Bayesian analysis for constraining neutron-proton effective mass splitting

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Workshop on EoS of Dense Nuclear Matter at RIBF and FRIB

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Outline

1. Background
2. The ImQMD Model
3. Computational Challenges and Gaussian Emulation
4. Bayesian Inference on Experimental Data
5. Closure Tests
6. Summary

Background – EOS and effective masses

- *Density dependence* of the symmetry energy can be parametrized by Taylor expansion around the saturation density ρ_0

$$S(\rho) = S_0 + \frac{1}{3} L \cdot \frac{\rho - \rho_0}{\rho_0} + \mathcal{O}(\rho^2)$$

- *Momentum dependence* of the nuclear mean-field potential arises from the Fock exchange term, finite range, and correlation effects. It can be characterized by **effective mass**,

$$\frac{m^*}{m_N} = \left(1 + \frac{m}{p}\frac{\partial U}{\partial p}\right)^{-1}$$

- *Isoscalar effective mass* m_s^* is reduced from the free nucleon mass m_N

- *Isovector effective mass* m_v^* leads to discrepancy between the neutron and proton effective masses, m_n^* and m_p^*.

Background – Observables

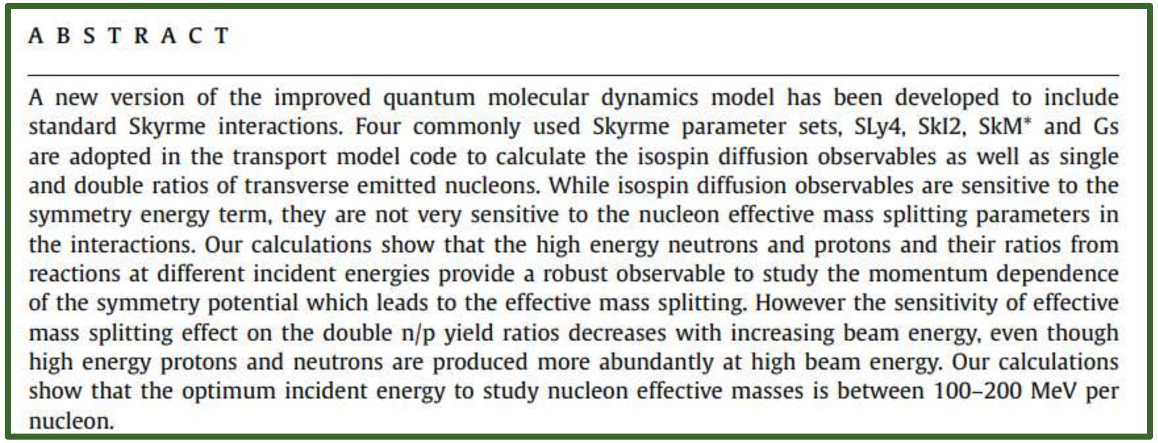
- In this study, we compare the neutron-proton spectrum ratio between experiment and simulations to constrain S_0, L, m_s^*, m_v^*.

- Particularly, an experiment at NSCL (currently known as FRIB) was performed, and data from the following two reaction systems were analyzed:
  - System 1: Ca40 + Ni58 at 140 MeV/u
  - System 2: Ca48 + Ni64 at 140 MeV/u

- We look at the **double ratio** of the transverse momentum spectra, ${}^{p_T}/_{A}$ [MeV/c], with particle cut at mid-rapidity, i.e.

$$\frac{Y_2(n)/Y_2(p)}{Y_1(n)/Y_1(p)}$$

The ImQMD Model



ABSTRACT

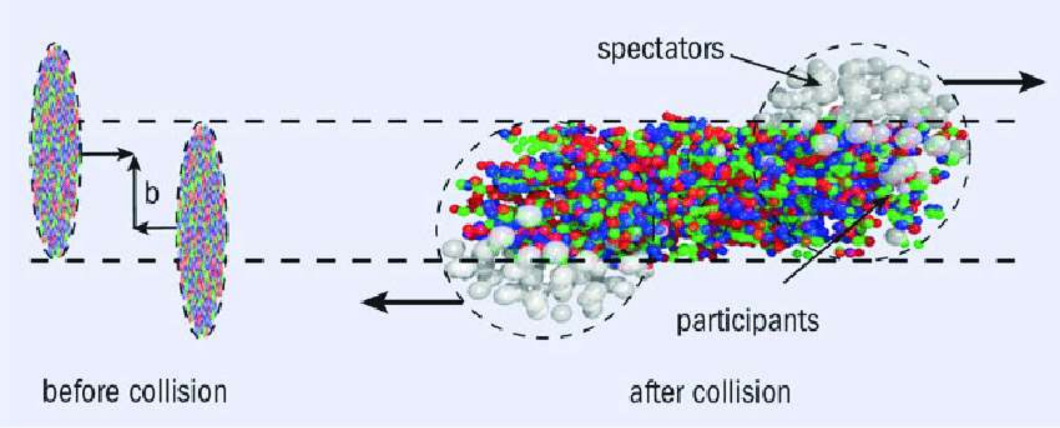
A new version of the improved quantum molecular dynamics model has been developed to include standard Skyrme interactions. Four commonly used Skyrme parameter sets, SLy4, SkI2, SkM* and Gs are adopted in the transport model code to calculate the isospin diffusion observables as well as single and double ratios of transverse emitted nucleons. While isospin diffusion observables are sensitive to the symmetry energy term, they are not very sensitive to the nucleon effective mass splitting parameters in the interactions. Our calculations show that the high energy neutrons and protons and their ratios from reactions at different incident energies provide a robust observable to study the momentum dependence of the symmetry potential which leads to the effective mass splitting. However the sensitivity of effective mass splitting effect on the double n/p yield ratios decreases with increasing beam energy, even though high energy protons and neutrons are produced more abundantly at high beam energy. Our calculations show that the optimum incident energy to study nucleon effective masses is between 100–200 MeV per nucleon.

Y. Zhang, et al. / PLB 732 (2014) 186-190

- ImQMD (Improved QMD) model was developed to study nuclear reactions, particularly heavy-ion collisions (HICs).
- Able to simulate the time evolution of nuclear collisions.
- Incorporates Skyrme energy density functional, which provides an effective interaction for nuclear systems.
- In this study, we focus on S_0 and L, which characterize the density dependence, as well as m_s^* and m_v^*, which characterize the momentum dependence.

The ImQMD Model – The simulation code

1. Specify input parameters such as S_0, L, m_s^*, m_v^*, and run the executable program.
2. The program simulates the collisions event by event.
3. For each event, the program outputs the final $\vec{r}$ and $\vec{p}$ of each nucleon.
4. Nucleons with relative momenta smaller than $p_0 = 200$ MeV/c and relative distances smaller than $R_0 = 3$ fm are *coalesced into one cluster* (nuclei).



Taken from Toia A 2013 CERN Courier April 31

Due to the stochastic nature of the simulation, a large number of events ($\sim 10^4$ or more) are collected to reliably determine the observables. Importantly, each event is independent of others, making this process inherently suitable for parallel computing. Consequently, it can be efficiently executed on high-performance computing (HPC) or high-throughput computing (HTC) platforms.

The ImQMD Model – The limitation of coalescence model

- Most transport models have difficulty reproducing the relative abundances of light isotopes produced as the system expands and disassembles.

- Following D.D.S. Coupland, et al., we calculated the ***coalescence invariant (CI)*** neutron and proton spectra by combining the nucleons with those bound in light isotopes with $A \leq 4$ and $Z \leq 2$.

- CI-p spectrum is given by

$$Y_{\mathrm{CI}}(p) = \sum_{A\leq 4, Z\leq 2} Z \cdot Y(A,Z) = Y(\,^{1}\mathrm{H}) + Y(\,^{2}\mathrm{H}) + Y(\,^{3}\mathrm{H}) + 2 \cdot Y(\,^{3}\mathrm{He}) + 2 \cdot Y(\,^{4}\mathrm{He})$$

- CI-n spectrum is given by

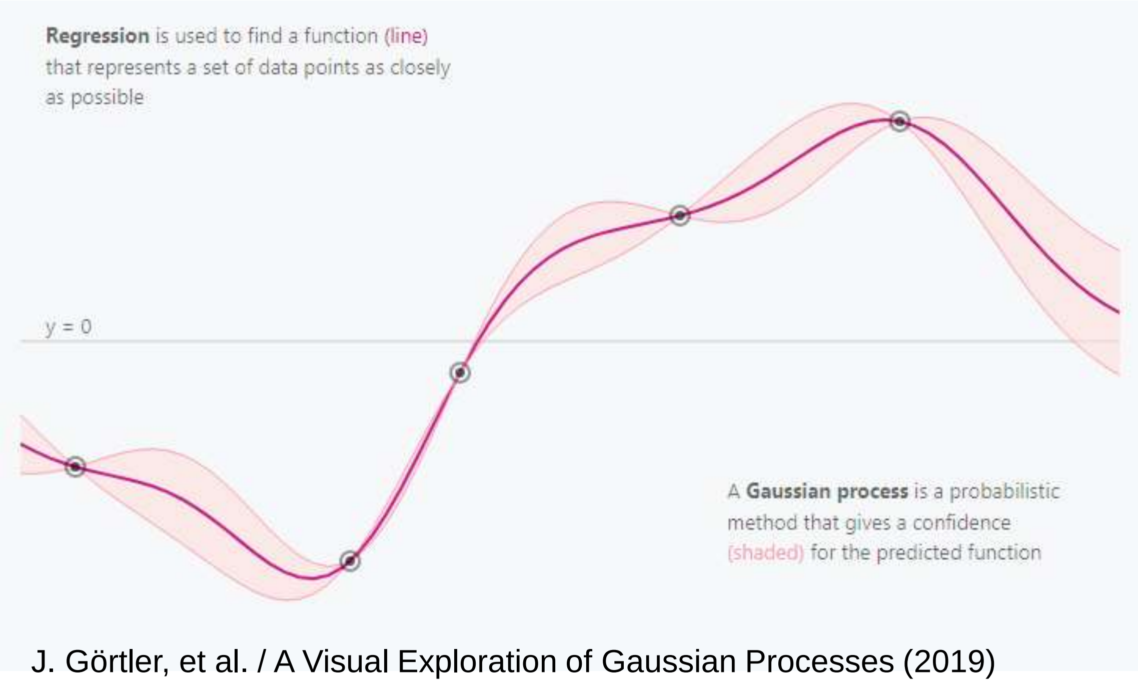
$$Y_{\mathrm{CI}}(n) = \sum_{A\leq 4, Z\leq 2} (A-Z) \cdot Y(A,Z) = Y(n) + Y(\,^{2}\mathrm{H}) + 2Y(\,^{3}\mathrm{H}) + Y(\,^{3}\mathrm{He}) + 2 \cdot Y(\,^{4}\mathrm{He})$$

D.D.S. Coupland, et al. / PRC 94 (2016) 011601(R)

Computational Challenges and Gaussian Emulation

- A large number of events need to be simulated to obtain reliably double ratio spectrum. Even with HPC / HTC, it takes ~hours to ~days for just one set of (S_0, L, m_s^*, m_v^*) input.

- Gaussian Emulation / Gaussian Process Regression (GPR):
  - ✓ Non-parametric
  - ✓ Good at interpolation
  - ✓ Handles noise and probabilistic predictions
  - ✓ Efficient with small dataset
  - ✓ Allow multivariate inputs



Regression is used to find a function (line) that represents a set of data points as closely as possible

A **Gaussian process** is a probabilistic method that gives a confidence (shaded) for the predicted function

J. Görtler, et al. / A Visual Exploration of Gaussian Processes (2019)
https://distill.pub/2019/visual-exploration-gaussian-processes/

Computational Challenges and Gaussian Emulation

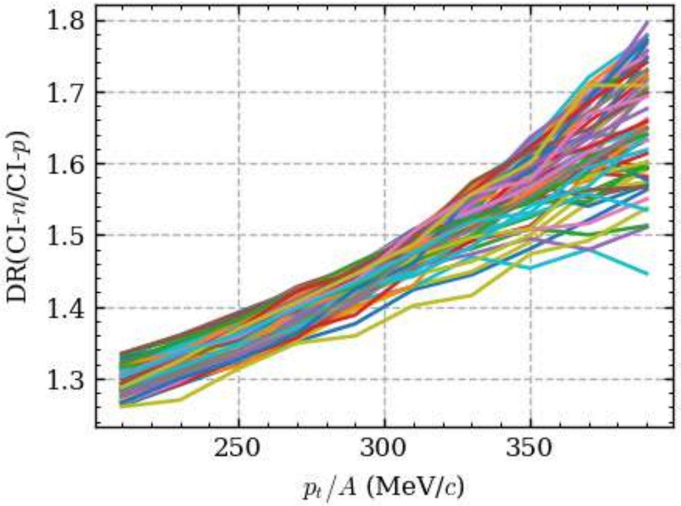
- Goal: Find $\mathcal{GP}$ such that $\mathcal{GP}(S_0, L, m_s^*, m_v^*) \approx \mathrm{DR}$

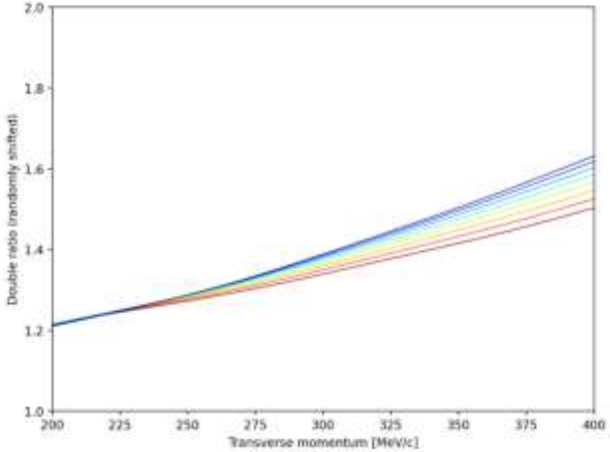
- Step 1: Run ImQMD on 70 sets of (S_0, L, m_s^*, m_v^*) values, chosen randomly according to Latin Hypercube sampling.

- Step 2: Calculate the Double Ratio values for each set, and estimate the associated uncertainties.

- Step 3: Train the Gaussian Emulator.

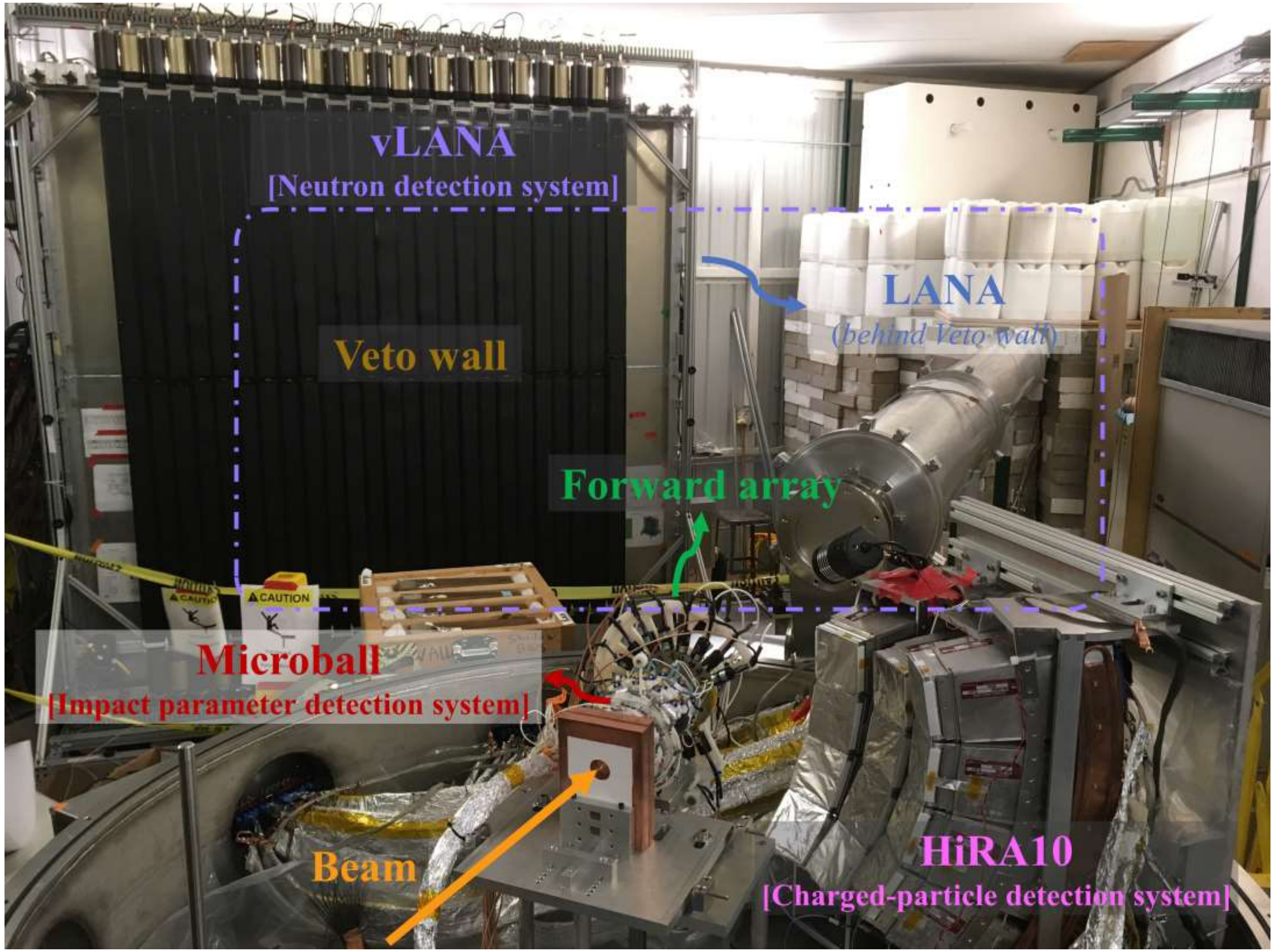
*Leave-one-out cross-validation (LOOCV) has been performed to check the validity of Gaussian Emulator.

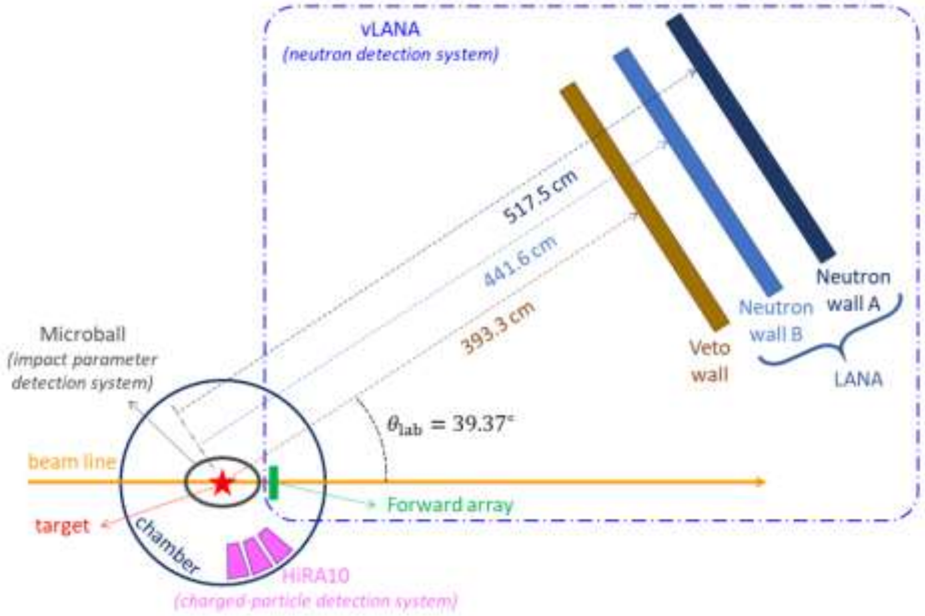




https://fanurs-simple-bayesian-example.streamlit.app/

Bayesian Inference on Experimental Data





An experiment at NSCL (currently known as FRIB) was performed, and data from the following two reaction systems were analyzed:

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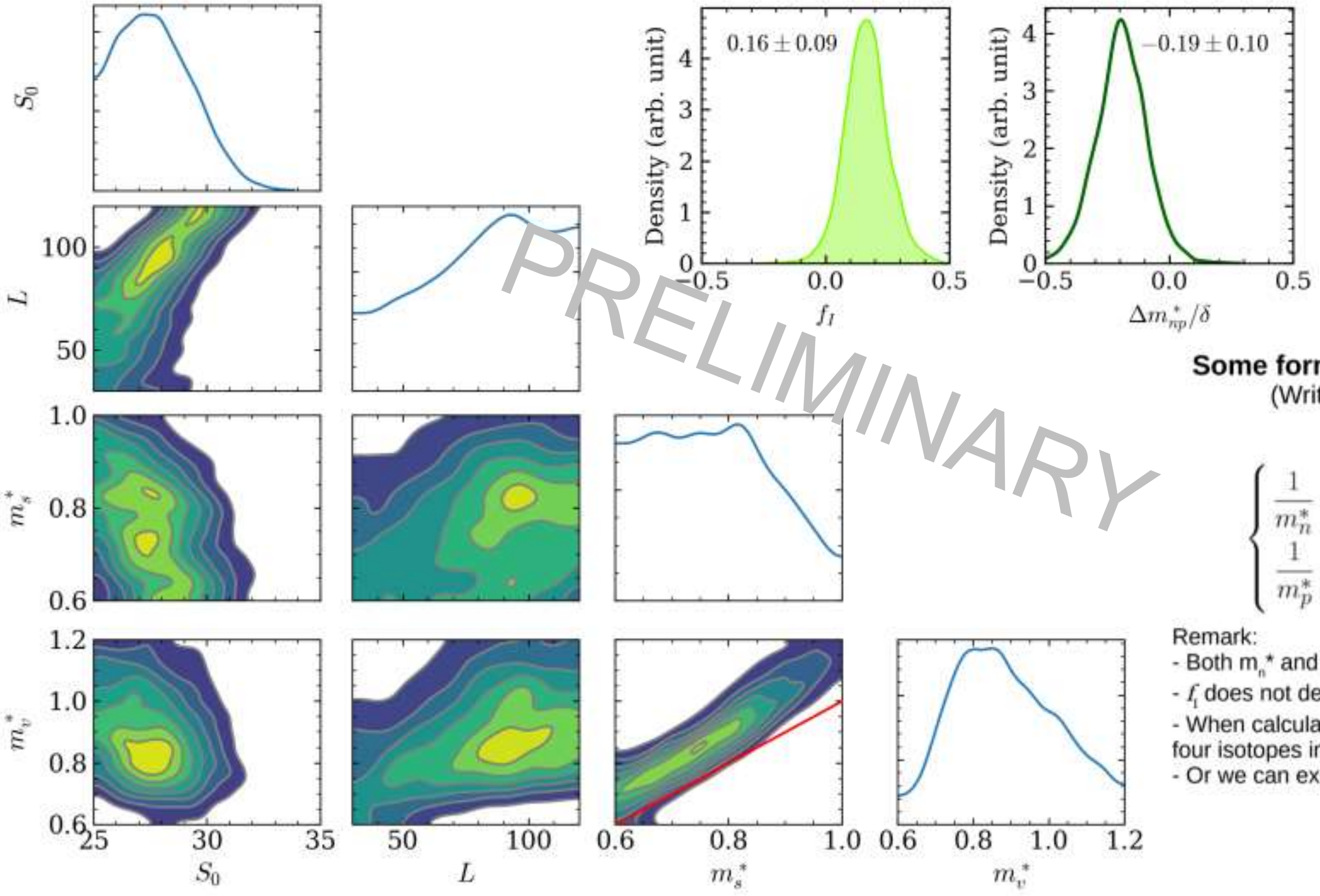
Bayesian Inference on Experimental Data



Some formulas involving effective masses

(Written in unit of nucleon mass, m_{N})

Conversion formulas

$$\begin{cases} \dfrac{1}{m_n^*} = (1+\delta)\left(\dfrac{1}{m_s^*} - \dfrac{1}{m_v^*}\right) + \dfrac{1}{m_v^*} \\ \dfrac{1}{m_p^*} = (1-\delta)\left(\dfrac{1}{m_s^*} - \dfrac{1}{m_v^*}\right) + \dfrac{1}{m_v^*} \end{cases}$$

Splittings

$$f_I \equiv \frac{1}{m_s^*} - \frac{1}{m_v^*}$$

$$\Delta m_{np}^* \equiv m_n^* - m_p^*$$

Remark:
- Both m_n^* and m_p^* depend on asymmetry (delta)
- f_I does not depend on asymmetry
- When calculating delta, I combine all Z and N from the four isotopes in Ca48+Ni64 and Ca40+Ni58: ~0.0857
- Or we can express Δm_{np}^* in unit of δ

$$\Delta m_{np}^* = -2\delta f_I \left[\frac{1}{m_s^{*2}} - \delta^2 \left(\frac{m_s^* - m_v^*}{m_s^* m_v^*}\right)^2\right]^{-1}$$

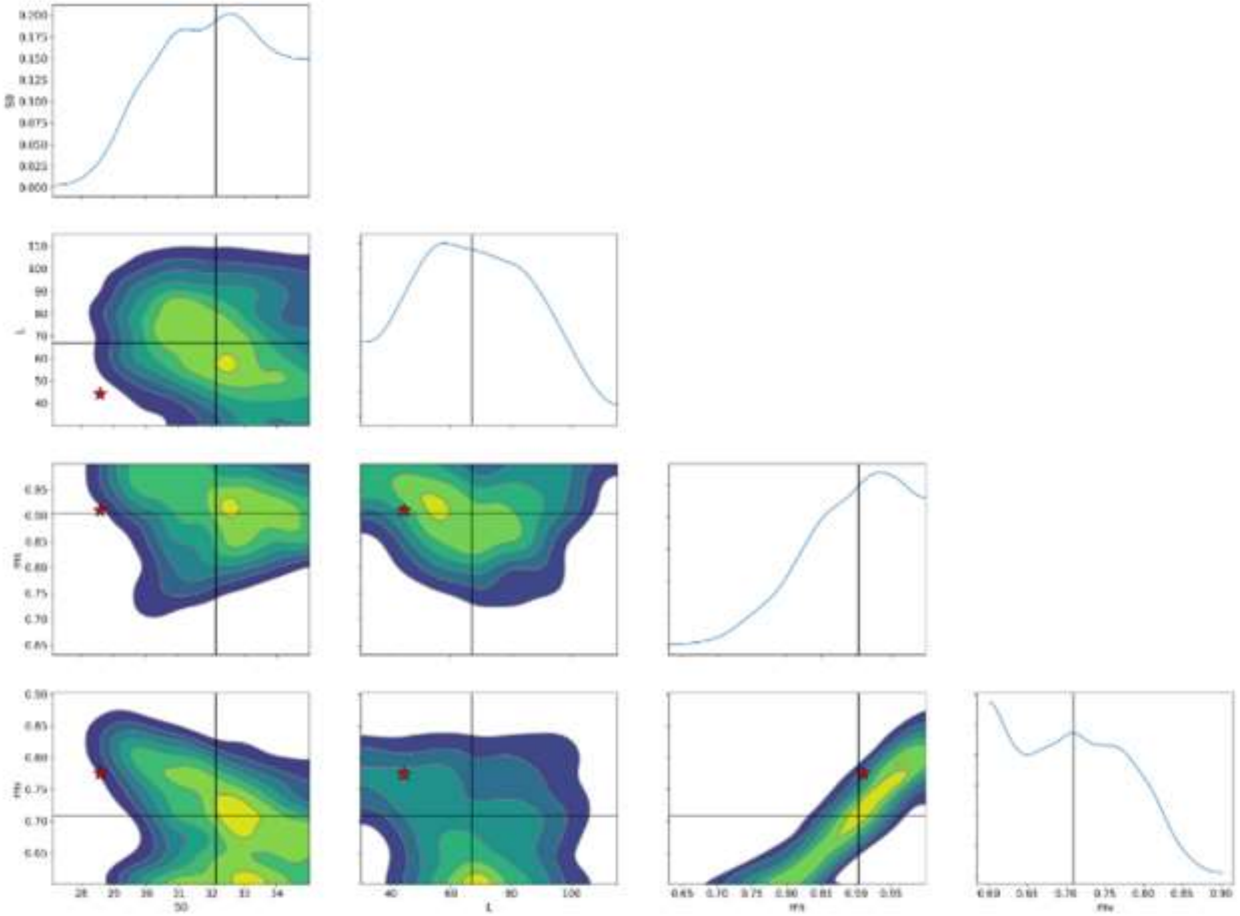
$$\approx -2\delta f_I \cdot m_s^{*2}$$

Closure Tests

- It is important to check if the same analysis procedure can *reproduce* similar results on synthetic data with known (S_0, L, m_s^*, m_v^*)

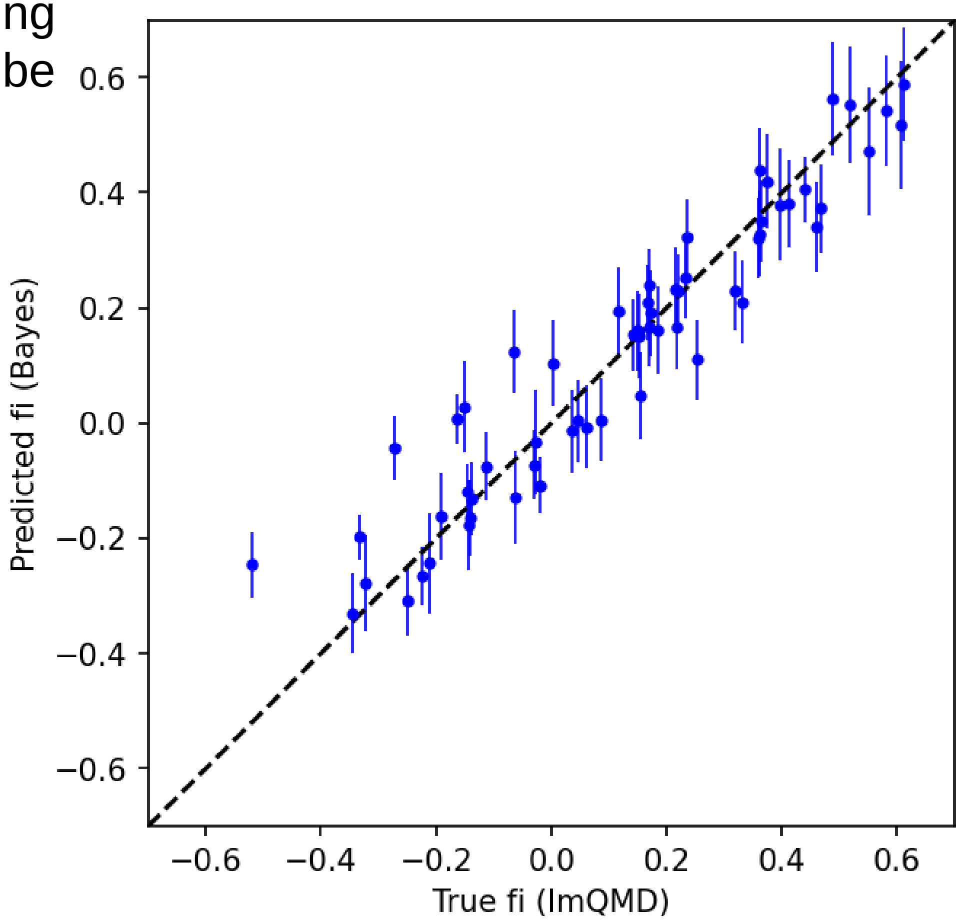
- We use the 70 sets of Skyrme simulation data for **closure test**. Each set is used as a synthetic data, with the remaining 69 sets used for training the Gaussian emulator.

- Using just the *Double Ratio* as observable cannot reliably reproduce all the parameters. However, the *effective mass splitting* (difference) seems to be consistently reproduced.

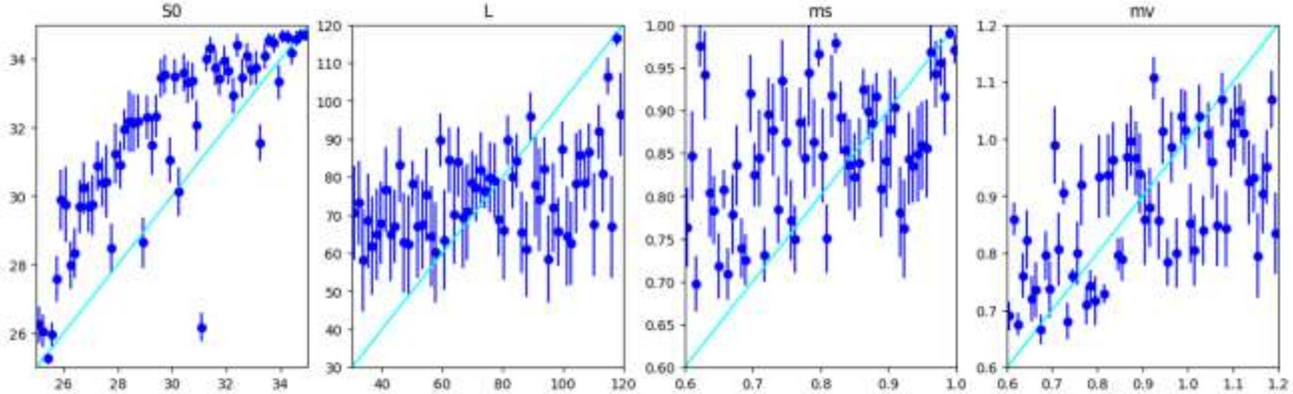


Closure Tests

If we take the “median” of the Bayesian sampling as “predicted” value, a strong correlation can be established on the effective mass splitting, f_I.



Individually, the parameters do not exhibit much, if any, reproducibility.



Summary

- An efficient way to “interpolate” transport models using Gaussian Processes.

- A potential way to constrain effective mass splitting with limited number of observables.

- The results may be model-dependent. It will be interesting to do the same analysis using transport models other than ImQMD and other reaction systems.

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