パルサー星雲におけるシンクロトロン放射偏光成分のモデル計算
(Model calculation for synchrotron polarization of pulsar wind nebulae)

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This work was supported by JSPS KAKENHI Grant Number 23H01186.

Abstract

Pulsar wind nebulae (PWNe) are non thermal sources (radio ~ gamma rays) powered by pulsars. The Crab nebula has been studied in detail. The X ray observations show detailed structures such as the torus, the jet, the knot and the wisps.

A standard picture of the Crab nebula was given by Kennel & Coroniti (1984, KC hereafter). According to their picture, a superfast magnetohydrodynamic (MHD) wind, which is generated by the central pulsar, terminates at a shock, with the nebula identified as a post-shock flow shining in synchrotron radiation.

Shibata et al.(2003) calculated images of the Crab nebula based on the KC model and compared them with the Chandra image. They found that the assumption of a pure toroidal field does not reproduce the ring-like structure seen on the Chandra image, and found that there must be random magnetic fields at least comparable to the toroidal field in order to reproduce the ring-like image. Furthermore, the flow velocity must be fast (~0.2c) in order to reproduce the surface brightness contrast between the front and the back sides of the ring.

Polarization measurements are a key to understand the magnetic field structure. In 2022, the Crab nebula and the pulsar were observed by the Imaging X-ray Polarimetry Explorer(IXPE) (Bucciantini et al. (2023)). They found that the nebula has a total space integrated polarised degree of ~20%, which is comparable to the previous observations (Weisskopf et al.(1978)) .

Nakamura and Shibata (2007) (NS07) calculated the synchrotron polarization properties for a relativistically expanding disc that has both the toroidal magnetic field and the random magnetic fields. They compared their results with observations of the Crab Nebula and found that the random magnetic fields contribute 60 per cent of the total magnetic energy in order to reproduce the observed polarization.

In this poster, we calculate the synchrotron polarization properties following the similar method of NS07, and examine whether statistical distributions of the random fields affect the polarization properties. We consider following three cases of the distributions: a) the random magnetic fields with a fixed amplitude and random orientations (NS07) , b) the random fields described by an isotropic random Gaussian (Bucciantini et al. (2017)(B17)), and c) the random fields with a uniform distribution function. For these three distributions we compute 1) the integrated polarization degree *P* as a function of the degree of randomness *b* and the inclination angle *i* (same as figure 3 in NS07), and 2) the spatial distribution of the polarization degree. Comparisons of results for the three distributions show no significant differences among them. This may suggest that the polarization properties are determined by the mean magnetic field and the random field amplitude and do not depend on the distribution function.

Mizuno et al. (2023) discuss the magnetic field structures in the Crab nebula through comparisons between the polarization properties by the IXPE and results obtained by using our simulation.

1． Introduction   Pulsar Wind Nebula(PWN)

Pulsar wind nebulae (PWNe) are non thermal sources (radio ~ gamma rays) powered by pulsars. The Crab nebula has been studied in detail. The X ray observations show detailed structures such as the torus, the jet, the knot and the wisps.

A standard picture of the Crab nebula was given by Kennel & Coroniti (1984, KC hereafter). According to their picture, a superfast magnetohydrodynamic (MHD) wind, which is generated by the central pulsar, terminates at a shock, with the nebula identified as a post-shock flow shining in synchrotron radiation.

〇Shibata et al.(2003)

Shibata et al.(2003) calculated images of the Crab nebula based on the KC model and compared them with the Chandra image.

- brightness contrast => flow velocity ~0.2c
- the ring-like image => random magnetic fields

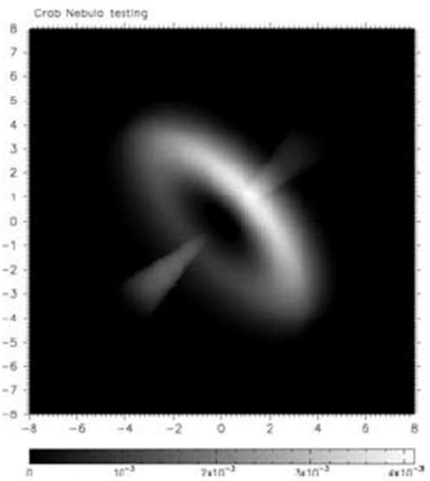


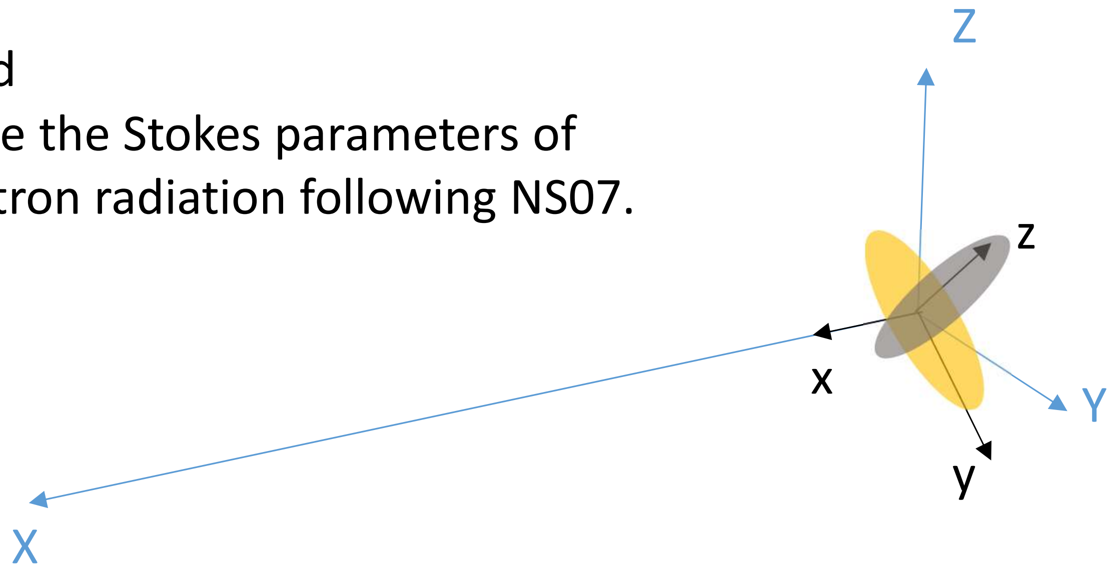
Figure 4. An image reproduced with assumptions of a turbulent field and a high speed flow. See text for details.

Polarization measurements are a key to understand the magnetic field structure.

2　Calculation of synchrotron polarization properties

1）Method

We compulte the Stokes parameters of the synchrotron radiation following NS07.



observer

(X,Y,Z)　observer frame

(x,y,z)　object frame

$$I_\omega(Y,Z) = \int_{-\infty}^{\infty} j_\omega(X,Y,Z,\boldsymbol{n})\,\mathrm{d}X.$$

The Stokes parameters *I*, *Q*, *U* for the observer can be obtained by integraing the contribution from each volume element (grid) in the nebula in the flow frame comoving with the post shock flow and transforming to the observer frame.

flow frame

volume emissivity

$$\begin{pmatrix} \mathrm{d}I'/\mathrm{d}s \\ \mathrm{d}Q'/\mathrm{d}s \\ \mathrm{d}U'/\mathrm{d}s \\ \mathrm{d}V'/\mathrm{d}s \end{pmatrix} = \begin{pmatrix} j'_{\mathrm{tot}} \\ j'_{\mathrm{pol}} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{p+7/3}{p+1}\,\Phi(\omega', p)\,(B'\sin\theta')^{\frac{p+1}{2}} \\ \Phi(\omega', p)\,(B'\sin\theta')^{\frac{p+1}{2}} \\ 0 \\ 0 \end{pmatrix} \tag{1}$$

total intensity
polarized intensity

θ' : the pitch angle

(Rybicki & Lightman 1979), where

$$\Phi(\omega', p) = \frac{\sqrt{3}e^3 K}{8\pi mc^2}\left(\frac{3e}{mc\omega'}\right)^{\frac{p-1}{2}} \Gamma\left(\frac{p}{4}+\frac{7}{12}\right)\Gamma\left(\frac{p}{4}-\frac{1}{12}\right), \tag{2}$$

p : $N(\gamma) = K\gamma^{-p}$

observer frame

$$
\begin{pmatrix} dI_\omega/ds \\ dQ_\omega/ds \\ dU_\omega/ds \\ dV_\omega/ds \end{pmatrix} = \mathcal{D}^2 \begin{pmatrix} j'_{tot} \\ -\cos 2(\chi_0 - \xi)\, j'_{pol} \\ -\sin 2(\chi_0 - \xi)\, j'_{pol} \\ 0 \end{pmatrix},
$$

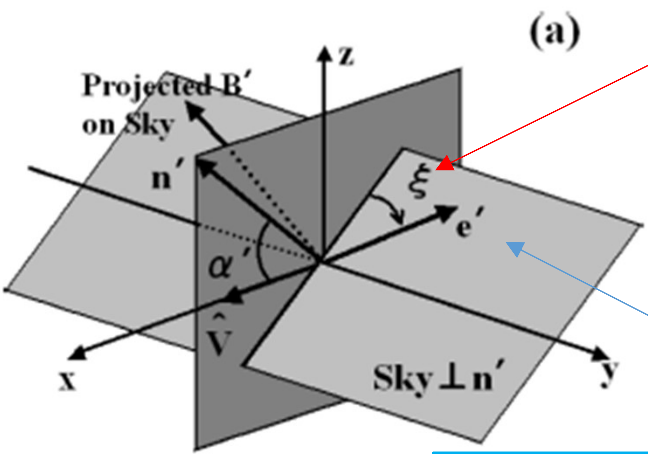
the Doppler effect

$$
D^2 = \frac{1}{\Gamma^2(1-\beta\mu)^2}
$$

The observed position angle ξ

$$
\tan\xi = -\frac{B'_z}{B'_y}\cos\alpha' + \frac{B'_x}{B'_y}\sin\alpha',
$$

(Nakamura & Shibata 2007))



ξ： the position angle is measured from the projected z axis.

e'：the electric field in the proper frame

$$
\boldsymbol{e}' \propto \boldsymbol{n}' \times \boldsymbol{B}'
$$

$\boldsymbol{n}'$ ：the direction of the observer in the proper frame

$\boldsymbol{B}'$ ：magnetic fields in the proper frame

$\boldsymbol{\alpha}'$ ：the angle between between $\boldsymbol{n}'$ and the flow velocity V

2） distributions of the random fields

a) Nakamura & Shibata 2007 (NS07)

a fixed amplitude and random orientations

$$j'_{\rm pol} = \Phi(\omega', p)\,\frac{1}{\Delta s}\int_{s}^{s+\Delta s} \times \left(B'\sqrt{1-\left(\boldsymbol{n}'\cdot\frac{\boldsymbol{B}'}{B'}\right)^{2}}\right)^{\frac{p+1}{2}} \cos 2\chi \, ds', \qquad (10)$$

polarized intensity

cos2χ : a correction factor representing the depolarization by random fields (Korchakov & Syrovatskii(1962))

○PWN

The nebula torus is modeled with a simple disc, which represents the post shock flow with a inner radius Rs located at the termination shock and with a constant semi-opening angle.

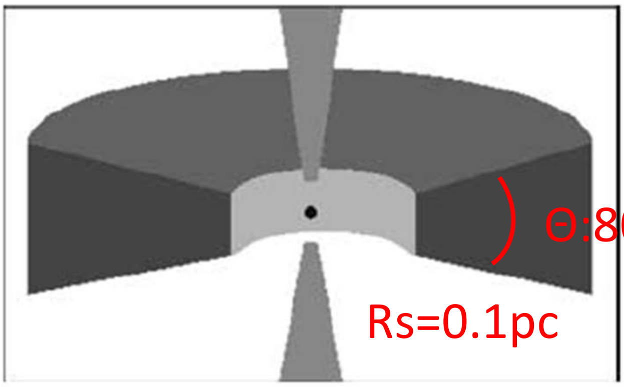


Figure 1. The three-dimensional structure we assumed for reproduction of image. The spherical flow by Kennel & Coroniti (1984a) is picked up for the disc and polar flows.

KC model

$L_{sd} = 5\times 10^{38}$ erg/s

$u_u = 3.3\times 10^{6}$

$r_s = 0.1$pc

$\sigma = 0.0038$

○The magnetic fields： the toroidal field and the random field

・The radial profile of the total magnetic fields： KC model

・The magnetic field randomness parameter *b* ：

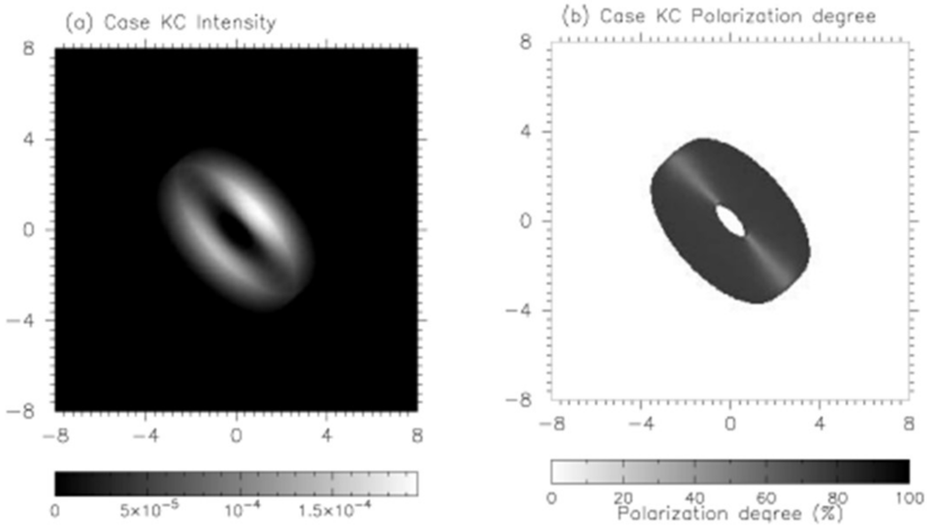
$$b = \frac{\langle B_1'^{2}\rangle}{B_0'^{2} + \langle B_1'^{2}\rangle},$$

the random fields

the total fields

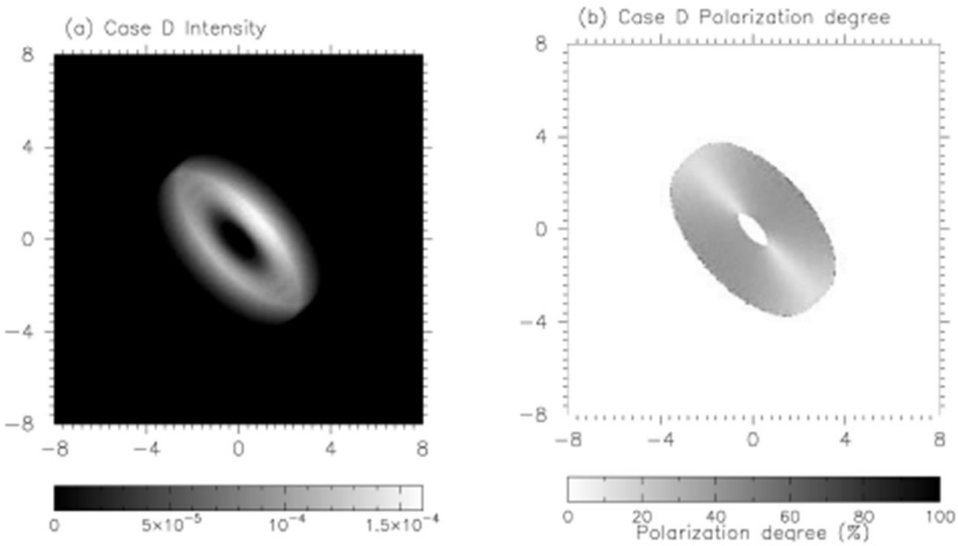
○ A pure toroidal field

- lip-shaped
- the maximum polarization degree $P_{max} \sim 75\%$
- the integrated polarization degree $P \sim 52.3\%$



○ The toroidal ＋ random fields（b=60%）

- Lip ⇒ Ring
- $P_{max} \sim 40\%$
- $P \sim 20\%$



- inclination angle = 28°
- energy　5.2 keV

b) Bucciantini et al. (2017) (B17)

Random fields described by an isotropic random Gaussian field

$$I(\nu, Y, Z) = \int_{-\infty}^{\infty} \xi(\alpha, B'_{\perp}, \sigma) j(\nu, X, Y, Z)\, \mathrm{d}X \tag{16}$$

$$Q(\nu, Y, Z) = \frac{\alpha+1}{\alpha+5/3} \int_{-\infty}^{\infty} \zeta(\alpha, B'_{\perp}, \sigma) j(\nu, X, Y, Z) \cos 2\chi\, \mathrm{d}X \tag{17}$$

$$U(\nu, Y, Z) = \frac{\alpha+1}{\alpha+5/3} \int_{-\infty}^{\infty} \zeta(\alpha, B'_{\perp}, \sigma) j(\nu, X, Y, Z) \sin 2\chi\, \mathrm{d}X. \tag{18}$$

B': the comoving magnetic field

$$\xi(\alpha, B_{\perp}, \sigma) = \Gamma\left(\frac{3+\alpha}{2}\right) {}_1F_1\left(-\frac{1+\alpha}{2}, 1, -\frac{B_{\perp}^2}{2B'^2\sigma^2}\right) \times \left(\frac{B_{\perp}}{\sqrt{2}B'\sigma}\right)^{-(1+\alpha)} \tag{14}$$

$$\zeta(\alpha, B_{\perp}, \sigma) = \frac{1}{2}\Gamma\left(\frac{5+\alpha}{2}\right) {}_1F_1\left(\frac{1-\alpha}{2}, 3, -\frac{B_{\perp}^2}{2B'^2\sigma^2}\right) \times \left(\frac{B_{\perp}}{\sqrt{2}B'\sigma}\right)^{(1-\alpha)}, \tag{15}$$

Two correction coefficients obtained by Bandiera & Petruk(2016)

3　Comparison of results for three random distributions

We calculate the synchrotron polarization properties following the similar method of NS07, and examine whether statistical distributions of the random fields affect the polarization properties.

- PWN model： KC model
- $N(\gamma) \propto \gamma^{-3}$
- nebula outer radius = 0.5 pc

Three cases of the distributions of the random fields:
a) a fixed amplitude with random orientations (NS07)
b) an isotropic random Gaussian (B17)
c) a uniform distribution function.

the randomness parmeter

$$b = \frac{\langle \boldsymbol{B}_1'^2 \rangle}{B_0'^2 + \langle \boldsymbol{B}_1'^2 \rangle},$$

the random fields

the total fields

We compute
1) the integrated polarization degree *P* as a function of the degree of randomness *b* and the inclination angle *i* (same as figure 3 in NS07),
2) the spatial distribution of the polarization degree.
As seen on following pages

◎No significant differences.

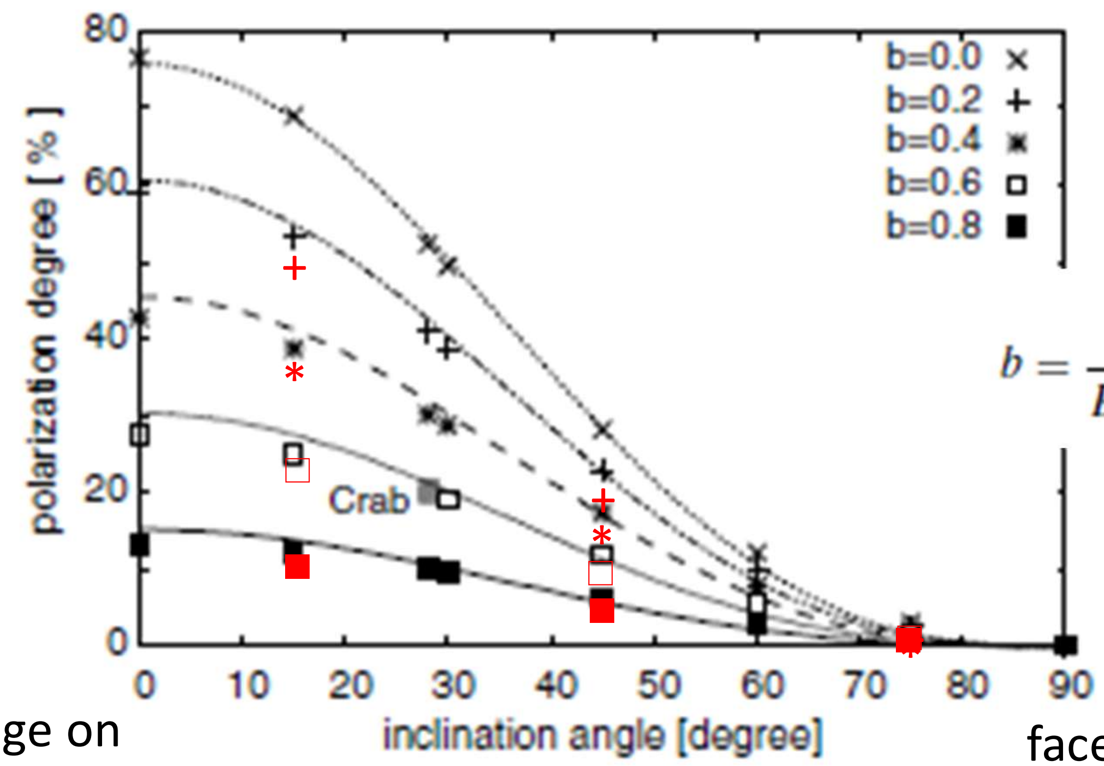
This may suggest that the polarization properties are determined by the mean magnetic field and the random field amplitude and do not depend on the distribution function.

1) The integrated polarization degree *P* as a function of the degree of randomness *b* and the inclination angle *i*.

a) a fixed amplitude with random orientations (NS07)

P ：overlaid on the figure3 in NS07

P

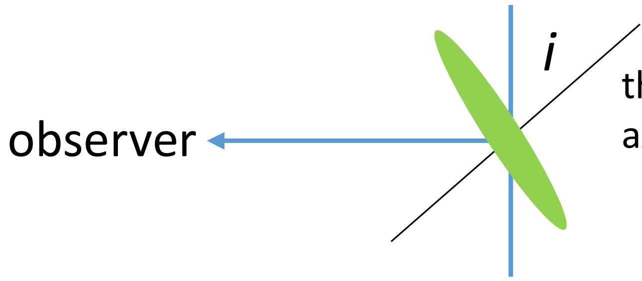


$$b = \frac{\langle B_1'^2 \rangle}{B_0'^2 + \langle B_1'^2 \rangle},$$

edge on

face on

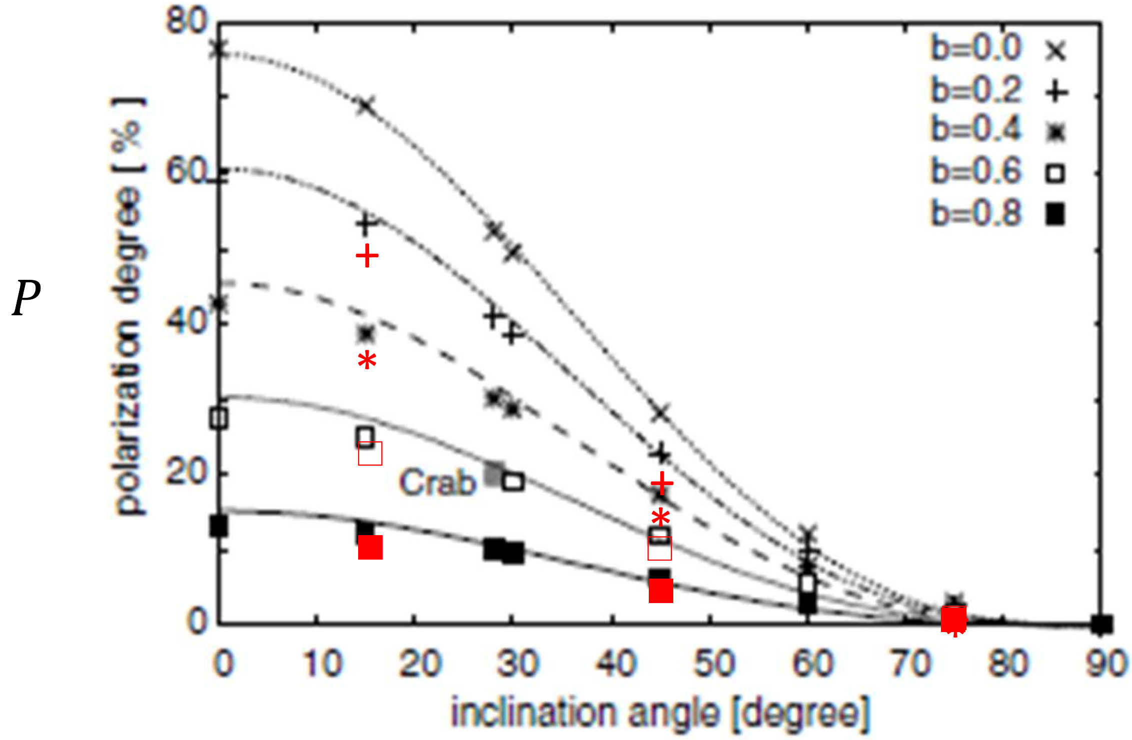
Figure 3. Polarization of the entire nebula as a function of the inclination angle i between the axis of the disc and the line of sight for $b = 0.0, 0.2, 0.4, 0.6$ and 0.8. The empirical curves $P = 75.8(1 - b)(\cos i)^{2.84}$ are overlaid.



the inclination angle of the symmetry axis from the plane of the sky

〇Simulated *P*s reproduce the results of NS07.

b) an isotropic random Gaussian



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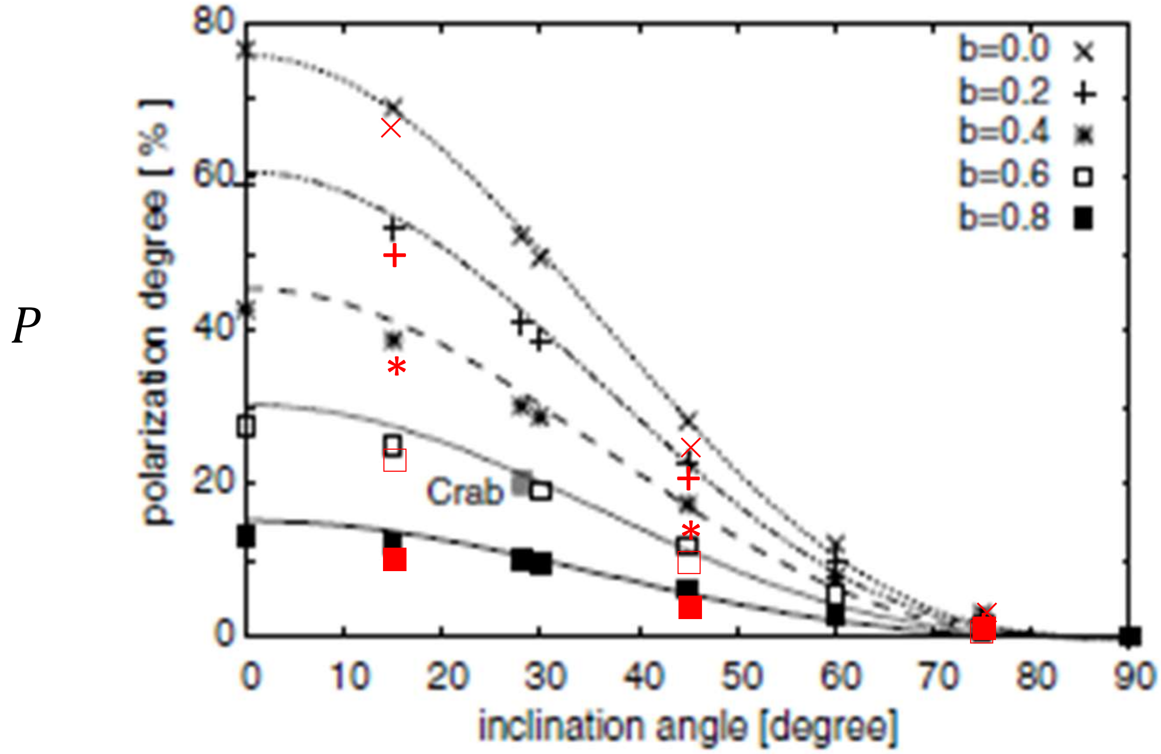
c) a uniform distribution function



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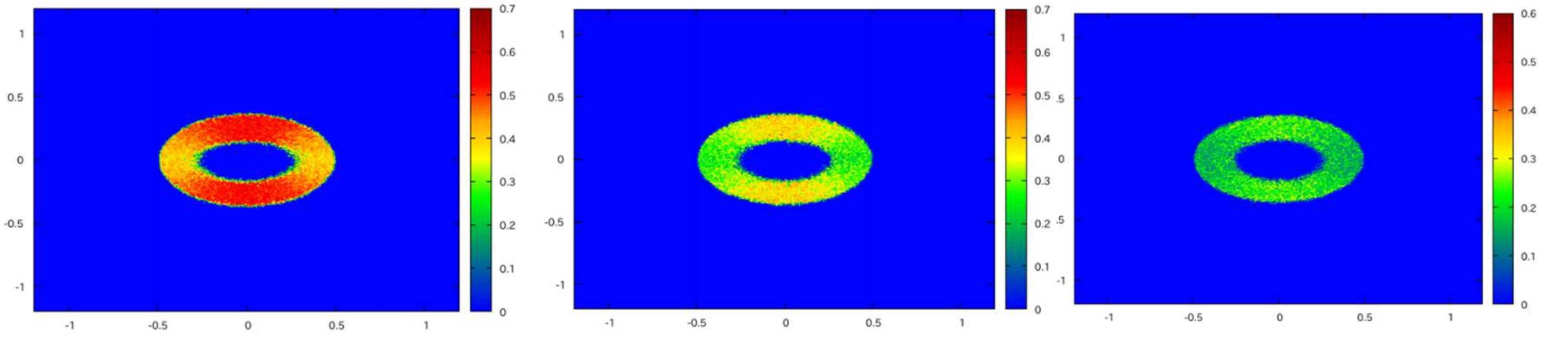
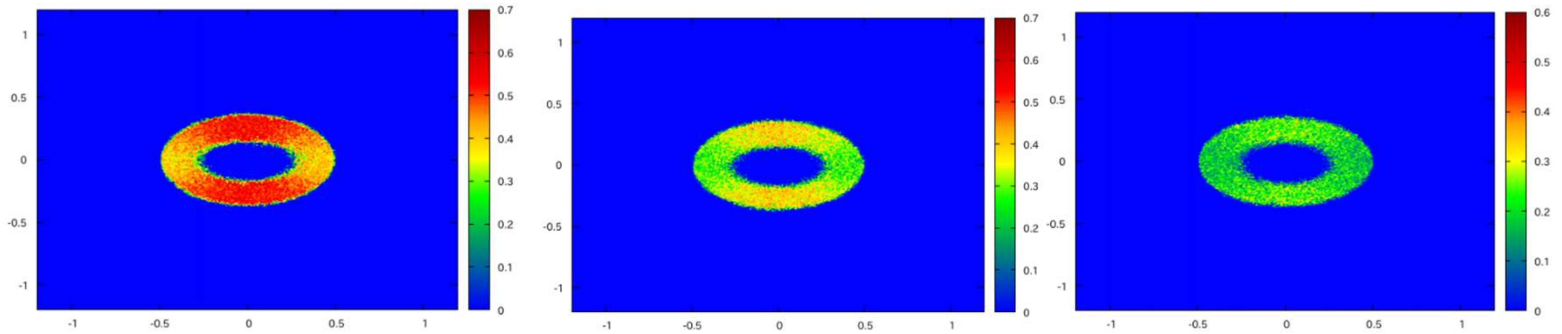
2) the spatial distribution of the polarization degree

b=40% | *b*=60% | *b*=80%

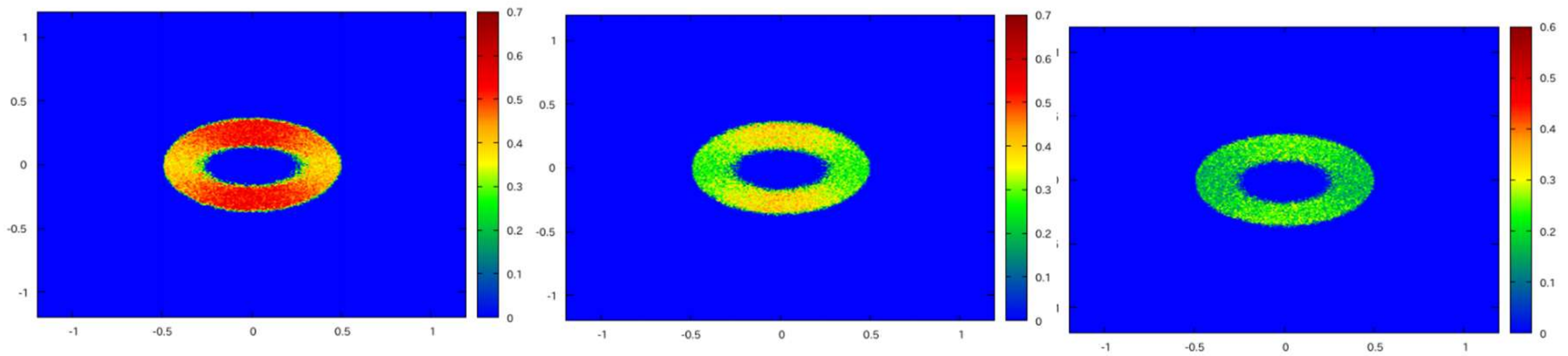
a) a fixed amplitude with random orientations (NS07)



b) an isotropic random Gaussian



c) a uniform distribution function



i=45deg.

No differences.

4 Summary

○We calculate the synchrotron polarization properties following the similar method of Nakamura and Shibata(2007).

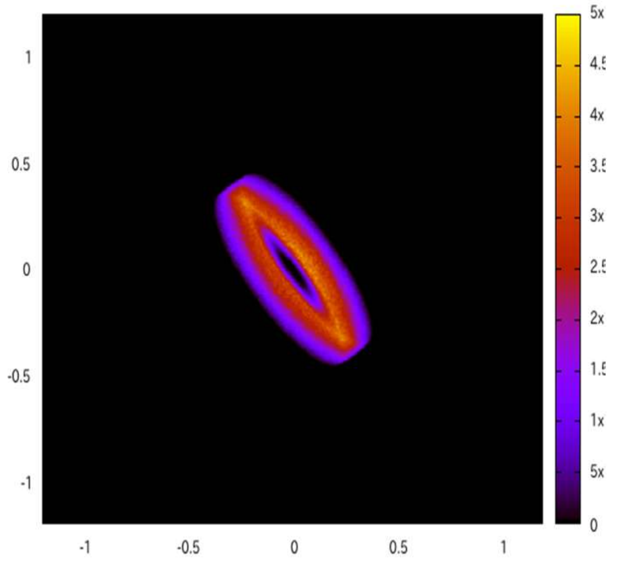
Ex. The Crab nebula

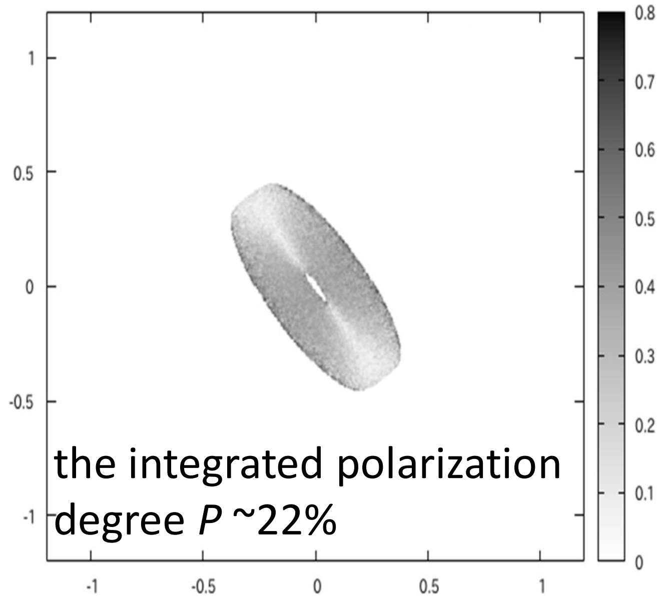
●KC model, The toroidal ＋ random fields（*b*=60%）
(same as Figure 4 in NS07)

Total intensity

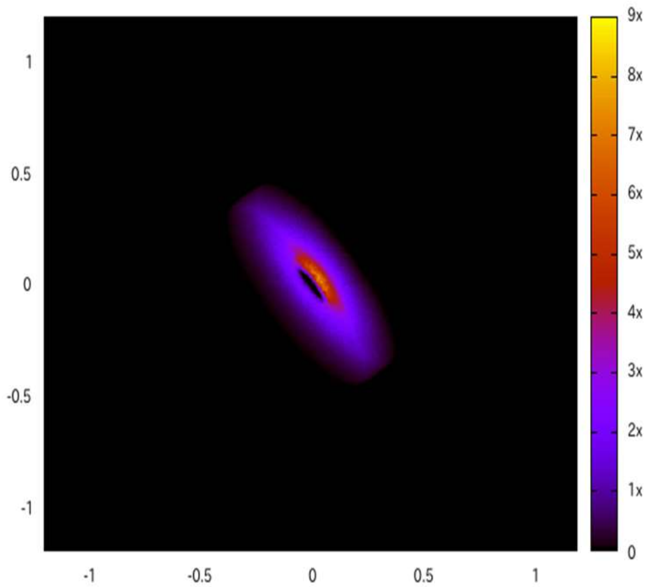
Polarization degree

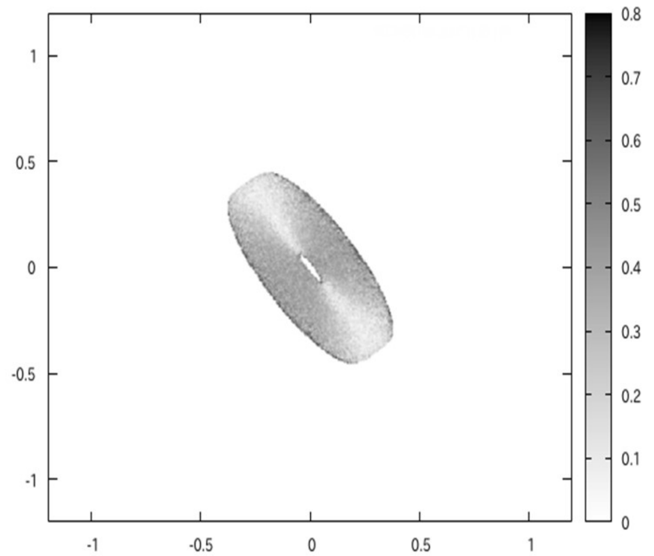
[pc]





● β=0.2, The toroidal ＋ random fields（*b*=60%）
(same as Figure 5 in NS07)





○We examine whether statistical distributions of the random fields affect the polarization properties. Comparisons of the results for the three distributions show no significant differences among them. This may suggest that the polarization properties are determined by the mean magnetic field and the random field amplitude and do not depend on the distribution function.

○Mizuno et al. (2023) discuss the magnetic field structures in the Crab nebula through comparisons between the polarization properties observed by the IXPE and results obtained by using our simulation.

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