Curved domain-wall fermion and its anomaly inflow

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Introduction

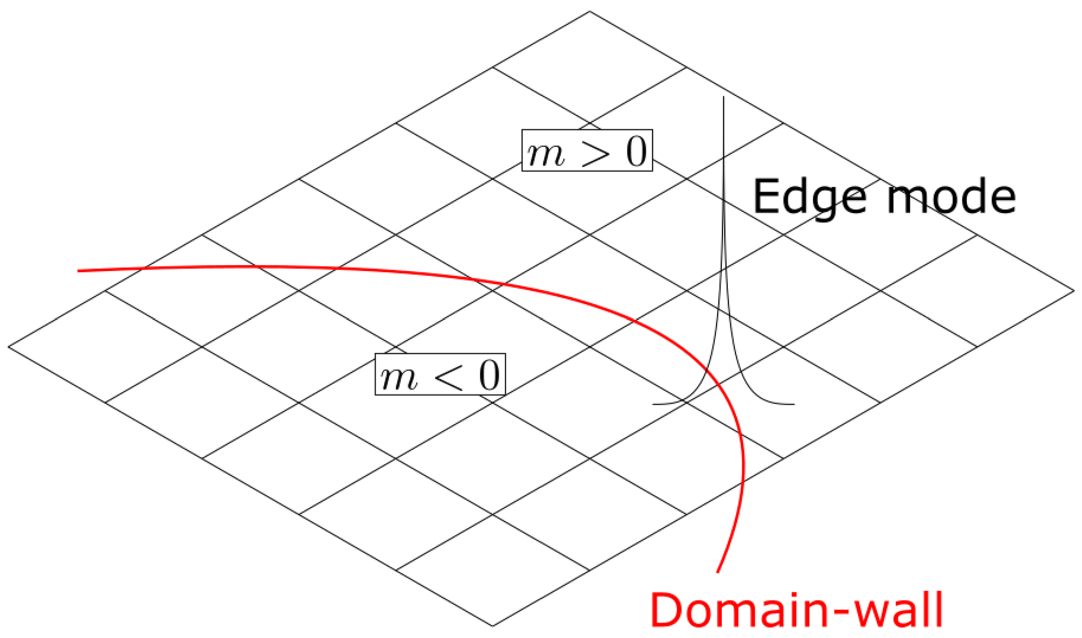
It is difficult to consider a gravitational effect on lattice system.

[Our idea]: Curved domain-wall [1]

Domain-wall [2] is a boundary where a sign of mass is flipped

→ Low energy states appear at the wall.

→ **They feel gravity by the equivalence principle.**



Curved domain-wall system

Hermitian Dirac operator:

$$H = \bar{\gamma}\left(\sum_{i=1}^{n+1}\gamma^i\frac{\partial}{\partial x^i} + m\mathrm{sign}(f)\right) = \bar{\gamma}(\not{D} + m\mathrm{sign}(f))$$

$$\gamma^a = -\sigma_2\otimes\tilde{\gamma}^a,\ \gamma^{n+1} = \sigma_1\otimes 1,\ \bar{\gamma} = \sigma_3\otimes 1$$

$$\{\tilde{\gamma}^a, \tilde{\gamma}^b\} = 2\delta^{a,b},\ (a, b = 1, \cdots, n)$$

$f = 0$ defines a domain-wall Y. In the large m limit,

$$H \to i\not{D}^Y = i\sum_{a=1}^{n}\tilde{\gamma}^a\left(e_a + \frac{1}{4}\sum_{bc}\omega_{bc,a}\tilde{\gamma}^b\tilde{\gamma}^c\right).$$

Spin connection!

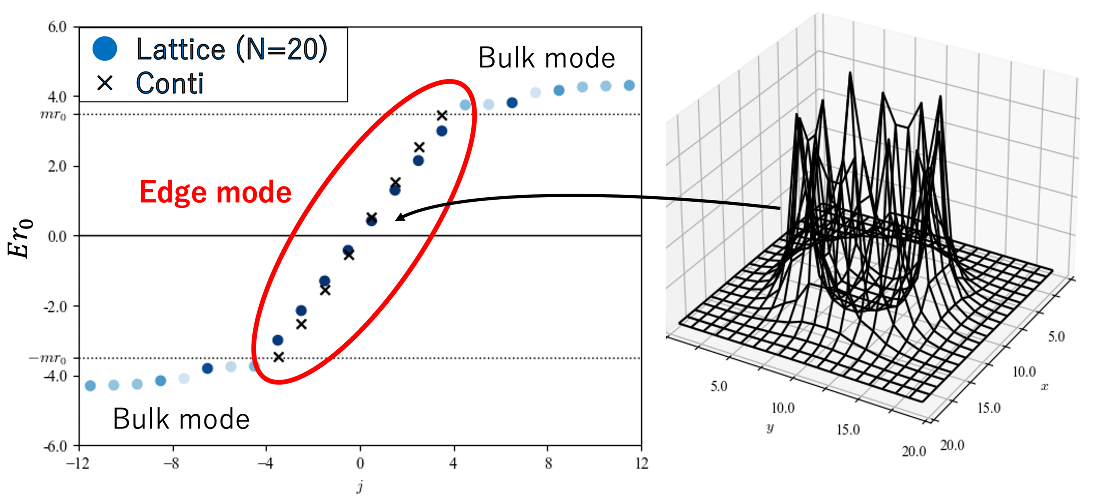
- Edge modes appear at the wall.
- They are chiral: $\gamma_{\text{normal}} = \boldsymbol{n}\cdot\boldsymbol{\gamma}$
- They feel gravity via the spin connection.
- We can also consider this system on square lattice.

S^1 and S^2 domain-wall on square lattice

$$H = \frac{\bar{\gamma}}{a}\left(\sum_i\left[\gamma^i\frac{\nabla_i - \nabla_i^\dagger}{2} + \frac{1}{2}\nabla_i\nabla_i^\dagger\right] + \epsilon ma\right),\quad \begin{pmatrix}(\nabla_i\psi)_x = \psi_{x+a} - \psi_x\\ (\nabla_i^\dagger\psi)_x = \psi_{x-a} - \psi_x\end{pmatrix}$$

[S^1 case]: $H \to \frac{1}{r_0}(-i\frac{\partial}{\partial\theta} + \frac{1}{2})$

Spin^c connection

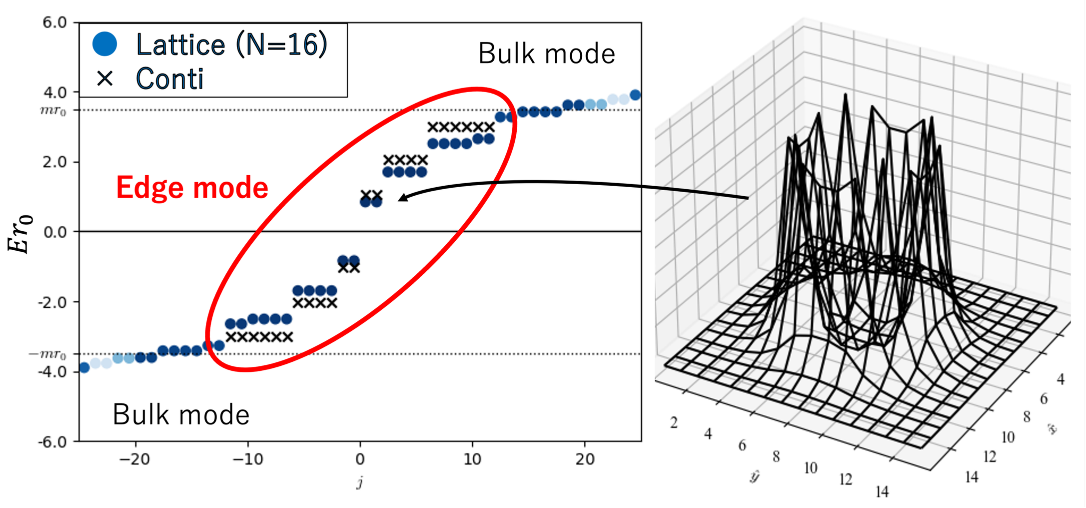


$$Er_0 = n + \frac{1}{2}$$

$$\gamma_{\text{normal}} = \frac{x}{r}\sigma_1 + \frac{y}{r}\sigma_2 \simeq 1$$

[S^2 case]: $H \to -\frac{\sigma_3}{r_0}\left(\sigma_1\frac{\partial}{\partial\theta} + \sigma_2\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\phi} + \frac{i}{2\sin\theta} - i\frac{\cos\theta}{2\sin\theta}\sigma_3\right)\right)$

Spin^c connection



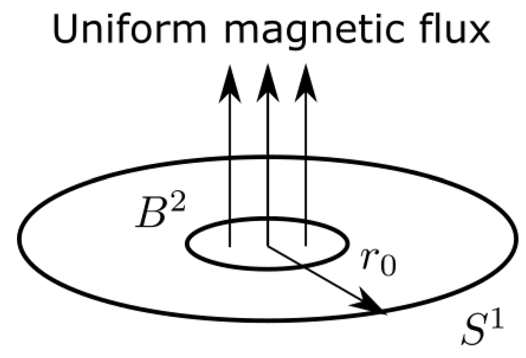
$$Er_0 = n + 1$$

$$\gamma_{\text{normal}} = \sum_{i=1}^{3}\frac{x^i\gamma^i}{r} \simeq 1$$

Agree well with the continuum prediction!

S^1 domain-wall with $U(1)$ flux

[Continuum]



$U(1)$ connection:

$$A = \begin{cases}\alpha\left(\frac{r}{r_1}\right)^2 d\theta & (r < r_1)\\ \alpha d\theta & (r_1 < r \leq r_0)\end{cases}$$

Chiral anomaly (Bulk) : $\frac{1}{2\pi}\int_{B^2} dA = \alpha$

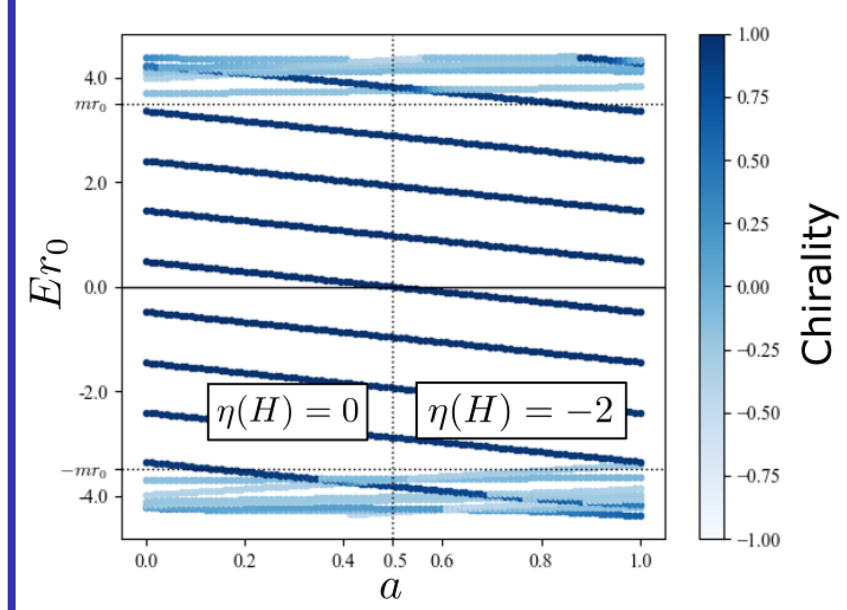
T-anomaly (Edge) : $-\frac{1}{2}\eta(i\not{D}^{S^1}_{eff}) = -\alpha + [\alpha + \frac{1}{2}]$

There exists an anomaly in Bulk and Edge, but they cancel each other [3].

$$\frac{1}{2\pi}\int_{S^1} dA - \frac{1}{2}\eta(i\not{D}^{S^1}_{eff}) = [\alpha + \frac{1}{2}] \in \mathbb{Z},\ (\eta(H) = \mathrm{tr}\frac{H}{|H|})$$

APS index

[Lattice]



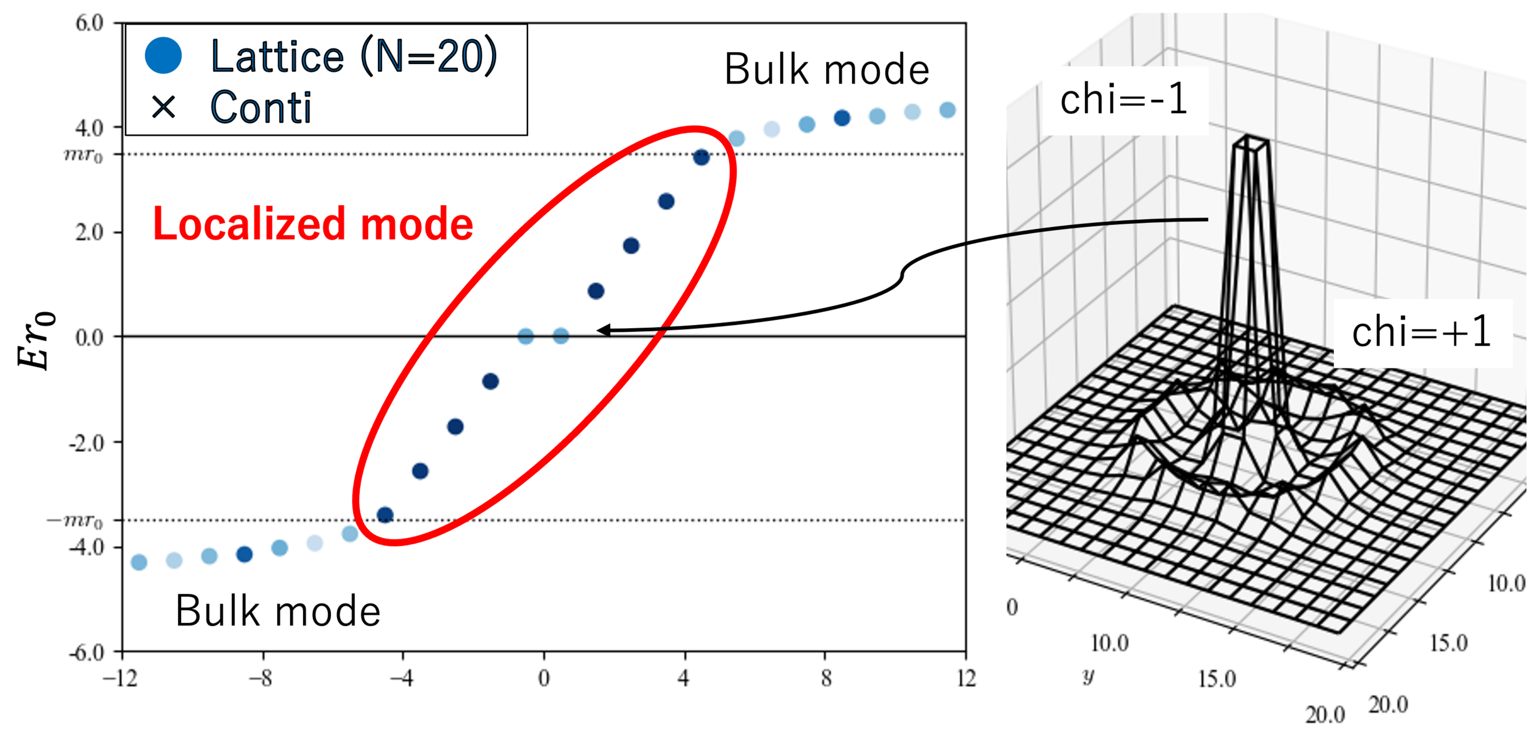
$$H = \frac{\sigma_3}{a}\sum_{i=1,2}\left[\sigma_i\frac{\nabla_i - \nabla_i^\dagger}{2} + \frac{1}{2}\nabla_i\nabla_i^\dagger\right] + \sigma_3\epsilon m$$

$$((\nabla_i\psi)_x = e^{-iA_ia}\psi_{x+a} - \psi_x)$$

$$\mathrm{Ind}_{APS} = -\frac{1}{2}\eta(H) = [\alpha + \frac{1}{2}]$$

Agree well with the continuum prediction!

When $\alpha = 1/2$ and $r_1 \to 0$, we get another zero-mode at $U(1)$ flux.



This system has TRS ($TT^* = -1$)! → $\mathbb{Z}_2$ skin effect [4]?

Conclusion

[Summary]

In cases S^1 and S^2, we embodied Nash's thm in domain-wall.

- Massless chiral edge sates appear on the domain-wall.
- Edge states feel gravity through the induced spin connection.
- We can see "Anomaly inflow" !

[Outlook]

- Gravitational anomaly inflow
- Index theorem with a nontrivial curvature
- Formulate real projective space

Reference

[1] K.-I. Imura *et al.* Spherical topological insulator. *Phys. Rev. B*, 86:235119, Dec 2012.

[2] D. B. Kaplan. A method for simulating chiral fermions on the lattice. *Physics Letters B*, 288(3):342–347, 1992.

[3] E. Witten. Fermion path integrals and topological phases. *Reviews of Modern Physics*, 88(3), jul 2016.

[4] N. Okuma *et al.* Topological origin of non-hermitian skin effects. *Physical Review Letters*, 124(8), feb 2020.