Lattice study of the trace anomaly contribution to glueball mass using renormalized energy-momentum tensor

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Introduction

The existence of composite states consisting solely of gluons, called **glueballs**, is one of the most important predictions of QCD. It is predicted that even the mass of lightest glueball state is much heavier than the nucleon mass. Since there is no mass scale in the pure gauge Yang-Mills theory at the classical level, the origin of the glueball masses can be attributed to the **trace anomaly**. We estimate **the trace anomaly contribution to glueball mass** through the mass decomposition based on the decomposition of the energy-momentum tensor.

Methods

Mass decomposition

In 1995, X. Ji proposed[1] an idea that the nucleon mass can be decomposed in terms of contributions from gluons and quarks, based on the traceful and traceless parts of the energy-momentum tensor (EMT) operator.

$$T^{\mu\nu} = \underbrace{\bar{T}^{\mu\nu}}_{\text{traceless}} + \underbrace{\hat{T}^{\mu\nu}}_{\text{traceful}} \Rightarrow \underbrace{H}_{\text{Hamiltonian}} = \bar{H} + \hat{H}\ (T^{00} = H) \Rightarrow M = \bar{M} + \hat{M}$$

This idea can be applied to glueballs, and it is much simpler conceptionally because glueball contains no quarks. However, there are both of technical and theoretical difficulties in such a numerical simulations. To overcome these difficulties, we adopt following two techniques.

Construction of the renormalized EMT operator using the gradient flow (Suzuki method)

It is hard to construct the renormalized energy-momentum tensor (EMT) on the lattice, where the loss of translational invariance is inevitable due to the discretization of the space-time. To overcome this problem, H. Suzuki proposed that **the gradient flow** approach[2] can be utilized to construct the renormalized EMT operator from the flowed fields[3]. Based on the small flow-time expansion, the operator is constructed on a small finite flow time τ, and by taking the $\tau \to 0$ limit, one can obtain the renormalized energy-momentum tensor operator.

An improvement of glueball correlation functions

It is notorious that the glueball correlation functions obtained in lattice QCD significantly suffers from ultraviolet noise from the gauge fields. To reduce this unphysical noise the techniques of **smearing** of are commonly used. In Ref. [4], we proposed a new smearing technique, **the spatial gradient flow**. The spatial gradient flow is an application of the gradient flow in which only the spatial links are evolved.

Spatial gradient flow

<< (statistics)
>> (costs)

Conventional smearing

Numerically identical
(at small a)[4]

Stout smearing

Comparable
computational costs

Numerical results

In the construction of the EMT operator using the Suzuki method we adopt the clover-shape definition. The diffusion radius R_d of the gradient flow is restricted to the region $2a \ll R_d \ll 1/\Lambda_{\overline{\text{MS}}}$ due to the size of the clover. We calculate $M_{\text{EMT}} = \langle G | T^{00} | G \rangle$ for glueball through the ratio of the 3pt. function C_{3pt} and the 2pt. function C_{2pt}, $C_{3pt}(t, t')/C_{2pt}(t)$, where t is the time separation and t' is the insertion time of the operator. We improve the glueball operators using the stout smearing. The calculations are carry out at the single cut-off in this study as the first calculation. The pure Yang-Mills gauge configurations listed on the Table below is used. The mass decomposition is performed as $M_{\text{EMT}} = \bar{M} + \hat{M} = M_{\text{TL}} + M_{\text{TA}}$.

$\beta(=6/g^2)$	$L^3 \times T$	Meas.	$1/a$[GeV]
6.4	$32^3 \times 32$	12000	3.84

In the left Fig., the ratio of M_{EMT} and the glueball mass M_G obtained from $C_{2pt} \sim e^{-M_G}$ (denoted as M_{2pt}) is shown. The $\tau \to 0$ result is

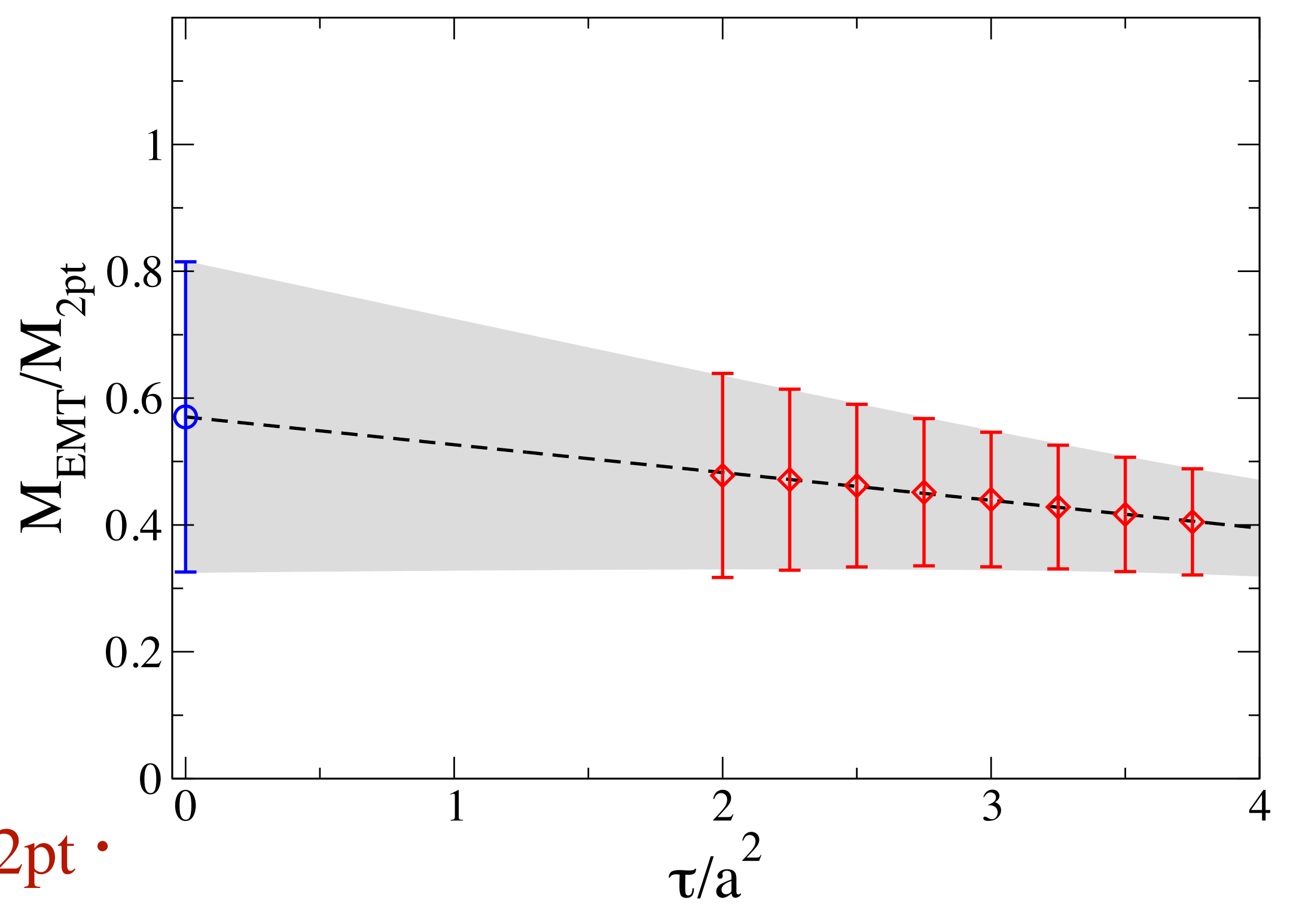
$$\boxed{M_{\text{EMT}}/M_{2\text{pt}} = 0.57(25)}$$

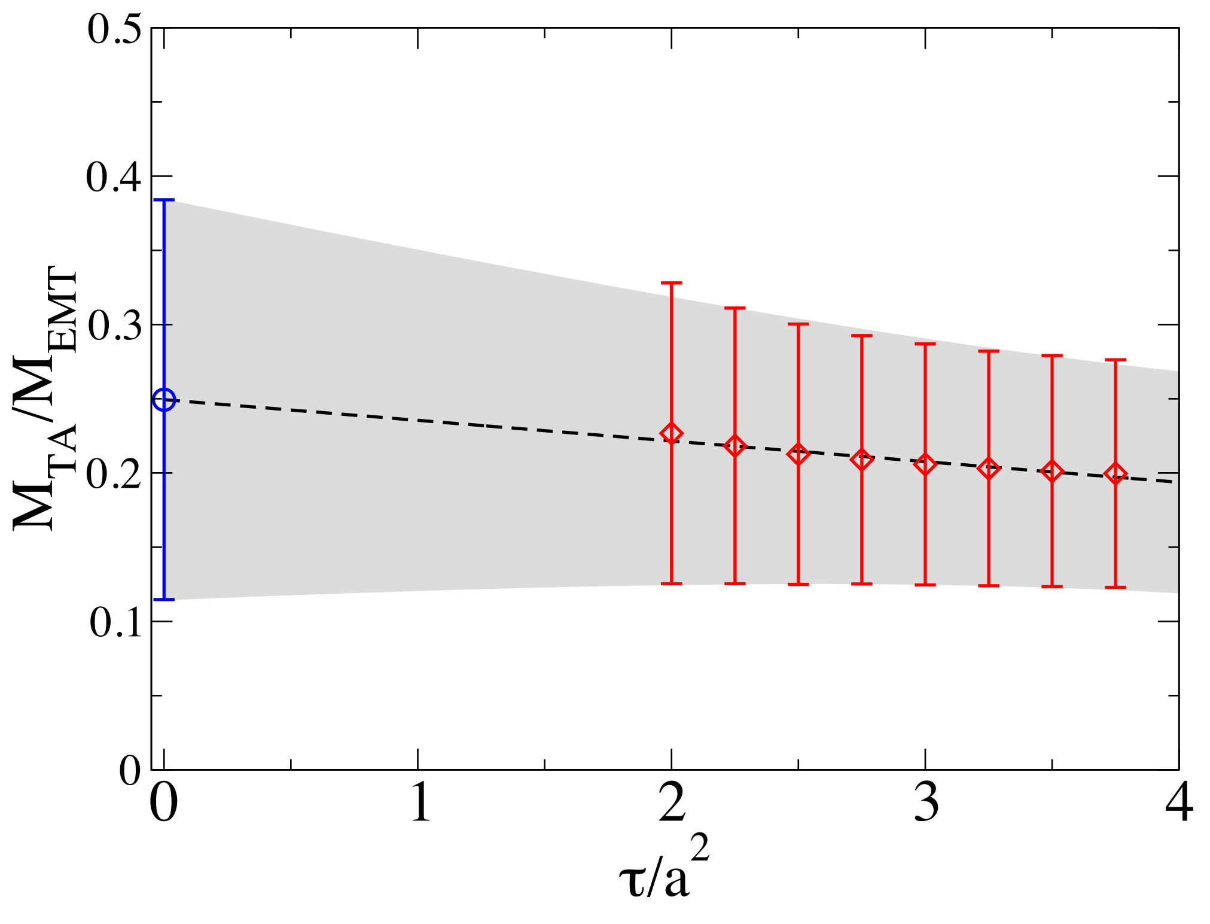
This ratio must be 1 in the continuum. Even at the single cut-off, it is consistent to 1 within 2σ. In the right Fig., the ratio of of M_{EMT} and its anomaly part is shown. The $\tau \to 0$ result is

$$\boxed{M_{\text{TA}}/M_{\text{EMT}} = 0.25(14)}$$

This ratio must be 1/4 if the EMT is correctly renormalized. From these results, we obtain

Trace anomaly $\langle G | T^{\mu}_{\mu} | G \rangle = 4M_{\text{TA}} \simeq M_{\text{EMT}} \sim M_{2\text{pt}}$.





Summary

We carried out **the first mass decomposition for glueball** by adopting the Suzuki method, which is utilizing the gradient flow to construct the renormalized EMT, and the spatial gradient flow, which is numerically equivalent at a small a, as a new smearing method. It became possible to evaluate the trace anomaly contribution to the glueball mass with the use of these 2 methods, and the **numerical correctness of the renormalized EMT from Suzuki method** is confirmed from the results.

References

[1] X. D. Ji, Phys. Rev. Lett. 74, 1071-1074 (1995).
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[3] H. Suzuki, PTEP 2013, 083B03 (2013) [erratum: PTEP 2015, 079201 (2015)].
[4] KS and S. Sasaki, arXiv:hep-lat/0508002 (soon will be published from PRD).