Parametrized quantum circuit for weight-adjustable quantum loop gas

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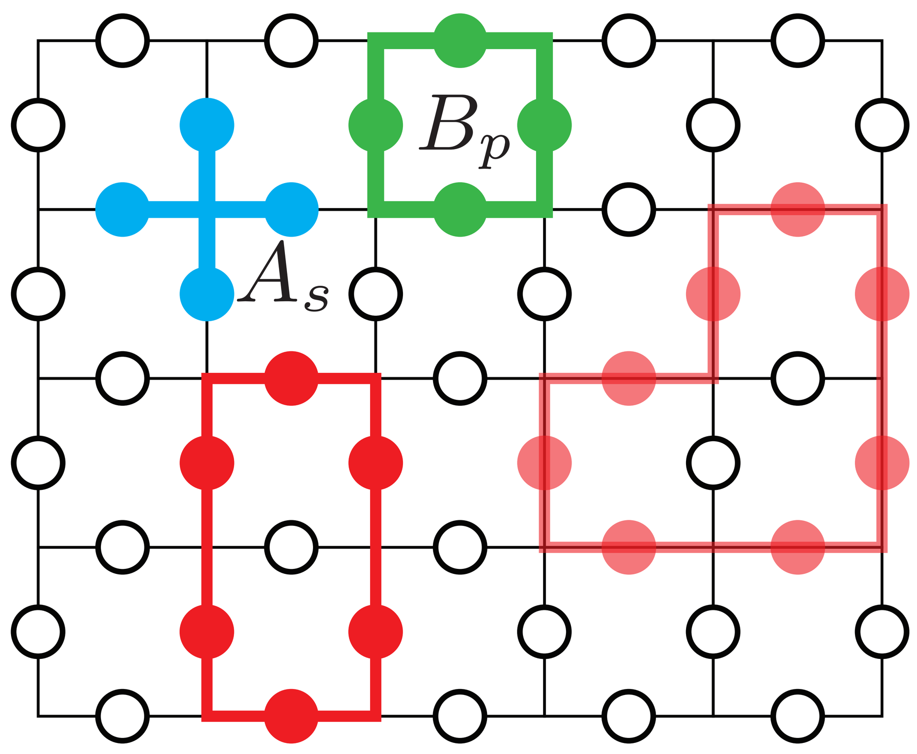
Motivated by the recent success of realizing the topologically ordered ground state of the exactly solvable toric code model by a quantum circuit on the real quantum device, here we propose a parametrized quantum circuit (PQC) with the same real-device-performable optimal structure to represent quantum loop gas states with adjustably weighted loop configurations. Combining such a PQC with the variational quantum eigensolver (VQE), we obtain the accurate quantum circuit representation for the toric code model in an external magnetic field with any field strength, where the system is not exactly solvable. The topological quantum phase transition in this system is further observed in the optimized circuits by measuring the magnetization and topological entanglement entropy (TEE).

Toric code model in a magnetic field



$\odot$ B

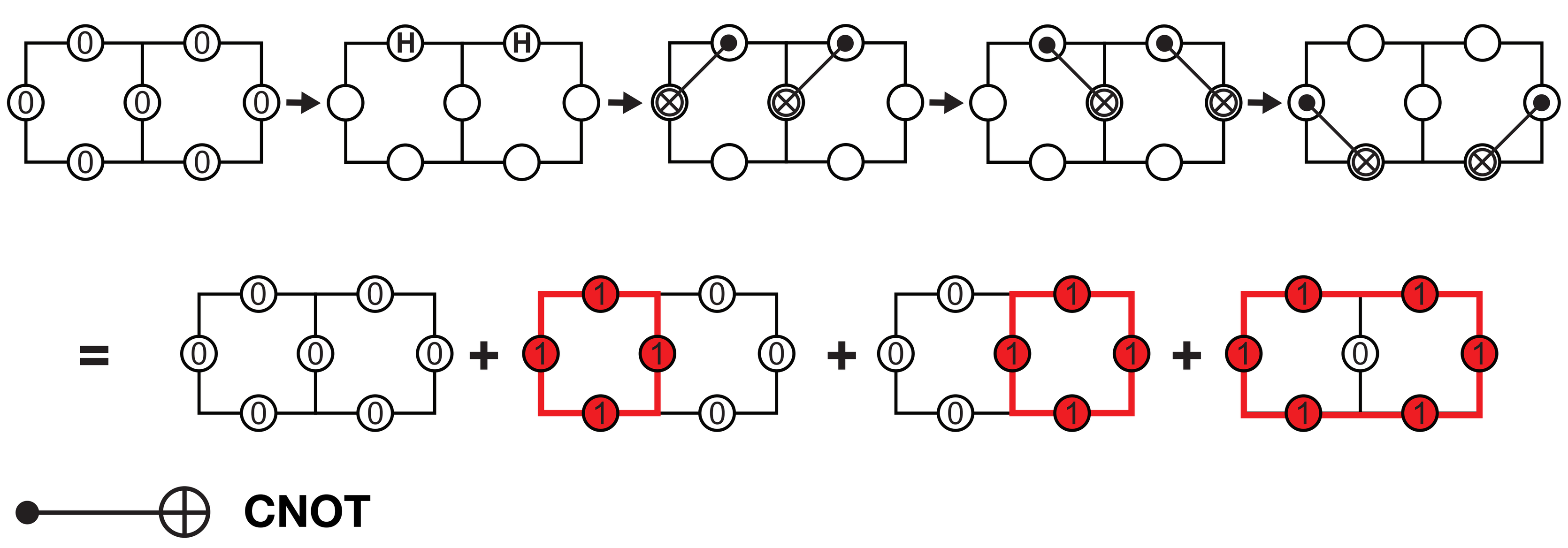
$$A_s = \prod_{i\in s}\sigma_i^z$$

$$B_p = \prod_{i\in p}\sigma_i^x$$

Toric code (TC) model

$$H_{\mathrm{TC}} = -\sum_s A_s - \sum_p B_p$$

- Exactly solvable
- Ground state
  - Intrinsic topological order / quantum loop gas
  - OBC: $|\Psi_0\rangle = \prod_{p=1}^{N_p}\left(\frac{1}{\sqrt{2}}I_p + \frac{1}{\sqrt{2}}B_p\right)|00\cdots0\rangle$
  - Realizable on real quantum devices:

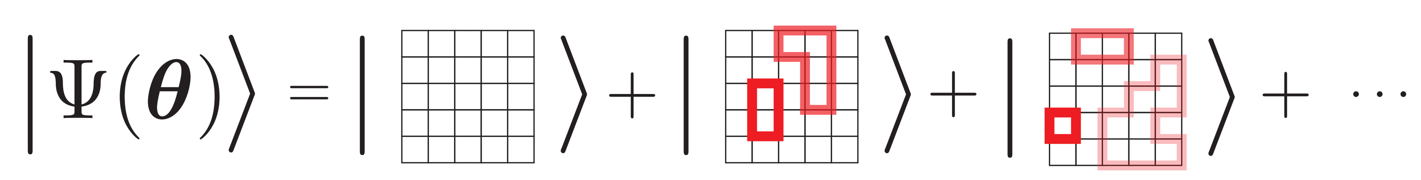


TC in a magnetic field

$$H_{\mathrm{TCM}}(x) = (1-x)H_{\mathrm{TC}} - x\sum_{i=1}^{N}\sigma_i^z$$

- Non-exactly solvable
- $|1\rangle_i$ loop with tension proportional to perimeter
- Topological order to ferromagnetic order: $x_c \sim 0.25$

Parametrized loop gas circuit (PLGC)

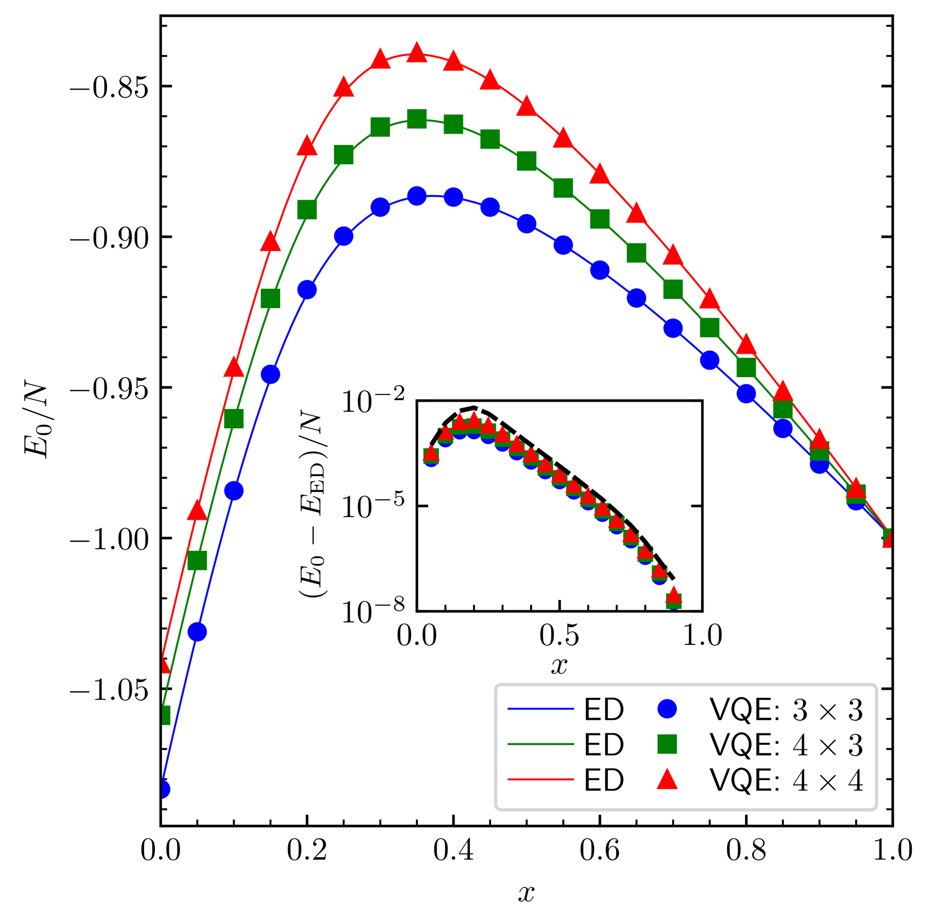


- $|\Psi(\boldsymbol{\theta})\rangle = \prod_{p=1}^{N_p}\left(\cos(\theta_p/2)I + \sin(\theta_p/2)B_p\right)|00\cdots0\rangle$
- Represent loop gas with loop tensions
- Realizable on real quantum devices:
  - By replacing Hadamard gate with Rotation-y gate

$$H|0\rangle_i = R_y(\pi/2)|0\rangle_i$$

Variational quantum simulation

- Calculate the ground state of $H_{\mathrm{TCM}}(x)$
  - Algorithm: VQE
  - Ansatz: PLGC



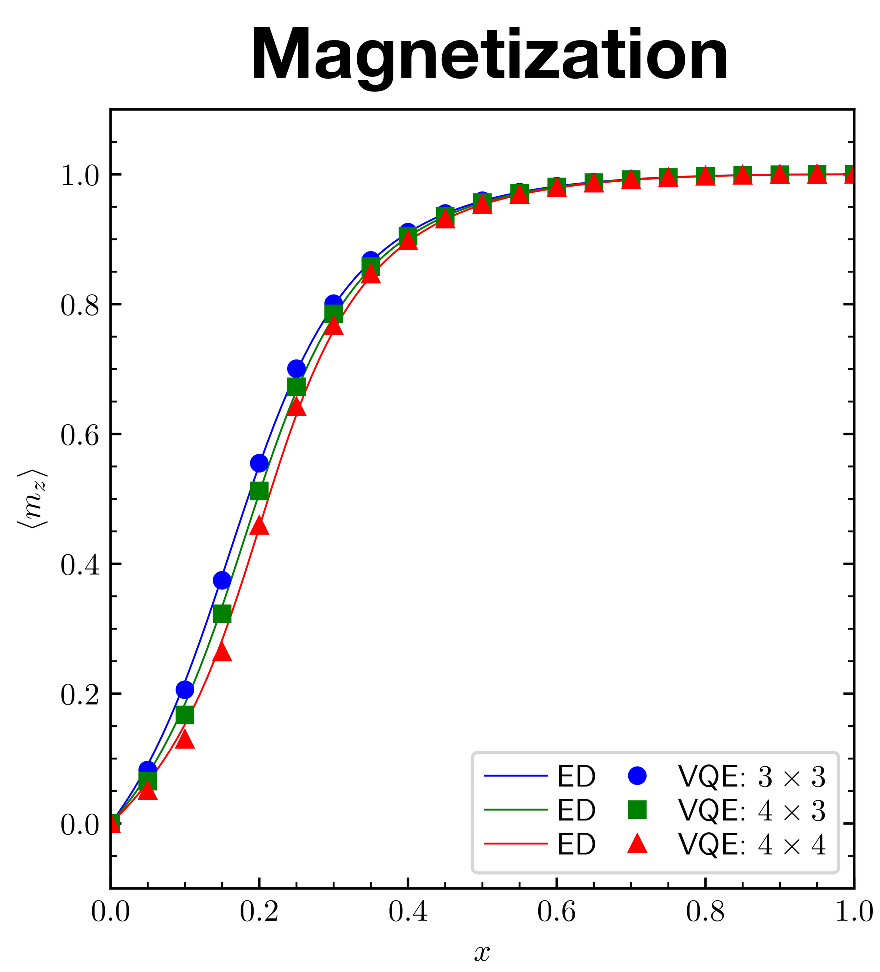
- Observables
  - Energy
  - Magnetization

$$\langle m_z\rangle = \frac{1}{N}\sum_{i=1}^{N}\langle\sigma_i^z\rangle$$

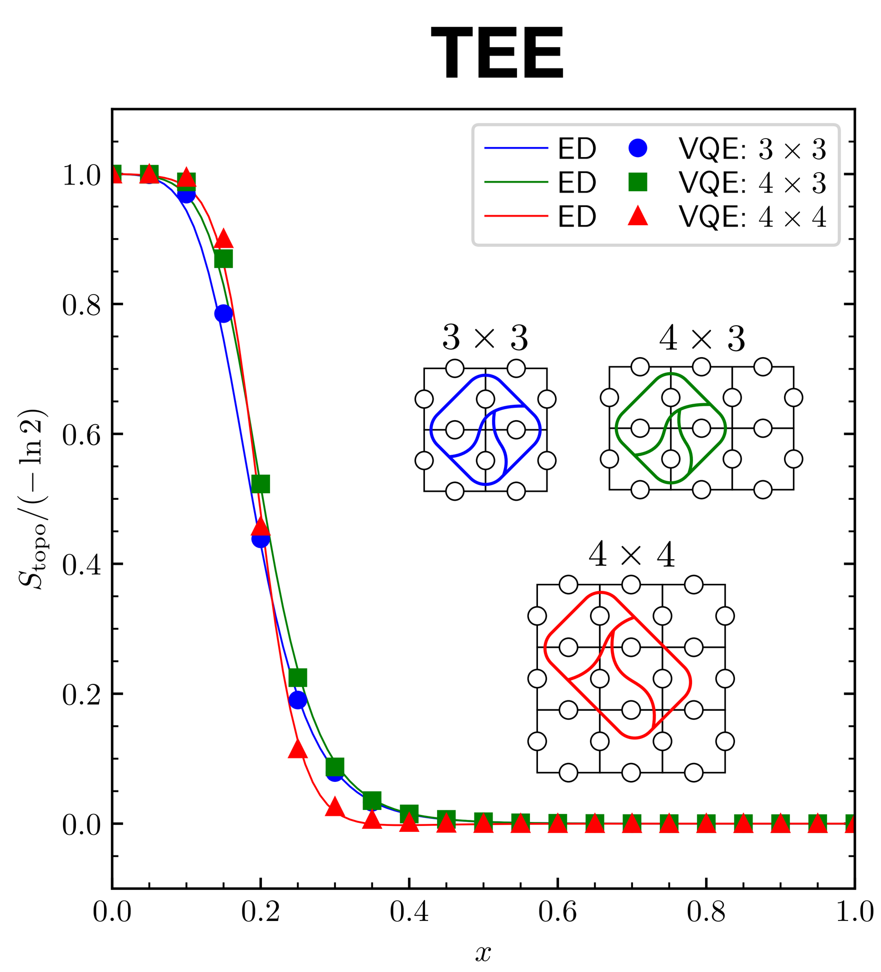
  - TEE

$$S_{\mathrm{topo}} = S_{\mathrm{A}} + S_{\mathrm{B}} + S_{\mathrm{C}} - S_{\mathrm{AB}} - S_{\mathrm{BC}} - S_{\mathrm{AC}} + S_{\mathrm{ABC}}$$

Magnetization



TEE



For more details: *PRB **107**, L041109 (2023)* arXiv: *2210.14662*