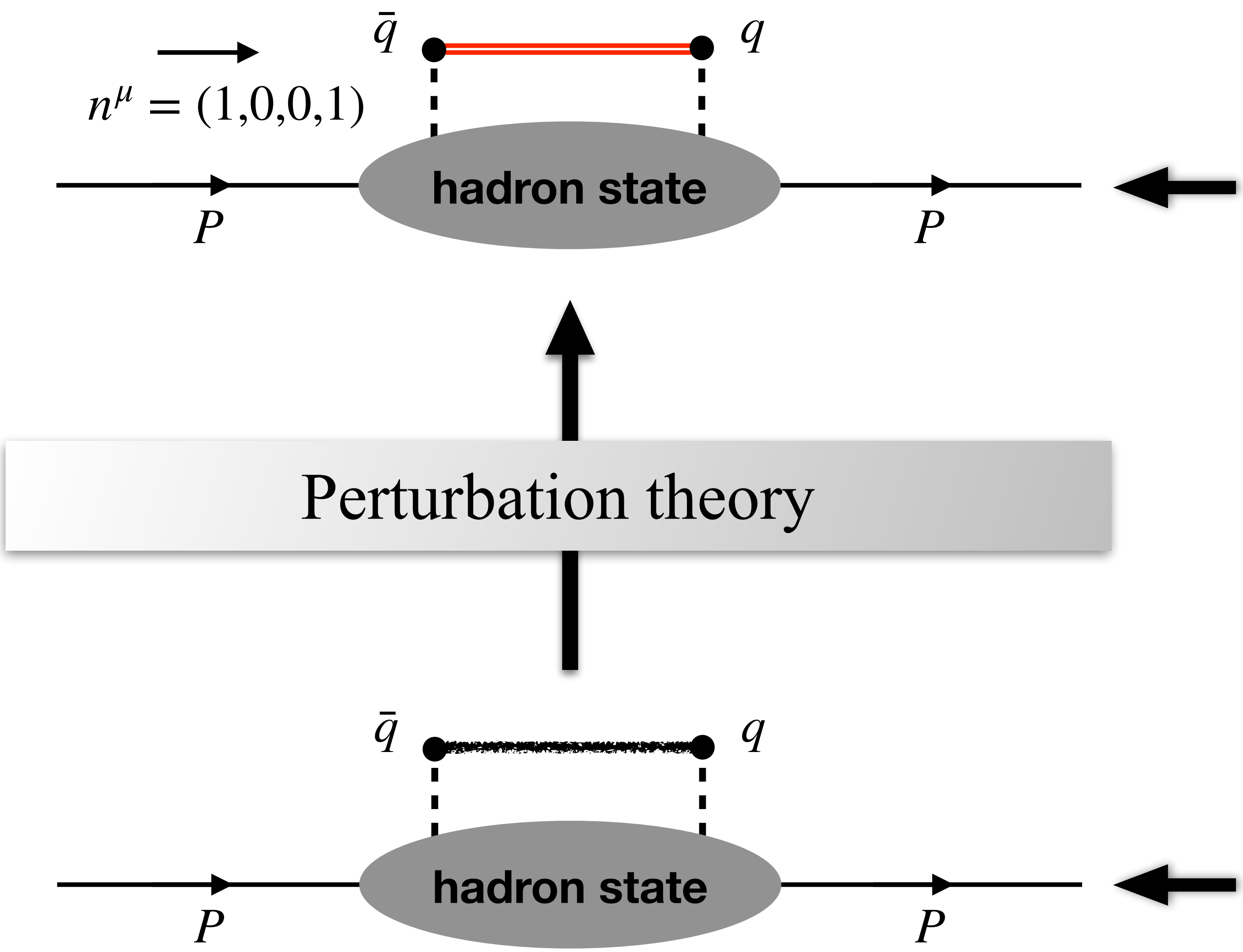
Octahedral symmetry

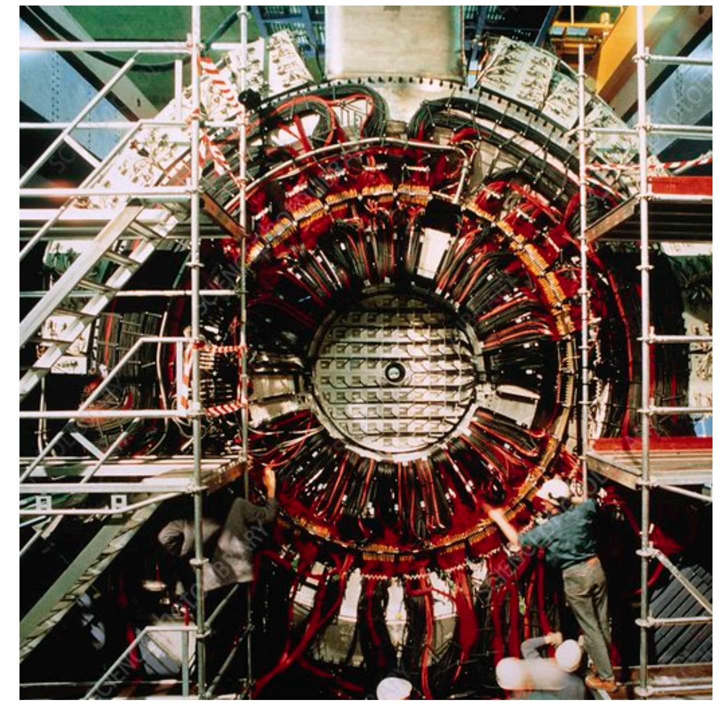
~~Spin~~

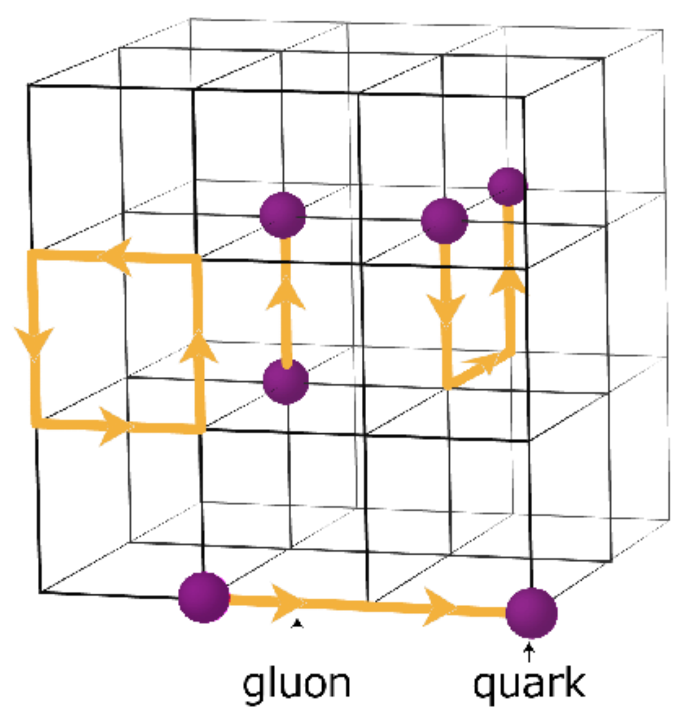
Operator mixing under renormalisation, power ($1/a^n$) divergence

→ Only the first few moments can be extracted in practice

“Novel” LQCD approach

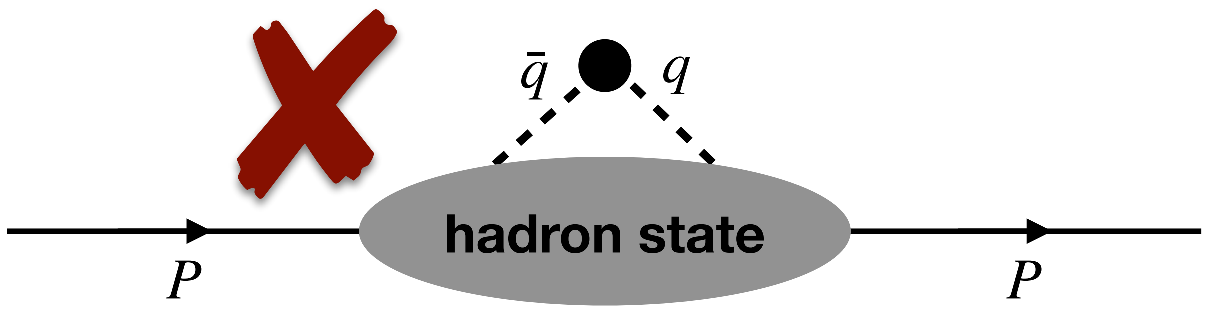


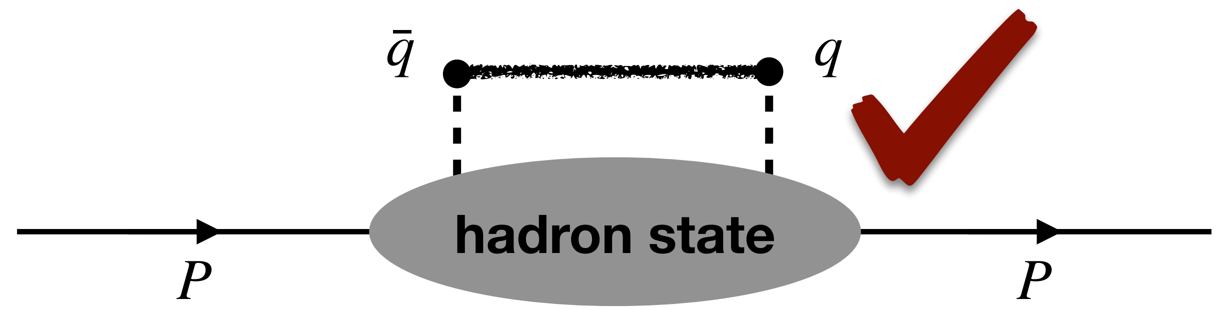




Parton distribution from lattice QCD

through *unphysical* non-local operators





- ★ A space-like Wilson line (quasi-PDF and pseudo-PDF)
  X. Ji, PRL 110 (2013); A. Radyushkin, PRD 96 (2017)
- ★ Two currents separated by space-like distance
  V. Braun and D. Mueller, EPJC 55 (2008)
- ★ Two flavour-changing currents with valence heavy quark (HOPE method)
  W. Detmold and CJDL, PRD 73 (2006)
- ★ More A. Chambers *et al.*, PRL 118 (2017); Y. Ma & J.-W. Qiu, PRL 120 (2018)…

The HOPE method for higher moments
and
pion light-cone distribution amplitude (LCDA)

Pion LCDA: definition and OPEs

$$\langle 0|\bar{d}(z)\gamma_\mu\gamma_5 W[z,-z]u(-z)|\pi^+(\mathbf{p})\rangle = ip_\mu f_\pi \int_{-1}^{1} d\xi\; e^{-i\xi p\cdot z}\phi_\pi(\xi,\mu)$$

Gegenbauer (conformal) OPE in the isospin limit

$$\phi_\pi(\xi,\mu) = \frac{3}{4}(1-\xi^2)\sum_{n=0,\text{even}}^{\infty}\phi_n(\mu)\mathcal{C}_n^{3/2}(\xi) \xrightarrow[\text{RG}]{\mu\to\infty} \frac{3}{4}(1-\xi^2)$$

Gegenbauer moments $\phi_n(\mu) = \dfrac{2(2n+3)}{3(n+1)(n+2)}\displaystyle\int_{-1}^{1} d\xi\; \mathcal{C}_n^{3/2}(\xi)\phi_\pi(\xi,\mu)$

Light-cone OPE

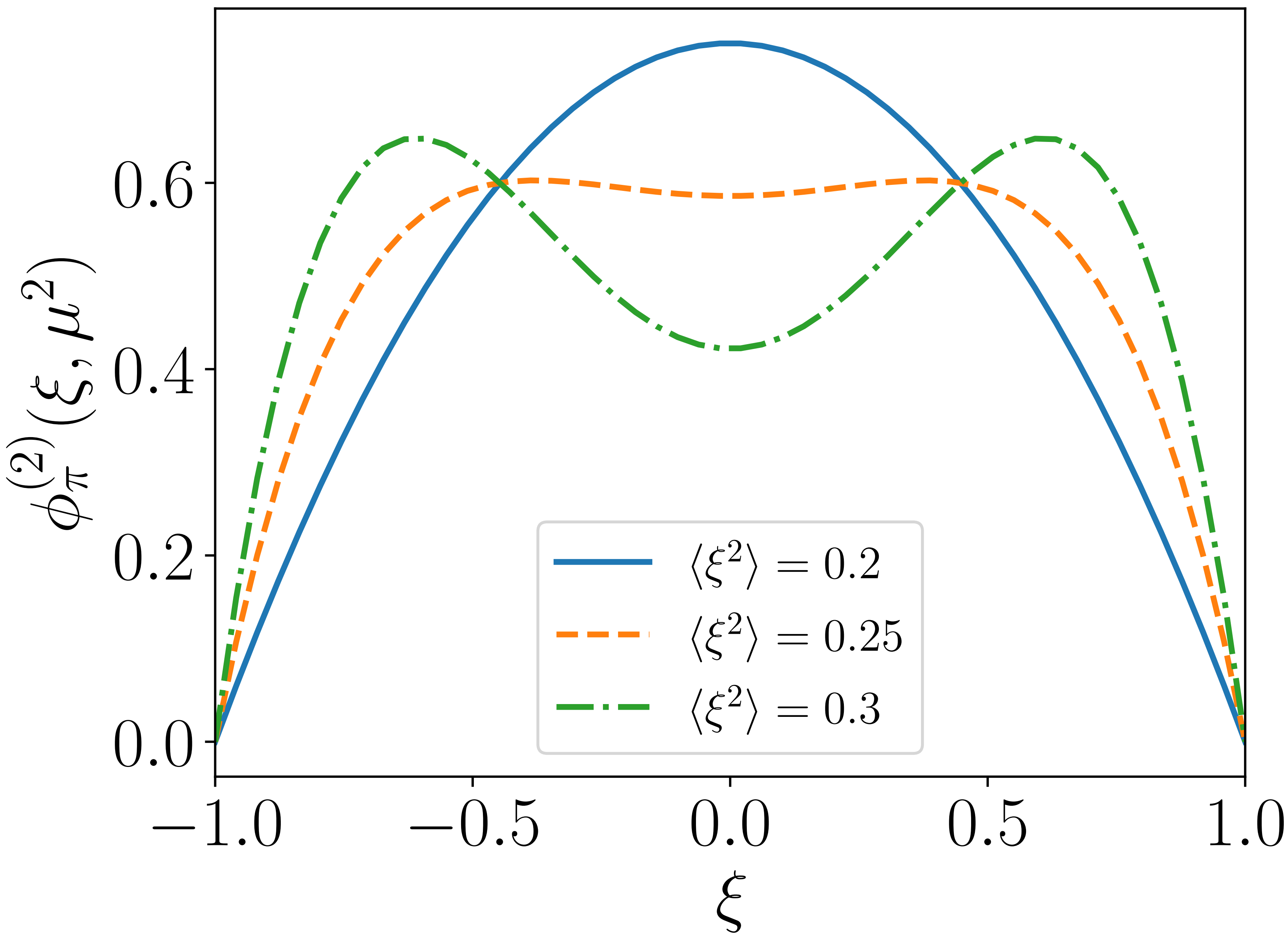
$$\langle 0|\left[\bar{d}\gamma^{\{\mu_0}\gamma_5(i\overleftrightarrow{D}^{\mu_1})\ldots(i\overleftrightarrow{D}^{\mu_n\}})u - \text{traces}\right]|\pi^+(\mathbf{p})\rangle$$
$$= f_\pi\,\langle\xi^n\rangle\,(\mu^2)\,[p^{\mu_0}p^{\mu_1}\ldots p^{\mu_n} - \text{traces}]$$

Mellin moments $\langle\xi^n\rangle(\mu) = \displaystyle\int_{-1}^{1} d\xi\; \xi^n\phi_\pi(\xi,\mu)$

$$\phi_0 = \langle\xi^0\rangle = 1\,,\;\; \phi_2 = \frac{7}{12}\left(5\langle\xi^2\rangle - \langle\xi^0\rangle\right)\,,\;\; \phi_4 = \frac{11}{24}\left(21\langle\xi^4\rangle - 14\langle\xi^2\rangle + \langle\xi^0\rangle\right)\,,\ldots.$$

OPE and ξ-dependence

ξ: the fraction of p_π carried by one of the valence quarks (parton limit)



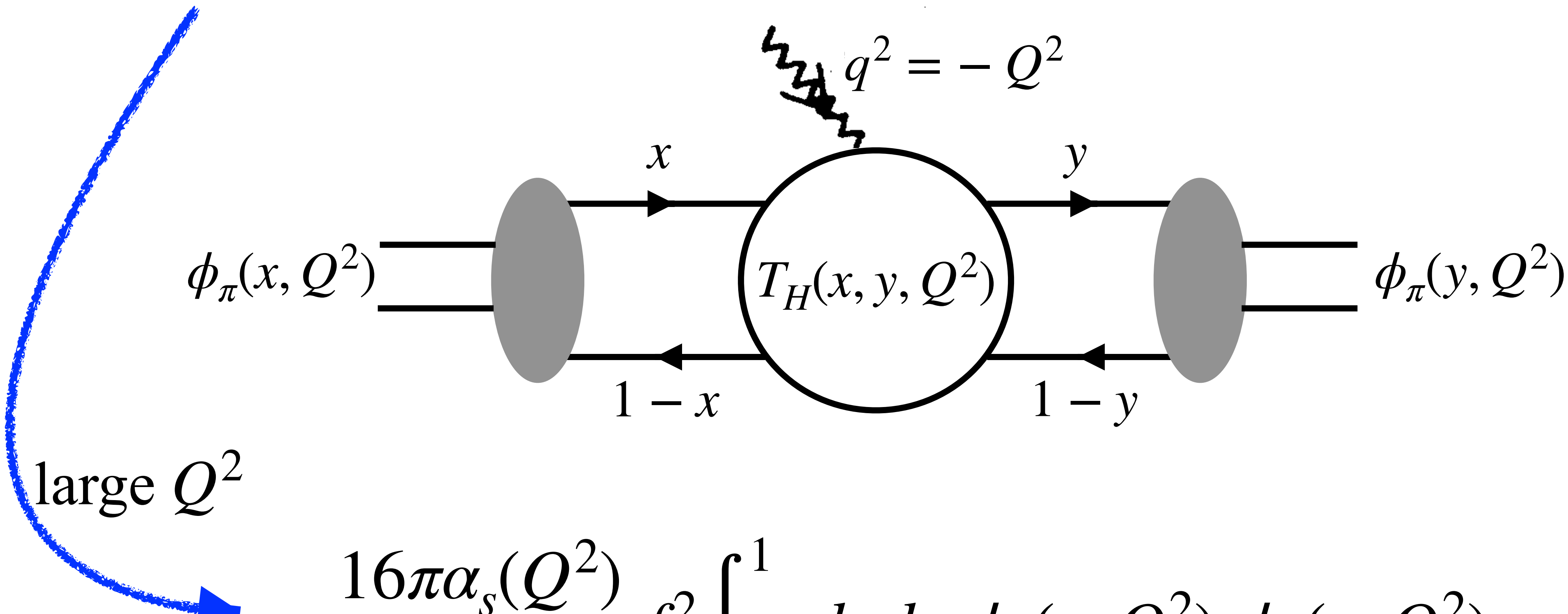
★ Power divergence already shows up in LQCD calculation for $\langle \xi^2 \rangle$

Phenomenological relevance

Pion form factor in QCD exclusive processes

G.P. Lepage and S.J. Brodsky, 1979

$$F_\pi(Q^2) = f_\pi^2 \int_0^1 dxdy\ \phi_\pi(x, Q^2)\ T_H(x, y, Q^2)\ \phi_\pi(y, Q^2)$$



$$= \frac{16\pi\alpha_s(Q^2)}{Q^2} f_\pi^2 \int_{-1}^1 dxdy\ \phi_\pi(x, Q^2)\ \phi_\pi(y, Q^2)$$

Phenomenological relevance

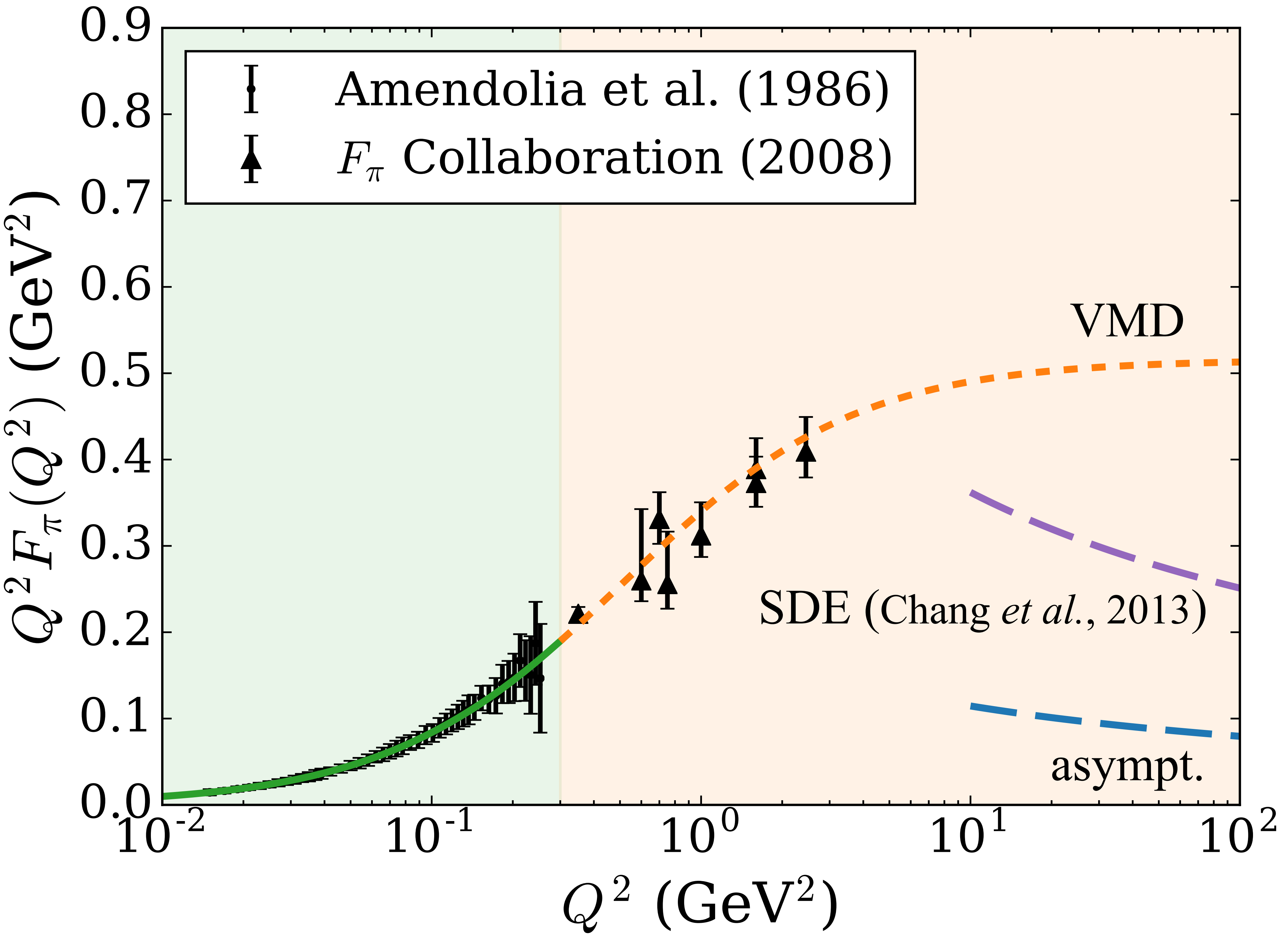
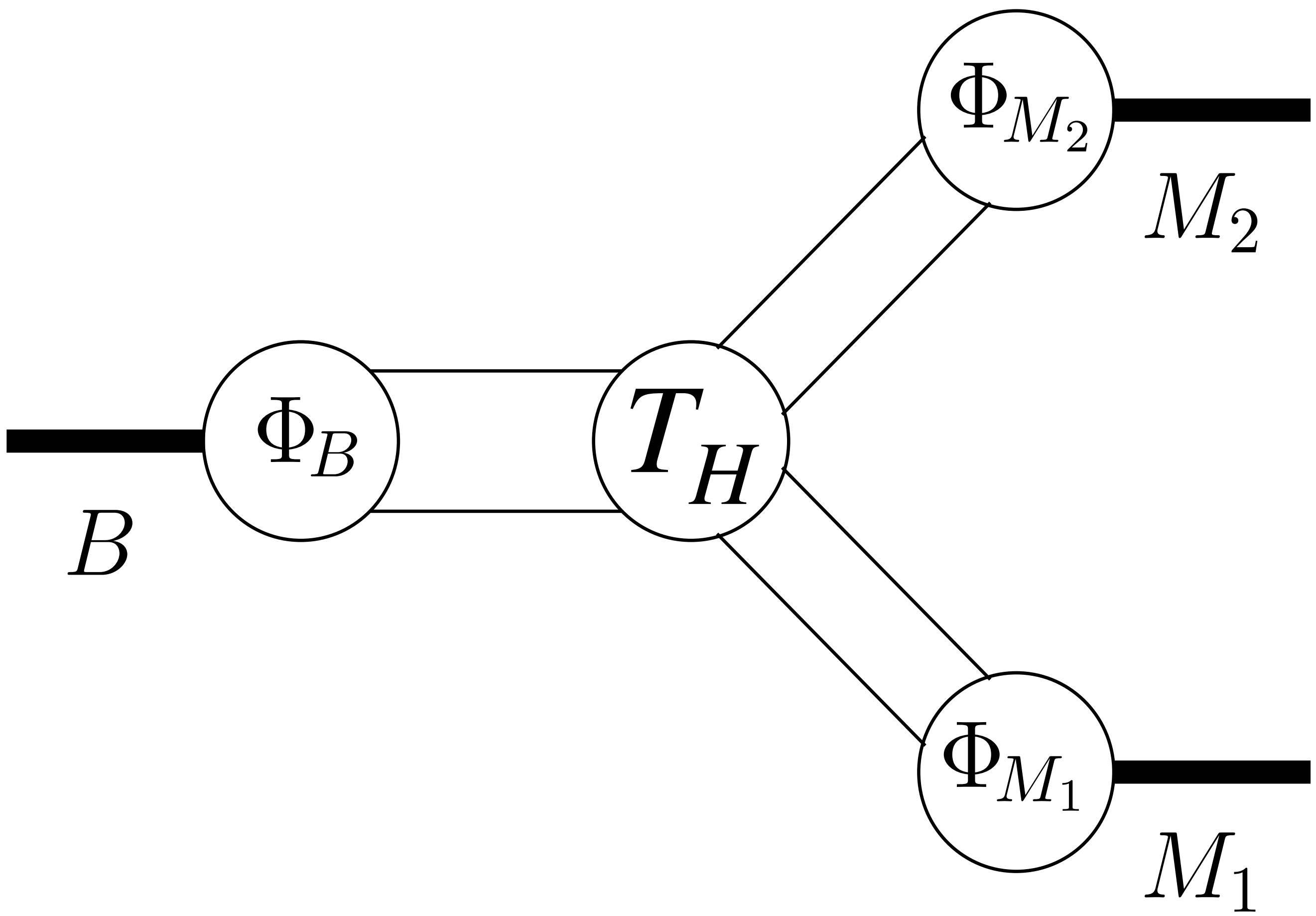


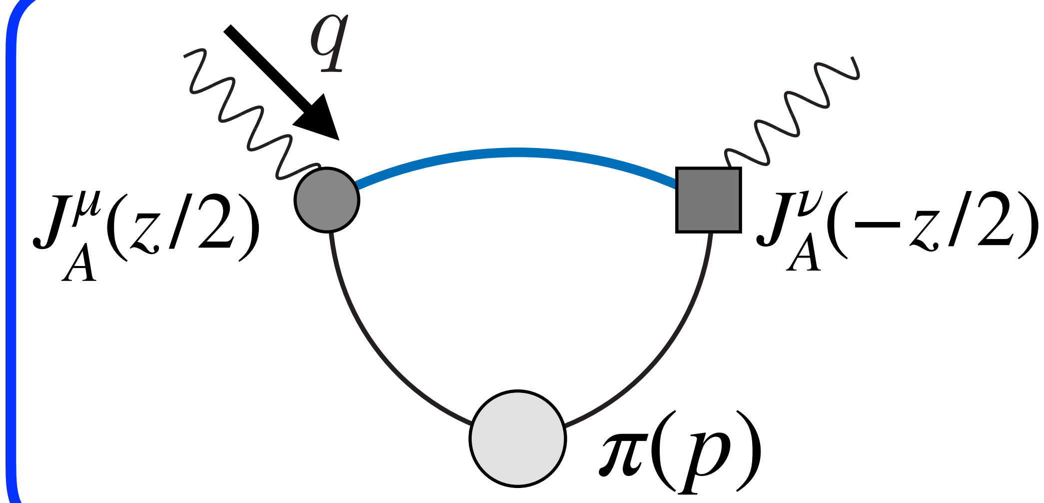
Figure from R.J. Perry *et al.*, PLB 807 (2020) 135581

Phenomenological relevance

Important input for flavour physics



HOPE amplitude for computing pion LCDA



+ the cross term

$$V^{\mu\nu}(p,q) = \int d^4z\, e^{iq\cdot z} \langle 0 \,|\, T[J_A^\mu(z/2) J_A^\nu(-z/2)] \,|\, \pi(\mathbf{p}) \rangle$$

$$J_A^\mu = \bar{\Psi}\gamma^\mu\gamma^5\psi + \bar{\psi}\gamma^\mu\gamma^5\Psi$$

Ψ is the valence, relativistic heavy quark

$$V^{[\mu\nu]}(p,q) = \frac{1}{2}\left[V^{\mu\nu}(p,q) - V^{\nu\mu}(p,q)\right]$$

OPE for HOPE amplitude

$$V^{[\mu\nu]}(p,q) = \frac{2\epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta}{\tilde{Q}^2} \sum_{n \text{ even}}^{\infty} \frac{\zeta^n \mathcal{C}_n^2(\eta)}{2^n(n+1)} C_W^{(n)}(\tilde{Q}^2) f_\pi \langle \xi^n \rangle + \mathcal{O}(1/\tilde{Q}^3)$$

higher-twist

$$\tilde{Q}^2 = q^2 + m_\Psi^2$$

$$\eta = \frac{p \cdot q}{\sqrt{p^2 q^2}}, \quad \zeta = \frac{\sqrt{p^2 q^2}}{\tilde{Q}^2}$$

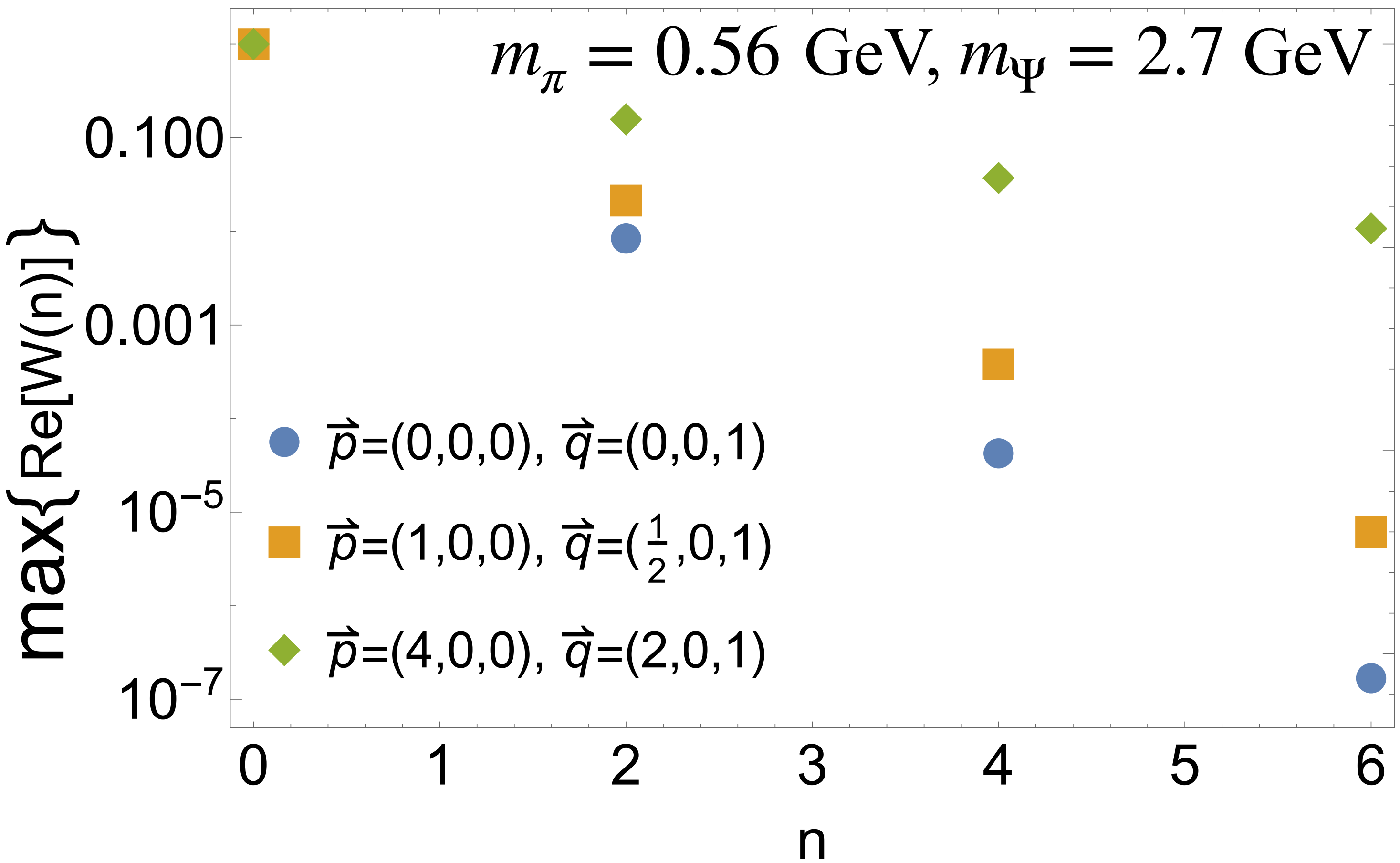
$\mathcal{C}_n^2(\eta)$: target-mass effect

- tree-level OPE
- one-loop
- fit lattice data

HOPE for $V^{[\mu\nu]}$: issue in fitting higher moments

$$V^{[\mu\nu]}(p,q) = \frac{2\epsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta}{\tilde{Q}^2}\sum_{n\ \text{even}}^{\infty}\frac{\zeta^n \mathcal{C}_n^2(\eta)}{2^n(n+1)}C_W^{(n)}(\tilde{Q}^2)f_\pi\langle\xi^n\rangle + \mathcal{O}(1/\tilde{Q}^3)$$

$$= \frac{2\epsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta}{\tilde{Q}^2}\sum_{n\ \text{even}}^{\infty}W(n)C_W^{(n)}(\tilde{Q}^2)f_\pi\langle\xi^n\rangle + \mathcal{O}(1/\tilde{Q}^3)$$



In general, need large **p** to access non-leading moments

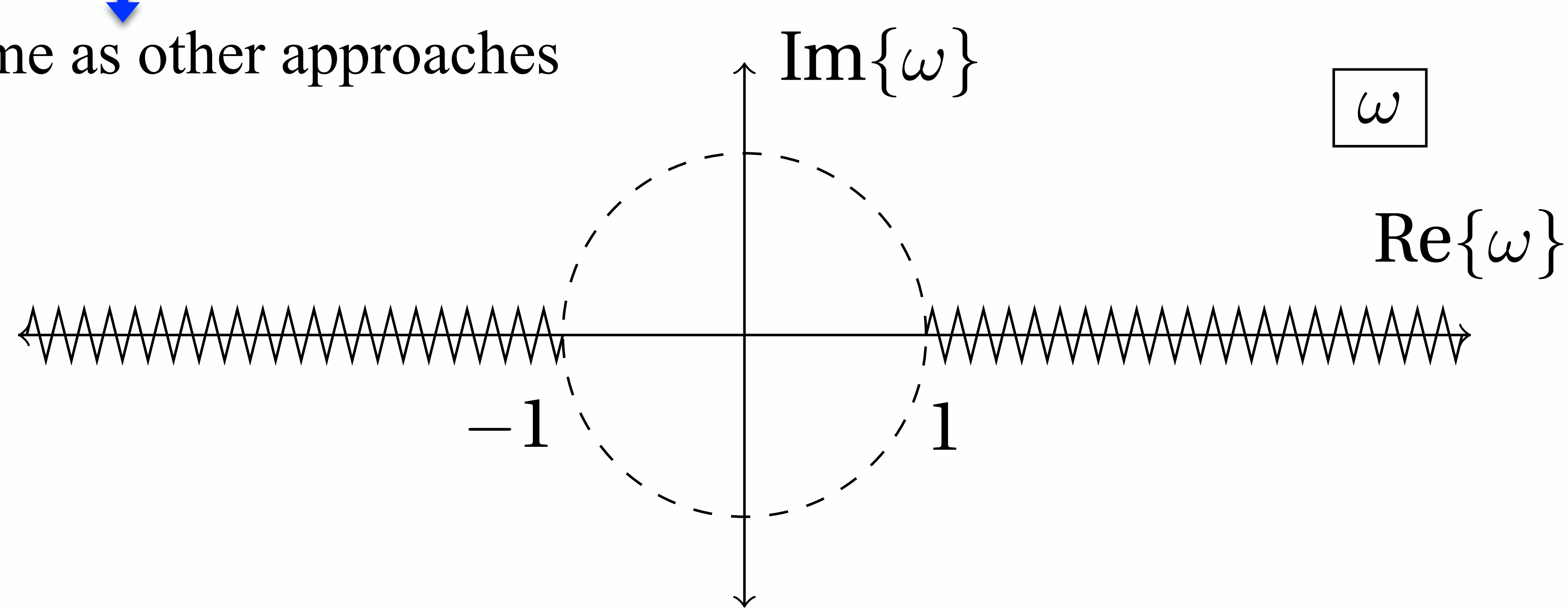
Generic issue in HOPE for higher moments

$$T^{\mu\nu}_{\Psi,\psi}(p,q) \sim \sum_{n=0}^{\infty} \langle \xi^n \rangle \omega^n + \text{higher twist},\ \omega = \frac{2p\cdot q}{\tilde{Q}^2} = \frac{2\mathbf{p}\cdot\mathbf{q} + 2iE_\pi q_4}{q_4^2 + \mathbf{q}^2 + m_\Psi^2}$$

simulate fit

- ★ Need large $\tilde{Q}^2$ to suppress higher-twist effects [$\sim (\Lambda_{\rm QCD}/\tilde{Q})^m$]
- ★ Need large **p** to make $|\omega| \to 1$ (sensitivity to higher moments)

Same as other approaches



Strategy for enhancing sensitivity to $\langle \xi^n \rangle$

$$V^{[12]}(p,q) = \frac{2\epsilon^{12\alpha\beta} q_\alpha p_\beta}{\tilde{Q}^2} \sum_{n\ \mathrm{even}}^{\infty} \frac{\zeta^n \mathcal{C}_n^2(\eta)}{2^n(n+1)} C_W^{(n)}(\tilde{Q}^2) f_\pi \langle \xi^n \rangle + \mathcal{O}(1/\tilde{Q}^3)$$

$$= \frac{2(q_3 p_4 - q_4 p_3)}{\tilde{Q}^2} \left[C_W^{(0)}(\tilde{Q}^2) f_\pi + \frac{6(p\cdot q)^2 - p^2 q^2}{6(\tilde{Q}^2)^2} C_W^{(2)}(\tilde{Q}^2) f_\pi \langle \xi^2 \rangle + \dots \right] + \mathcal{O}(1/\tilde{Q}^3)$$

$p_4 = iE_\pi$

choose $\mathbf{p}\cdot\mathbf{q}\neq 0$ while $p_3=0$, $q_3 \neq 0$ and q_4 being real

$$= \frac{2iq_3 E_\pi}{\tilde{Q}^2} \left[C_W^{(0)}(\tilde{Q}^2) f_\pi + \frac{6(p\cdot q)^2 - p^2 q^2}{6(\tilde{Q}^2)^2} C_W^{(2)}(\tilde{Q}^2) f_\pi \langle \xi^2 \rangle + \dots \right] + \mathcal{O}(1/\tilde{Q}^3)$$

imaginary — real — complex

→ The largest contribution to $\mathrm{Re}[V^{[12]}]$ is from $\langle \xi^2 \rangle$

Analysis strategy

★ Momentum space

$$V^{[\mu\nu]}(p,q) \equiv \int d^4z \, \mathrm{e}^{iq\cdot z} \, \langle 0 \,|\, T[J^{[\mu}(z/2)J^{\nu]}(-z/2)] \,|\, \pi(\mathbf{p})\rangle$$

★ Time-momentum representation (TMR)

$$R^{[\mu\nu]}(\tau; \mathrm{p}, \mathrm{q}) = \int dz_4 \, \mathrm{e}^{-iq_4 z_4} \, V^{[\mu\nu]}(p,q)$$

$$= \int d^3\mathbf{z} \, \mathrm{e}^{\mathbf{q}\cdot\mathbf{z}} \, \langle 0 \,|\, T[J^{[\mu}(z/2)J^{\nu]}(-z/2)] \,|\, \pi(\mathbf{p})\rangle$$

➡ Fourier transform of Wilson coeff numerically

Quenched calculation @ $M_\pi \approx 560$ MeV

- Proof-of-principle nature
- 4 lattice spacings: 0.04 to 0.08 fm
- Learn how to control errors
- Good result for $\langle \xi^2 \rangle$
- Reasonable exploratory result for $\langle \xi^4 \rangle$
- 64 Intel KNL nodes

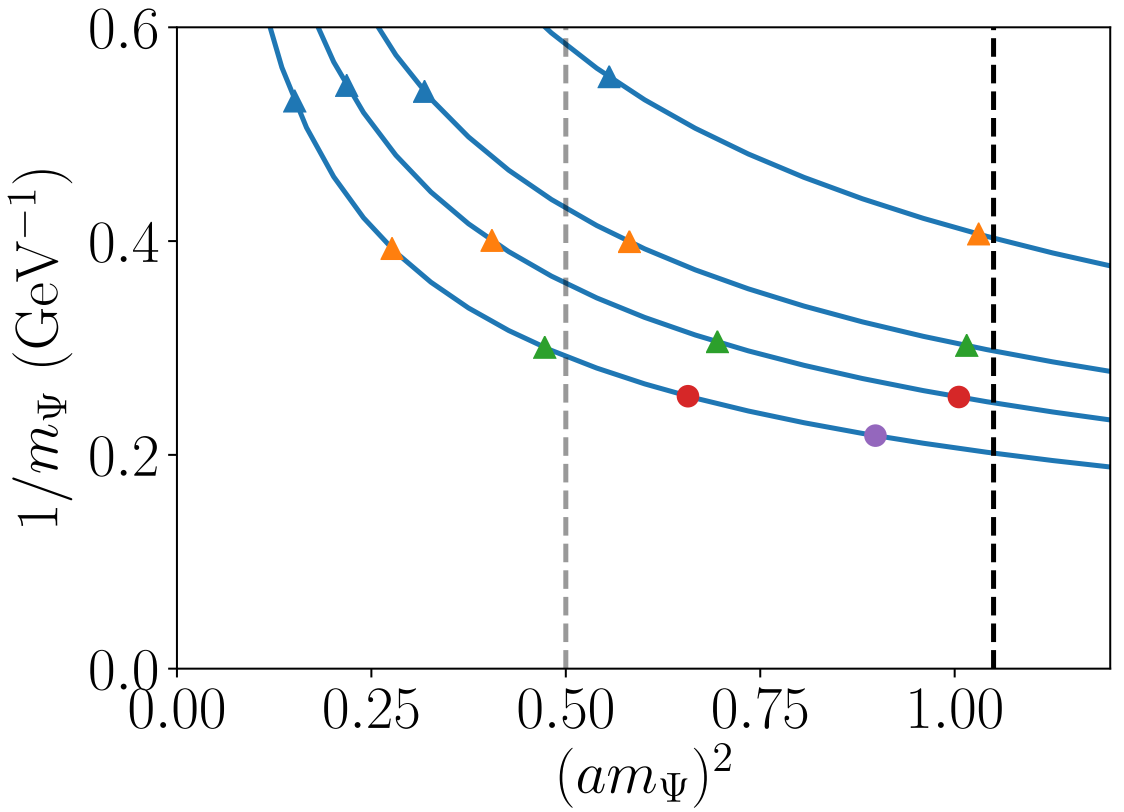
Quenched calculation for $\langle \xi^2 \rangle$ @ $M_\pi \approx \mathbf{560}$ MeV

Lattice setting for determining $\langle \xi^2 \rangle$

Wilson plaquette and non-perturbatively improved clover actions

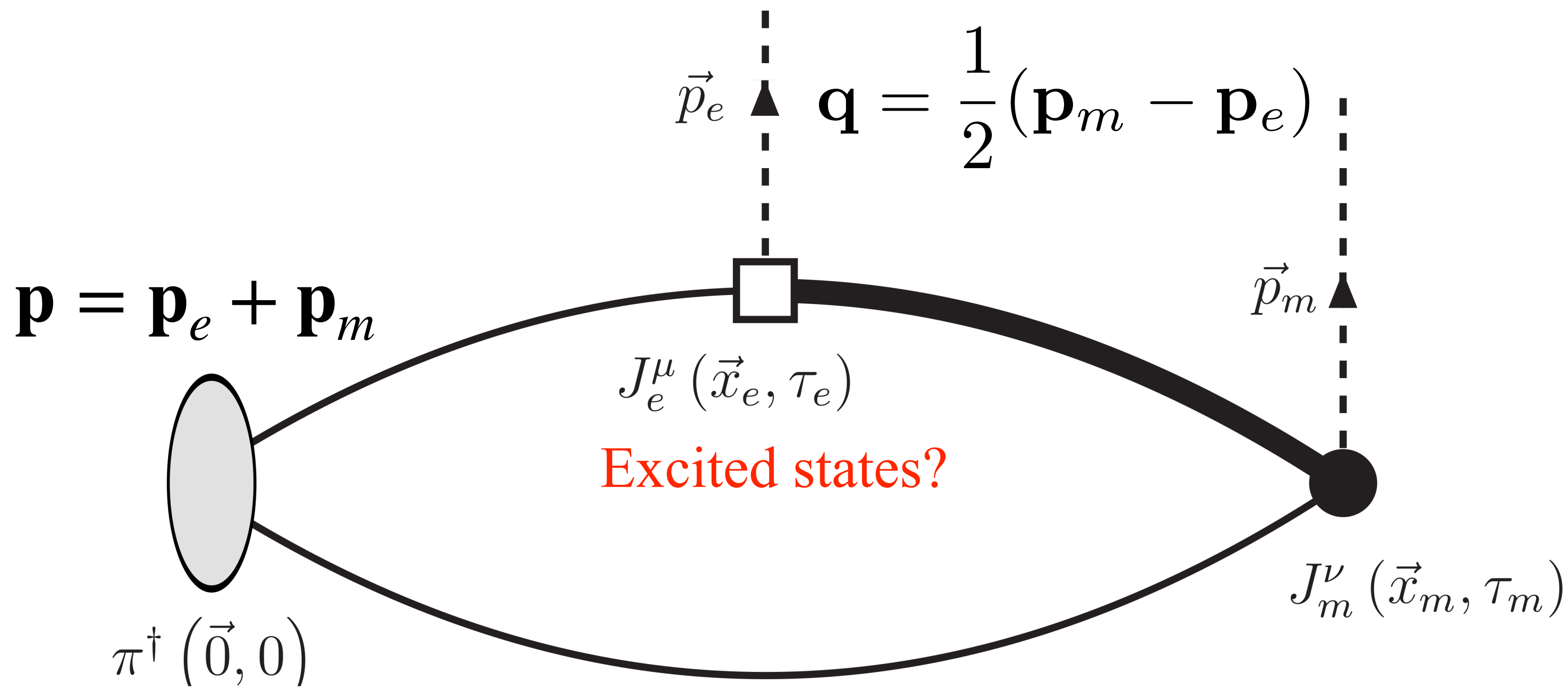
a (fm)	$\hat{L}^3 \times \hat{T}$	$N_{\rm config}$	$N_{\rm src}$
0.081	$24^3 \times 48$	650	12
0.060	$32^3 \times 64$	450	10
0.048	$40^3 \times 80$	250	6
0.041	$48^3 \times 96$	341	10

$L \sim 2$ fm



- **p** = (1,0,0) **q** = (1/2,0,1) in units of $2\pi/L \sim 0.64$GeV
- $V^{\mu\nu}$ is $O(a)$ improved without improving the axial current

Correlators for lattice calculation



$$C_3^{\mu\nu}(\tau_e, \tau_m; \mathbf{p}_e, \mathbf{p}_m) = \int d^3x_e\, d^3x_m\, e^{i\mathbf{p}_e\cdot\mathbf{x}_e} e^{i\mathbf{p}_m\cdot\mathbf{x}_m} \langle 0|\mathcal{T}\left[J_A^{\mu}(\tau_e, \mathbf{x}_e) J_A^{\nu}(\tau_m, \mathbf{x}_m) \mathcal{O}_{\pi}^{\dagger}(\mathbf{0})\right]|0\rangle$$

$$= R^{\mu\nu}(\tau_e - \tau_m; \mathbf{p}, \mathbf{q}) \frac{Z_{\pi}(\mathbf{p})}{2E_{\pi}(\mathbf{p})} e^{-E_{\pi}(\mathbf{p})(\tau_e+\tau_m)/2}$$

$z = x_e - x_m$

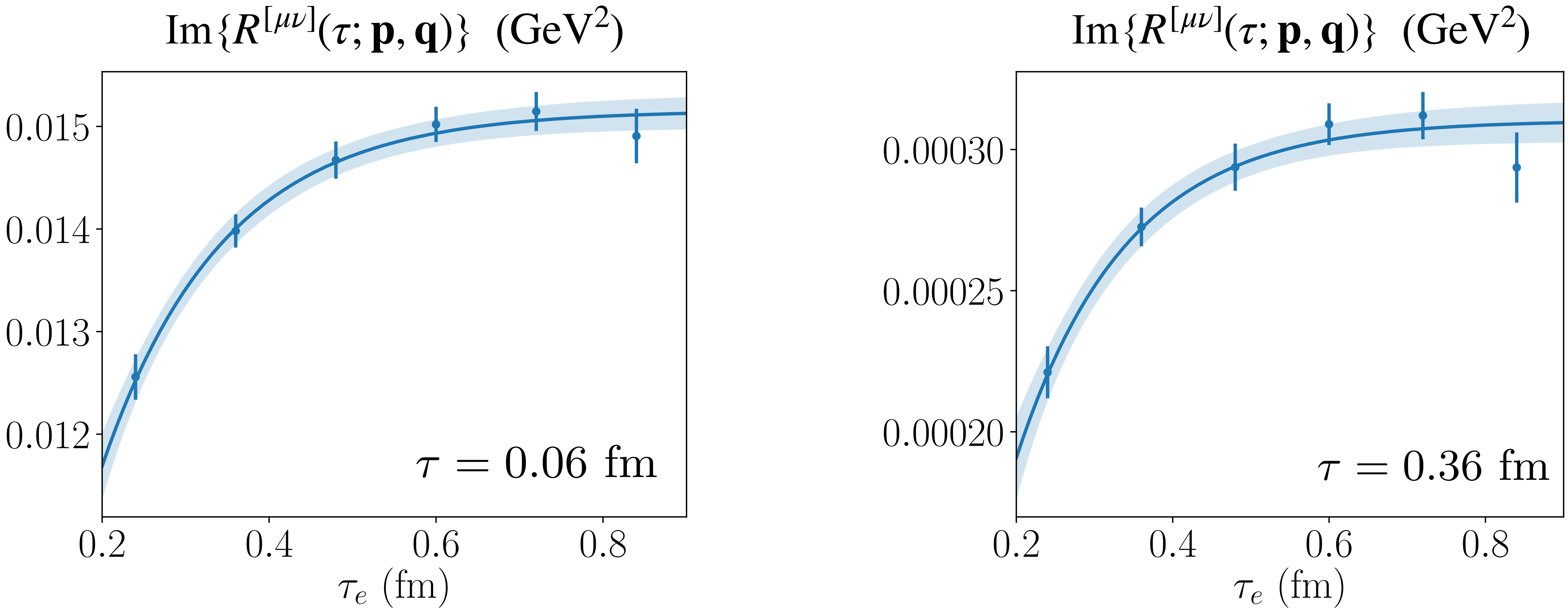
$$\int d^3z\, e^{iq\cdot z}\langle 0|\, T[J_A^{\mu}(z/2) J_A^{\nu}(-z/2)]\,|\,\pi(p)\rangle$$

HOPE hadronic amplitude in TMR

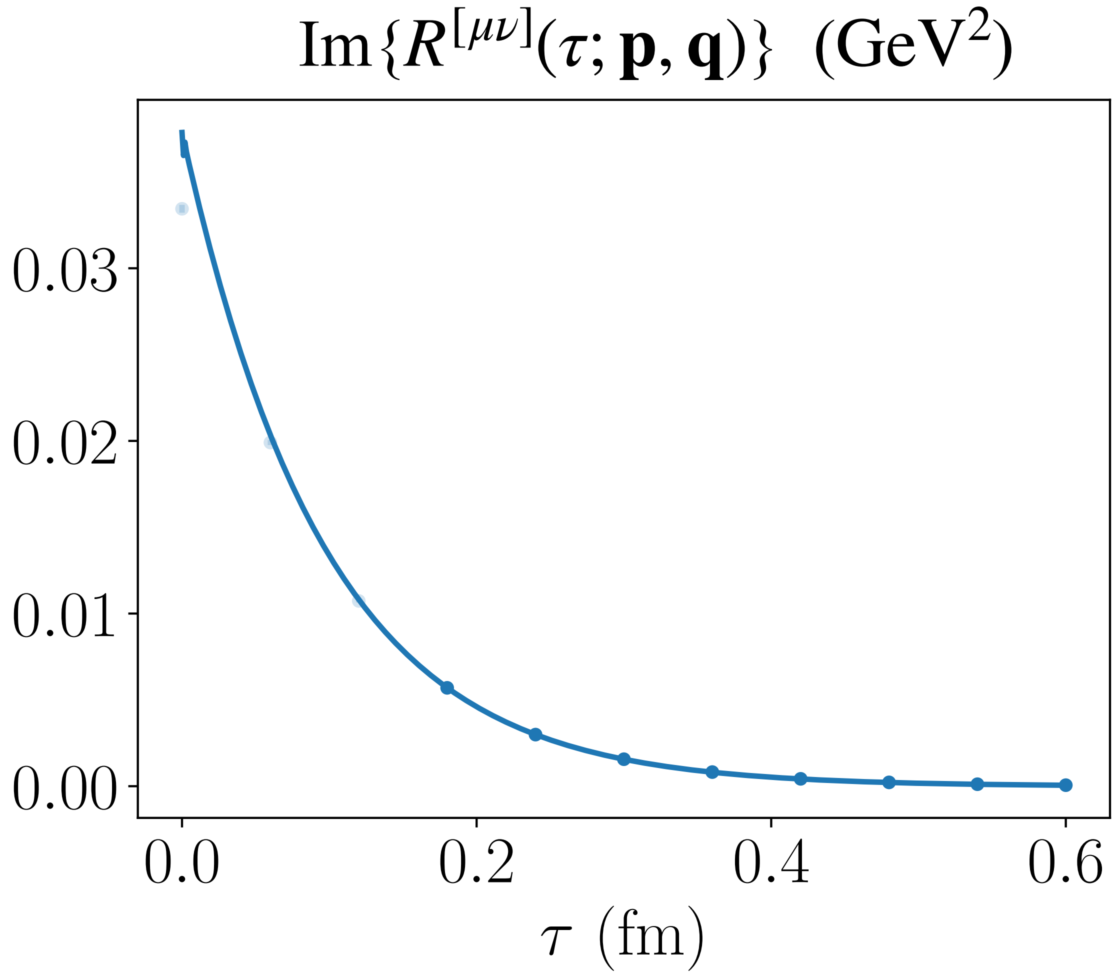
Fit from

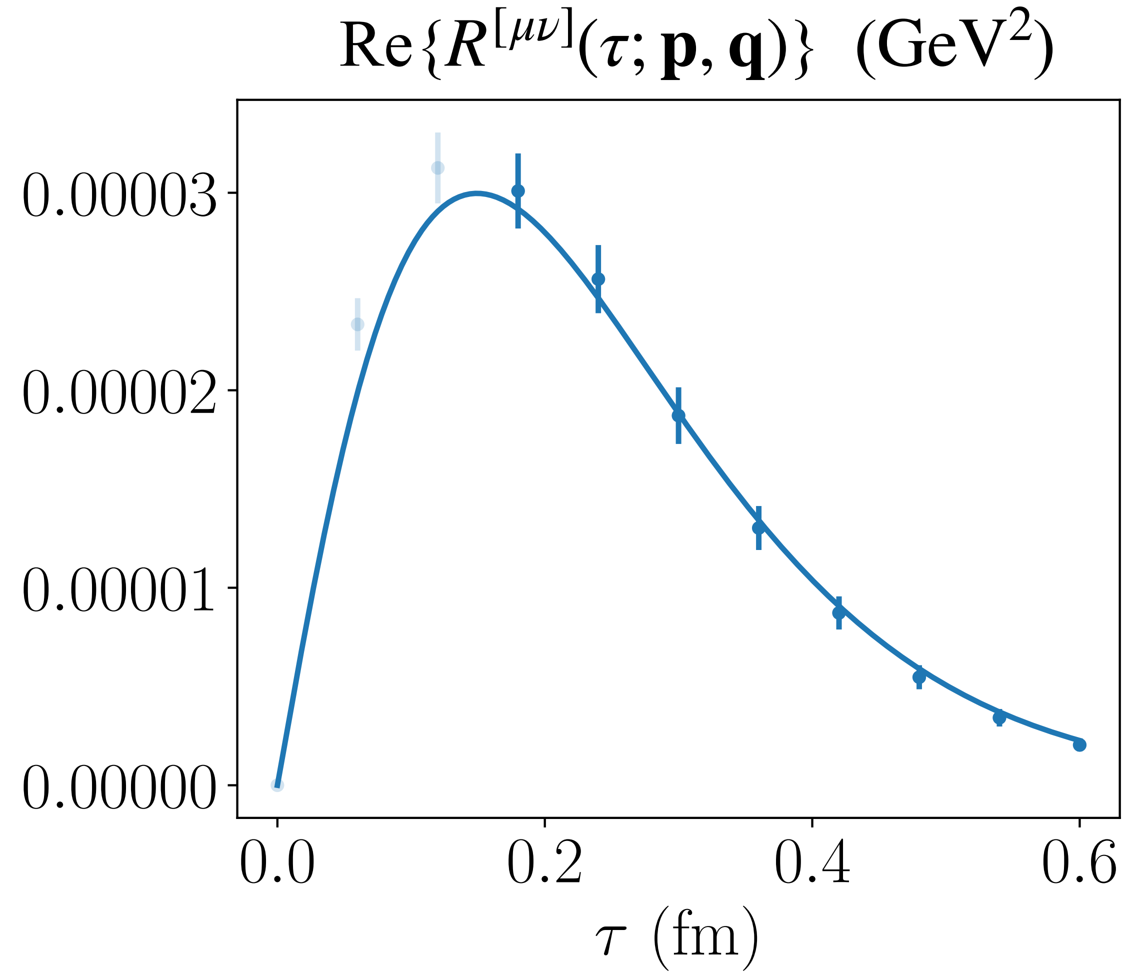
$$C_2(\tau_{\pi}, \mathbf{p}) = \int d^3\mathbf{x}\, e^{i\mathbf{p}\cdot\mathbf{x}} \langle 0|\mathcal{O}_{\pi}(\mathbf{x}, \tau_{\pi}) \mathcal{O}_{\pi}^{\dagger}(\mathbf{0}, 0)|0\rangle$$

Excited state contamination in $R^{[\mu\nu]}(\tau; \mathbf{p}, \mathbf{q})$



Extracting $\langle \xi^2 \rangle$ from HOPE formula

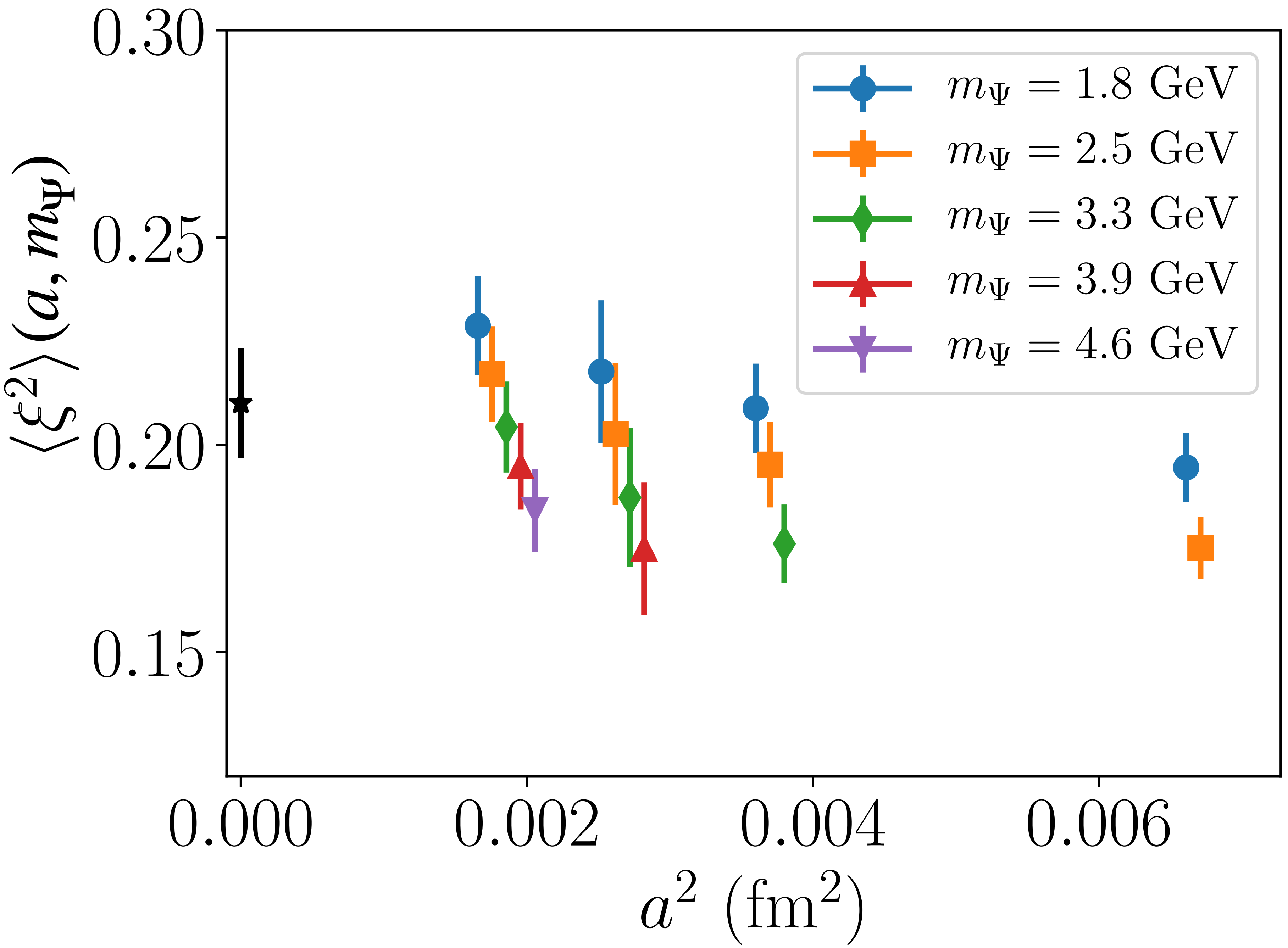




fit f_π and m_Ψ

fit $\langle \xi^2 \rangle$

Lattice artefacts and higher-twist effects in $\langle \xi^2 \rangle(a, m_\Psi)$

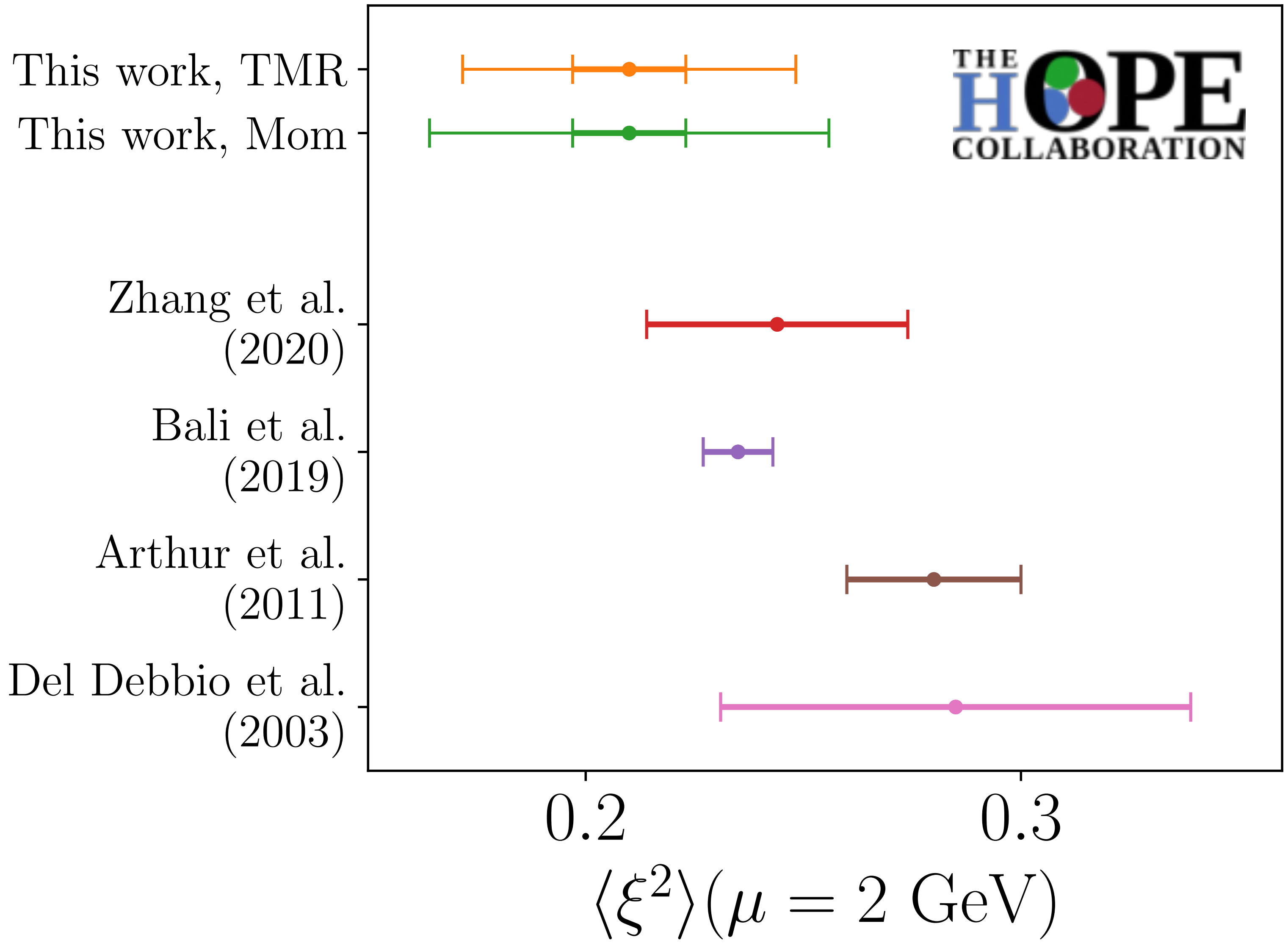


$$\langle \xi^2 \rangle(a, m_\Psi) = \langle \xi^2 \rangle + \frac{A}{m_\Psi} + Ba^2 + Ca^2 m_\Psi + Da^2 m_\Psi^2$$

Result for $\langle \xi^2 \rangle$

TMR analysis errors

Source of error	Size
Statistical	0.013
Continuum extrapolation	0.016
Higher-twist	0.025
Excited-state contamination	0.002
Unphysical m_π	0.014
Fit range	0.002
Running coupling	0.008
Total (exc. quenching)	**0.036**



$$\langle \xi^2 \rangle_{\text{TMR}} (\mu = 2 \text{ GeV}) = 0.210 \pm 0.013 \text{ (stat.)} \pm 0.034 \text{ (sys.)} = 0.210 \pm 0.036$$

$$\langle \xi^2 \rangle_{\text{Mom}} (\mu = 2 \text{ GeV}) = 0.210 \pm 0.013 \text{ (stat.)} \pm 0.044 \text{ (sys.)} = 0.210 \pm 0.046$$

Quenched calculation … @ $M_\pi \approx 560$ MeV

Exploratory hitherto

Lattice setting for determining $\langle \xi^4 \rangle$

Wilson plaquette and non-perturbatively improved clover actions

$L^3 \times T$	a (fm)	N_{cfg}	N_Ψ
$24^3 \times 48$	0.0813	6500	2
$32^3 \times 64$	0.0600	5000	3
$40^3 \times 80$	0.0502	$\mathcal{O}(5000)$	4
$48^3 \times 96$	0.0407	$\mathcal{O}(5000)$	5

$m_\Psi = 1.3 \sim 2.3$ GeV

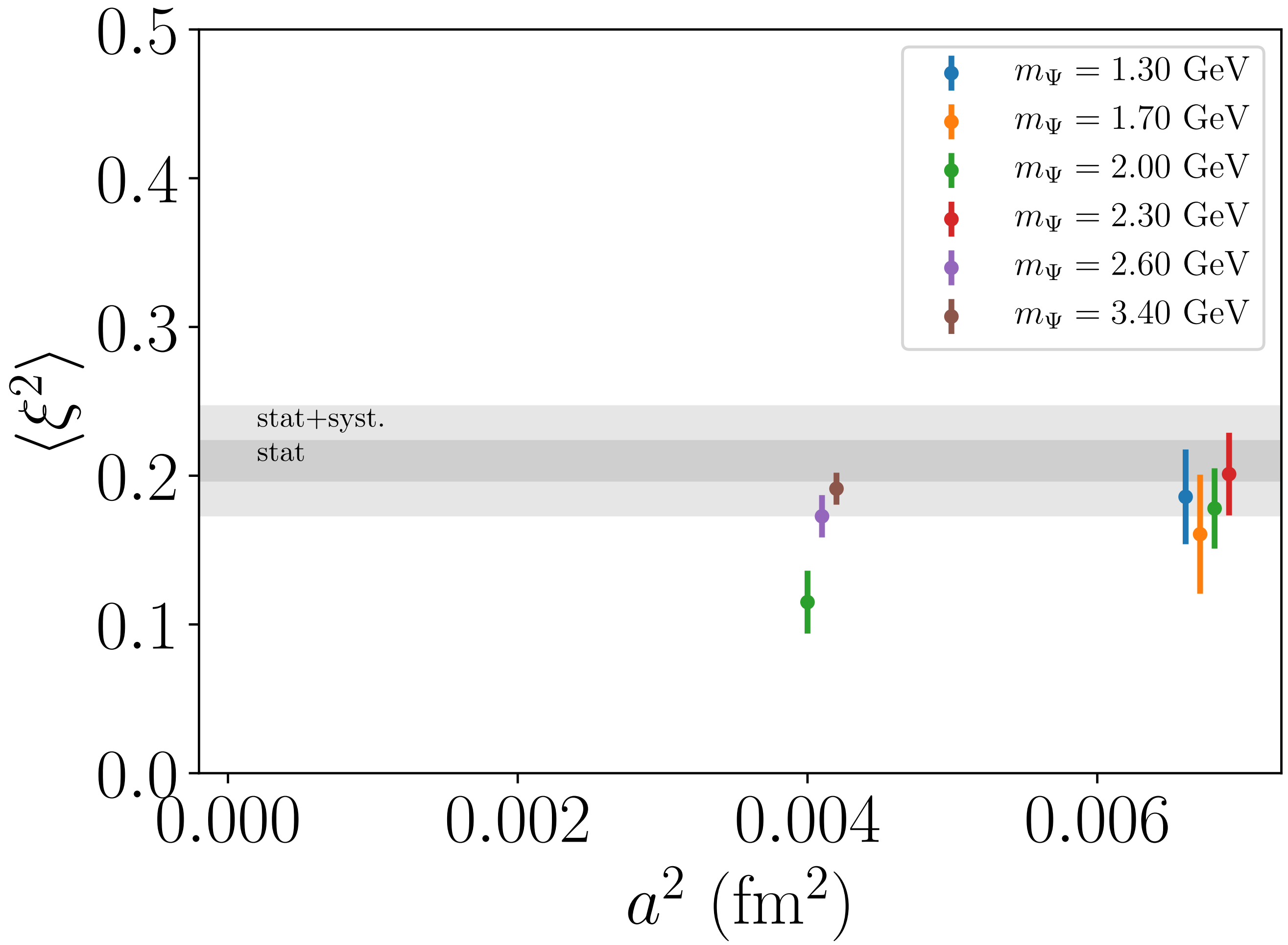
$m_\Psi = 2.0 \sim 3.4$ GeV

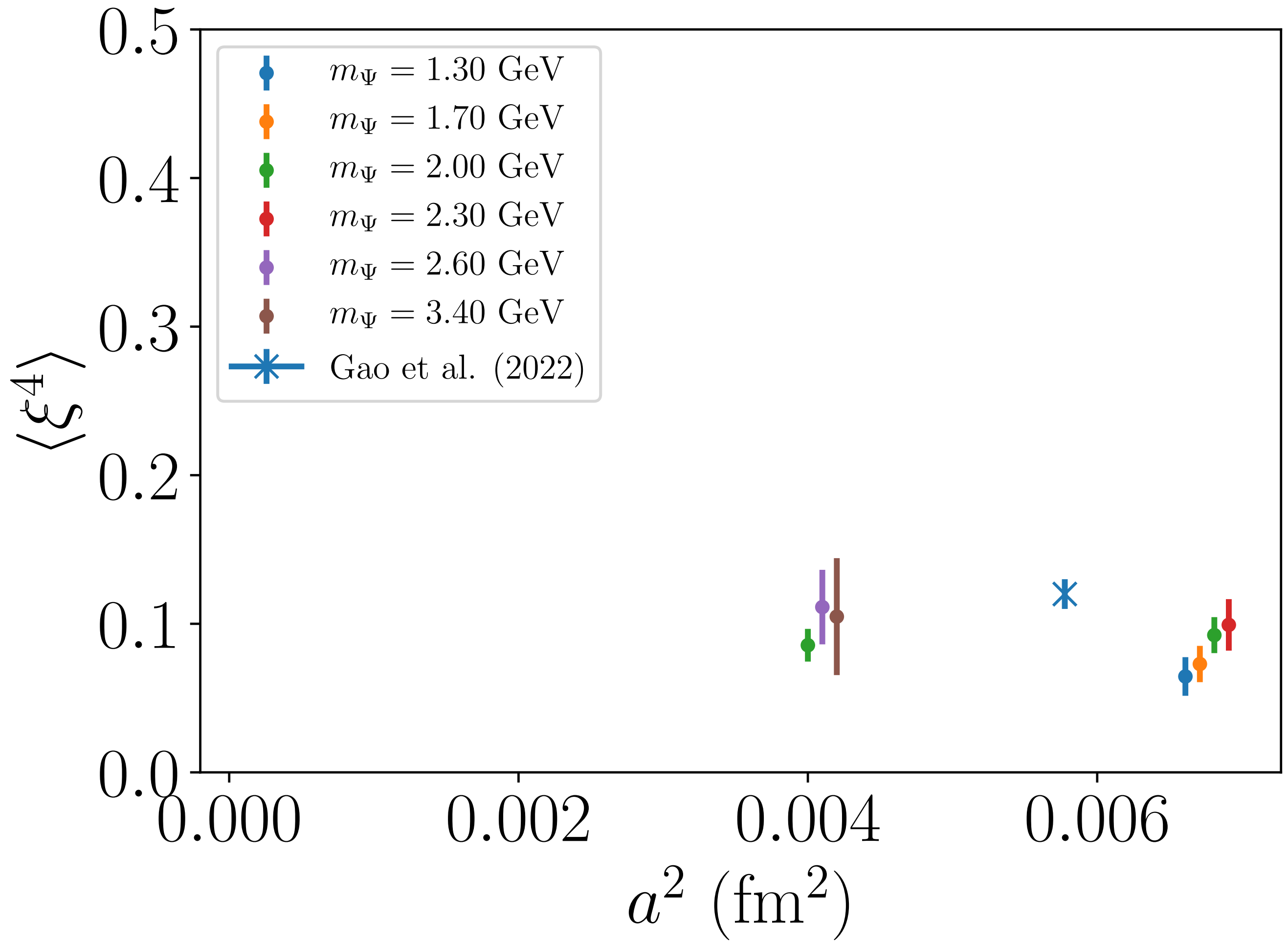
Work in progress ($40^3 \times 80$ and $48^3 \times 96$)

- $\mathbf{p} = (2,0,0)$ $\mathbf{q} = (1/2,0,1)$ in units of $2\pi/L \sim 0.64$GeV
- Variational analysis with pion interpolating operators $\bar{\psi}\gamma_5\psi$ and $\bar{\psi}\gamma_\mu\gamma_5\psi$
- Momentum smearing for pion interpolators

G.S. Bali *et al.*, PRD93 (2016) 9, 094515

Status of $\langle \xi^4 \rangle$ calculation with GEVP and a "double-ratio" strategy





Work in progress ⇒ More data and reduced error

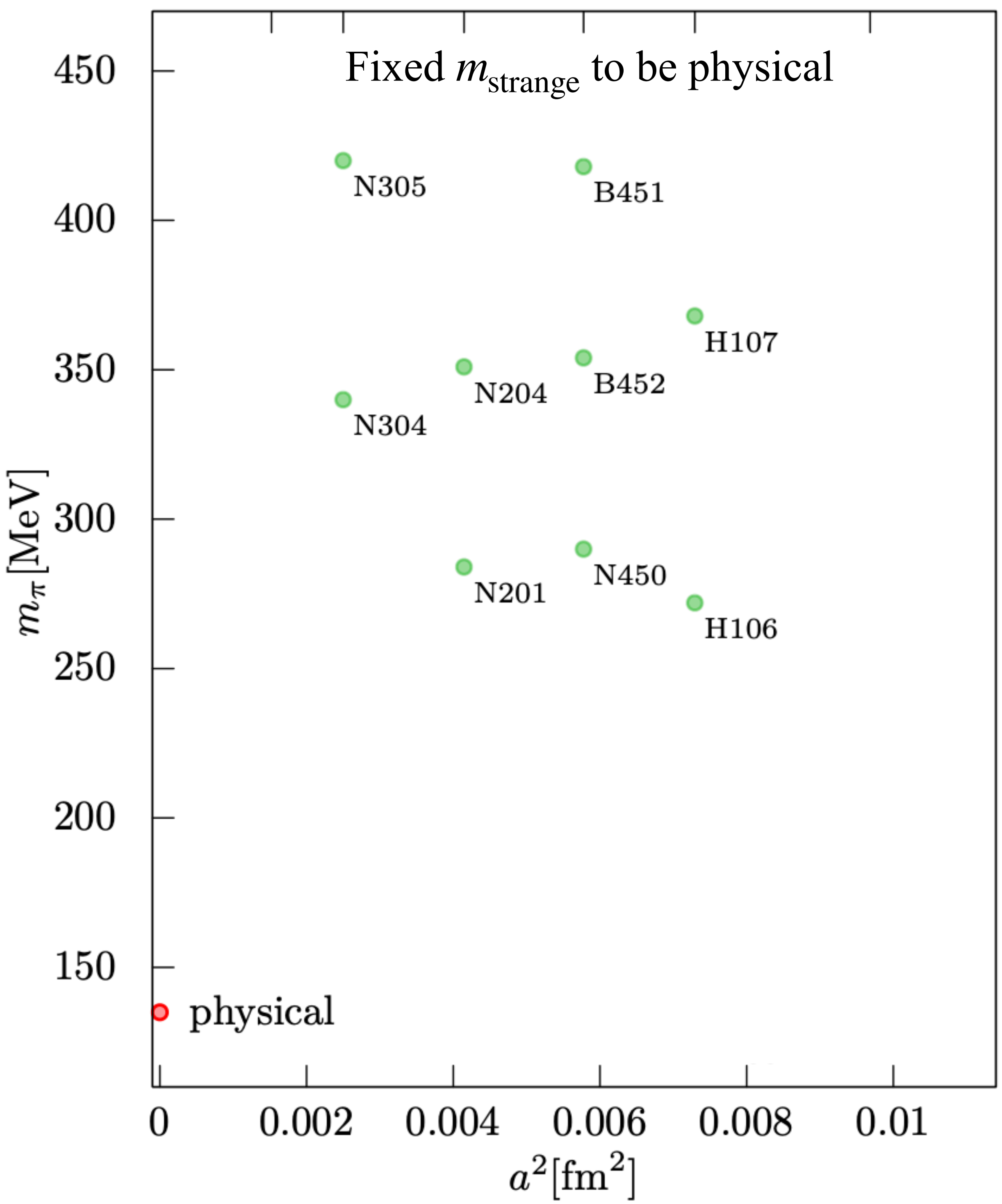
Conclusion and outlook: testing the method

- HOPE method facilitates high-moments calculations
- Numerically well tested *via* $\langle \xi^2 \rangle$ of $\phi_\pi(\xi, \mu)$
- Reasonable exploratory result of $\langle \xi^4 \rangle$ of $\phi_\pi(\xi, \mu)$
- Other parton-physics quantities planned for the future
- Direct calculation for ξ-dependence from HOPE

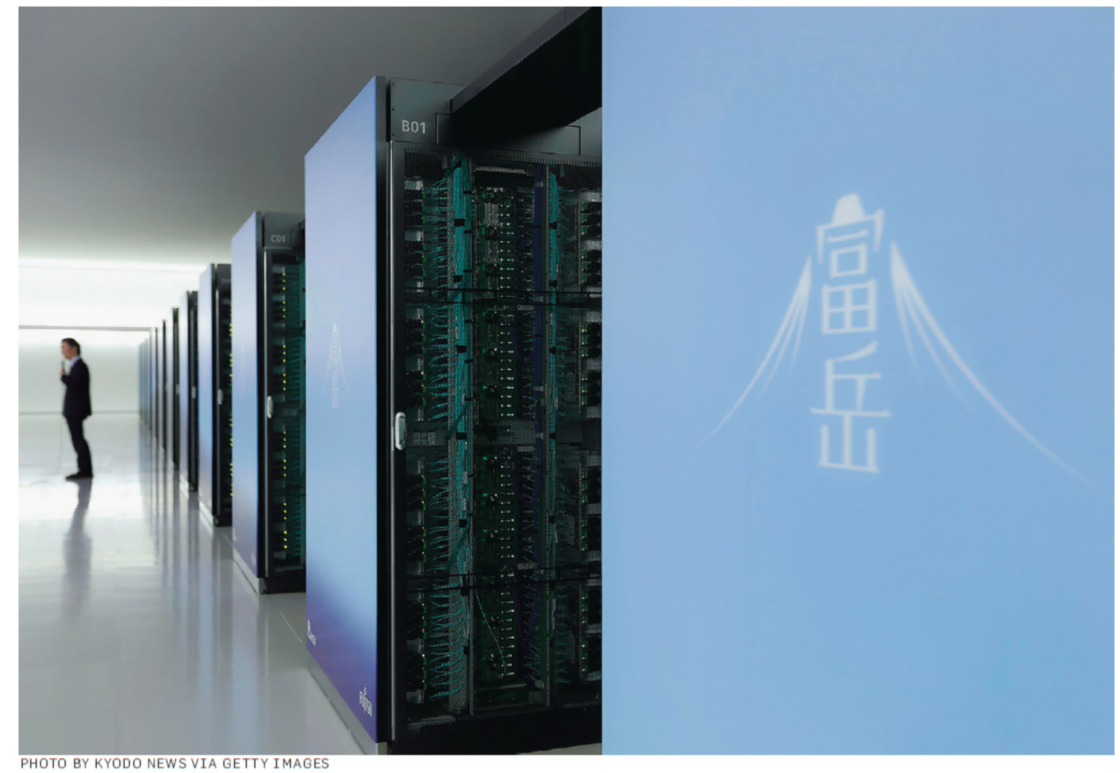
HOPE Collaboration, W. Detmold *et al*., Phys. Rev. **D 104** (2021) 7, 074511

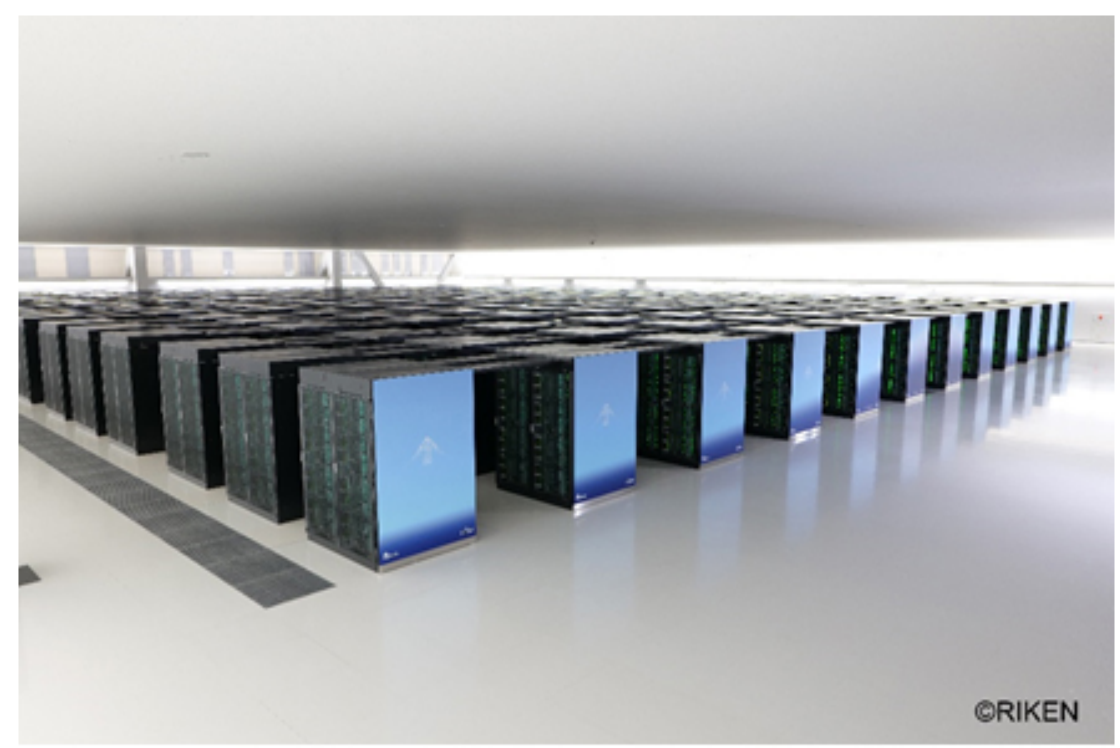
Outlook: precision computations

- Full-QCD dynamical calculation for $\phi_\pi(\xi, \mu)$ commenced



Planned our calculations on these CLS ensembles





Pictures from RIKEN RCCS

Backup slide

Introducing the valence heavy quark

- ★ Valence ⟶ Not in the action
- ★ The "heavy quark" is relativistic ⟶ Propagating in both space and time
- ★ The current for computing the even moments of the PDF

$$J^{\mu}_{\Psi,\psi}(x) = \overline{\Psi}(x)\gamma^{\mu}\psi(x) + \overline{\psi}(x)\gamma^{\mu}\Psi(x)$$

↓ Compton tensor

$$T^{\mu\nu}_{\Psi,\psi}(p,q) \equiv \sum_S \langle p,S|t^{\mu\nu}_{\Psi,\psi}(q)|p,S\rangle = \sum_S \int d^4x\ \mathrm{e}^{iq\cdot x}\langle p,S|T\left[J^{\mu}_{\Psi,\psi}(x)J^{\nu}_{\Psi,\psi}(0)\right]|p,S\rangle$$

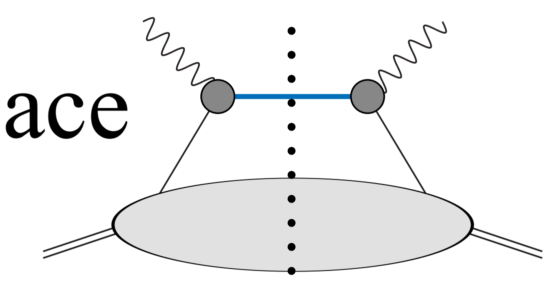
Strategy for extracting the moments

$$T^{\mu\nu}_{\Psi,\psi}(p,q) \equiv \sum_S \langle p,S| t^{\mu\nu}_{\Psi,\psi}(q) |p,S\rangle = \sum_S \int d^4x \; e^{iq\cdot x} \langle p,S| T\left[J^{\mu}_{\Psi,\psi}(x) J^{\nu}_{\Psi,\psi}(0) \right] |p,S\rangle$$

simulate

$$J^{\mu}_{\Psi,\psi}(x) = \overline{\Psi}(x)\gamma^\mu \psi(x) + \overline{\psi}(x)\gamma^\mu \Psi(x)$$

- ★ Simple renormalisation for quark bilinears
- ★ Work with the hierarchy of scales $\Lambda_{\rm QCD} << \sqrt{q^2} \leq m_\Psi << \frac{1}{a}$
  - → Heavy scales for short-distance OPE
  - → Avoid branch point in Minkowski space at $(q+p)^2 \sim (m_N + m_\Psi)^2$



- ★ Extrapolate to the continuum limit
  - → Match to the short-distance OPE results
  - → Extract the moments without power divergence

Enhancing the signal: the need

We work with $|\omega| = \left|\frac{2p \cdot q^2}{\tilde{Q}}\right| < 1$

Leading contribution in $\mathrm{Im}[V^{12}]$ is $\sim\langle \xi^0 \rangle$

Leading contribution in $\mathrm{Re}[V^{12}]$ is $\sim\langle \xi^2 \rangle \omega^2$

Much noisier compared to $\mathrm{Im}[V^{12}]$

Enhancing the signal: the idea

We work with $|\omega| < 1$ where Minkowskian $V^{\mu\nu}$ is imaginary.

From $V^{\mu\nu}_{\text{Minkowski}}(p,q) = \int_{-\infty}^{\infty} d\tau\, e^{-q_0\tau} R^{\mu\nu}(\tau;\mathbf{p},\mathbf{q})$.

→ $R^{\mu\nu}$ is imaginary.

Back to Euclidean space:

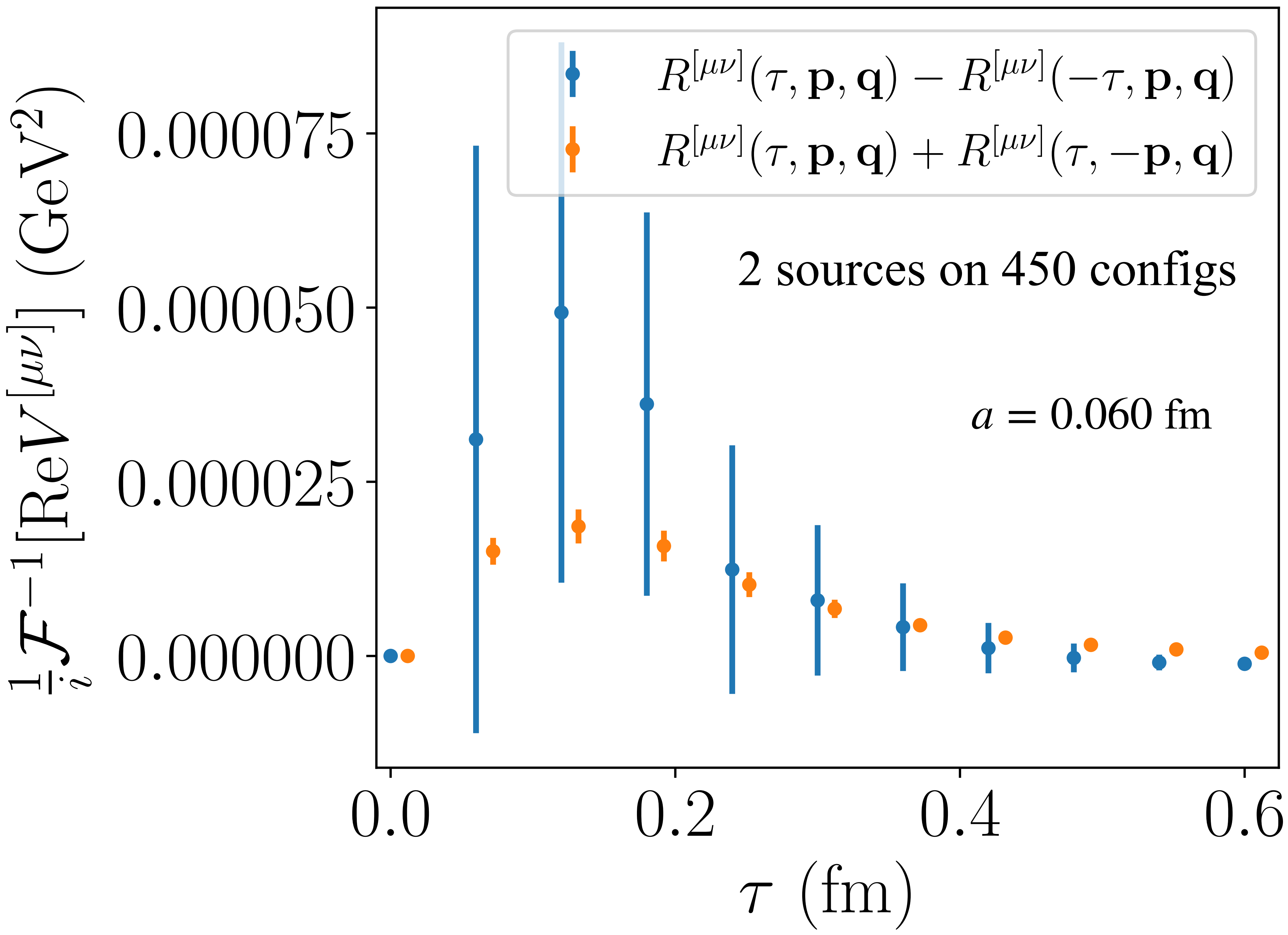
$$\mathrm{Re}[U^{\mu\nu}(\mathbf{p},q)] = \mathrm{Re}\left[\int_{-\infty}^{\infty} d\tau\, R^{\mu\nu}(\tau;\mathbf{p},\mathbf{q}) e^{-iq_4\tau}\right]$$

$$\propto \int_0^{\infty} d\tau\, [R^{\mu\nu}(\tau;\mathbf{p},\mathbf{q}) - R^{\mu\nu}(-\tau;\mathbf{p},\mathbf{q})] \sin(q_4\tau)$$

γ_5 hermiticity → $= R^{\mu\nu}(\tau;\mathbf{p},\mathbf{q}) + R^{\mu\nu}(\tau;-\mathbf{p},\mathbf{q})$

More correlated → reduced error

Enhancing the signal: the result



The “double ratio” method

★ Propagation of the excited states depends only on $\tau_e + \tau_m$

$$C_3^{\mu\nu}(\tau_e, \tau_m; \mathbf{p}_e, \mathbf{p}_m) = R^{\mu\nu}(\tau_e - \tau_m; \mathbf{p}, \mathbf{q}) \frac{Z_\pi(\mathbf{p})}{2E_\pi(\mathbf{p})} e^{-E_\pi(\mathbf{p})(\tau_e+\tau_m)/2} ,$$

★ Construct the ratio

$$\mathcal{R} = \frac{C_3^{\mu\nu}(\tau_e - 1, \tau_m + 1; \mathbf{p}_e, \mathbf{p}_m)}{C_3^{\mu\nu}(\tau_e, \tau_m; \mathbf{p}_e, \mathbf{p}_m)} = \frac{R^{\mu\nu}(\tau_e - \tau_m - 2; \mathbf{p}, \mathbf{q})}{R^{\mu\nu}(\tau_e - \tau_m; \mathbf{p}, \mathbf{q})} \left[1 + \ldots\right]$$

★ No need for two-point function

★ No need for renormalisation

Excited-state contamination for **p** = (2,0,0)