Pion production in heavy-ion collisions and the symmetry energy

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Equation of State of Dense Nuclear Matter at RIBF and FRIB
RIKEN, 23-26 May 2023

Overview

(Instead of) Motivation

Model Details

dcQMD – model details
Threshold Effects
Nucleon and Δ(1232) interactions
Model Benckmarking

Empirical Determination of the Δ(1232) potential

Observables of interest
Isoscalar and Isovector Component

Studying the Symmetry Energy

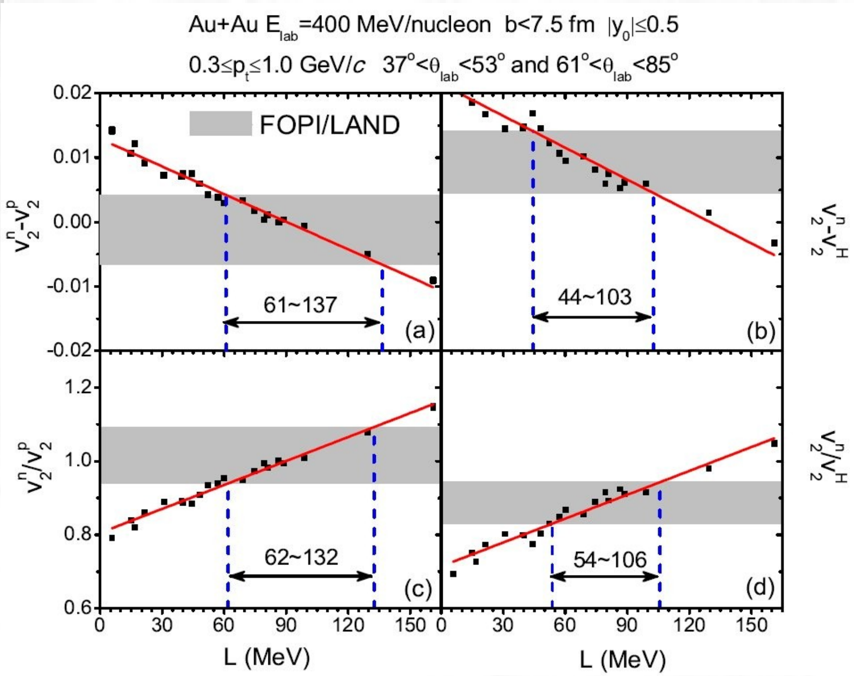
Approach
FOPI/SΠRIT/HADES

Summary & Conclusions

Elliptic Flow vs. Pion ratios

$$\frac{dN}{d\phi} \sim 1 + 2 v_1 \cos\phi + 2 v_2 \cos 2\phi$$

UrQMD - Y. Wang et al. PRC 89, 044603 (2014)



TuQMD - linear/moderately stiff

M.D. Cozma et al. PRC 88, 044912 (2013)

UrQMD – linear

P. Russotto et.al PLB 697, 491 (2011)

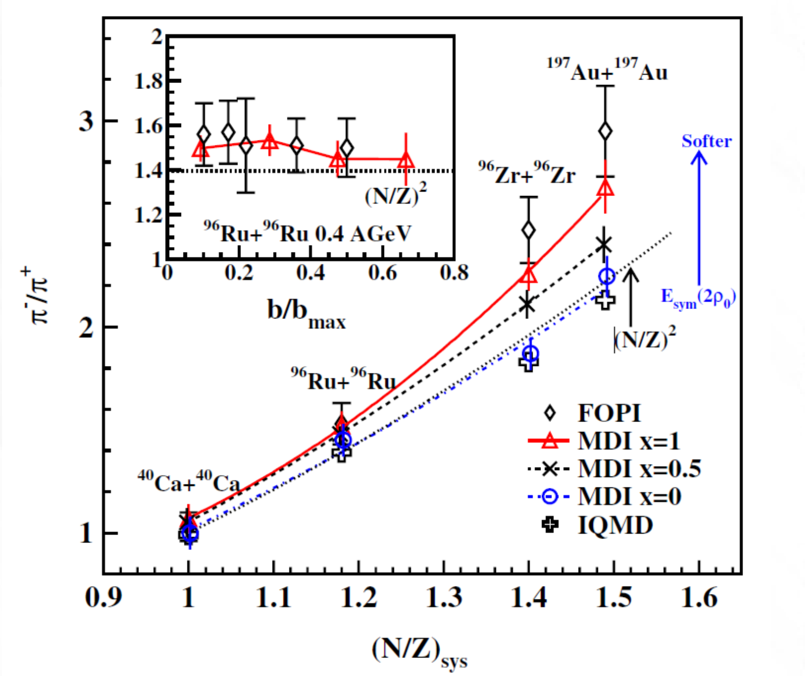
IBUU - linear/moderately stiff

G.-C. Yong private communication

Isobar model (no symmetry potential)

$$\pi^-/\pi^+ = (5N^2+NZ)/(5Z^2+NZ)$$

IBUU - Z.Xiao et al. PRL 102,062502 (2009)



LQMD – stiff

Z.-Q. Feng et al., PLB 683, 140 (2010)

Boltzmann-Langevin – super-soft

W.-J. Xie et al., PLB 718, 1510 (2013)

TuQMD (VEC) – super-soft

M.D. Cozma, PLB 753, 166 (2016)

pBUU – no sensitivity to SE

J. Hong et al. PRC 90, 024605 (2014)

Model Details & Benchmarking

dcQMD transport model: newest version EPJA 57, 309 (2021)

an upgraded version of TuQMD, see H. Wolter et al.

Prog.Part.Nucl.Phys. 125, 103962 (2022)

Interaction (nucleonic d.o.f.)

momentum dependent potential **MDI2** -generalization of MDI of

Das, Das Gupta, Gale, Li PRC67, 034611 (2003)

$$\frac{E}{N}(\rho,\beta,x,y)=\frac{1}{2}A_1 u+\frac{1}{2}A_2(x,y)u\beta^2+\frac{Bu^{\sigma}}{\sigma+1}(1-x\beta^2)+\frac{Du^2}{3}(1-y\beta^2)$$

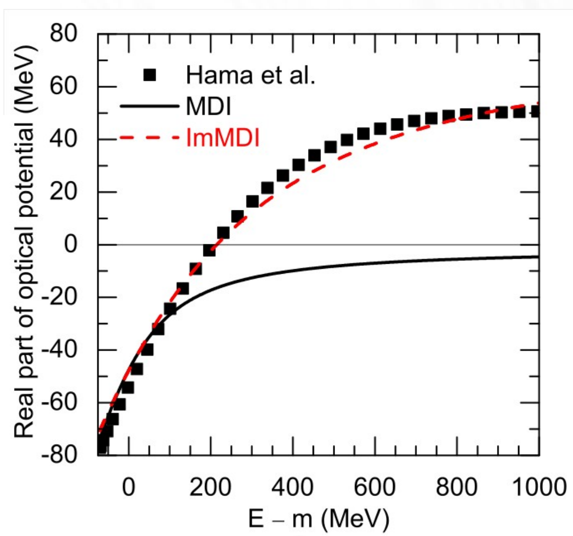
$$+\frac{1}{u\rho_0^2}\sum_{\tau,\tau'}C_{\tau\tau'}\int\int d^3p\,d^3p'\,\frac{f_\tau(p,p')f_{\tau'}(p,p')}{1+(\vec{p}-\vec{p}\,')^2/\Lambda^2}$$

$$A_2(x,y)=A_2^0+\frac{2xB}{\sigma+1}\bar{u}^{\sigma-1}+\frac{2yD}{3}\bar{u} \qquad u=\frac{\rho}{\rho_0}$$

Fit:
U_∞,K,J_0,m* -isoscalar
S($\tilde{u}$),L,K_{sym},δm_{isv} -isovector

$C_l - C_u$

momentum dependent part: similar with that of J. Xu et al. PRC 91, 014611 (2015)
(see also C. Hartnack, J. Aichelin PRC 49, 2801 (1994))
used previously to test model dependence: flow ratio PRC 88, 44912 (2013)
pion multiplicity ratio PLB 753, 166 (2016)
independent part: extra term (vary L vs. K_{sym} and also J_0 vs. K independently)



from J. Xu et al. PRC 91, 014611 (2015)

Input		Parameters	
ρ_0 [fm^{-3}]	0.16	A [MeV]	708.001
E_B [MeV]	-16.0	C_l [MeV]	-13.183
m_s^*/m	0.70	C_u [MeV]	-140.405
δ^*_{n-p} (ρ_0, β =0.5)	0.165	B [MeV]	137.305
K_0 [MeV]	245.0	σ	1.2516
J_0 [MeV]	-350.0	$\tilde{A}_l$ [MeV]	-130.495
$\tilde{\rho}$ [fm^{-3}]	0.10	$\tilde{A}_u$ [MeV]	-8.828
S($\tilde{\rho}$) [MeV]	25.4	D [MeV]	7.357

Collision Term

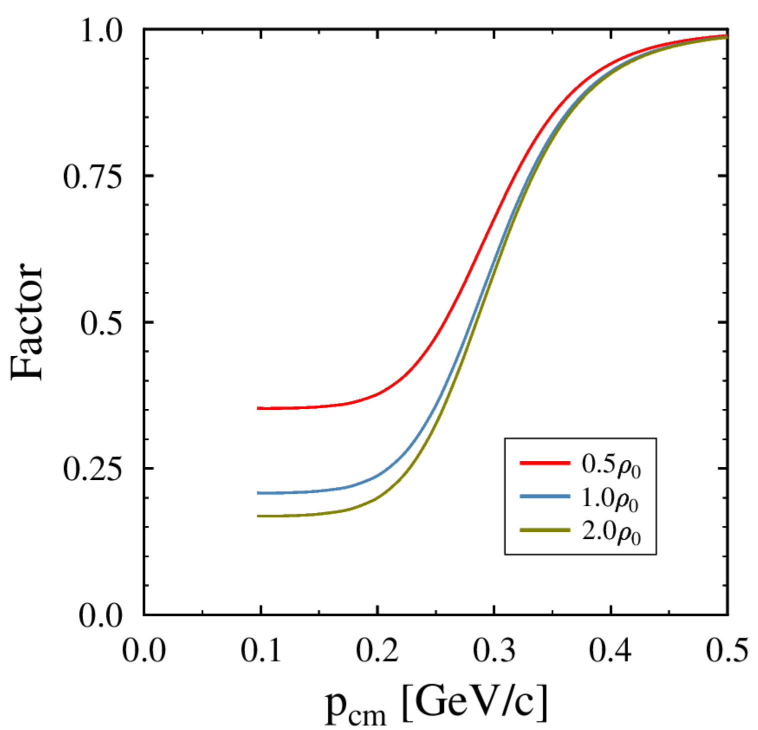
Elastic baryon-baryon collisions

below pion production threshold: Li-Machleidt

Li, Machleidt PRC 48, 1702 (1993),
Li, Machleidt PRC 49, 566 (1994)

above pion production threshold: Cugnon

in-medium modification factor



elastic scattering: empirical reduction factor
Y.Wang et al. PRC 89, 034606 (2014)

Inelastic baryon-baryon, meson-baryon collisions (related only to pion production)

two step process:

- resonance excitation in baryon-baryon collisions
  parametrization of the OBE model of
  S.Huber et al., NPA 573, 587 (1994)
- resonance decay:
  Breit-Wigner shape of the resonance spectral function J. Weil et al, PRC 94, 054905 (2016)

- charge exchange reactions: NR->NR'

pion absorption:

-resonance model (all 4* resonances below 2 GeV)
K. Shekhter, PRC 68, 014904 (2003)

inelastic channels: mass scaling formula

$$\sigma_{N\Delta}(\rho,\beta,p)=\sigma_{N\Delta}^{vac}(p)\frac{\mu_{ini}(\rho,p)}{\mu_{ini}^{vac}(p)}\frac{\mu_{fin}(\rho,p)}{\mu_{fin}^{vac}(p)}$$

See also Larionov et al., NPA 728, 135 (2003)

Threshold Effects (dcQMD)

- direct consequence of imposing (total) energy conservation in the medium

$$\sqrt{p_1^2+m_1^2}+U(p_1)+\sqrt{p_2^2+m_2^2}+U(p_2)=\sqrt{p'^2_1+m'^2_1}+U(p'_1)+\sqrt{p'^2_2+m'^2_2}+U(p'_2)$$

- rarely considered in transport models below 1 AGeV, with a few exceptions:
  RBUU: G. Ferini et al. PRL 97, 202301 (2006), RVUU: T. Song, C.M. Ko PRC 91, 014901 (2015);
  χBUU: Z. Zhang et al, PRC 98, 054614 (2018)
- required for thermodynamical consistency of the model
  Z.Zhang et al, PRC 97, 014610 (2018)
- reactions: NN↔NR, R↔Nπ (R↔Nππ not corrected)

- assumptions (dcQMD): - two-body collisions are part of N-body one
  - in-medium two-body collisions modeled as a succession of bare (vacuum-like) collisions followed/preceded by energy exchanges with the fireball, while momentum is conserved
  - reaction with highest probability: corresponds to the one which included the bare collision of highest probability

Example: NN->NΔ

$$\sigma^{(med)}_{NN\to N\Delta}(s^*) = \frac{\mu^{(ini)*}}{\mu^{(ini)}}\frac{\mu^{(fin)*}}{\mu^{(fin)}}\sigma^{(vac)}_{NN\to N\Delta}(s^*)$$

$s^*=Max\{s^{ini},s^{fin}\}$

Introduced in TuQMD/dcQMD in DC, PLB 753, 166 (2016)

Δ(1232) Potential

nucleonic resonance of isospin T and isospin projection τ

$$U_R^{\tau}(\rho,\beta,p)=\frac{1}{2}(1-\tau/T)\,U_{-1/2}(\rho,\beta,p)+\frac{1}{2}(1+\tau/T)\,U_{1/2}(\rho,\beta,p)$$

particular case of a T=3/2 resonance

$$\begin{aligned}
U_{\Delta^-} &= U_{-\frac{1}{2}} &&= U_{is} + U_{iv}\\
U_{\Delta^0} &= \tfrac{2}{3}U_{-\frac{1}{2}} + \tfrac{1}{3}U_{\frac{1}{2}} &&= U_{is} + \tfrac{1}{3}U_{iv}\\
U_{\Delta^+} &= \tfrac{1}{3}U_{-\frac{1}{2}} + \tfrac{2}{3}U_{\frac{1}{2}} &&= U_{is} - \tfrac{1}{3}U_{iv}\\
U_{\Delta^{++}} &= U_{\frac{1}{2}} &&= U_{is} - U_{iv}
\end{aligned}$$

expressions for the isoscalar and isovector potentials

$$U_{is}(\rho,\beta,p)=\frac{A_u+A_l}{2}\frac{\rho}{\rho_0}+B\left(\frac{\rho}{\rho_0}\right)^{\sigma}(1-x\beta^2)+D\left(\frac{\rho}{\rho_0}\right)^{2}(1-y\beta^2)+\frac{C_l+C_u}{\rho_0}[I(p,p_F^n)+I(p,p_F^p)]$$

$$U_{iv}(\rho,\beta,p)=\frac{A_l-A_u}{2}\frac{\rho}{\rho_0}\beta-2x\frac{B}{\sigma+1}\left(\frac{\rho}{\rho_0}\right)^{\sigma}\beta-\frac{2y}{3}\left(\frac{\rho}{\rho_0}\right)^{2}\beta+\frac{C_l-C_u}{\rho_0}[I(p,p_F^n)-I(p,p_F^p)]$$

$$\text{with } I(p,p_F^{\tau})=\int d^3p'\frac{f_{\tau}(r,p')}{1+(p-p')^2/\Lambda^2}$$

Δ(1232) parameters for

$$U^{\Delta}_{is/iv}\approx U^{nucleon}_{is/iv}$$

Input		Parameters	
m^*_Δ	0.65	Λ	700.98
$U_0^\Delta(\rho_0, p=0)$	-67.0	C_l+C_u	-153.82
$U_0^\Delta(2\rho_0, p=0)$	-55.0	A_l+A_u	-26.15
$U_0^\Delta(\rho_0, p=\infty)$	+75.0	B	88.08
	.	D (fixed)	0.0
	.	σ (fixed)	1.465
δm^*_Δ	0.175	C_l-C_u	125.50
$U^\Delta_{1,sym}(\rho_0, p=0)$	+45.0	A_l-A_u	-109.97
$U^\Delta_{1,sym}(2\rho_0, p=0)$	+67.5	x	0.140
	.	y (fixed)	0.0

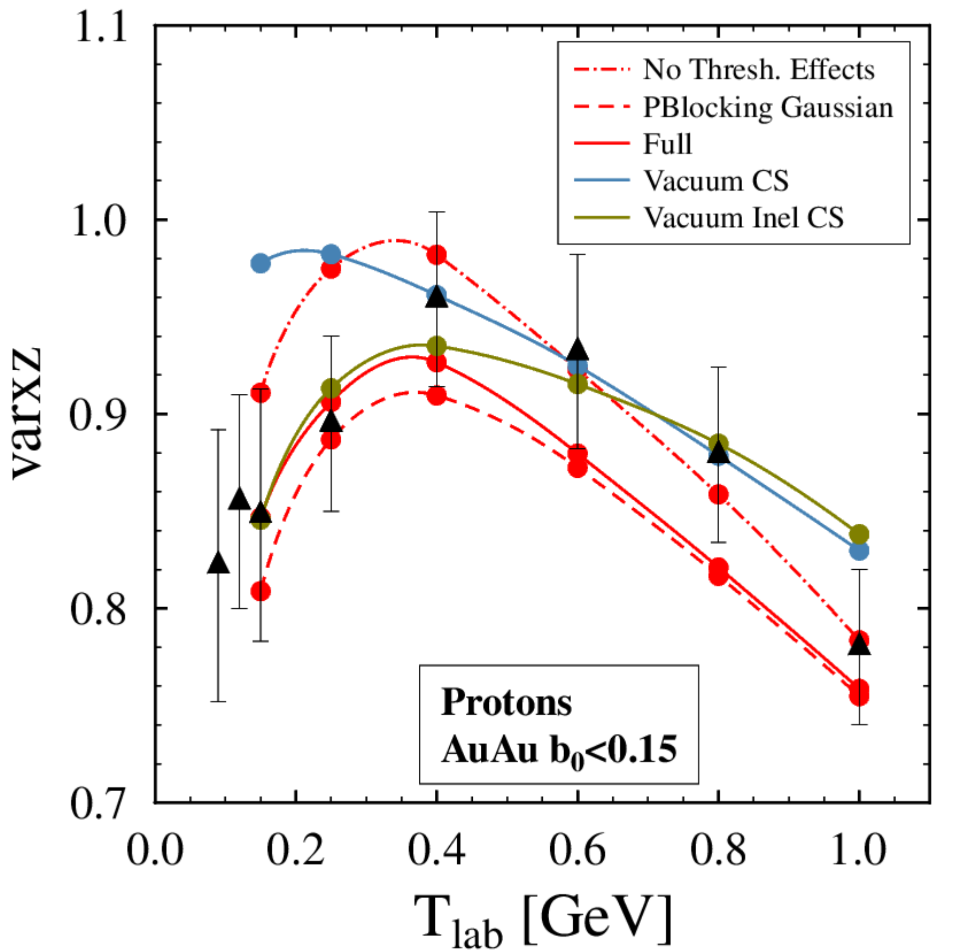
Model Benchmarking (nucleonic sector)

Proton - stopping (varxz)
- directed flow
- elliptic flow

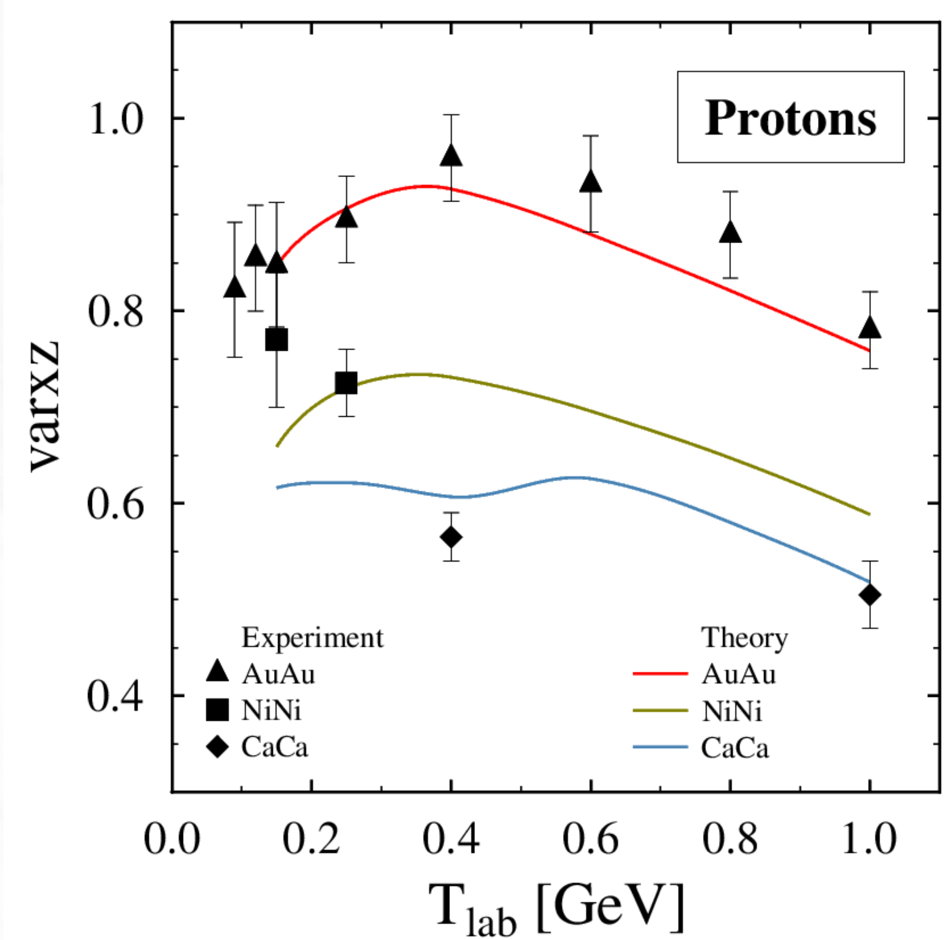
System size dependence
FOPI impact energies

Stopping

Model dependence



System size dependence

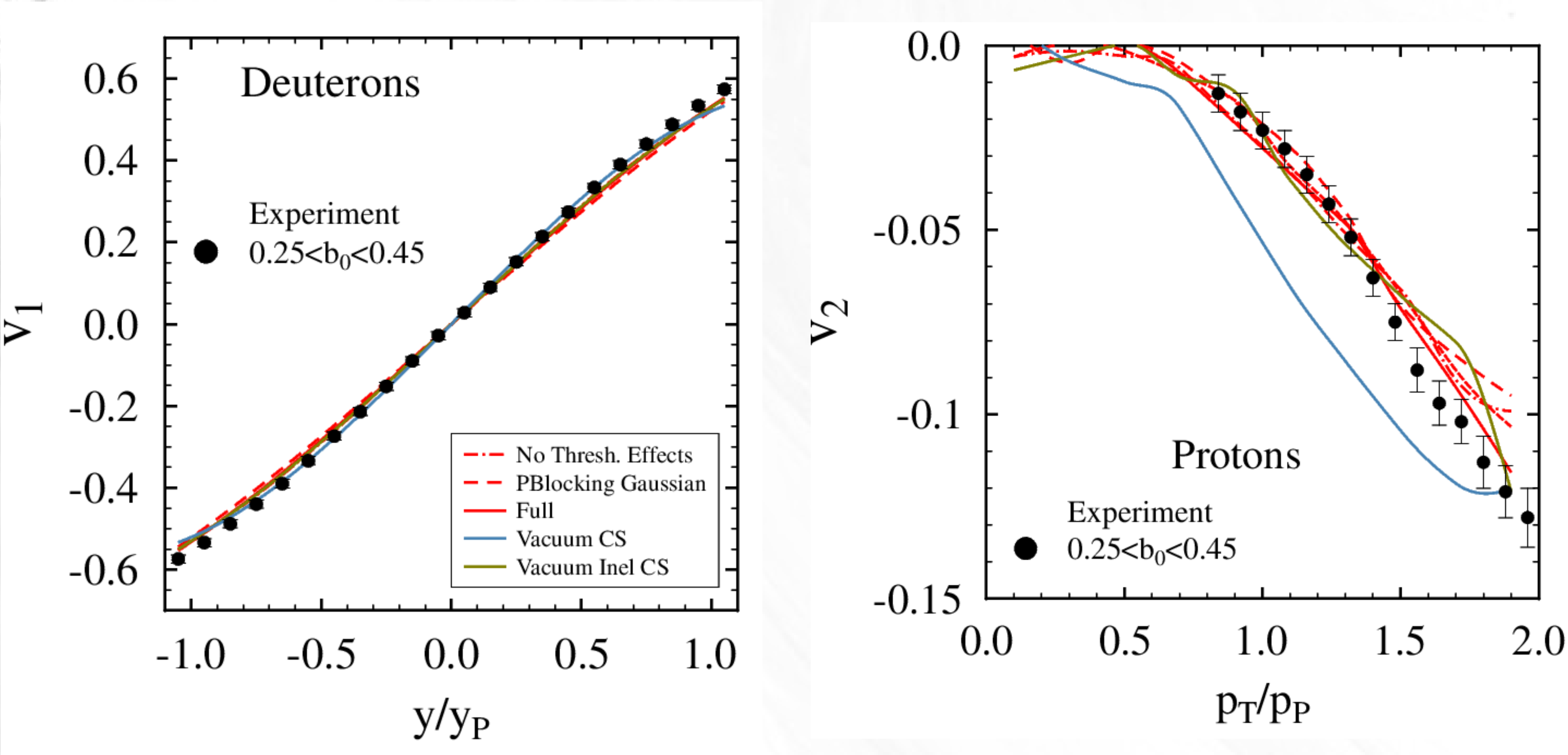


light cluster experimental stopping underestimated: deuterons (moderately), tritons (severely)

Experimental data: W. Reisdorf et al. (FOPI) NPA 848, 366 (2010)

Directed & Elliptic FLow

AuAu@250 MeV/nucleon



- similar quality of description of exp. data for p,d,t,α in the energy range 150-800 MeV/nuc
- system size dependence: some deviations for lighter systems (RuRu and CaCa)

Experimental data: W. Reisdorf et al. (FOPI) NPA 876, 1 (2012)

Pionic Observables

This study: Total charged pions multiplicity (PM)

Ratio of PM between systems with different isospin asymmetry

Charged Pions Multiplicity Ratio (PMR)

Experimental data: - FOPI collaboration (CaCa @ 400, 600, 800 MeV
RuRu,ZrZr, AuAu @ 400 MeV)

W. Reisdorf et al., NPA 848, 366 (2010)

- SΠRIT collaboration (^{108}Sn^{112}Sn,^{132}Sn^{124}Sn @ 270 MeV)

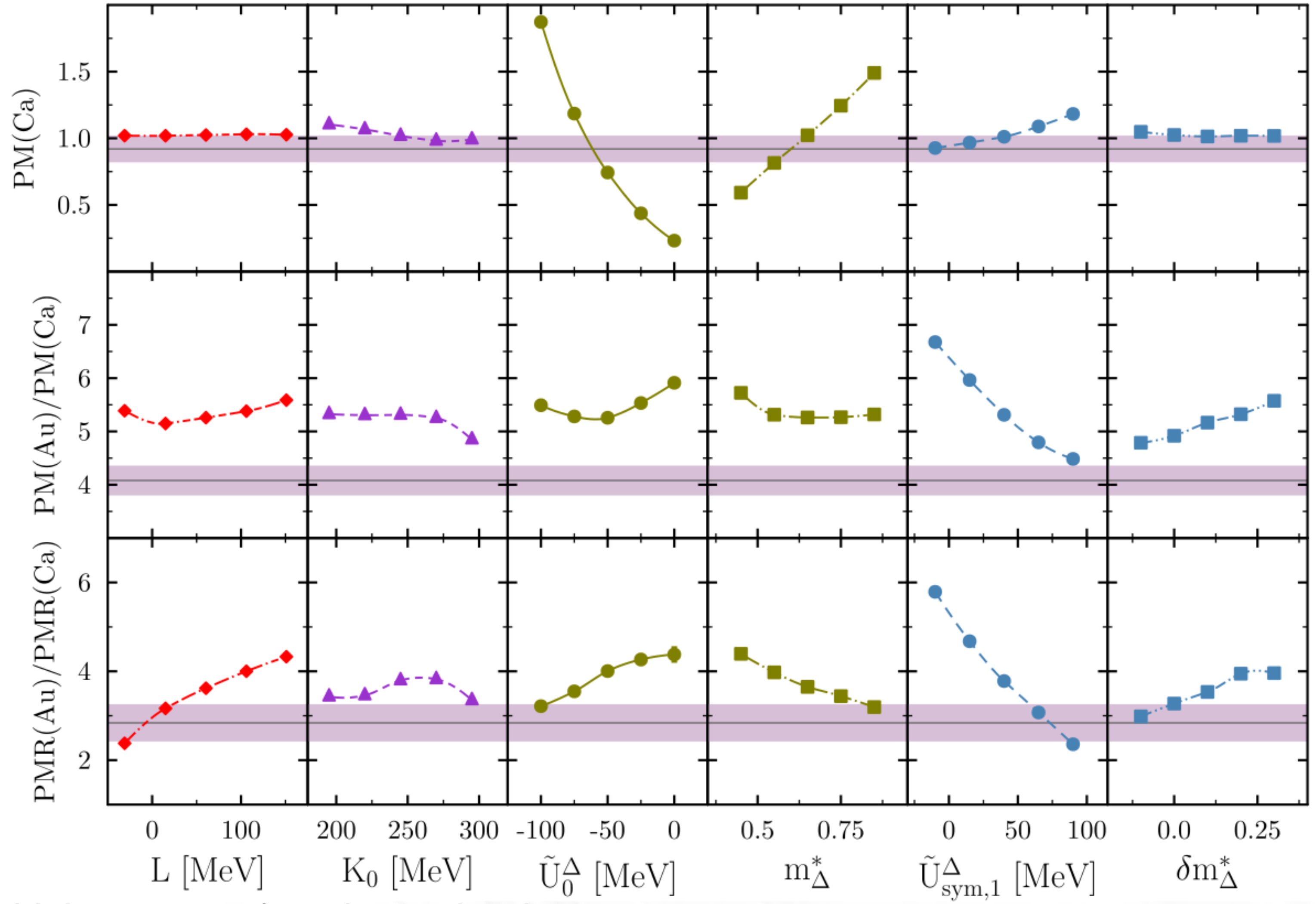
G. Jhang et al., PLB 813, 136016 (2020)

- HADES collaboration (AuAu @ 1.23 GeV)

J. Adamczewski-Musch et al. (HADES), EPJA 56, 259 (2020)

Impact of Δ(1232) at saturation

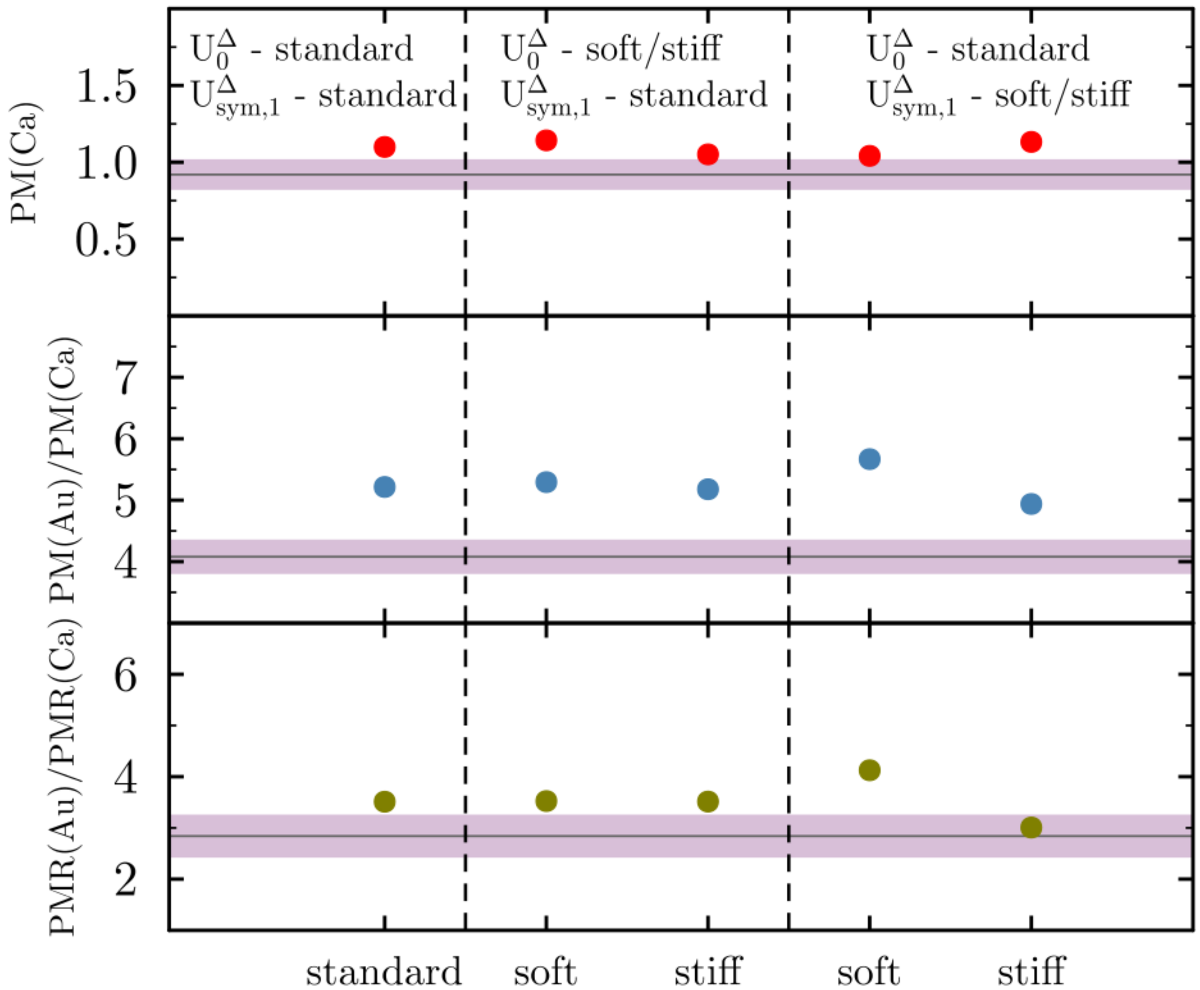
CaCa, AuAu @ 400 MeV/nuc, b_0<0.15 (FOPI)



- sensitivity w.r.t. U_0^Δ and m^*_Δ dominant
- sensitivity w.r.t. to isovector potential depends on system's isospin asymmetry: two contributions of opposite sign due to average asymmetry and fluctuations

=> closest to zero for RuRu rather than CaCa

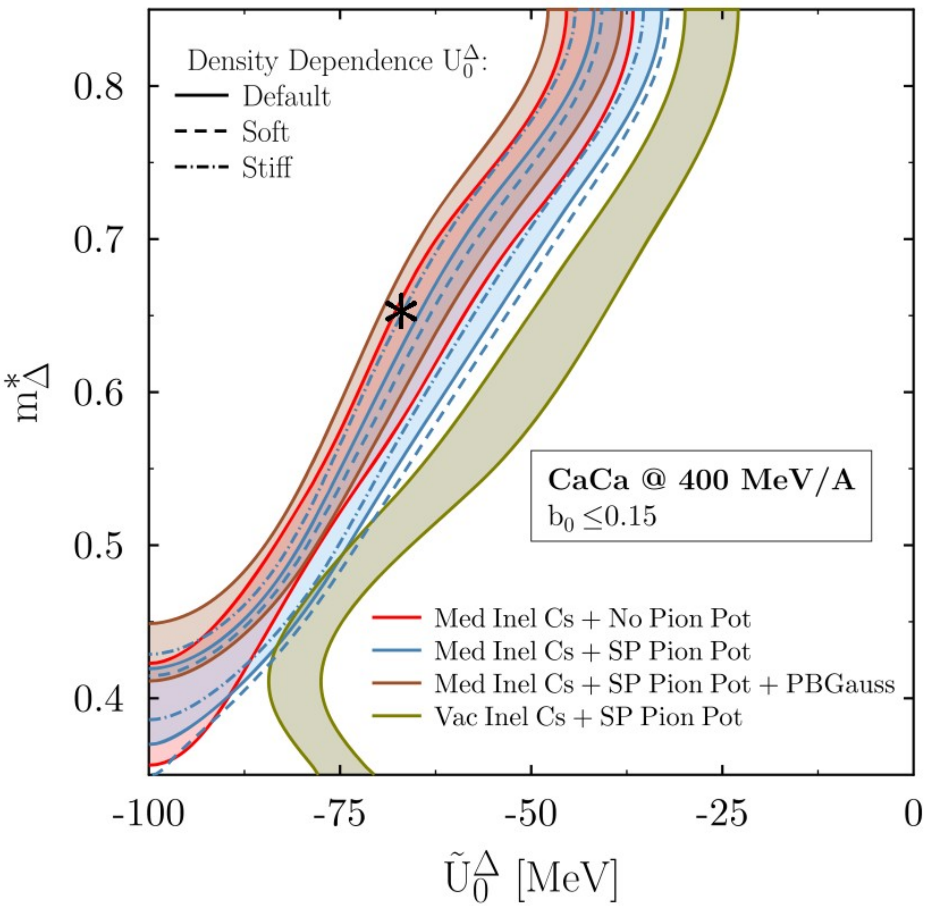
The high density Δ(1232) potential

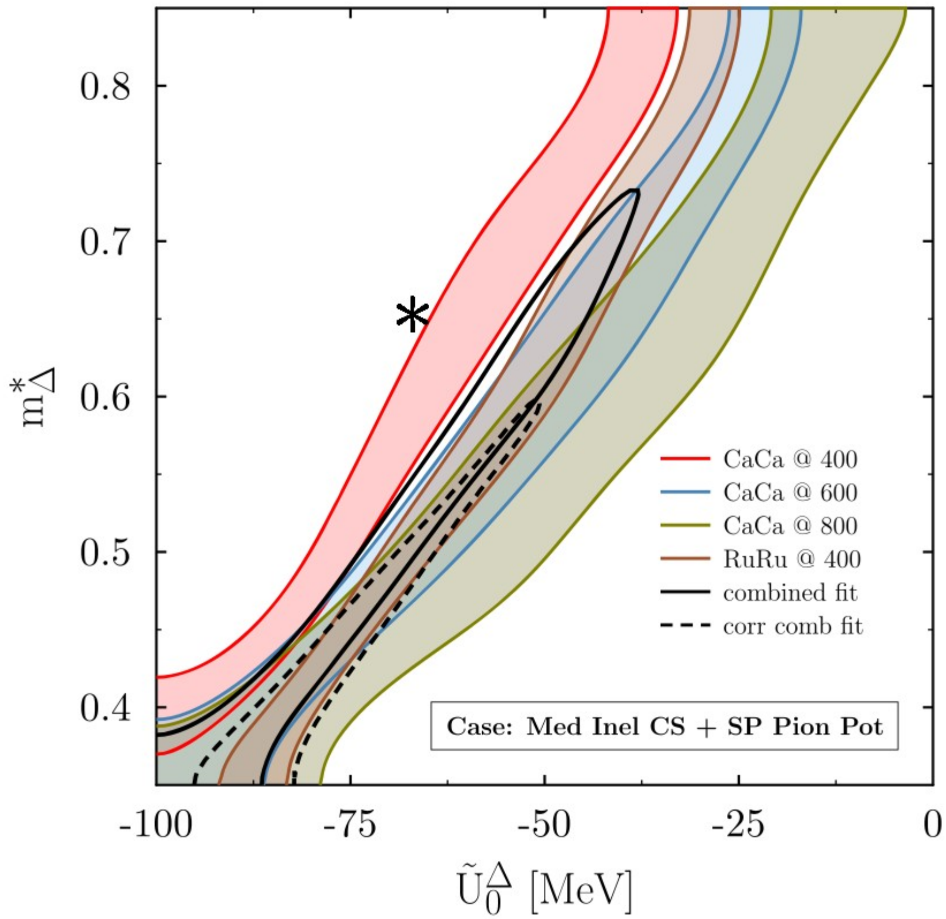


- sensitivity w.r.t. high density part of U_0^Δ ~ 10%

$U^\Delta_{sym,1}$ ~ 30%

Constraints for the Isoscalar Δ Potential

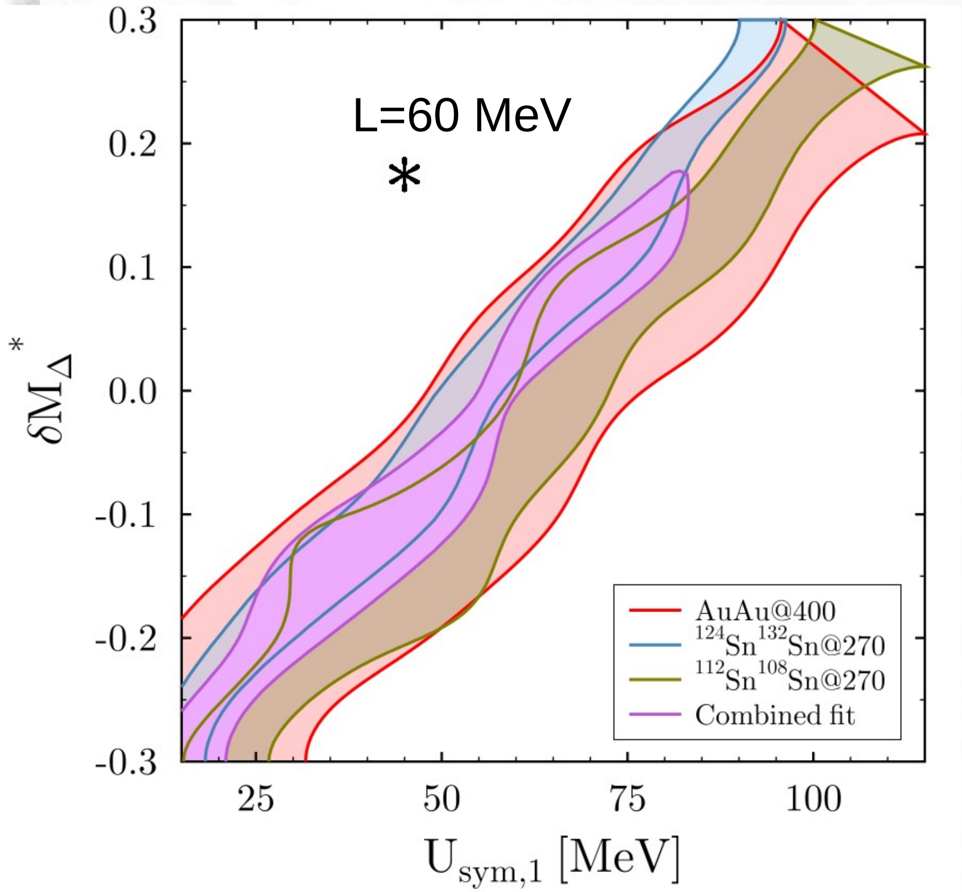


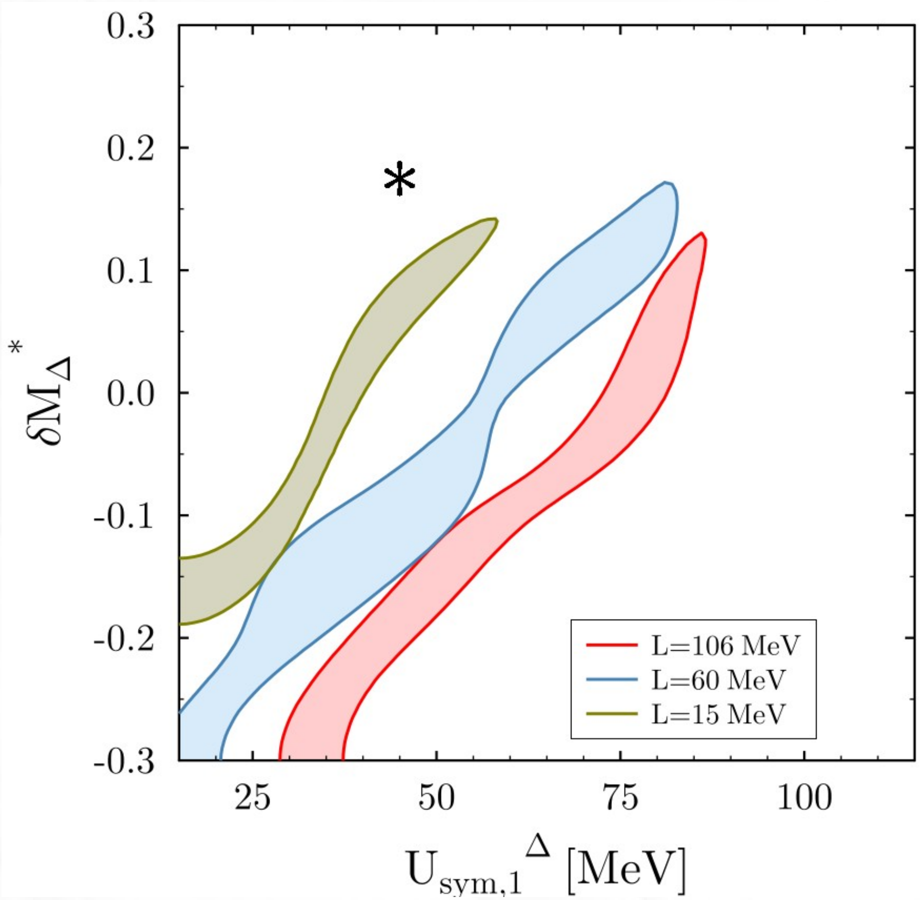


Constraints for the Isovector Δ Potential

π^-/π^+ multiplicity ratio

$\overline{U}_0^{\Delta}$=-78 MeV and m^*_Δ=0.45 adopted





- higher impact energy systems (AuAu 0.6, 0.8 GeV/nucleon) not effective
- non-negligible dependence on the value of L

How about the Symmetry Energy?

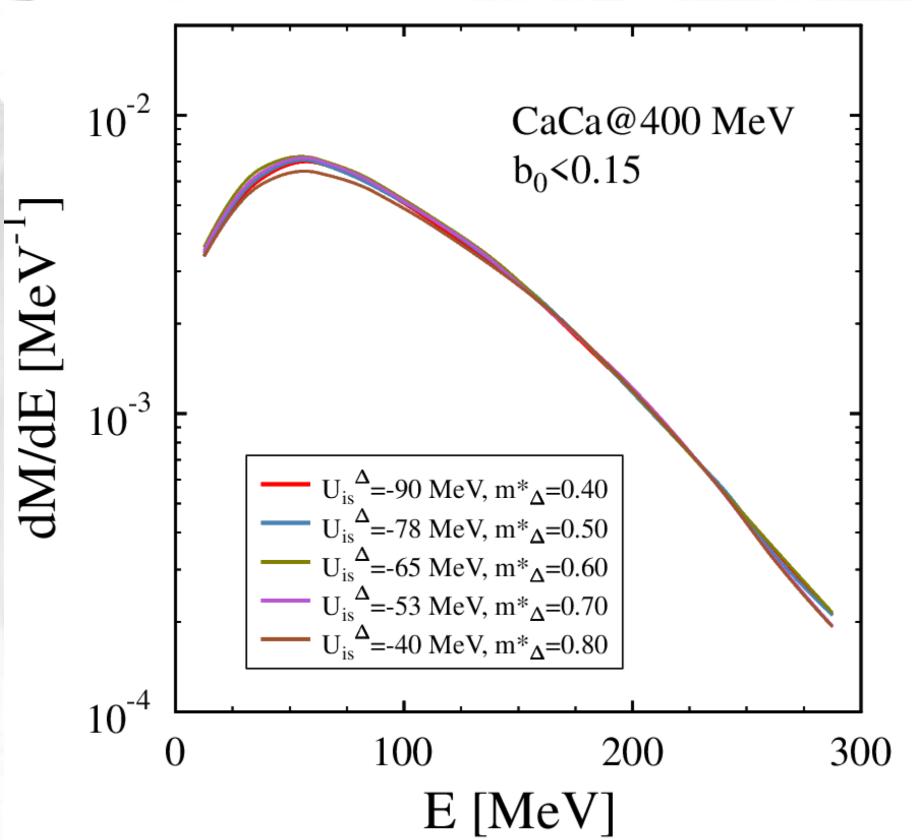
Observation: Sensitivity to L is sizable, but its extraction is obstructed by the unknown Δ(1232) potential

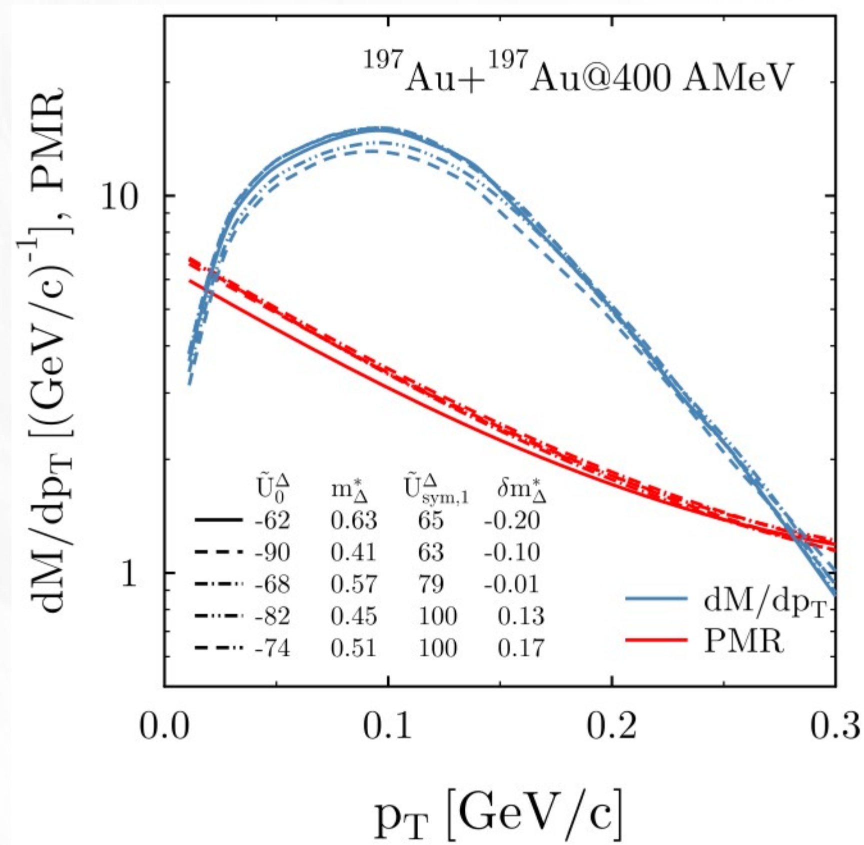
Neglected Fact: Close to threshold Δ and pion production is perturbative => pion spectra do not depend on Δ(1232) potential parameters if multiplicities are fixed (or the experimental values described). But they do depend on parameters of the nucleon interaction (and hence on L, $\delta m^*_{n\text{-}p}$, etc)

Consequences:

- The symmetry energy can be studied by a comparison theory-experiment for spectra, **once** the values of the Δ(1232) potential parameters lie on the subspace that allows the description of integrated multiplicities => normalization constants
- The computational effort is reduced considerably

Numerical check of pertubative production





Isovector Δ potential set equal to that
of nucleon; choice is in principle important,
shown simulation are however for CaCa

PM within 5% of the exp value

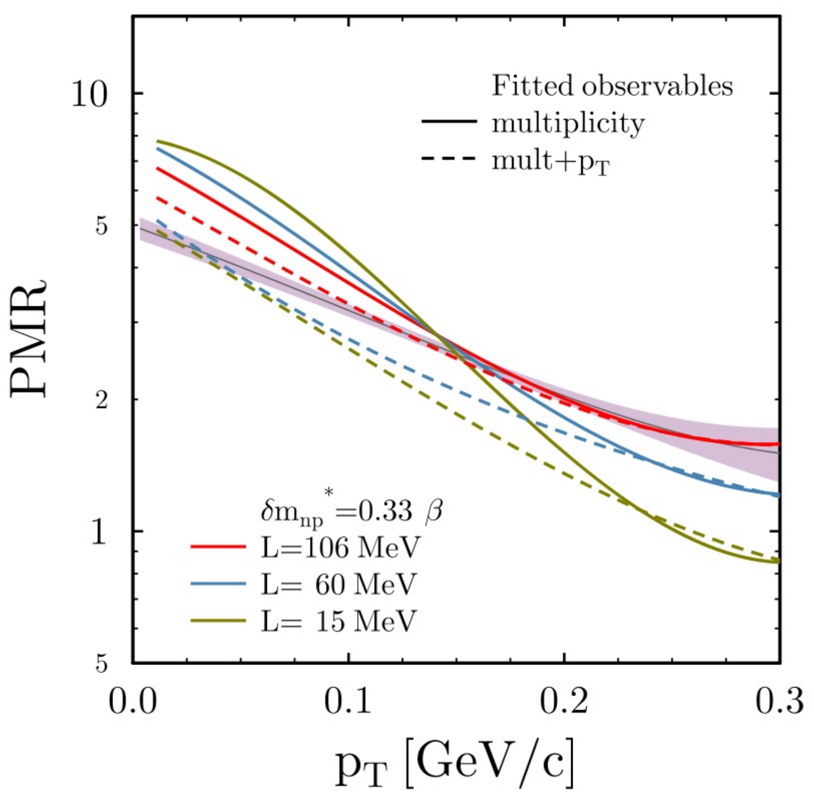
- fit in the full 4 dim parameter space
- deviations due to finite accuracy of experimental data and relatively few points in parameter space available for interpolation

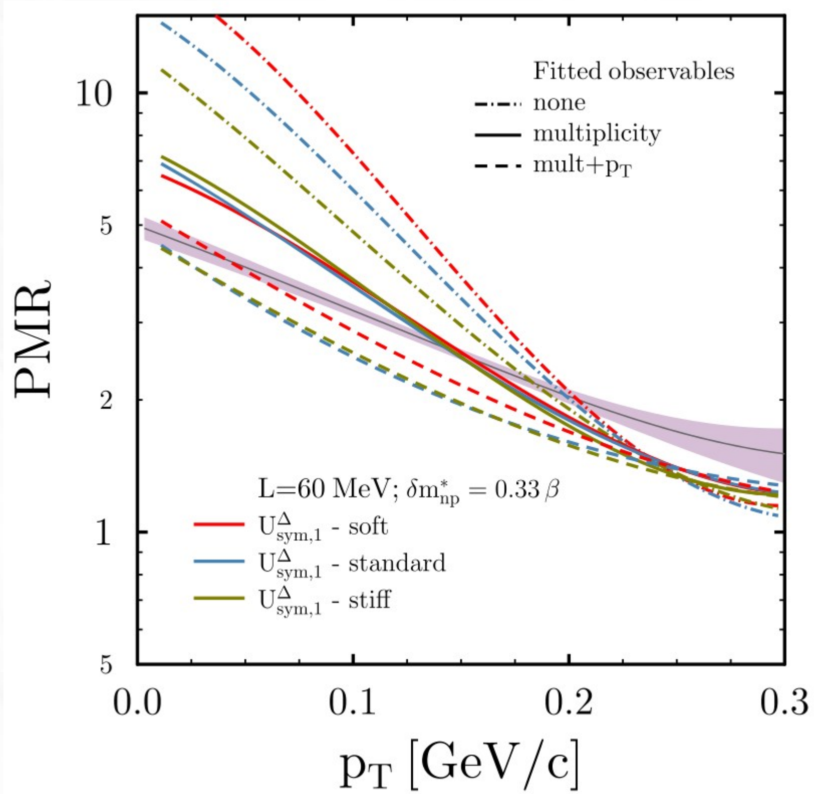
Constraining the SE: FOPI data

AuAu @ 400 MeV/nucleon 3.35 fm<b<6.0 fm

- Fitted observables: PM, PMR
- Fix parameters: m^*_Δ=0.45, δm^*_Δ=0.0
- Fitted parameters: $\overline{U}^\Delta_0$,$\overline{U}^\Delta_{sym,1}$

- Observables described within 3%
- Assumed exp uncertainty: the same relative error as for central collisions



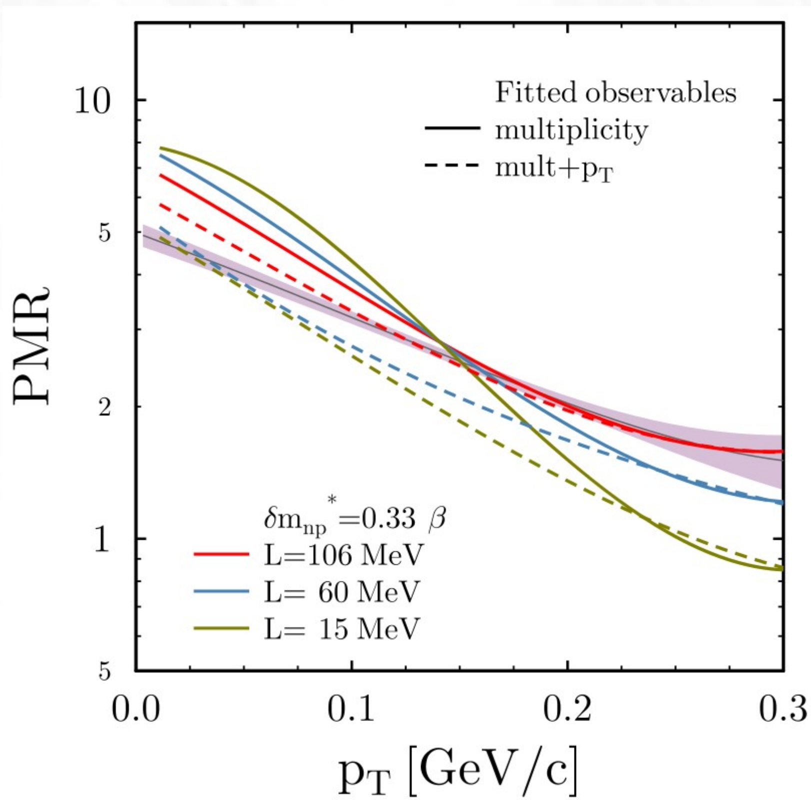


- FOPI spectra: missing low p_T pions (W. Reisdorf, only statistical error available)
- model-dependence sources: δm^*_{n-p} (here 0.33β was used)
  S_0 -value of symmetry energy at saturation

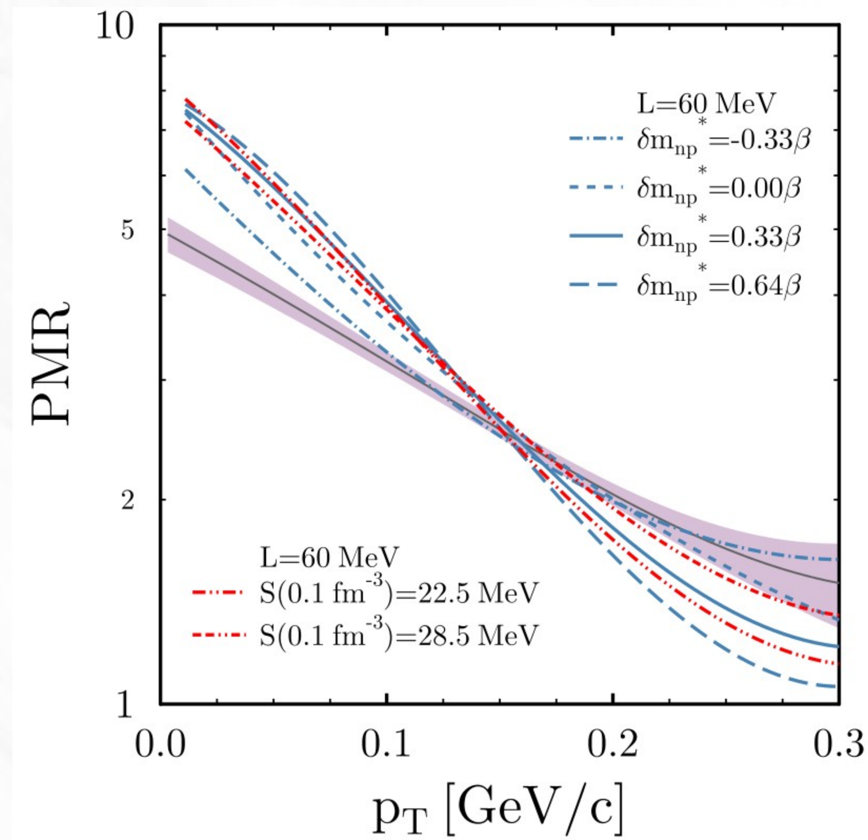
Constraining the SE: model dependence

AuAu @ 400 MeV/nucleon 3.35 fm<b<6.0 fm

sensitivity to slope of SE



- neutron-proton effective mass difference (npEMD)
- magnitude of symmetry energy at ρ=0.1 fm^{-3}



Additional model dependence (<10% effect at large p_T)

- in-medium modification of inelastic cross-sections
- pion S and P wave optical potentials

(see EPJA 57, 309 (2021))

Constraining the SE: SπRIT data

J. Estee et al., PRL 126, 162701 (2021)

Model Features:

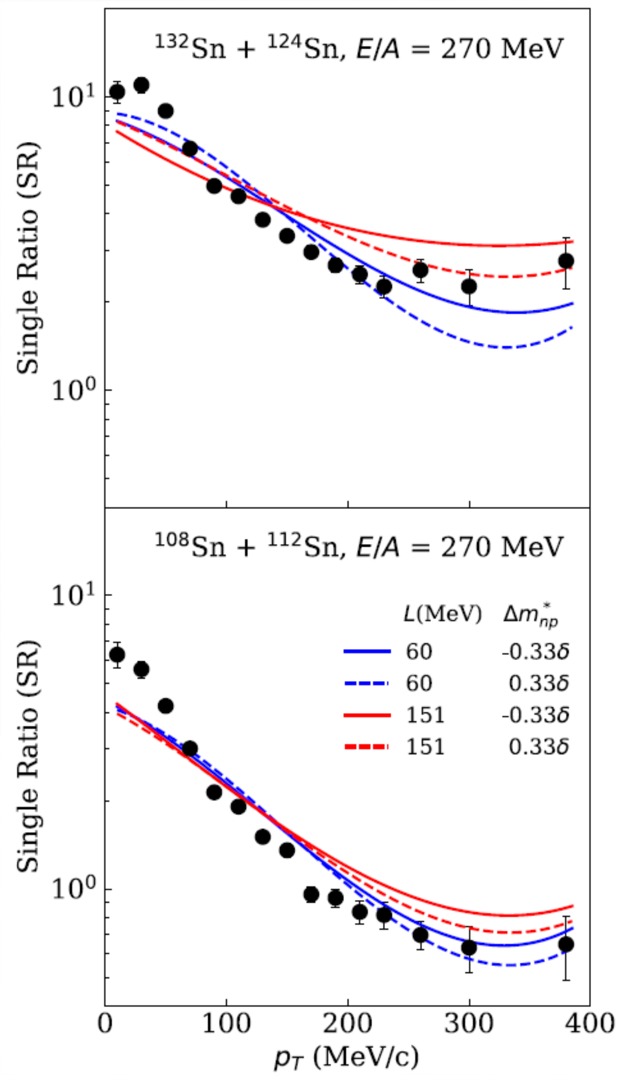
- pion production mechanism in dcQMD: excitation of baryonic resonances, predominantly Δ(1232)
- pion production close to threshold: dominated by non-resonant production channels (meson exchange currents etc)
- tails of spectra (p_T>200 MeV/c) correspond to impact energies > 600 MeV

Assumpsions:

K_{sym}=-488+6.728 L [MeV]

Constraint:

L=79.9±37.6 MeV S_0=35.3±2.8 MeV



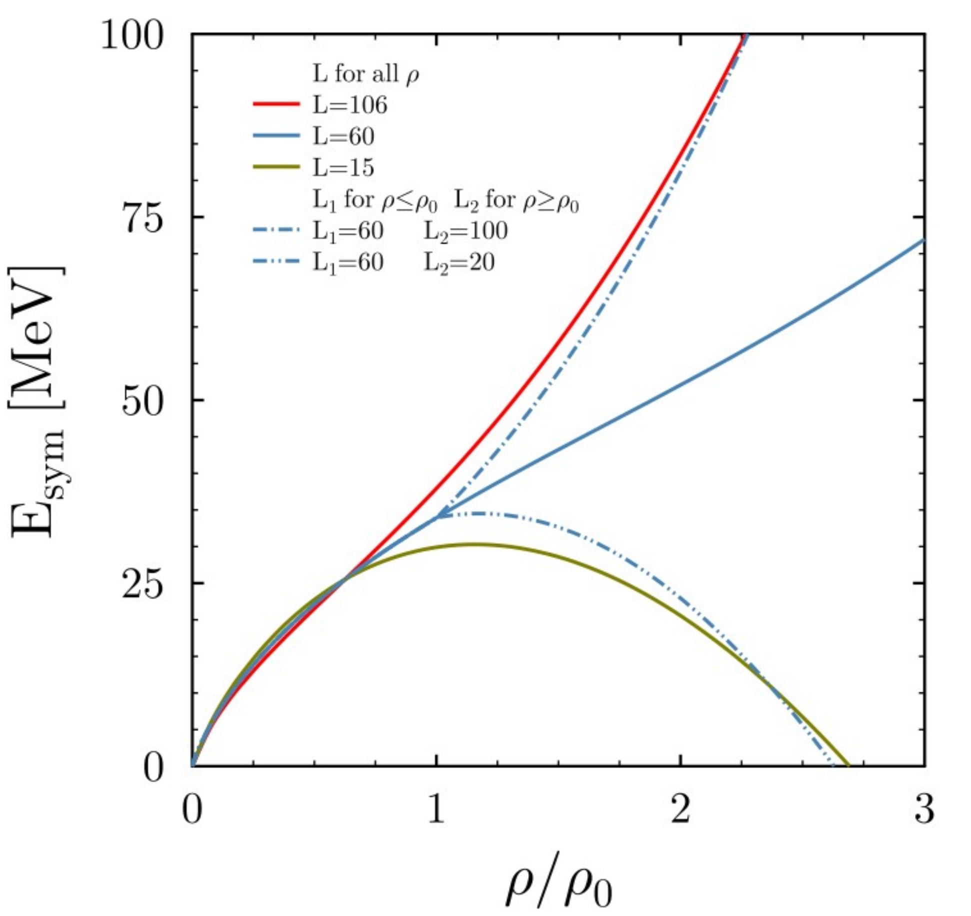
Analysis of ASYEOS+FOPILAND (n/ch and n/p v_2 ratios)

D.C., EPJA 54, 40 (2018)

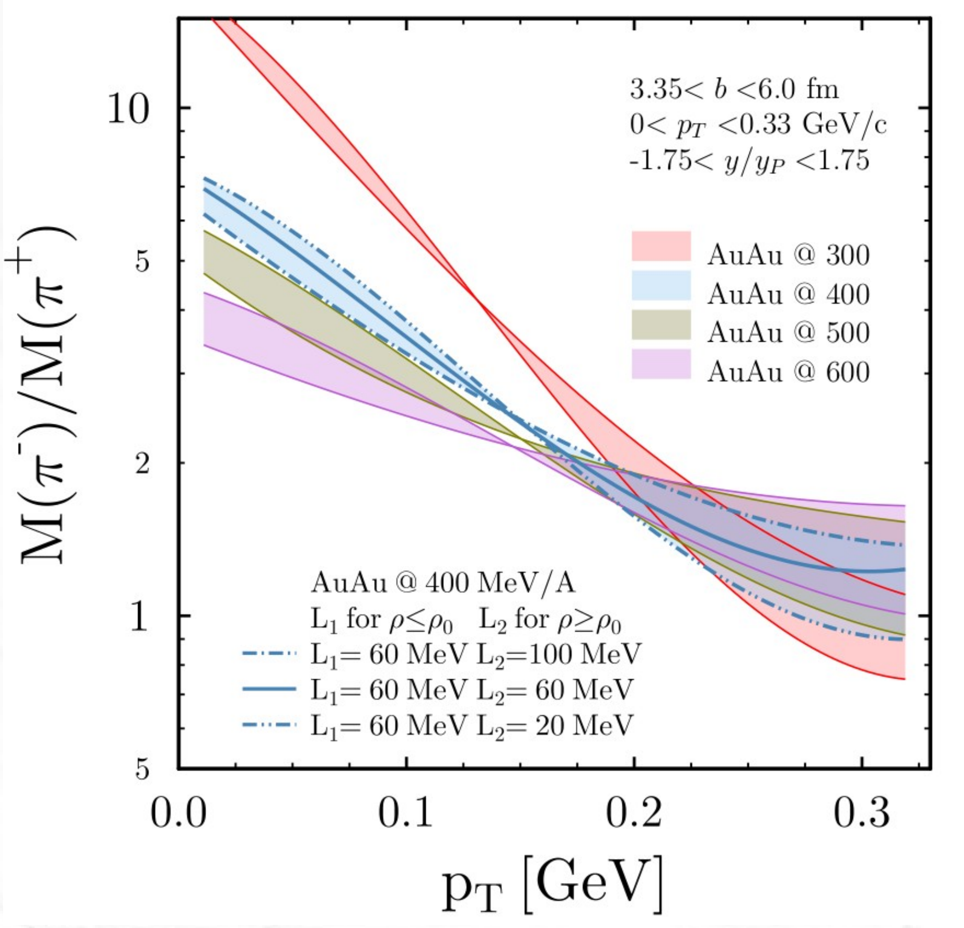
$$L = 85 \pm\ 22(\mathrm{exp}) \pm\ 20(\mathrm{th}) \pm\ 12(\mathrm{sys})\ \mathrm{MeV}$$
$$K_{sym} = 96 \pm 315(\mathrm{exp}) \pm 170(\mathrm{th}) \pm 166(\mathrm{sys})\ \mathrm{MeV}$$

High Density Probe

- density dependence above saturation



sensitivity of the probe (PMR for p_T>0.22) at different impact energies



High Density Constraint (SΠRIT)

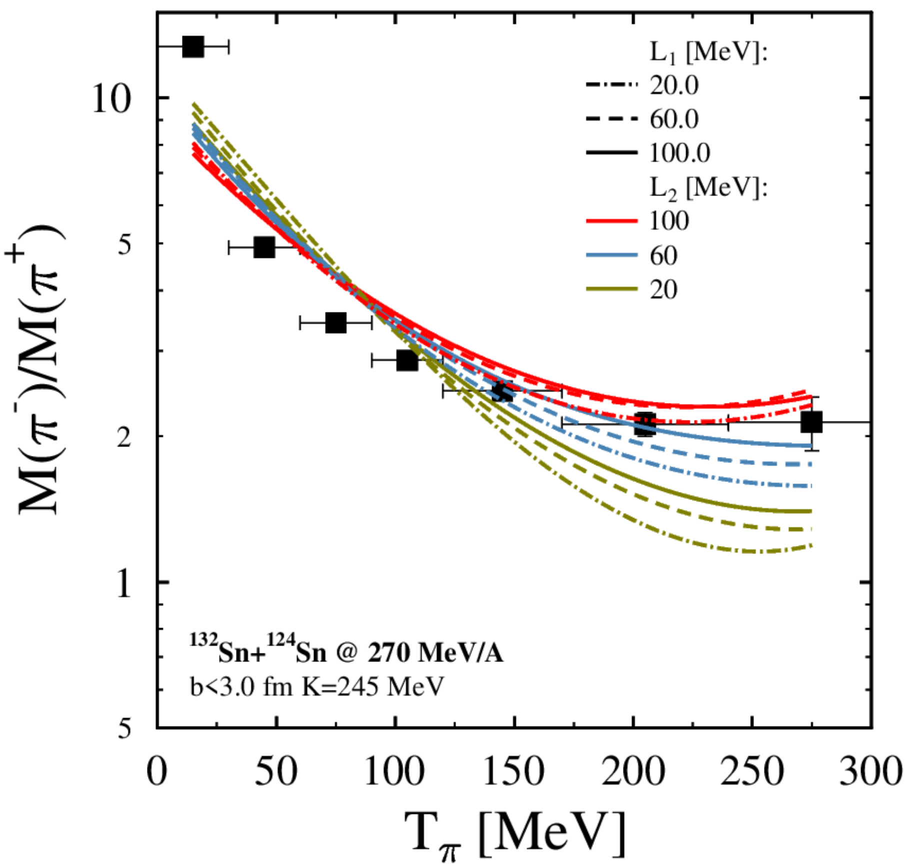
Complementary to the published SΠRIT result: L=79±37 MeV, S_0=35.3±2.8 MeV

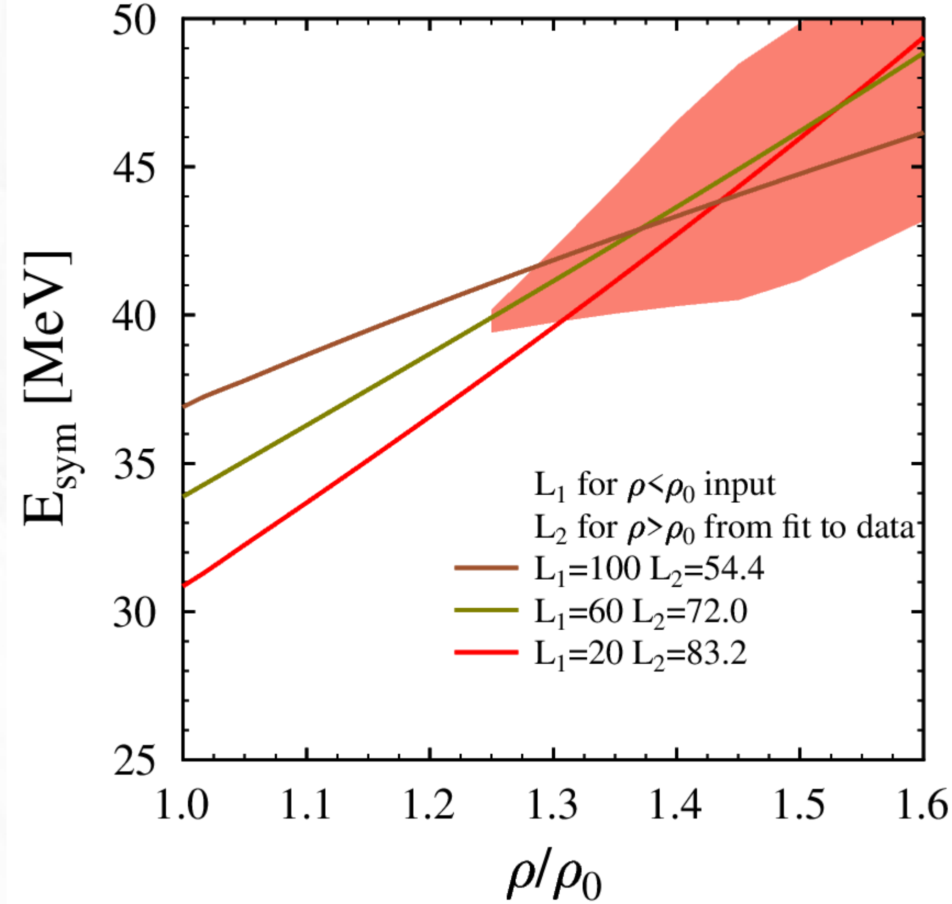
J. Estee et al., PRL 126, 162701 (2021)

npEMD= -0.14 β

T. Malik et al., PRC 98, 064316 (2018)

Constraint: S(1.45±0.05 ρ_0)= 45 ± 4 MeV





Additional uncertainties: - fluctuations modify density dependence of EoS (- 1.0-1.5 MeV)

npEMD (+0.06 β => -1.0 MeV)

- uncertainties in m* and m^*_v => ± 1.0 MeV

ChPT result:
S(1.45 ρ_0)= 38 ± 1 MeV
C. Drischler, PRC 102, 054315 (2020)

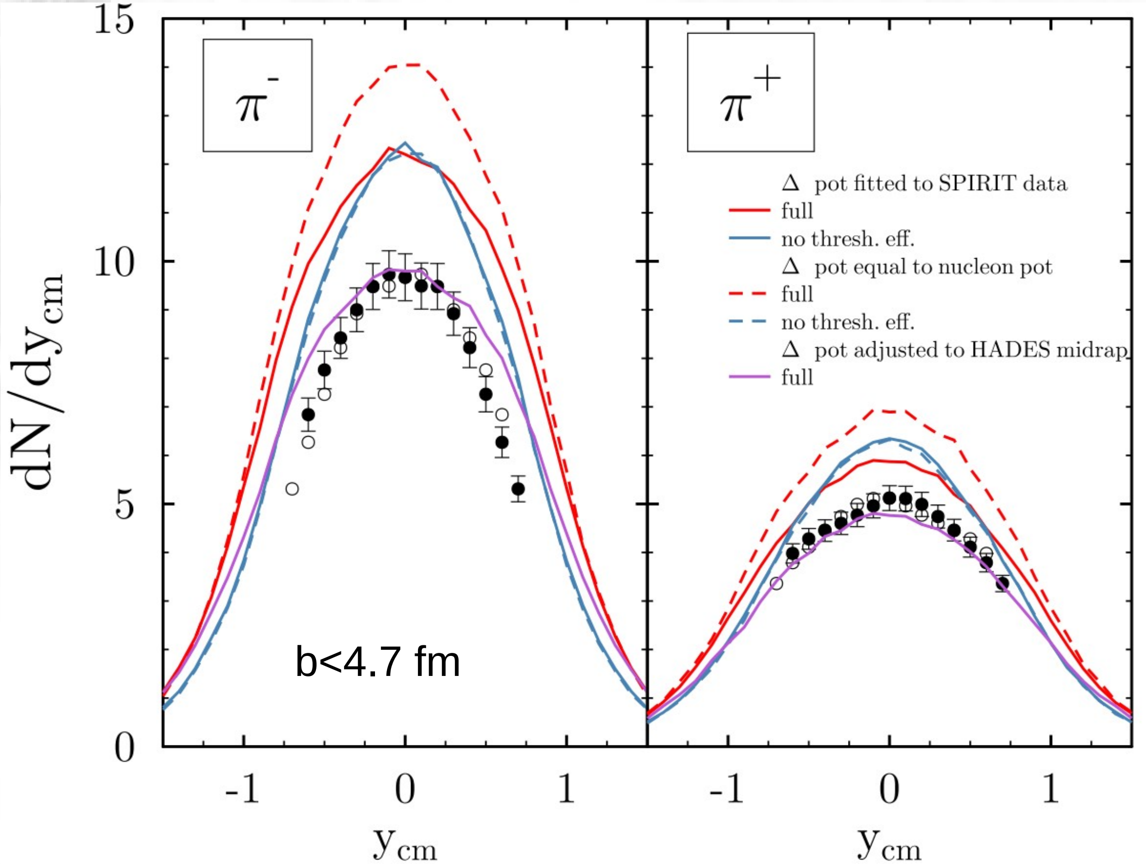
Constraint: S(1.45±0.1 ρ_0)= 43 ± 4 MeV

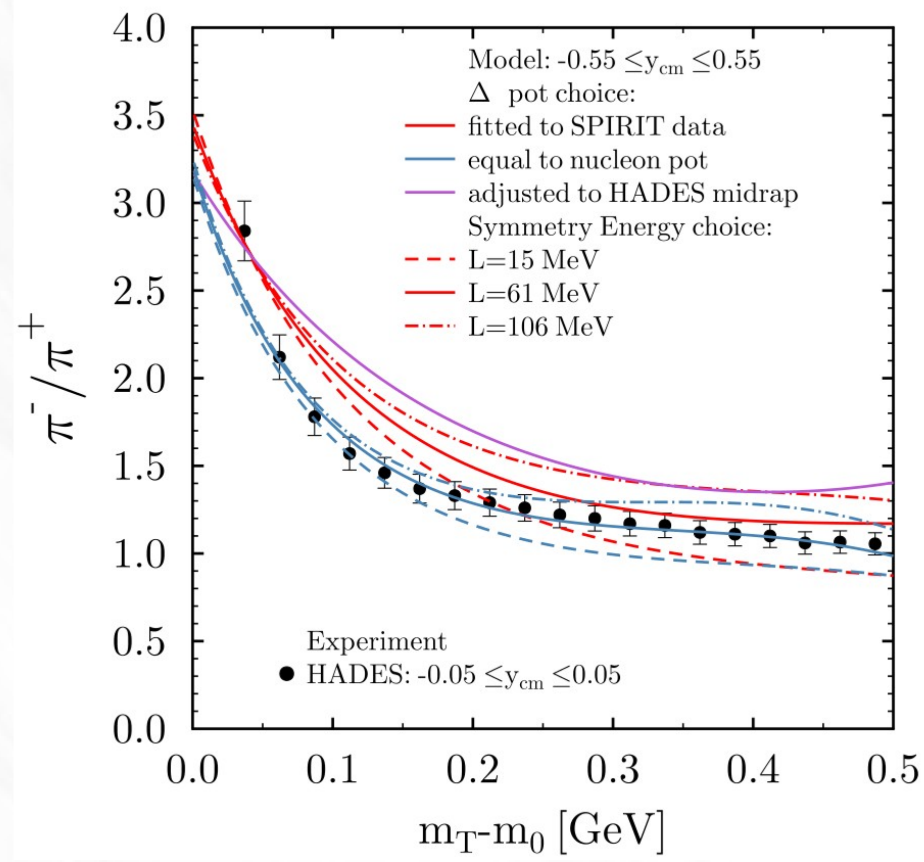
Pion production in AuAu at 1.23 AGeV

J. Adamczewski-Musch et al. (HADES), EPJA 56, 259 (2020)

Note: HADES pion total yield are larger then FOPI yields at 1.2 GeV by about 30%

very preliminary calculation addressing the feasibility of studying the symmetry energy



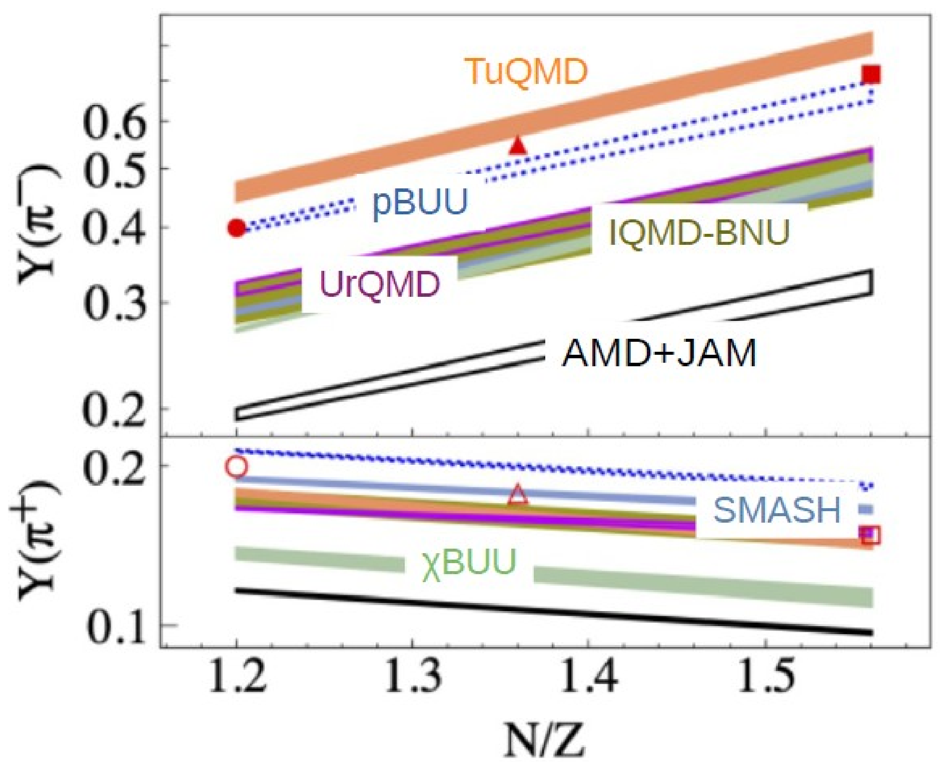


-alternative approach to a systematic description of HADES rapidity and transverse mass spectra: K. Godbey et al. PLB 829, 137134 (2022)

Outlook

G. Jhang et al., PLB 813, 136016 (2021)

locally extendedTuQMD renamed dcQMD



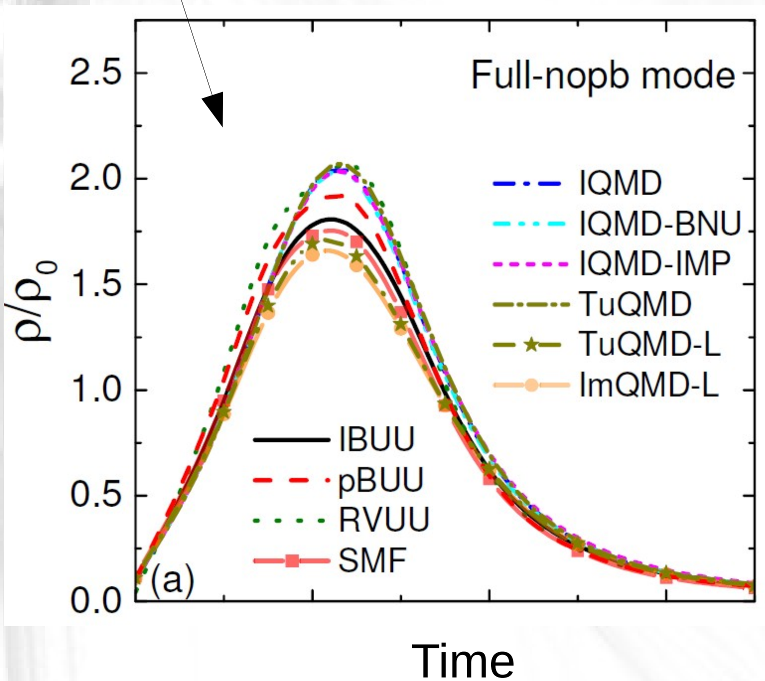
Total charged multiplicities vary by a factor of 2.5
Pion multiplicity ratios and double ratios vary by factors of 2.0 and 1.2 respectively

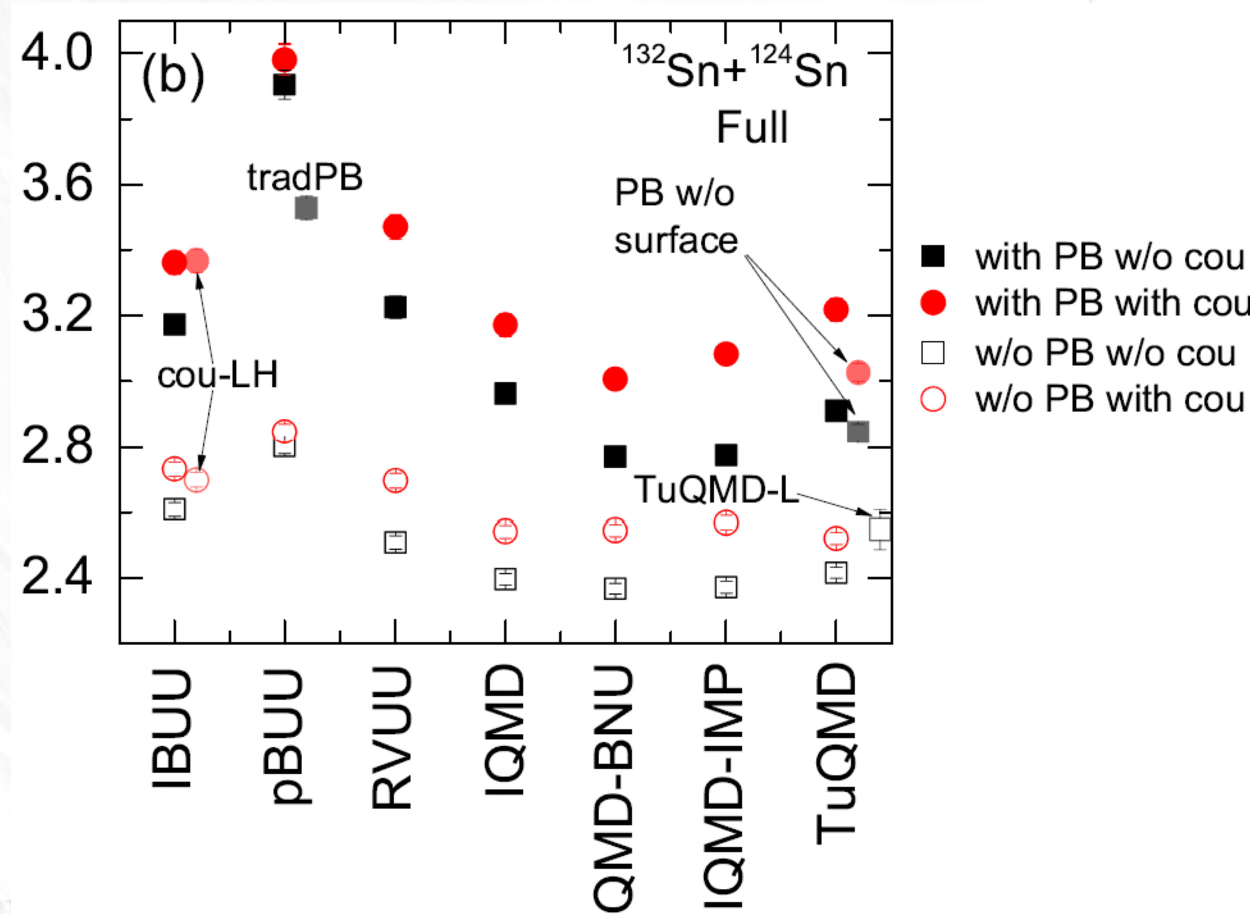
Model	Features
χBUU	Threshold Effects; MDI Skχm* ; Bertsch elastic CS; Huber Inelastic CS;
TuQMD	Threshold Effects; MDI2 (Gogny); Li-Machleidt + Cugnon+ med corrections; Huber inelastic CS+ med corr
pBUU	MDI – Landau EF; medium modified elastic CS (stopping); Huber inelastic CS
AMD+JAM	SLy4+mom dependence; cluster correlations; med modified elastic CS; Randrup inelastic CS
IQMD-BNU	MI interaction; Cugnon elastic CS +Li-Machleidt med corr; Bertsch inelastic CS
SMASH	MI interaction; Cugnon elastic CS; Dmitriev OBE inelastic CS; no Coulomb
UrQMD	MDI interaction (old HA); PDG elastic CS+med corrections; modified Huber OBE model (mass dependent amplitudes)

Transport Model Evaluation Project (TMEP)

J. Xu et al. (in preparation):
- 3 BUU + 4 QMD transport models
- common initialization
- momentum independent interaction
- realistic HIC: SnSn @ 270 MeV/nucleon (b=4.0 fm)
- test mean-field propagation, collision term, Coulomb

average density in a
5x5x5 fm^3 cube centered at r=0



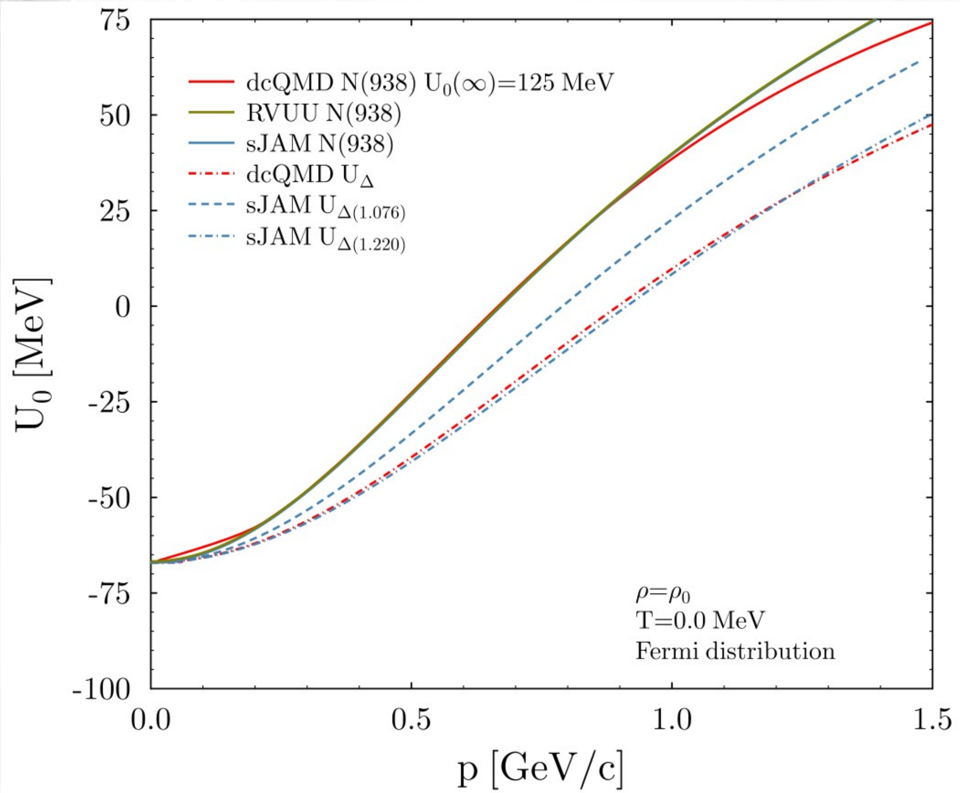


convergence of BUU and QMD-L predictions for the single ratio at 5% level can be achieved

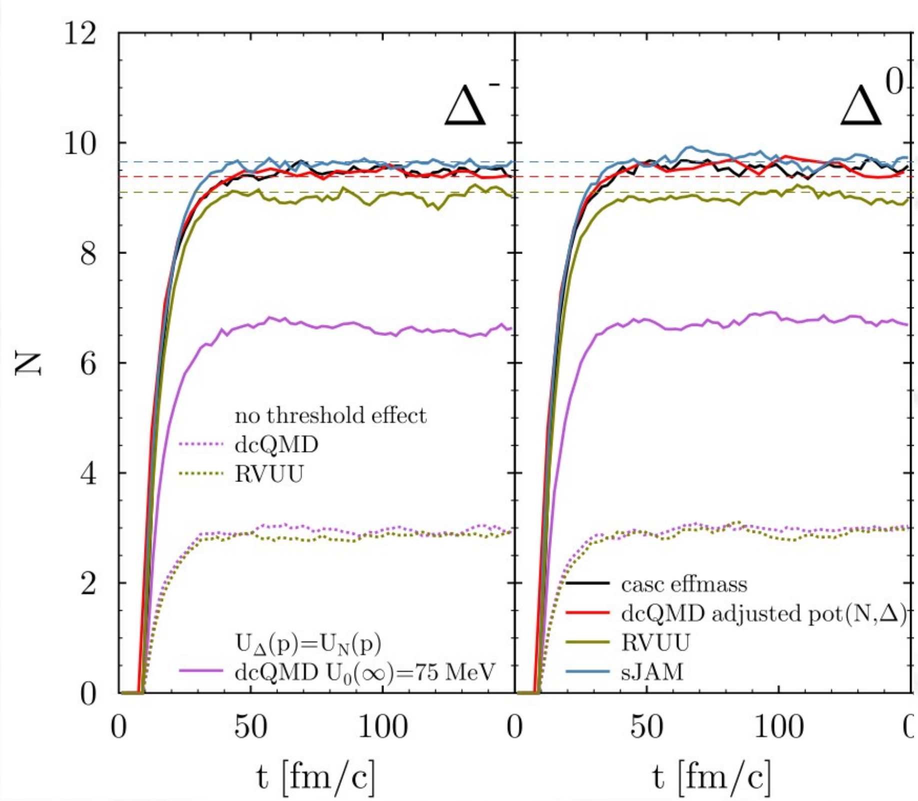
Transport Model Evaluation Project (TMEP)

D. Cozma et al. (work in progress):
- 1 BUU and 2 QMD models
- box calculation (symm and asym, T=60 MeV)
- test momentum dependence of the interaction
- test implementation of threshold effects

Momentum dependence of the interaction



Multiplicities



results compared among codes and to thermal models: very similar up to small differences in the MDI, finite time step size (where applicable) and possibly higher order correlations in the collision term

see also Z.Zhang et al, PRC 97, 014610 (2018)

Summary & Conclusions

Study of the symmetry energy:

- Integrated pion multiplicities highly sensitive to quantities besides EoS (can be used to fix Δ(1232) potentials in nuclear matter)
- possible by studying the high energy tail of the spectral ratio
- pion probe supra-saturation densities and HIC significantly above threshold can be used to study the symmetry energy
- extraction of the magnitude of symmetry energy **$S(1.45 \pm 0.1\ \rho_0) = 43 \pm 4$ MeV**
- consistency with result of a similar analysis of FOPI-LAND+ASYEOS data for v2 ratio of n/p and n/ch
- proof of feasibility of studying the symmetry energy above $2\rho_0$ using pion production (HADES)

Outlook:

Question: Can a consistent interpretation of experimental data using several transport models be accomplished ?

Answer: TMEP It appears feasible but additional (substantial) work required as pion observables highly sensitive to even small differences/approximations (treatment of the three-body term, momentum dependence, threshold effects)