arXiv:2306.03327 [nucl-th]

自己無撞着超流動バンド計算を用いた中性子星内殻の物性の探索

Neutron Star Workshop 2023

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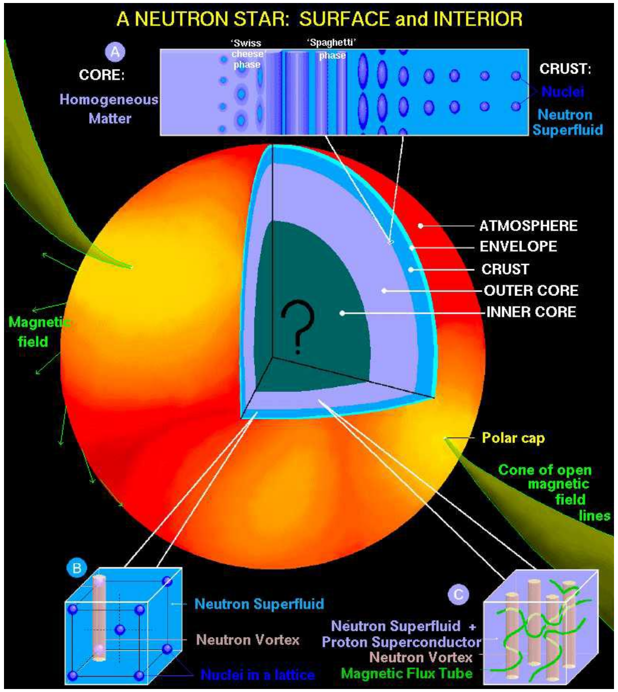
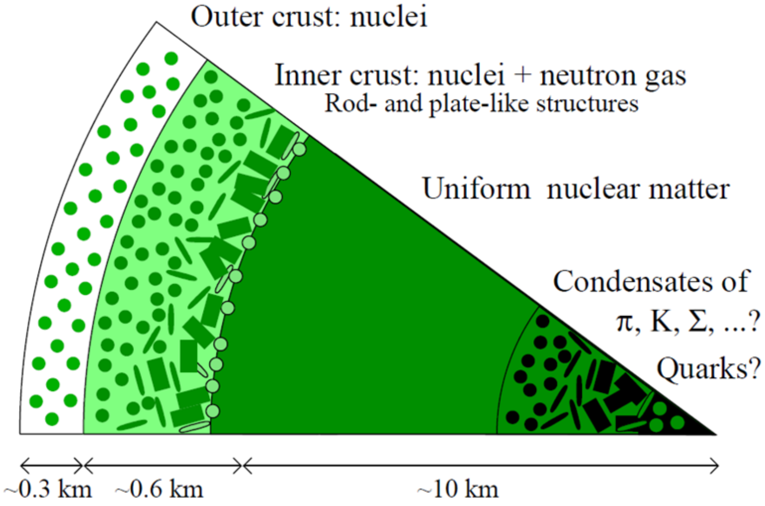
Outline

- Motivation
  - What is Neutron Star?
  - Pasta Structure
- Formulation
  - DFT and HFB Formalism
  - Band Theory Introduction
- Calculation Result
- Summary
- Preliminary Results of ongoing extension

What is Neutron Star?



J.M.Lattimer & M.Prakash, Science 304, 536 (2004)



Neutron star?

- Center density : ~ $5\rho_0$
- Temperature : ~ keV
- Rotation period : ~ ms
- Magnetic Field : ~ 10^{18}G

Structure?

- **Pasta Structure**
- Meson condensates?
- Quark gluon plasma?

Phenomena?

- Supernovae
- Nucleosynthesis (r-process)
- Quasi-periodic oscillation
- Pulsar glitches

Foremost line of Theoretical Physics

Pasta Structure

Pasta Phases?

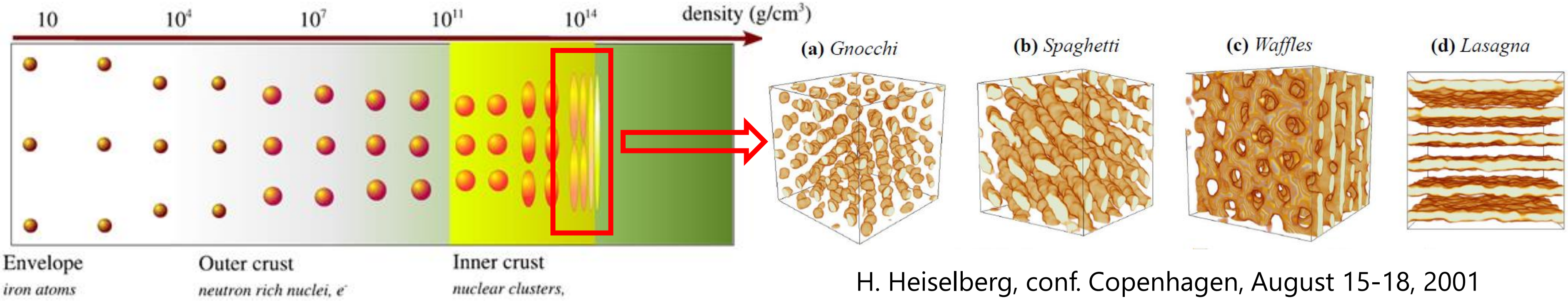
- Crystalline structures of nuclear matter
- Realized at the bottom(high density region) of inner crust
- Immersed in the *sea* of superfluid neutrons

Related Topics

- Pulsar Glitch
- Neutrino cooling of proto-neutron star
- Symmetric Energy in nuclear EoS

How to investigate?

- Numerical simulation with ***Band theory***



H. Heiselberg, conf. Copenhagen, August 15-18, 2001

N. Chamel and P. Haensel, Living Rev. Relativity 11, 10 (2008)

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Easiest DFT Introduction

- *Quantum many-body systems* $|\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N)\rangle$

but…
- We don't know the many-body interaction
- We don't know the entity of nuclear force

3D arguments

Density Functional Theory

- Energy Density Functional (EDF) : $E[\rho] = \langle\Psi[\rho]|\hat{H}|\Psi[\rho]\rangle$
- The ***exact*** ground state is given by : $E_{\min} = E[\rho_{\text{g.s.}}]$

DFT and Time-Dep. DFT formalism

Kohn-Sham Equation : $\left[\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + v_{\text{KS}}(\boldsymbol{r})\right]\phi_i(\boldsymbol{r}) = \epsilon_i\phi_i(\boldsymbol{r}) \qquad v_{\text{KS}}(\boldsymbol{r}) = \frac{\delta E[\rho]}{\delta\rho}$

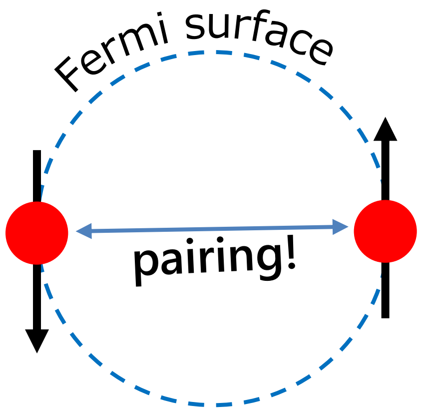
TDDFT Equation: $i\hbar\frac{\partial\phi(\boldsymbol{r})}{\partial t} = \hat{H}(\boldsymbol{r})\phi(\boldsymbol{r})$

Point

- By DFT we can significantly reduce comp. cost
- We can also perform time-develop self-consistently

Pairing Inclusion

Nucleons form ***cooper pairs*** near Fermi surface



BCS Theory

- BCS wave function : $|\mathrm{BCS}\rangle = \sum_{k>0}(u_k + v_k \hat{a}_k^\dagger \hat{a}_{-k}^\dagger)\,|0\rangle$
- Gap Equation : $\Delta = \dfrac{G}{2}\sum_{k>0}\dfrac{\Delta}{\sqrt{(\varepsilon_k-\lambda)^2+\Delta^2}}$

BCS generalized by the ***Bogoliubov transformation***

Hartree-Fock-Bogoliubov (HFB) Theory

- Quasi-particle operators : $\hat{\alpha}_\mu = \sum_k \left(U^*_{k\mu}\hat{a}_k + V^*_{k\mu}\hat{a}_k^\dagger\right)$ $\hat{\alpha}_\mu^\dagger = \sum_k \left(U_{k\mu}\hat{a}_k^\dagger + V_{k\mu}\hat{a}_k\right)$
- HFB wave function : $|\mathrm{HFB}\rangle = \prod_\mu \hat{\alpha}_\mu\,|0\rangle$
- HFB equation : $\begin{pmatrix} \hat{h}-\lambda I & \Delta \\ \underbrace{-\Delta^*}_{\text{Pairing “field”}} & -\hat{h}^*+\lambda I \end{pmatrix}\begin{pmatrix} U_\mu \\ V_\mu \end{pmatrix} = \begin{pmatrix} U_\mu \\ V_\mu \end{pmatrix}\underbrace{E_\mu}_{\text{“Quasi-particle” energy}}$

Combination with Band Theory

In the coordinate representation,

$$\begin{pmatrix} \hat{h}(\boldsymbol{r}) - \lambda & \Delta(\boldsymbol{r}) \\ -\Delta^*(\boldsymbol{r}) & -\hat{h}^*(\boldsymbol{r}) + \lambda \end{pmatrix} \begin{pmatrix} u_{\mu\uparrow}(\boldsymbol{r}) \\ v_{\mu\downarrow}(\boldsymbol{r}) \end{pmatrix} = E_\mu \begin{pmatrix} u_{\mu\uparrow}(\boldsymbol{r}) \\ v_{\mu\downarrow}(\boldsymbol{r}) \end{pmatrix}$$

Band theory

- Bloch's Theorem : $\phi(\boldsymbol{r}) = \tilde{\phi}(\boldsymbol{r})e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$ with $\tilde{\phi}(\boldsymbol{r}+\boldsymbol{T}) = \tilde{\phi}(\boldsymbol{r})$
- in our case : $u_{\mu\uparrow}(\boldsymbol{r}) = \tilde{u}_{\mu\uparrow}(z)e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$ $v_{\mu\downarrow}(\boldsymbol{r}) = \tilde{v}_{\mu\downarrow}(z)e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$

Substituting those relations, finally we have :

$$\nabla(\psi(\boldsymbol{r})e^{i\boldsymbol{k}\cdot\boldsymbol{r}}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}}(\nabla + i\boldsymbol{k})\psi(\boldsymbol{r})$$

$$\begin{pmatrix} \hat{h}(z) + \hat{h}_{\boldsymbol{k}}(z) - \lambda & \Delta(z) \\ \Delta^*(z) & -\hat{h}^*(z) - \hat{h}^*_{-\boldsymbol{k}}(z) + \lambda \end{pmatrix} \begin{pmatrix} u_{\mu\boldsymbol{k}\uparrow}(z) \\ v_{\mu\boldsymbol{k}\downarrow}(z) \end{pmatrix} = E_{\mu\boldsymbol{k}} \begin{pmatrix} u_{\mu\boldsymbol{k}\uparrow}(z) \\ v_{\mu\boldsymbol{k}\downarrow}(z) \end{pmatrix}$$

$$\rho^{(0)}(z) \Rightarrow \hat{H}(z) \Rightarrow u_\mu, v_\mu \Rightarrow \rho(z) \Rightarrow \hat{H}(z) \Rightarrow \cdots$$

Spatial coordinate : $N_z \sim 50$
Bloch wave : $N_{k_z} = 80$
Plain wave : $N_{k_\parallel} = 150$
(u,v), (p,n) : 2×2

2400000 orbitals!?

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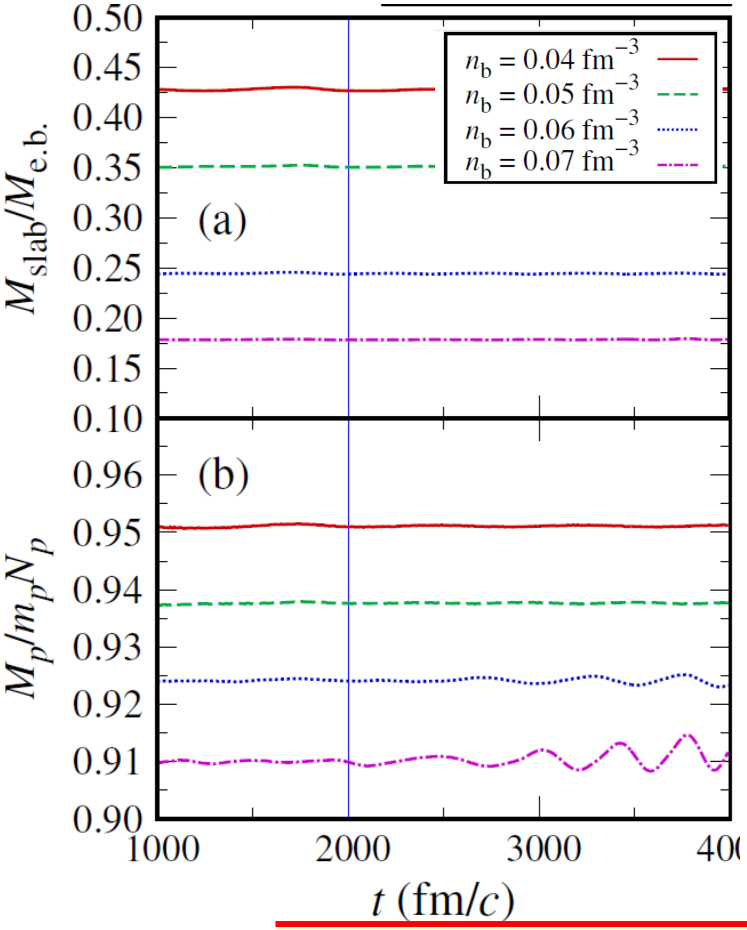
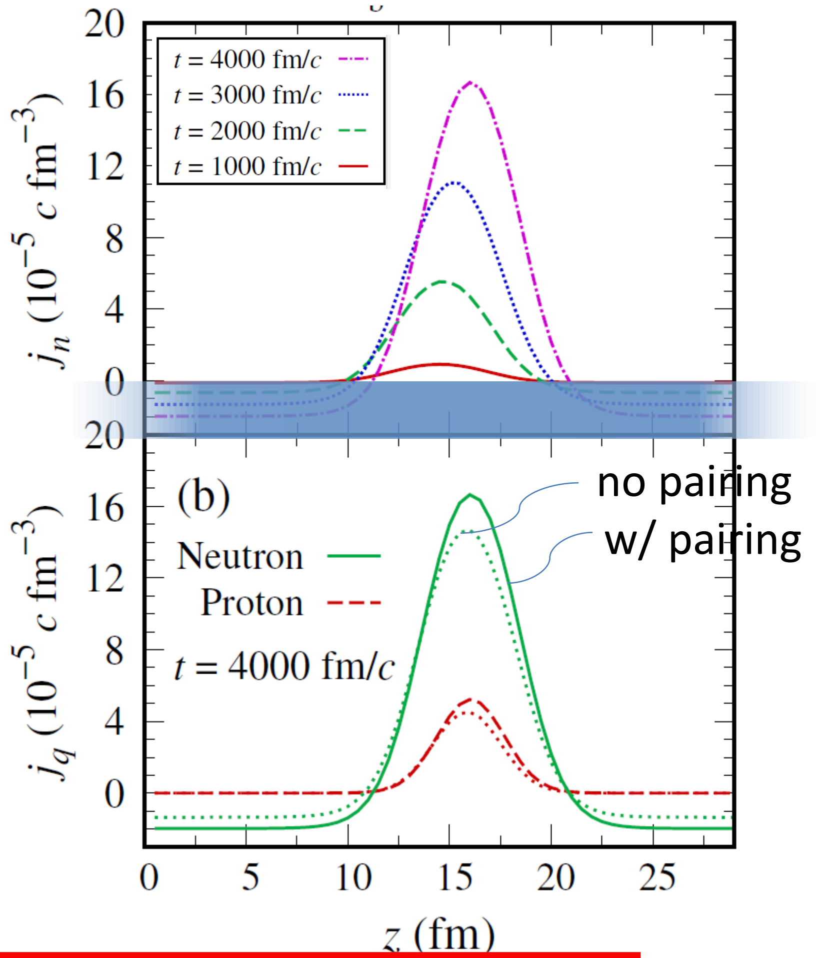
Main Result 2

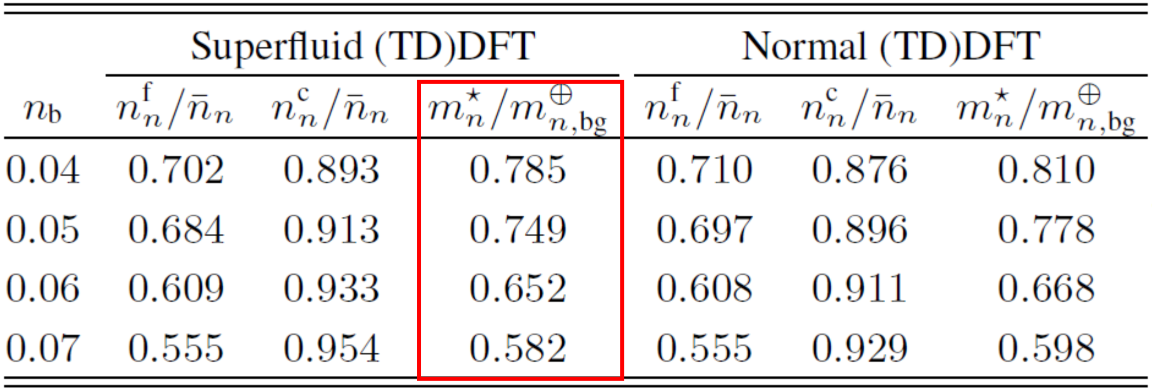




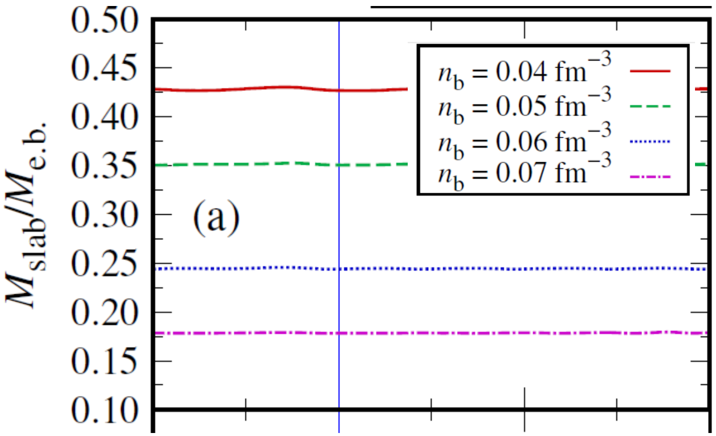
Point

- Effective mass significantly reduced!!
- ”Counterflow” of neutrons (***anti-entrainment***?)
- Superfluidity “enhances” anti-entrainment!!

Summarized Result



	Superfluid (TD)DFT			Normal (TD)DFT		
n_b	$n_n^\mathrm{f}/\bar{n}_n$	$n_n^\mathrm{c}/\bar{n}_n$	$m_n^\star/m_{n,\mathrm{bg}}^\oplus$	$n_n^\mathrm{f}/\bar{n}_n$	$n_n^\mathrm{c}/\bar{n}_n$	$m_n^\star/m_{n,\mathrm{bg}}^\oplus$
0.04	0.702	0.893	0.785	0.710	0.876	0.810
0.05	0.684	0.913	0.749	0.697	0.896	0.778
0.06	0.609	0.933	0.652	0.608	0.911	0.668
0.07	0.555	0.954	0.582	0.555	0.929	0.598



Summary

- We've shown eff. masses "always" less than bare masses
- Superfluidity little affects anti-entrainment, but enhances it.

Next step

- Finite-temperature extension (almost done)
- Finite-magnetic field extension (for *magnetar*)
- 2D, and 3D crystalline phase? (our final goal)

Next we will show preliminary results of f.t. calcs…

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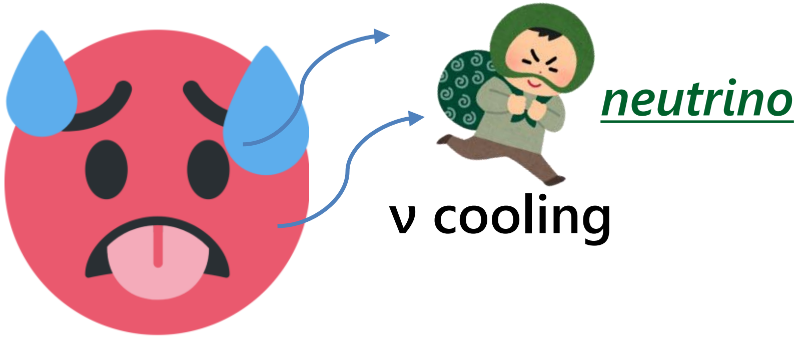
Why Finite Temperature?

Why F.T. is important?

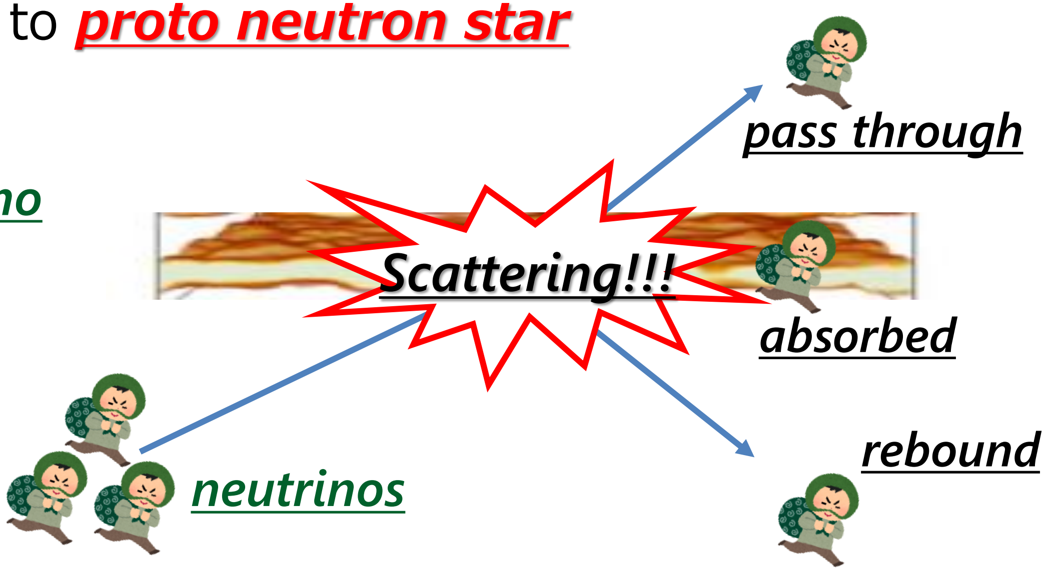
It is deeply related to ***proto neutron star***



neutrino

ν cooling

~MeV
Hot P.N.S



pass through

Scattering!!!

absorbed

rebound

neutrinos

Our question is :

①Pasta structures in high temperature situation?

②Phase transition from superfluid systems?

③Neutrino-pasta scattering?

We can do it now!!

Finite Temperature Extension

Finite temperature extension

zero temperature

$$\rho(z) = \sum_{\mu} |v_\mu(z)|^2$$

$$\kappa(z) = \sum_{\mu} v_\mu^*(z) u_\mu(z)$$

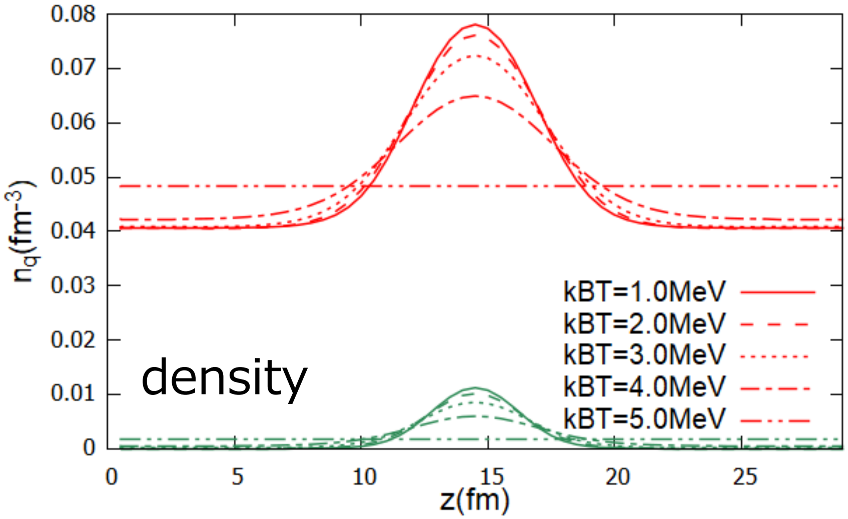
finite temperature

$$\rho(z) = \sum_{\mu>0} f(-E_\mu)|v_\mu(z)|^2 + f(E_\mu)|u_\mu(z)|^2$$

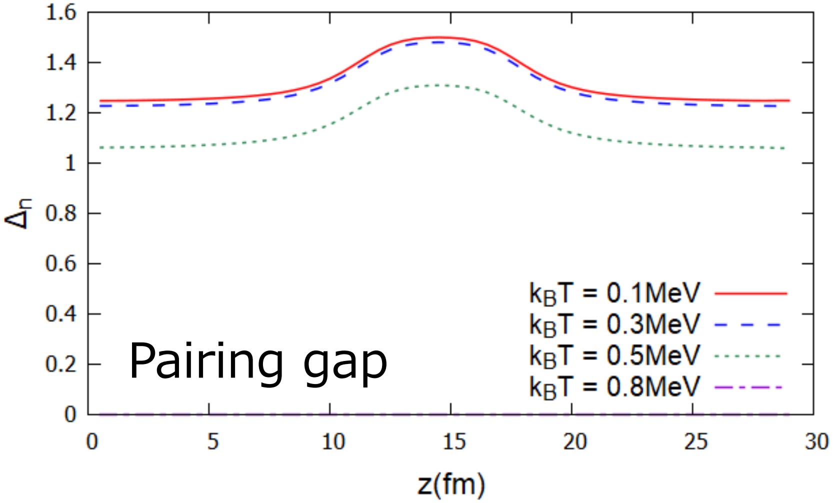
$$\kappa(z) = \sum_{\mu>0} v_\mu^*(z) u_\mu(z) \frac{f(-E_\mu) - f(E_\mu)}{2}$$

$$\Delta(z) = g_{\text{eff}}(z)\kappa(z)$$

g_{eff} ⋯coupling constant



Nuclear melting

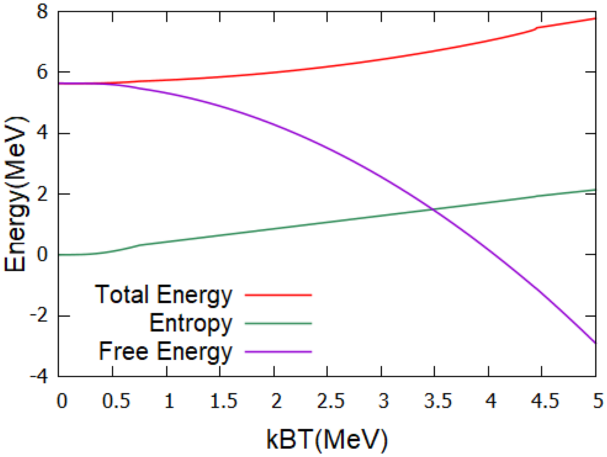


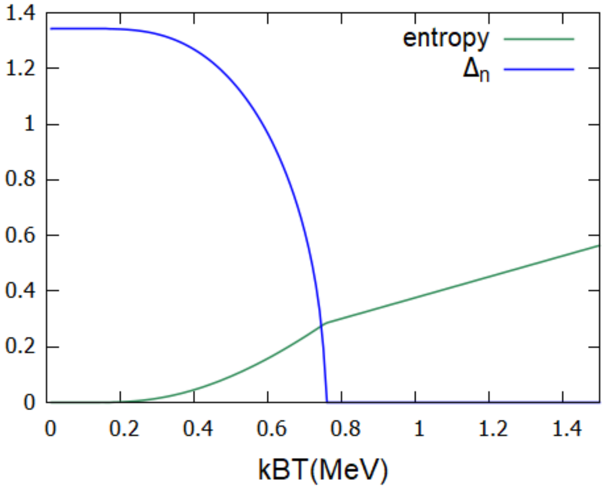
Phase transition?

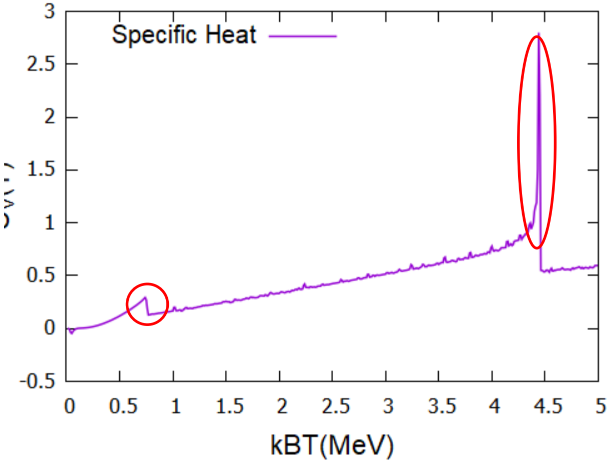
Finite Temperature Extension

Energy Calculation

- ***Free Energy*:** $F(T) = E[\rho(T)] - S(T)T$
- ***Entropy*:** $S(T) = k_B \sum_{\mu} [f(E_\mu) \ln f(E_\mu) + f(-E_\mu) \ln f(-E_\mu)]$
- ***Specific heat*:** $C_V(T) = \dfrac{\partial E(T)}{\partial T}$







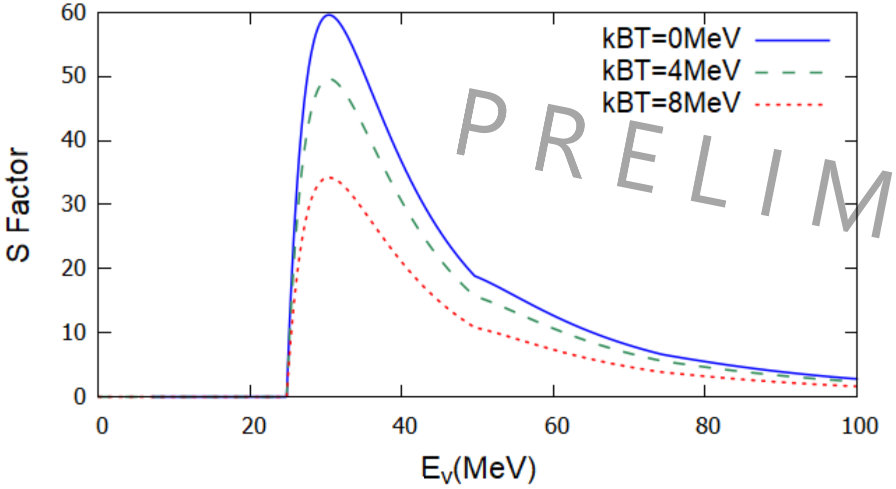
Point

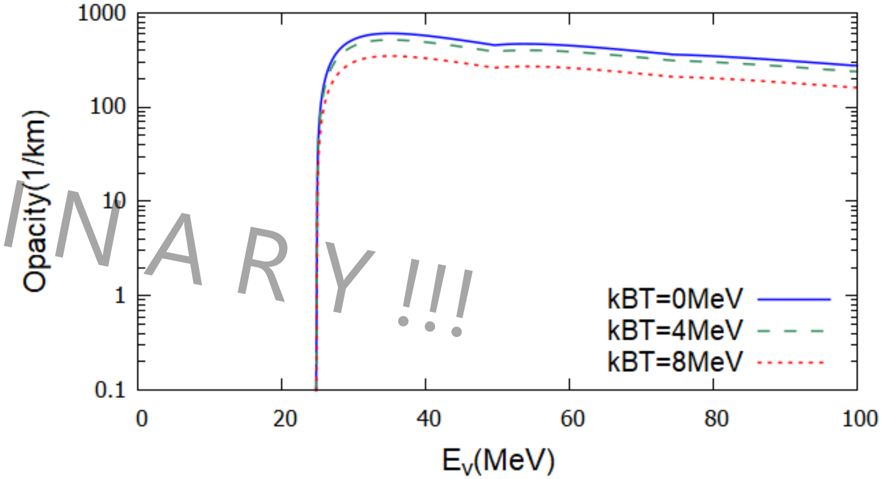
- We can straightforwardly extend into f.t. systems
- We can see two phase transitions :
  -- ①superfluid to normal fluid ② nuclear melting

ν -pasta scattering

ν-pasta scattering

- diff. cross section : $\frac{1}{V}\frac{\mathrm{d}\sigma(E_\nu)}{\mathrm{d}(\cos\theta)} = \frac{G_F^2}{4\pi^2} n E_\nu^2 \left[c_V^2(1+\cos\theta)S_V(\boldsymbol{q}) + c_A^2(3-\cos\theta)S_A(\boldsymbol{q})\right]$
- neutrino ***opacity*** : $\chi_T(E_\nu) = \frac{2G_F^2E_\nu^2}{3\pi} n \left[c_V^2\left\langle S_V(E_\nu)\right\rangle + 5c_A^2\left\langle S_A(E_\nu)\right\rangle\right]$





Point

- Structure factors show strong peaks
- Characteristic of pasta structure?

Summary and Outlook

Summary

1. We have realized ①superfluid ②self-consistent ③band calculations and performed them for 1 dimensional crystalline structures.
2. We found "anti-entrainment" remains with superfluidity
3. We have extended our calculations into finite-temperature systems and are investigating phase transitions, and v-pasta scattering

Future Work

1. Extension into finite-magnetic field systems (magnetar)
2. Simulations of N.S. crust vibration (Quasi-Periodic Oscillations)
3. QP resonances? FFLO phase?
4. Extension into 2D and 3D crystalline structures?

Final Goal: *to derive the Equation of State, and superfluid density of all the inner crust phases, with all possible situations.*

Thank you for your careful attention!

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