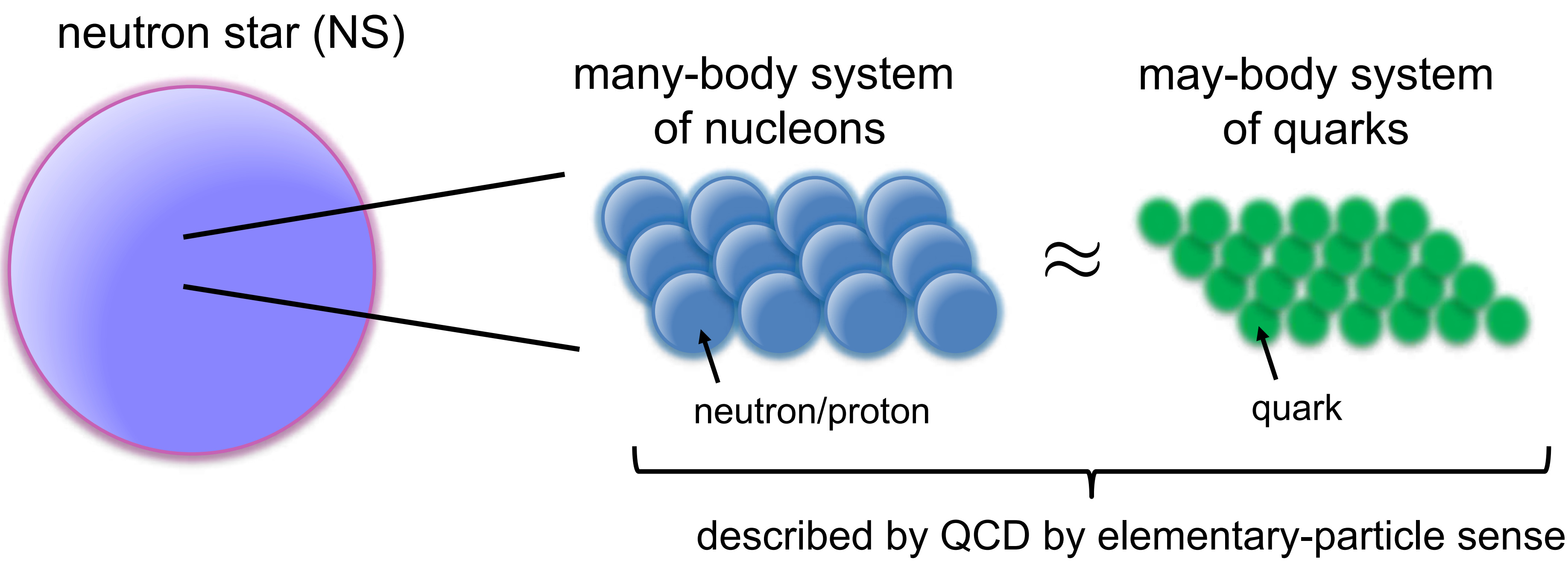
ハドロン有効理論を用いた音速ピーク構造の再現:

有限密度系2カラーカイラル有効理論による平均場解析

末永大輝 (理研 仁科センター)

Introduction

- **Neutron star and QCD**

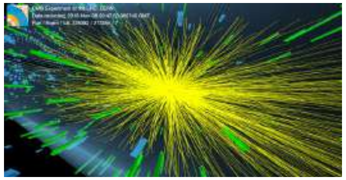


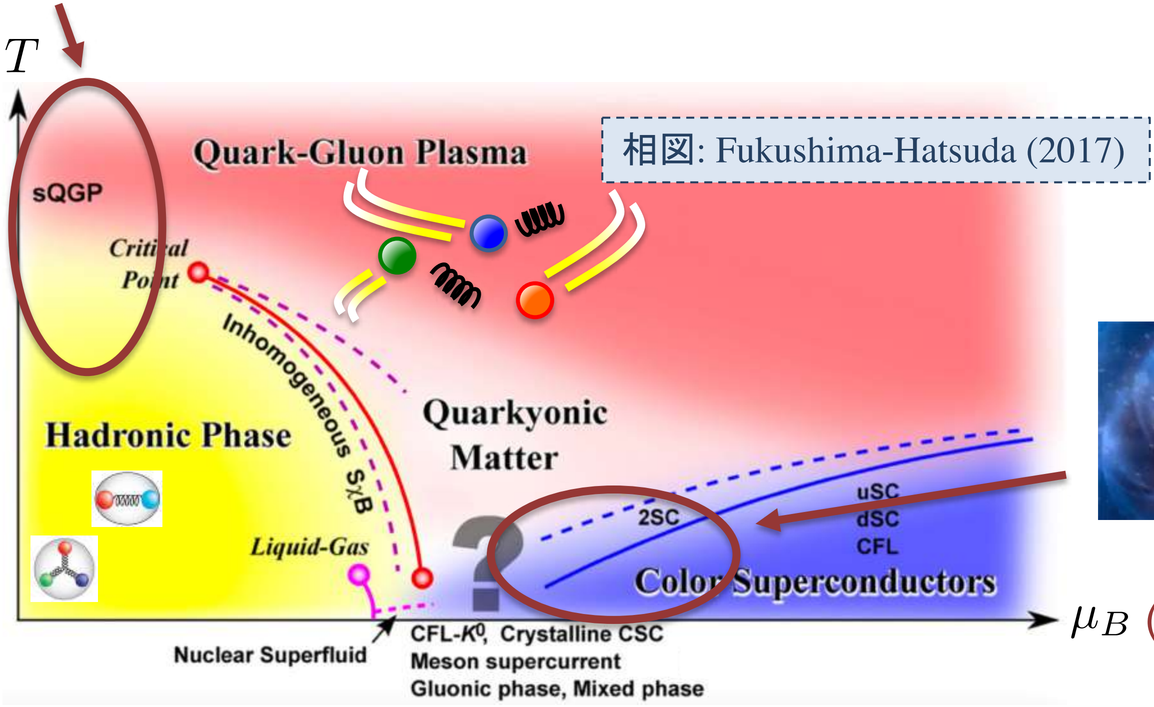
- Delineation from QCD is inevitable for understanding of center of NS

Introduction

- **QCD phase diagram**

Heavy-ion collision at LHC etc. (high T and low μ)





μ_B (low μ and high T)

NS etc.

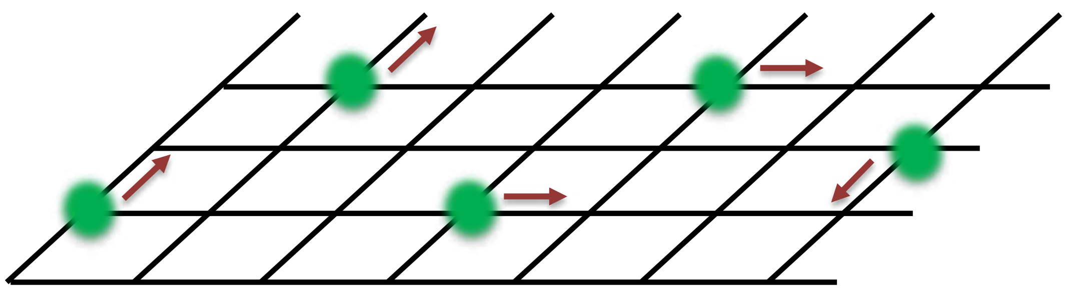
QCD at temperature and density

- Prediction of various states: quark-gluon plasma, color superconductivity, chiral restoration, $U(1)_A$ anomaly change, etc.

Introduction

- **Lattice QCD simulation**
  - Lattice QCD is a first-principle numerical simulation for delineating QCD

$$Z = \int DAD\psi D\bar{\psi}\, \exp\left[i \int d^4x \left(-\frac{1}{2}\mathrm{tr}[G_{\mu\nu}G^{\mu\nu}] + \bar{\psi}(i\not{D} - m)\psi\right)\right]$$



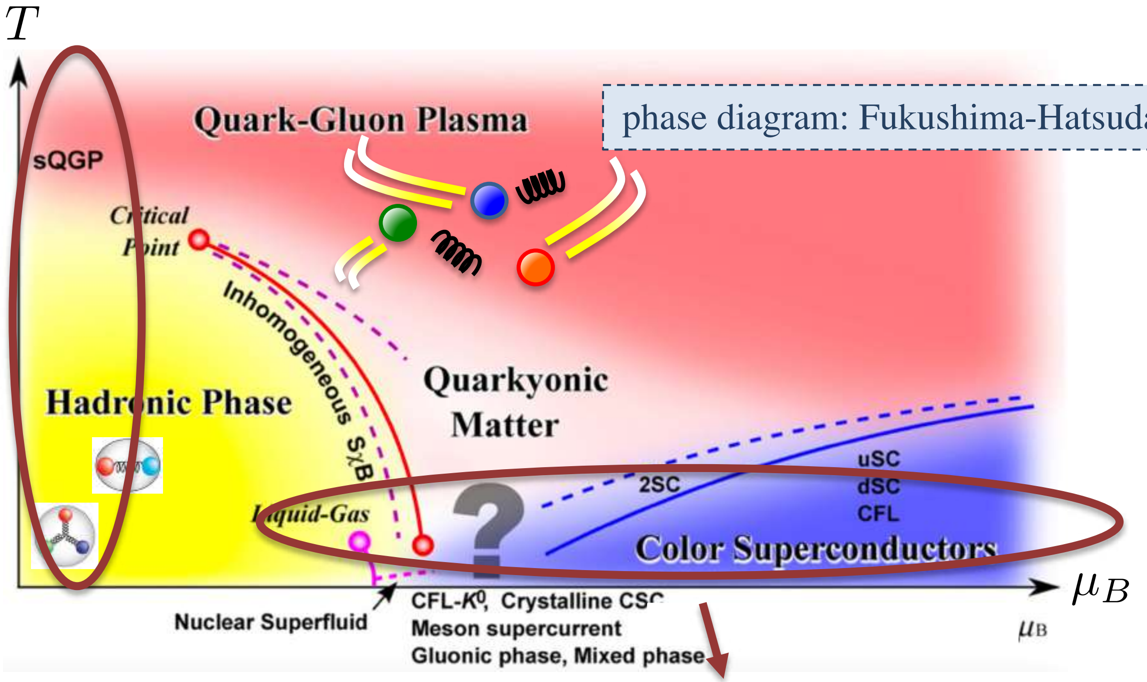
first-principle simulation of path integral

super computers (富岳 etc.)

Introduction

- **QCD phase diagram**: from simulation aspects

lattice QCD: OK!



Problem: lattice QCD cannot apply ! (sign problem)

technical problem

- Our understanding in finite-density system is limited compered to in temperature system

Introduction

• Two-color QCD world

three-color QCD (our world)

- Lattice QCD at density is not easy

  (sign problem) ✕

- Baryon is made of three quarks

proton/neutron

- chiral symmetry is $SU(N_f)_L \times SU(N_f)_R$

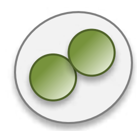
⋮

two-color QCD (imaginary world)

- Lattice QCD at density is possible!

  (sign problem disappears) ○

- Baryon is made of two quarks



diquark baryon

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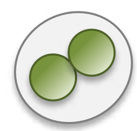

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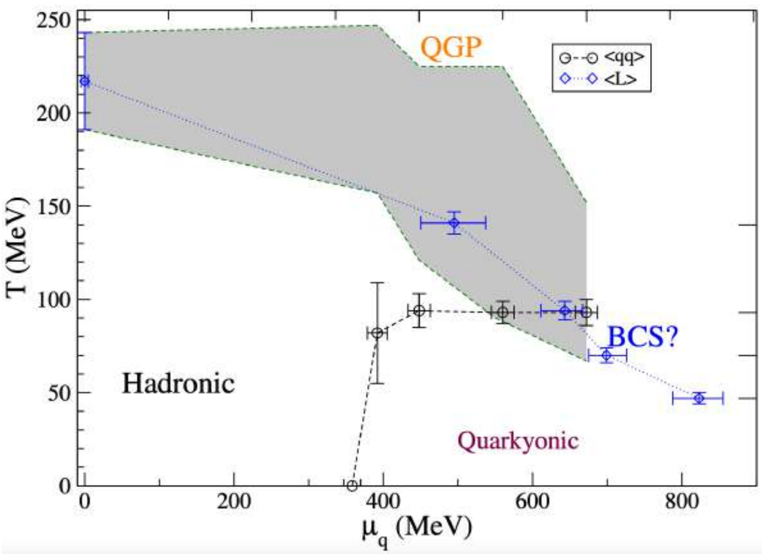
Pursue understanding of dense matter in three-color QCD via two-color QCD world

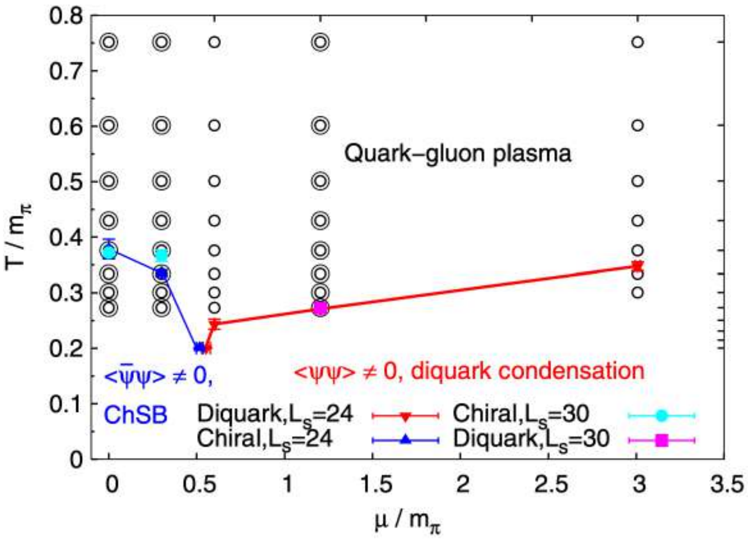
Introduction

- **Phase diagram in two-color QCD**

- Examples of simulation results of phase diagram in two-color QCD



Boz-Cotter-Fister-Mehta-Skullerud (2013)



Buividovich-Smith-Smekal (2020)

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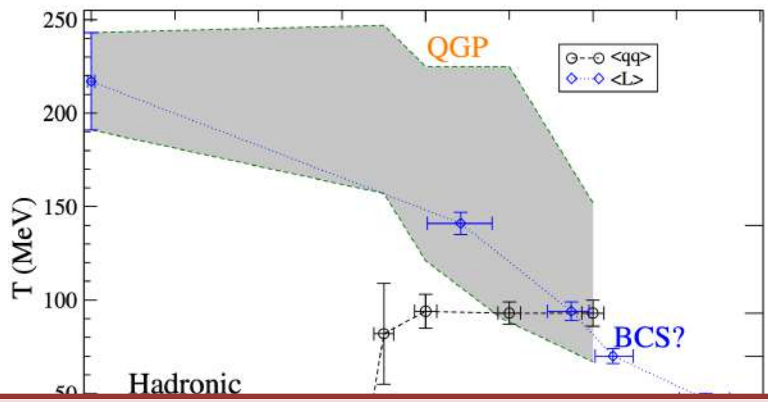
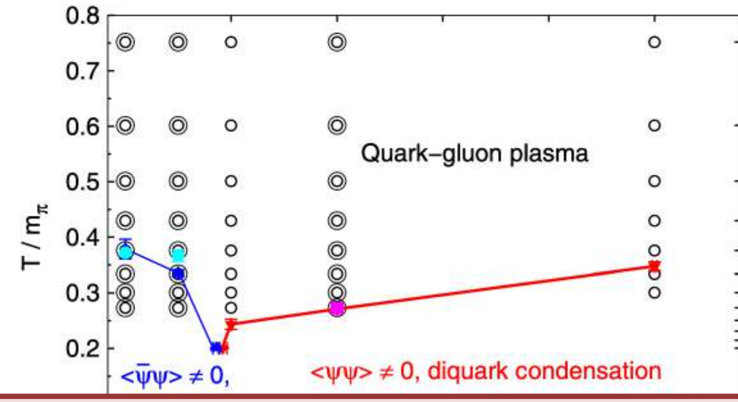




…

My approach

- (i) Regard lattice QCD in two-color QCD as useful "numerical experiments" of dense QCD, and (ii) give interpretation based on effective models

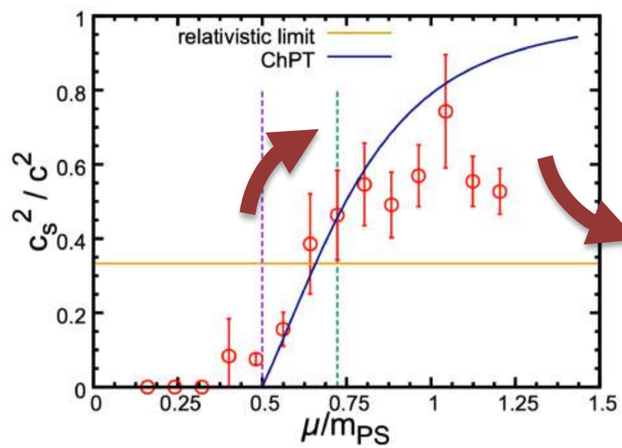
my previous works on two-color QCD

gluon propagator: Suenaga-Kojo(2019), Kojo-Suenaga(2021), CSE effect: Suenaga-Kojo(2021), peak of sound wave: Kojo-Suenaga(2022), hadron mass: Suenaga-Murakami-Itou-Iida (2023) topological susceptibility: Kawaguchi-Suenaga(2023)

Introduction

- **Peak of sound velocity**

  - Lattice result in two-color QCD



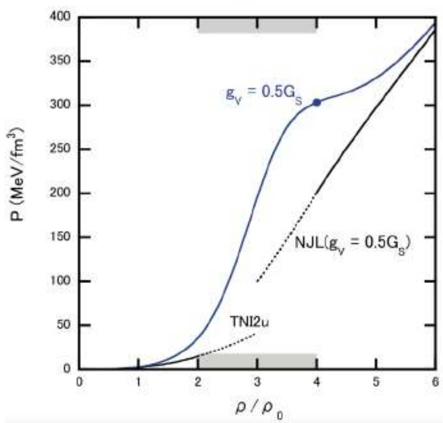
Iida-Itou (2022)

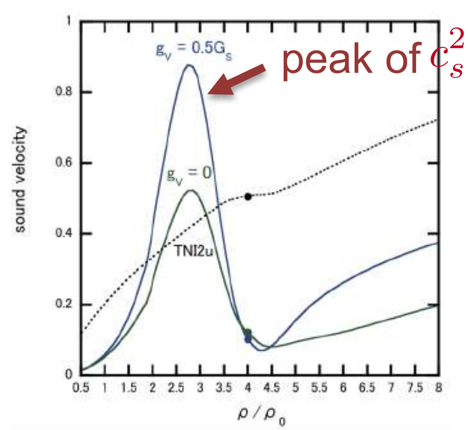
finally converges to this limit

  - This result shows c_s^2 exceeds $1/3$

⇨ suggests the existence of a peak structure

three-color QCD





peak of c_s^2

Relation with continuity from hadron matter to quark matter

Masuda-Hatsuda-Takatsuka(2012)

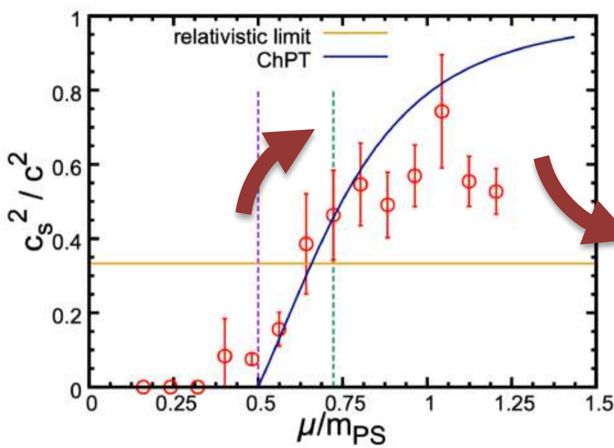
<u>This talk</u>

- I will discuss sound velocity in two-color QCD from effective model approach

Introduction

- **Peak of sound velocity**

  - Lattice result in two-color QCD



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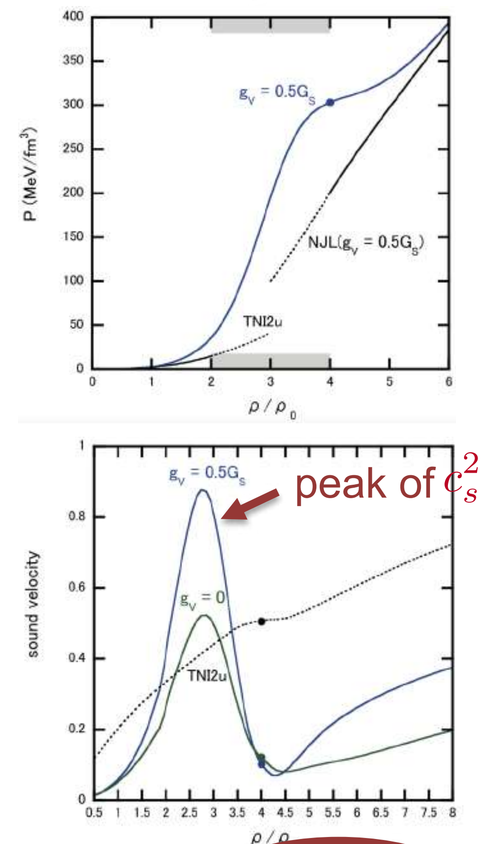

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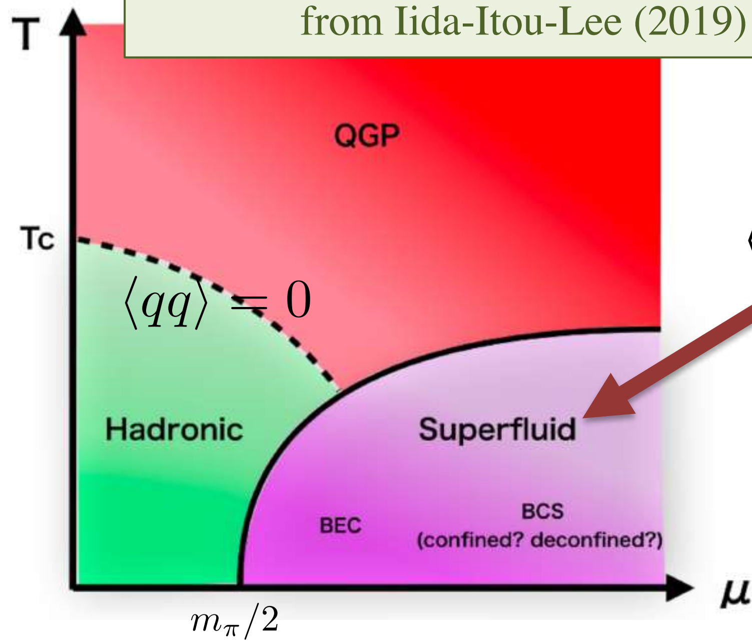
Quark saturation

- **Baryon superfluidity**

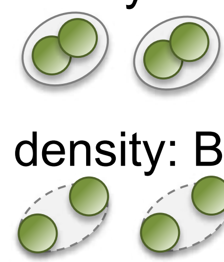
- Baryon superfluidity emerges in cold dense two-color medium

schematic phase diagram in two-color QCD
from Iida-Itou-Lee (2019)



$\langle qq \rangle \neq 0$ (violation of quark number)

- low density: BEC (hadronic)
- high density: BCS (quark cooper pair)



cf, color superconductivity in three-color QCD,
Alford-Schmitt-Rajagopal-Schäfer (2008)

BEC-BCS crossover, eg, He(2010)

Quark saturation

- **NJL model** **(explicit quark d.o.f)**

- NJL model having four-point interactions of meson and diquark channels is

$$\mathcal{L}_{\rm NJL} = \bar{q}(i\partial - m + \mu\gamma_0)q + G[(\bar{q}\tau_a q)^2 + (\bar{q}i\gamma_5\tau_a q)^2] + H[|\bar{q}i\gamma_5\tau_2\sigma_2 q_c|^2 + |\bar{q}\tau_2\sigma_2 q_c|^2]$$

mean field: $M = m - 4G\langle\bar{q}q\rangle \quad \Delta = 2H\langle\bar{q}_c\gamma_5\tau_2\sigma_2 q\rangle$

- Thermodynamic potential at MF level reads

$$\Omega_{\rm MF} = N_c N_f \sum_{\xi={\rm p,a}} \int_{\boldsymbol{k}} [-\epsilon^{\xi}_{\boldsymbol{k}} - 2T\ln(1 + e^{-\epsilon^{\xi}_{\boldsymbol{k}}/T})]$$
$$+2G\langle\bar{q}q\rangle^2 + H|\langle\bar{q}_c\gamma_5\tau_2\sigma_2 q\rangle|^2$$

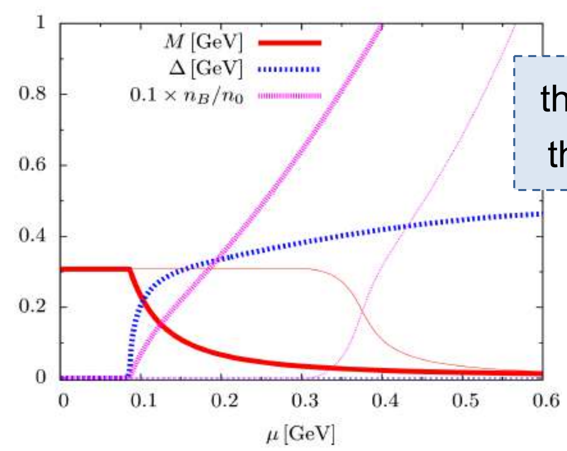
with $\epsilon^{\xi}_k = \sqrt{(E_k - \eta_\xi\mu)^2 + |\Delta|^2}$

$E_k = \sqrt{k^2 + M^2}$

$\eta_{\rm p/a} = \pm 1$

- M, Δ and baryon density n_B at μ ⇨

input
$$\begin{cases} \Lambda = 1.0\,\mathrm{GeV} \\ G\Lambda^2 = 2.8 \\ m = 5\,\mathrm{MeV} \\ (T = 0) \end{cases}$$



thick $H = G$

thin $H = 0$

Quark saturation

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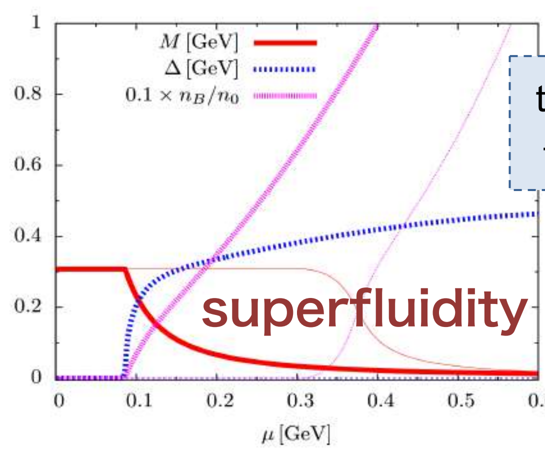


thick $H = G$
thin $H = 0$

superfluidity for $H = G$

Quark saturation

- **NJL model** **(explicit quark d.o.f)**

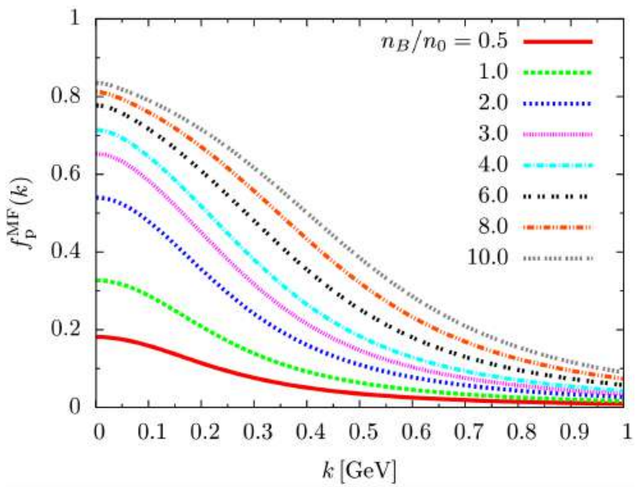
- Occupation probability for quasiparticle(≒quark) is

$$f_\xi^{\mathrm{MF}}(k) = \frac{1}{2}\left(1 - \frac{E_k - \eta_\xi \mu}{\epsilon_k^\xi}\right)$$

“number” of quarks per momentum

quark density: $n_q^{\mathrm{MF}} = 2N_{\mathrm{f}} \int_{\boldsymbol{k}} (f_{\mathrm{p}}^{\mathrm{MF}}(\boldsymbol{k}) - f_{\mathrm{a}}^{\mathrm{MF}}(\boldsymbol{k}))$

- $f_{\mathrm{p}}^{\mathrm{MF}}(k)$ at several densities $(H = G)$



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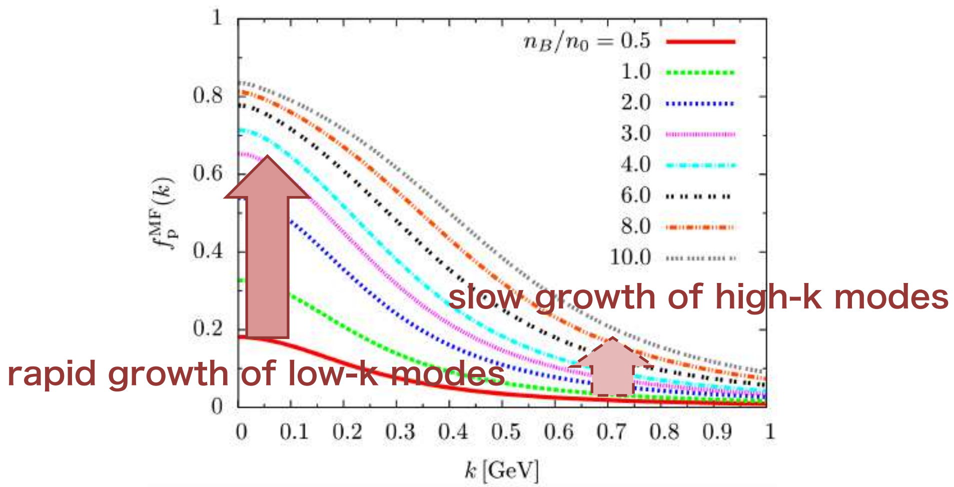

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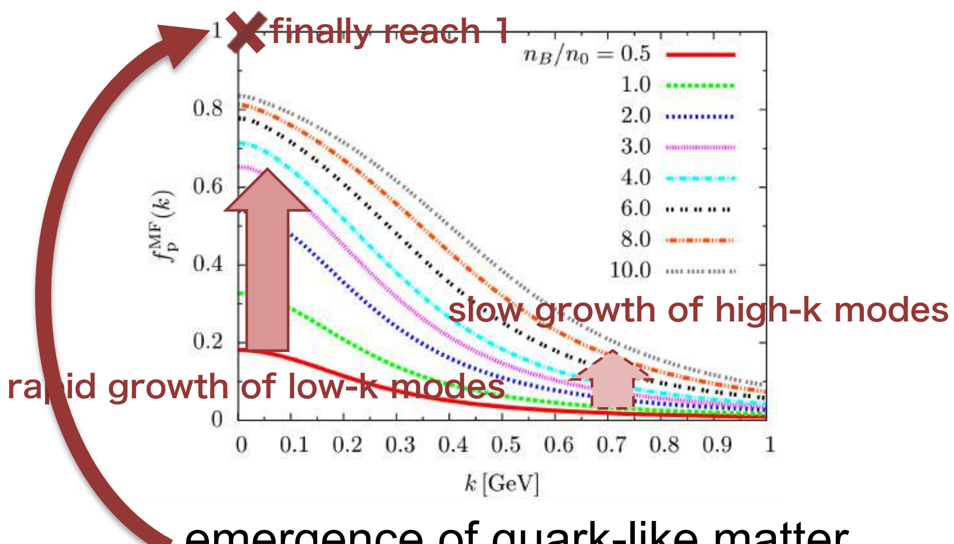

emergence of quark-like matter

= quark saturation

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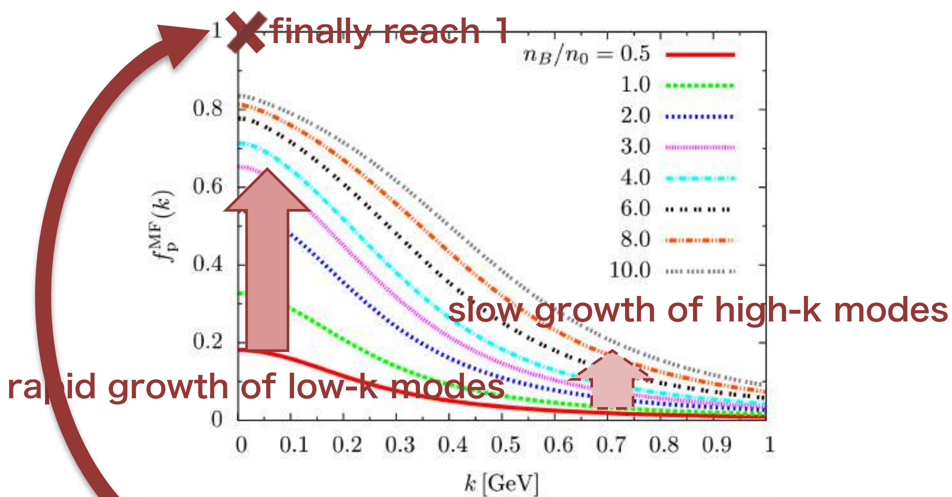

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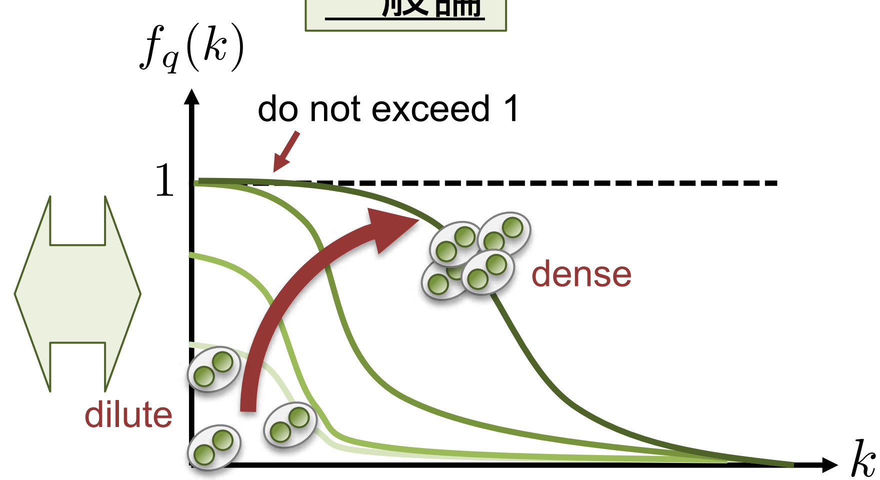
一般論

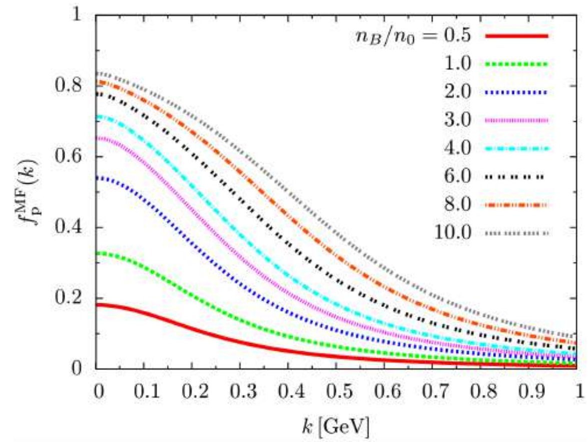


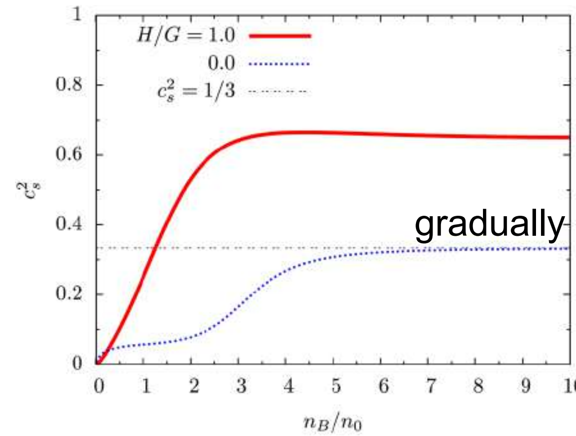
$f_q(k) \approx 1$ from low-k modes

Quark saturation

- **NJL model** **(explicit quark d.o.f)**

- Relation between occupation probability and sound velocity





gradually converges to 1/3

- c_s^2 exceeds 1/3 as $f_{\mathrm{p}}^{\mathrm{MF}}(k)$ approaches quark saturation $(n_B/n_0 \gtrsim 2)$
- c_s^2 gradually converges to 1/3

⇨ emergence of a very gentle peak (bump)

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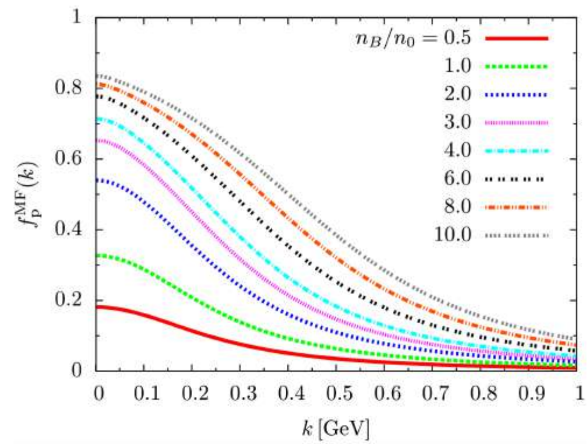

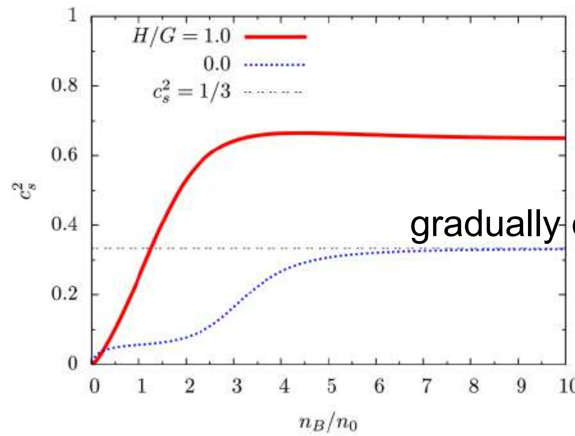

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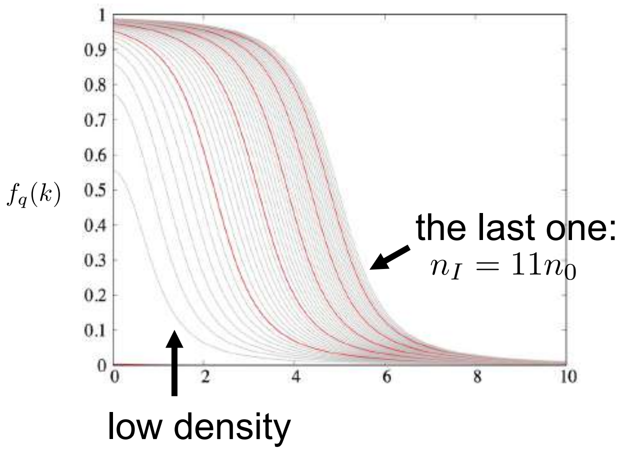
- Quark degrees of freedom may be essential to explain the peak

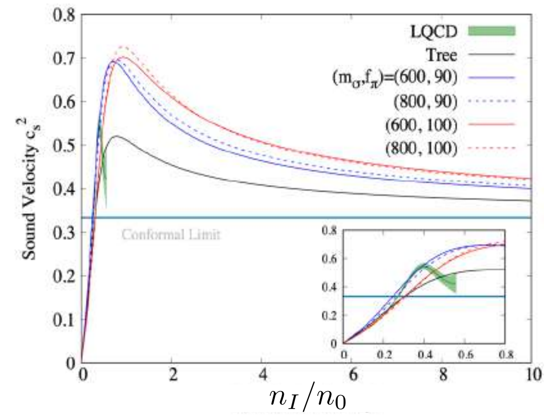
Kojo-Suenaga (2022)

Quark saturation

- **Similar other consideration**

 - Results in isospin matter (essentially same as two-color QCD matter)





Quark-meson model:
Chiba-Kojo (2023)

(explicit quark d.o.f)

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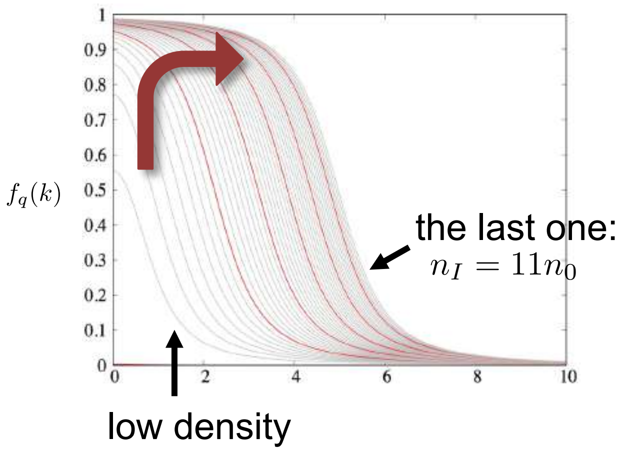

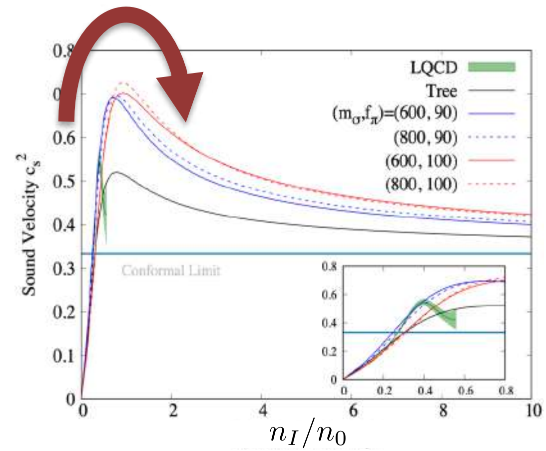

Quark-meson model:
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(explicit quark d.o.f)

- When the evolution becomes “right angled”, peak of c_s^2 gets sharp

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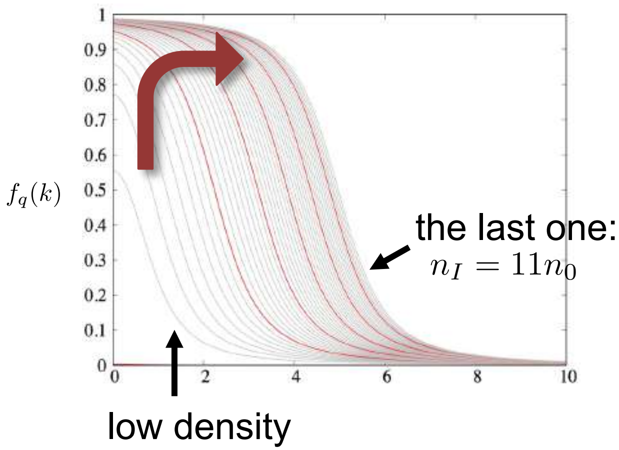

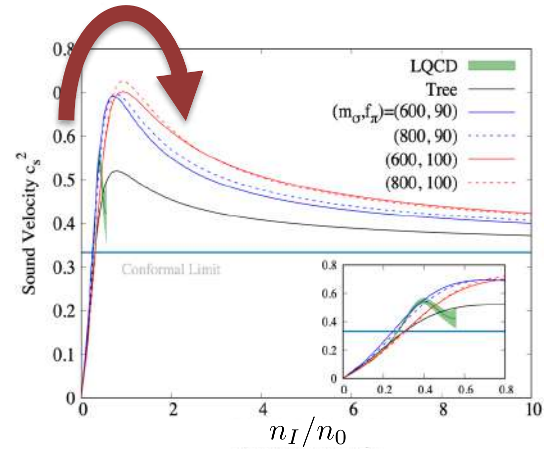

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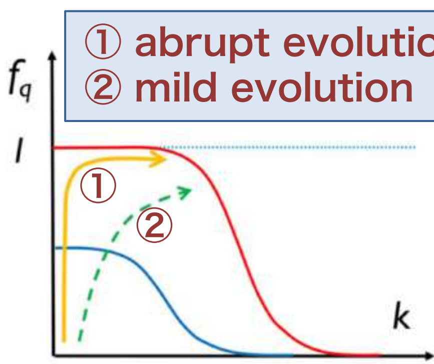
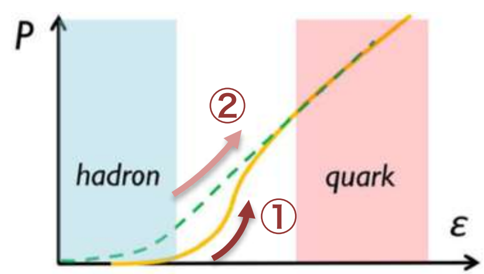
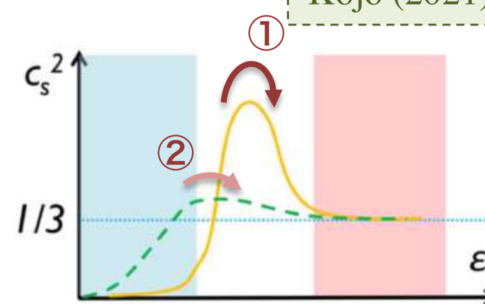




Kojo (2021)



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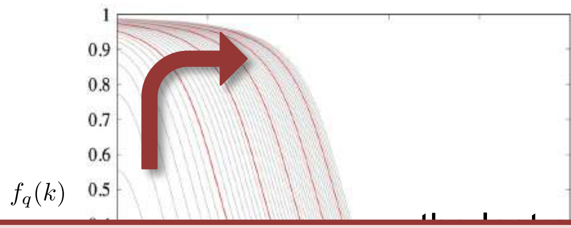
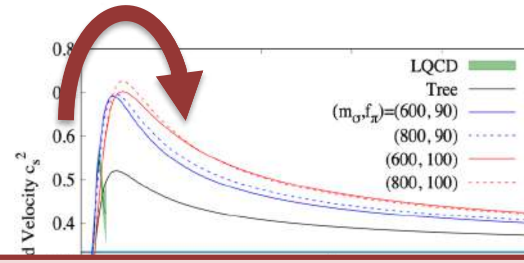


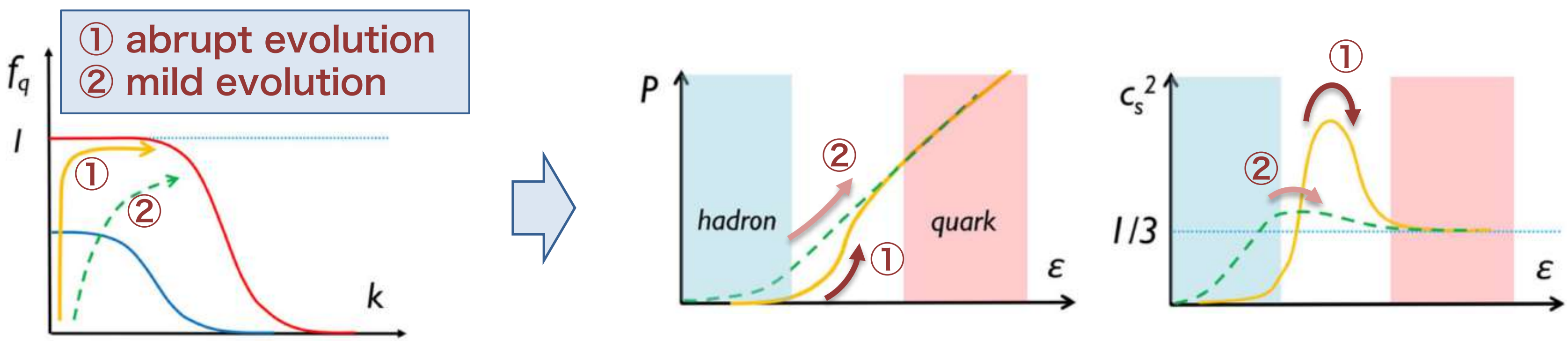


Quark-meson model:
Chiba-Kojo (2023)

(explicit quark d.o.f)

Q: Do we need explicit quark degrees of freedom to discuss the peak of sound velocity?

- When the evolution becomes “right angled”, peak of c_s^2 gets sharp



Hadronic model

- **Reconstruction from ChPT (no quark d.o.f.)**

- Chiral perturbation theory (ChPT) in two-color QCD with $\mathcal{O}(p^4)$ derivatives is

$$\mathcal{L}_{\rm ChPT} = \frac{f_\pi^2}{4}{\rm tr}[D_\mu\Sigma^\dagger D^\mu\Sigma] + \frac{f_\pi^2 m_\pi^2}{4}{\rm tr}[E\Sigma + \Sigma^\dagger E^T]$$
$$+L_1({\rm tr}[D_\mu\Sigma^\dagger D^\mu\Sigma])^2 + L_2{\rm tr}[D_\mu\Sigma^\dagger D_\nu\Sigma]{\rm tr}[D^\mu\Sigma^\dagger D^\nu\Sigma] + L_3{\rm tr}[D_\mu\Sigma^\dagger D^\mu\Sigma D_\nu\Sigma^\dagger D^\nu\Sigma] + \underbrace{\cdots}_{\propto (\mu^2/\Lambda_\chi^2)^n \to 0}$$

$$\begin{cases} D_\mu\Sigma = \partial_\mu\Sigma - i\mu\delta_{\mu 0}(J\Sigma + \Sigma J^T) \\ \Sigma = \xi E^T \xi^T \\ \xi = \exp(i\Pi/f_\pi) \end{cases}$$

with $J = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $E = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$$\Pi \ni (\underbrace{\pi^0, \pi^+, \pi^-}_{\text{pions}}, \underbrace{B, \bar{B}}_{\text{diquark baryons}})$$

- This is a hadronic model (NO compositeness due to quarks)

- Reconsider sound wave based on ChPT at mean-field level

Hadronic model

- ChPT at mean-field level

- Mean field describing chiral and diquark condensates reads

$$\langle \Sigma \rangle = e^{i2\sqrt{2}X_5\beta}E^T = \left(\cos\beta + i2\sqrt{2}X_5\sin\beta\right)E^T$$

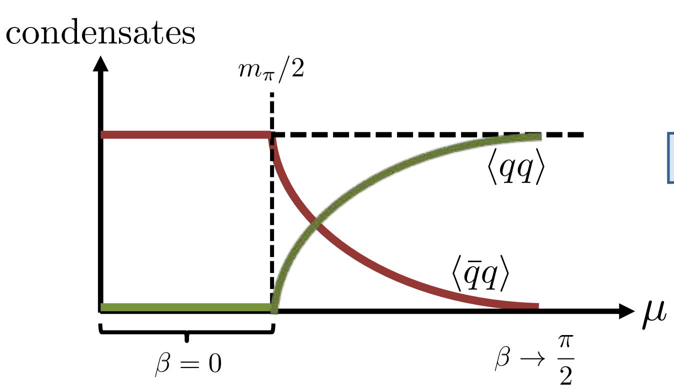
with $X_5 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & i\tau_2 \\ -i\tau_2 & 0 \end{pmatrix}$

$\langle \bar{q}q \rangle \propto f_\pi \cos\beta$
$\langle qq \rangle \propto f_\pi \sin\beta$

⇨ Mean-field Lagrangian: $\mathcal{L}^{\mathrm{MF}}_{\mathrm{ChPT}} = 2f_\pi^2\left[\mu^2(1-\cos 2\beta) + m_\pi^2\cos\beta\right] + \frac{C_4}{4\pi f_\pi}\mu^4\sin^4\beta$

with $C_4/(4\pi f_\pi) = 16^2(L_1+L_2) + 64L_3$

- There remain three parameters f_π, m_π, C_4 $\leftrightarrow$ β is determined by gap eq.



⇨ - Superfluidity for $\mu > \frac{m_\pi}{2}$ is reproduced

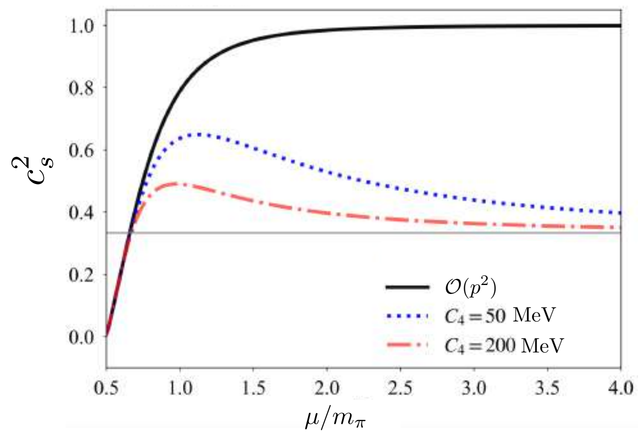
Kogut-Stephanov-Toublan (1999)

Hadronic model

- **Sound velocity from ChPT**

- Inputs: $f_\pi = 177\,\mathrm{MeV}$ and $m_\pi = 738\,\mathrm{MeV}$

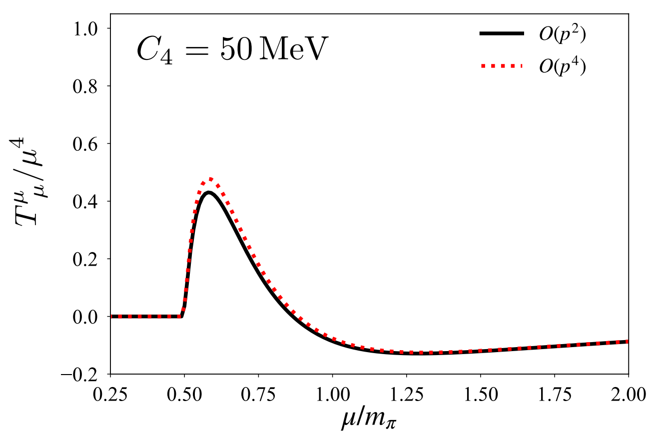
Suenaga-Murakami-Itou-Iida (2023)



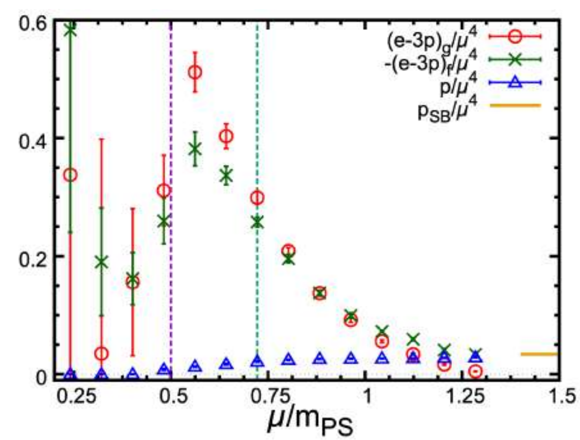
- Peak of c_s^2 is obtained even at hadronic model (no quark d.o.f.)

(so does linear sigma model)

- Trace anomaly



Lattice QCD, Iida-Itou (2022)



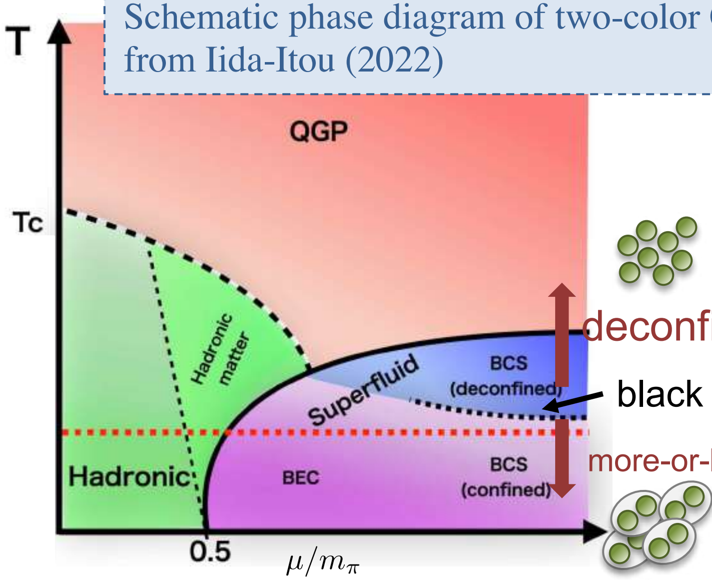
Hadronic model

- **Discussion**

- Do hadrons play essential role even at higher density in two-color QCD

may be possible!

Schematic phase diagram of two-color QCD from Iida-Itou (2022)



deconfined

black dotted: Tc curve

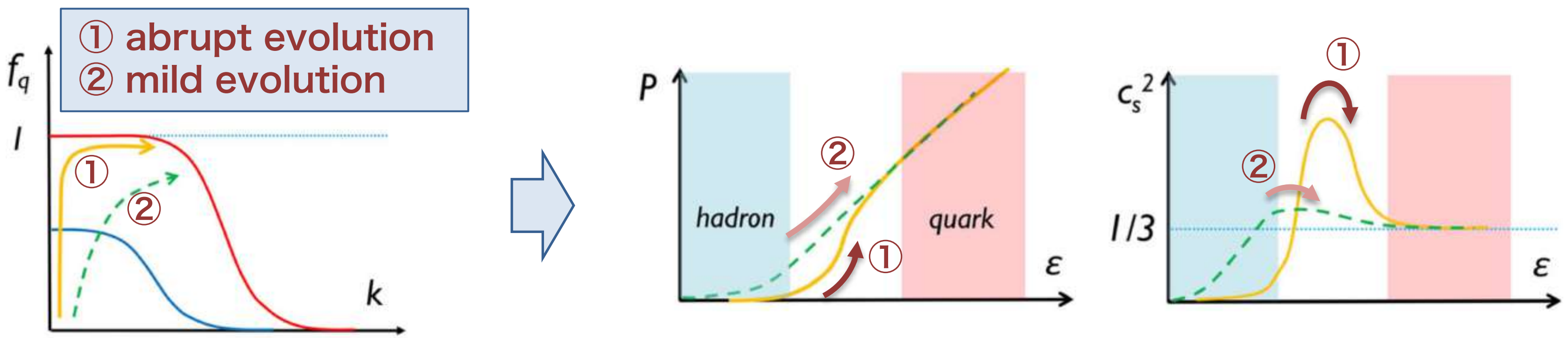
more-or-less confined-like

From Polyakov loop simulation:
Braguta-Ilgenfritz-Kotov-Molochkov-Nikolaev (2016)
Boz-Giudice-Hands-Skullerud (2019)
Iida-Itou-Lee (2020)

- Is there quark saturation effect?

Conclusions

- I discussed the peak of sound velocity from quark saturation



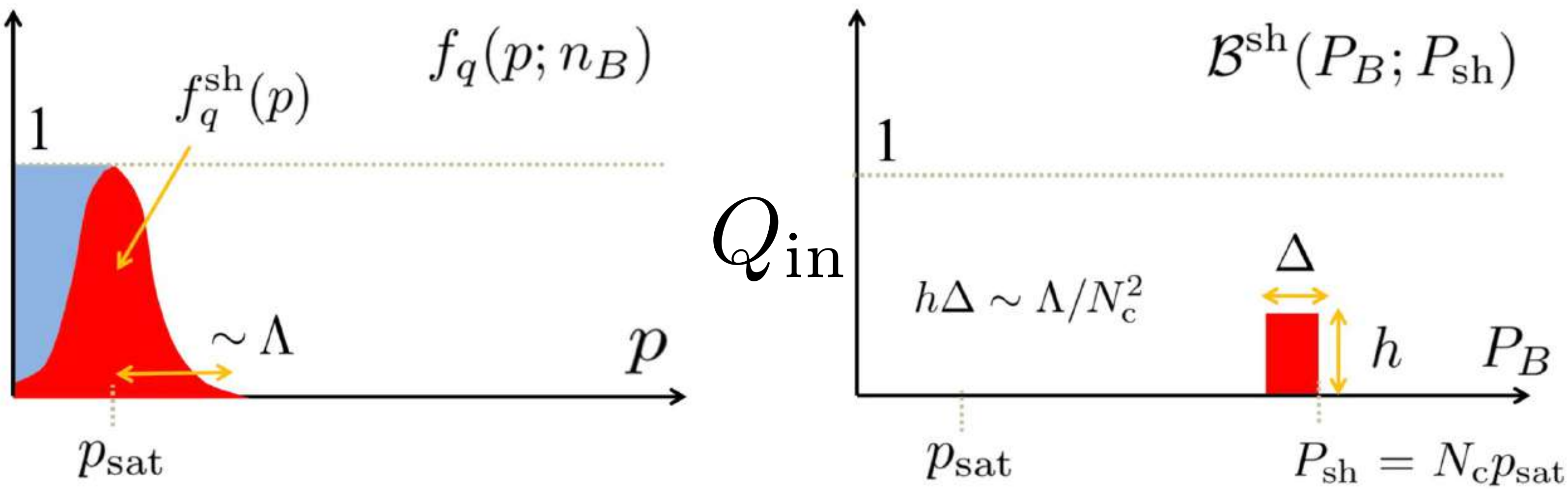
- In two-color QCD, **hadronic model can also lead to the peak**

  (NO quark degrees of freedom)

  ⇨ Consistent with the confined phase at high density

- How about in three-color QCD?

- **Back up**



$$f_q(p; n_B) = \int_{\boldsymbol{P}_B} \mathcal{B}(P_B; n_B) Q_{\rm in}(\boldsymbol{p}, \boldsymbol{P}_B)$$

with $Q_{\rm in}$ a probability density of the quark inside hadrons