Superfluid and Finite-Temperature Extensions of Self-Consistent Band Theory for the Inner Crust of Neutron Stars

Kenta Yoshimura Tokyo Institute of Technology
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INTRODUCTION

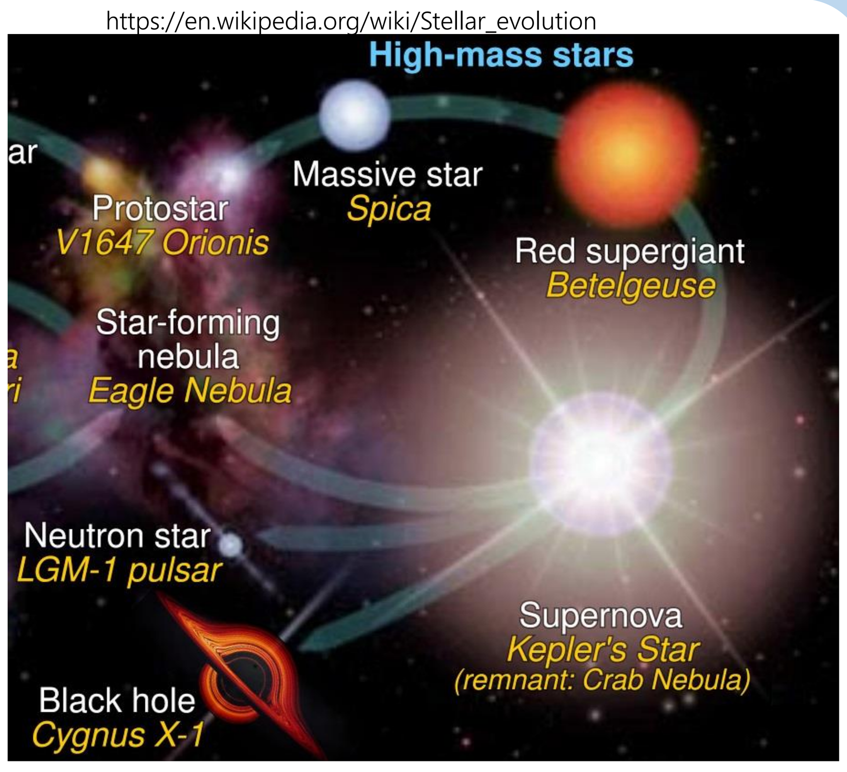
Neutron star is what?

Neutron star : a remnant of supernova

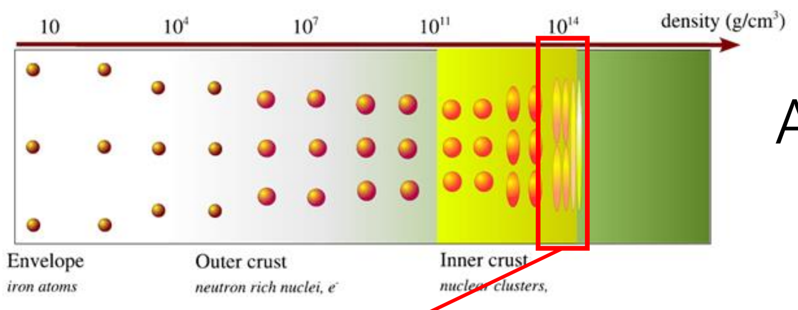
- densities : $\sim 5\rho_0$
- temperatures : ~keV
- mag. field : $\sim 10^{18}$G
- rotation period : ~ms

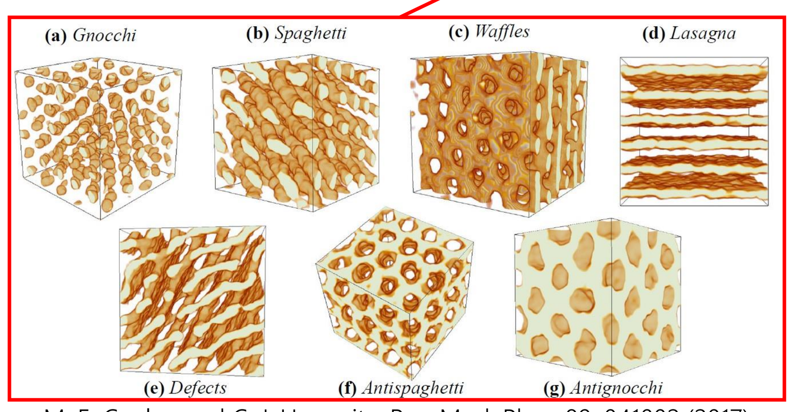
A frontier of theoretical physics

https://en.wikipedia.org/wiki/Stellar_evolution



N. Chamel and P. Haensel, Living Rev. Relativity 11, 10 (2008)





M. E. Caplan and C. J. Horowitz, Rev. Mod. Phys. 89, 041002 (2017)

The "Pasta Structures"

At the bottom of "crust" area, nuclei form crystalline structures?

They are called "pasta" phases, and we need micro. Calculations using band theory.

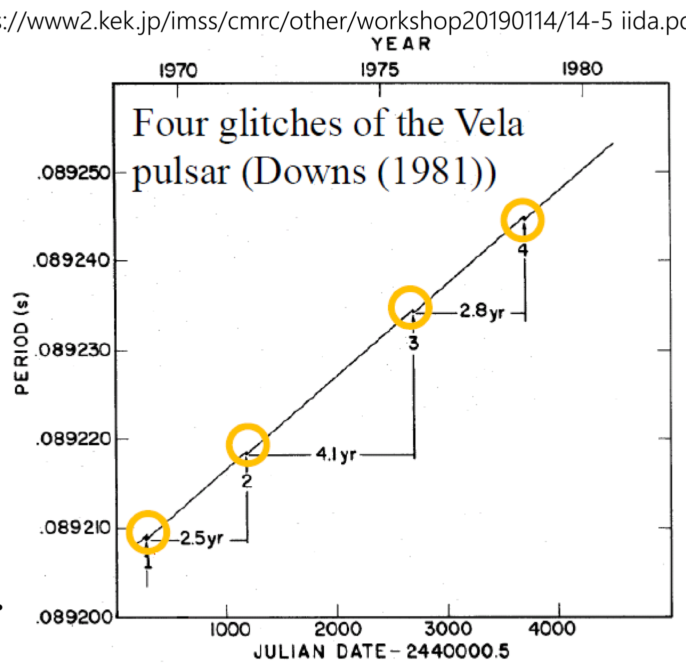
Nuclear pasta is said to be deeply related to "pulsar glitch".

The "Pulsar Glitch" mystery

Observation data indicates that periods of neutron stars irregularly rebound, which is called "glitch" phenomena.

It is advocated that pasta phases and there dripped neutrons are the cause.

https://www2.kek.jp/imss/cmrc/other/workshop20190114/14-5 iida.pdf



Our Research Target

- Formulate superfluid self-consistent calculations combined with band theory, for quantitative interpretation of glitch.
- For now, we focus on 1-dim. crystalline structure (slab), for them perform calculations, and tackle several extensions.

FORMALISM

Density Functional Theory (DFT)

Assuming that we have the total energy as a function of density :

$$\mathcal{E}_{\text{total}} = \int d\boldsymbol{r}\mathcal{H}(\boldsymbol{r})$$

and the g.s. is given by variational condition : $\dfrac{\delta\mathcal{H}}{\delta\phi^*(\boldsymbol{r})} = 0$

It returns to Sch. like Eq. : $\left[\dfrac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + v_{\text{KS}}[n]\right]\psi_k(\boldsymbol{r}) = \varepsilon_k\psi_k(\boldsymbol{r})$

Hartree-Fock-Bogoliubov Theory (HFB)

For description of pairing, we consider those "quasi-particle" :

$$\hat{\beta}_\mu = \sum_i\left(U^*_{i\mu}\hat{a}_i + V^*_{i\mu}\hat{a}^\dagger_i\right) \qquad \hat{\beta}^\dagger_\mu = \sum_i\left(U_{i\mu}\hat{a}^\dagger_i + V_{i\mu}\hat{a}_i\right)$$

and employ the Bogoliubov-de-Gennes equation :

$$\begin{pmatrix}\hat{h}_q(\boldsymbol{r}) - \lambda & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -\hat{h}^*_q(\boldsymbol{r}) + \lambda\end{pmatrix}\begin{pmatrix}u^{(q)}_\nu(\boldsymbol{r}) \\ v^{(q)}_\nu(\boldsymbol{r})\end{pmatrix} = E_\nu\begin{pmatrix}u^{(q)}_\nu(\boldsymbol{r}) \\ v^{(q)}_\nu(\boldsymbol{r})\end{pmatrix}$$

Band Theory

According Bloch's theorem, the wfs are decomposed into

$$\phi(\boldsymbol{r}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\tilde{\phi}(\boldsymbol{r}) \quad \text{where} \quad \tilde{\phi}(\boldsymbol{r}+\boldsymbol{T}) = \tilde{\phi}(\boldsymbol{r})$$

For 1-dimensional crystalline structures, it turns to be

$$\tilde{\phi}(\boldsymbol{r}) = \tilde{\phi}(z) \qquad \boldsymbol{k} = (k_\parallel, k_z)$$

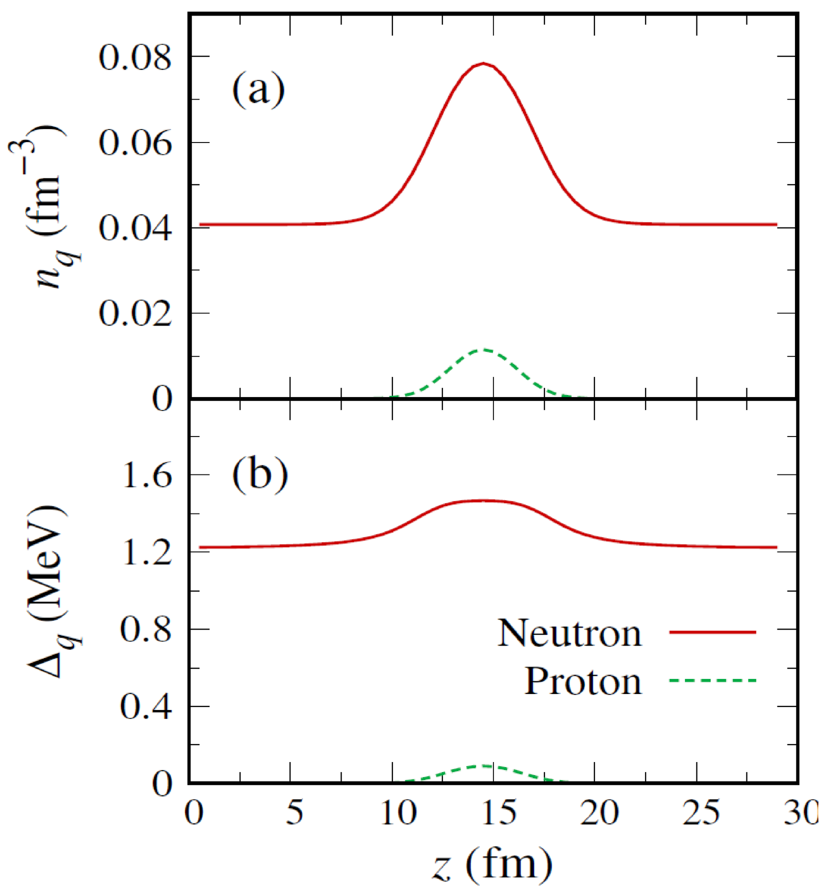
Combining these 3 theories, finally we have

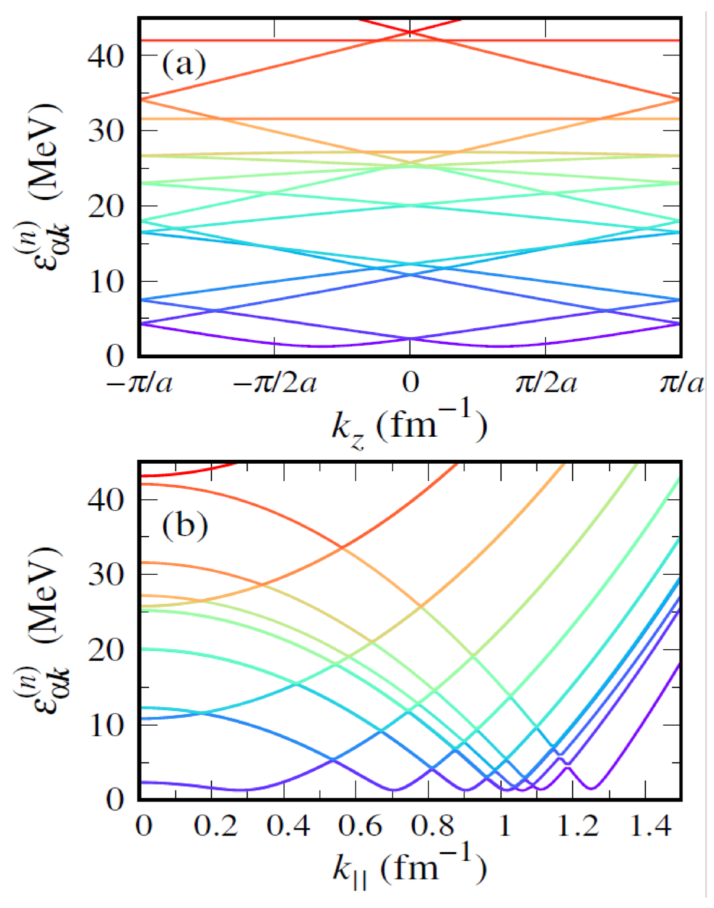
$$\begin{pmatrix}\hat{h}_q(z) + \hat{h}_{\boldsymbol{k}}(z) - \lambda & \Delta(z) \\ \Delta^*(z) & -\hat{h}^*_q(z) - \hat{h}_{\boldsymbol{k}}(z) + \lambda\end{pmatrix}\begin{pmatrix}u^{(q)}_{\nu\boldsymbol{k}}(z) \\ v^{(q)}_{\nu\boldsymbol{k}}(z)\end{pmatrix} = E^{(q)}_{\nu\boldsymbol{k}}\begin{pmatrix}u^{(q)}_{\nu\boldsymbol{k}}(z) \\ v^{(q)}_{\nu\boldsymbol{k}}(z)\end{pmatrix}$$

NUMERICAL RESULT

From typical results, we can see

①Neutrons are excessively dripped and distributed uniformly
②n's pairing is almost uniform, p have superconductivity.
③Quasi-particle energies form band structure



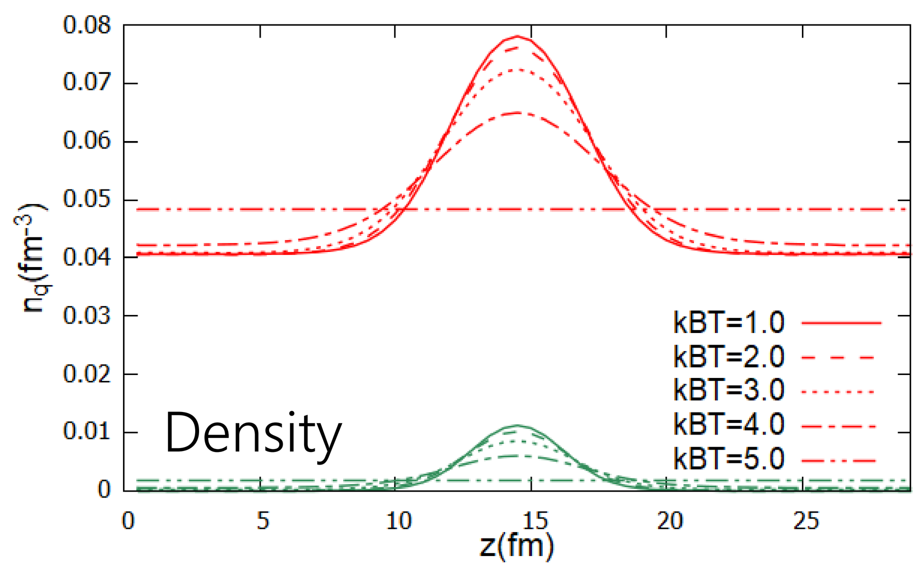


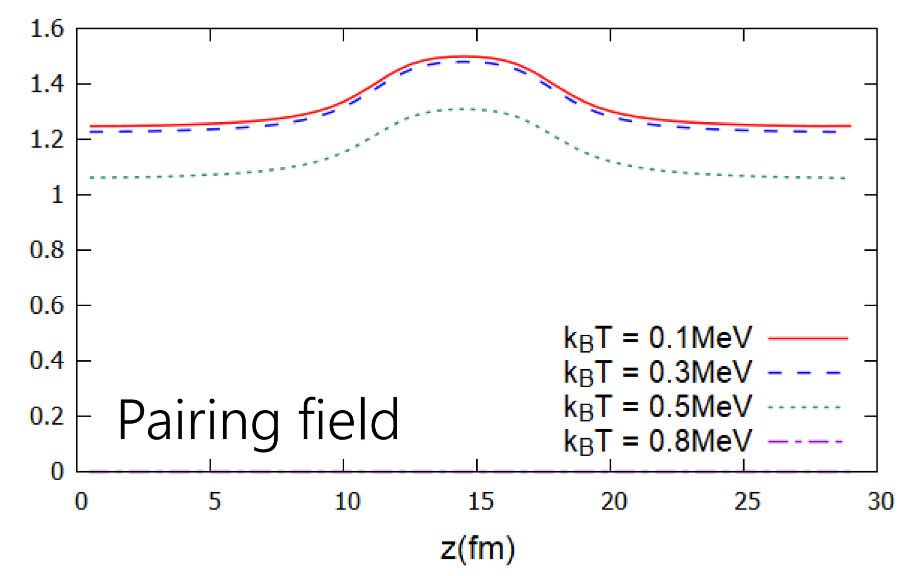
Neutrons' "effective masses" are always less than bare mass, which means conduction of neutrons is "enhanced" by band structures, which is called "anti-entrainment".

	Superfluid (TD)DFT			Normal (TD)DFT		
n_b	$n^f_n/\bar{n}_n$	$n^c_n/\bar{n}_n$	$m^\star_n/m^\oplus_n$	$n^f_n/\bar{n}_n$	$n^c_n/\bar{n}_n$	$m^\star_n/m^\oplus_n$
0.04	0.702	0.893	0.785	0.710	0.876	0.810
0.05	0.684	0.913	0.749	0.697	0.896	0.778
0.06	0.609	0.933	0.652	0.608	0.911	0.668
0.07	0.555	0.954	0.582	0.555	0.929	0.598

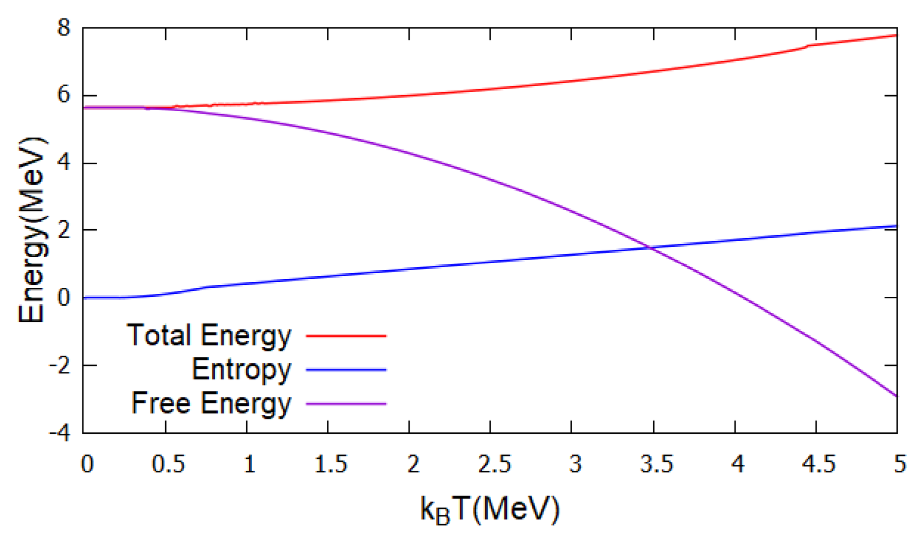
FURTHER EXTENSION

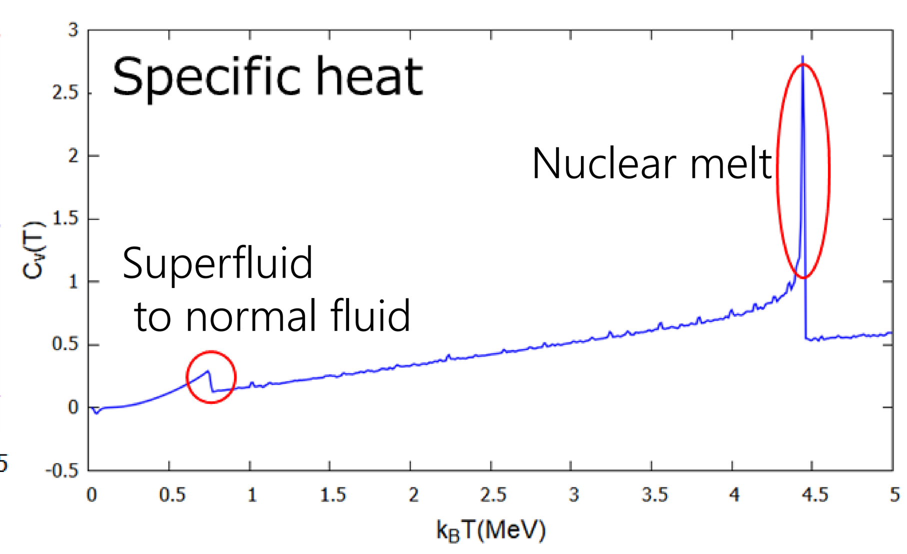
We can straightforwardly extend those calculations into finite temperature systems, and investigate their thermal properties.





Tracing energies with respect to temperatures, we can get the "specific heat" of slab phase, and critical temperatures.





SUMMARY and PROSPECT

Summary

- We realized ①superfluid②self-consistent③band calculations and perform them for various systems under β equilibrium.
- We clarify diverse properties of pasta nuclei, e.g. anti-entrainment, phase transitions and nuclear melting.

Future Works

- Investigation of cooling of "proto neutron stars" by calculations of "Neutrino-pasta scattering" (now trying!)
- Calculations of structures of "magnetars" with $\sim 10^{18}$ G, by extensions into finite-magnetic field systems (almost done!)
- Final goal : applications for all pasta structures, and reveal anti-entrainment and Equation of State systematically.