Analyzing Gravitational Wave Signals with the Stacked Hilbert-Huang Transform

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Abstract

The Hilbert-Huang transform (HHT) is a novel technique for tracking frequency evolution in detail. However, the mode-mixing problem makes it difficult to characterize signals that vary over a wide frequency range, such as the gravitational wave (GW) signal. In this context, I propose an improved model-independent time-frequency analysis based on the HHT. Instead of the HHT algorithm, which obtains intrinsic mode functions through ensemble empirical mode decomposition and yields instantaneous frequencies, the new algorithm computes the ensemble mean on the time-frequency map. This method has been successfully applied to trace the first GW events in the initial GW transient catalog, simulate GW signals from core-collapse supernovae (CCSNe), and analyze X-ray superorbital modulation in SMC X-1. The time-frequency maps generated by the new algorithm exhibit significantly greater detail compared to wavelet spectra. Furthermore, the oscillations in the instantaneous frequency caused by mode-mixing can be greatly reduced. In the case of CCSNe data, the initial stages of different modes of oscillations can be clearly distinguished. Finally, detailed frequency modulation of the superorbital modulation in SMC X-1 can be detected. With the assistance of the spin period detected using MAXI, a potential spin-superorbital connection is observed. These findings offer new insights for further refinement of the detection algorithm and serve as new avenues for investigating the underlying physical mechanisms.

1. HHT, Mode Mixing, and Gravitational Waves

- The HHT (Huang 1998) has been suggested as one of the best tools for tracing GW signals (e.g., Camp + 2007, Kaneyama + 2016, Sakai + 2017). The HHT consists of two core components:
  - EMD – Decomposing a signal into a finite number of intrinsic mode functions (IMFs).
  - Calculating instantaneous frequency using the normalized Hilbert transform, quadrature, or generalized zero-crossing.
- However, the mode-mixing problem prevents us from tracing the frequency evolution in detail.
  - Mode-mixing (or splitting): a signal may be decomposed into multiple IMFs.
  - This can be avoided with EEMD if the frequency does not change by a factor > 2 (Wu + 2009).
  - Mode-mixing may cause spurious frequency modulation, especially at the boundary when a signal jumps from one IMF to another (Fig. 1).

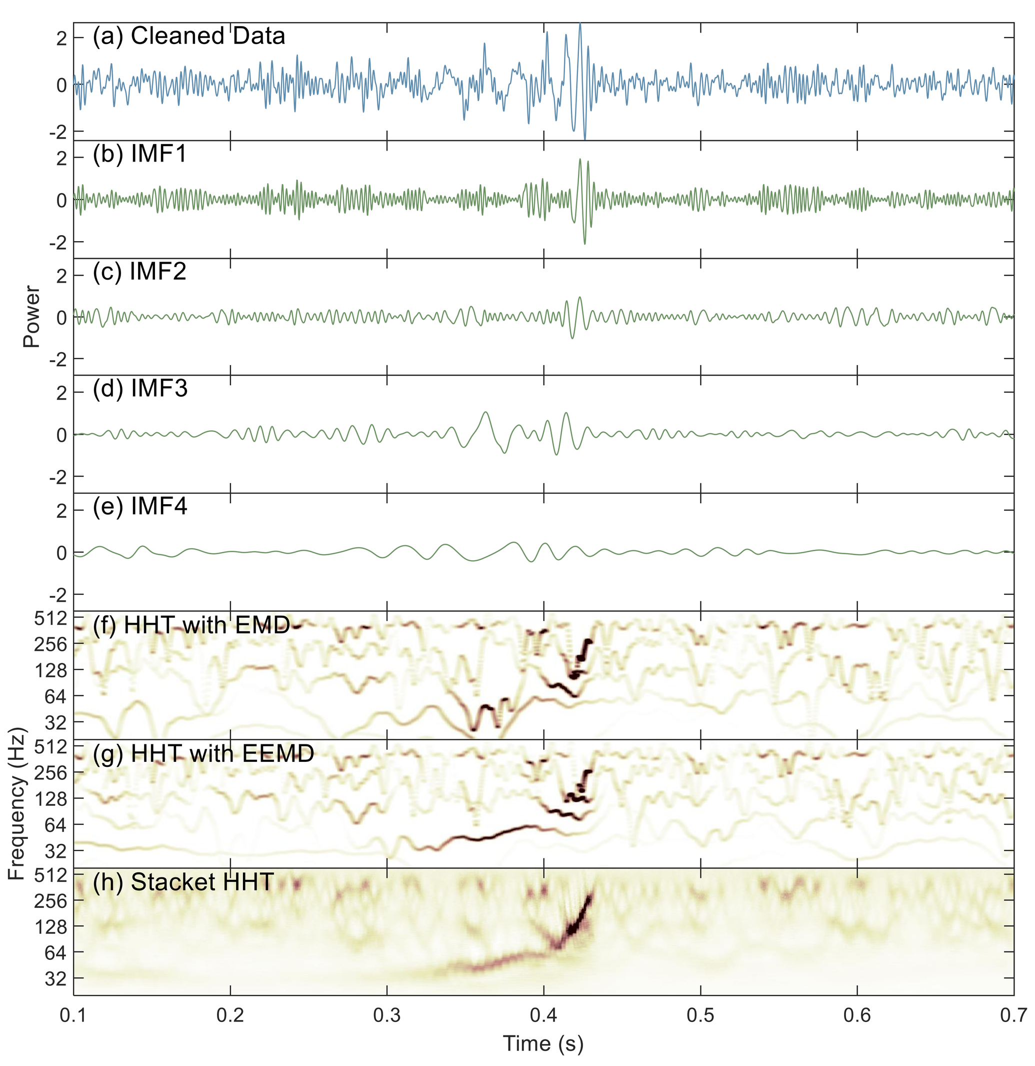

Figure 1. Gravitational wave signal of GW 150914 observed with Livingston observatory. Panel (a) shows the whitened data, wherr panels (b) – (e) show first four IMFs obtained using EMD. Panels (f) – (h) show the Hilbert spectra estimated using EMD and EEMD, and Stacked HHT, respectively.

2. New Algorithm

- Instead of taking the ensemble mean of individual IMFs, I propose a new technique that directly stacks the time-frequency map (Fig. 2).

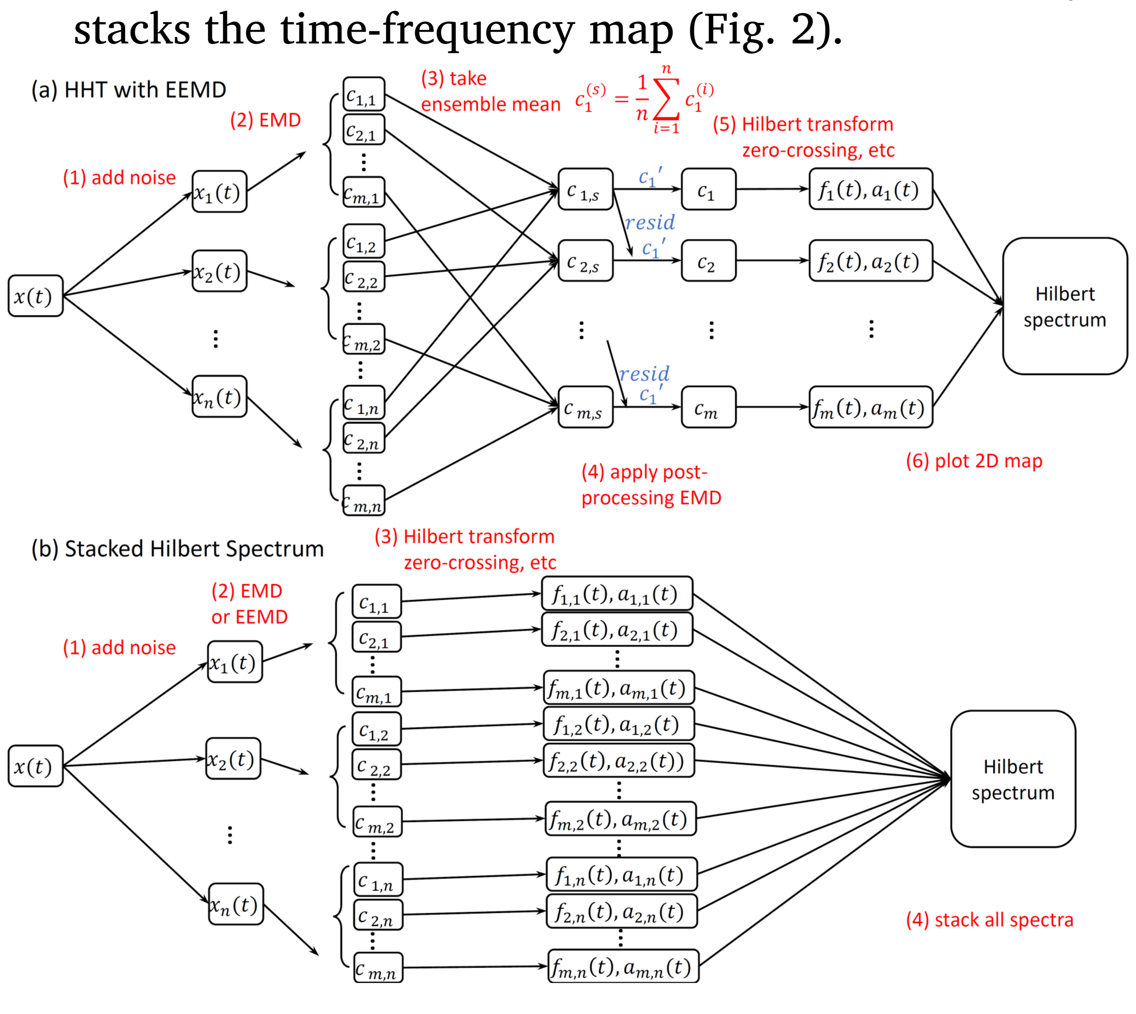

Figure 2. (a) The procedure of EEMD in this paper. Note that a post-processing EMD is applied from the first IMF. (b) Stacked Hilbert analysis algorithm that is optimized for visualization of the time-frequency map.

Paper 1:
a comprehensive analysis of gravitational wave data using Stacked HHT

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3. Binary Black Hole Coalescence

- The new algorithm successfully traces the gravitational wave signal from binary black hole (BBH) coalescence in GWTC-1 (Abbott + 2019).
  - However, it failed to trace the evolution of GW170817 (neutron star coalescence).
  - The modulation of GW170817 is too subtle to be detected by an algorithm that is sensitive to instantaneous changes.

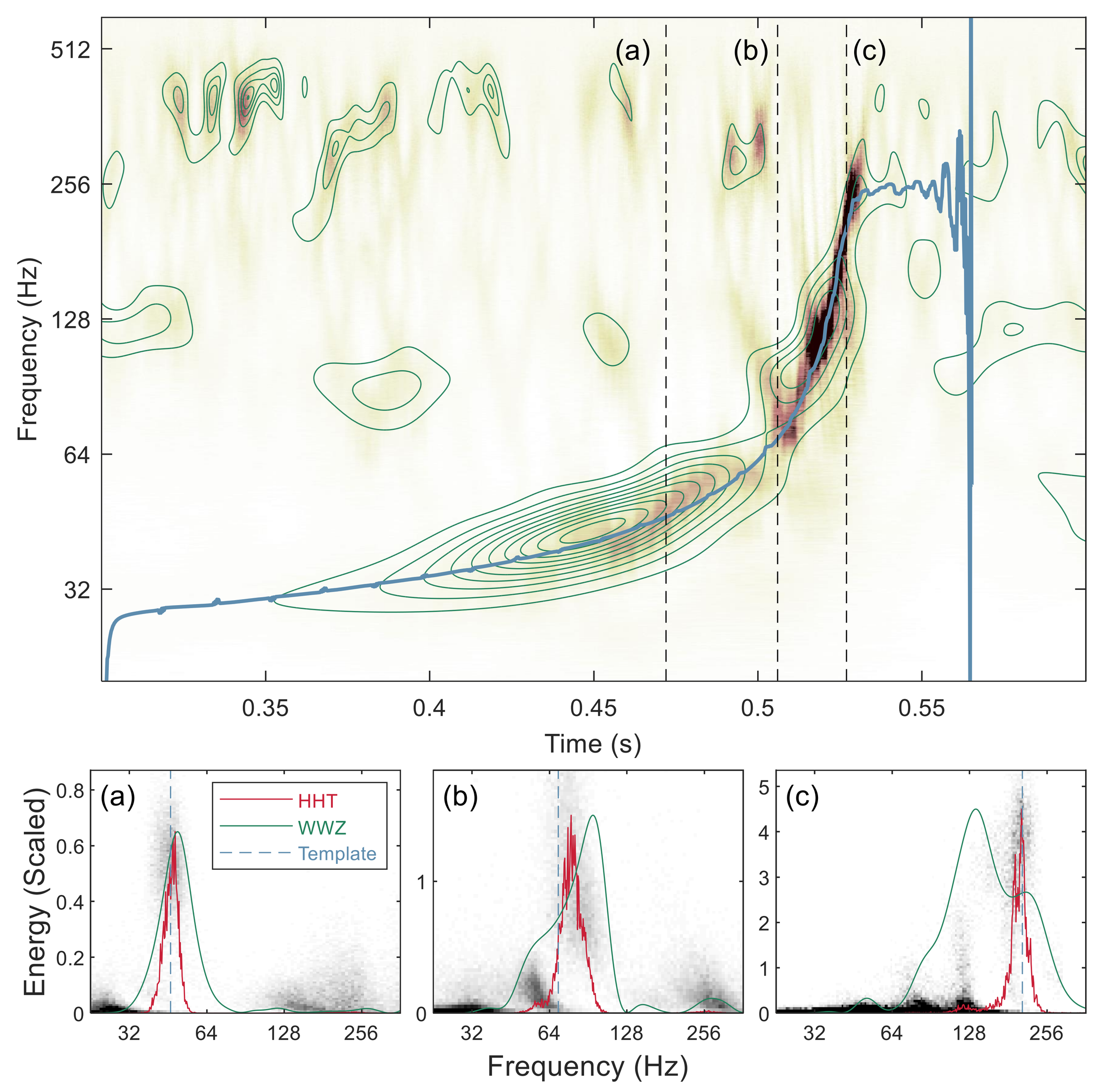

Figure 3. Comparison between the stacked HHT spectrum (color map), WWZ spectrum (contour) and the instantaneous frequency of the template (blue line) of GW 150914. Three slices are labeled for low-frequency (a), intermediate-frequency (b), and high-frequency (c) regimes. The amplitude versus frequency plot are plotted in the lower panels. Density maps of the instantaneous amplitude and frequency of the specific time of all simulation are shown as black dots, where the red profile is the averaged profile for the instantaneous frequency. The green curve denotes the result from the WWZ, where the blue dashed line are frequency obtained from the template.

4. Core-Collapse Supernovae

- The new stacked HHT algorithm has also been applied to simulated gravitational wave signals from core-collapse supernovae.
  - Waveforms are generated using three-dimensional self-consistent CCSN simulations with the Isotropic Diffusion Source Approximation for neutrino transport, employing the FLASH code (Pan + 2021).
  - The stacked HHT spectrum reveals much more detail than wavelet analysis spectra (Fig. 4).

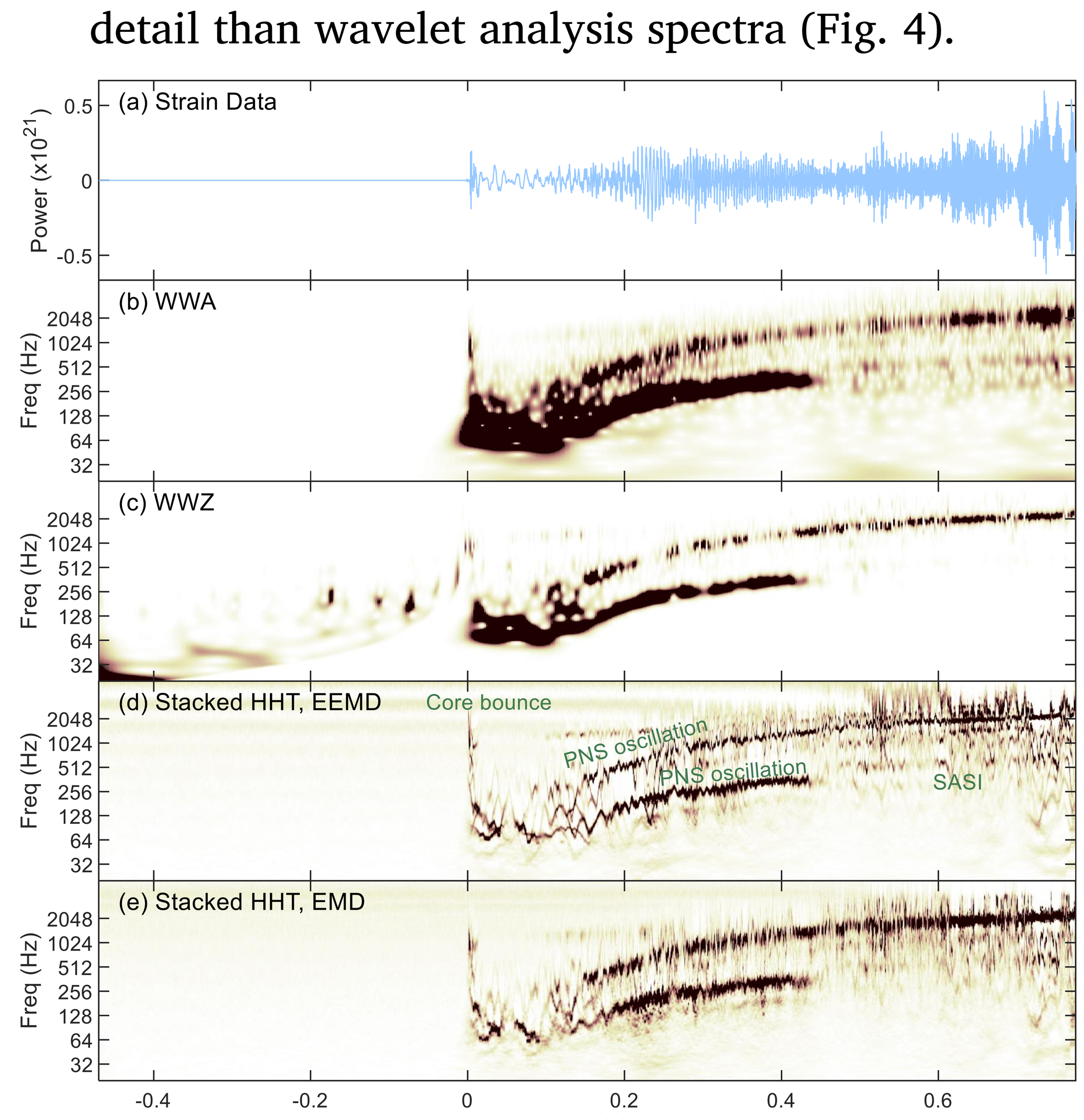

Figure 4. (a) The cross-component of the gravitational waveform of a CCSN seen from the equatorial plane with an initial stellar mass of 40~$M_{\odot}$ and without rotation. (b)--(c) The WWA and WWZ map of the gravitational signal. (d)--(e) Stacked Hilbert spectra based on EEMD (d) and EMD (e).

5. Superorbital Modulation of SMC X-1

- SMC X-1 is a HMXB showing superorbital modulation
  - Period varies between ~40 (excursion epochs) to ~65 days (regular, or non-excursion epochs).
  - Caused by radiation-driven warp (Pringle 1996, Ogilvie + 2001)?
  - The stacked HHT can trace the superorbital modulation period in detail.
- The spin frequency (~1.4 Hz) evolution can be traced with MAXI with the cadence of a superorbital cycle.
  - The spin-up rate remained stable during the third excursion (in 2017-2018)
  - However, the spin-up rate increased a year before the fourth superorbital excursion (2020-2021)
  - An inside-out process with a threshold?
  - The pulsed fraction increased during the superorbital excursion
  - Detailed monitoring using NinjaSat may help exploring underlying physics.

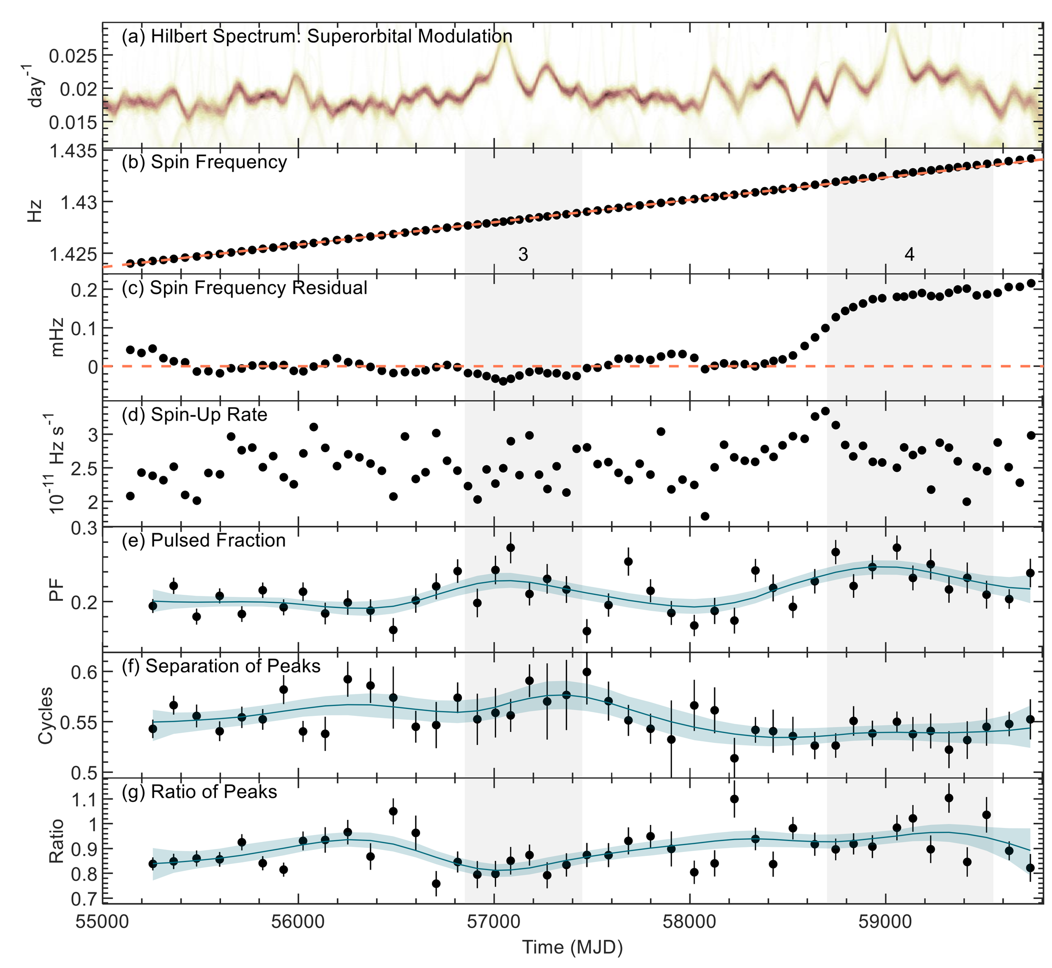

Figure 5. The stacked HHT spectrum obtained with Swift BAT is shown in panel (a). Panel (b) shows the spin period in each superorbital high state, where the orange dashed line denotes the best-fit linear model obtained using data before MJD 58,484. The residual after subtracting the linear trend and the spin-up rate are shown in panels (c) and (d), respectively. The evolution of the pulsed fraction (e), phase separation of two peaks (f), and the ratio of the two peaks (g) are shown in the following panels. Gray shaded area denotes the epochs of the 3rd and 4th excursions. Blue curves are obtained from EEMD band-pass filtered data, where the light blue areas correspond to 1-sigma confidence intervals.

6. Ongoing Work

- Building an EMD-CNN Model for Gravitational Wave Detection
  - The finite number of IMFs effectively transfers the 1-D gravitational wave data into a 2-D image, which is suitable for convolutional neural work.
  - Preliminary results show that EMD + CNN is feasible.
- Investigate the mathematical properties of stacked HHT

References

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Paper 2:
Monitoring superorbital period variation and spin period evolution of SMC X-1

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