Controlling residual chiral symmetry breaking effects of domain wall fermions in QCD thermodynamics

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based on and updated from Lattice 2022 talk by Y. Aoki [9]

Abstract:

Investigation of QCD thermodynamics for $N_f = 2+1$ along the lines of constant physics with Möbius domain wall fermions is underway. At our coarsest lattice $N_t = 12$, reweighting to overlap fermions is not successful. To use domain wall fermions with the residual mass larger than average physical ud quarks, careful treatments of the residual chiral symmetry breaking are necessary. One of the examples is the chiral condensate where a UV power divergence associated with the residual chiral symmetry breaking emerges with a coefficient not known a priori. In this presentation we introduce first the setup of the computations and then discuss methodologies to overcome potential problems towards the continuum limit in this setup.

Formulations and codes

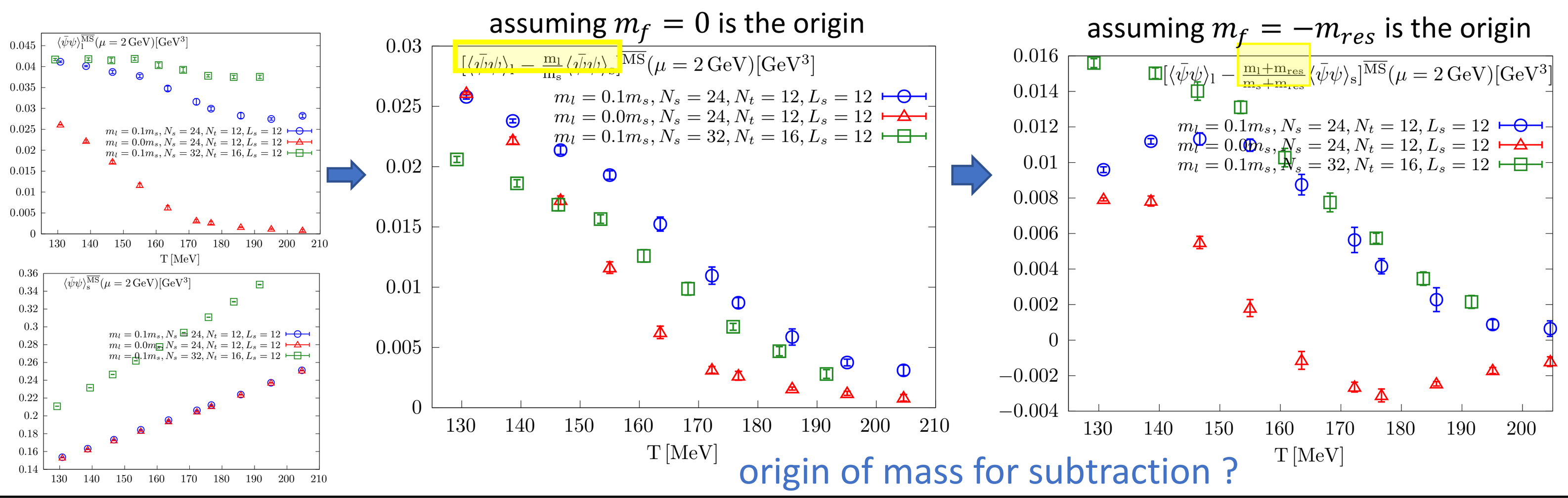
for N_f=2+1 T>0 and related T=0 studies:

- *Gauge formulation*: Tree-level Symanzik → ↓ stout smear
- *Quark formulation*: Möbius domain wall fermion (scale factor 2 Shamir type) w/ L_s=12
- ***Line of constant physics: $a(\beta)$, $m_s(\beta)$ obtained from fine T=0 data***
- *Numerical Method*: Rational Hybrid Monte Carlo (HMC), Conjugate Gradient solver for HMC and measurements
- *Codes*: Grid [2](HMC) *tuned* [6] *for A64FX by Regensburg group* [7], BQCD [3], Hadrons [4], Bridge++[5]

Light quark $\Sigma = -\langle\bar\psi\psi\rangle$

- Two step UV renormalization necessary (naively)
  - Logarithmic divergence (multiplicative): $Z_S(\overline{MS}, 2\text{ GeV})$
  - Power divergence (additive): $\propto m_f\, a^{-2}$
  - Subtracted using $\langle\bar s s\rangle$: $\langle\bar\psi\psi\rangle_l - \frac{m_l}{m_s}\langle\bar\psi\psi\rangle_s$

$m_\pi^2 \propto (m_f - m_{res}) + h.o.$



origin of mass for subtraction ?

- $\Sigma|_{DWF} \sim \frac{m_f + x m_{res}}{a^2} + \Sigma|_{cont.} + \cdots$ S. Sharpe [8]

$m_{res} \neq x m_{res}$; $x = O(1) \neq 1$

"Since x is not known, this term gives an uncontrolled error in the condensate. It can be studied and reduced only by increasing L_s - a very expensive proposition." – S. Sharpe.

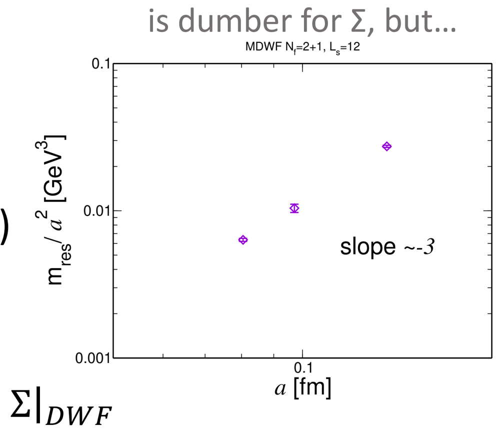
- We propose another way to estimate $x m_{res}$ using m'_{res}

"Forget about m_{res}" is dumber for Σ, but…

If chiral symmetry is restored → $\Sigma|_{cont.} = 0$

$-x m_{res}$ is a zero of $\Sigma|_{DWF}$ which is related with

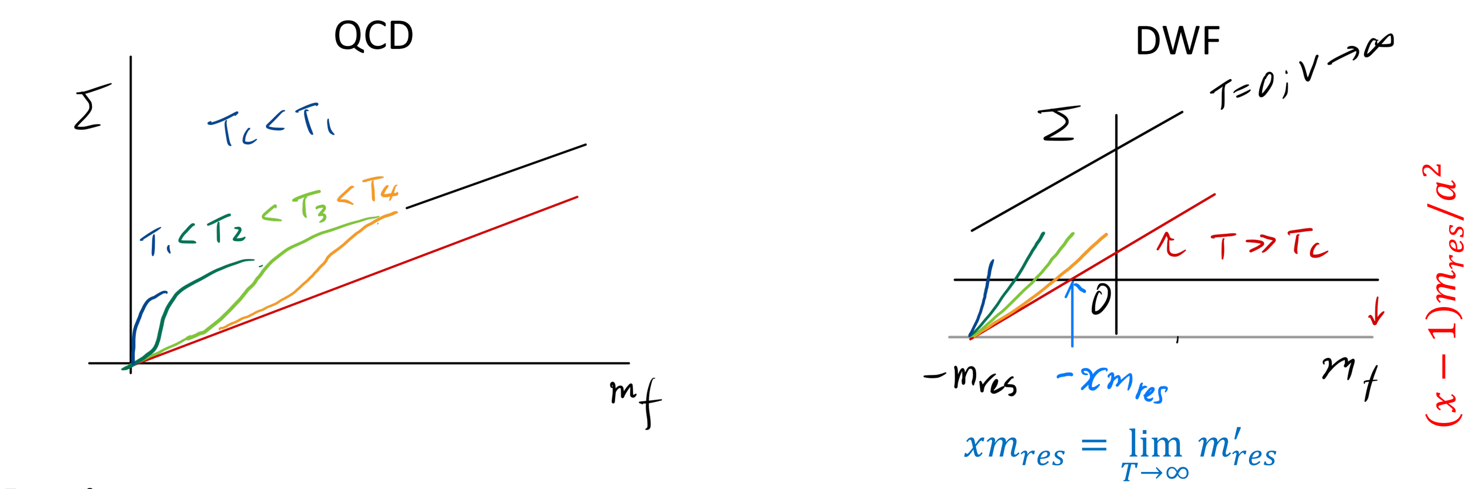
$$m'_{res} = \frac{\sum_x \langle J_{5q}(x) P(0)\rangle}{\sum_x \langle P(x) P(0)\rangle} \quad \left(\leftrightarrow\ m_{res} = \frac{\sum_{\vec x}\langle J_{5q}(\vec x,t) P(0)\rangle}{\sum_{\vec x}\langle P(\vec x,t) P(0)\rangle} \text{ at large } t\right)$$



$m_f = -m'_{res}$ is a zero of $\Sigma|_{DWF}$

due to Axial WT identity: $(m_f + m'_{res})\sum_x \langle P(x) P(0)\rangle = \Sigma|_{DWF}$

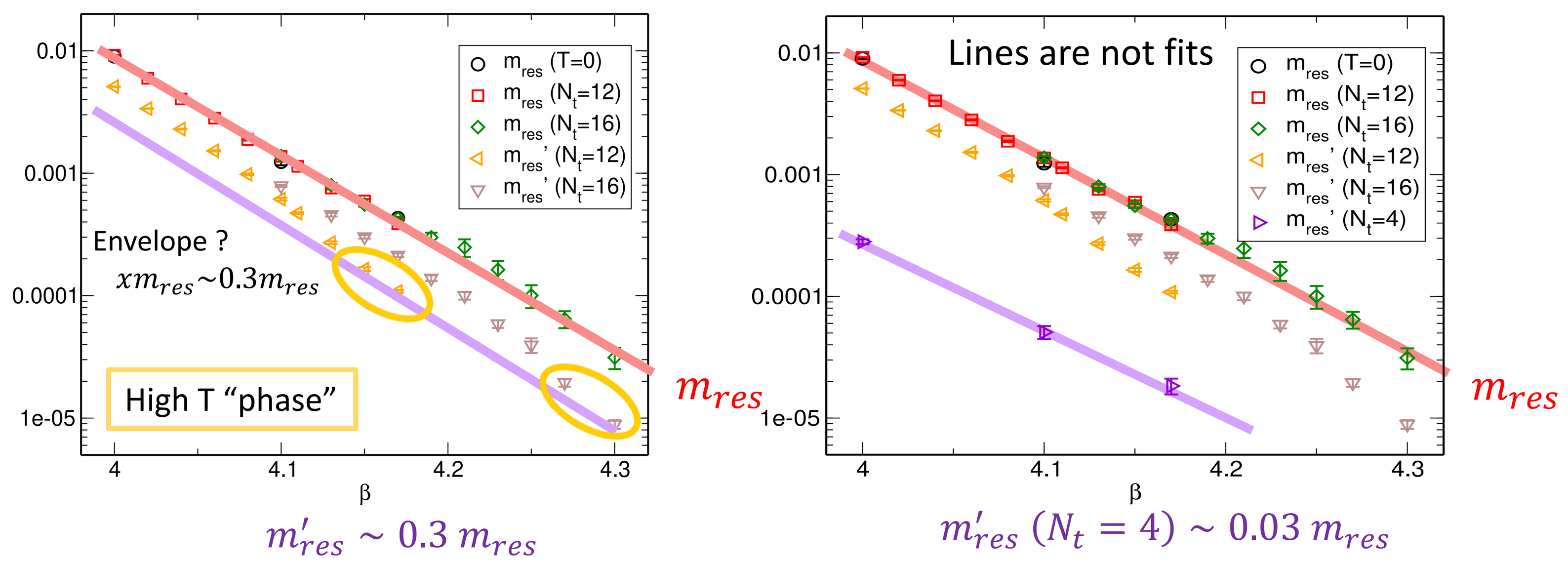
From: $\Delta_\mu \langle A_\mu(x) P(0)\rangle = 2m_f \langle P(x) P(0)\rangle + 2\langle J_{5q}(x) P(0)\rangle - 2\Sigma\,\delta_{x,0}$



Recipe:

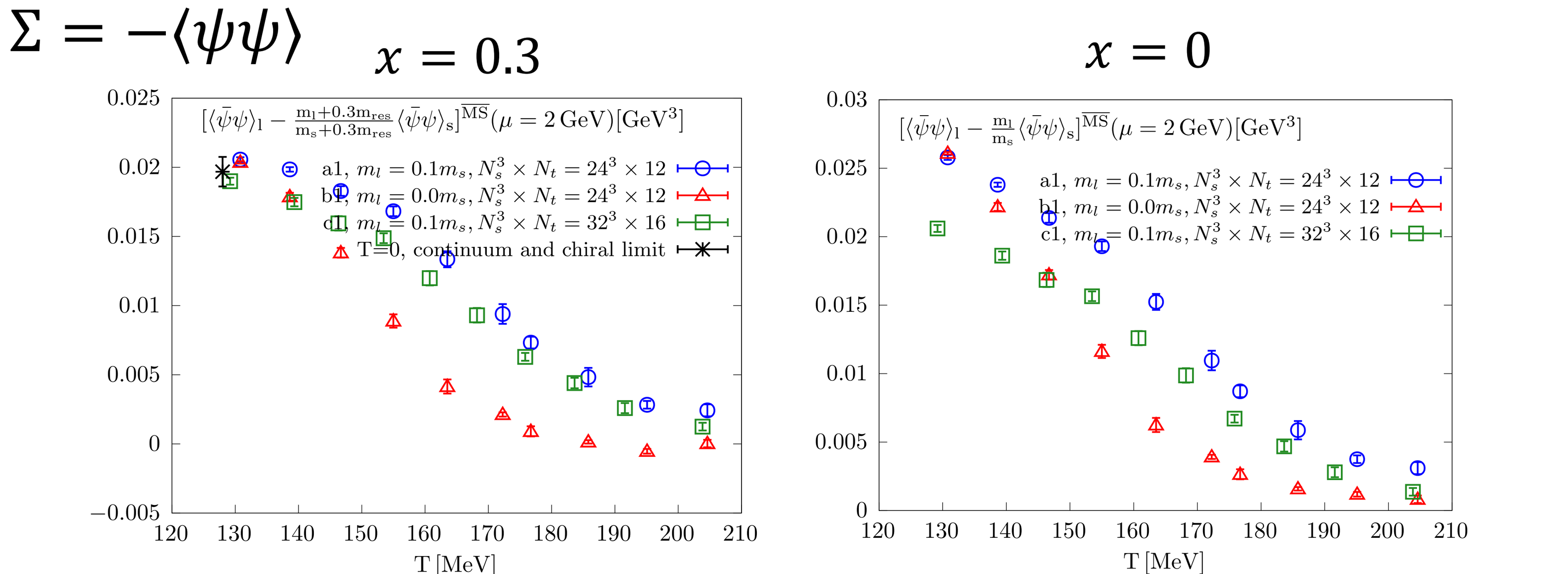
1. measure m'_{res} for fixed N_t; make sure high enough T
2. increase T (decrease N_t) to check stability
3. check stability against m_f
4. $\to x m_{res} = \lim_{T\to\infty} \lim_{m_f \to -m'_{res}} m_{res}'$
5. subtract $c(m_f + x m_{res})/a^2$ from $\Sigma = -\langle\bar\psi\psi\rangle$

m_{res} and m_{res}' for $N_f = 2+1$ (on LCP for $T>0$)



$\boxed{x < 0.03}$ is likely

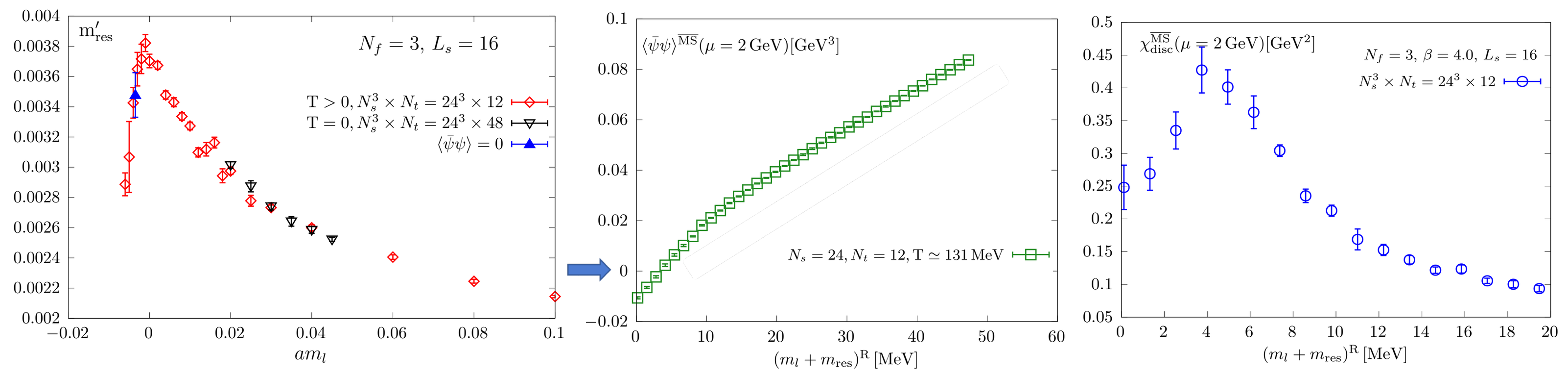
$\Sigma = -\langle\bar\psi\psi\rangle$



- Remark: quark mass tuning of simulation points wrt m_{res} yet to be done to ensure "Constant Physics"

m'_{res}: example in N_f=3 case

- Checked: $m_f = -m'_{res}$ indeed coincides with a zero of the chiral condensate

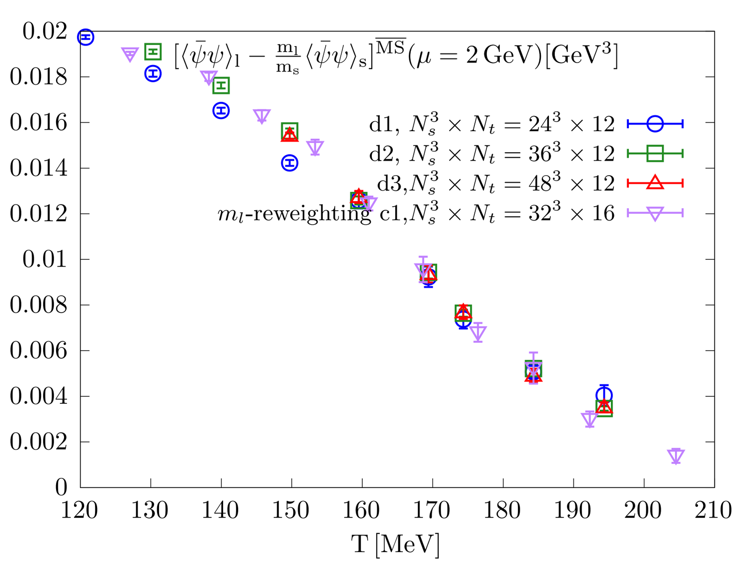


Lessons

- m'_{res} can be non-monotonic near (pseudo) critical point
- m_f dependence may be rather strong even in symmetric "phase"

Prelim. results of chiral condensate for $m_l = 0.1 m_s$

- After quark mass tuning of simulation points
- Assuming $x = 0$ (because we only know $x < 0.03$)
- Combining with quark mass tuning w or w/o reweighting



- d1($N_t = 12$) - c1 (16) : cutoff diff
  - ➢ Good continuum scaling
- d1, d2, d3: volume diff
  - ➢ No significant volume dep. except d1 (R=2)
- For more details, see the poster by J. Goswami

Summary

- m_{res} effects need to be addressed for $\Sigma = -\langle\bar\psi\psi\rangle$
- Promising economic procedure: utilization of m'_{res}

Outlook

- $m'_{res}(N_t, m_f)$ needs better understanding
- will give answer as to whether we can/should refine the subtraction scheme of chiral condensate
- More data (eg. more T-points, $N_t = 8$, on different LCP) will help

References

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6. Grid configured with --enable-simd=A64FX --enable-gen-simd_width=64
7. N. Meyer et al, PoS LATTICE2018 (2019) 316.
8. S. Sharpe arXiv: 0706.0218, "Future of Chiral Extrapolations with Domain Wall Fermions"
9. Y. Aoki et al (JLQCD), talk at Lattice 2022, "Thermodynamics with Möbius domain wall fermions near physical point I"