Quantum simulation for correlated quantum many-body systems on noisy quantum devices

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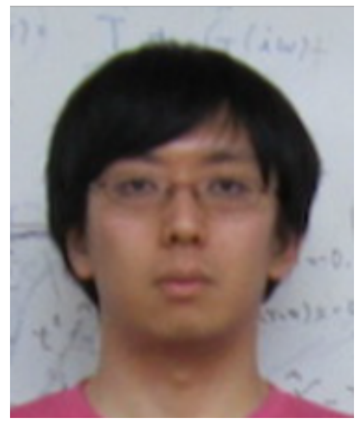

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LQCD Workshop
2023/02/16

Outline

- Introduction: Quantum simulation & quantum computer
  - Quantum simulation by quantum computer
  - Near-term noisy intermediate-scale quantum (NISQ) devices
- Variational quantum algorithm in NISQ era
  - Variational quantum eigensolver (VQE)
  - Issues and challenges in VQE
- Applications to correlated quantum many-body systems
  - To gapless fermion: *T. Shirakawa, K. Seki & S. Yunoki, PRResearch* ***3****, 013004 (2021)*
  - To Heisenberg model: *K. Seki, T. Shirakawa & S. Yunoki, PRA* ***101****, 052340 (2020)*
  - To topological order: *R.-Y. Sun, T. Shirakawa & S. Yunoki, PRB* ***107****, L041109 (2023)*
- Summary

Introduction:
Quantum simulation & quantum computer

Quantum simulation by quantum computer

"…Nature isn't classical, dammit, and **if you want to make a simulation of nature, you'd better make it quantum mechanical**, and by golly it's a wonderful problem, because it doesn't look so easy…"

—— *R. P. Feynman, Int. J. Theor. Phys.* ***21****, 467 (1982)*

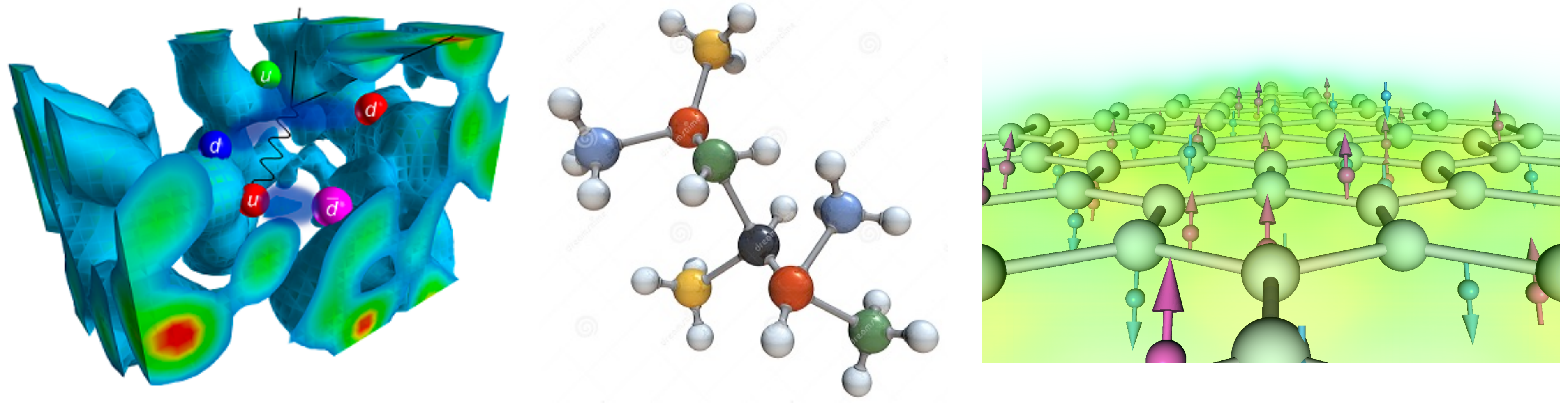
R. P. Feynman

classical computer

exponentially large resource required

↔

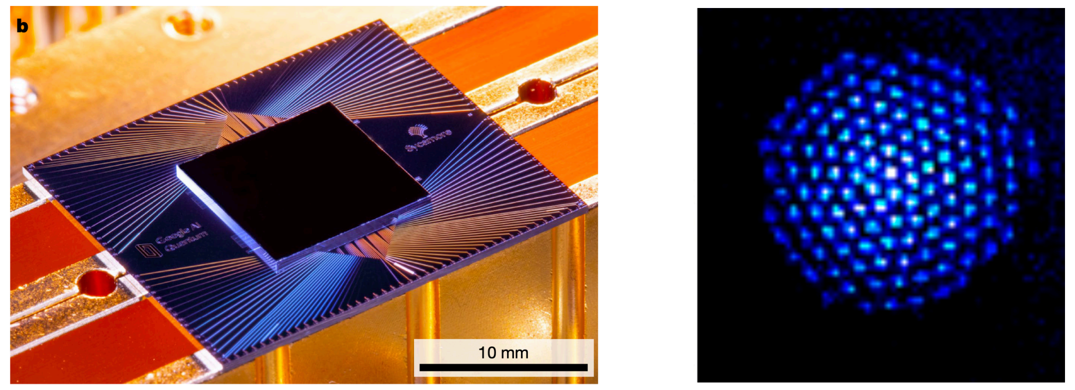
quantum many-body systems



exponentially large degrees of freedom

↔

other quantum systems
(quantum computer / simulator)



exponentially large resource available

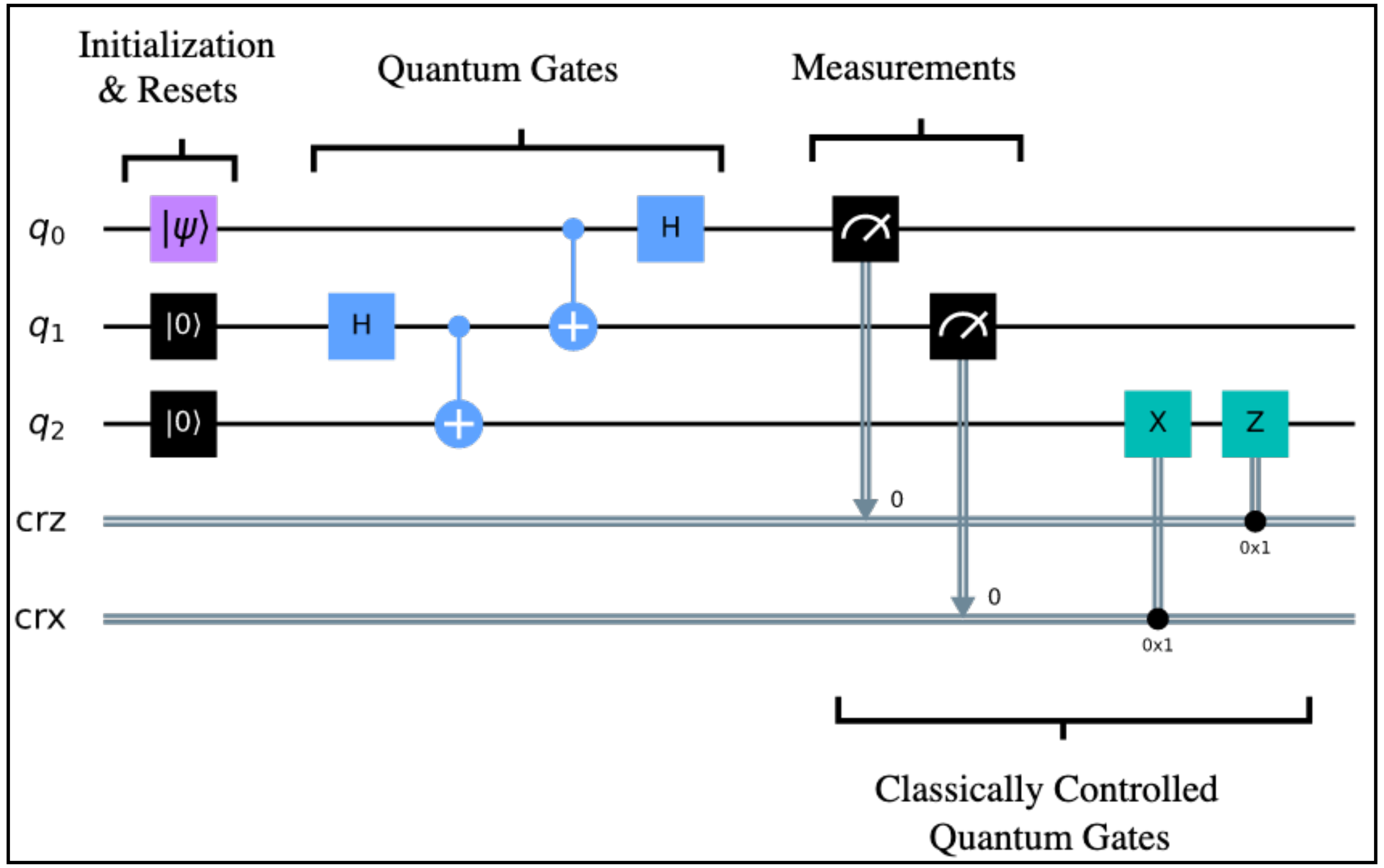
Simulating a quantum system by using other quantum systems (i.e. controllable quantum devices)

Quantum simulation by quantum computer

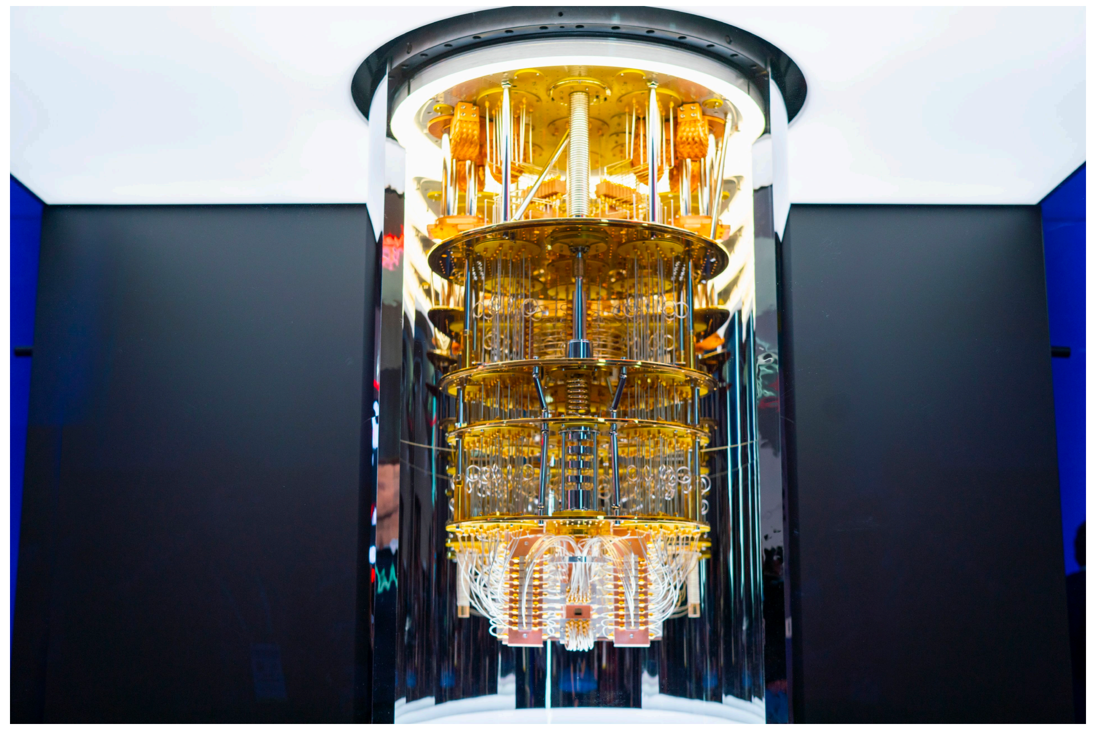
Circuit-based quantum computer

- Quantum circuit model
  - **Qubit**
    - form a quantum state
    - Two-level quantum mechanical system: $|0\rangle$ and $|1\rangle$
  - **Quantum logic gate**
    - update the quantum state
    - basic quantum circuit operating on a small number of qubits

      One-qubit gate / two-qubit gate / …
  - **Measurement**
    - information readout: classical bit strings



Google



IBM

Quantum simulation by quantum computer

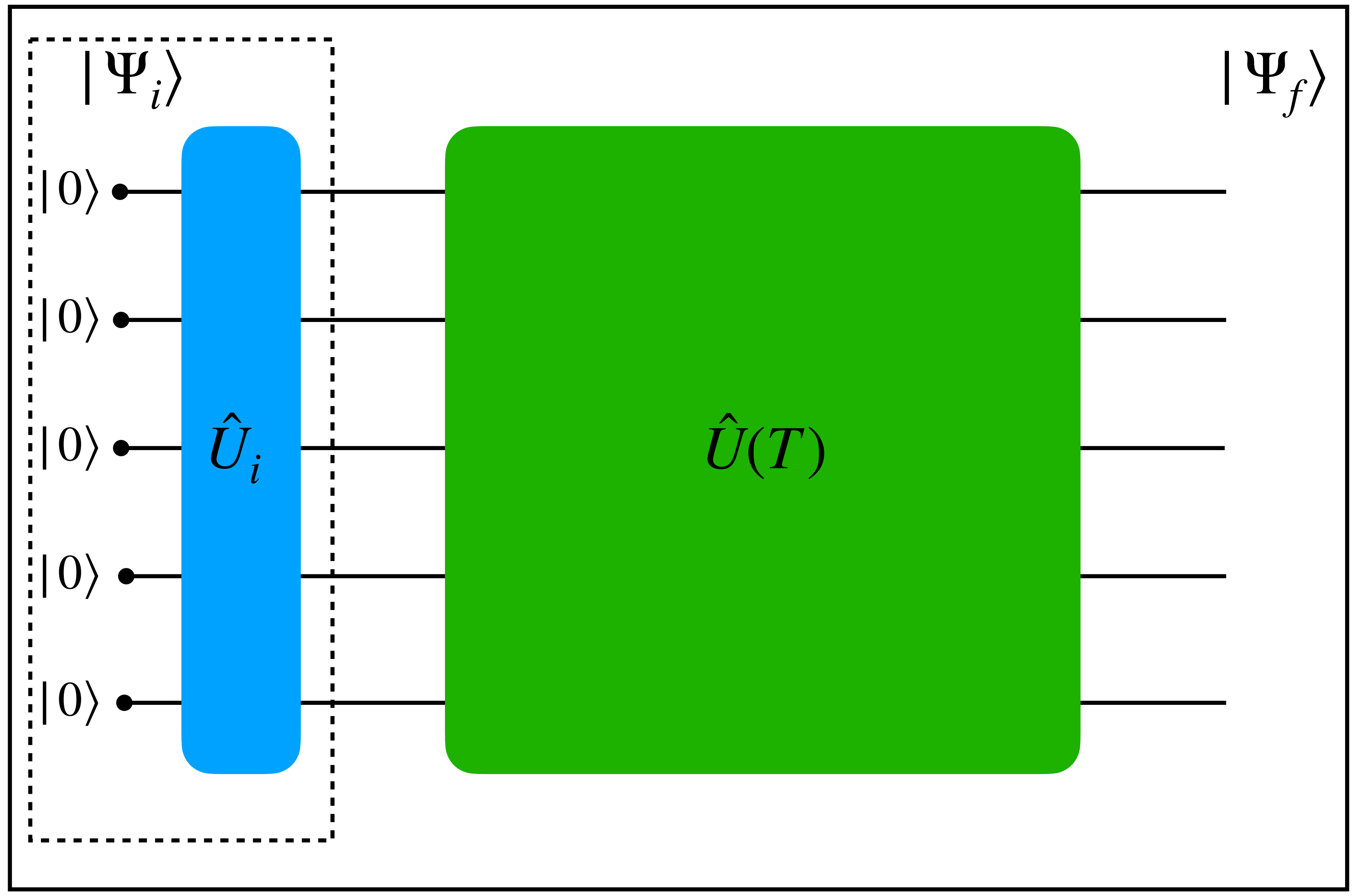
In principle: quantum adiabatic process

- Goal: find the ground state $|\Psi_f\rangle$ of Hamiltonian $\hat{H}_f$
- Start from $\hat{H}_i$
  - Simple Hamiltonian with the known ground state $|\Psi_i\rangle = \hat{U}_i|0\rangle$
- Construct the mixture Hamiltonian
  - $\hat{H}(t) = s_i(t)\hat{H}_i + s_f(t)\hat{H}_f$
  - Evolution operator: $\hat{U}(T) = \mathcal{T}\exp\{-i\int_0^T \hat{H}(t)\mathrm{d}t\}$

$s_i(0) = s_f(T) = 1$
$s_i(T) = s_f(0) = 0$
$\hat{H}(0) = \hat{H}_i,\ \hat{H}(T) = \hat{H}_f$

- Perform adiabatic time evolution
  - $|\Psi(T)\rangle = \hat{U}(T)|\Psi_i\rangle$
- Quantum adiabatic theorem
  - For sufficiently long T, $|\Psi(T)\rangle = |\Psi_f\rangle$

Realization on quantum device



Fault-tolerant quantum computer ~ solve quantum many-body problem

Issues:
- **How to design mixture hamiltonian?**
- **How to decompose the evolution operator on quantum device?**

Fatal weakness:
Long time evolution on current accessible quantum devices is impossible!

Near-term noisy quantum devices

- Noisy intermediate-scale quantum (NISQ) era
  - A few $\mathcal{O}(10^1 \sim 10^2)$ qubits without error correction
  - A few $\mathcal{O}(10^0 \sim 10^1)$ depths circuit evolution
    - E.g. "Google's quantum supremacy" circuit: 53 qubits with 40 depths

Quantum Computing in the NISQ era and beyond

John Preskill

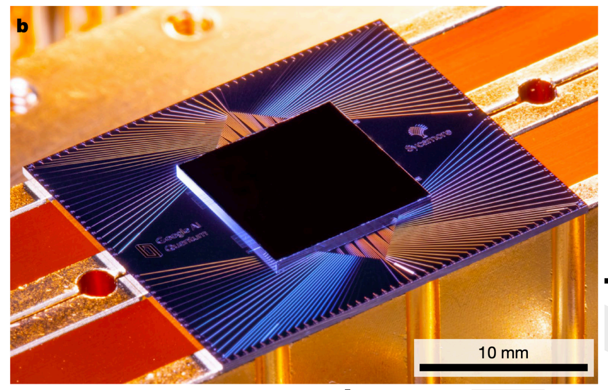
Institute for Quantum Information and Matter and Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena CA 91125, USA

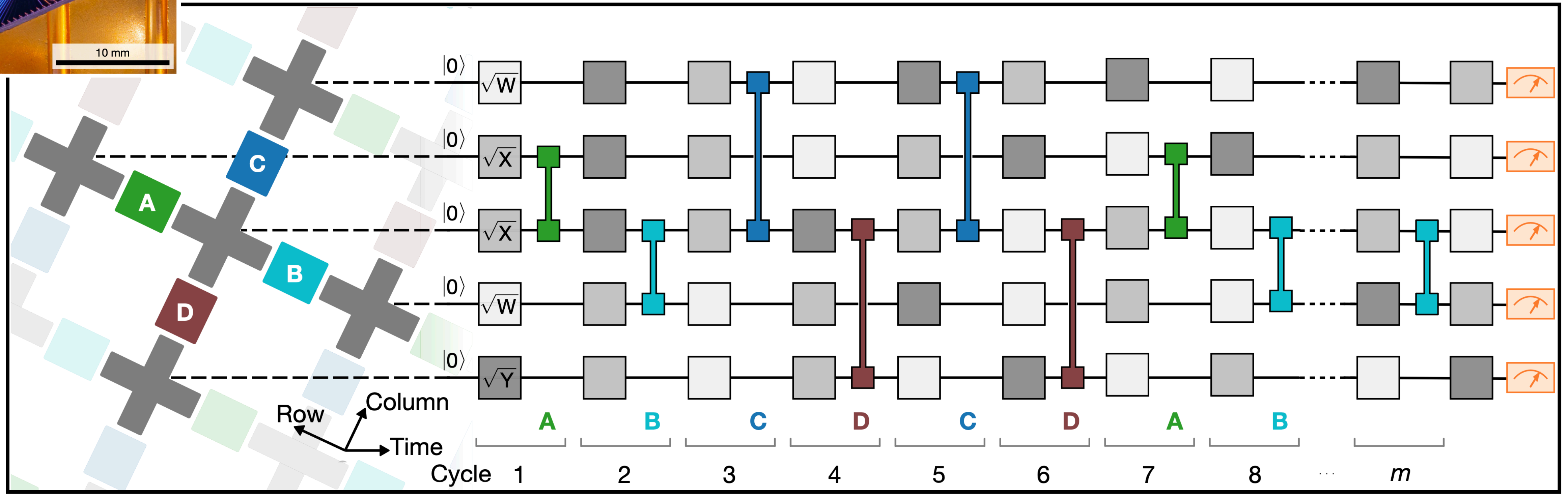
30 July 2018

Quantum **2**, *79 (2018)*

J. Preskill

In comparison:
Construct $\hat{U}(\delta t) = e^{-i\delta t \hat{H}}$ use ~ 20 (40) depths in 1D (2D) case when δt is very small





Nature **574**, *7779 (2019)*

Quantum adiabatic evolution is hopeless in NISQ era
Special algorithms for NISQ devices are required
Key point: Only shallow circuits are considered

Near-term aim: achieve useful quantum advantage on NISQ devices

Useful example: understand correlated quantum many-body problems

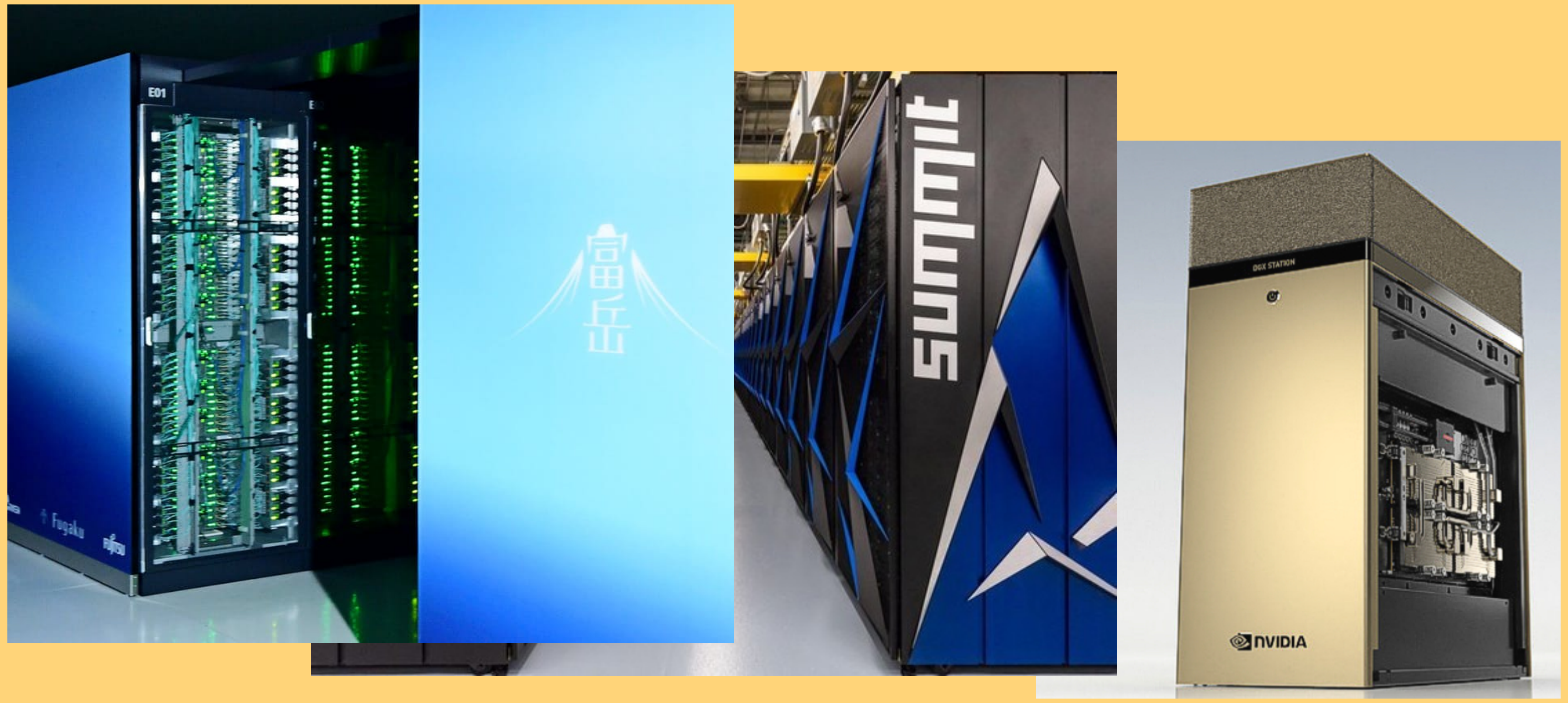
Variational quantum algorithm in NISQ era

Quantum-classical hybrid paradigm in NISQ era

Quantum-classical loop

quantum computer

classical computer



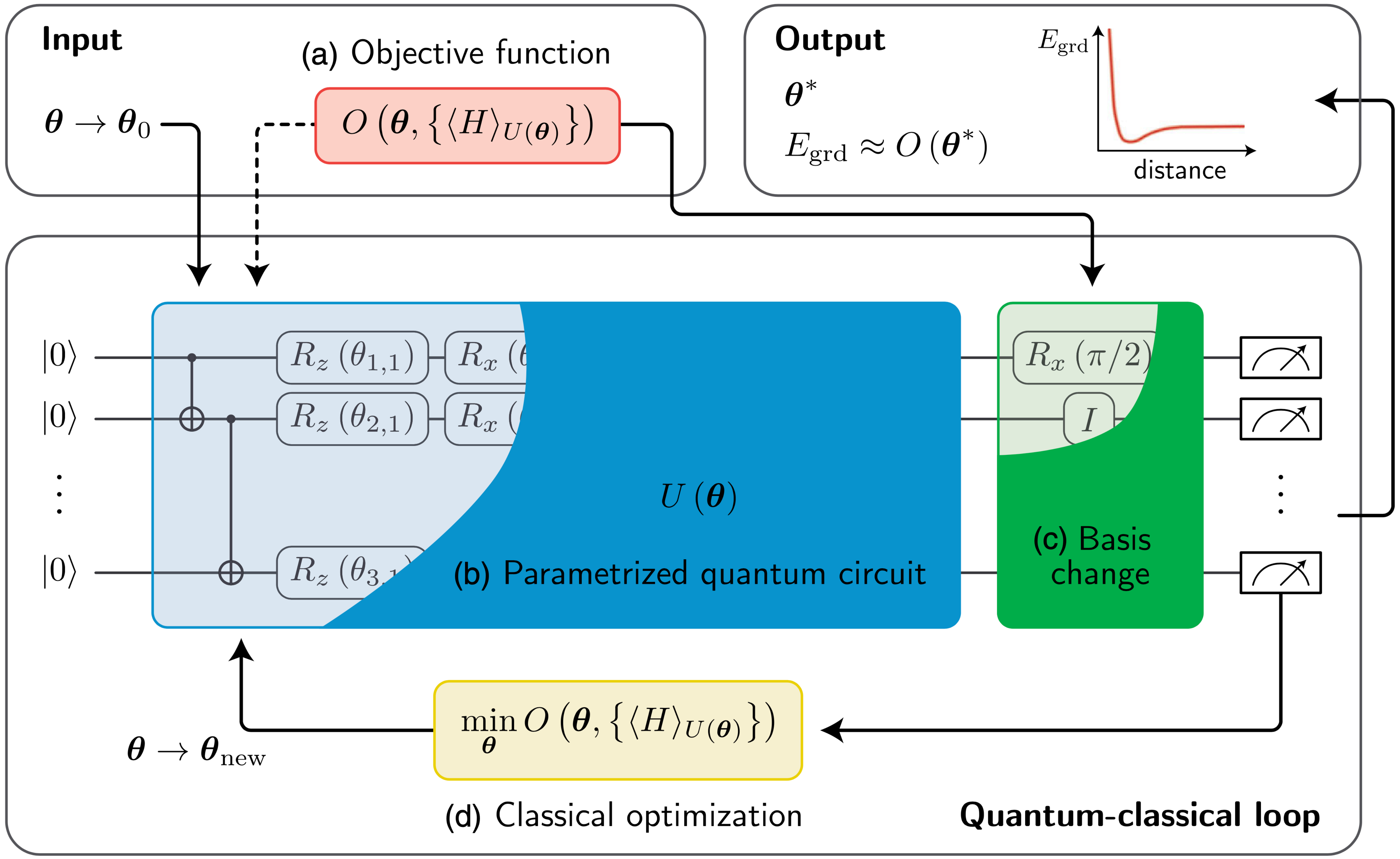
describe a quantum state
& extract information

pre/post processing
less demanding tasks

……

Variational quantum algorithm (VQA)

- Objective function (OF)
  - With parameters related to quantum circuit
  - Goal: minimize the OF
  - Example: system energy
- Parametrized quantum circuit (PQC)
  - Take parameters in OF
  - Be performed on quantum computer
  - Example: wavefunction ansatz to describe ground state
- Classical optimization
  - Classical optimizer for updating parameters in PQC
  - Based on values of OF and functions related to OF (for instance, derivatives)
  - Be performed on classical computer
  - Example: gradient descent optimizer



Rev. Mod. Phys. **94**, 015004 (2022)

Variational quantum eigensolver (VQE)

A VQA for finding the ground state of quantum many-body system

$$\hat{H}|\Psi(\{\theta_i\})\rangle = E(\{\theta_i\})|\Psi(\{\theta_i\})\rangle$$

- Objective function
  - Energy
- Parametrized quantum circuit (PQC)
  - Parametrized circuit ansatz state

variational quantum state:
parametrized circuit ansatz

$$|\Psi(\{\theta_i\})\rangle = \hat{U}(\{\theta_i\})|0\rangle =$$

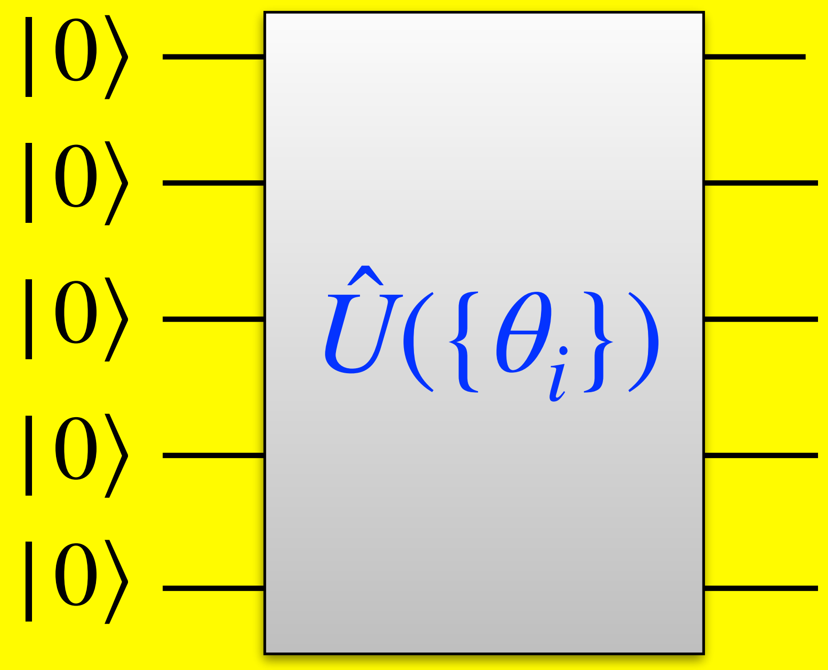
unitary operator

$|\Psi(\{\theta_i\})\rangle$: 2^N dimensional vector
(variational state)
$\{\theta_i\}$: poly(N) dimensional vector
(variational parameters)

N: number of qubits

quantum computer

quantum circuit



expectation value of energy

$$E(\{\theta_i\}) = \sum_k \langle \hat{H}_k \rangle_{\{\theta_i\}}$$

classical computer

optimization of variational parameters $\{\theta_i\}$

$$\theta_i \leftarrow \theta_i - \lambda \partial E(\{\theta_i\})/\partial \theta_i$$

A path to achieve useful quantum advantage (find the ground state of quantum many-body system) on NISQ devices

Nat. Commun. **5**, *4213 (2014)*

Variational quantum eigensolver (VQE)

Issues & challenges

- Parametrized circuit ansatz
  - Scalability on NISQ devices
  - Expressibility to the ground state
  - Optimizable or not
- Classical optimization
  - Capability to get the global minimal
  - Efficiency for NISQ device
- Suitable problem
  - Lower spacial dimension (1D and 2D)
  - Simple system (spins)

Scalable VQE (up to 500~1000 qubits)

$\approx$

Solve a small group of quantum many-body problems

$\approx$

Achieve useful quantum advantage in the near-term

Applications to correlated quantum many-body systems

Quantum simulation for gapless fermion

T. Shirakawa, K. Seki & S. Yunoki, Phys. Rev. Research ***3****, 013004 (2021)*

Discretized quantum adiabatic process (DQAP) ansatz

Let us assume $\hat{H}_f = \hat{V}_1 + \hat{V}_2$, where $[\hat{V}_1, \hat{V}_2] \neq 0$,

and $\hat{H}_i = \hat{V}_1$

The time-evolution operator is

$$\hat{U}(T) = \lim_{M\to\infty} \hat{U}_d(\boldsymbol{\theta}_M)\hat{U}_d(\boldsymbol{\theta}_{M-1})\cdots\hat{U}_d(\boldsymbol{\theta}_1)$$

where

$$\hat{U}_d(\boldsymbol{\theta}_m) = \mathrm{e}^{-\mathrm{i}\theta_1^{(m)}\hat{V}_1}\mathrm{e}^{-\mathrm{i}\theta_2^{(m)}\hat{V}_2}$$

$$\left.\begin{aligned}\theta_1^{(m)} &= [s_i(\tau_m) + s_f(\tau_m)]\delta\tau \\ \theta_2^{(m)} &= s_f(\tau_m)\delta\tau\end{aligned}\right\}$$

→ e.g., linear scheduling

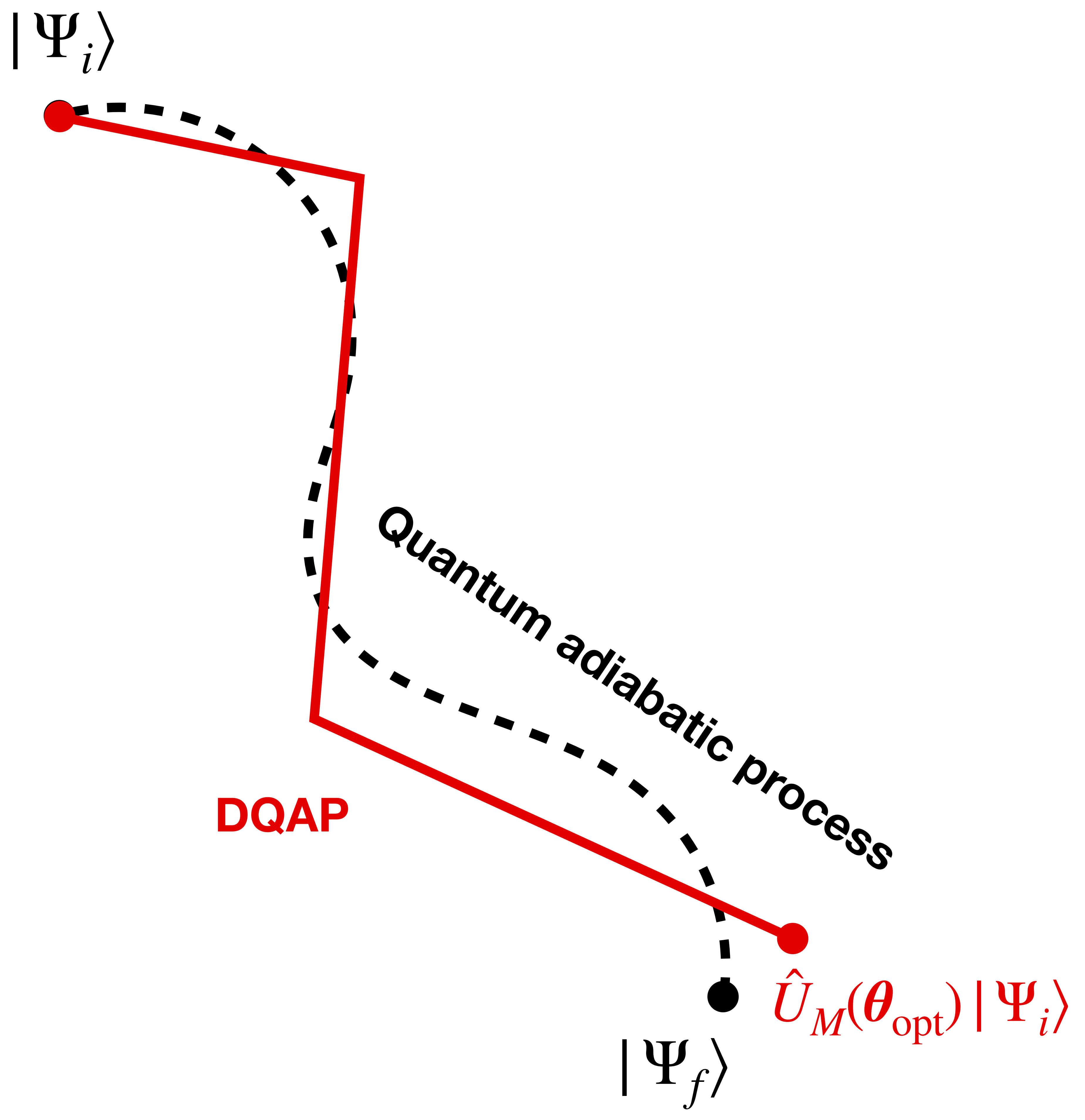
$$s_i(\tau) = 1 - \frac{\tau}{T}$$

$$s_f(\tau) = \frac{\tau}{T}$$

with

$$\delta\tau = T/M$$

$$\tau_m = m\delta\tau$$



Set M to be finite and consider $\{\theta_1^{(m)}, \theta_2^{(m)}\}$ as variational parameters

A parametrized circuit ansatz $\hat{U}_M(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \cdots, \boldsymbol{\theta}_M)|\Psi_i\rangle$ with M depths

DQAP ansatz or quantum approximate optimization ansatz (QAOA)

A quantum adiabatic evolution path itself is to be optimized

Question:
How many depths are required to represent the exact ground state?

DQAP ansatz for 1D free fermions

*T. Shirakawa, K. Seki & S. Yunoki, Phys. Rev. Research **3**, 013004 (2021)*

1D free fermions (spinless) with L sites at half filling (gapless ground state)

$$\mathcal{H} = -t\sum_{x=1}^{L-1}(\hat{c}_{x+1}^\dagger \hat{c}_x + \hat{c}_x^\dagger \hat{c}_{x+1}) - t\gamma(\hat{c}_1^\dagger \hat{c}_L + \hat{c}_L^\dagger \hat{c}_1) = \hat{\mathcal{V}}_1 + \hat{\mathcal{V}}_2$$

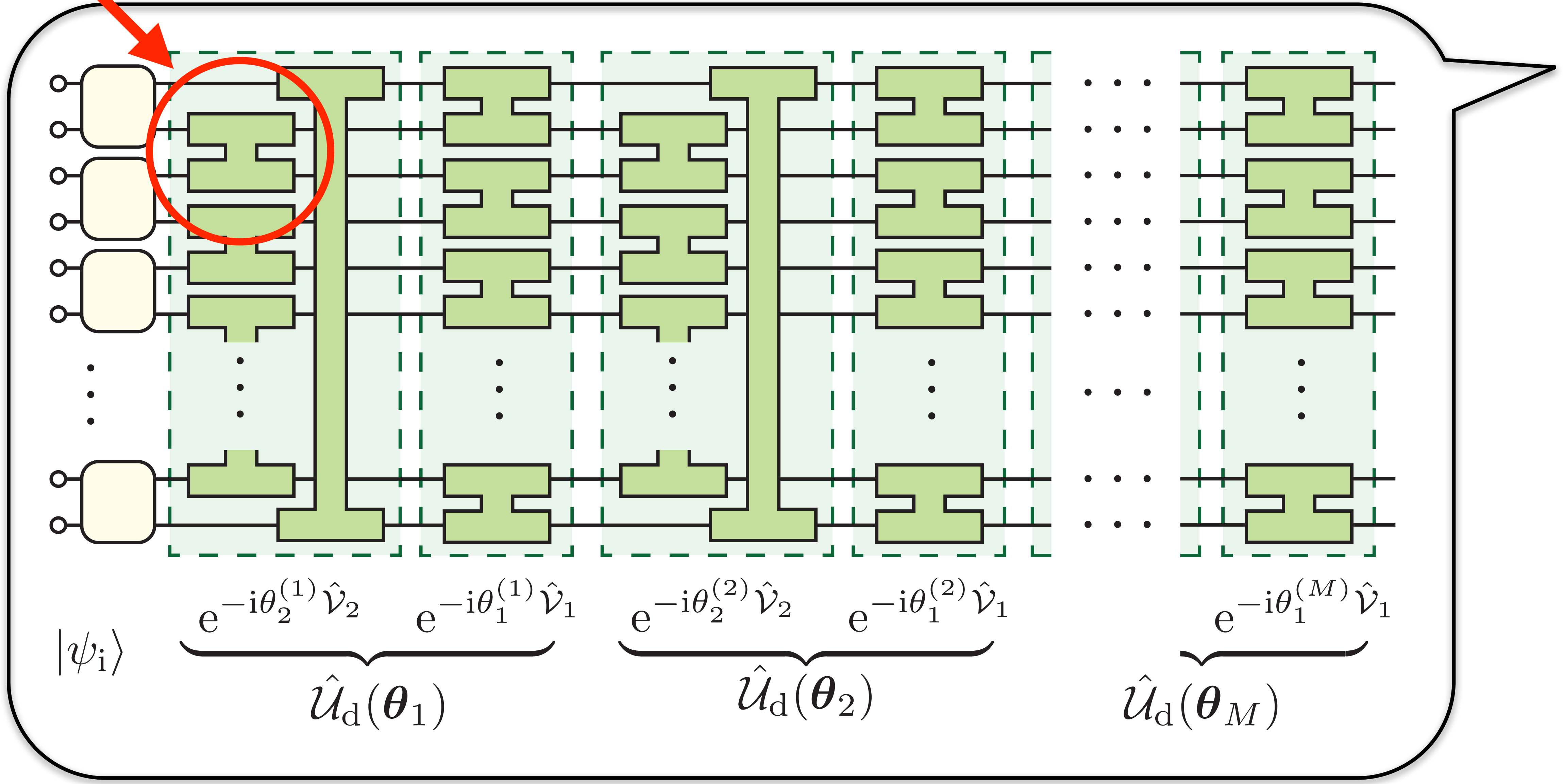
"even" bond: $\mathcal{V}_1 = -t\sum_{x=1}^{L/2}(\hat{c}_{2x}^\dagger \hat{c}_{2x-1} + \hat{c}_{2x-1}^\dagger \hat{c}_{2x})$

$$\gamma = \begin{cases} 1 & \text{periodic} \\ -1 & \text{antiperiodic} \end{cases}$$

"odd" bond: $\mathcal{V}_2 = -t\sum_{x=1}^{L/2-1}(\hat{c}_{2x+1}^\dagger \hat{c}_{2x} + \hat{c}_{2x}^\dagger \hat{c}_{2x+1}) - \gamma t(\hat{c}_1^\dagger \hat{c}_L + \hat{c}_L^\dagger \hat{c}_1)$

local time-evolution operator

$e^{-i\theta_2^{(1)}(-t)(\hat{c}_3^\dagger \hat{c}_2 + \text{h.c})}$



$$|\psi_M(\boldsymbol{\theta})\rangle = \hat{\mathcal{U}}_M(\boldsymbol{\theta})|\psi_\mathrm{i}\rangle$$

$$\hat{\mathcal{U}}_M(\boldsymbol{\theta}) = \hat{\mathcal{U}}_d(\boldsymbol{\theta}_M)\hat{\mathcal{U}}_d(\boldsymbol{\theta}_{M-1})\cdots\hat{\mathcal{U}}_d(\boldsymbol{\theta}_1)$$

$$\hat{\mathcal{U}}_d(\boldsymbol{\theta}_m) = e^{-i\theta_1^{(m)}\hat{\mathcal{V}}_1}e^{-i\theta_2^{(m)}\hat{\mathcal{V}}_2}$$

$$\boldsymbol{\theta} = \{\boldsymbol{\theta}_m\}_{m=1}^{M} \qquad \boldsymbol{\theta}_m = \{\theta_1^{(m)}, \theta_2^{(m)}\}$$

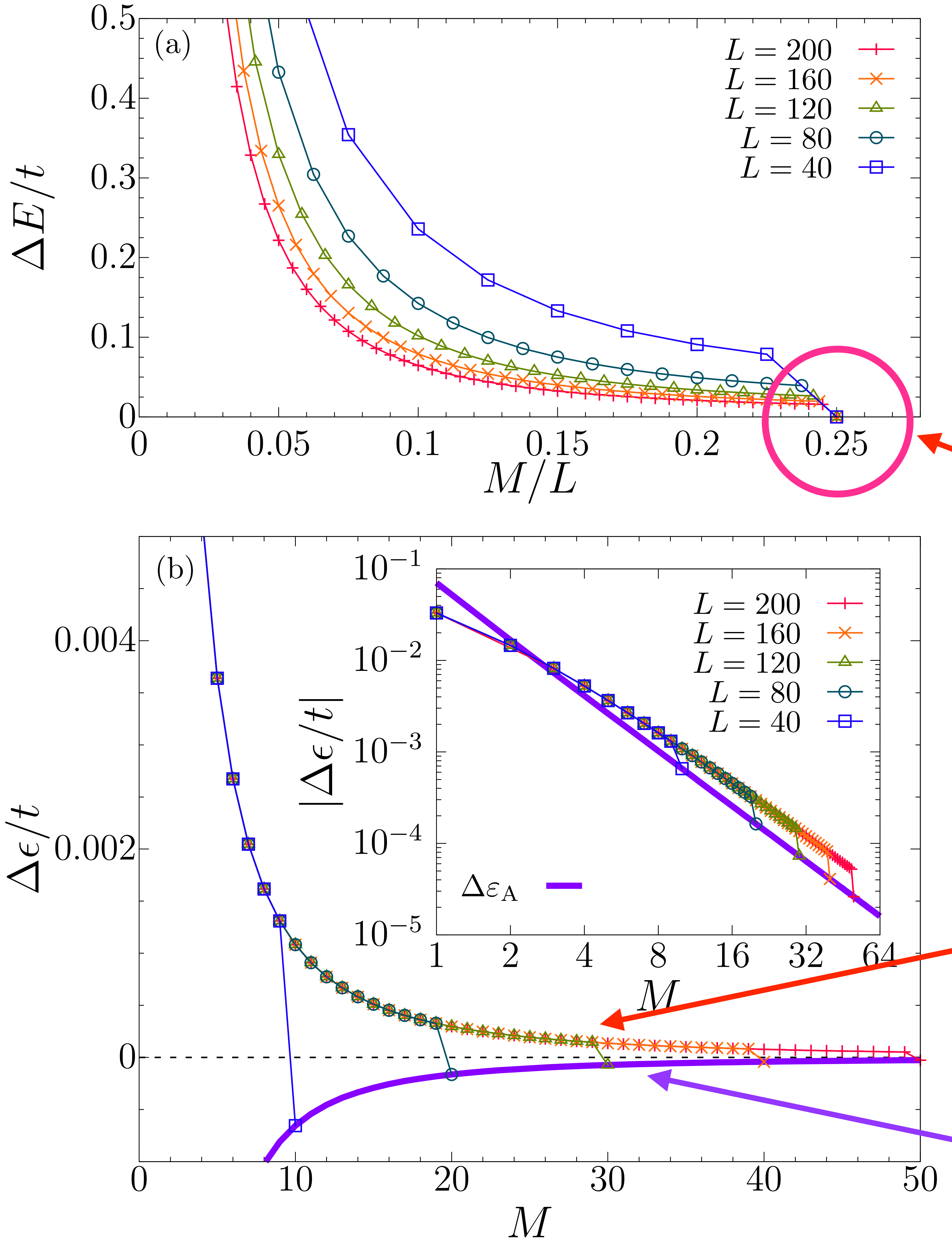
→ variational parameters

\# of parameters: *2M*

■ Natural gradient decent method in classical computer

Ground state energy

*T. Shirakawa, K. Seki & S. Yunoki, Phys. Rev. Research **3**, 013004 (2021)*



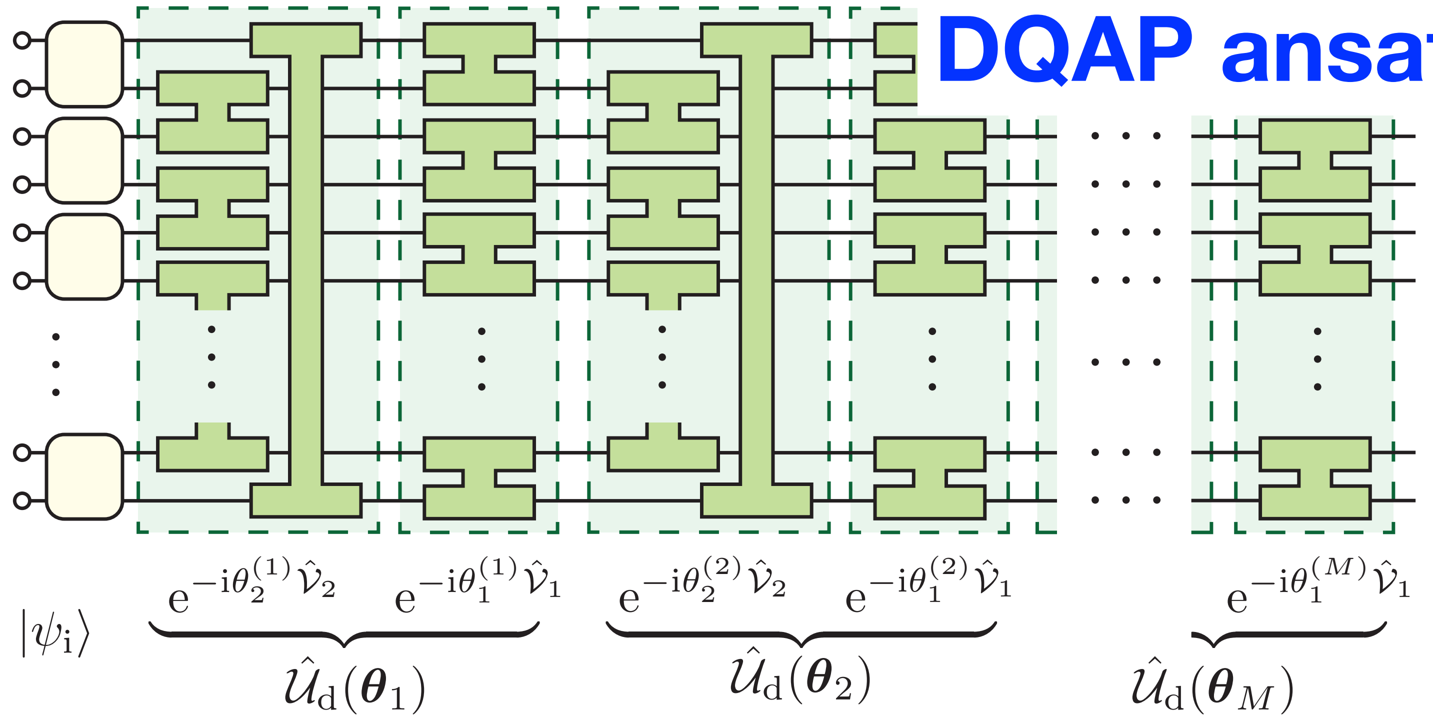
$$\Delta E = E(\boldsymbol{\theta}) - E_{\text{exact}}(L)$$

$$E(\boldsymbol{\theta}) = \langle \psi_M(\boldsymbol{\theta}) | \hat{\mathscr{H}} | \psi_M(\boldsymbol{\theta}) \rangle$$

$E_{\text{exact}}(L)$: exact ground state energy for system size L

exact ground state at *M=L/4*

DQAP ansatz



$$\Delta\varepsilon = E(\boldsymbol{\theta})/L - \lim_{L\to\infty} E_{\text{exact}}(L)/L$$

states $|\psi_M(\boldsymbol{\theta})\rangle$ with $M < L/4$ are independent of system size L

purple line: $E_{\text{exact}}(L)/L - \lim_{L\to\infty} E_{\text{exact}}(L)/L$ plotted at $M = L/4$

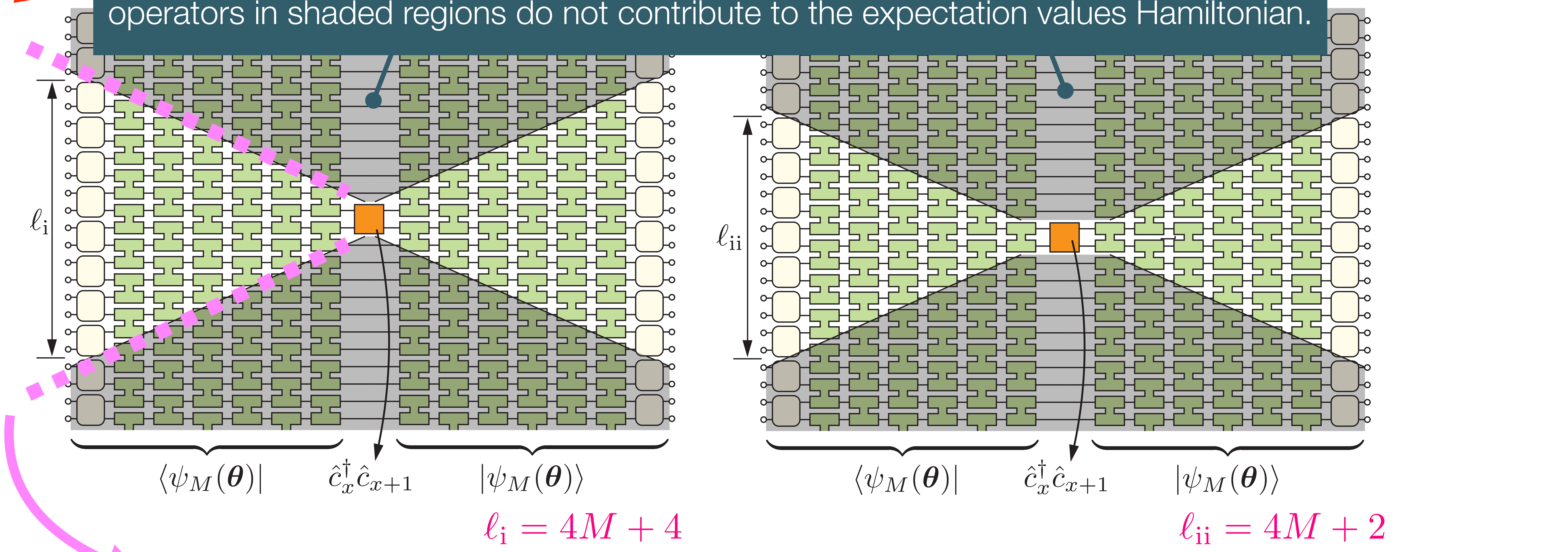
Causality cone and Lieb-Robinson bound

T. Shirakawa, K. Seki & S. Yunoki, Phys. Rev. Research **3**, 013004 (2021)

$$E(\boldsymbol{\theta}) = \langle \psi_M(\boldsymbol{\theta}) | \hat{\mathscr{H}} | \psi_M(\boldsymbol{\theta}) \rangle$$

$$\hat{\mathscr{H}} = \sum_{k=1}^{L/2} (\hat{\mathscr{V}}_1^{(k)} + \hat{\mathscr{V}}_2^{(k)}) \quad \hat{\mathscr{V}}_1^{(k)} = -t\left(\hat{c}_{2k}^\dagger \hat{c}_{2k-1} + \mathrm{h.c}\right) \quad \hat{\mathscr{V}}_2^{(k)} = -t\left(\hat{c}_{2k+1}^\dagger \hat{c}_{2k} + \mathrm{h.c}\right)$$

Due to the unitarity of the local time-evolution operators, these ***local*** time-evolution operators in shaded regions do not contribute to the expectation values Hamiltonian.



causality cone set by **Lieb-Robinson bound**

M=L/4 corresponds to the point where the causality-cone exceeds the system size *L*.

The DQAP ansatz can provide the circuit with the minimum number of depths (*M=L/4*) to describe the exact ground state

Quantum simulation for spin-1/2 Heisenberg model

K. Seki, T. Shirakawa & S. Yunoki, Phys. Rev. A ***101****, 052340 (2020)*

$S = 1/2$ antiferromagnetic Heisenberg model

K. Seki, T. Shirakawa & S. Yunoki, *Phys. Rev. A* **101**, 052340 (2020)

$$\hat{\mathcal{H}} = \frac{J}{4}\sum_{\langle i,j\rangle}\left(\hat{X}_i\hat{X}_j + \hat{Y}_i\hat{Y}_j + \hat{Z}_i\hat{Z}_j\right) = \frac{J}{2}\sum_{\langle i,j\rangle}\left(\hat{\mathcal{P}}_{ij} - \frac{\hat{I}}{2}\right)$$

$\hat{X}_i, \hat{Y}_i, \hat{Z}_i$: Pauli matrices

$\hat{\mathcal{P}}_{ij}$: Permutation (SWAP) operator s.t. $\hat{\mathcal{P}}_{ij}|a\rangle_i|b\rangle_j = |b\rangle_i|a\rangle_j$

In 1D

$$\hat{H} = \hat{H}_1 + \hat{H}_2 \qquad \hat{H}_1 = \frac{J}{2}\sum_{i:\ \mathrm{odd}}\left(\hat{P}_{i,i+1} - \frac{\hat{I}}{2}\right),\ \hat{H}_2 = \frac{J}{2}\sum_{i:\ \mathrm{even}}\left(\hat{P}_{i,i+1} - \frac{\hat{I}}{2}\right)$$

DQAP ansatz:

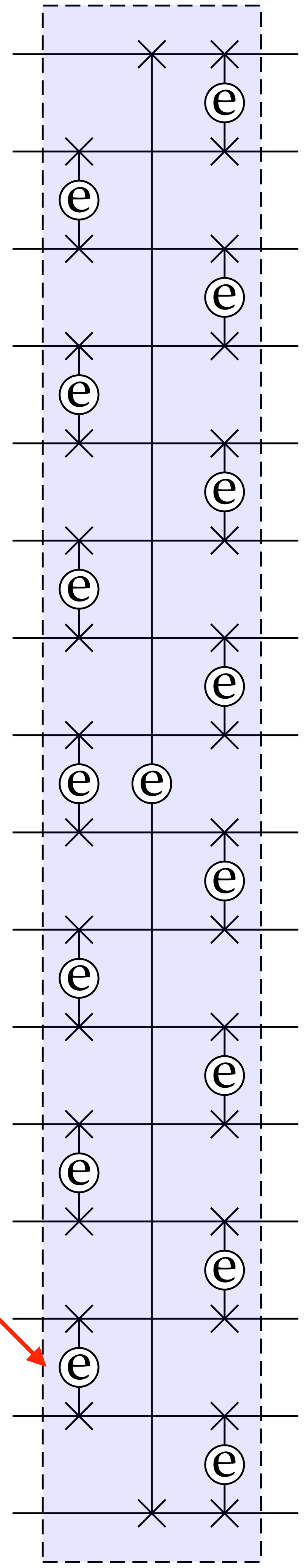
$$|\psi_M(\{\theta_1^{(m)}, \theta_2^{(m)}\})\rangle = \hat{\mathcal{U}}_d(\boldsymbol{\theta}_M)\hat{\mathcal{U}}_d(\boldsymbol{\theta}_{M-1})\cdots\hat{\mathcal{U}}_d(\boldsymbol{\theta}_1)|\psi_{\mathrm{i}}\rangle$$

singlet product state: GS of $\hat{H}_1$

$$\hat{\mathcal{U}}_d(\boldsymbol{\theta}_m) = \mathrm{e}^{-\mathrm{i}\theta_1^{(m)}\hat{H}_1}\mathrm{e}^{-\mathrm{i}\theta_2^{(m)}\hat{H}_2} \qquad \mathrm{e}^{-\mathrm{i}\theta_2^{(m)}\hat{H}_2} = \prod_{i:\mathrm{even}} \mathrm{e}^{-\mathrm{i}\theta_2^{(m)}\hat{P}_{i,i+1}/2}$$

(ESWAP circuit ansatz)

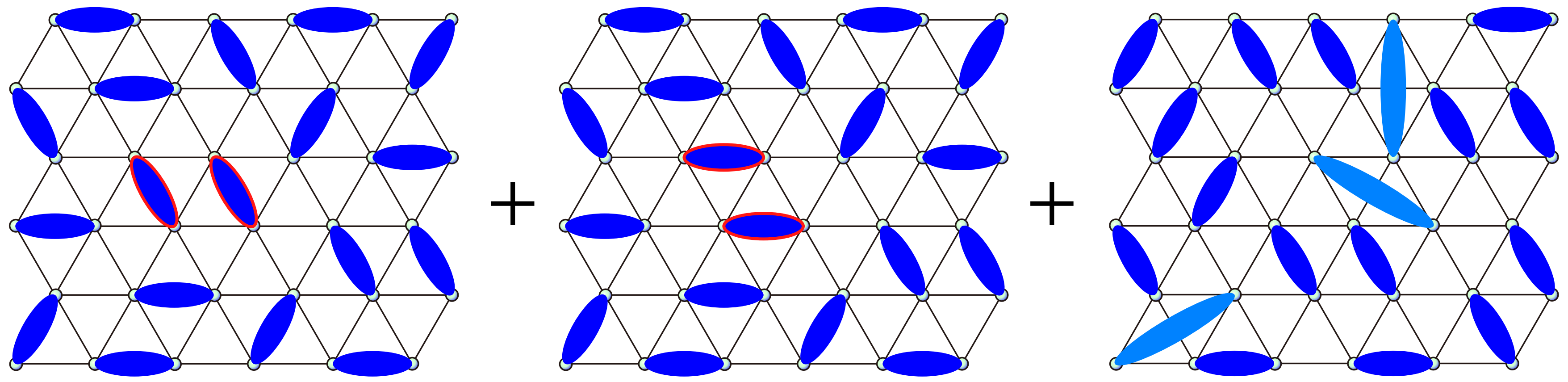
exponential-swap gate



RVB-type parametrized circuit ansatz

K. Seki, T. Shirakawa & S. Yunoki, Phys. Rev. A **101**, 052340 (2020)

- Resonating valence bond (RVB) state
  - Represent any spin state
    - Magnetic order / topological order / valence bond solid …

$|\mathrm{RVB}\rangle =$  $+ \dots$

P. W. Anderson, Mater. Res. Bull. **8**, 153 (1973)

- DQAP ansatz constructs RVB states
  - $|\Phi\rangle$ represents a nearest neighboring dimer covering
  - $|\Psi(\theta)\rangle$ represents a linear combination of dimer coverings

$$|\Psi(\boldsymbol{\theta})\rangle = \hat{\mathcal{U}}(\boldsymbol{\theta})|\Phi\rangle$$

Exponential-SWAP gates

$$\hat{\mathcal{U}}(\boldsymbol{\theta}) = \prod_{\langle ij\rangle} \hat{U}_{ij}(\theta_{ij})$$

$$\hat{U}_{ij}(\theta) = \exp(-i\theta\hat{\mathcal{P}}_{ij}/2) = \hat{I}\cos\frac{\theta}{2} - i\hat{\mathcal{P}}_{ij}\sin\frac{\theta}{2}$$

$\boldsymbol{\theta}$: variational parameters

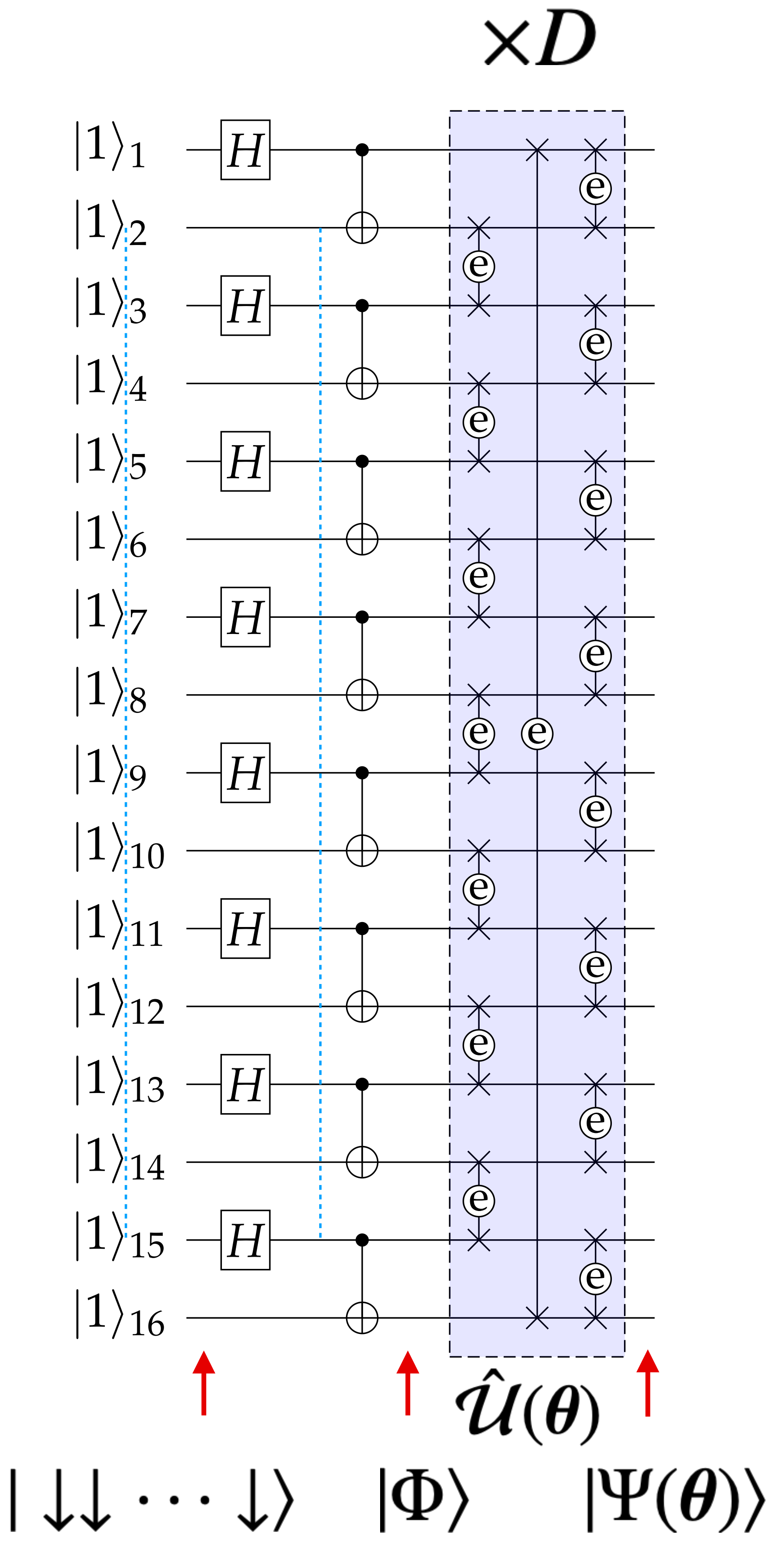
Singlet product state

$$|\Phi\rangle = \bigotimes_{i\in 2\mathbb{Z}+1} |[i, i+1]\rangle$$

$|[i,j]\rangle$: spin-singlet state

$$|[i,j]\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_i|1\rangle_j - |1\rangle_i|0\rangle_j\right)$$

ESWAP circuit ansatz:

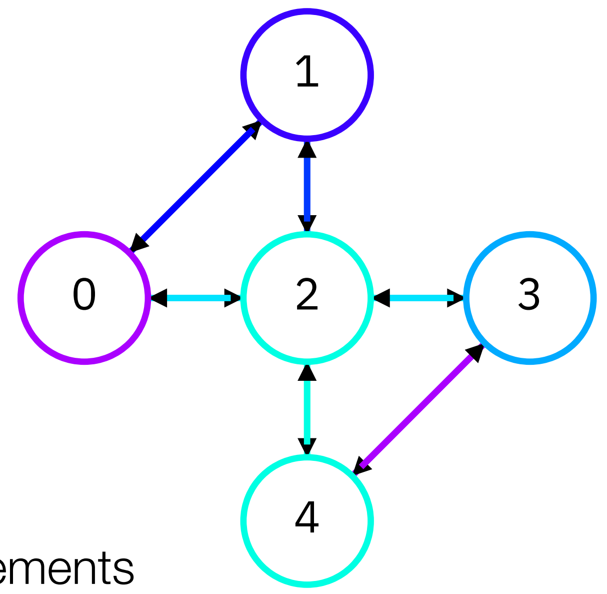


Experiment with a quantum device

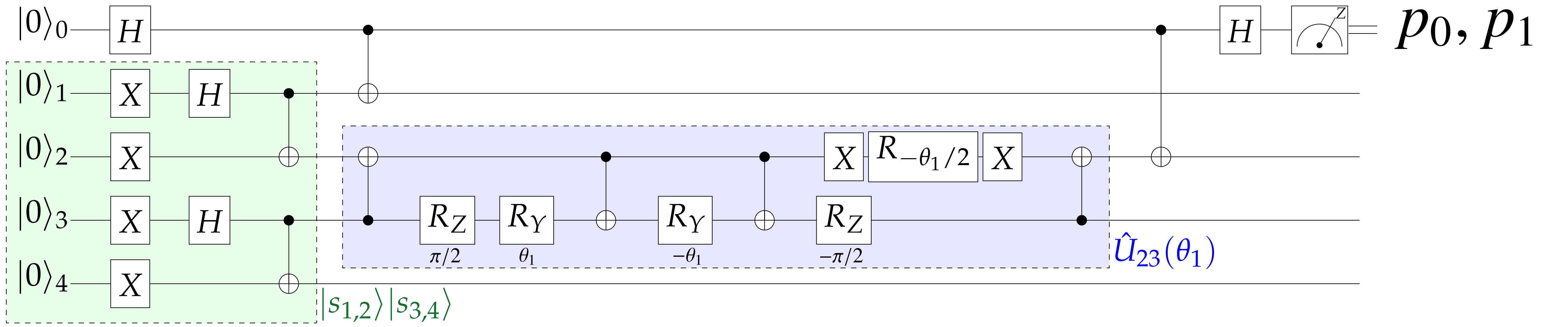
K. Seki, T. Shirakawa & S. Yunoki, Phys. Rev. A **101**, 052340 (2020)

S=1/2 Heisenberg ring with 4 sites

ibmq_5_yorktown



Quantum circuit for evaluating the spin correlation function $\mathrm{Re}\langle\Psi_0|\hat{X}_1\hat{X}_2|\Psi_0\rangle$



$$p_0 - p_1 = \mathrm{Re}\langle\Psi_0|\hat{X}_1\hat{X}_2|\Psi_0\rangle$$

each sample contains 1,024 measurements

Sample	$p_0(\%)$	$p_1(\%)$	$\mathrm{Re}\langle\Psi_0\vert\hat{X}_1\hat{X}_2\vert\Psi_0\rangle$
1	15.430	84.570	-0.69140
2	17.969	82.031	-0.64062
3	15.625	84.375	-0.68750
4	16.309	83.691	-0.67382
5	16.016	83.984	-0.67968
6	15.430	84.570	-0.69140
7	17.578	82.422	-0.64844
8	18.457	81.543	-0.63086
9	17.090	82.910	-0.65820
10	17.969	82.031	-0.64062
11	16.602	83.398	-0.66796
12	17.090	82.910	-0.65820
13	16.992	83.008	-0.66016
14	15.527	84.473	-0.68946
15	16.113	83.887	-0.67774
16	14.648	85.352	-0.70704
Mean	16.553(274)	83.447(274)	-0.66894(549)
Ideal	16.667	83.333	-0.66667

standard error of the mean

data taken on April 6, 2020 (EST)

Exact $E_0/J = -2$

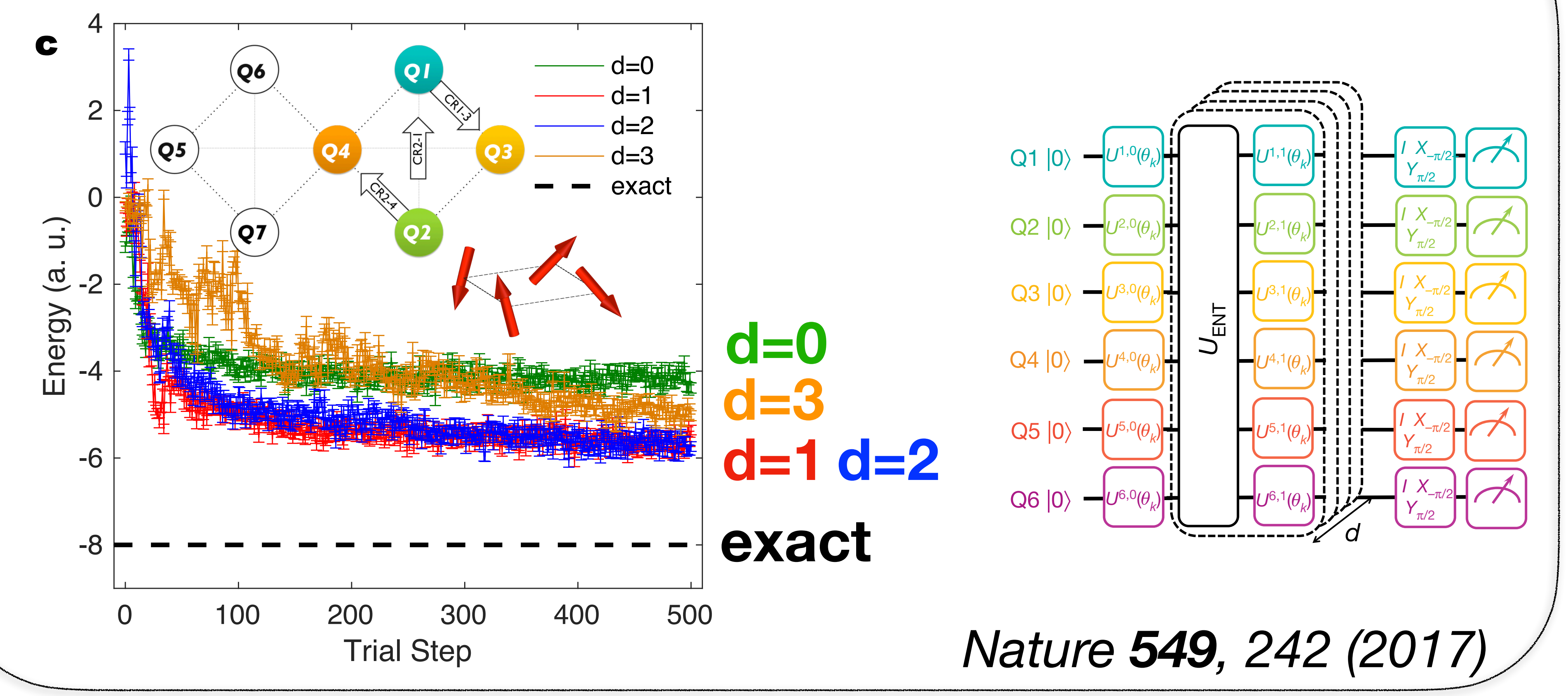
Estimated $E_0/J = -2.007(17)$

Exact energy is obtained within the statistical error

RVB-type circuit ansatz catches the physics!!

vs.

Hardware efficient ansatz



Nature **549**, 242 (2017)

Symmetry-adapted VQE

K. Seki, T. Shirakawa & S. Yunoki, Phys. Rev. A **101**, 052340 (2020)

ESWAP circuit state breaks the translational symmetry, but can be restored by:

$$|\psi^{(q)}(\vec{\theta})\rangle = \frac{\hat{P}^{(q)}|\psi(\vec{\theta})\rangle}{\sqrt{\langle\psi(\vec{\theta})|\hat{P}^{(q)}|\psi(\vec{\theta})\rangle}}$$

Symmetry-restored state

Symmetry-broken state

Projection operator (linear combination of unitaries)

$$\hat{P}^{(q)} = \frac{1}{N}\sum_{n=0}^{N-1}\chi^{(q)}(\hat{T}_n)^*\hat{T}_n$$

$\hat{T}_n$:translation operator（product of SWAPs or f-SWAPs）

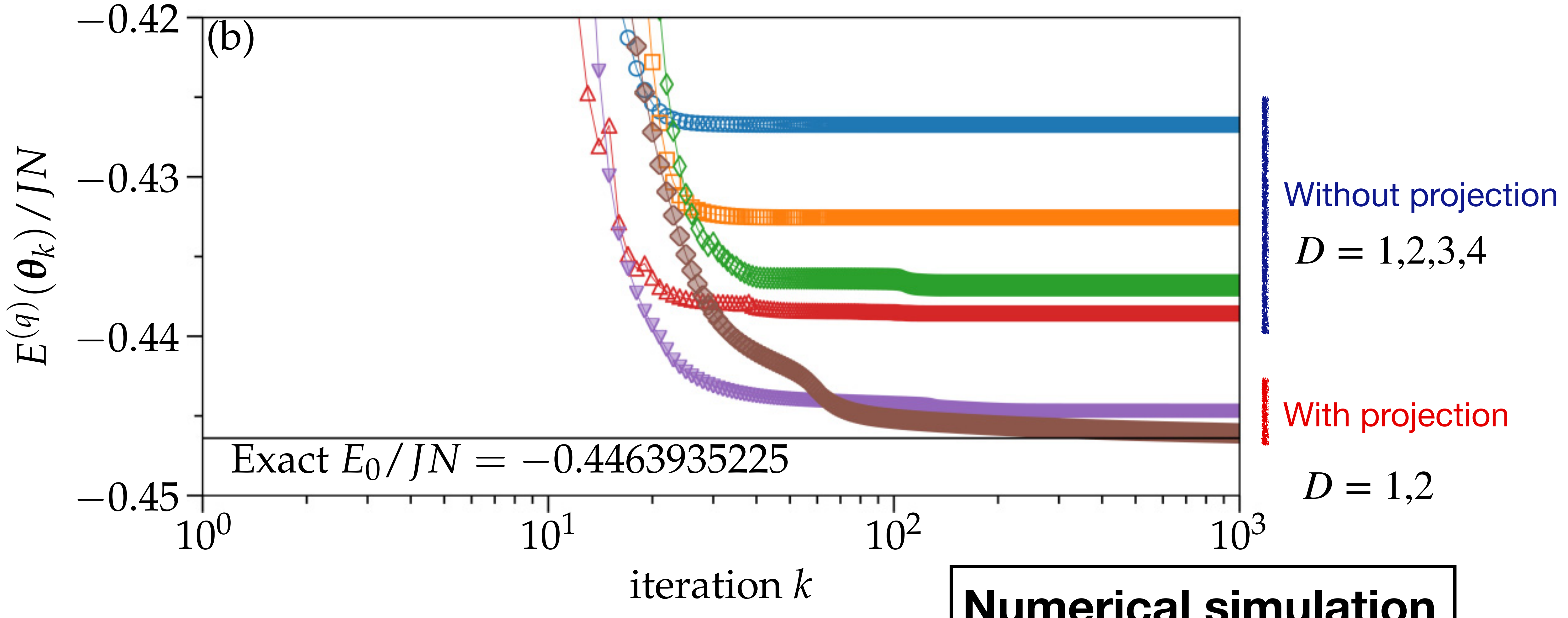
$\chi^{(q)}(\hat{T}_n)$:character, N :number of translation operations

Energy expectation value:

$$E^{(q)}(\vec{\theta}) = \frac{\langle\psi(\vec{\theta})|\hat{H}\hat{P}^{(q)}|\psi(\vec{\theta})\rangle}{\langle\psi(\vec{\theta})|\hat{P}^{(q)}|\psi(\vec{\theta})\rangle} = \frac{\sum_{n=0}^{N-1}\chi^{(q)}(\hat{T}_n)^*\langle\psi(\vec{\theta})|\hat{H}\hat{T}_n|\psi(\vec{\theta})\rangle}{\sum_{n=0}^{N-1}\chi^{(q)}(\hat{T}_n)^*\langle\psi(\vec{\theta})|\hat{T}_n|\psi(\vec{\theta})\rangle}$$

Evaluate the matrix elements on quantum computer

Variational energy per site for 16-site Heisenberg model:



Numerical simulation

Quantum simulation for topological order state

R.-Y. Sun, T. Shirakawa & S. Yunoki, Phys. Rev. B ***107****, L041109 (2023)*

See Poster 70

Toric code model in a magnetic field

R.-Y. Sun, T. Shirakawa & S. Yunoki, Phys. Rev. B **107**, L041109 (2023)

- Toric code model

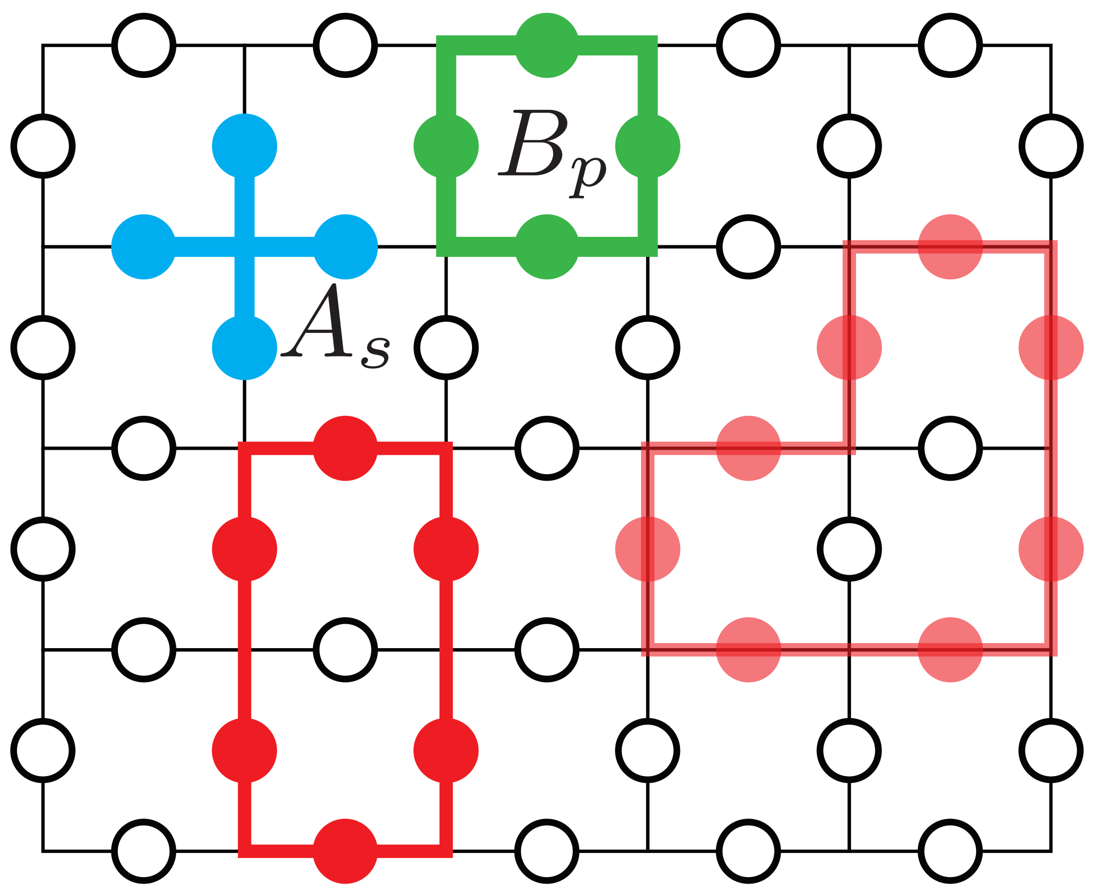
$$H_{\mathrm{TC}} = -\sum_s A_s - \sum_p B_p$$

  - Exactly solvable
  - Intrinsic topological order / equally-weighted quantum loop ($|1\rangle$) gas
  - OBC ground state
    - $|\Psi_0\rangle = \prod_{p=1}^{N_p} \left(\frac{1}{\sqrt{2}} I_p + \frac{1}{\sqrt{2}} B_p \right) |00\cdots 0\rangle$
  - Realizable on real quantum devices with high fidelity
    - Only need **Hadamard gate** and **CNOT gate**

- Toric code in a magnetic field

$$H_{\mathrm{TCM}}(x) = (1-x)H_{\mathrm{TC}} - x\sum_{i=1}^{N} \sigma_i^z$$

  - Non-exactly solvable
  - Topological order to ferromagnetic order: $x_c \sim 0.25$



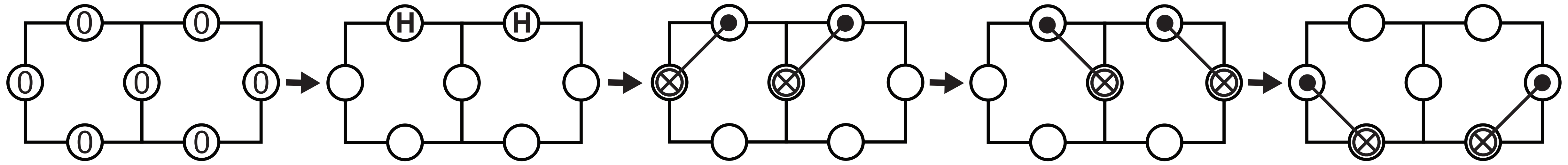
⊙ B

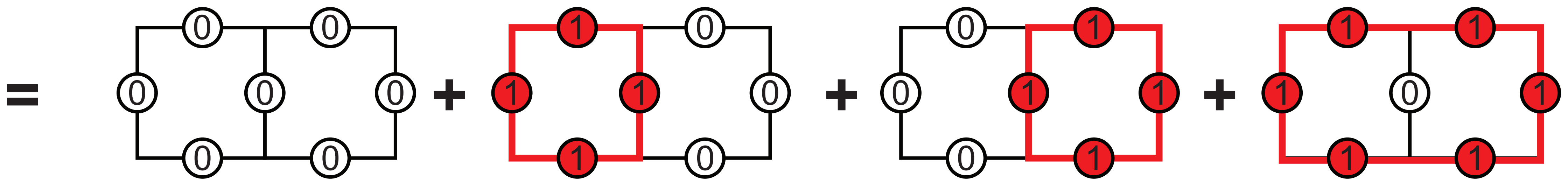
$$A_s = \prod_{i \in s} \sigma_i^z$$

$$B_p = \prod_{i \in p} \sigma_i^x$$

Hadamard gate:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$





CNOT

Science **374**, 1237 (2021)

Goal: simulate the ground state of $H_{\mathrm{TCM}}(x)$ on quantum computer

Parametrized loop gas circuit (PLGC)

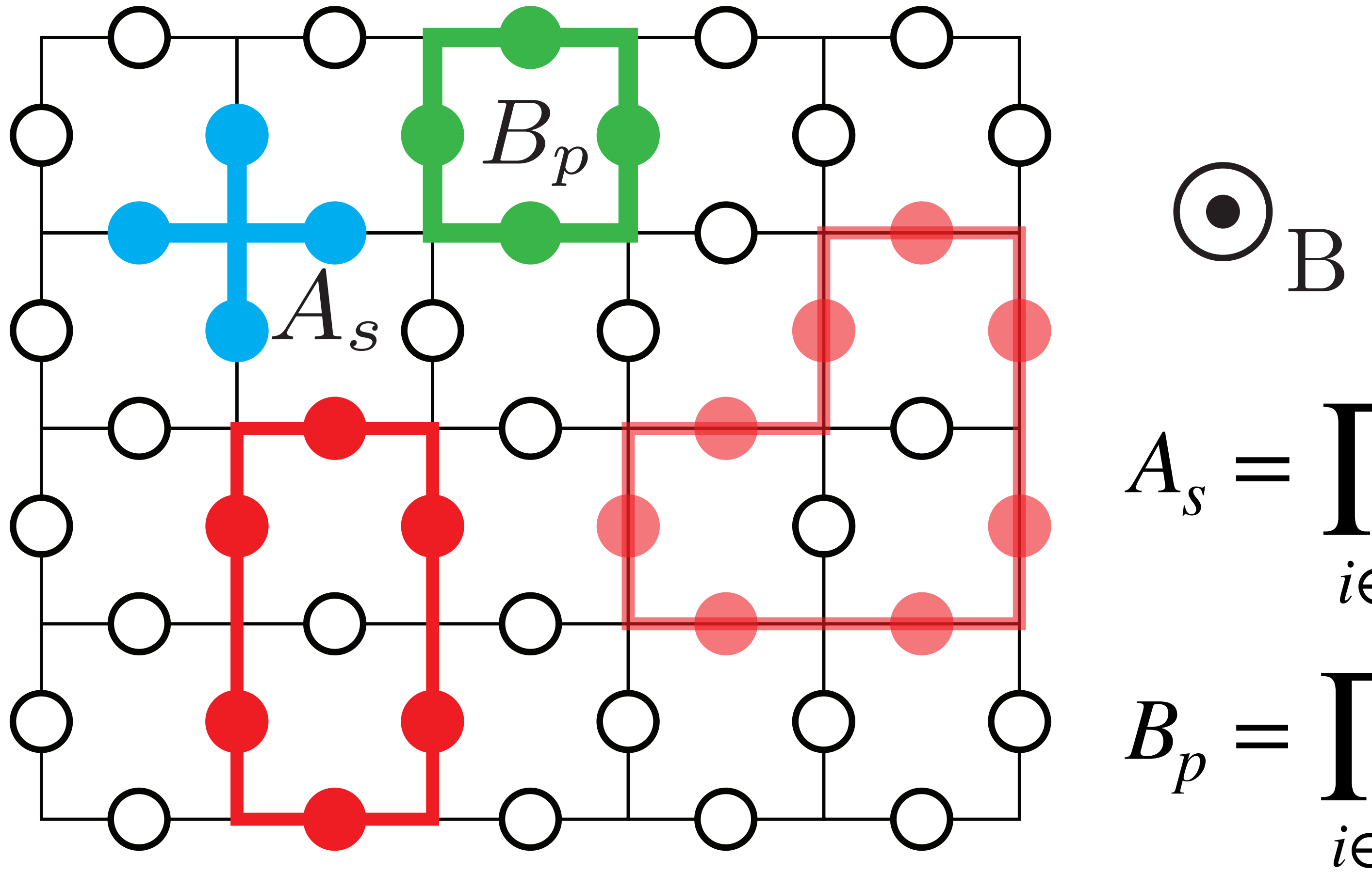
R.-Y. Sun, T. Shirakawa & S. Yunoki, Phys. Rev. B **107**, L041109 (2023)

- To simulate loop gas with loop tensions
  - Design a parametrized circuit to represent weight-adjustable quantum loop gas
- Dope parameter

$$H|0\rangle = R_y(\pi/2)|0\rangle$$

$$R_y(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$

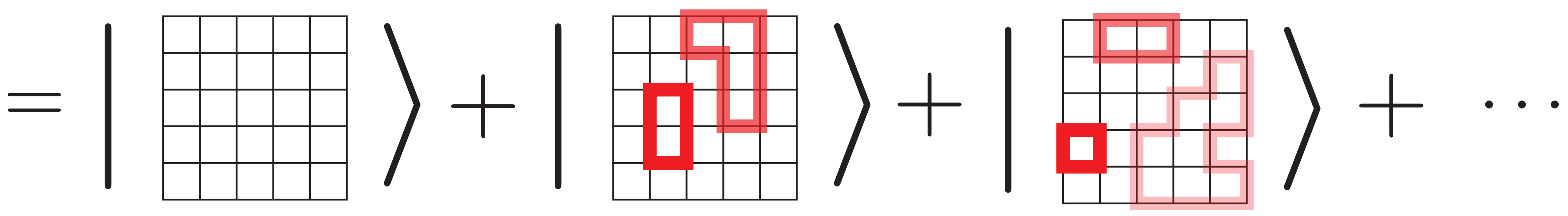
  - Replace Hadamard gate with Rotation-y gate
  - Number of parameters: N_p
- Realizable on real quantum devices
  - Rotation-y gate can be realized with high fidelity
  - With the same complexity



$$A_s = \prod_{i\in s} \sigma_i^z$$

$$B_p = \prod_{i\in p} \sigma_i^x$$

$$|\Psi(\boldsymbol{\theta})\rangle = \prod_{p=1}^{N_p} \left(\cos(\theta_p/2)I + \sin(\theta_p/2)B_p\right)|00\cdots0\rangle$$



VQE simulation using PLGC

- Ground state energy by VQE calculations with PLGC
  - Difference difference with the numerical exact energy $< 10^{-2}$
  - Keep the accuracy for arbitrary magnetic field strength
  - Keep the accuracy in the systems with varying sizes
- Observing topological quantum phase transition
  - Magnetization:

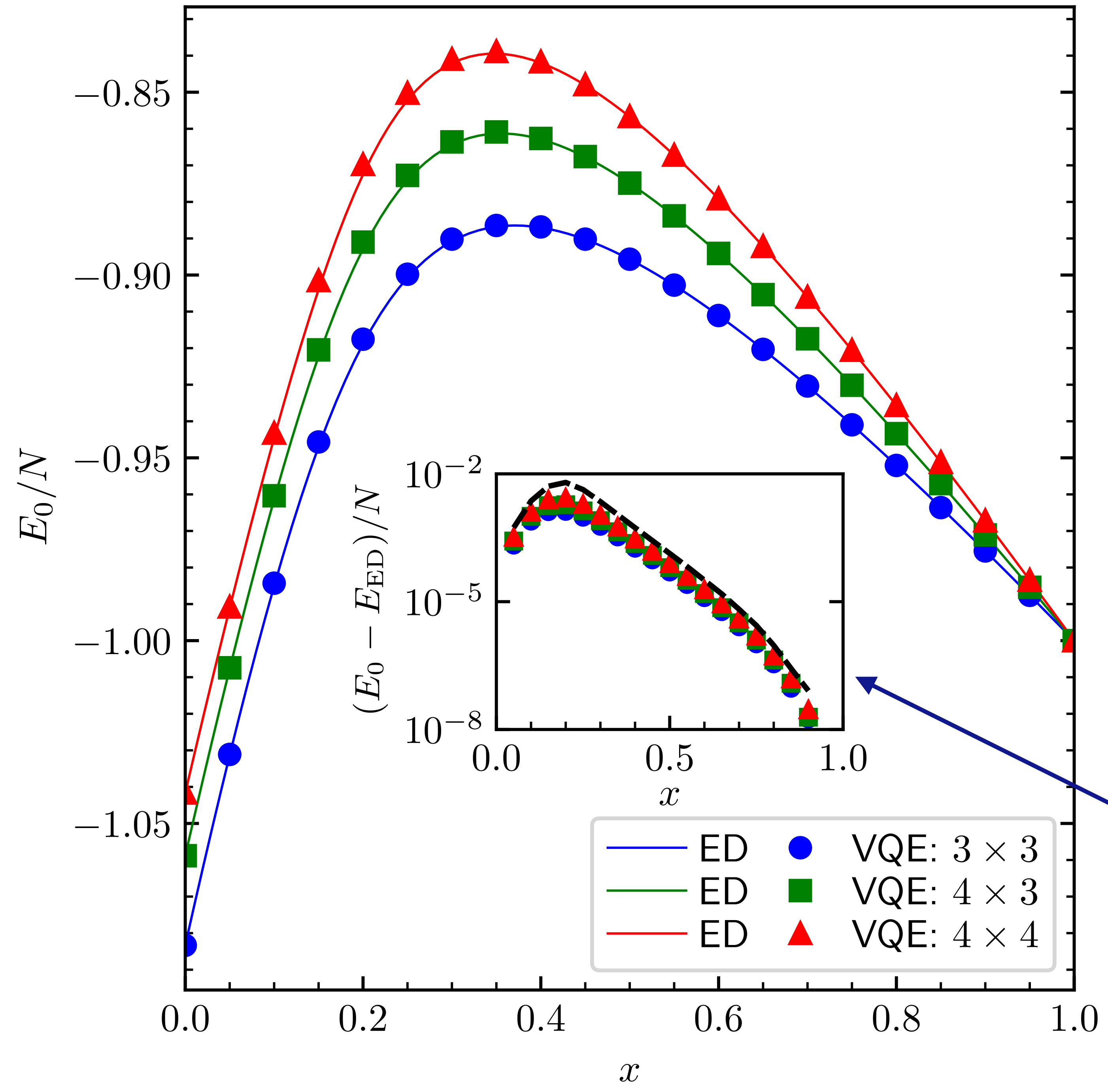
$$\langle m_z \rangle = \frac{1}{N} \sum_{i=1}^{N} \langle \sigma_i^z \rangle$$

  - Topological entanglement entropy (TEE):

Entanglement entropy of the subsystem

$$S_{\text{topo}} = S_{\text{A}} + S_{\text{B}} + S_{\text{C}} - S_{\text{AB}} - S_{\text{BC}} - S_{\text{AC}} + S_{\text{ABC}}$$

  - VQE calculations are consistent with numerical exact results



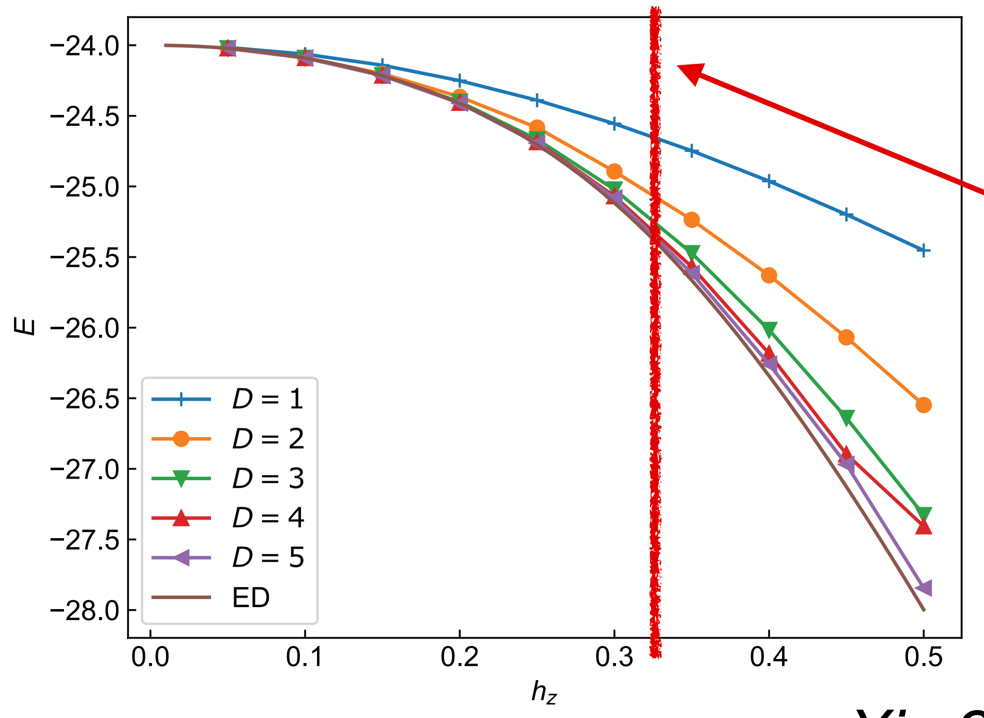
Energy difference

PLGC ansatz catches the physics!!

vs.

QAOA-type

$$|\Psi(\theta_l^0, \theta_l^1)\rangle = \prod_{l=1}^{D} \left[e^{-i\theta_l^0 H_{TC}} e^{-i\theta_l^1 H_h} \right] |\Psi_0\rangle$$



Phase transition

arXiv:2202.02909

VQE simulation using PLGC

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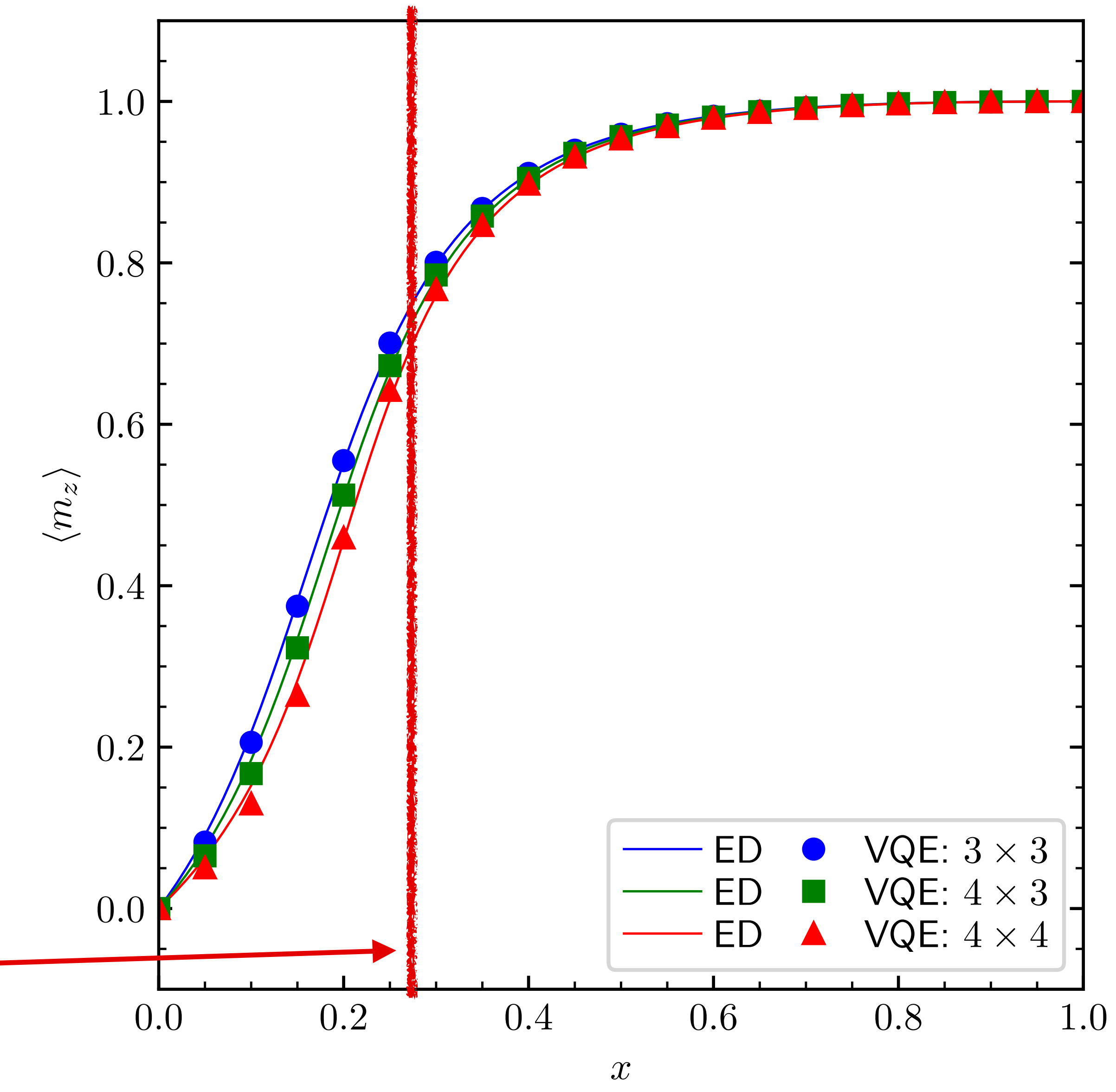
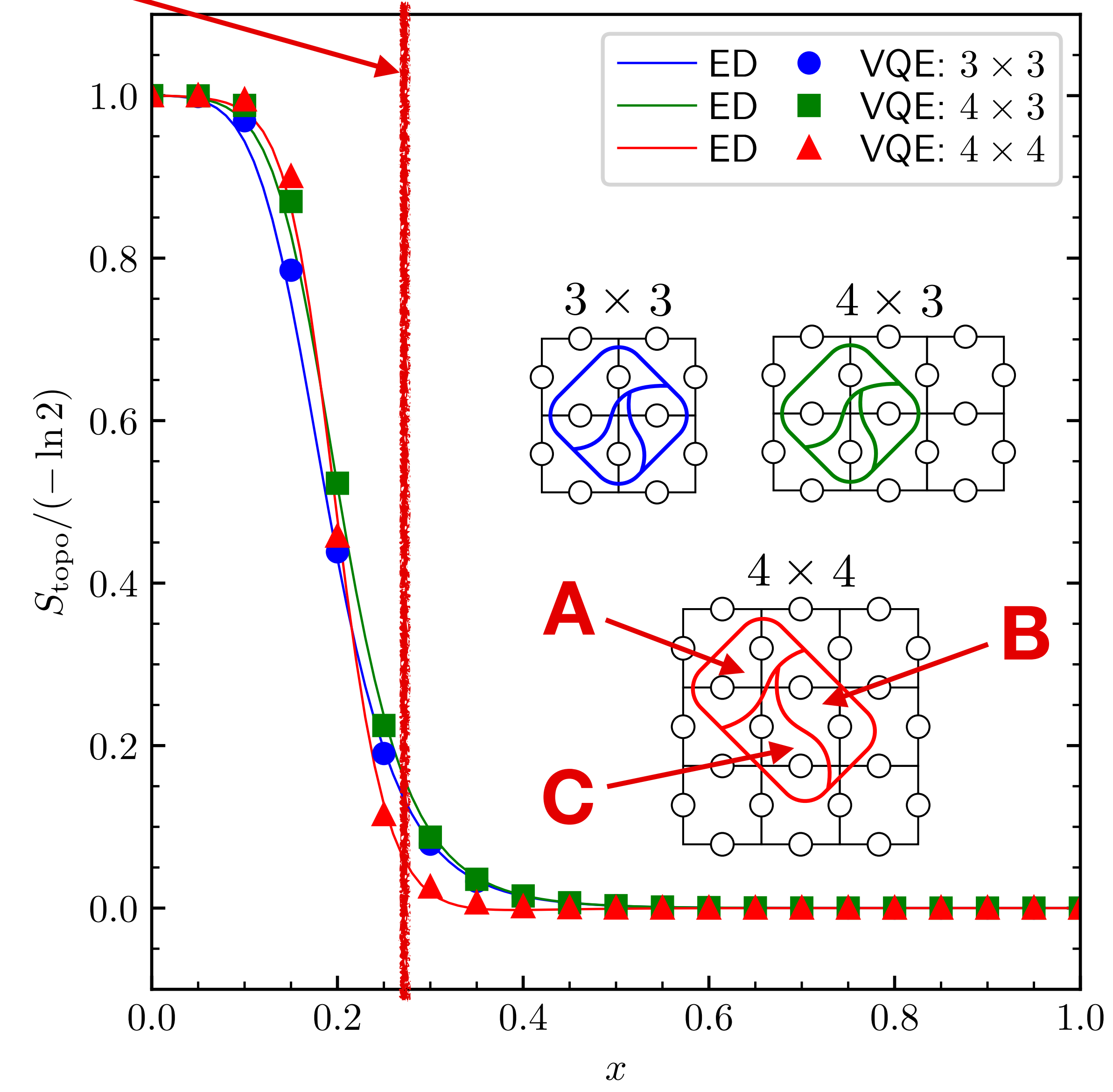
Magnetization:



Phase transition

TEE:



Summary

- Quantum computing provides a new way to study correlated quantum many-body systems
  - Quantum simulation: simulate quantum system of interest by another controllable quantum system
- We are in the NISQ era
  - Noisy quantum devices with quantum-classical hybrid algorithms
  - Apply to quantum many-body physics: variational quantum eigensolver (VQE, promising but with challenges)
- VQE study of correlated quantum many-body systems
  - Discretized quantum adiabatic process (DQAP) ansatz provides the exact ground state with $L/4$ depths: *PRResearch **3**, 013004 (2021)*
  - RVB-type circuit ansatz (emerged from DQAP) catches the ground state of Heisenberg model; Broken symmetry can be restored by post processing (symmetry-adapted VQE): *PRA **101**, 052340 (2020)*
  - Real-device-realizable PLGC ansatz describes the topological order in non-exactly solvable cases and can go through the phase transition: *PRB **107**, L041109 (2023)*

Thanks for your attention!