COMPOSITE OCTET BARYONS IN NEUTRON STAR MATTER

KAITO NORO[A,B], WOLFGANG BENTZ[C], IAN C. CLOËT[D], TERUYUKI KITABAYASHI[C]

GRADUATE SCHOOL OF SCIENCE AND TECHNOLOGY, TOKAI UNIVERSITY[A],
MICRO/NANO TECHNOLOGY CENTER, TOKAI UNIVERSITY[B],
DEPARTMENT OF PHYSICS, SCHOOL OF SCIENCE, TOKAI UNIVERSITY[C],
PHYSICS DIVISION, ARGONNE NATIONAL LABORATORY[D]

arXiv:2308.13179

2023年9月7日

Introduction

- Structure of neutron stars

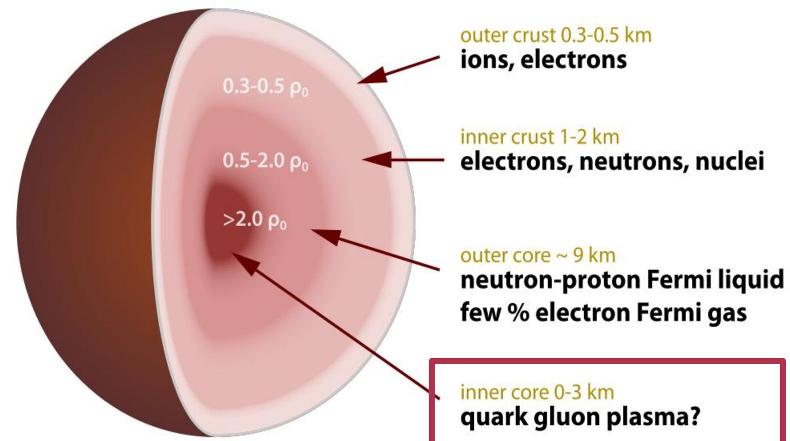
✓ Outer Core：Neutrons, Protons, Electrons

✓ Inner Core：Quarks?, Hyperons? Pions?

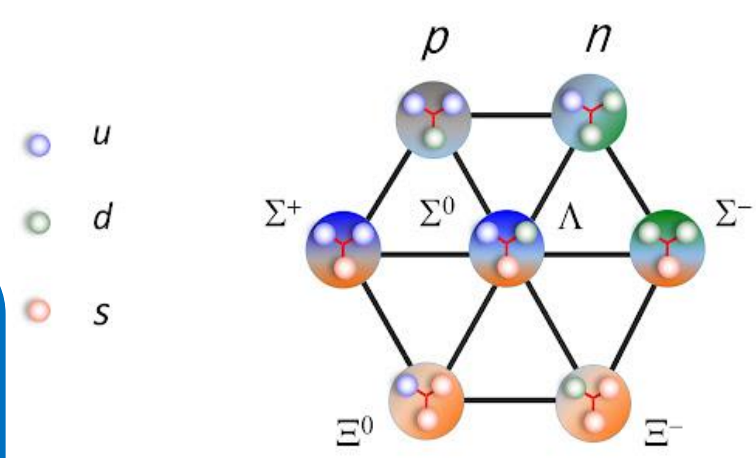
Baryons : A composite particle made of three quarks.

Baryons we will consider in this study:

$$p, n, \Sigma^-, \Sigma^0, \Sigma^+, \Lambda, \Xi^0, \Xi^-$$



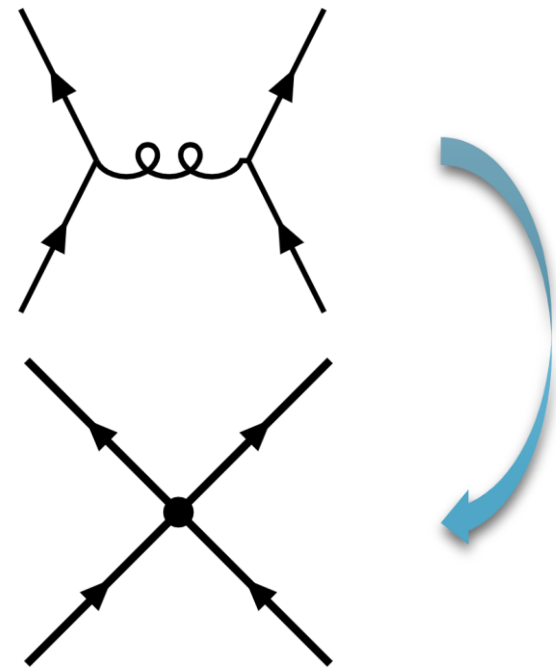
(https://en.wikipedia.org/wiki/Neutron_star)



(http://kakudan.rcnp.osaka-u.ac.jp/jp/overview/world/Flavor.html)

Our Study：
We use a quark-diquark description of octet baryons based on the Faddeev framework to examine the equation of state of neutron star matter in the relativistic mean field approximation.

Nambu-Jona-Lasinio (NJL) Model



- A quark model based on relativistic field theory.
- Contact interactions between quarks.
- We can describe hadrons (nucleons, mesons) as bound state of quarks.

Flavor SU(3) NJL Model Lagrangian → 4-fermi interaction

$$\mathcal{L} = \bar{q}(i\not{\partial} - \hat{m})q$$ … Kinetic term for quarks (u, d, s)

$$+G_\pi[(\bar{q}\lambda_a q)^2 - (\bar{q}\lambda_a \gamma_5 q)^2]$$ … Lorentz scalar + pseudoscalar $\bar{q}q$ channels

$$-G_v[(\bar{q}\lambda_a \gamma^\mu q)^2 + (\bar{q}\lambda_a \gamma^\mu \gamma_5 q)^2]$$ … Lorentz vector + pseudovector $\bar{q}q$ channels

$$+\mathcal{L}_I^{(qq)}$$ … Chiral invariant qq interaction channels

$\hat{m}$: current quark mass matrix $(= \mathrm{diag}(\mathrm{m_u}, \mathrm{m_d}, \mathrm{m_s}))$
λ_a : Gell-Mann flavor matrices $(a = 0,1,2,\cdots,8)$

qq interaction Lagrangian

$$\mathcal{L}_I^{(qq)} = G_S\left[\left(\bar{q}\gamma_5 C\lambda_a\lambda_A^{(c)}\bar{q}^T\right)\left(q^T C^{-1}\gamma_5\lambda_a\lambda_A^{(c)}q\right) - \left(\bar{q}C\lambda_a\lambda_A^{(c)}\bar{q}^T\right)\left(q^T C^{-1}\lambda_a\lambda_A^{(c)}q\right)\right]$$

$$+G_A\left[\left(\bar{q}\gamma_\mu C\lambda_s\lambda_A^{(c)}\bar{q}^T\right)\left(q^T C^{-1}\gamma_\mu\lambda_s\lambda_A^{(c)}q\right) - \left(\bar{q}\gamma_\mu\gamma_5 C\lambda_a\lambda_A^{(c)}\bar{q}^T\right)\left(q^T C^{-1}\gamma^\mu\gamma_5\lambda_a\lambda_A^{(c)}q\right)\right]$$

G_S term : Scalar and pseudoscalar diquark channels

We include only scalar diquark : spin 0, antisymmetric in flavor $\bar{3}_f$.

G_A term : Axial-vector and vector diquark channels

We include only axial-vector diquark : spin 1, symmetric in flavor 6_f.

λ_a : antisymmetric Gell-Mann flavor matrices ($a = 2,5,7$)
λ_s : symmetric Gell-Mann flavor matrices ($s = 0,1,3,4,6,8$)
$\lambda_A^{(c)}$: antisymmetric Gell-Mann color matrices ($A = 2,5,7$)
C : charge conjugation Dirac matrix ($C = i\gamma_2\gamma_0$)

Flavor SU(3) NJL Model Lagrangian

$$\mathcal{L} = \bar{q}(i\not{\partial} - \hat{m})q$$ ⋯ Kinetic term for quarks (u, d, s)

$$+G_\pi[(\bar{q}\lambda_a q)^2 - (\bar{q}\lambda_a \gamma_5 q)^2]$$ ⋯ Lorentz scalar + pseudoscalar $\bar{q}q$ channels

$$-G_v[(\bar{q}\lambda_a \gamma^\mu q)^2 + (\bar{q}\lambda_a \gamma^\mu \gamma_5 q)^2]$$ ⋯ Lorentz vector + pseudovector $\bar{q}q$ channels

$$+\mathcal{L}_I^{(qq)}$$ ⋯ Chiral invariant qq interaction channels

- G_π: Reproduce pion decay constant $f_\pi = 93$ [MeV] and the pion mass $m_\pi = 140$ [MeV].
  $\rightarrow G_\pi = 19.04\mathrm{GeV}^{-2}$

- G_v: Reproduce binding energy per-nucleon in symmetric nuclear matter $E_B/A = 16[\mathrm{MeV}]$ at saturation density of $\rho_{B_0} = 0.15[\mathrm{fm}^{-3}]$. $\rightarrow G_v = 6.03\ \mathrm{GeV}^{-2}$

- G_S, G_A: Reproduce the masses in vacuum of the nucleon (940 MeV) and the delta particle (1232 MeV) by using the Faddeev equation.
  $\rightarrow G_S = 8.76\ \mathrm{GeV}^{-2}, G_A = 7.36\ \mathrm{GeV}^{-2}$

✓ Diquark

Two quarks which form a bound state inside a baryon.

ex）Proton（uud）

- [ud]u
  → Scalar diquark (spin 0, isospin 0)
- {ud}u, {uu}d
  → Axial-vector diquark (spin 1, isospin 1)

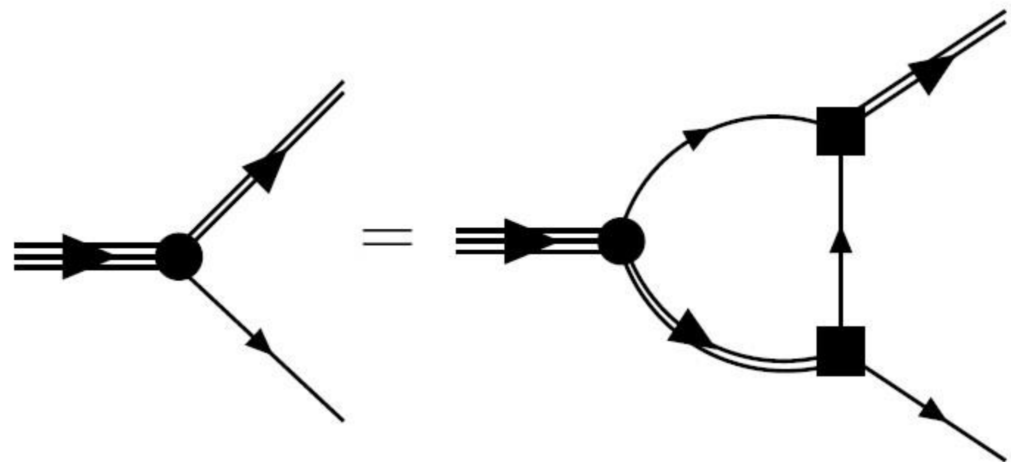


Figure 1. Feynman diagram of the Faddeev equation

Table 1.　Mass of the baryons in vacuum [MeV]

	p	n	Σ^+	Σ^0	Σ^-	Λ	Ξ^0	Ξ^-
Calc.	940.0	940.0	1168.5	1168.5	1168.5	1124.6	1318.7	1318.7
Obs.	938.3	939.6	1189.4	1192.6	1197.7	1115.7	1314.9	1321.7

Extended NJL Model

$$\mathcal{L} = \bar{q}(i\partial - \hat{m})q$$
$$+G_\pi[(\bar{q}\lambda_a q)^2 - (\bar{q}\lambda_a \gamma_5 q)^2]$$
$$-G_v[(\bar{q}\lambda_a \gamma^\mu q)^2 + (\bar{q}\lambda_a \gamma^\mu \gamma_5 q)^2]$$
$$+\mathcal{L}_I^{(qq)}$$

Standard 4-fermi interactions

Extend to include 6-fermi and 8-fermi interactions

6-fermi interaction Lagrangian (‘t Hooft, PRD (1976))

$$\mathcal{L}_6 = G_6 \det\left[\bar{q}_\alpha(1-\gamma_5)q_\beta + \bar{q}_\alpha(1+\gamma_5)q_\beta\right]$$

- G_6 ：Reproduce the mass difference of η' and η mesons as 0.41 GeV.
  $\rightarrow G_6 = 1260\ \mathrm{GeV}^{-5}$

8-fermi interaction Lagrangian (Osipov et al. , Ann. Phys. (2007))

$$\mathcal{L}_8 = G_8^{(ss)}(\mathcal{L}_s\mathcal{L}_s) - G_8^{(sv)}(\mathcal{L}_s\mathcal{L}_v) - G_8^{(vv)}(\mathcal{L}_v\mathcal{L}_v)$$

- $G_8^{(ss)}, G_8^{(sv)}, G_8^{(vv)}$ ：Free parameters for now, but not to spoil the saturation properties of isospin symmetric nuclear matter.

Extended NJL Model

$$\mathcal{L} = \bar{q}(i\partial - \hat{m})q$$
$$+G_\pi[(\bar{q}\lambda_a q)^2 - (\bar{q}\lambda_a\gamma_5 q)^2]$$
$$-G_v[(\bar{q}\lambda_a\gamma^\mu q)^2 + (\bar{q}\lambda_a\gamma^\mu\gamma_5 q)^2]$$
$$+\mathcal{L}_I^{(qq)}$$

Standard 4-fermi interactions

Case 1

Extend to include 6-fermi and 8-fermi interactions

6-fermi interaction Lagrangian (‘t Hooft, PRD (1976))

$$\mathcal{L}_6 = G_6 \det\left[\bar{q}_\alpha(1-\gamma_5)q_\beta + \bar{q}_\alpha(1+\gamma_5)q_\beta\right]$$

- G_6 ：Reproduce the mass difference of η' and η mesons as 0.41 GeV.
  $\rightarrow G_6 = 1260\ \mathrm{GeV}^{-5}$

Case 2

8-fermi interaction Lagrangian (Osipov et al. , Ann. Phys. (2007))

$$\mathcal{L}_8 = G_8^{(ss)}(\mathcal{L}_s\mathcal{L}_s) - G_8^{(sv)}(\mathcal{L}_s\mathcal{L}_v) - G_8^{(vv)}(\mathcal{L}_v\mathcal{L}_v)$$

Case 3

- $G_8^{(ss)}, G_8^{(sv)}, G_8^{(vv)}$ ：Free parameters for now, but not to spoil the saturation properties of isospin symmetric nuclear matter.

Equation of state

From the first law of thermodynamics,

$$\mathcal{E} = -P + \sum_{\alpha=b,l} \mu_\alpha \rho_\alpha$$

$\mathcal{E}$: energy density, P : pressure
α : Baryons and leptons (e, μ)
μ_α : Chemical potentials, ρ_α : Density for each particle

4-fermi interaction only

The total energy density in the mean field approximation is expressed as

$$\mathcal{E} = \mathcal{E}_{\text{vac}} - \frac{\omega_q^2}{8G_v} + 2G_v\rho_q^2 + \mathcal{E}_B + \mathcal{E}_l$$

$\mathcal{E}_{\text{vac}}$:Vacuum term of constituent quarks (u, d, s)
$\mathcal{E}_B$: Baryon kinetic term (Baryons moving in mean scalar and vector fields)
$\mathcal{E}_l$: Lepton kinetic terms
ω_q : vector mean fields $(\omega_u, \omega_d, \omega_s)$, $\omega_q = 4G_v\langle q^\dagger q\rangle$
ρ_q : quark densities (ρ_u, ρ_d, ρ_s)
G_v : 4-fermi coupling constant

Equation of state

4+6+8-fermi interaction

The total energy density in the mean field approximation is expressed as

$$\mathcal{E} = \mathcal{E}_{\text{vac}} - \frac{\omega_q^2}{8G_v} + 2G_v\rho_q^2 + \mathcal{E}_B + \mathcal{E}_l + \mathcal{E}_6 + \mathcal{E}_8$$

The new contributions from 6-fermi and 8-fermi interactions to the energy density are

$$\mathcal{E}_6 = -\frac{G_6}{16G_\pi^3}(\sigma_u\sigma_d\sigma_s - \sigma_{u0}\sigma_{d0}\sigma_{s0})$$

$$\mathcal{E}_8 = \frac{3G_8^{(ss)}}{64G_\pi^4}\left(\sigma_q^2\sigma_{q'}^2 - \sigma_{q0}^2\sigma_{q'0}^2\right) - \frac{3G_8^{(sv)}}{64G_\pi^2G_v^2}\sigma_q^2\omega_{q'}^2 - \frac{3G_8^{(vv)}}{64G_v^4}\omega_q^2\omega_{q'}^2$$

σ_q : scalar mean fields $(\sigma_u, \sigma_d, \sigma_s)$, $\sigma_q = 4G_\pi\langle\bar{q}q\rangle$

σ_{q0} : vacuum values of scalar mean fields $(\sigma_{u0}, \sigma_{d0}, \sigma_{s0})$

Scalar and vector mean fields are determined by the conditions

$$\frac{\partial\mathcal{E}}{\partial\sigma_q} = \frac{\partial\mathcal{E}}{\partial\omega_q} = 0$$

under the requirements of chemical equilibrium and charge neutrality.

Numerical results for equation of state

4+6+8-fermi interactions

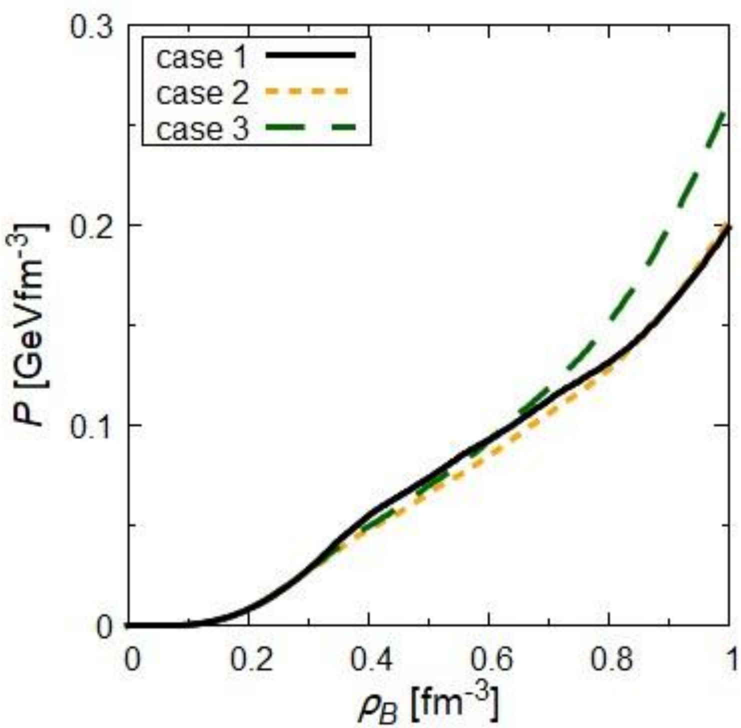


Figure 2. Relation between baryon density and pressure in neutron star matter.

Table 2. Values of the 6-fermi coupling constant G_6 in units of GeV^{-5}, and the 8-fermi coupling constants $G_8^{(ss)}$ and $G_8^{(vv)}$ in units of GeV^{-8}. The coupling $G_8^{(sv)}$ is set to zero in all three cases.

	G_6	$G_8^{(ss)}$	$G_8^{(vv)}$
Case 1	0	0	0
Case 2	1260	0	0
Case 3	1260	2330	1220

Numerical results for equation of state

4+6+8-fermi interaction

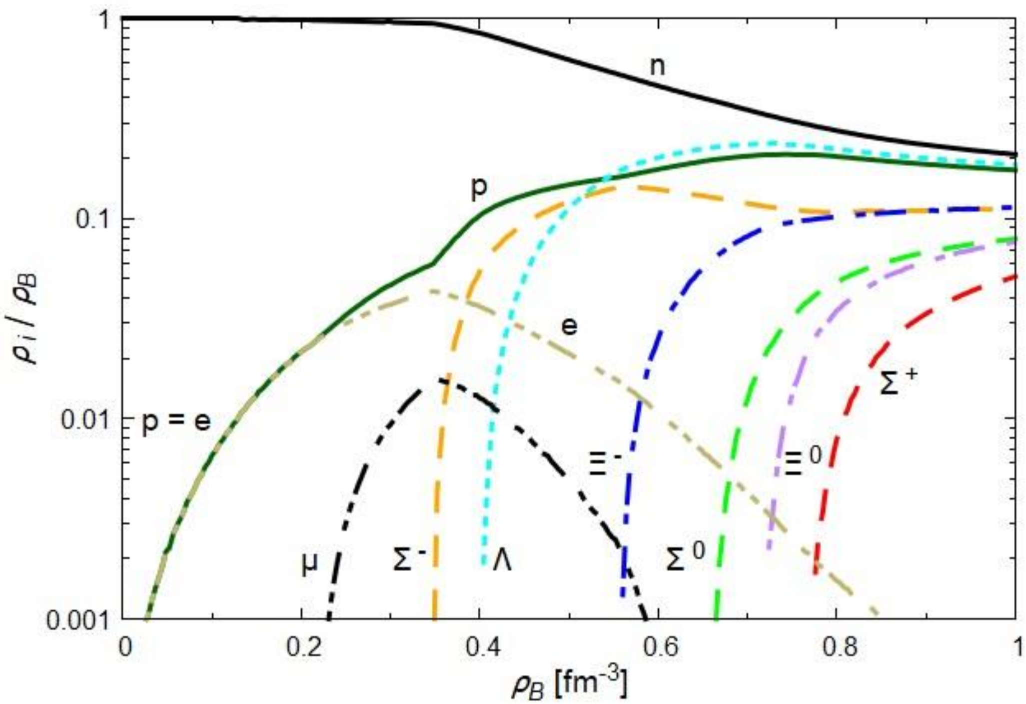


Figure 3. Particle densities for case 1.

Threshold densities

μ : 0.22 fm^{-3}

Σ^- : 0.35 fm^{-3}

Others : higher densities

Numerical results for neutron stars

4+6+8-fermi interactions

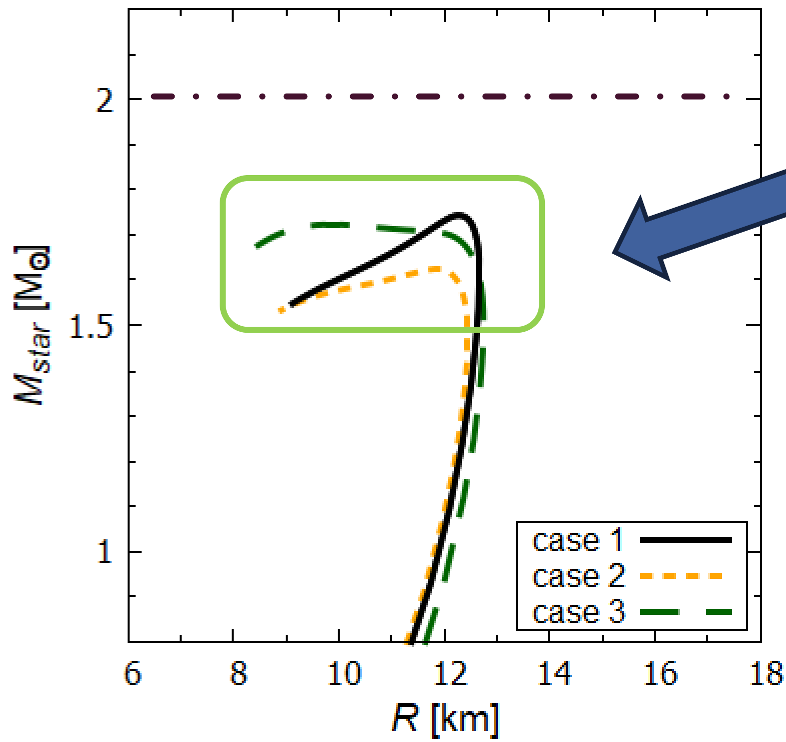


Figure 4. Relation between star mass and radii.

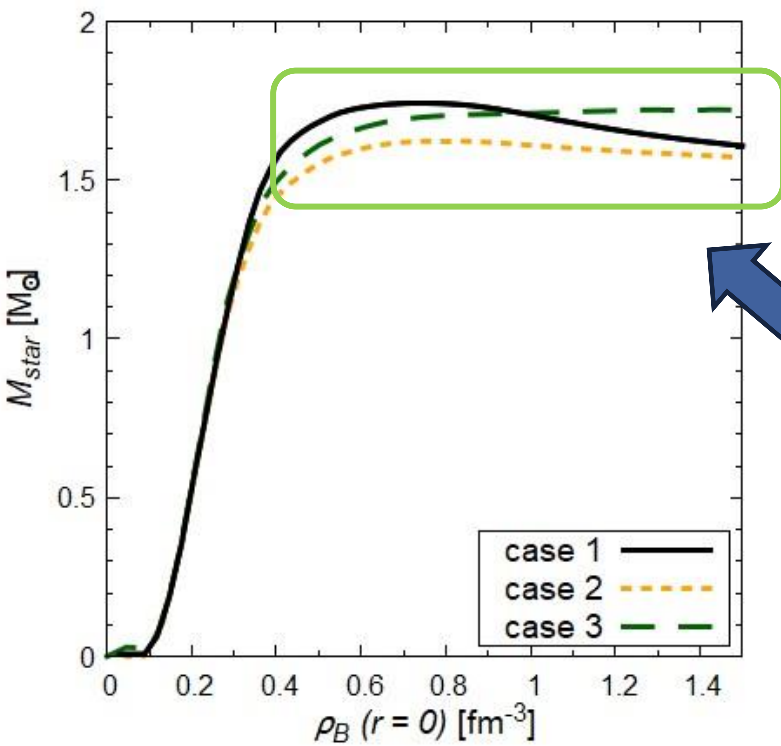
Predicted star mass including hyperons is too low compared to the observed mass.
→ "Hyperon Puzzle"

Table 3. Values of the central baryon densities $\rho_B^{max}(r=0)$ in units of fm^{-3}, star masses M_{star}^{max} in units of $\mathrm{M}_\odot$, and the radii of the stars R in units of km, which gives the maximum star mass.

	$\rho_B^{max}(r=0)$	M_{star}^{max}	R
Case 1	0.72	1.73	12.3
Case 2	0.8	1.62	11.9
Case 3	1.4	1.72	9.8

Numerical results for neutron stars

4+6+8-fermi interactions



Star mass for case 3 is stable for large region of densities.

Figure 5. Relation between central baryon density and star mass.

Summary

- We designed our mean field approximation so that it reflects the basic symmetries of the model and their dynamical breakings, regardless of possible disagreements with the observations.
- Hyperon puzzle persists in the NJL model for composite octet baryons in the mean field approximation, and 6-fermi and 8-fermi interactions do not solve the problem.
- A special kind of 8-fermi interaction can support stable stars up to 1.7 solar masses over a large region of densities.

Backup Slides

Chiral invariant interaction Lagrangian

$$\mathcal{L}_I^{(qq)} = G_S\left[\left(\bar{q}\gamma_5 C\lambda_a\lambda_A^{(C)}\bar{q}^T\right)\left(q^T C^{-1}\gamma_5\lambda_a\lambda_A^{(C)}q\right) - \left(\bar{q}C\lambda_a\lambda_A^{(C)}\bar{q}^T\right)\left(q^T C^{-1}\lambda_a\lambda_A^{(C)}q\right)\right]$$

$$+G_A\left[\left(\bar{q}\gamma_\mu C\lambda_s\lambda_A^{(C)}\bar{q}^T\right)\left(q^T C^{-1}\gamma_\mu\lambda_s\lambda_A^{(C)}q\right) - \left(\bar{q}\gamma_\mu\gamma_5 C\lambda_a\lambda_A^{(C)}\bar{q}^T\right)\left(q^T C^{-1}\gamma^\mu\gamma_5\lambda_a\lambda_A^{(C)}q\right)\right]$$

G_S term : Scalar and pseudoscalar diquark channels

We include only scalar diquark : spin 0, antisymmetric in flavor $\bar{3}_f$.

G_A term : Axial-vector and vector diquark channels

We include only axial-vector diquark : spin 1, symmetric in flavor 6_f.

✓ Diquark

→ Two quarks which bind inside a baryon

ex) Proton (uud)

- [ud]u
  → Scalar diquark (spin 0, isospin 0)
- {ud}u, {uu}d
  → Axial-vector diquark (spin 1, isospin 1)

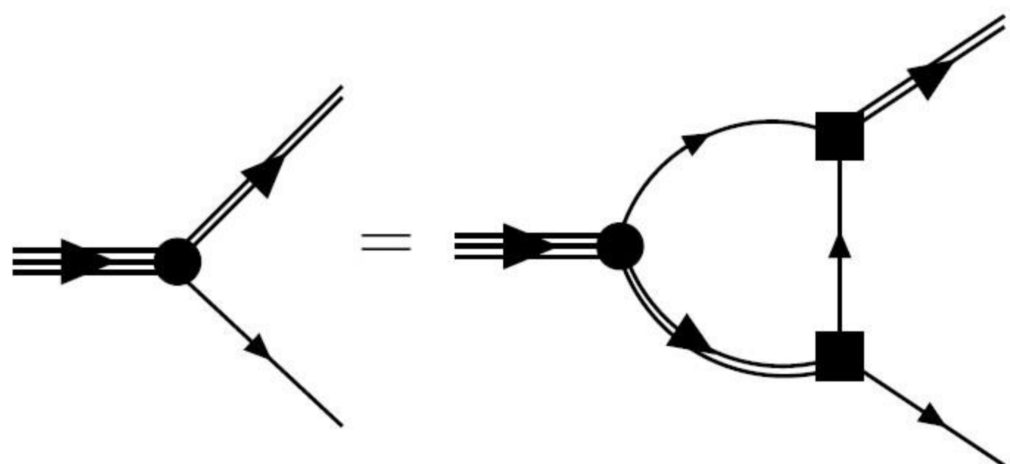


Figure 1. Feynman diagram of the Feddeev equation

Chiral invariant interaction Lagrangian

$$\mathcal{L}_I^{(qq)} = G_S\left[\left(\bar{q}\gamma_5 C\lambda_a\lambda_A^{(C)}\bar{q}^T\right)\left(q^T C^{-1}\gamma_5\lambda_a\lambda_A^{(C)}q\right) - \left(\bar{q}C\lambda_a\lambda_A^{(C)}\bar{q}^T\right)\left(q^T C^{-1}\lambda_a\lambda_A^{(C)}q\right)\right]$$

$$+G_A\left[\left(\bar{q}\gamma_\mu C\lambda_s\lambda_A^{(C)}\bar{q}^T\right)\left(q^T C^{-1}\gamma_\mu\lambda_s\lambda_A^{(C)}q\right) - \left(\bar{q}\gamma_\mu\gamma_5 C\lambda_a\lambda_A^{(C)}\bar{q}^T\right)\left(q^T C^{-1}\gamma^\mu\gamma_5\lambda_a\lambda_A^{(C)}q\right)\right]$$

G_S term : Scalar and pseudoscalar diquark channels

We include only scalar diquark : spin 0, antisymmetric in flavor $\bar{3}_f$.

G_A term : Axial-vector and vector diquark channels

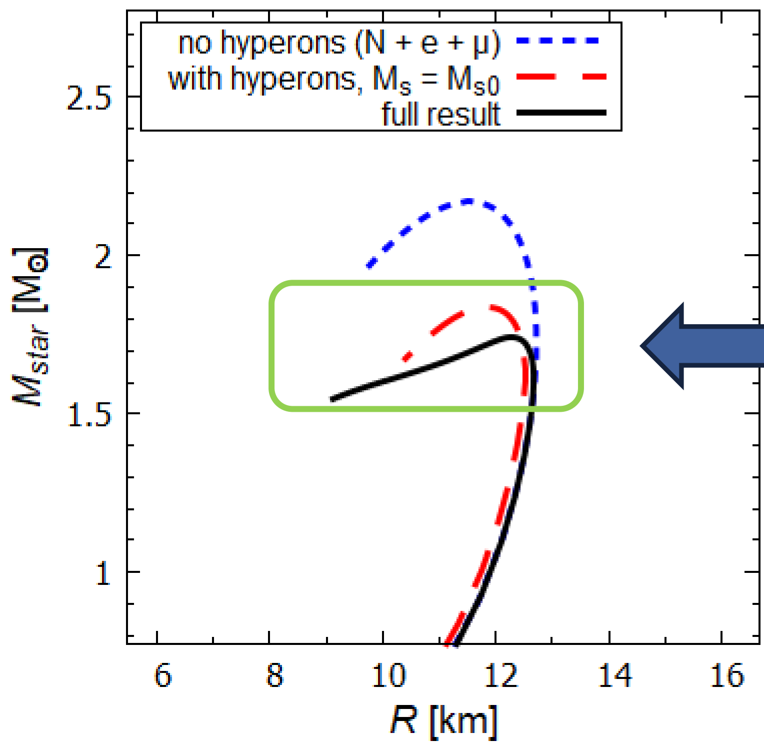
We include only axial-vector diquark : spin 1, symmetric in flavor 6_f.

Table 1. Mass of the baryons in vacuum [MeV]

	p	n	Σ^+	Σ^0	Σ^-	Λ	Ξ^0	Ξ^-
Calc.	940.0	940.0	1168.5	1168.5	1168.5	1124.6	1318.7	1318.7
Obs.	938.3	939.6	1189.4	1192.6	1197.7	1115.7	1314.9	1321.7

Numerical Results

4-fermi interaction only



Star mass with hyperons is too low compared to the observation of heavy star.
→ "Hyperon Puzzle"

Figure 7. Relation between star mass and radii.