Magnetic fields in Magnetar and CCO

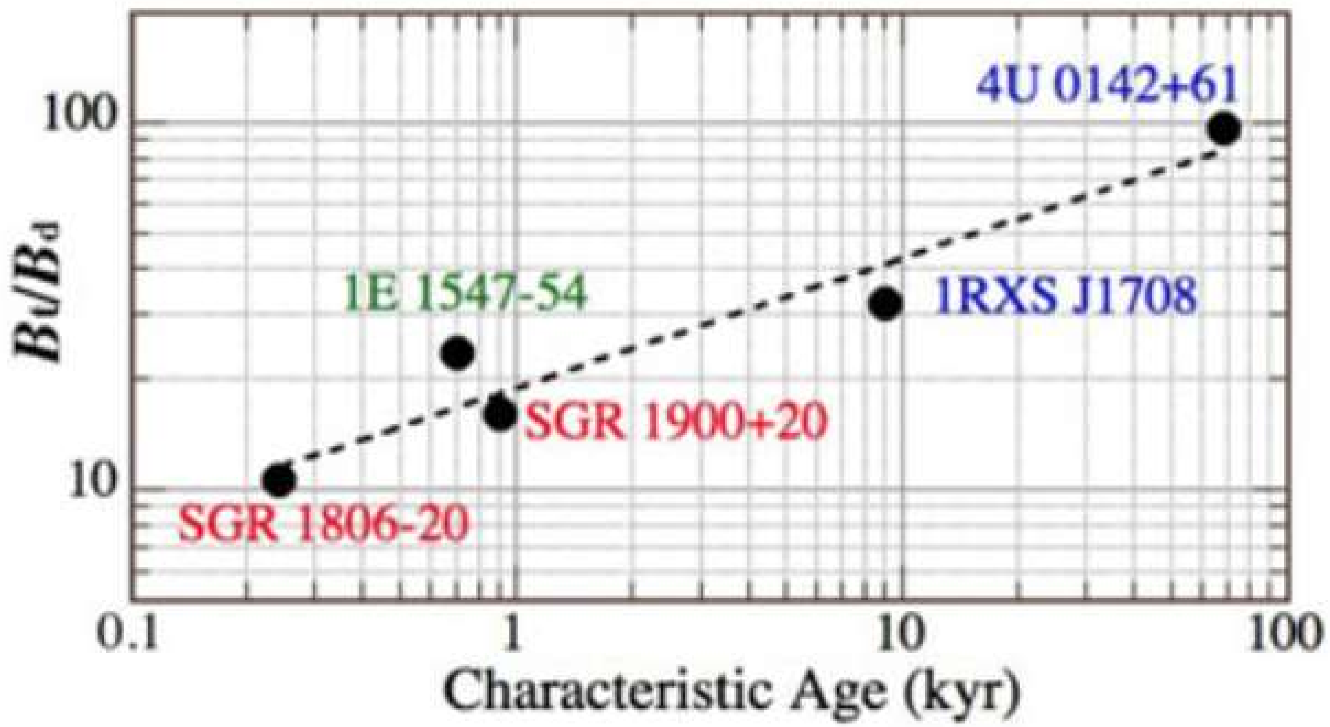
Yasufumi Kojima
Hiroshima University

NS2023 @ Kyoto 2023/Sept/06-08

‘Evolution’ of B-field

K. Makishima

IAU Symposium No. 363, 2023



Magnetic field in magnetar

- $B_d \sim 10^{14} G$ (surface dipole) from spin down $(P, \dot{P})$
- $B_t \sim 10^{16} G$ (toroidal) inferred by free precession
- Magnetic field 'evolution'

Observation Sources plotted as a function of characteristic age

-> Toroidal component is constant with time

-> Surface dipole decays (not simple exponential but power law form)

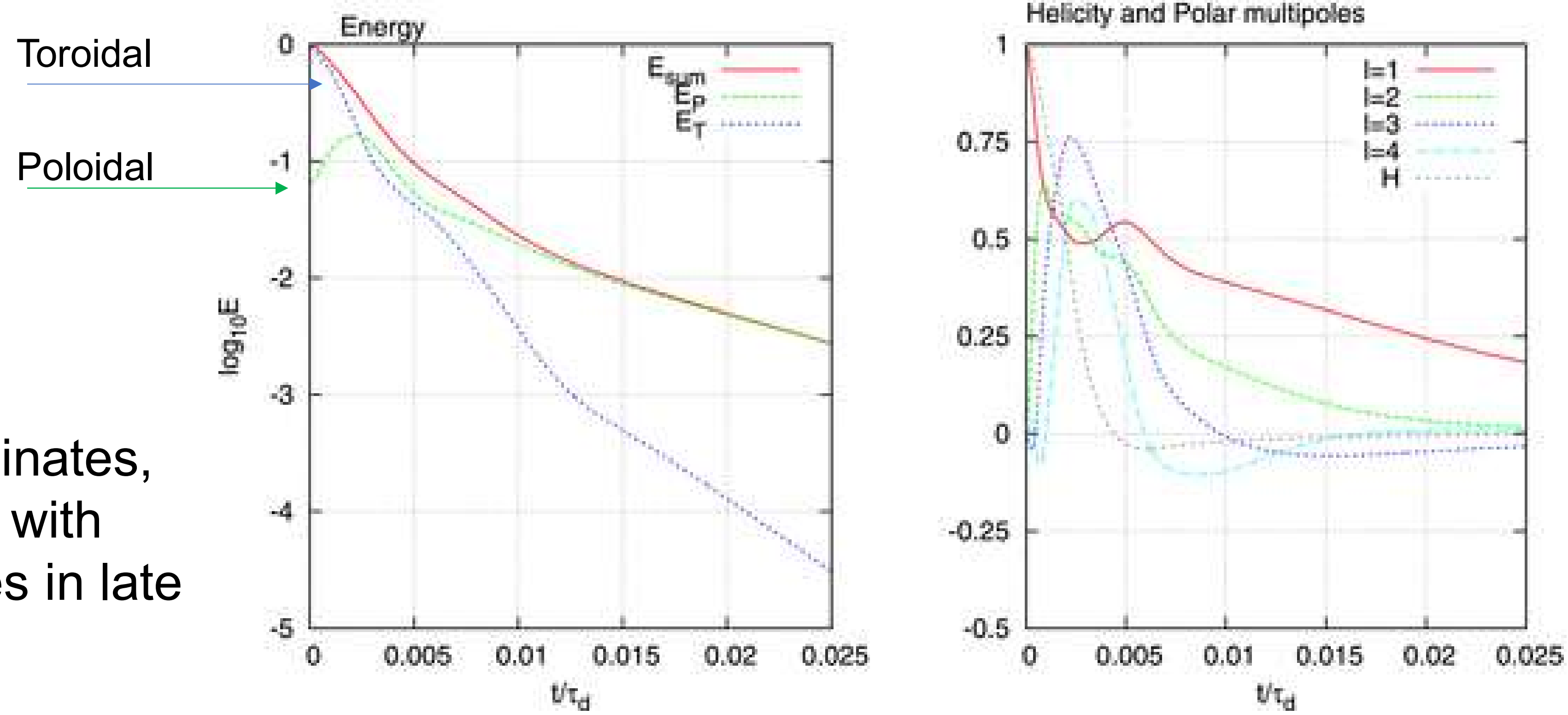
Hall evolution in crust (theory)

Intense toroidal component decays fast!

Energy transfers due to non-linear coupling, so that the decay of dipole is not simple exponential but complicated (likely slow power-law).

Hall evolution in crust

YK, Kisaka(12) MNRAS 421, 2722



Initially toroidal component dominates, but poloidal one with small l dominates in late phase

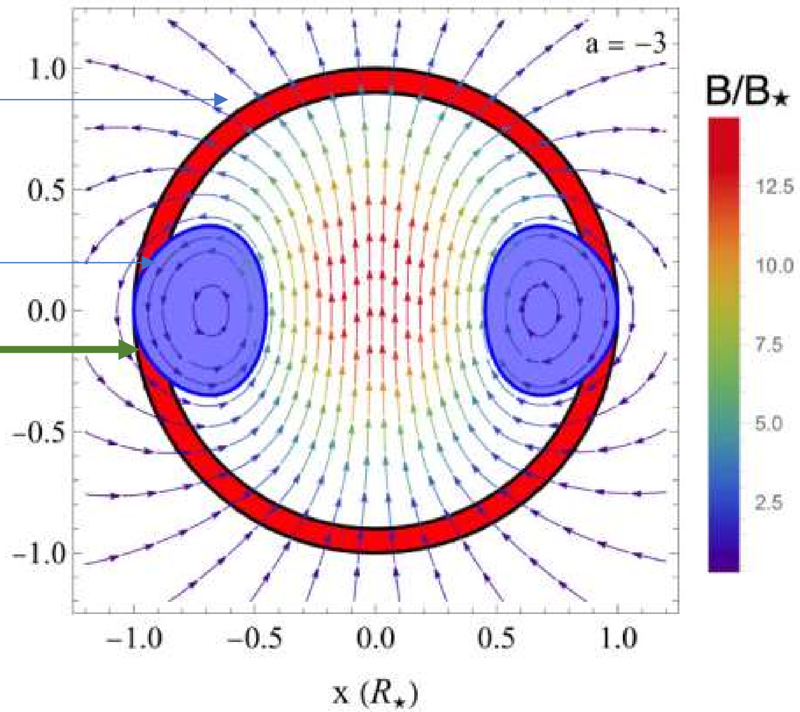
Model
Crustal fracture relevant to magnetic field

Dipole

$B_d \sim 10^{14} G$ @ R

Strong troidal

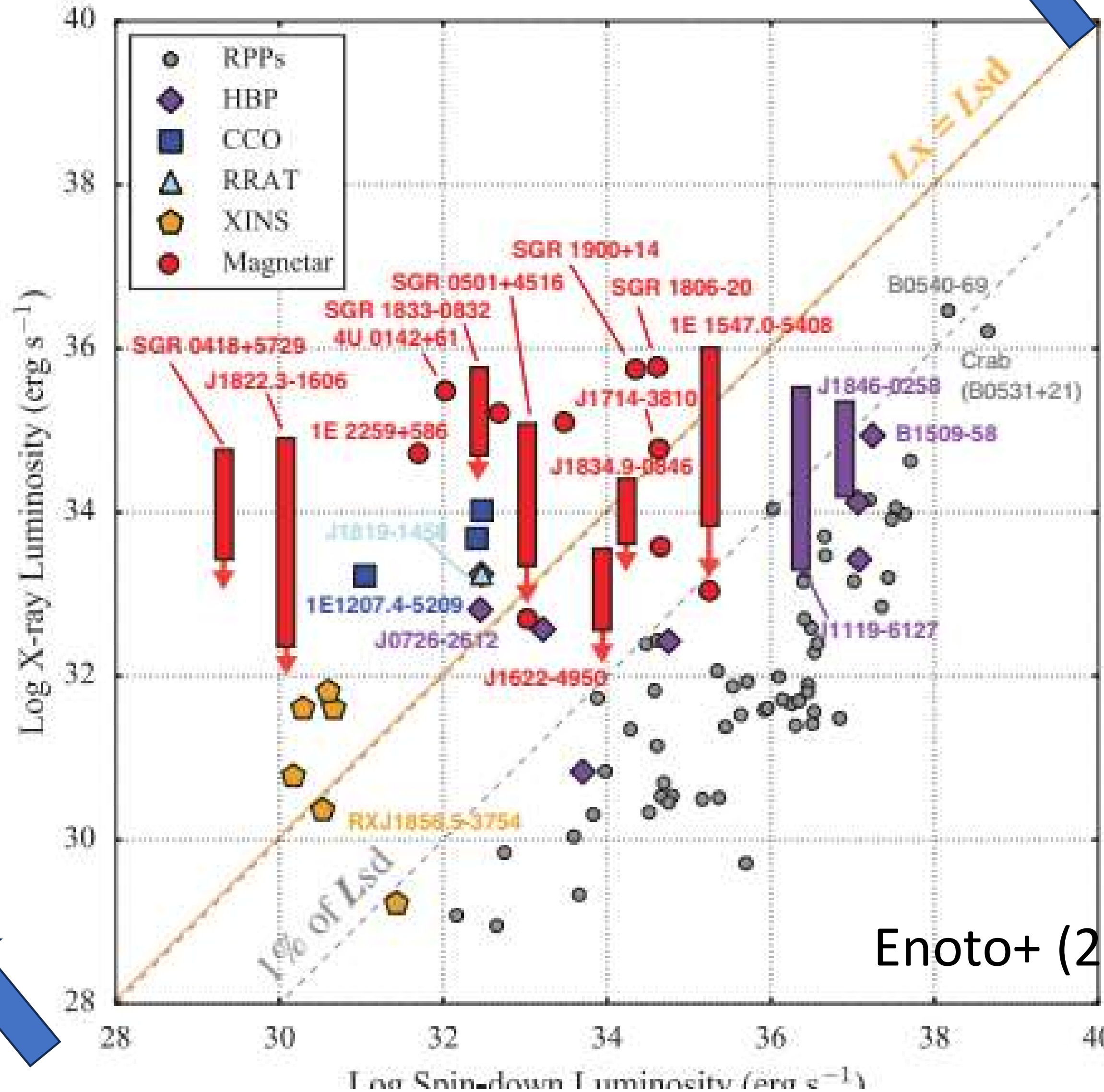
$B_t < 10^{14} G$ @ r_c



Magnetic field geometry is assumed, but the field strength is constrained by the magneto-elastic equilibrium

Savorov, A(23) MNRAS 523, 4089

Lx vs
Spin-down Luminosity



Enoto+ (2019)

Sources except for
Rot. E. in
Magnetar, CCO, XINS

$B_d \sim 10^{14} G$

$B_d \sim 10^{11} G$

Working hypothesis (biased view) for B-field

Two components for B-field

- $B_t \sim 10^{16} G$ (toroidal) confined deep in core, Ferromagnet?
  $B_t \sim 10^{14} G$ at core-crust interface (K. Fujisawa+(23); YK, & S. Yoshida (23) in prep.)
- $B \quad \sim 10^{14} G$ (weak poloidal and toroidal components) in crust
  Relevant electric current decays to produce Joule heat

Crustal magnetic field $\sim 10^{14} G$ ($B \sim 10^{14-15} G$)

Magnetic energy stored in crust for magnetars and CCOs

$E_B \sim 2 \times 10^{45} (B_{14.5})^2 erg$

$\mathrm{L} \sim E_B/\tau \sim 3 \times 10^{33} (B_{14.5})^2 (\tau_{10kyr})^{-1}\ erg/s$

τ magnetic dissipation timescale ~ life time of the activity $\sim 10^4 - 10^5$ yr

Giant flare (E$\sim 10^{45} erg$)? B-field in core/entire star (e.g., Ioka 01)

Interior B-field is uncertain, but the consequence is examined below

Crustal fracture by the Hall magnetic field evolution

YK (22) arXiv:2209.04139; ApJ(22)938,id.91
YK, Kisaka & Fujisawa(23) arXiv:2303.02312; ApJ(23) 946 ,id.75

Magnetic field strength is same, but geometry is different

$$B \sim 10^{14} G$$

Elastic deformation by the Hall evolution

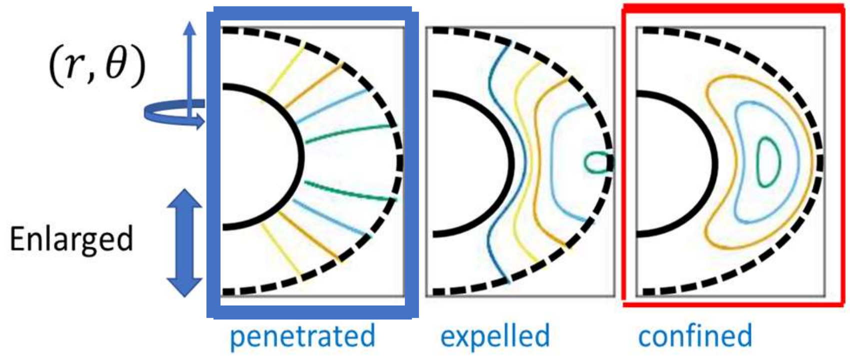
Dynamical equilibrium

$\neq$ Chemical (compositional) equilibrium

-> evolution in a long timescale

Elastic limit -> crustal fracture

- Breakup time?
- Elastic energy?



Magnetar **CCO**

Magnetic field evolution in crust by Hall effect

Dynamical equilibrium (barotropic)

$$-\frac{1}{\rho}\vec{\nabla}P - \vec{\nabla}\Phi_{\rm g} + \frac{1}{c\rho}\vec{j} \times \vec{B} = 0,$$

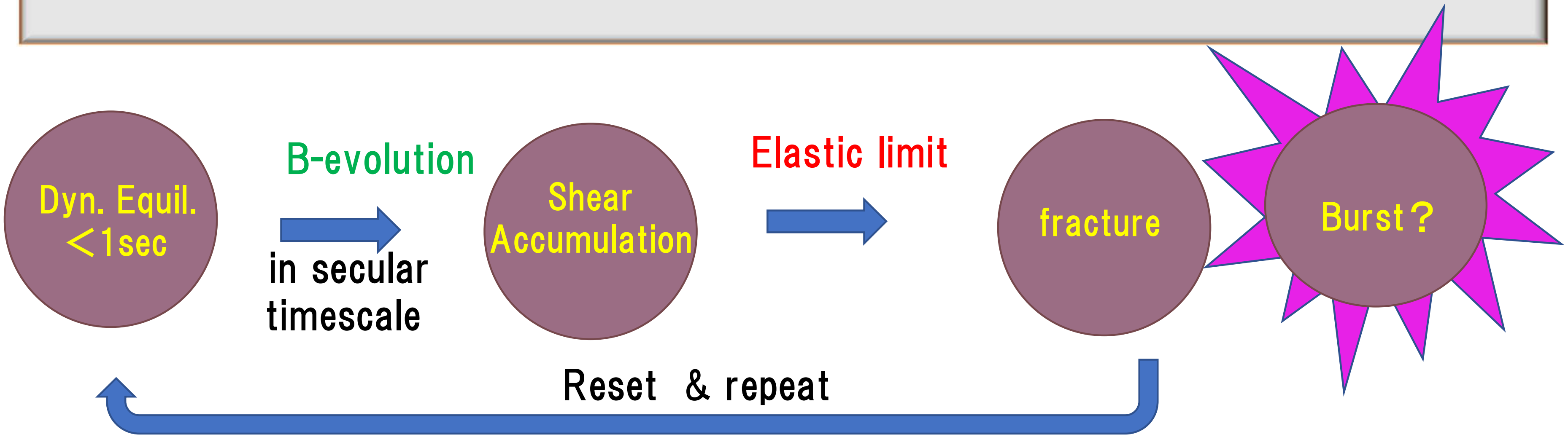
Evolution driven by electron fraction gradient for barotropic case

Calculation in a Hall timescale (secular timescale $\sim 10^2 - 10^5$ yr)

$$\frac{\partial}{\partial t}\vec{B} = -\vec{\nabla} \times \left[\frac{1}{en_{\rm e}}\vec{j} \times \vec{B}\right] = -\vec{\nabla}\chi \times \vec{a} - \chi\vec{\nabla} \times \vec{a},$$

$$\chi \equiv \frac{c\rho}{en_e} = \frac{4\pi\rho_c(\Delta r)^2}{\tau_{\rm H}B_0}\hat{\chi}. \quad \vec{j} \times \vec{B}/(c\rho) = \vec{a}$$

Outline of calculation



$$\delta B \rightarrow \delta f \; (= c^{-1}\vec{j} \times \vec{B}) \rightarrow \sigma, \xi_\phi$$

$$\nabla_j \left(2\mu\sigma^j_\phi\right) + \delta f_\phi = 0,$$

$$\sigma_{ij} = \frac{1}{2}(\nabla_i \xi_j + \nabla_j \xi_i),$$

Shear strain limited by a cond.

Beyond a threshold, solid crust is broken <-> Bursts in SGR?

Breakup time, the maximum elastic energy are calculated

Result for barotropic model

- Breakup time

$$t \le t_* \equiv 1.5 \times 10^{-3} \left(\frac{\sigma_c}{0.1}\right) \left(\frac{v_s}{v_a}\right)^2 \tau_{\rm H},$$

$$= 4.2 \times 10^3 \left(\frac{\sigma_c}{0.1}\right) \left(\frac{B_0}{10^{14}{\rm G}}\right)^{-3} {\rm yr.}$$

$t_* \propto B^{-3}$

a few years for $B = 10^{15} G$

- Elastic energy

$$\Delta E_{\rm ela} = 2\pi \int_{r_c}^{R} r^2 dr \int_0^{\pi} \sin\theta d\theta \ \mu \sigma_{ij} \sigma^{ij}$$

$$= 5.8 \times 10^{-7} \ \mu_c R^3 \left(\frac{\sigma_c}{0.1}\right)^2 \left(\frac{t}{t_*}\right)^2,$$

E$\sim 10^{41}$ erg

- Change of magnetic energy

$$\Delta E_{\rm mag} = 2\pi \int_{r_c}^{R} r^2 dr \int_0^{\pi} \sin\theta d\theta \ \frac{(\delta B_\phi)^2}{8\pi}$$

$$= 2.0 \times 10^{-4} \ B_0^2 R^3 \left(\frac{\sigma_c}{0.1}\right)^2 \left(\frac{v_s}{v_a}\right)^4 \left(\frac{t}{t_*}\right)^2$$

E$\sim 10^{41}$ erg

$\sim 10^{-7} \ E_B \ (B = 10^{15} G$

Main results and summray

Breakup time and elastic energy estimated

✓ a few years for $B = 10^{15} G$ and E$\sim 10^{41}$ erg in a simple magnetar model

comparable to repeat time and energy scale

in observed bursts (not giant flare) (YK 22)

✓Timescale increases by $\sim 10^2$ for confined field in CCOs (YK+ 23)

No (or quite rare) bursts in CCOs

Further improvement

irregular magnetic field, coupling with magnetosphere, thermal effect T$\sim 10^8$K , …