～中性子星の観測と理論～研究活性化ワークショップ 2023 @Kyoto
6-8 Sept. 2023

中性子星の熱的進化とパルサーの回転進化

野田常雄 (久留米工業大学)

KURUME INSTITUTE OF TECHNOLOGY

Introduction
Neutron star cooling and rotation

Superfluidity affects Neutron Star Cooling

+

Superfluidity also affects the rotation of the star

⬇

Concurrent calculations of cooling and rotation of neutron stars allows comparison with pulsar spin-downs.

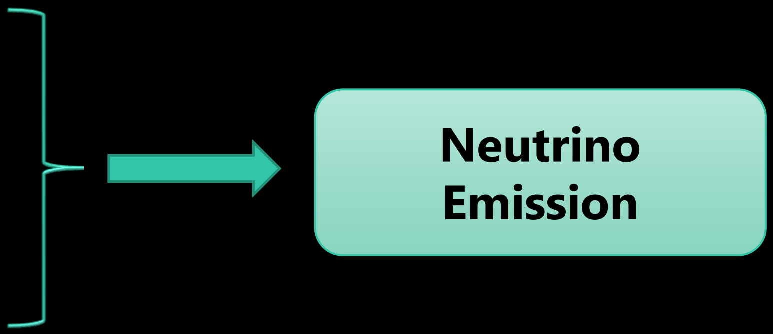
Adding rotational effects with cooling models

Cooling of Neutron Stars

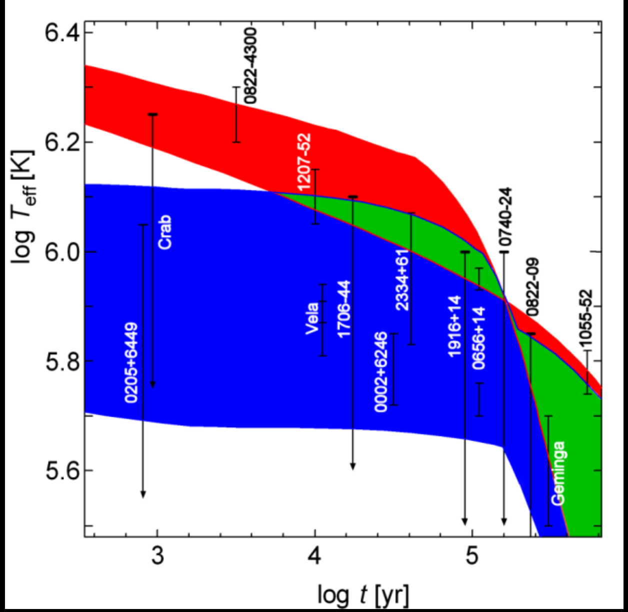
Cooling processes of NS strongly depend on the interior state

- **Normal nuclear matter**
- π condensation
- K condensation
- **Quark matter**
- **Superfluidity**

etc…



Neutrino Emission



(TN+ 2006)

Exotic phase appears in high density state, **cools star rapidly**

Central density above the threshold density = Heavy NS

Comparing the calculation results and isolated NS observation

⇒ Constraining the high-density state

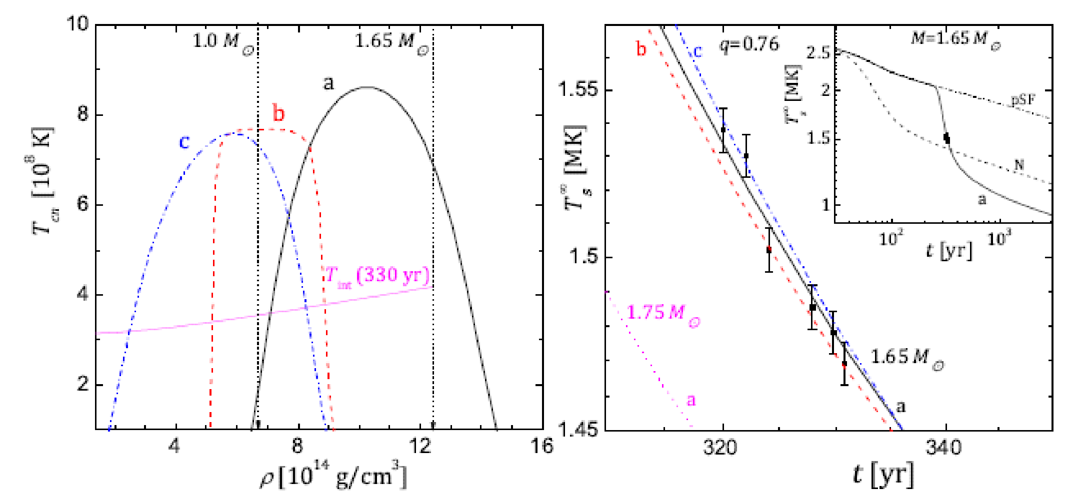
Neutrino Emission

- Occurring in all NS (**Standard**)
  - Modified URCA: **Weak**
  - Nucleon superfluidity: **Marginally Strong** (at transition)
- Occurring in heavy NS (**Exotic**)
  - Quark β-decay (Considering **Quark Matter**): **Strong**
  - Direct URCA ($y_p < 1/9$): **Strong**

- Superfluidity, Superconductivity
  - Neutrino emission at transition
  - Superfluid state **suppresses** **Other Neutrino Emission**
    → mild the “Strong” emission

Effects of Superfluidity on cooling

- Superfluidity has 2 effects on cooling
  - Transition from Normal to Super: Neutrino emission (Accelerate cooling)
  - After the transition: Suppression other Neutrino emission processes (Decelerate cooling)
- Neutron $^3P_2$
  - The density dependence of the critical temperature determine the cooling history of neutron stars.



Shternin et al. 2011

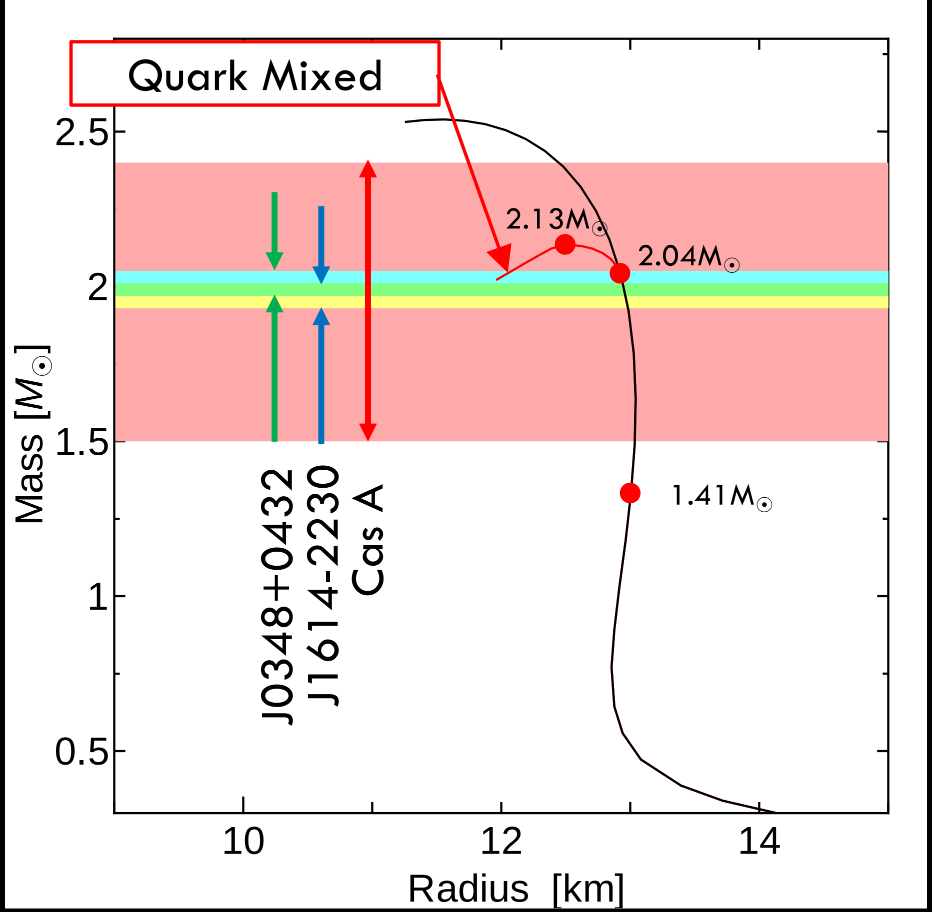
Models

- EoS with Maximum mass above $2M_\odot$
  - Brueckner-Hartree-Fock (HM) + Dyson-Schwinger (QM)
  - Mixed phase between HM-QM (Yasutake+ 2016)

Even at the maximum mass, the centre is mixed phase

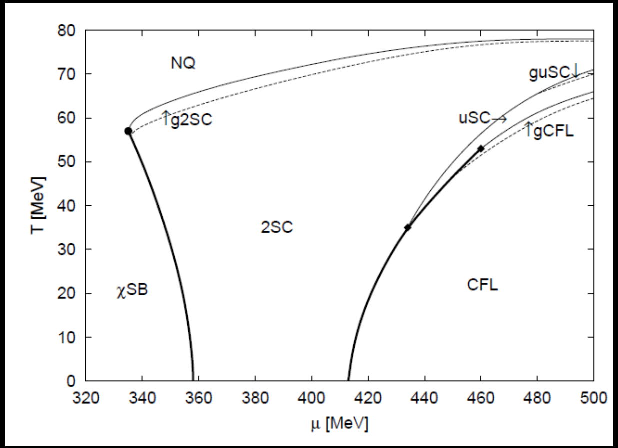
  - Surface composition: ^{56}Fe

- Cooling processes
  - Modified URCA + Bremsstrahlung
  - n-Super($^1S_0$, $^3P_2$), p-Super($^1S_0$)
  - **Direct URCA (y_p>1/9)**
  - **Quark Cooling** with Colour Superconductivity (CSC)
- Parameters
  - **Masses**
  - **n, d- $^3P_2$ Superfluidity** Critical Temperature
  - **CSC Paring** (CFL / 2SC / 2SC+X)



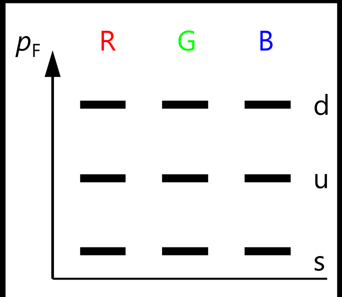
Pairing of Quarks

- CSC in quark matter has **Multiple parings**
  - Degrees of freedom of colour and flavour
- **CFL** (Colour Flavour Locking・**Higher density**✗)
  - All colous and flavours can make pairs
  - **All quarks (RGB) in superconducting** → **Suppressing neutrino emission**✓
- **2SC** (Two Flavour Superconductivity・**Lower density**✓)
  - 2 of colours/flavours can make pairs
  - **1/3 of normal quark remains**→**Strong neutrino emission**✗

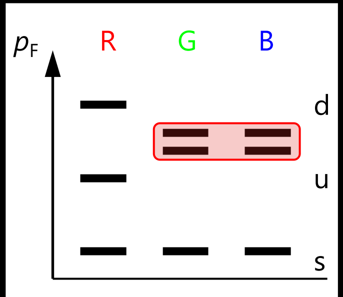


Rüster et al. (2006)

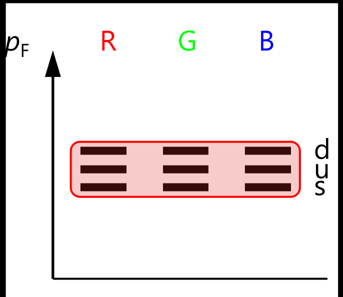
Unpaired



2SC Pairing



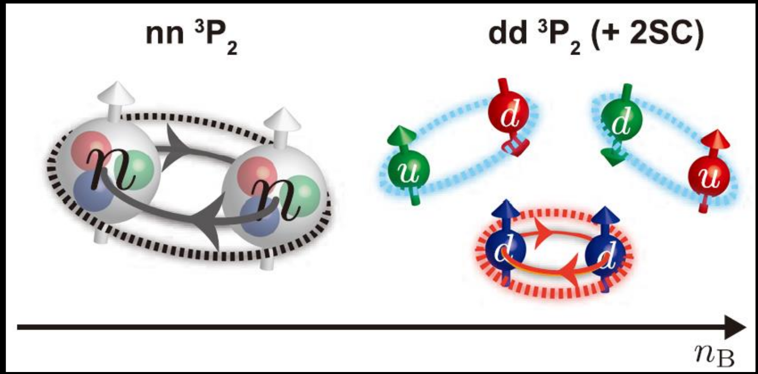
CFL Pairing



Quark-Hadron Continuity

Neutron $^3P_2$ →Quark $^3P_2$+2SC Continuous transition (Fujimoto+ PRD 101, 094009, (2020))

- Neutron $^3P_2$ has been continued by d-quarks → Other can make 2SC



- **All quarks can make pairs**
- **Suppressing neutrino emission in 2SC (2SC+X)**

- Assumption: **Critical temperature of Neutron $^3P_2$ is carried by d-quark's $^3P_2$**
    - No effects for proton $^1S_0$
    - Δ of 2SC / CFL are few tens of MeV
    - No s-quarks

Nucleon Superfluidity

- Neutrons and protons become superfluid

Neutron: **$^1S_0$**, **$^3P_2$** Proton: **$^1S_0$**

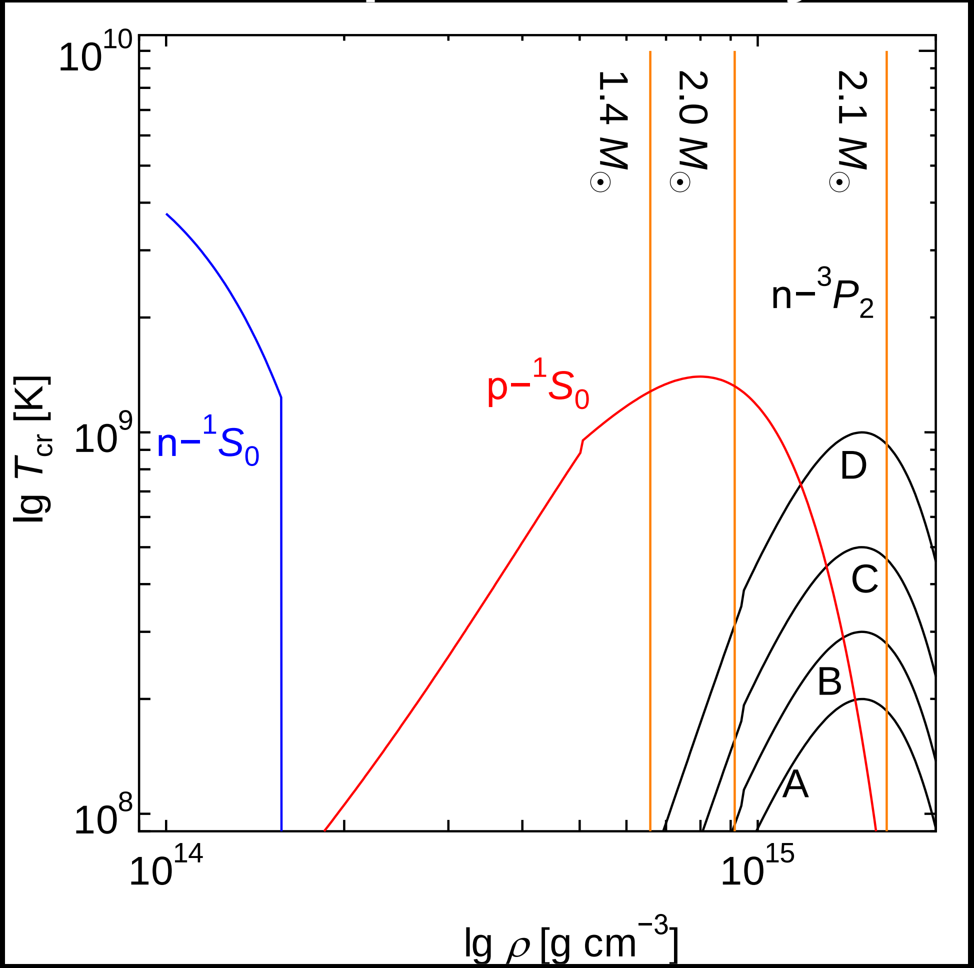
- Critical temperature (T_{cr})
  - Functionated density dependence

Effects on Cooling

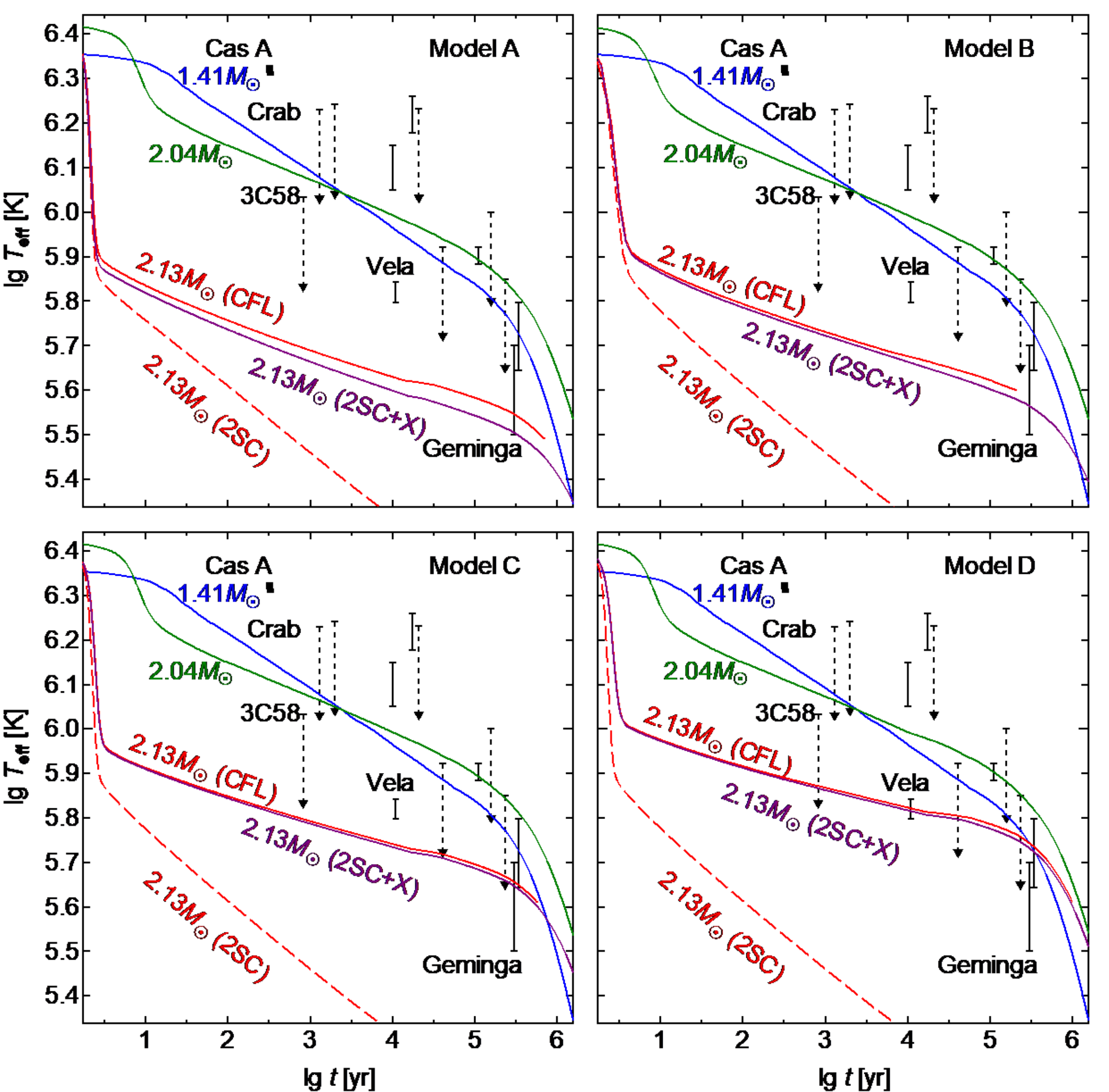
Superfluid transition: **Strong cooling (PBF)**

(Page+ 2004)

Superfluid state: **Suppresses other neutrino emission**



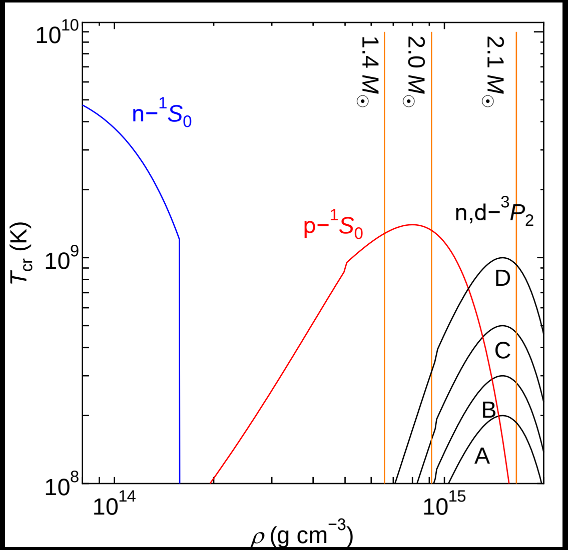
- n- **$^3P_2$** critical temperature is continued by 2SC+X
- Calculating with changing the n, d- **$^3P_2$** model



Cooling Results

- Cooling behaviour depends on hadron superfluidity models
- Quark pairing affects cooling curves.

Checking superfluid cooling models with other superfluid phenomena.



Summary of Neutron Star Cooling

- Rapid cooling in 2SC → Too cold ✗
    - Appearing density ✓
- Marginal cooling with CFL (depending n- ${}^3P_2$ critical temperature) → Observation ✓
    - Appearing density does NOT match ✗
- **2SC+X is similar to CFL** (depending n, d ${}^3P_2$ critical temperature) → **Observation** ✓
    - Appearing density ✓

- **Show the effect on rotation following the method of Ho & Andersson(2012)**

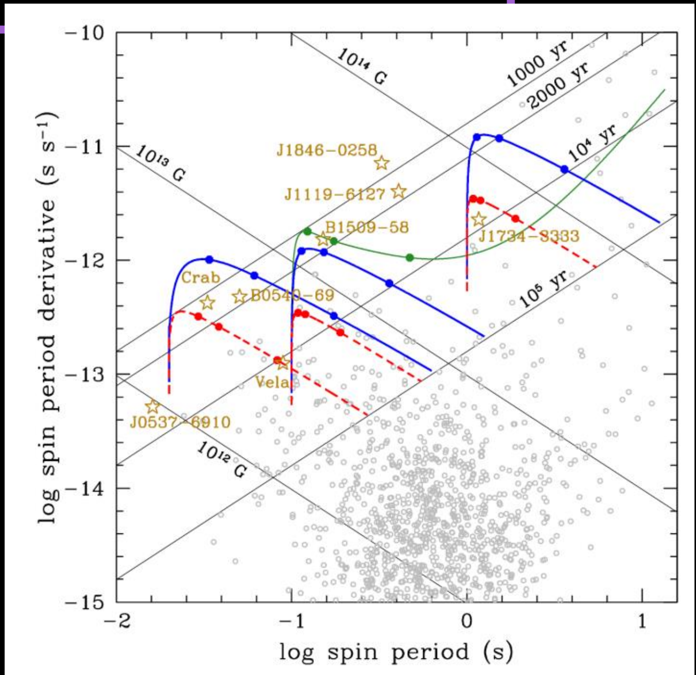
nature physics

LETTERS

PUBLISHED ONLINE: 30 SEPTEMBER 2012 | DOI: 10.1038/NPHYS2424

Rotational evolution of young pulsars due to superfluid decoupling

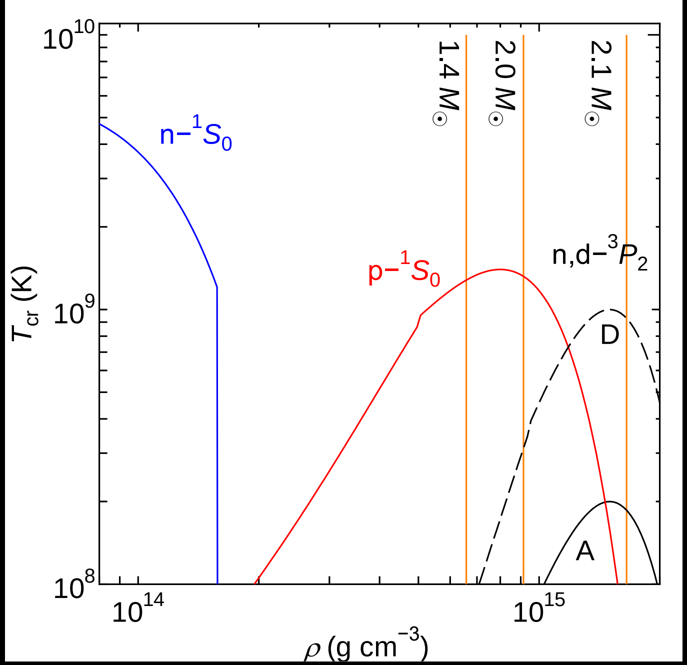
Wynn C. G. Ho* and Nils Andersson*



Cooling Pulsar Evolution

1. As the neutron star cools, the region of superfluidity increases
2. Decreased moment of inertia in the normal state region
3. Spin-down due to magnetic dipole radiation changes with decreasing moment of inertia
4. Line in $P - \dot{P}$ Diagram bends

- Settings
  - Superfluid Model **A(Solid)** and **D(Dashed)**
  - Superfluid state region does not affect rotation
  - Friction is small and constant between super and normal
  - Color superconducting regions are treated in the same way as nucleon superfluidity



Variation of moment of inertia / rotation

Moment of inertia

- General relativistic sphere moment of inertia
  - Exact relation: Ravenhall & Pethick (1994)

$$I = \frac{8\pi}{3}\int_0^R (\rho + P/c^2)\Lambda r^4 dr$$

$$\Lambda = \left(1 - \frac{2Gm}{c^2 r}\right)^{-1}$$

Calculate normal layer only

Rotation

- Angular velocity variation considering magnetic dipole radiation

$$\frac{d\Omega}{dt} = (\Omega_{SF} - \Omega)\frac{1}{I}\frac{dI}{dt} - \frac{\beta\Omega^3}{I}$$

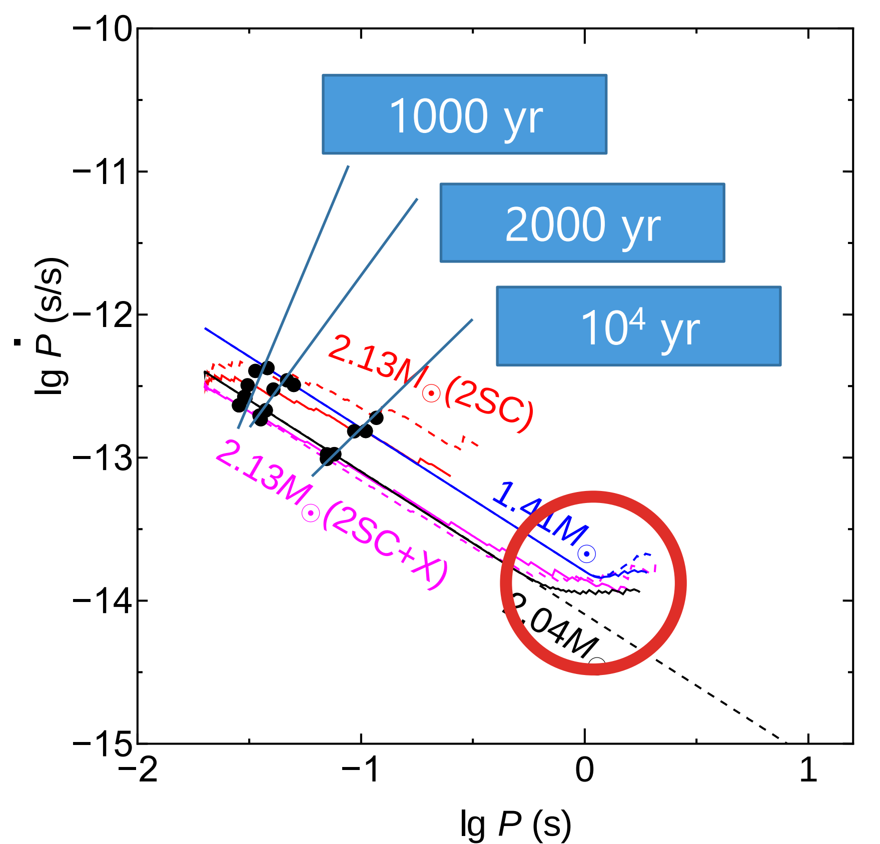
$$\beta \simeq B^2R^6/6c^3$$

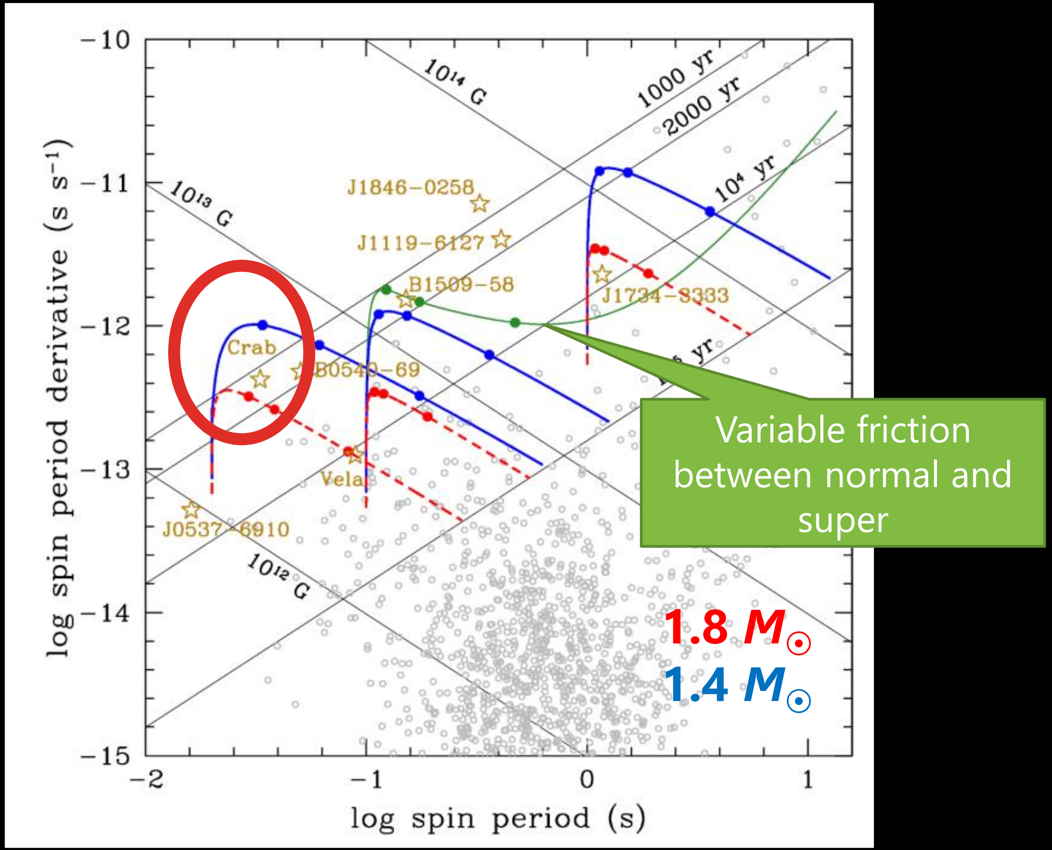
$$\Omega_{SF} - \Omega \leq 1.0 \times 10^{-6}$$

Parameters: Initial value of B & Ω

$$(P, B) = (0.02\,\mathrm{s}, 5 \times 10^{12}\,\mathrm{G})$$

$P - \dot{P}$ Diagram

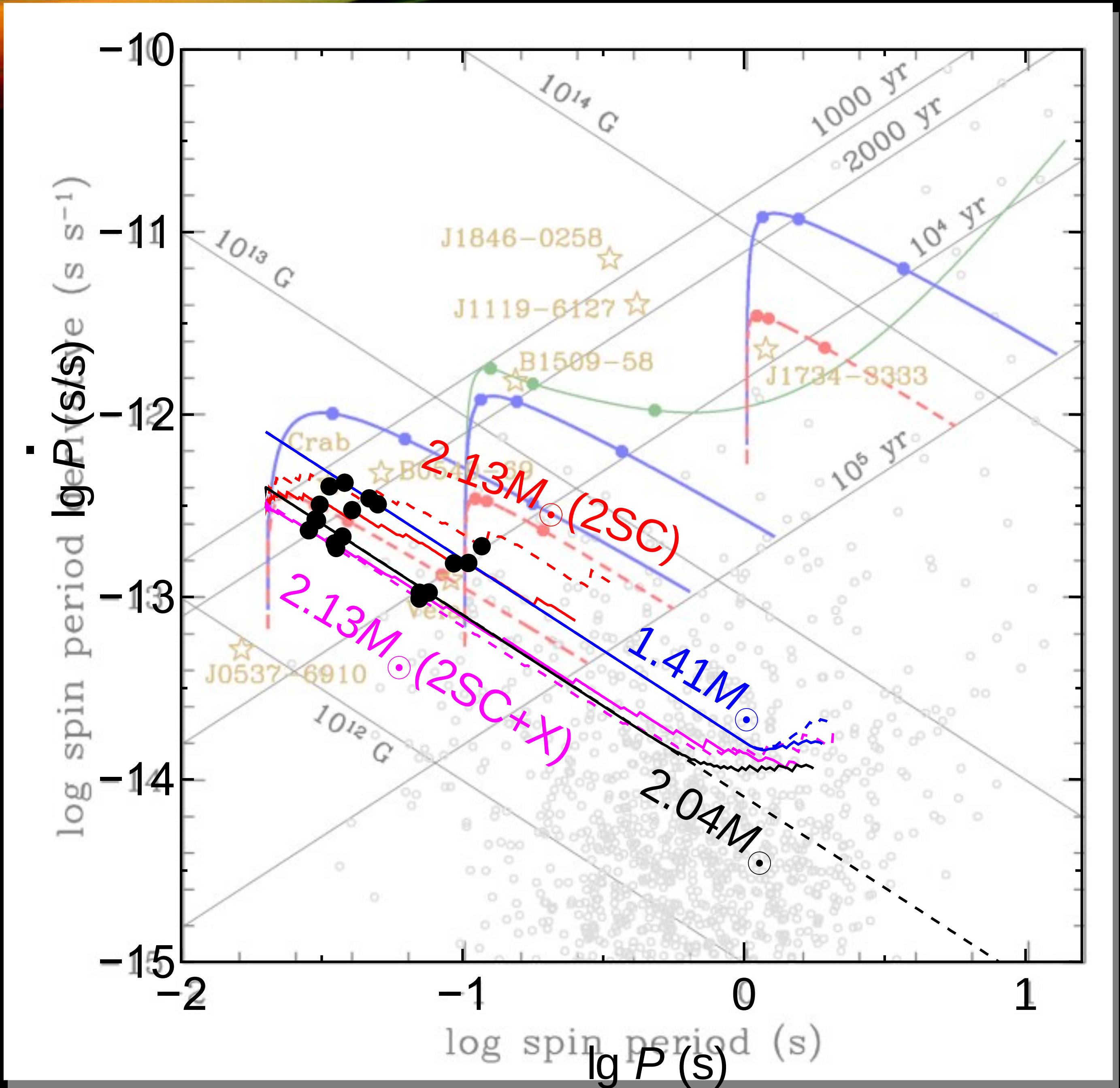




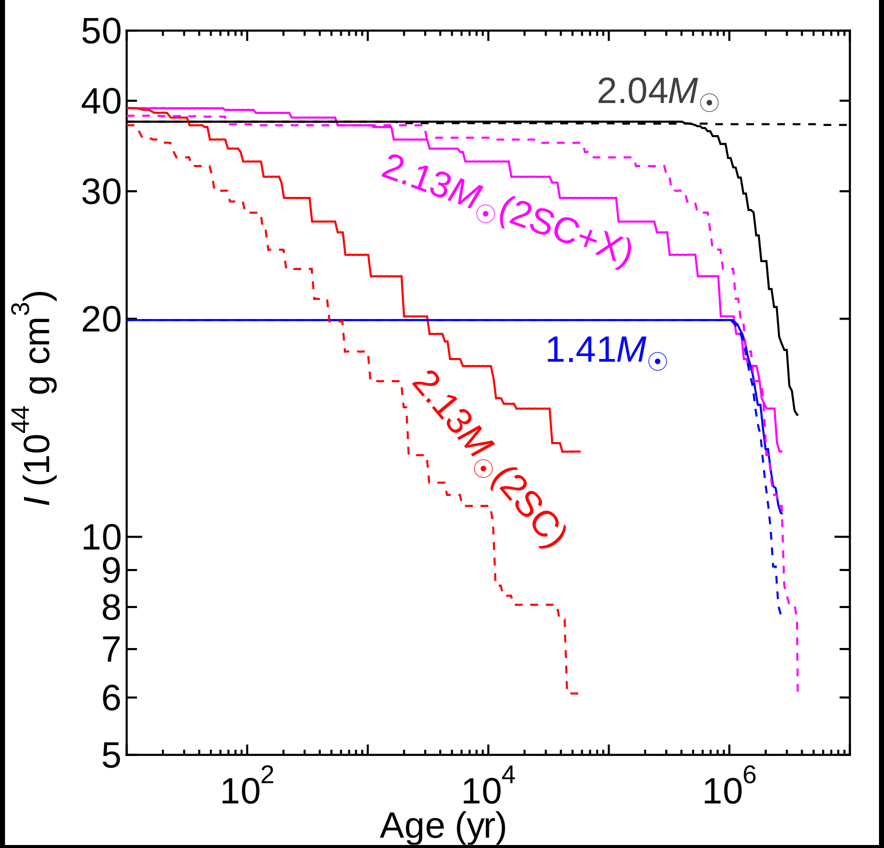
Variable friction between normal and super

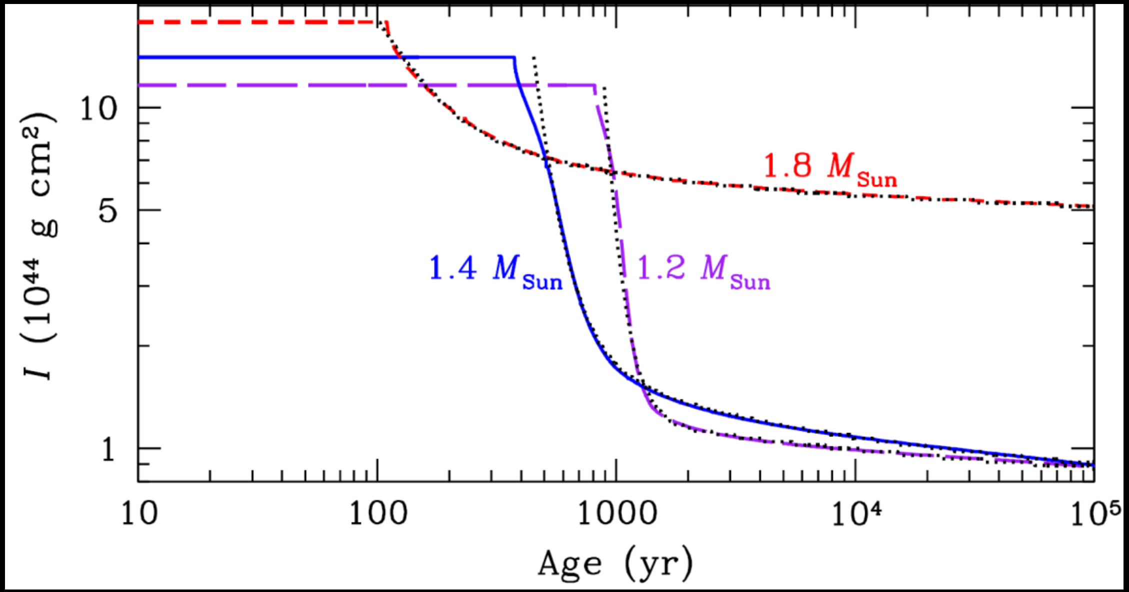
$P - \dot{P}$ Diagram differs to previous research (Ho&Andersson)

- Rise timing of $\dot{P}$


−10
−11
−12
−13
−14
−15
−2
−1
0
1
lg Ṗ (s/s)
lg P (s)
log spin period derivative (s s^{-1})
log spin period (s)
10^{14} G
10^{13} G
10^{12} G
1000 yr
2000 yr
10^4 yr
10^5 yr
J1846−0258
J1119−6127
B1509−58
J1734−3333
Crab
B0540−69
Vela
J0537−6910
2.13M$_\odot$(2SC)
1.41M$_\odot$
2.13M$_\odot$(2SC+X)
2.04M$_\odot$

Time variation of moment of inertia





Evolutional tracks of the momentum inertia differ to the previous research (Ho&Andersson)

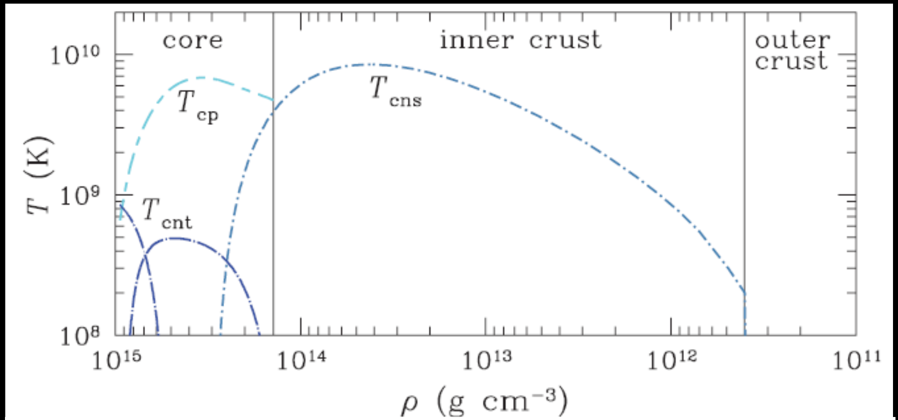
The reason of the time variation of moment of inertia

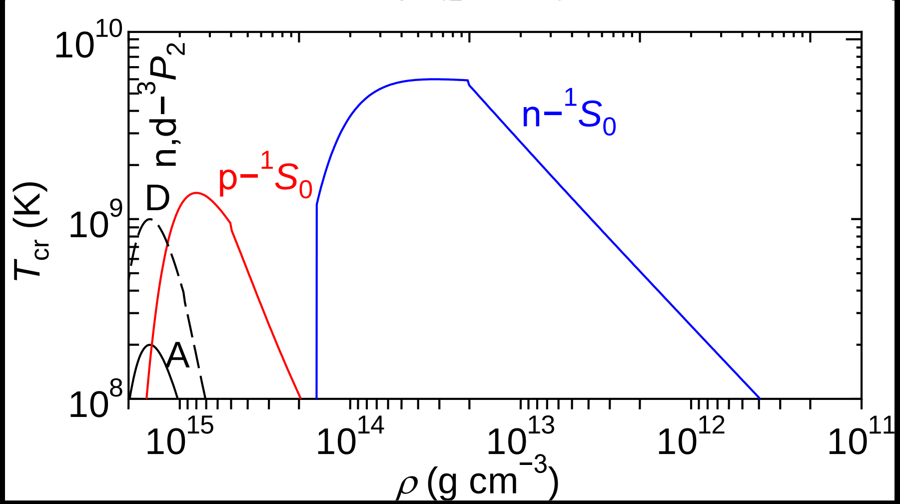
Ho & Andersson (2012)

- Superfluid model with “shallow” component
- Outer layer of core: **Superfluid from the beginning**
    - **Moment of inertia decreases in early stage**
- Curves rise in $P-\dot{P}$ Diagram in early stage

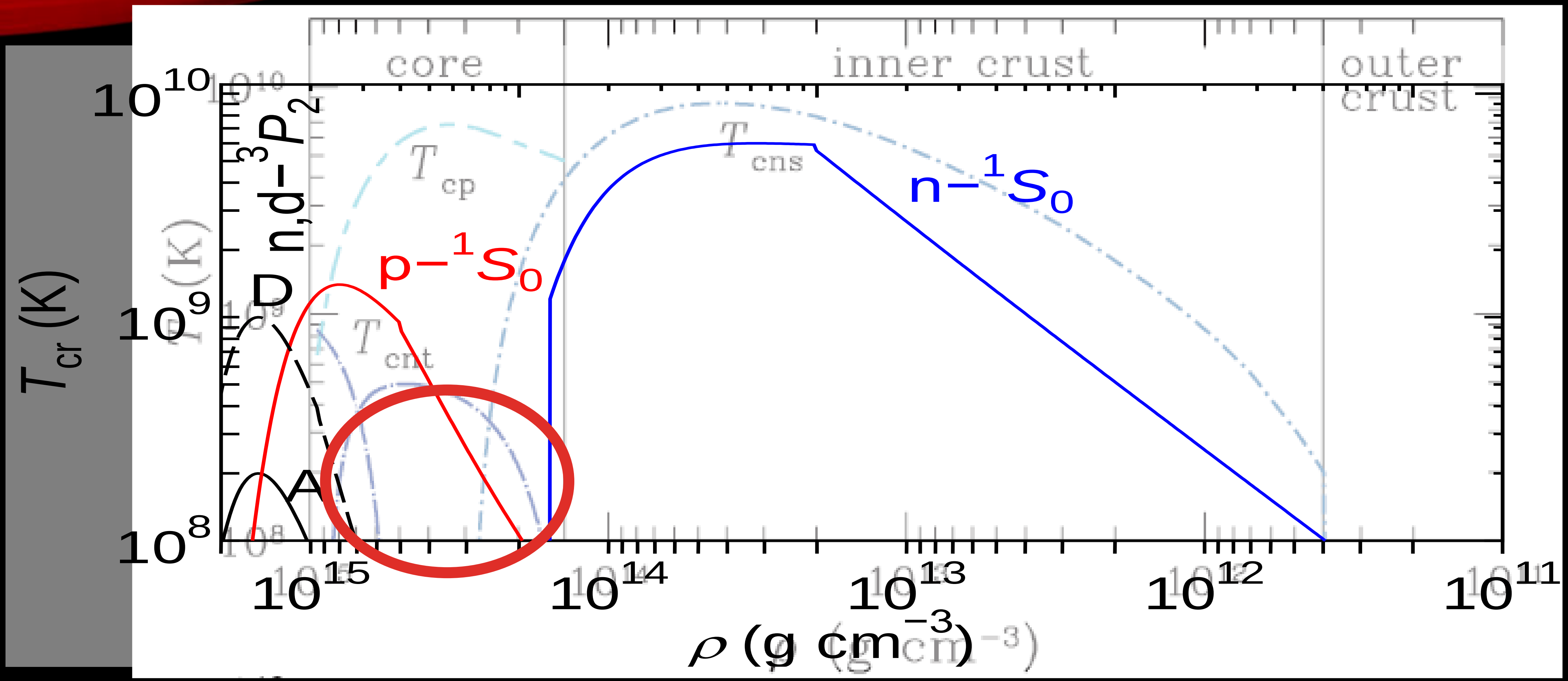
Ours

- Superfluid transition starts from the centre of the core
- Superfluid region increases by time
- When the superfluid region expands to a certain level, the moment of inertia is greatly reduced.
- Curves rise in $P-\dot{P}$ Diagram in late stage



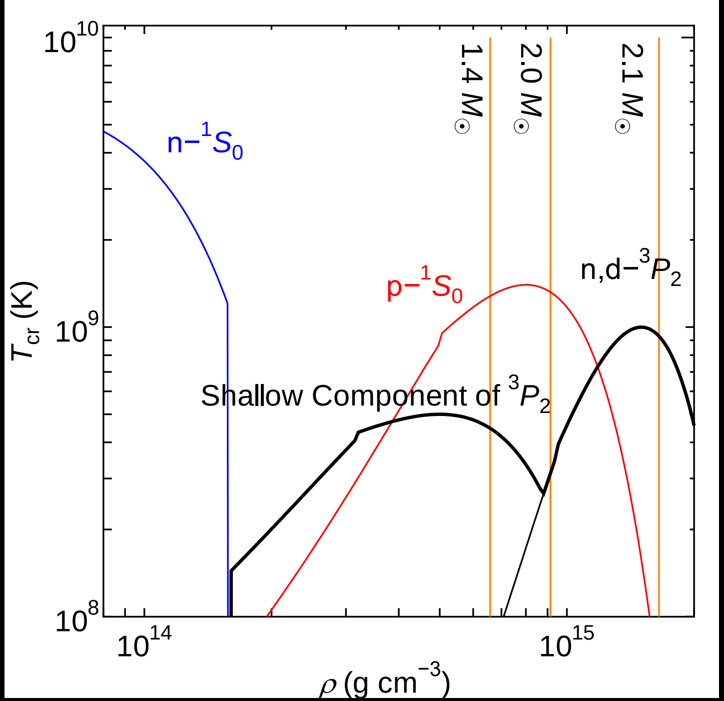


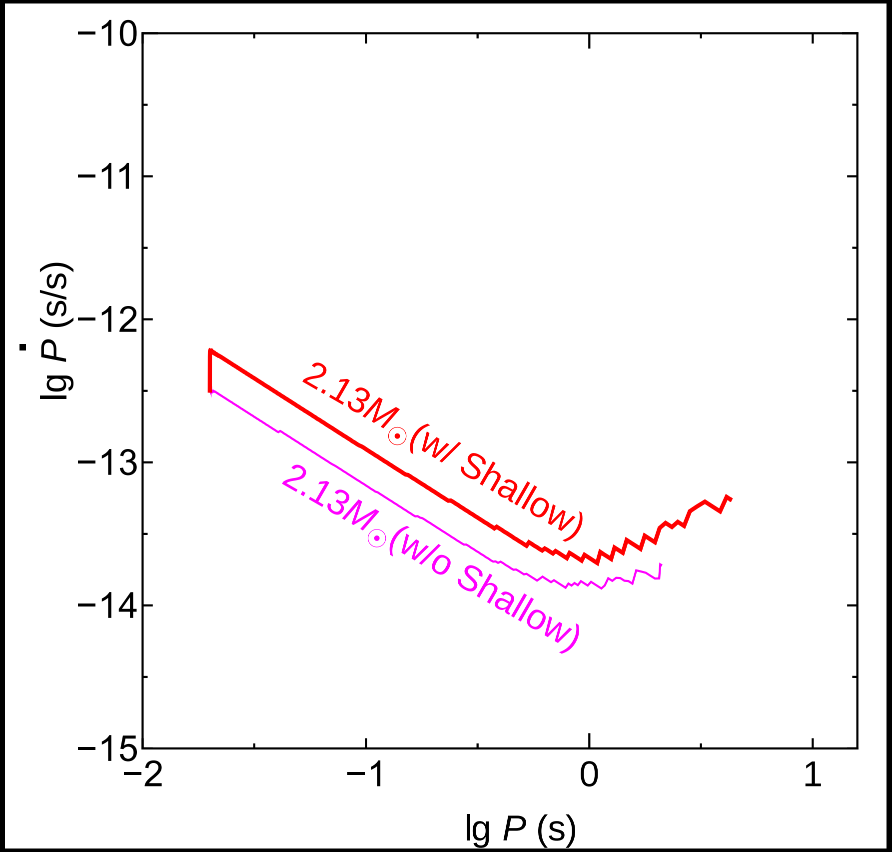
Critical temperatures slightly outside the centre are effective on moment of inertia

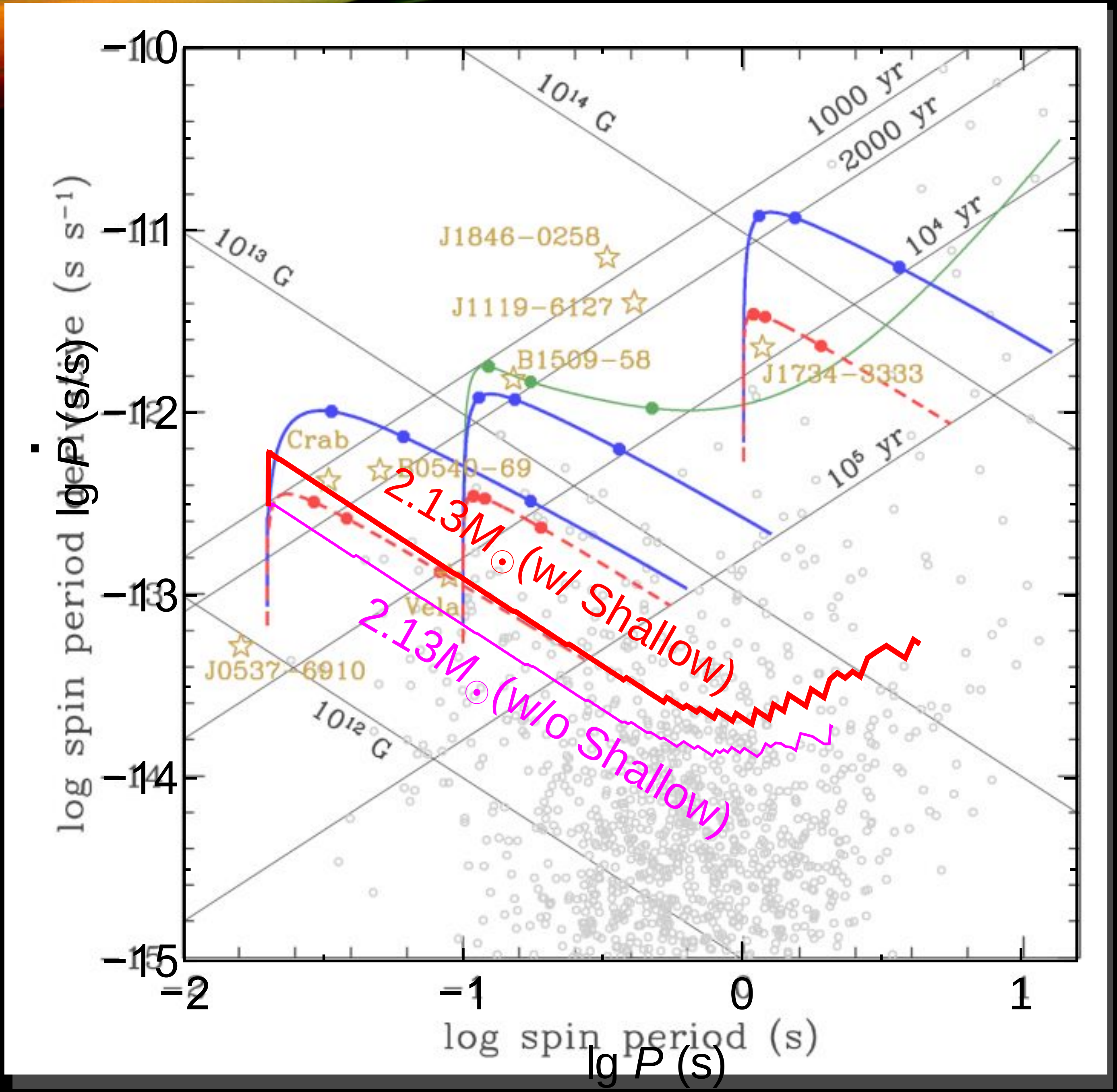

core
inner crust
outer crust
T_{cp}
T_{cns}
T_{cnt}
n,d−$^3P_2$
p−$^1S_0$
n−$^1S_0$
D
A
T_{cr} (K)
10^{10}
10^9
10^8
10^{15}
10^{14}
10^{13}
10^{12}
10^{11}
ρ (g cm^{-3})

Shallow Component of $^3P_2$

- Including "Shallow Component" to $^3P_2$ superfluidity
- Rise of $P - \dot{P}$ curve can be replicated

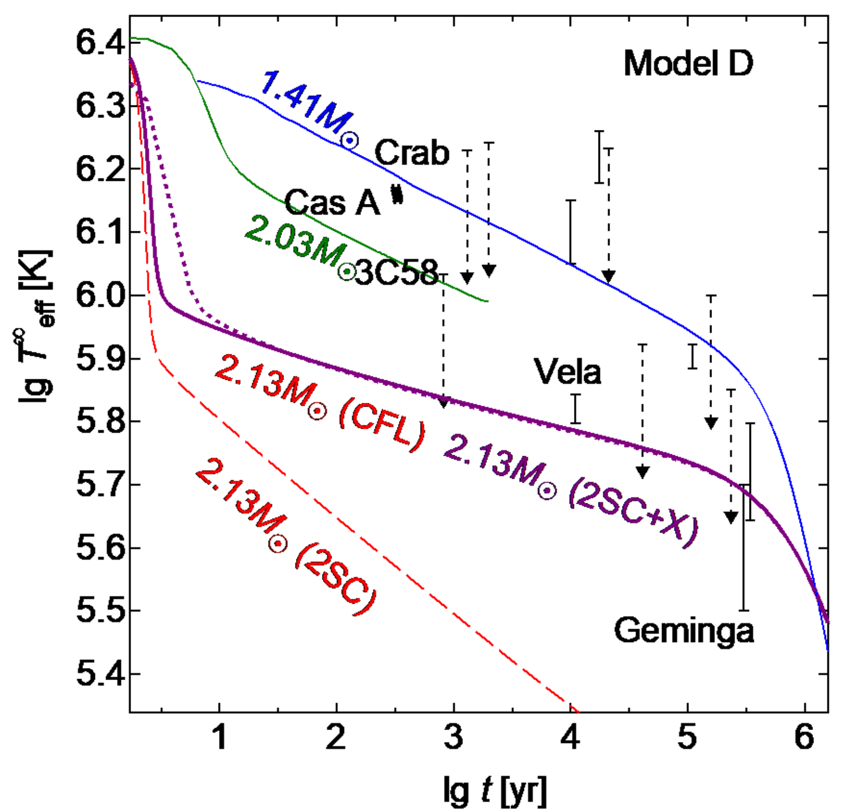


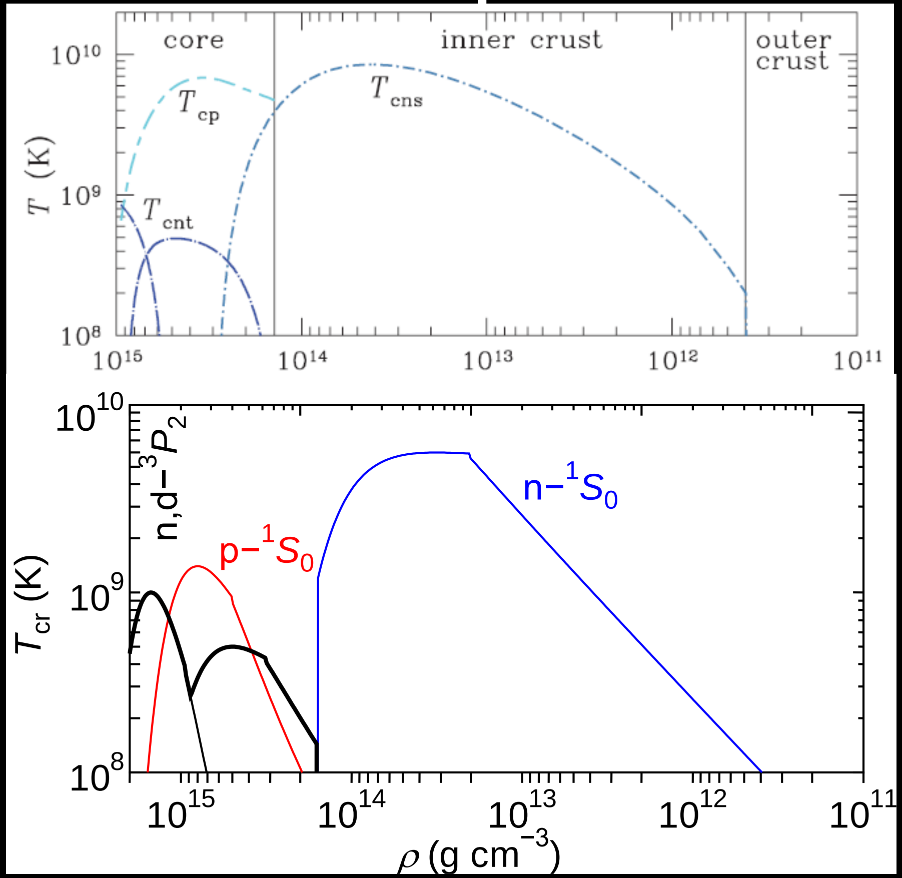



−10
−11
−12
−13
−14
−15
−2
−1
0
1
lg $\dot{P}$ (s/s)
lg P (s)
log spin period derivative (s s^{-1})
log spin period (s)
10^{14} G
10^{13} G
10^{12} G
1000 yr
2000 yr
10^4 yr
10^5 yr
J1846−0258
J1119−6127
B1509−58
J1734−3333
Crab
B0540−69
Vela
J0537−6910
2.13$M_\odot$(w/ Shallow)
2.13$M_\odot$(w/o Shallow)

Shallow Component of $^3P_2$

- With Shallow component, $P-\dot{P}$ curve rises at early stage.
- Cooling curves also change





Summary of Pulsar Evolution

- Calculate **the pulsar evolution** based on the cooling calculation of neutron stars
  - Magnetic dipole radiation
  - Time variation of Momentum of Inertia by superfluid transition ⇒ **$P - \dot{P}$ Diagram**
- Results of current model slightly differ from Ho & Andersson (2012)
  - $P - \dot{P}$ Diagram: Variation timing of $\dot{P}$
  - Momentum of Inertia: Decrease timing of the momentum of inertia → $P - \dot{P}$ Diagram
  - **Difference of superfluid model**

- Variation timing of the momentum of inertia of entire star
  - Depends on neutron $\mathbf{{}^3P_2}$ superfluid critical temperature in outer layer, not centre
  - **Observation of rise timing of $P - \dot{P}$ Diagram to upper right may constrain the “tail” of the critical temperature of neutron $\mathbf{{}^3P_2}$?**