Collision terms with energy conservation in AMD and sJAM

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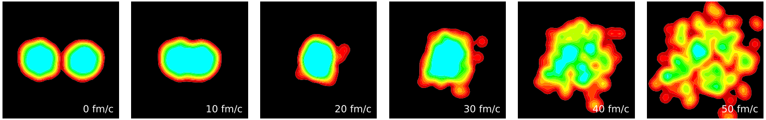
- Energy conservation $\delta(E_\mathrm{f} - E_\mathrm{i}) \Rightarrow \sigma(\boldsymbol{p}_1, \boldsymbol{p}_2, \text{environment})$ in AMD and sJAM
- Effects of the momentum dependence and symmetry energy
- Cluster observables and pion observables

Transport equation for heavy-ion collisions



Sn+Sn @300 MeV/u

One-body distribution function $f = \left\{f_\alpha; \alpha = p, n \text{ (and } \Delta^-, \Delta^0, \Delta^+, \Delta^{++}, \pi^-, \pi^0, \pi^+)\right\}$

$$f_\alpha(\boldsymbol{r}, \boldsymbol{p}, t) = \text{Wigner transform of} \sum_{i\in\alpha} \psi_i(\boldsymbol{r}, t)\psi_i^*(\boldsymbol{r}', t)$$

Transport equation (Boltzmann/BUU equation): e.g., Wolter et al., PPNP 125 (2022) 103962

$$\frac{\partial f_\alpha(\boldsymbol{r},\boldsymbol{p},t)}{\partial t} = \underbrace{\frac{\partial h_\alpha[f]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_\alpha}{\partial \boldsymbol{p}} - \frac{\partial h_\alpha[f]}{\partial \boldsymbol{p}} \cdot \frac{\partial f_\alpha}{\partial \boldsymbol{r}}}_{\text{mean-field propagation}} + \underbrace{\int |\boldsymbol{v}| \frac{d\sigma}{d\Omega}\{f_3 f_4 (1-f)(1-f_2) - f f_2 (1-f_3)(1-f_4)\} \frac{d\boldsymbol{p}_2 d\Omega}{(2\pi\hbar)^3}}_{\text{collision term}}$$

Single-particle Hamiltonian:

$$h_\alpha(\boldsymbol{r},\boldsymbol{p};f) = \frac{\boldsymbol{p}^2}{2m_\alpha} + U_\alpha(\boldsymbol{r},\boldsymbol{p};f) \qquad U_\alpha(\boldsymbol{r},\boldsymbol{p};f)\text{: Momentum-dependent mean-field potential}$$

$$\approx \frac{\boldsymbol{p}^2}{2m_\alpha^*(\boldsymbol{r};f)} + U_\alpha(\boldsymbol{r};f) \qquad m_\alpha^*(\boldsymbol{r};f)\text{: effective masses } m_n^* \text{ and } m_p^*$$

Potentials $U_\alpha(\boldsymbol{r},\boldsymbol{p})$ enter in both the mean-field propagation and the collision term (in principle).

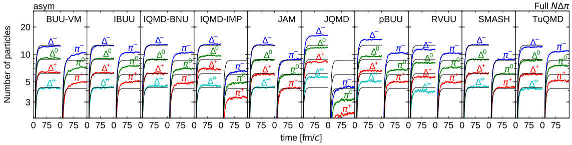
Collision term and equilibrium/EOS/composition

Boltzmann/BUU equation for heavy-ion collisions:

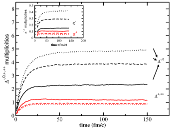
$$\frac{\partial f}{\partial t} = \underbrace{\frac{\partial h}{\partial \boldsymbol{r}}\cdot\frac{\partial f}{\partial \boldsymbol{p}} - \frac{\partial h}{\partial \boldsymbol{p}}\cdot\frac{\partial f}{\partial \boldsymbol{r}}}_{\text{mean-field propagation}} + \underbrace{\int |\boldsymbol{v}|\frac{d\sigma}{d\Omega}\{f_3 f_4(1-f)(1-f_2) - f f_2(1-f_3)(1-f_4)\}\frac{d\boldsymbol{p}_2 d\Omega}{(2\pi\hbar)^3}}_{\text{collision term, e.g., } nn\to nn,\ p\Delta^-,\ n\Delta^0}$$

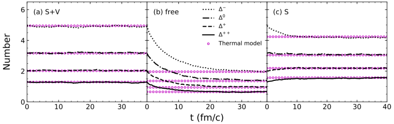
Tests of collision term in a box:

- TMEP comparison for $N\Delta\pi$ system without mean-field interaction
  Ono et al. (TMEP), PRC 100, 044617 (2019).



- RMF-type transport models with threshold effect, using cross section by Huber et al.
  - Ferini et al., NPA 762 (2005).
  - Zhen Zhang et al., PRC 97, 014610 (2018).





Collision term is essential for equilibrium ~ EOS

- Energy conservation under potentials
- Detailed balance
- Other problems in the treatment of collision term

For HIC dynamics

We need to know the cross section $\sigma(\boldsymbol{p}_1, \boldsymbol{p}_2; \text{env.})$ under the presence of potentials.

Collision term under potential

Boltzmann/BUU equation for heavy-ion collisions:

$$\frac{\partial f}{\partial t} = \underbrace{\frac{\partial h}{\partial \boldsymbol{r}} \cdot \frac{\partial f}{\partial \boldsymbol{p}} - \frac{\partial h}{\partial \boldsymbol{p}} \cdot \frac{\partial f}{\partial \boldsymbol{r}}}_{\text{mean-field propagation}} + \underbrace{\int |\boldsymbol{v}| \frac{d\sigma}{d\Omega} \{ f_3 f_4 (1-f)(1-f_2) - f f_2 (1-f_3)(1-f_4) \} \frac{d\boldsymbol{p}_2 d\Omega}{(2\pi\hbar)^3}}_{\text{collision term}}$$

Initial state:

$+\boldsymbol{p}_i$ → 1 ← $-\boldsymbol{p}_i$ 2

Final state:

$+\boldsymbol{p}_f$ ↗ 3

$-\boldsymbol{p}_f$ ↙ 4

Collision rate/cross-section: $\sigma(\boldsymbol{p}_1, \boldsymbol{p}_2; \text{environment})$ for $1 + 2 \to 3 + 4$

$$v_i \frac{d\sigma}{d\Omega} \propto \int |M|^2 \delta(E_f - E_i) p_f^2 dp_f = |M|^2 \frac{p_f^2}{dE_f/dp_f} \qquad \therefore \quad \frac{d\sigma}{d\Omega} = \left(\frac{p_i}{v_i} \frac{p_f}{v_f} \right) \times |M|^2 \times \frac{p_f}{p_i}$$

Potentials at the space-time point of the collision enter in the energies:

$$E_i = \sqrt{m_1^2 + p_1^2} + U_1 + \sqrt{m_2^2 + p_2^2} + U_2, \quad E_f = \sqrt{m_3^2 + p_3^2} + U_3 + \sqrt{m_4^2 + p_4^2} + U_4$$

We must solve

$$E_f(\boldsymbol{p}_f) = E_i$$

- $\mu_i^* = p_i/v_i$ and $\mu_f^* = p_f/v_f$ are the effective reduced masses in the initial and final states. They are *reduced* by the momentum dependence of the mean field.
- We assume the matrix element $|M|^2$ is not strongly affected by the potential.
- The final momentum factor p_f is determined by the energy conservation, which can be strongly affected by the potential. E.g, the threshold of $NN \to N\Delta$ (endothermic reaction) is determined by $p_f = 0$.

Energy density and potential in our model (AMD + sJAM)

Interaction energy density: ~ Skyrme

$$\mathcal{E}_{\rm int}(\mathbf{r}) = \sum_{\alpha\beta}\Big\{U^{t_0}_{\alpha\beta}\rho_\alpha(\mathbf{r})\rho_\beta(\mathbf{r}) + U^{t_3}_{\alpha\beta}\rho_\alpha(\mathbf{r})\rho_\beta(\mathbf{r})[\rho(\mathbf{r})]^\gamma + U^{\tau}_{\alpha\beta}\tilde{\tau}_\alpha(\mathbf{r})\rho_\beta(\mathbf{r}) + U^{\nabla}_{\alpha\beta}\nabla\rho_\alpha(\mathbf{r})\cdot\nabla\rho_\beta(\mathbf{r})\Big\}$$

Densities:

$$\rho_\alpha(\mathbf{r}) = \int\frac{d\mathbf{p}}{(2\pi\hbar)^3}f_\alpha(\mathbf{r},\mathbf{p}),\quad \tilde{\tau}_\alpha(\mathbf{r}) = \int\frac{d\mathbf{p}}{(2\pi\hbar)^3}\frac{[\mathbf{p}-\bar{\mathbf{p}}(\mathbf{r})]^2}{1+[\mathbf{p}-\bar{\mathbf{p}}(\mathbf{r})]^2/\Lambda_{\rm md}^2}f_\alpha(\mathbf{r},\mathbf{p}),\quad \bar{\mathbf{p}}(\mathbf{r}) = \frac{1}{\sum_\alpha\rho_\alpha(\mathbf{r})}\sum_\alpha\int\frac{d\mathbf{p}}{(2\pi\hbar)^3}\mathbf{p}f_\alpha(\mathbf{r},\mathbf{p})$$

Mean-field potential (in AMD):

$$U_\alpha(\mathbf{r},\mathbf{p}) = (2\pi\hbar)^3\frac{\delta}{\delta f_\alpha(\mathbf{r},\mathbf{p})}\int\mathcal{E}_{\rm int}(\mathbf{r})d\mathbf{r} = \frac{A_\alpha(\mathbf{r})[\mathbf{p}-\bar{\mathbf{p}}(\mathbf{r})]^2}{1+[\mathbf{p}-\bar{\mathbf{p}}(\mathbf{r})]^2/\Lambda_{\rm md}^2} + \tilde{C}_\alpha(\mathbf{r})$$

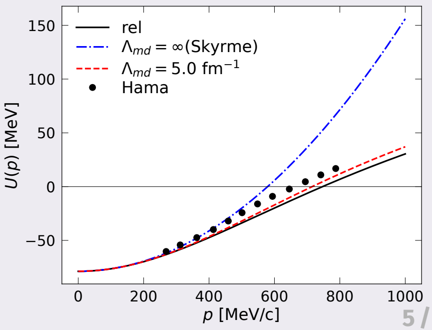
Relativistic version (in sJAM):

$$U_\alpha(\mathbf{r},\mathbf{p}) = \sqrt{[m_\alpha+\Sigma^s_\alpha(\mathbf{r})]^2+[\mathbf{p}-\boldsymbol{\Sigma}_\alpha(\mathbf{r})]^2} + \Sigma^0_\alpha(\mathbf{r}) - \sqrt{m_\alpha^2+\mathbf{p}^2}$$

- The p^2 dependence of Skyrme is modified by $\Lambda_{\rm md} = 5\ {\rm fm}^{-1}$.
  (Similar to Gale, Bertsch, Das Gupta, PRC 35, 1666 (1987))
- Similar momentum dependence in the relativistic version.

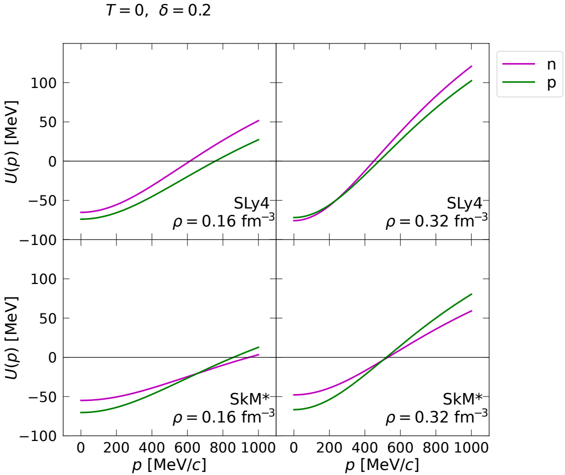
Potential in nuclear matter

$\rho = \rho_0,\ \rho_n = \rho_p,\ T = 0$



Nucleon potentials in nuclear matter, based on SLy4 and SkM*

In the zero temperature nuclear matter with the isospin asymmetry $\rho_n/\rho_p = 3/2$ ($\delta = 0.2$)



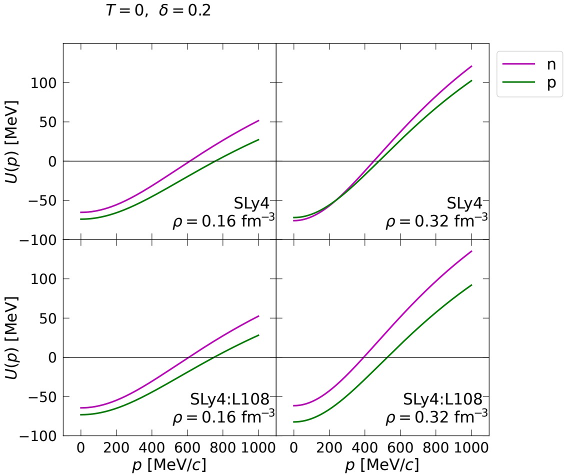
Similar EOS for SLy4 and SkM*

	SLy4	SkM*	
ρ_0	0.160	0.160	fm^{-3}
E_0	−15.97	−15.77	MeV
K	230	217	MeV
m^*/m	0.69	0.79	
S_0	32.0	30.0	MeV
L	46	46	MeV
$(m_n^* - m_p^*)/m$	−0.18	+0.33	δ
	$m_n^* < m_p^*$	$m_n^* > m_p^*$	in n-rich

But they are different in the momentum dependence of the symmetry potential:

$$U_{\text{sym}}(p) = [U_n(p) - U_p(p)]/2\delta$$

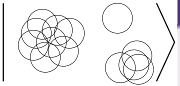
In the zero temperature nuclear matter with the isospin asymmetry $\rho_n/\rho_p = 3/2$ ($\delta = 0.2$)



Similar EOS for SLy4 and SLy4:L108

	SLy4	SkM*	
ρ_0	0.160	0.160	fm^{-3}
E_0	−15.97	−15.97	MeV
K	230	230	MeV
m^*/m	0.69	0.69	
S_0	32.0	32.0	MeV
L	46	108	MeV
$(m_n^* - m_p^*)/m$	−0.18	−0.18	δ
	$m_n^* < m_p^*$	$m_n^* < m_p^*$	in n-rich

But they are different in the density dependence of the symmetry energy (L).



AMD wave function

$$|\Phi(Z)\rangle = \det_{ij}\left[\exp\left\{-\nu\left(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\right)^2\right\}\chi_{\alpha_i}(j)\right]$$

$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = $(2.5\ \text{fm})^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Equation of motion for the wave packet centroids Z

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}} \quad + \quad \text{(NN collisions)}$$

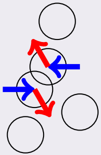
$\{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}}$: Motion in the mean field

$$\mathcal{H} = \frac{\langle\Phi(Z)|H|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle} + \text{(c.m. correction)}$$

H: Effective interaction (e.g. Skyrme force)

NN collisions

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$

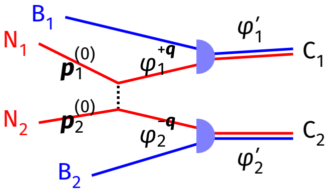


- $|V|^2$ or σ_{NN} (in medium)
- Pauli blocking

Ono, Horiuchi, Maruyama, Ohnishi, Prog. Theor. Phys. 87 (1992) 1185.

$$N_1 + B_1 \; + \; N_2 + B_2 \;\to C_1 + C_2$$

- N_1, N_2 : Colliding nucleons
- B_1, B_2 : Spectator nucleons/clusters (maybe empty)
- C_1, C_2 : N, $(2N)$, $(3N)$, $(4N)$ (up to α cluster)



Transition probability

$$W(\mathrm{NBNB} \to \mathrm{CC}) = \frac{2\pi}{\hbar} |\langle \mathrm{CC}|V|\mathrm{NBNB}\rangle|^2 \delta(E_\mathrm{f} - E_\mathrm{i})$$

$$v_\mathrm{i} d\sigma \propto |\langle \varphi_1' | \varphi_1^{+\boldsymbol{q}}\rangle|^2 |\langle \varphi_2' | \varphi_2^{-\boldsymbol{q}}\rangle|^2 |M|^2 \delta(E_\mathrm{f} - E_\mathrm{i}) p_{\mathrm{f,rel}}^2 dp_{\mathrm{f,rel}} d\Omega$$

$$\boldsymbol{p}_\mathrm{rel} = \tfrac{1}{2}(\boldsymbol{p}_1 - \boldsymbol{p}_2) = p_\mathrm{rel}\hat{\boldsymbol{\Omega}}$$

$$\boldsymbol{p}_1 = \boldsymbol{p}_1^{(0)} + \boldsymbol{q}$$

$$\boldsymbol{p}_2 = \boldsymbol{p}_2^{(0)} - \boldsymbol{q}$$

$$\varphi_1^{+\boldsymbol{q}} = \exp(+i\boldsymbol{q}\cdot\boldsymbol{r}_{\mathrm{N}_1})\varphi_1^{(0)}$$

$$\varphi_2^{-\boldsymbol{q}} = \exp(-i\boldsymbol{q}\cdot\boldsymbol{r}_{\mathrm{N}_2})\varphi_2^{(0)}$$

$$\frac{d\sigma_{\mathrm{C}_1\mathrm{C}_2}}{d\Omega} = P(\mathrm{C}_1\mathrm{C}_2, p_{\mathrm{f,rel}}, \Omega) \times \left(\frac{p_{\mathrm{i,rel}}}{v_\mathrm{i}} \frac{p_{\mathrm{f,rel}}}{v_\mathrm{f}}\right) \times \left|M(p_{\mathrm{i,rel}}, p_{\mathrm{f,rel}}, \Omega)\right|^2 \times \frac{p_{\mathrm{f,rel}}}{p_{\mathrm{i,rel}}}$$

$E_\mathrm{f}(p_\mathrm{f}) = E_\mathrm{i}$ and $v_\mathrm{f} = \dfrac{dE_\mathrm{f}}{dp_\mathrm{f}}$ are solved numerically with

$$E_\mathrm{i},\; E_\mathrm{f}(p_\mathrm{f}) = \frac{\langle\Phi|H|\Phi\rangle}{\langle\Phi|\Phi\rangle} = \text{hamil\_calculate\_eng(amd)}$$

Ono, J. Phys. Conf. Ser. 420 (2013) 012103.
Ikeno, Ono et al., PRC 93 (2016) 044612.
Ono, JPS Conf. Proc. 32 (2020) 010076.

Cross section of cluster production

$N_1 + B_1 + N_2 + B_2 \rightarrow C_1 + C_2$

$$\frac{d\sigma_{C_1C_2}}{d\Omega} = P(C_1C_2, p_{f,rel}, \Omega) \times \left(\frac{p_{i,rel}}{v_i}\frac{p_{f,rel}}{v_f}\right) \times \left|M(p_{i,rel}, p_{f,rel}, \Omega)\right|^2 \times \frac{p_{f,rel}}{p_{i,rel}}$$

- Phase space factors are determined after energy conservation.
- The matrix element $|M|^2$ (or the corresponding NN cross section) may depend on a kind of density ρ'.

$$\sigma_{NN}(\rho', \epsilon) = \sigma_0 \tanh\left(\sigma_{free}(\epsilon)/\sigma_0\right), \qquad \sigma_0 = 0.5 \times (\rho')^{-2/3} \qquad \text{(formula similar to P. Danielewicz)}$$

- Cluster formation may be switched off when $\rho' > \rho_c$. (e.g., $\rho_c = 0.125\ \mathrm{fm}^{-3}$)

Density ρ' with a momentum cut:

$$\rho_i'^{(ini/fin)} = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{2}} \sum_{k(\neq i)} \theta\left(p_{cut} > |\boldsymbol{P}_i^{(ini/fin)} - \boldsymbol{P}_k|\right) e^{-2\nu(\boldsymbol{R}_i - \boldsymbol{R}_k)^2}$$

An energy-dependent momentum cut was chosen, $p_{cut} = (375\ \mathrm{MeV}/c)\, e^{-\epsilon/(225\ \mathrm{MeV})}$, where ϵ is the collision energy (i.e. the sum of the kinetic energies of N_1 and N_2 in their c.m. frame).

Cross section of cluster production

$N_1 + B_1 + N_2 + B_2 \rightarrow C_1 + C_2$

$$\frac{d\sigma_{C_1C_2}}{d\Omega} = P(C_1C_2, p_{f,rel}, \Omega) \times \left(\frac{p_{i,rel}}{v_i}\frac{p_{f,rel}}{v_f}\right) \times \left|M(p_{i,rel}, p_{f,rel}, \Omega)\right|^2 \times \frac{p_{f,rel}}{p_{i,rel}}$$

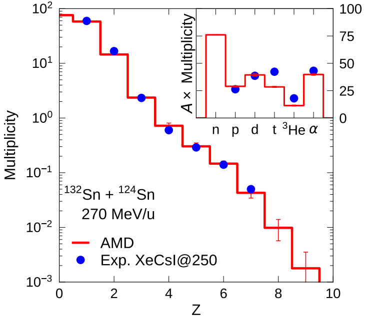
- Gaussian width $\nu_{cl} = 0.24\ \mathrm{fm}^{-2}$ for the overlap factors.
- There are a huge number of final cluster configurations (C_1, C_2).

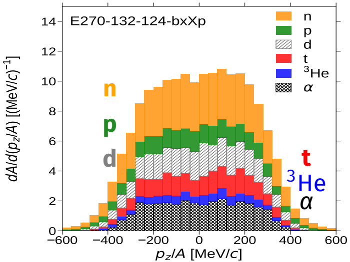
$$\sum_{C_1C_2} P(C_1C_2, p_{f,rel}, \Omega) = 1 \quad \text{for any fixed } (p_{f,rel}, \Omega)$$

- The energy-conserving final momentum $p_{f,rel} = p_{f,rel}(C_1C_2, \Omega)$ depends on the cluster configuration. When cluster(s) are formed, $p_{f,rel}$ tends to be large, and the effect of collisions will increase.
  - the phase space factor ↑
  - Pauli blocking ↓ (collision probability ↑)
  - momentum transfer ↑

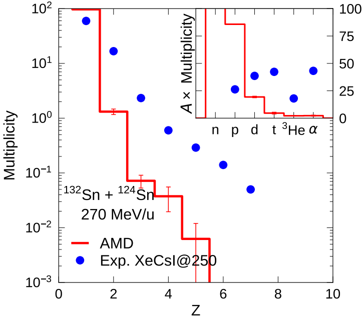
Effects of clusters in Xe + CsI collisions at 250 MeV/u

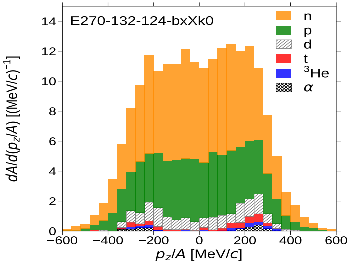
Cluster(0.125), $|M|^2_{\text{in-medium}}$





Cluster(0), $|M|^2_{\text{in-medium}}$





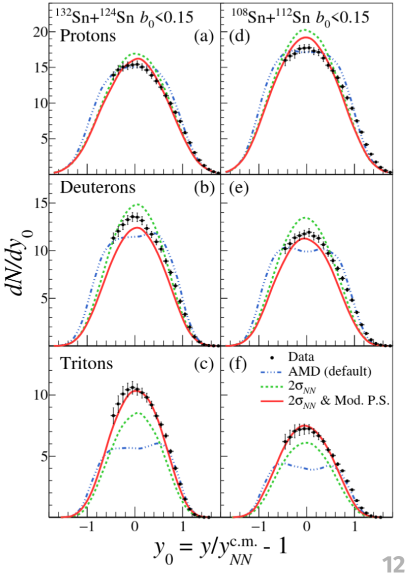
M. Kaneko, Murakami, Isobe, Kurata-Nishimura, Ono, Ikeno et al. (SπRIT), PLB 822 (2021) 136681.

neutron rich vs. neutron deficient

Rapidity distributions

for *p*, *d* and *t*, in the central collisions of

- ^{132}Sn + ^{124}Sn at 270 MeV/nucleon
- ^{108}Sn + ^{112}Sn at 270 MeV/nucleon

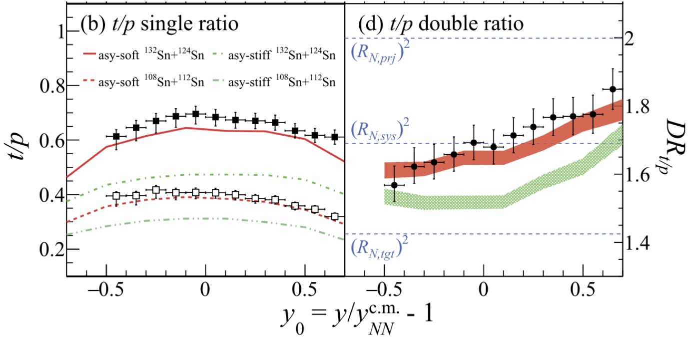


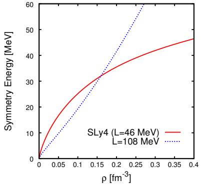
- Black points: SπRIT data
- Blue lines ("σ_{NN}"):
  AMD with a standard choice of $|M|^2_{\text{in-medium}}$.

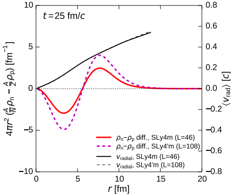
$$\sigma_{C_1C_2} = P_{C_1C_2} \times \left(\frac{p_i}{v_i}\frac{p_f}{v_f}\right) \times |M|^2_{\text{in-medium}} \times \frac{p_f}{p_i}$$

- Red and green lines ("$2\sigma_{NN}$"):
  AMD with $2 \times |M|^2_{\text{in-medium}}$.

t/p ratio and its implication on the symmetry energy

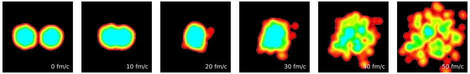




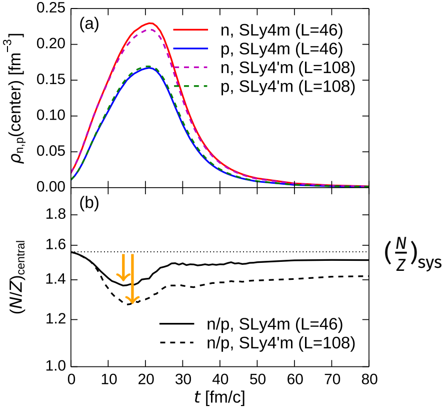


$$\frac{t/p \text{ in } {}^{132}\text{Sn} + {}^{124}\text{Sn}}{t/p \text{ in } {}^{108}\text{Sn} + {}^{112}\text{Sn}} = (t/p \text{ double ratio})$$

- In AMD calculation, the t/p ratio in the neutron-rich system is high for the asy-soft symmetry energy (L=46 MeV) compared to the asy-stiff symmetry energy (L=108 MeV).
- The above trend is consistent with the $\rho_n - \rho_p$ difference in the central region at compression.
- The SπRIT data (black points) favor SLy4 (L = 46 MeV) rather than SLy4:L108 (L = 108 MeV).
- The t/p double ratio is close to $(R_{N,sys})^2 = (N/N)^2$ (accidentally or for some reason?).
- Stopping and triton production need to be understood better systematically.



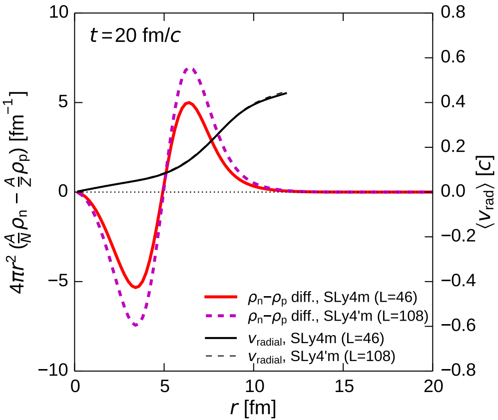
^{132}Sn + ^{124}Sn, E/A = 300 MeV, b ~ 0



- Neutron-proton density diff. (fn of r)

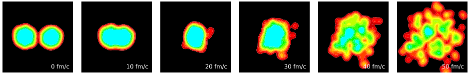
$$4\pi r^2\left[\frac{A}{N}\rho_n(r) - \frac{A}{Z}\rho_p(r)\right]$$

- Radial expansion velocity $v_{\mathrm{rad}}(r)$



Momentum dependence of Skyrme (SLy4) interaction has been corrected for high energy collisions, in a similar way to Gale, Bertsch, Das Gupta, PRC 35 (1987) 1666.

Compression and expansion in collisions at 300 MeV/nucleon



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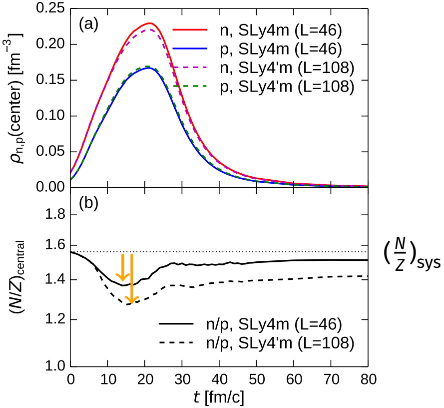

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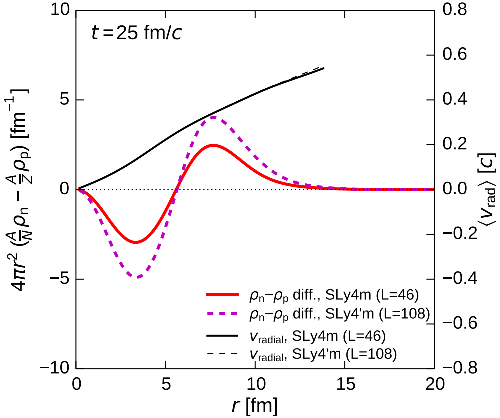
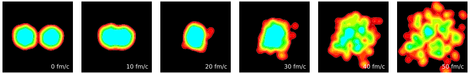
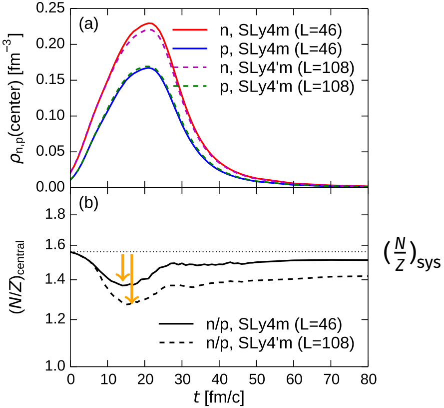
- Radial expansion velocity $v_{rad}(r)$



Compression and expansion in collisions at 300 MeV/nucleon



^{132}Sn + ^{124}Sn, E/A = 300 MeV, b ~ 0

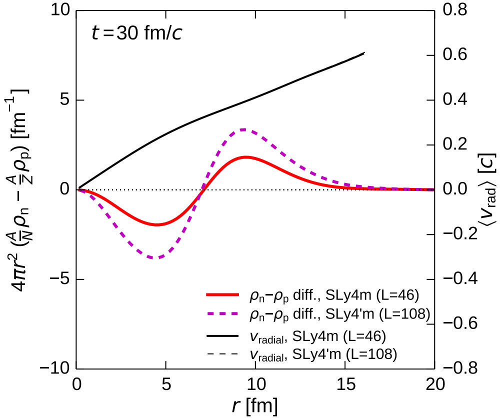


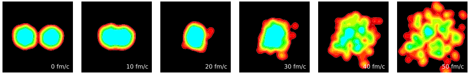
Momentum dependence of Skyrme (SLy4) interaction has been corrected for high energy collisions, in a similar way to Gale, Bertsch, Das Gupta, PRC 35 (1987) 1666.

- Neutron-proton density diff. (fn of r)

$$4\pi r^2\left[\frac{A}{N}\rho_n(r) - \frac{A}{Z}\rho_p(r)\right]$$

- Radial expansion velocity $v_{\rm rad}(r)$





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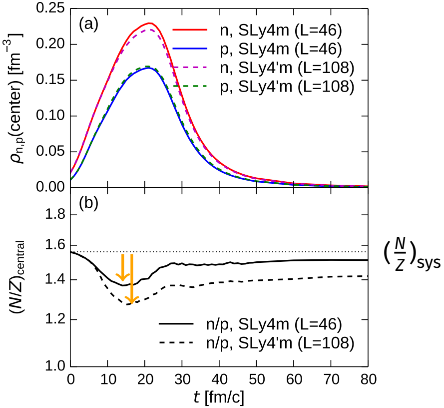

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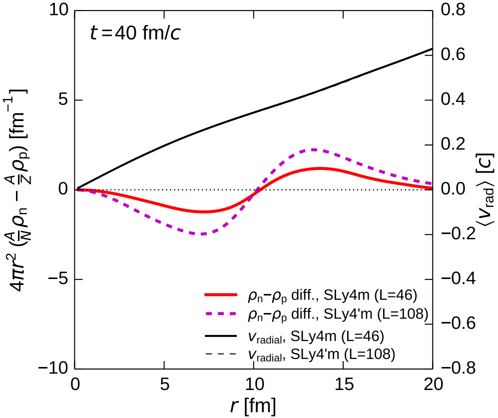

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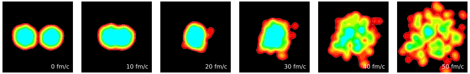

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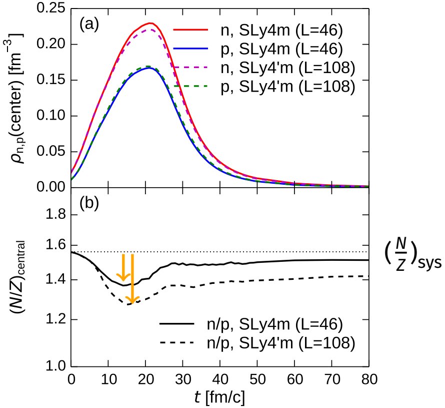

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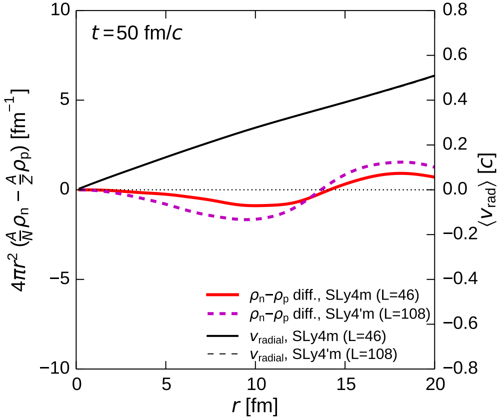
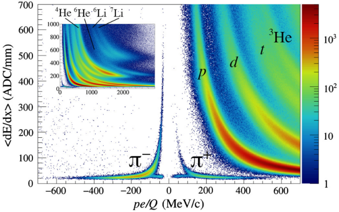


TMEP prediction of π^-/π^+ and comparison with the SπRIT data

G. Jhang et al. (SπRIT & TMEP), PLB 813 (2021) 136016.

SπRIT data at 270*A* MeV

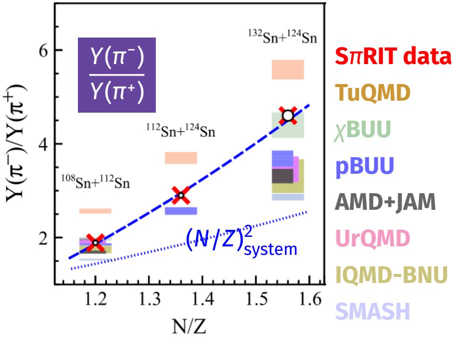


Pion yields and ratios from

- $^{132}Sn + ^{124}Sn$
- $^{112}Sn + ^{124}Sn$
- $^{108}Sn + ^{112}Sn$

J. Estee et al., PRL 126, 162701 (2021)

TMEP: "HIC prediction homework"



Height of each box: difference of soft and stiff symmetry energy

The predictions of π^-/π^+ by most transport models (including our AMD+JAM model) were lower than the SπRIT data.

How to solve this?
⇒ Treatment of the momentum dependence of the mean field.

AMD + sJAM

AMD wave function

$$|\Phi_{\rm AMD}\rangle = \det_{ij}\Big[\exp^{-\nu(\mathbf{r}_j-\frac{\mathbf{Z}_i(t)}{\sqrt{\nu}})^2}\chi_{\alpha_i}(j)\Big]$$

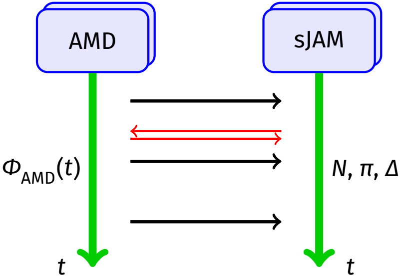
Effective interaction:

SLy4, SkM* etc. with $\Lambda_{\rm md}$

Collision term with clusters:

$N_1 + N_2 + B_1 + B_2 \to C_1 + C_2$

$$\frac{d\sigma}{d\Omega} = P(C_1, C_2)\Big(\frac{p_{\rm i}}{v_{\rm i}}\frac{p_{\rm f}}{v_{\rm f}}\Big)|M|^2\frac{p_{\rm f}}{p_{\rm i}}$$



Collisions including ...

- $NN \leftrightarrow NN$
- $NN \leftrightarrow N\Delta$
- $\Delta \leftrightarrow N\Delta$

E.g.,

$$\frac{d\sigma_{NN\to N\Delta}}{dm_\Delta} \propto f_{\rm i} f_{\rm f}|M|^2\frac{p_{\rm f}}{p_{\rm i}}A_\Delta(m_\Delta)$$

is calculated as a function of $(\boldsymbol{p}_1, \boldsymbol{p}_2;$ environment) for every possible collision.

The JAM code (without pot.) in the AMD+JAM model has been replaced by the new sJAM code (with pot.).

AMD+JAM: Ikeno, Ono, Nara, Ohnishi, PRC 93 (2016) 044612; Ikeno, Ono, Nara, Ohnishi, PRC 101 (2020) 034607.

$$\sigma(\boldsymbol{p}_1, \boldsymbol{p}_2; \text{environment}) \propto \left(\frac{p_\mathrm{i}}{v_\mathrm{i}}\frac{p_\mathrm{f}}{v_\mathrm{f}}\right)|M|^2\frac{p_\mathrm{f}}{p_\mathrm{i}}$$

Initial state:

$+\boldsymbol{p}_\mathrm{i}$ → 1 ← 2 $-\boldsymbol{p}_\mathrm{i}$

Final state:

$+\boldsymbol{p}_\mathrm{f}$ ↗ 3
$-\boldsymbol{p}_\mathrm{f}$ ↙ 4

AMD

$E_\mathrm{f}(p_\mathrm{f}) = E_\mathrm{i}$ and $v_\mathrm{f} = \dfrac{dE_\mathrm{f}}{dp_\mathrm{f}}$ are solved numerically with $E_\mathrm{i},\ E_\mathrm{f}(p_\mathrm{f}) = \dfrac{\langle\Phi|H|\Phi\rangle}{\langle\Phi|\Phi\rangle}$ = hamil_calculate_eng(amd)

sJAM

At each time t,
at the positions $\boldsymbol{r}_i$ of
all particles i,
we have to get
potentials.

$$\begin{pmatrix}
\Sigma^s_n(\boldsymbol{r}_i),\ \Sigma^0_n(\boldsymbol{r}_i),\ \boldsymbol{\Sigma}_n(\boldsymbol{r}_i) \\
\Sigma^s_p(\boldsymbol{r}_i),\ \Sigma^0_p(\boldsymbol{r}_i),\ \boldsymbol{\Sigma}_p(\boldsymbol{r}_i) \\
\Sigma^s_{\Delta^-}(\boldsymbol{r}_i),\ \Sigma^0_{\Delta^-}(\boldsymbol{r}_i),\ \boldsymbol{\Sigma}_{\Delta^-}(\boldsymbol{r}_i) \\
\Sigma^s_{\Delta^0}(\boldsymbol{r}_i),\ \Sigma^0_{\Delta^0}(\boldsymbol{r}_i),\ \boldsymbol{\Sigma}_{\Delta^0}(\boldsymbol{r}_i) \\
\Sigma^s_{\Delta^+}(\boldsymbol{r}_i),\ \Sigma^0_{\Delta^+}(\boldsymbol{r}_i),\ \boldsymbol{\Sigma}_{\Delta^+}(\boldsymbol{r}_i) \\
\Sigma^s_{\Delta^{++}}(\boldsymbol{r}_i),\ \Sigma^0_{\Delta^{++}}(\boldsymbol{r}_i),\ \boldsymbol{\Sigma}_{\Delta^{++}}(\boldsymbol{r}_i)
\end{pmatrix}$$

for $i = 1, 2, \ldots, A$.

⇒

Cross sections for each pair ij:

- $\sigma_{ij}(nn \to nn)$
- $\sigma_{ij}(nn \to p\Delta^-)$ or closed
- $\sigma_{ij}(nn \to n\Delta^0)$ or closed

Total cross section:

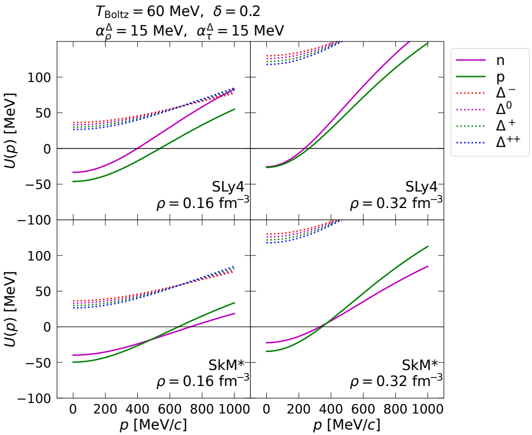
$$\sigma^\mathrm{tot}_{ij} = \sum_\mathrm{open} \sigma_{ij}(nn \to \text{final channel})$$

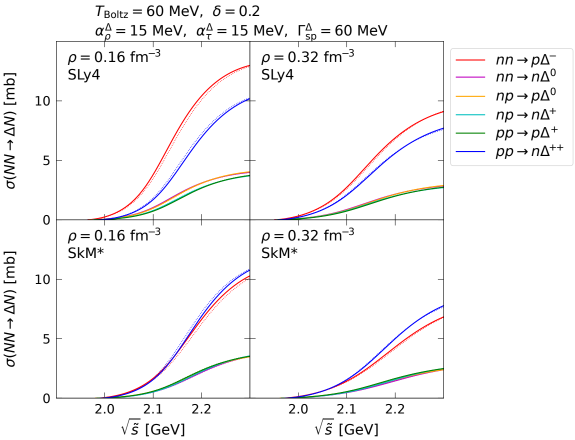
⇒

If $\pi d^2_{ij} < \sigma^\mathrm{tot}_{ij}$,
then i and j
may collide
...

An example: $NN \to N\Delta$ cross sections in neutron-rich nuclear matter

Production of a Δ resonance by a collision of two nucleons with momenta $+\boldsymbol{p}_N$ and $-\boldsymbol{p}_N$ in the nuclear matter, as a function of $\sqrt{\tilde{s}} = 2\sqrt{m_N^2 + \boldsymbol{p}_N^2}$.



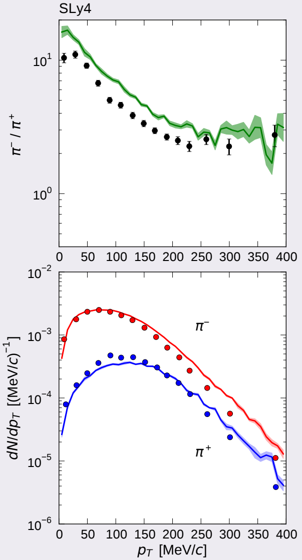


$$\sigma \sim p_f \sim \sqrt{\epsilon^*} \quad \text{with} \quad \epsilon^* = \underbrace{2\sqrt{m_N^2 + p_N^2} - m_N - m_\Delta}_{\text{same as in vaccum}} + \underbrace{U_1(p_N) + U_2(p_N) - U_3(0) - U_\Delta(0)}_{\text{effect of potentials}}$$

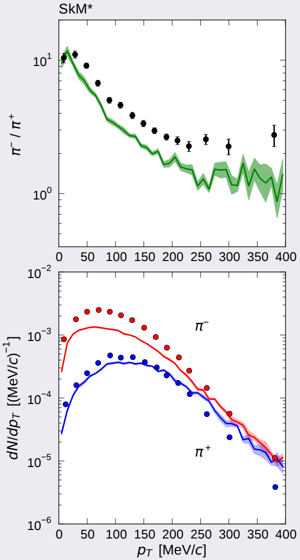
Strong impact on the isospin dependence of Δ production by e.g. $2U_n(p_N) - U_p(0)$. $p_N \gtrsim 400$–500 MeV/c.

Pion production from ^{132}Sn + ^{124}Sn at 270 MeV/u

SLy4 ($m_n^* < m_p^*$)



SkM* ($m_n^* > m_p^*$)



Spectra of π^- and π^+ (lower) and the π^-/π^+ ratio (upper)

Lines AMD+sJAM calculation with potentials in the collision term.

Points SπRIT experimental data. J. Estee et al., PRL 126, 162701 (2021)

The charged pion ratio π^-/π^+ is strongly affected by the momentum dependence of

$$U_n(p) - U_p(p) = 2\delta\, U_{\mathrm{sym}}(p)$$

The shape of spectrum at low momenta has been improved by considering the spreading width in the Δ spectral function.

$$\Gamma_\Delta(m_\Delta) = \Gamma_{\Delta\to N\pi}(m_\Delta) + \Gamma^\Delta_{\mathrm{sp}}\rho/\rho_0$$

J. Estee at al. (SπRIT), PRL 126, 162701 (2021)

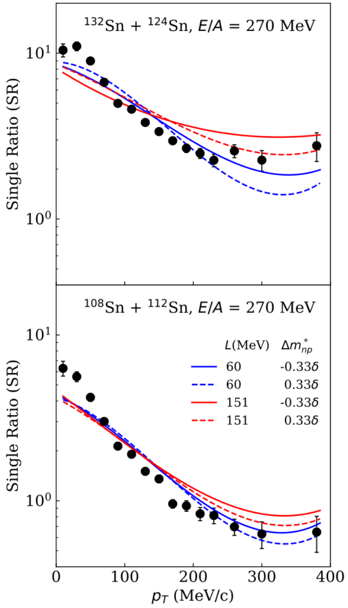
From energetic pions with $p_T > 200$ MeV/c, the constraint

$$42 < L < 117 \text{ MeV}$$

$$32.5 < S_0 < 38.1 \text{ MeV}$$

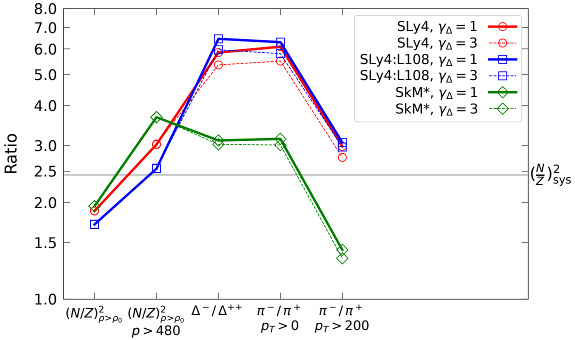
was obtained by the comparison of the SπRIT data and the dcQMD calculation.

Model dependence?



Effects of $E_{sym}(\rho)$ and $U_{sym}(\boldsymbol{p})$ in the compression stage

^{132}Sn + ^{124}Sn at 270 MeV/u



See Ikeno's talk.

Ikeno and Ono, in preparation.

- $(N/Z)^2$ in the high density region ($\rho > \rho_0$).

  [Soft E_{sym}: SkM*, SLy4] > [Stiff E_{sym}: SLy4:L108]

- $(N/Z)^2$ in the phase space of high density ($\rho > \rho_0$) and high momentum ($|\boldsymbol{p} - \boldsymbol{p}_{rad}| > 480$ MeV/c).

$$\langle \boldsymbol{p}^2\rangle_n > \langle \boldsymbol{p}^2\rangle_p \quad \text{for} \quad m_n^* > m_p^* \quad \text{(SkM*)}$$
$$\langle \boldsymbol{p}^2\rangle_n < \langle \boldsymbol{p}^2\rangle_p \quad \text{for} \quad m_n^* < m_p^* \quad \text{(SLy4)}$$

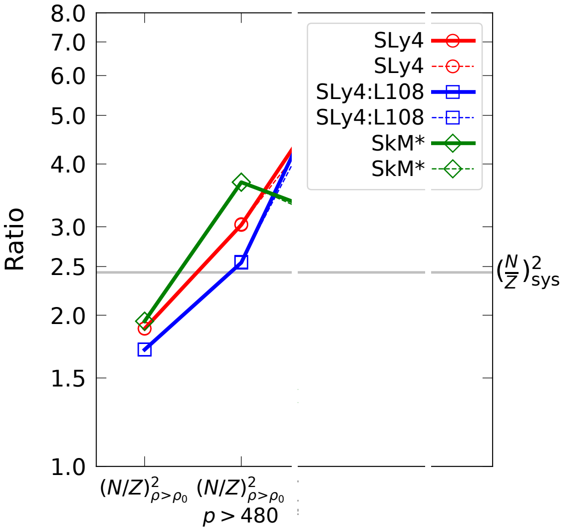
A simple argument for thermal equilibrium

When $U(\boldsymbol{p}) = A\boldsymbol{p}^2 + C$ and Pauli blocking is negligible,

$$f_n(\boldsymbol{p}) \propto \exp\left[\frac{-\boldsymbol{p}^2}{2m_n^* T}\right] \quad \text{and} \quad f_p(\boldsymbol{p}) \propto \exp\left[\frac{-\boldsymbol{p}^2}{2m_p^* T}\right]$$

$$\langle \boldsymbol{p}^2\rangle_n = 3m_n^* T \quad \text{and} \quad \langle \boldsymbol{p}^2\rangle_p = 3m_p^* T$$

^{132}Sn + ^{124}Sn at 270 MeV/u



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- $(N/Z)^2$ in the high density region ($\rho > \rho_0$).

  [Soft E_{sym}: SkM*, SLy4] > [Stiff E_{sym}: SLy4:L108]

- $(N/Z)^2$ in the phase space of high density ($\rho > \rho_0$) and high momentum ($|\boldsymbol{p} - \boldsymbol{p}_{rad}| > 480$ MeV/c).

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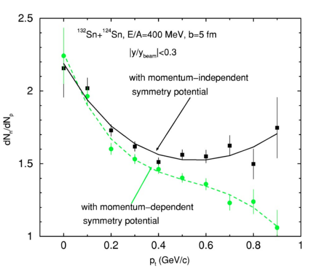
$$\langle \boldsymbol{p}^2 \rangle_n = 3m_n^* T \quad \text{and} \quad \langle \boldsymbol{p}^2 \rangle_p = 3m_p^* T$$

Consistent with predictions by other models?

IBUU model

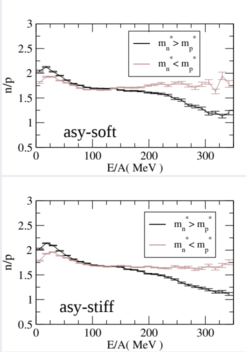
BA Li et al., PRC 69, 011603(R) (2004)



$^{132}Sn + ^{124}Sn$ at 400 MeV/u

SMM model

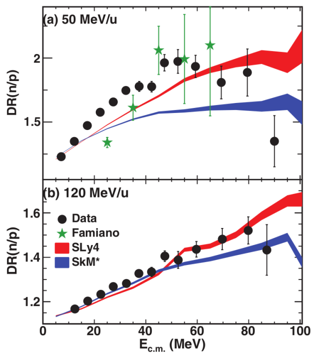
Giordano et al., PRC 81, 044661 (2010)



Au + Au at 400 MeV/u

ImQMD model

Coupland et al., PRC 94, 011601(R) (2016)



$(^{124}Sn + ^{124}Sn)/(^{112}Sn + ^{112}Sn)$

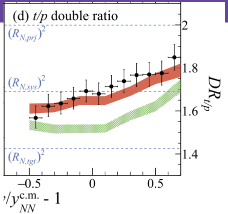
All these models predict energetic neutrons in neutron-rich systems when $m_n^* < m_p^*$.

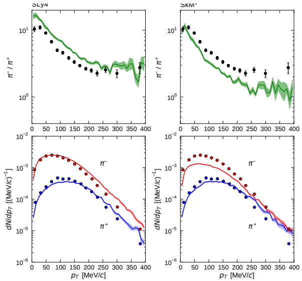
In AMD and sJAM, the collision term that is consistent with the mean-field potential (and EOS).

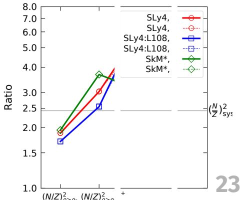
- Energy conservation
- Threshold condition e.g. for $NN \to N\Delta$
- $\sigma(\boldsymbol{p}_1, \boldsymbol{p}_2, \text{environment}) \propto \left(\frac{p_\mathrm{i}}{v_\mathrm{i}}\frac{p_\mathrm{f}}{v_\mathrm{f}}\right)|M|^2 \frac{p_\mathrm{f}}{p_\mathrm{i}}$
- Cluster production in AMD

Compressed neutron-rich systems, e.g., in Sn + Sn at 270 MeV/u.

- Symmetry energy effect in ρ_n/ρ_p of the cenral region of the compressed system ⇒ seen in cluster observables?
- The π^-/π^+ ratio is sensitive to the momentum-dependent potentials $U_n(p)$ and $U_p(p)$ at high momenta $p \sim 500$ MeV/c and $\rho = (1\text{-}2)\rho_0$.
- The momentum dependence of $U_n(p)$ and $U_p(p)$ is also reflected in $(\text{neutron}/\text{proton})_{\text{high density \& high momentum}}$.



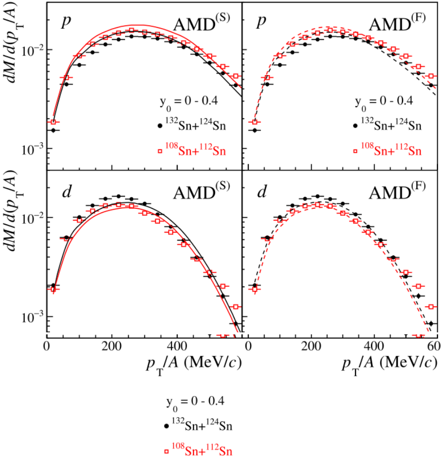




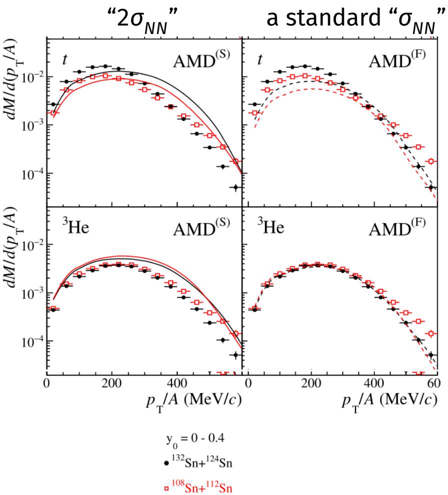
Transverse momentum spectra of protons and clusters

JW Lee et al. (SπRIT), EPJA (2022) 58:201

“$2\sigma_{NN}$” a standard “σ_{NN}”



JW Lee et al. (SπRIT), EPJA (2022) 58:201



Shape of t and ^{3}He spectra.

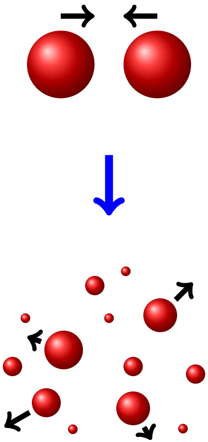
- In AMD, $t \approx {}^3$He.
- Experimental data show a large t yield at low p_T.

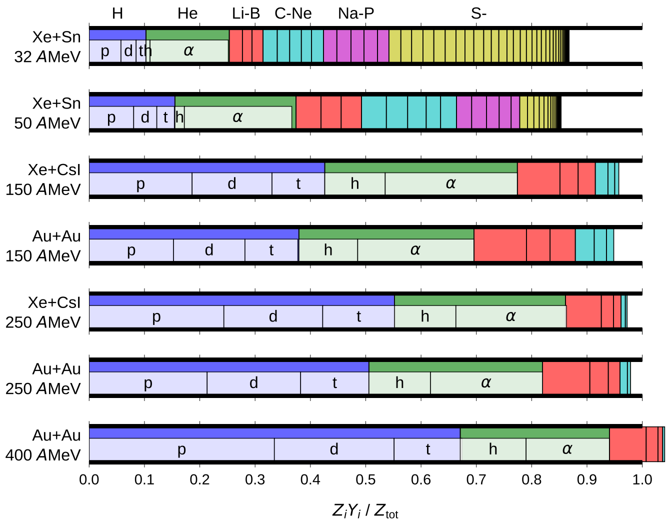
Difference between ^{132}Sn + ^{124}Sn and ^{108}Sn + ^{112}Sn.

- In AMD, triton production is enhanced in the neutron rich system simply by an overall factor.
- In experimental data, triton emission is enhanced at high p_T in the neutron-deficient system.

Experimental data suggest that high-momentum clusters are suppressed in the neutron-rich system compared to the more symmetric system.

Fraction of protons in clusters and fragments in heavy-ion collisions



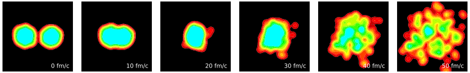


INDRA: Hudan et al., PRC67 (2003) 064613. FOPI: Reisdorf et al., NPA 848 (2010) 366.

Figure in Ono, PPNP 105 (2019) 139.

This is a challenge to theorists and transport models.

Link from nuclear EOS to observables in heavy-ion collisions

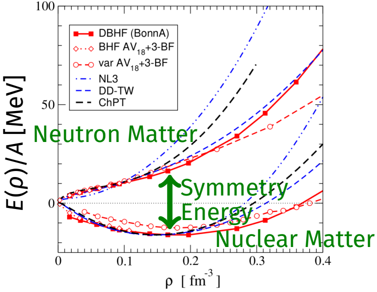


Nuclear matter EOS

$$\frac{E}{A}(\rho_p,\rho_n) = \left(\frac{E}{A}\right)_0(\rho) + E_{\mathrm{sym}}(\rho)\delta^2 + \cdots$$

$$\rho = \rho_p + \rho_n, \quad \delta = (\rho_n - \rho_p)/\rho$$

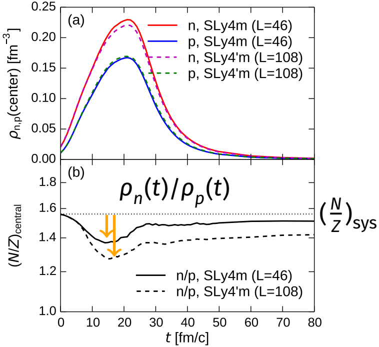
$$S_0 = E_{\mathrm{sym}}(\rho_0), \quad L = 3\rho_0(dE_{\mathrm{sym}}/d\rho)_{\rho=\rho_0}$$



Fuchs and Wolter, EPJA 30, 5 (2006)

Heavy-ion collision dynamics

$\rho_n(t)$ and $\rho_p(t)$ at $\boldsymbol{r} \sim 0$



^{132}Sn + ^{124}Sn, 300*A* MeV

Observables

How can we get information on ρ_n/ρ_p at $t = 20$ fm/c?

- π^-/π^+ ratio: $\sim (\rho_n/\rho_p)^2$

  B.A. Li, PRL 88 (2002) 192701.

  $$NN \leftrightarrow N\Delta, \quad \Delta \leftrightarrow N\pi$$

  $$nn \xrightarrow{\text{mainly}} p\Delta^- \cdots \rightarrow \pi^-$$

  $$pp \xrightarrow{\text{mainly}} n\Delta^{++} \cdots \rightarrow \pi^+$$

- Nucleons and clusters
  - Flow
  - ...

Relies on transport models. (mean field + collision term)

EOS and mean-field potential

Energy of the system in mean field models:

$$E[f] = \int \mathcal{E}(\mathbf{r}; f)d\mathbf{r} \quad = \cdots$$

as a functional of the one-body distribution function (in the mean-field theory)

$$f(\mathbf{r},\mathbf{p}) = \text{Wigner transform of } \sum_i \psi_i(\mathbf{r})\psi_i^*(\mathbf{r}'), \qquad \rho(\mathbf{r}) = \int f(\mathbf{r},\mathbf{p})\frac{d\mathbf{p}}{(2\pi\hbar)^3}$$

- Energy per nucleon (EOS) of nuclear matter:

$$E/A = E[f]/A = \mathcal{E}[f]/\rho \qquad \text{for} \quad f(\mathbf{r},\mathbf{p}) = \theta(p_F - |\mathbf{p}|) \quad \text{in the case of } T = 0$$

- Mean-field potential:

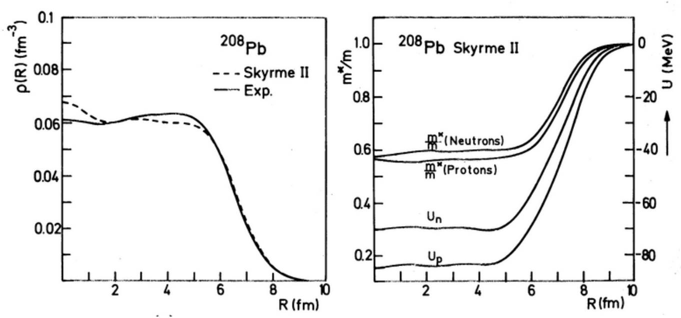
$$h(\mathbf{r},\mathbf{p}; f) = \frac{\mathbf{p}^2}{2m} + U(\mathbf{r},\mathbf{p}; f) = (2\pi\hbar)^3 \frac{\delta E[f]}{\delta f(\mathbf{r},\mathbf{p})}$$

Potential includes information more than EOS, e.g., when we consider particles with $|\mathbf{p}| > p_F$.

Skyrme force (effective interaction):

$$v_{12} = t_0(1 + x_0 P^\sigma)\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) + \tfrac{1}{2}t_1[\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\boldsymbol{k}^2 + \boldsymbol{k}^2\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)] + t_2\boldsymbol{k}\cdot\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\boldsymbol{k} + iW_0(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\cdot[\boldsymbol{k}\times\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\boldsymbol{k}]$$

$$\boldsymbol{k} = \frac{1}{2\hbar}(\boldsymbol{p}_1 - \boldsymbol{p}_2)$$



Vauterin and Brink, Phys. Rev. C 5 (1972) 626.

Mean field:

$$h(\boldsymbol{r},\boldsymbol{p}) = \frac{\boldsymbol{p}^2}{2m_\alpha} + U_\alpha(\boldsymbol{r},\boldsymbol{p}) = \frac{\boldsymbol{p}^2}{2m_\alpha^*(\boldsymbol{r})} + U_\alpha(\boldsymbol{r},0)$$

Effective mass:

$$m_\alpha^* = \frac{\boldsymbol{p}}{\boldsymbol{v}} = \frac{m}{1 + \frac{m}{p}\frac{\partial U_\alpha}{\partial p}} \quad (\text{at } p = p_F)$$

In a system with $N > Z$,

$$U_n(\boldsymbol{r}) > U_p(\boldsymbol{r}), \quad m_n^*(\boldsymbol{r}) \neq m_p^*(\boldsymbol{r})$$

Current topic

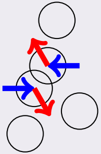
Is a neutron heavier or lighter than a proton?

E.g., B.A. Li et al., PPNP 99 (2018) 29.

NN collisions in AMD (without/with cluster correlations)

Without cluster correlation

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$



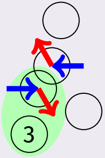
A collision of particles 1 and 2 will change only the two wave packets.

$$\{|\Psi_f\rangle\} = \{|\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,\dots)\rangle\}$$

(ignoring antisymmetrization for simplicity of presentation.)

With cluster correlation

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$

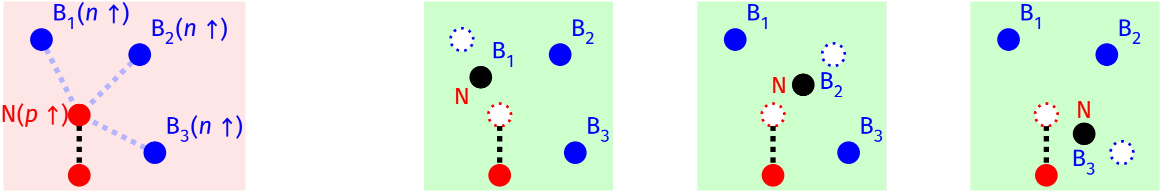


Include correlated states in the set of the final states of each NN collision.

$$\{|\Psi_f\rangle\} \ni |\varphi_{k_1}(1)\psi_d(2,3)\Psi(4,\dots)\rangle,\ \dots$$

(ignoring antisymmetrization for simplicity of presentation.)

Clusters (in the final states) are assumed to have $(0s)^N$ configuration.



$|\Phi^{\boldsymbol{q}}\rangle$ — After $\boldsymbol{p}^{(0)} \to \boldsymbol{p}^{(0)} + \boldsymbol{q}$

$|\Phi'_1\rangle$ — $N + B_1 \to C_1$

$|\Phi'_2\rangle$ — $N + B_2 \to C_2$

$|\Phi'_3\rangle$ — $N + B_3 \to C_3$

Final states are not orthogonal: $N_{ij} \equiv \langle \Phi'_i | \Phi'_j \rangle \neq \delta_{ij}$

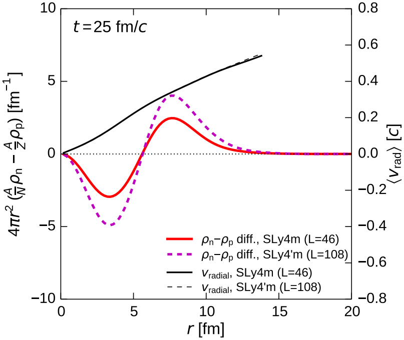
The probability of cluster formation with one of B's:

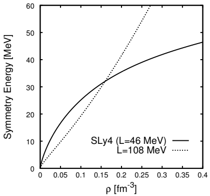
$$\hat{P} = \sum_{ij} |\Phi'_i\rangle N^{-1}_{ij} \langle \Phi'_j|, \qquad P = \langle \Phi^{\boldsymbol{q}} | \hat{P} | \Phi^{\boldsymbol{q}} \rangle \qquad \neq \sum_i |\langle \Phi'_i | \Phi^{\boldsymbol{q}} \rangle|^2$$

$$\begin{cases} P & \Rightarrow \text{Choose one of the candidates and make a cluster.} \quad \propto |\langle \Phi'_i | \Phi^{\boldsymbol{q}} \rangle|^{2\gamma} \\ 1 - P & \Rightarrow \text{Don't make a cluster (with any n}\uparrow\text{).} \end{cases}$$

A. Ono, EPJ Web of Conferences 117 (2016) 07003.

$\rho_n - \rho_p$ at the compression stage

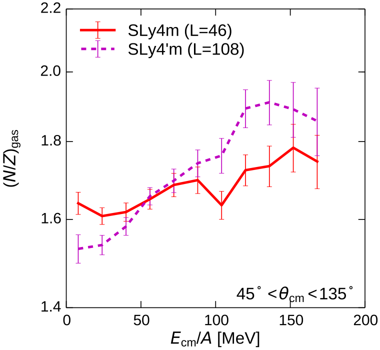




⇔

similar

N/Z of the spectrum of emitted particles



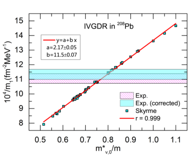
$$\left(\frac{N}{Z}\right)_{gas} = \frac{Y_n(\epsilon) + Y_d(\epsilon) + 2Y_t(\epsilon) + Y_h(\epsilon) + 2Y_\alpha(\epsilon)}{Y_p(\epsilon) + Y_d(\epsilon) + Y_t(\epsilon) + 2Y_h(\epsilon) + 2Y_\alpha(\epsilon)}$$

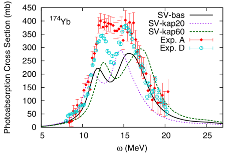
$$\epsilon = E_{cm}/A$$

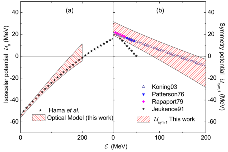
Impacts of neutron-proton effective mass splitting in nuclear physics and astrophysics

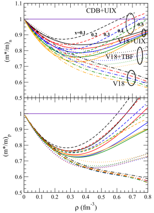
$$\Delta m^*_{n-p} = m^*_n - m^*_p = (???) \times \delta, \quad U_{n/p}(p;\rho,\delta) = U_0(p;\rho) \pm U_{sym,1}(p;\rho)\delta + \cdots, \qquad \delta = (N-Z)/A = (\rho_n - \rho_p)/\rho$$

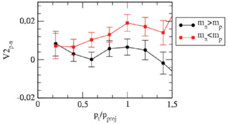
- Isovector Giant Dipole Resonance
  - Zhen Zhang, LW Chen, PRC 93, 034335 (2016); Zhen Zhang et al., Chin. Phys. C 45, 064104 (2021)
  - Oishi, Kortelainen, Hinohara, PRC 93, 034239 (2016)
- Nucleon–Nucleus Scattering (optical potential)
  - AJ Koning, JP Delaroche, NPA 713 (2003) 231 etc
  - BA Li, PRC 69, 064602 (2004)
  - XH Li, WJ Guo, BA Li, LW Chen et al., PLB 743 (2015) 408
- Stellar Neutrino Emission
  - Baldo et al., PRC 89, 048801 (2014)
- Heavy-Ion Collision Observables
  - Giordano, Colonna, Di Toro et al., PRC 81, 044661 (2010)
  - BA Li et al., PRC 69, 011603(R) (2004)
  - Coupland et al., PRC 94, 011601(R) (2016); Morfouace et al., PLB 799 (2019) 135045; YX Zhang et al., PLB 732 (2014) 186
  - Jun Su et al., PRC 94 (2016) 034619

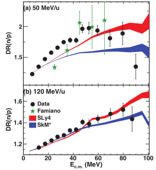








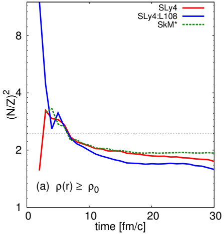




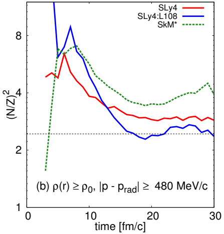
HIC can explore a wide range of density and momentum.

Detailed link from neutron/proton to π^-/π^+

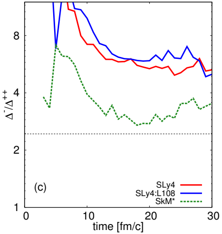
$(N/Z)^2$ at $\rho > \rho_0$



$\rho > \rho_0$ & high p

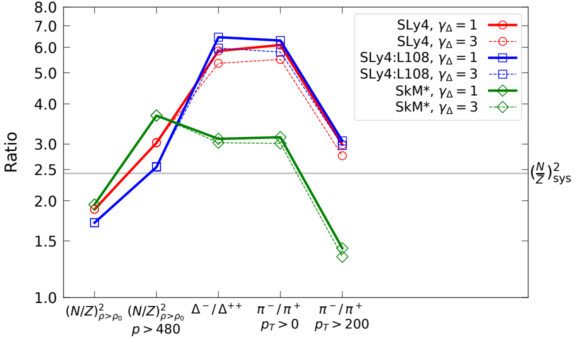


Δ^-/Δ^{++} production



As a function of time,

- $(N/Z)^2$ in the region of $\rho > \rho_0$.
- $(N/Z)^2$ in the phase-space region of $\rho > \rho_0$ and $|\boldsymbol{p} - \boldsymbol{p}_{\mathrm{rad}}| > 480$ MeV/c.
- Rate($nn \to p\Delta^-$)/Rate($pp \to n\Delta^{++}$)



Red vs Blue: effect of symmetry energy $E_{\mathrm{sym}}(\rho)$

interesting and important, but not discussed today.

Red vs Green: effect of p dependence of $U_{\mathrm{sym}}(\boldsymbol{p})$

- Clear effect in the $(N/Z)^2$ of high-p nucleons.
- Very strong effect in the Δ production.

$$(nn \to p\Delta^-)/(pp \to n\Delta^{++}) \Rightarrow \pi^-/\pi^+$$