How to fit our data?

Whatever we obtained in the (K^-, N) reactions

— Towards more realistic spectral fitting —

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Experiments we have done @ J-PARC K1.8BR

E31

$d(K^-, N)\bar{K}N$

- $d(K^-, n)\pi^{\mp,0}\Sigma^{\pm,0}$
- $d(K^-, p)\pi^-\Lambda$
- $d(K^-, p)\pi^-\Sigma^0$

E15

$^3\mathrm{He}(K^-, N)\bar{K}NN$

- $^3\mathrm{He}(K^-, n)\Lambda p$
- $^3\mathrm{He}(K^-, n)\pi^+\Lambda n$
- $^3\mathrm{He}(K^-, n)\pi^{\mp}\Sigma^{\pm}p$
- $^3\mathrm{He}(K^-, p)\pi^-\Lambda p$

T77

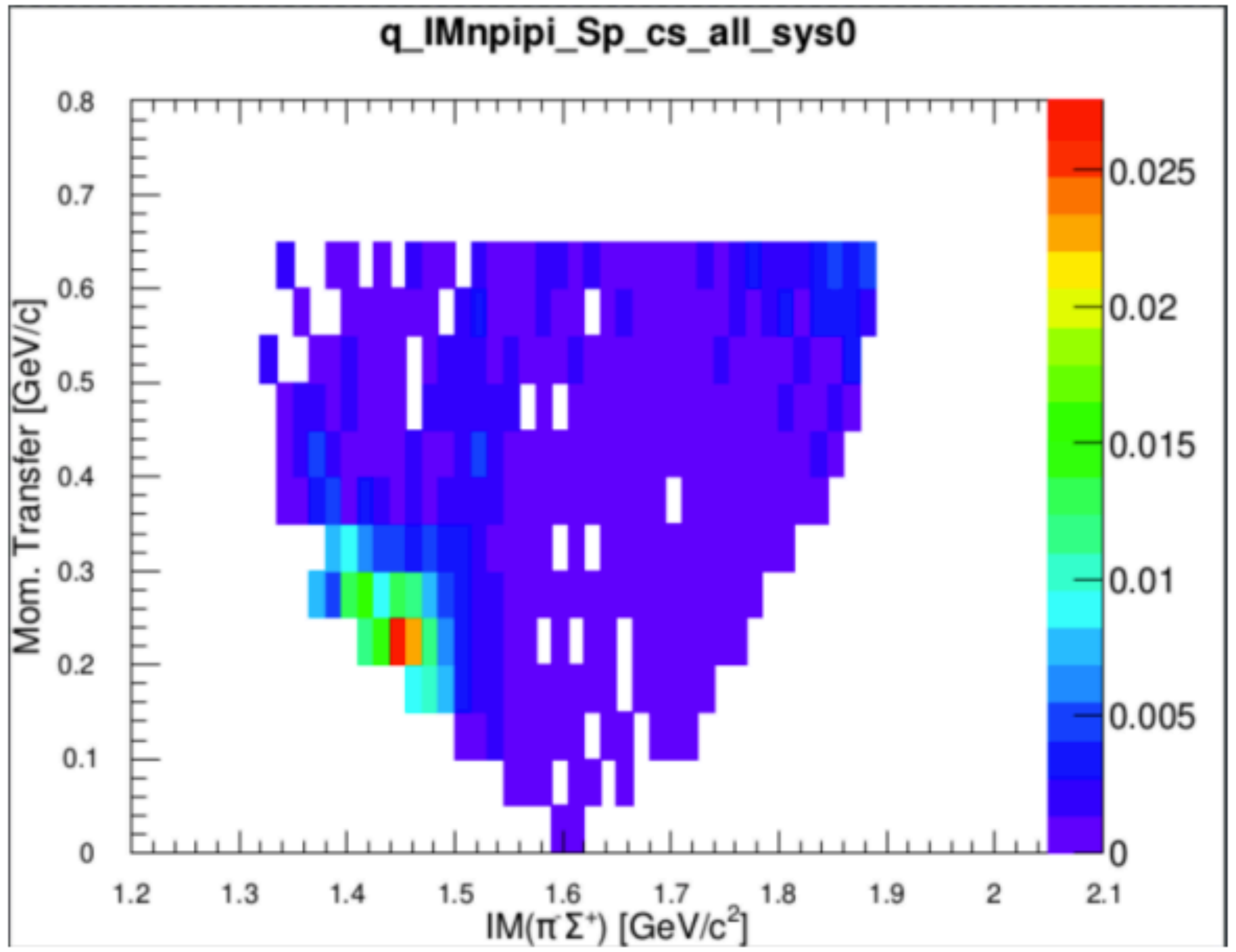
$^4\mathrm{He}(K^-, N)\bar{K}NNN$

- $^4\mathrm{He}(K^-, n)\Lambda d$

Experiments we have done @ J-PARC K1.8BR

E31

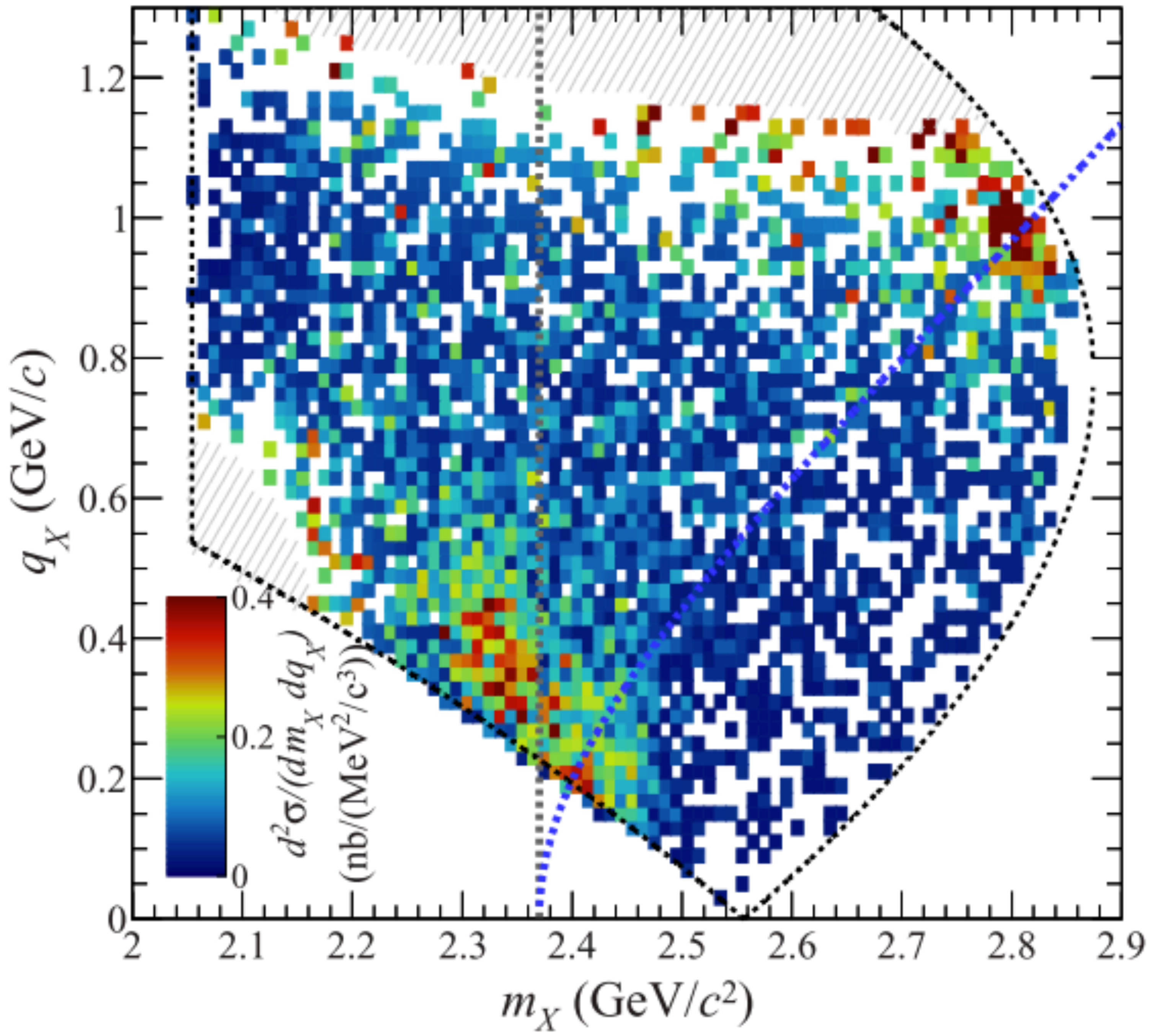
$d(K^-, n)\pi^{\mp}\Sigma^{\pm}$



Asano meeting report (2022.05.25)

E15

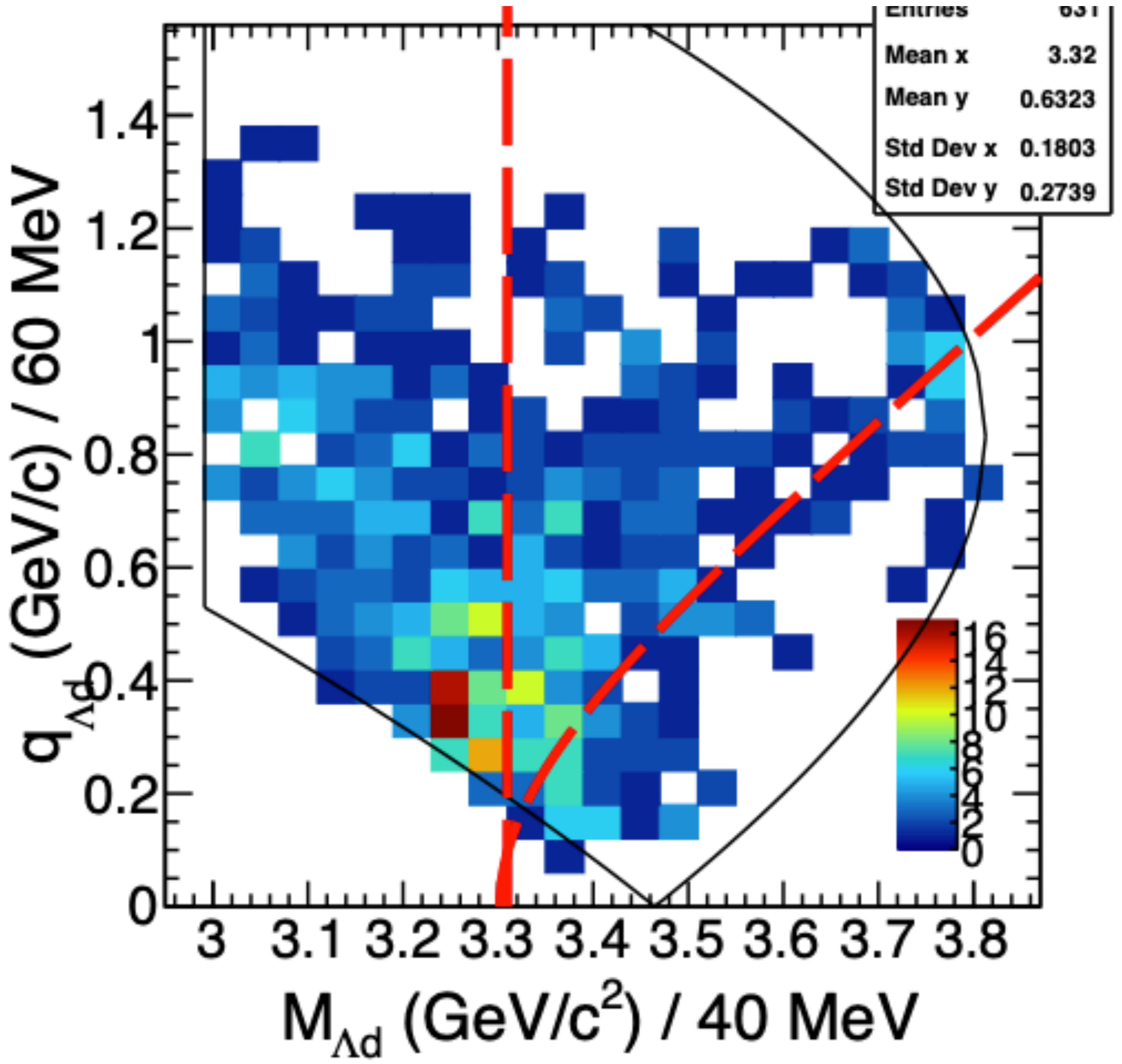
$^3\mathrm{He}(K^-, n)\Lambda p$



Phys. Rev. C **102**, 044002 (2020).

T77

$^4\mathrm{He}(K^-, n)\Lambda d$



Hashimoto meeting report (2022.06.25)

Experiments we have done @ J-PARC K1.8BR

E31	E15	T77
$d(K^-, n)\pi^{\mp}\Sigma^{\pm}$	$^3\mathrm{He}(K^-, n)\Lambda p$	$^4\mathrm{He}(K^-, n)\Lambda d$

2D distributions are quite similar!

Quasi free & Resonance are dominant components.

2-step process is dominant!

⇩

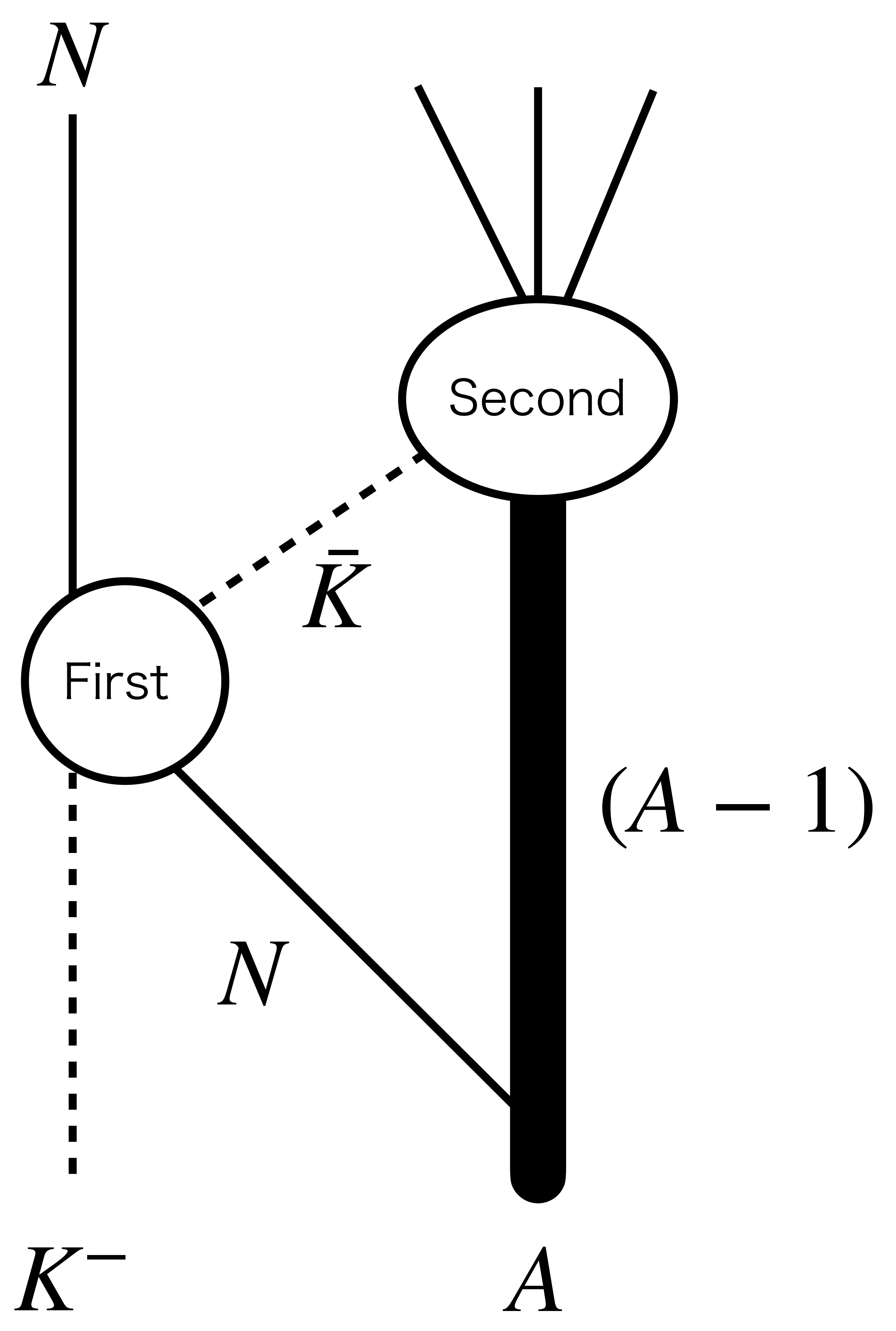
All of them could be fit by the same manner.

⇩

It would provide more precise information.

Form factor, Production mechanism, ⋯

2-step process in $A(K^-, N)$



- **First step**

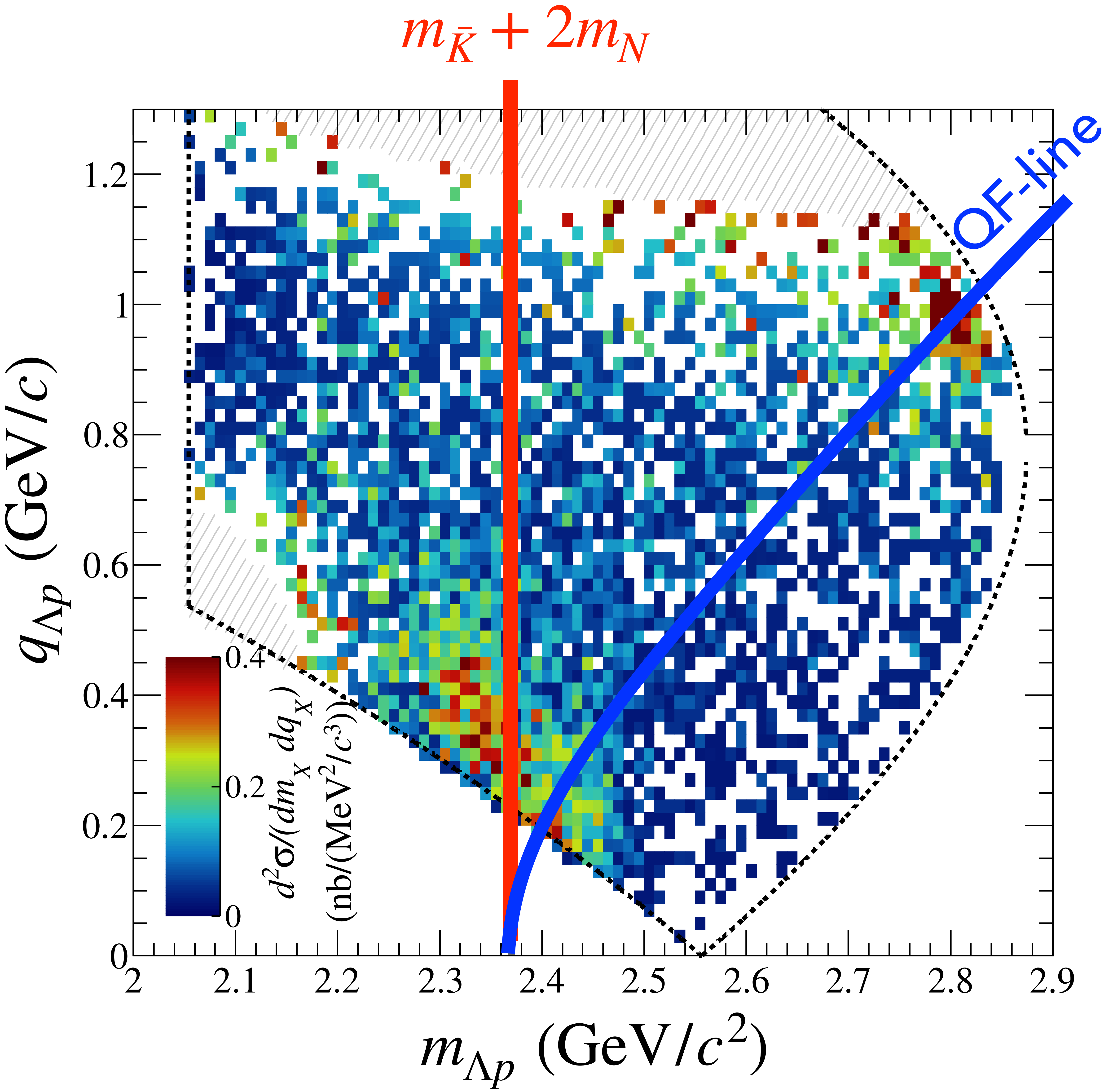
Elementary (K^-, N) reactions @ $\sqrt{s} \sim 1.8\ \mathrm{GeV}/c^2$

⇨
- $p(K^-, p)K^-$
- $n(K^-, n)K^-$
- $p(K^-, n)\bar{K}^0$

- **Second step**

Resonances $(Y^*, \bar{K}NN, \cdots)$

Fit for E15 data



Lorentz Invariant Phase Space

$$d\sigma \propto \left(f_{\bar{K}NN} + f_{QF}\right) d\Phi_{\Lambda pn}$$

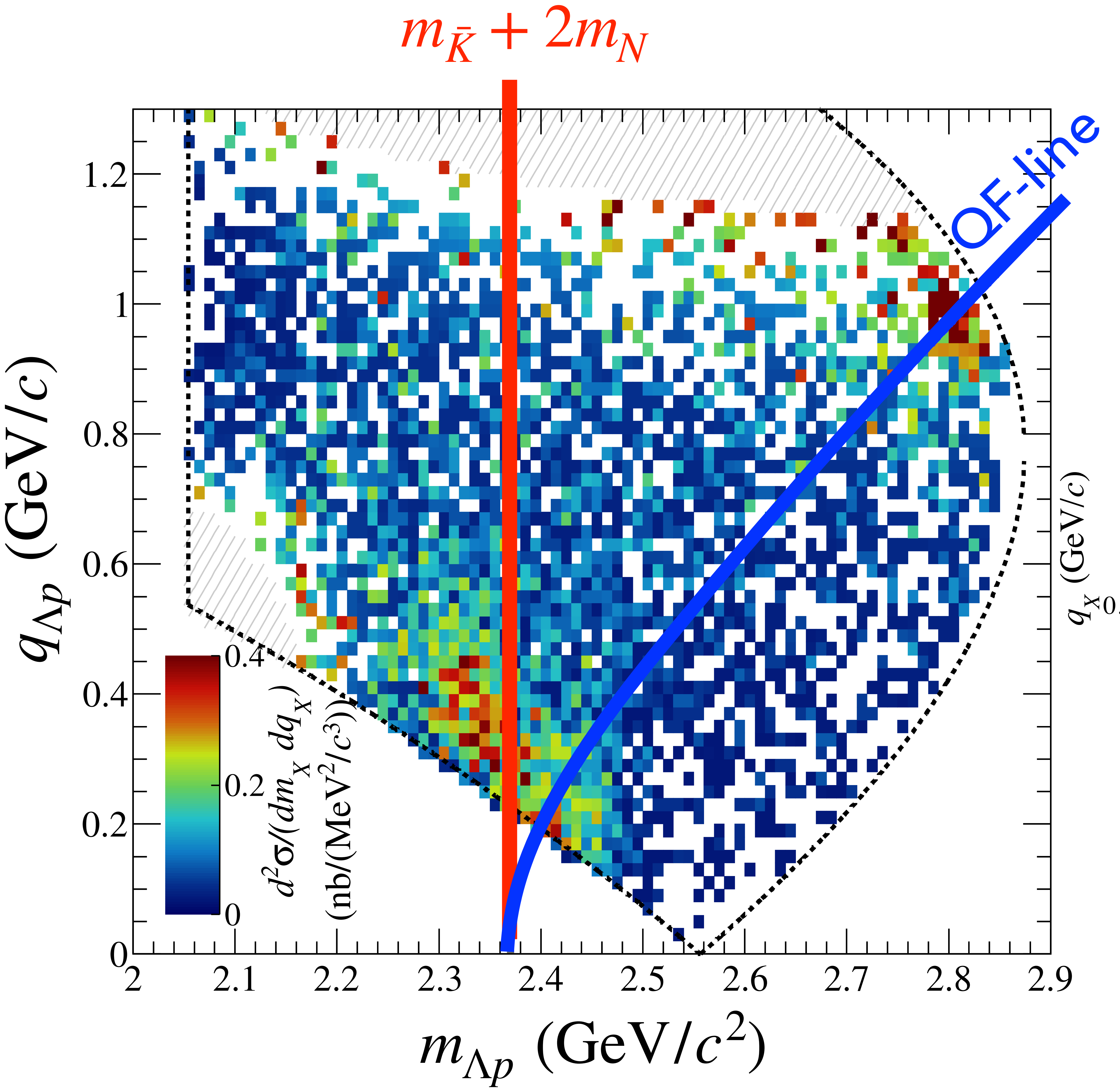
- $\bar{K}NN$ production

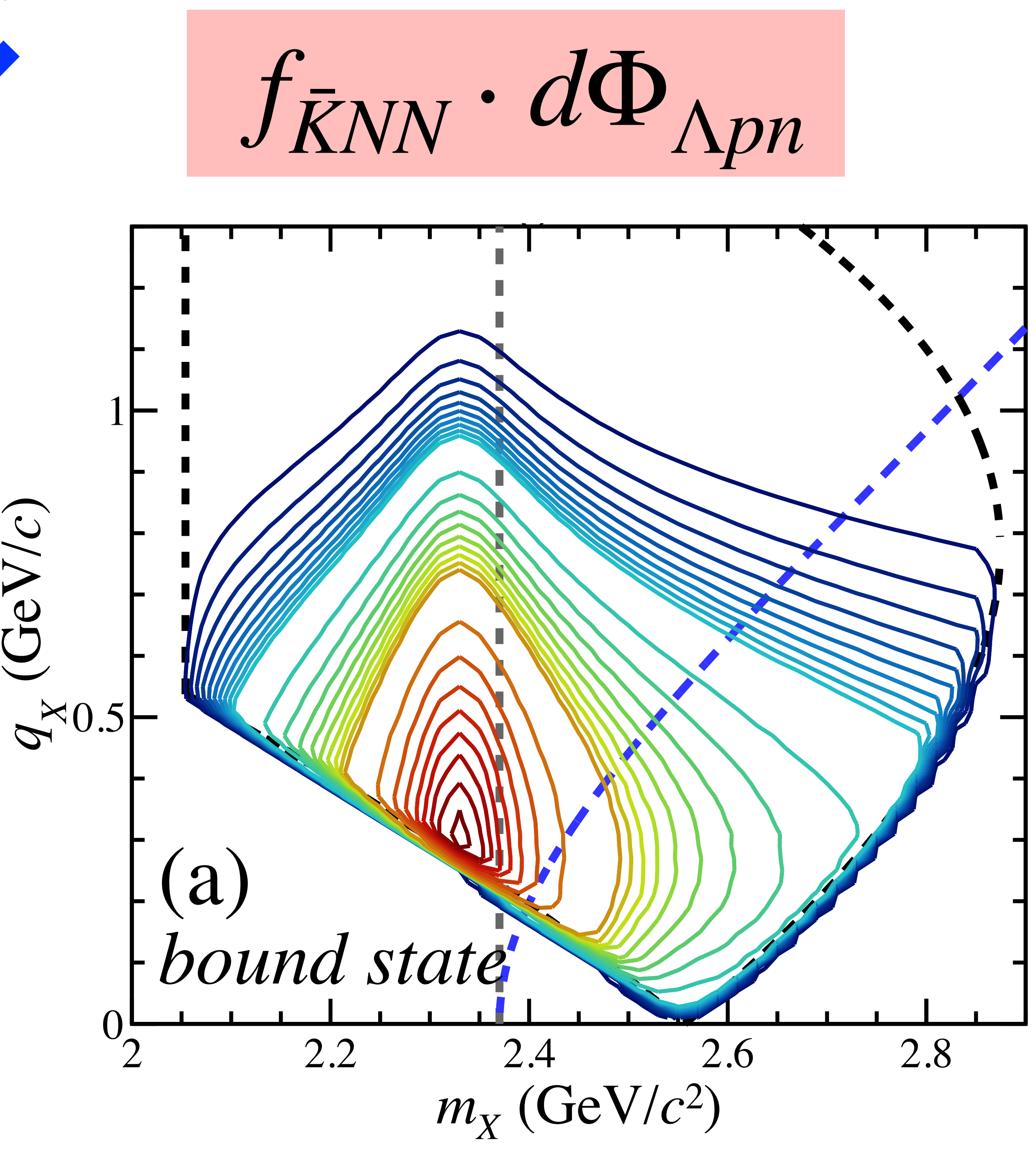
$$f_{\bar{K}NN} = \frac{\Gamma^2/4}{\left(m - M_R\right)^2 + \Gamma^2/4} \cdot \exp\left(-\frac{q^2}{Q^2}\right)$$

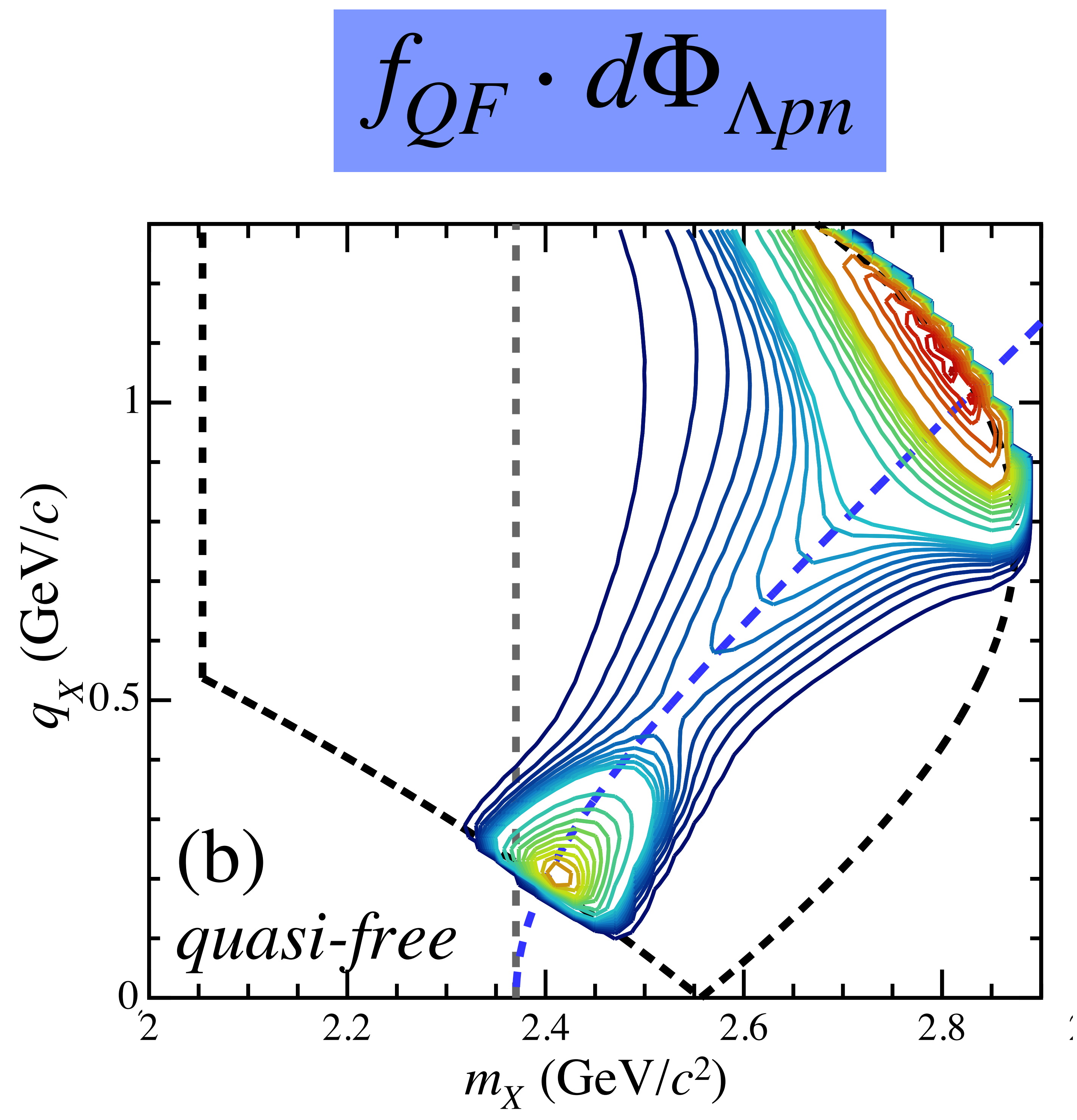
- Quasi-free process

$$f_{QF} = \exp\left(-\frac{\left(m - M_{QF}\right)^2}{\sigma_{QF}^2}\right) \cdot g(q)$$

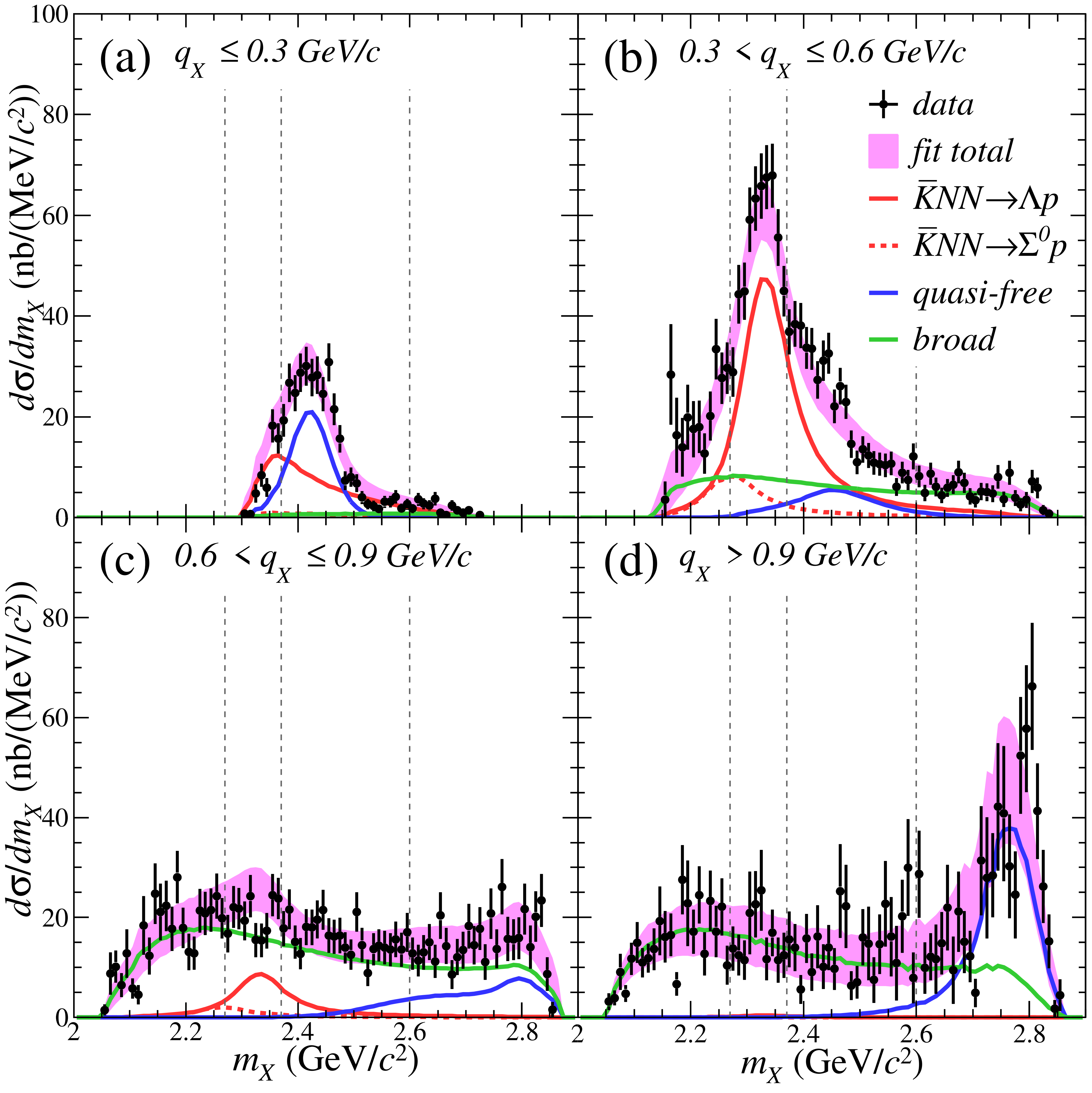
Fit for E15 data

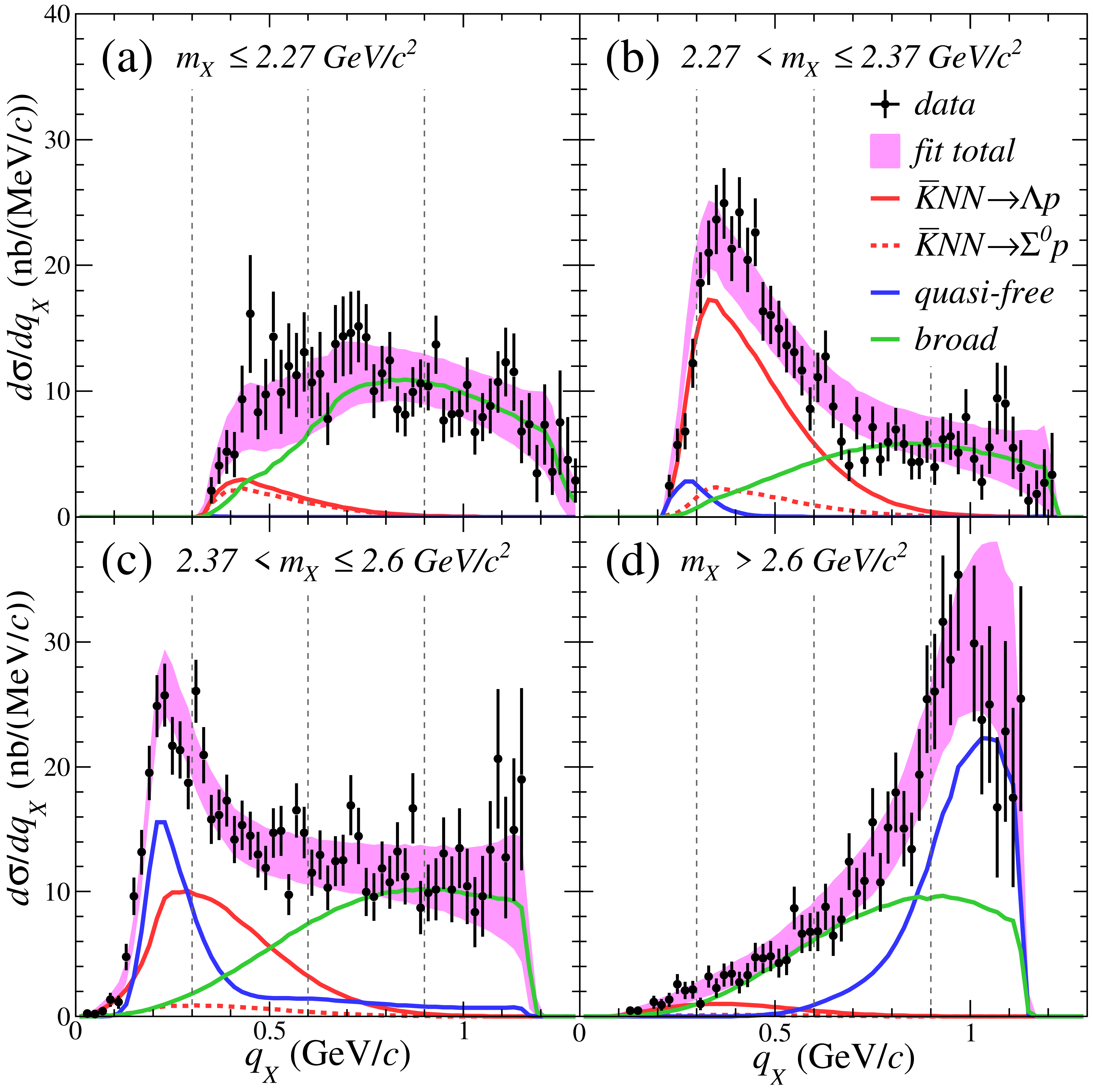






Fit for E15 data



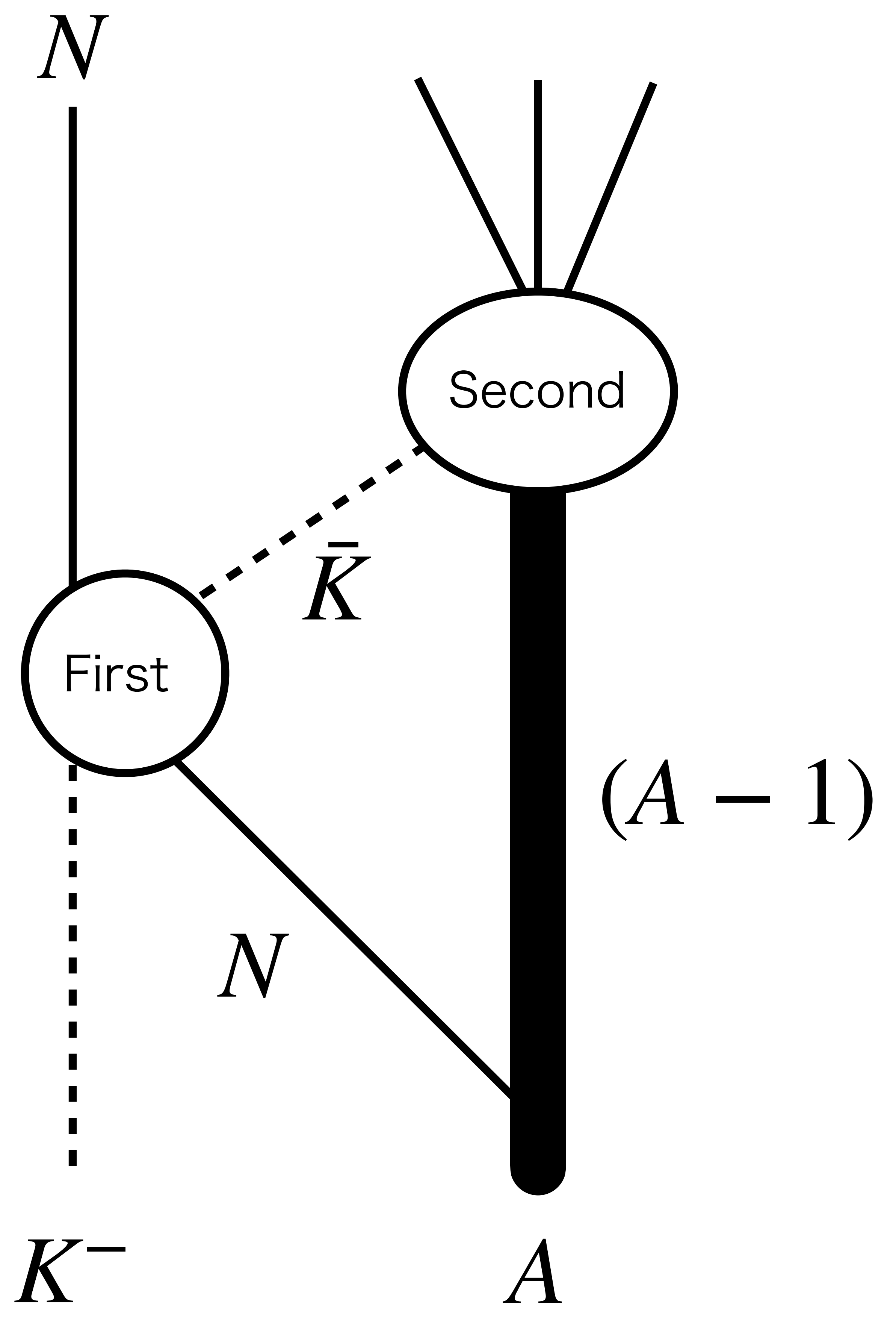


Fit for E15 data

- E15 data was well reproduced by the model functions.
  - The data can be explained mainly by Resonance & Quasi-free.

- However, the model functions are too phenomenological.
  - Momentum transfer (or angular) dependence does NOT contain the first step elementary (K^{-}, N) contribution, even we consider it is dominant.
    - Angular dependence of the model function is NOT related to that of the elementary processes at all.
  - Resonance $(\bar{K}NN)$ is considered to be simple Breit-Wigner formula which should contain threshold effects similar to $\Lambda(1405)$.

2-step process in $A(K^-, N)$



- **First step** ⇦ How to introduce this part?

Elementary (K^-, N) reactions @ $\sqrt{s} \sim 1.8\ \mathrm{GeV}/c^2$

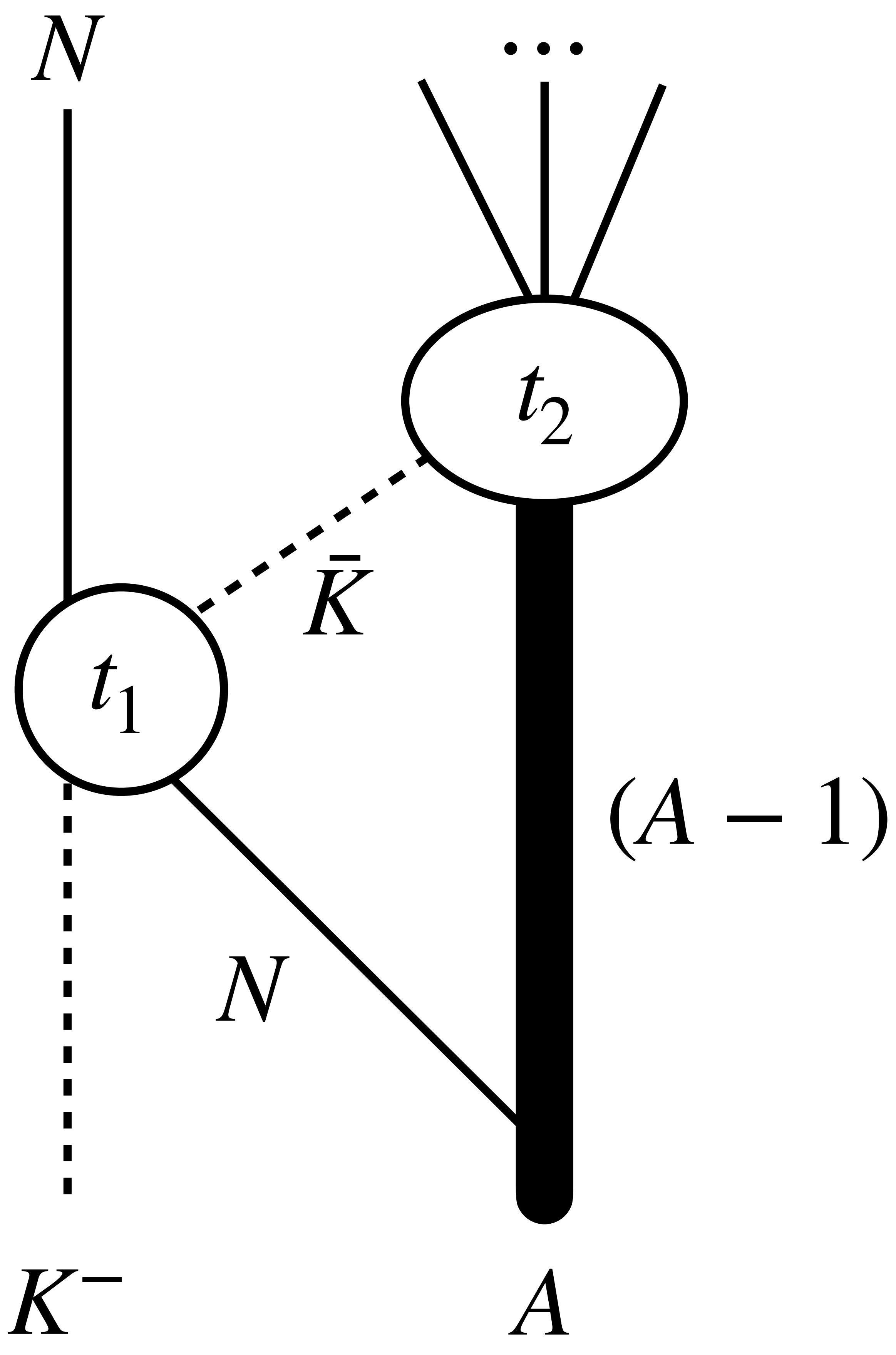
⇨
- $p(K^-, p)K^-$
- $n(K^-, n)K^-$
- $p(K^-, n)\bar{K}^0$

- **Second step** ⇦ How to treat threshold effects?

Resonances $(Y^*, \bar{K}NN, \cdots)$

Both problems are solved in the E31 fitting.

Fit for E31 data



Elementary (K^-, N)

$\bar{K}$ propagator

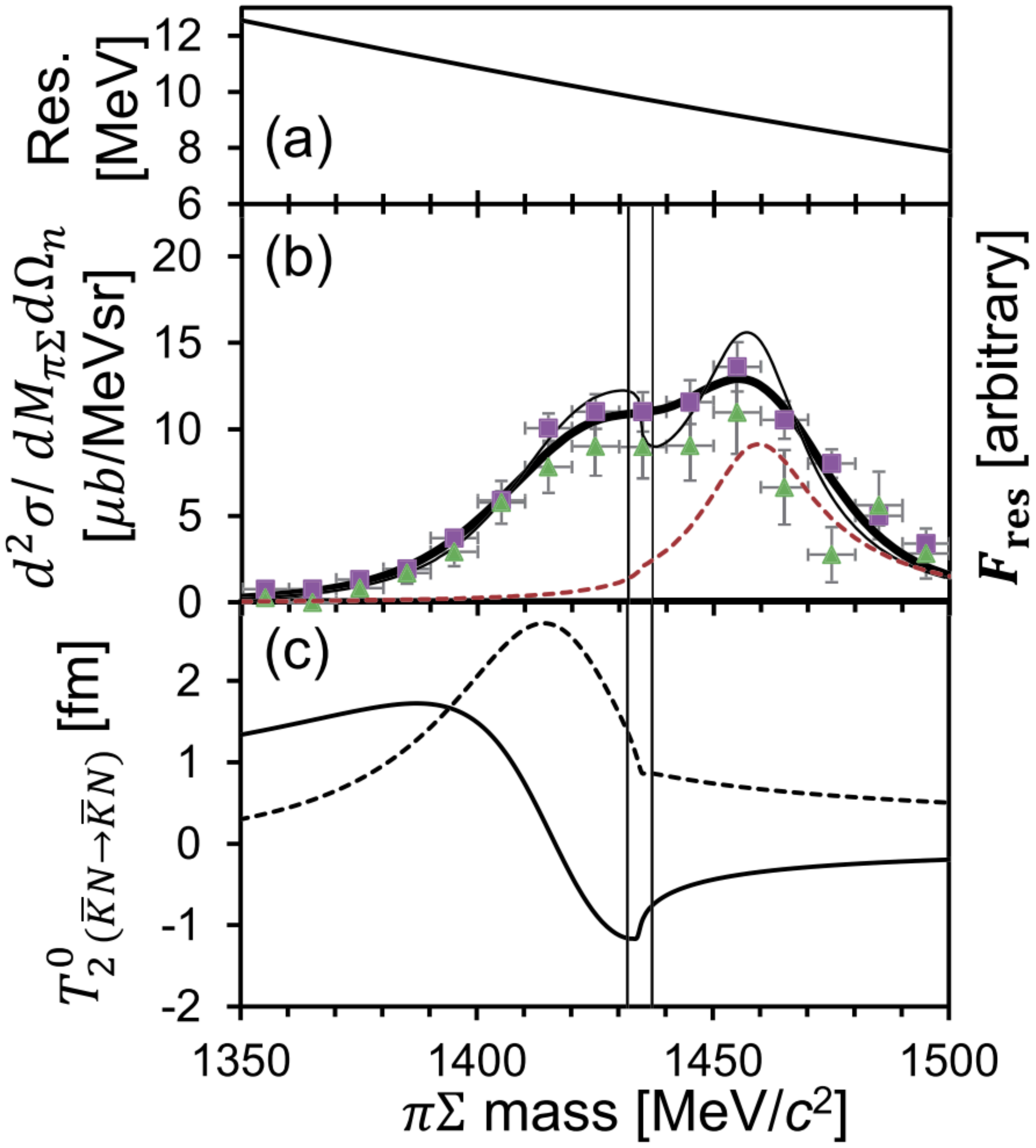
$$T = \left\langle final \left| t_2 G\left(\bar{K}, A-1\right) t_1 \right| K^- \Phi_A \right\rangle$$

Resonance; $\Lambda(1405)$

$$T_2^{I'}(\bar{K}N, \bar{K}N) = \frac{A^{I'}}{1 - iA^{I'}k_2 + \frac{1}{2}A^{I'}R^{I'}k_2^2},$$

$$T_2^{I'}(\bar{K}N, \pi\Sigma) = \frac{e^{i\delta^{I'}}}{\sqrt{k_1}} \frac{\sqrt{\mathrm{Im}A^{I'} - \frac{1}{2}|A^{I'}|^2 \mathrm{Im}R^{I'}k_2^2}}{1 - iA^{I'}k_2 + \frac{1}{2}A^{I'}R^{I'}k_2^2},$$

Fit for E31 data



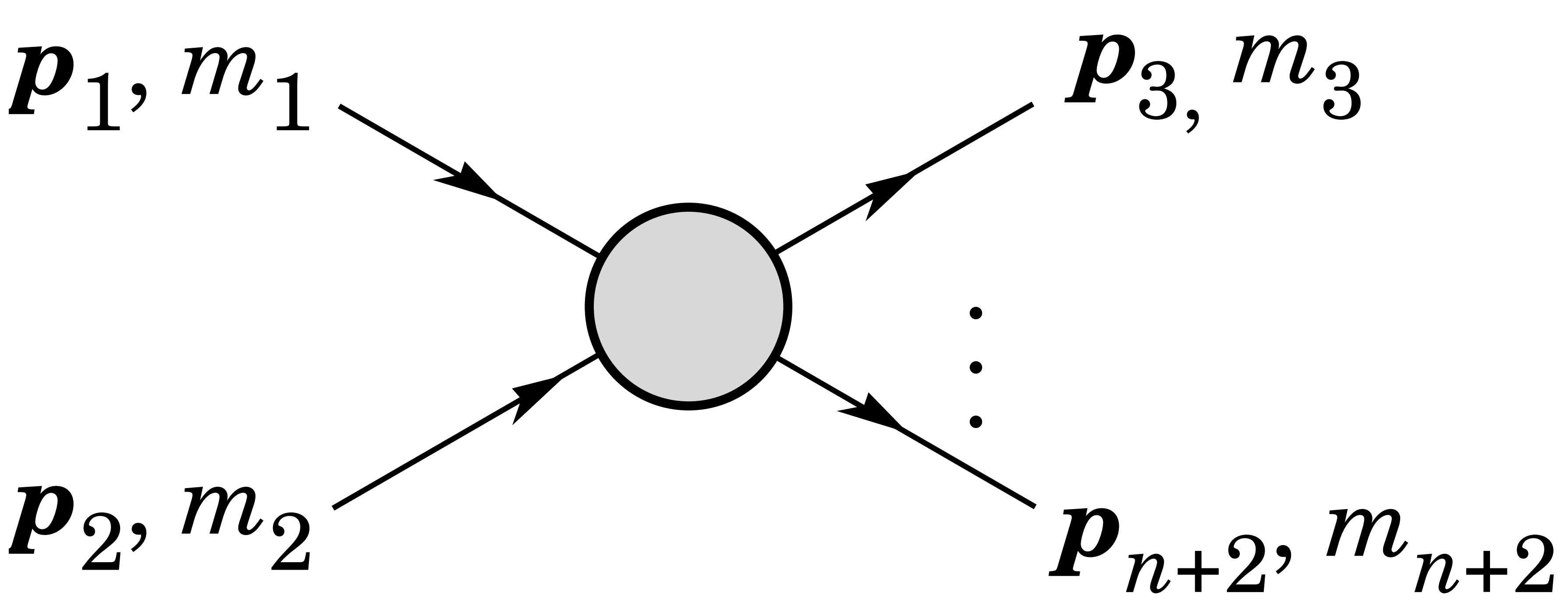
- Data is well explained.

Cross section

Taken from PDG

$$\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2} = p_{1\mathrm{cm}}\sqrt{s}\ .$$

Cross sections (taken from PDG “Kinematics”)

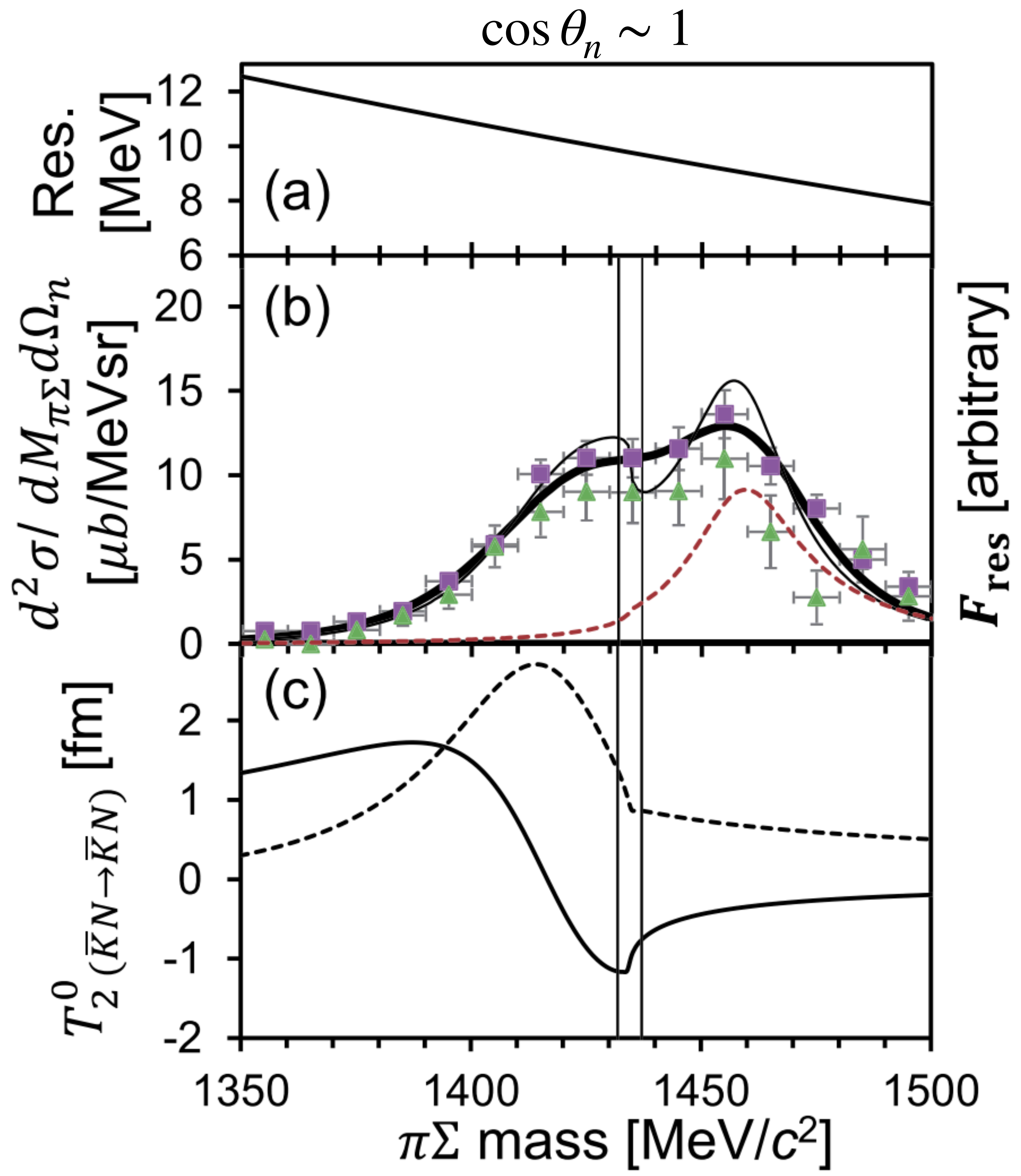


$$d\sigma = \frac{(2\pi)^4 |\mathscr{M}|^2}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} \times d\Phi_n(p_1 + p_2;\ p_3, \ldots, p_{n+2})\ .$$

Amplitude

n-body phase space

Fit for E31 data



- Data is well explained.
- Do we need multiply $d\Phi_{\pi\Sigma n}$?
  - Even it is needed, the result would not be changed so much, since $d\Phi_{\pi\Sigma n}$ is almost flat at this energy region.
- So, we would like to apply this model functions to other spectra.
  - How to extend angular region?
  - How to treat three(or more)-body threshold?

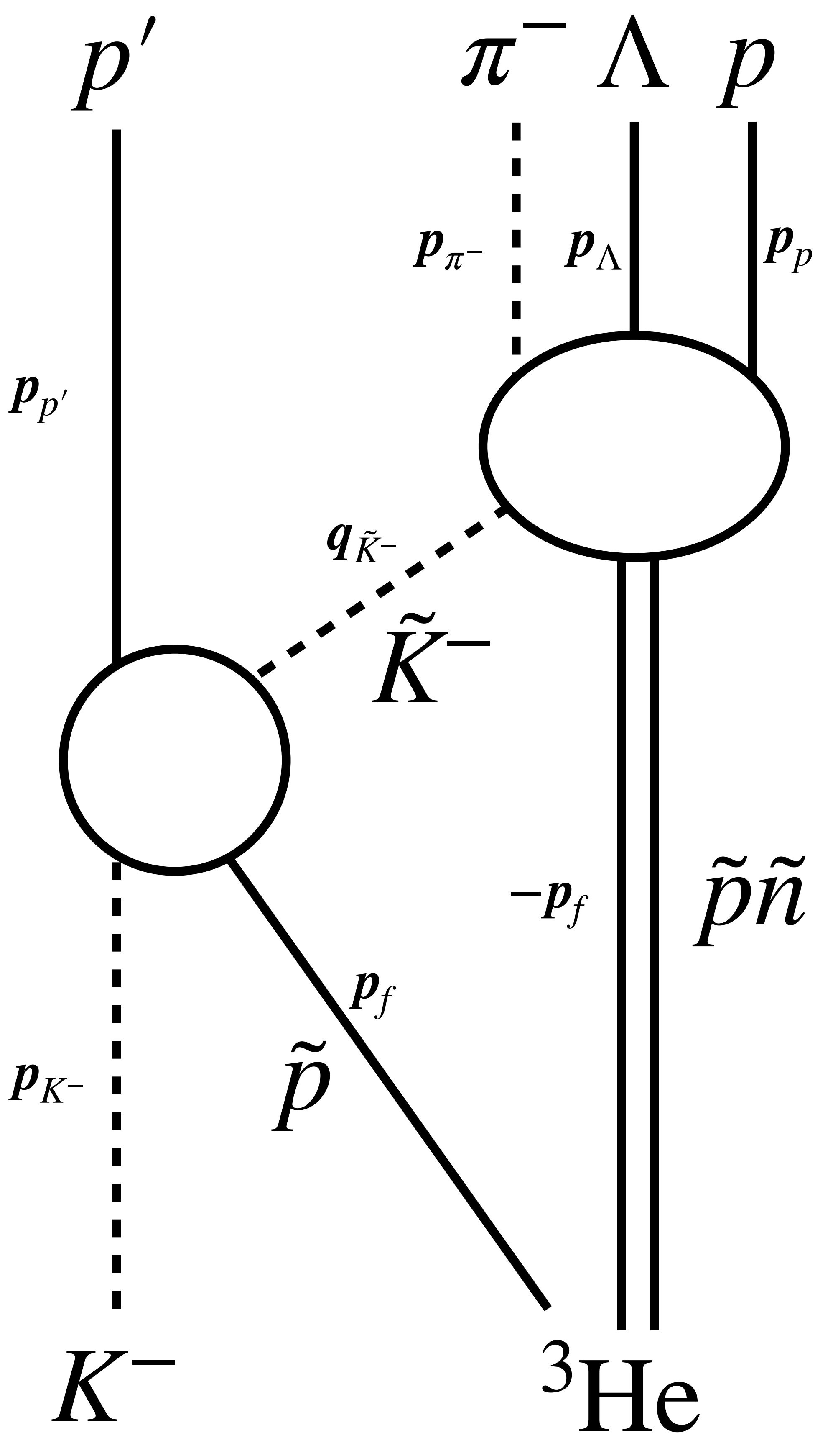
What we need to improve from E31 fit

- To extend angular region
  - We need to extend F_{res} to higher θ_N region.

$$\frac{d^2\sigma}{dM_{\pi\Sigma}d\Omega_n} \approx \left|T_2^{I'}\right|^2 F_{\mathrm{res}}(M_{\pi\Sigma}),$$

$$F_{\mathrm{res}}(M_{\pi\Sigma}) = \left|\int G_0 T_1^I \Phi_d(q_{N_2}) d^3 q_{N_2}\right|^2.$$

Extension of F_{res}



Center of mass energy of the final step

$$w_F = \left(\left(w - m_p\right)^2 - w\,T^*_{p'}\right)^{\frac{1}{2}} = m_{\pi^-\Lambda p}$$

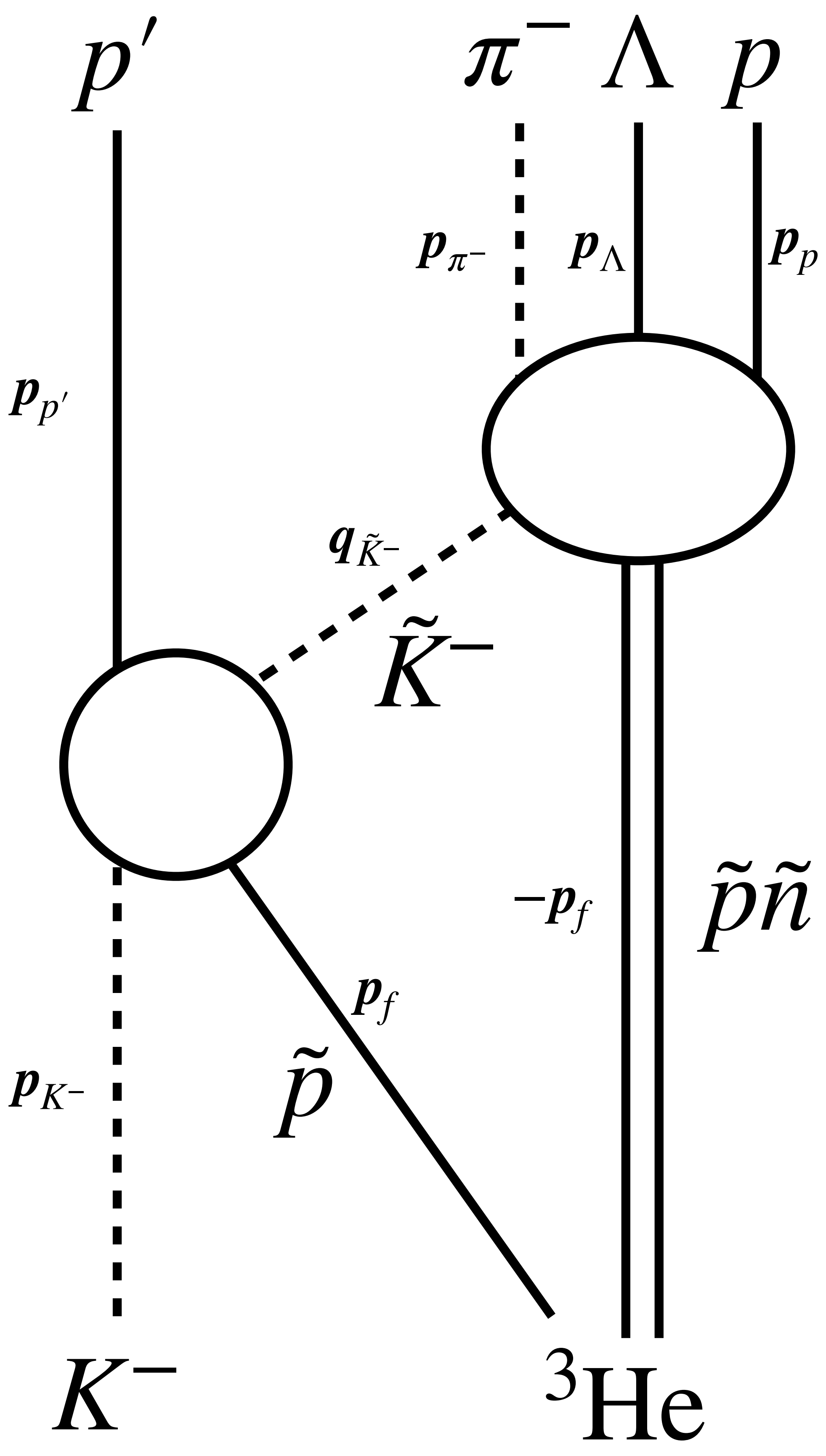
Distribution of F_{res}

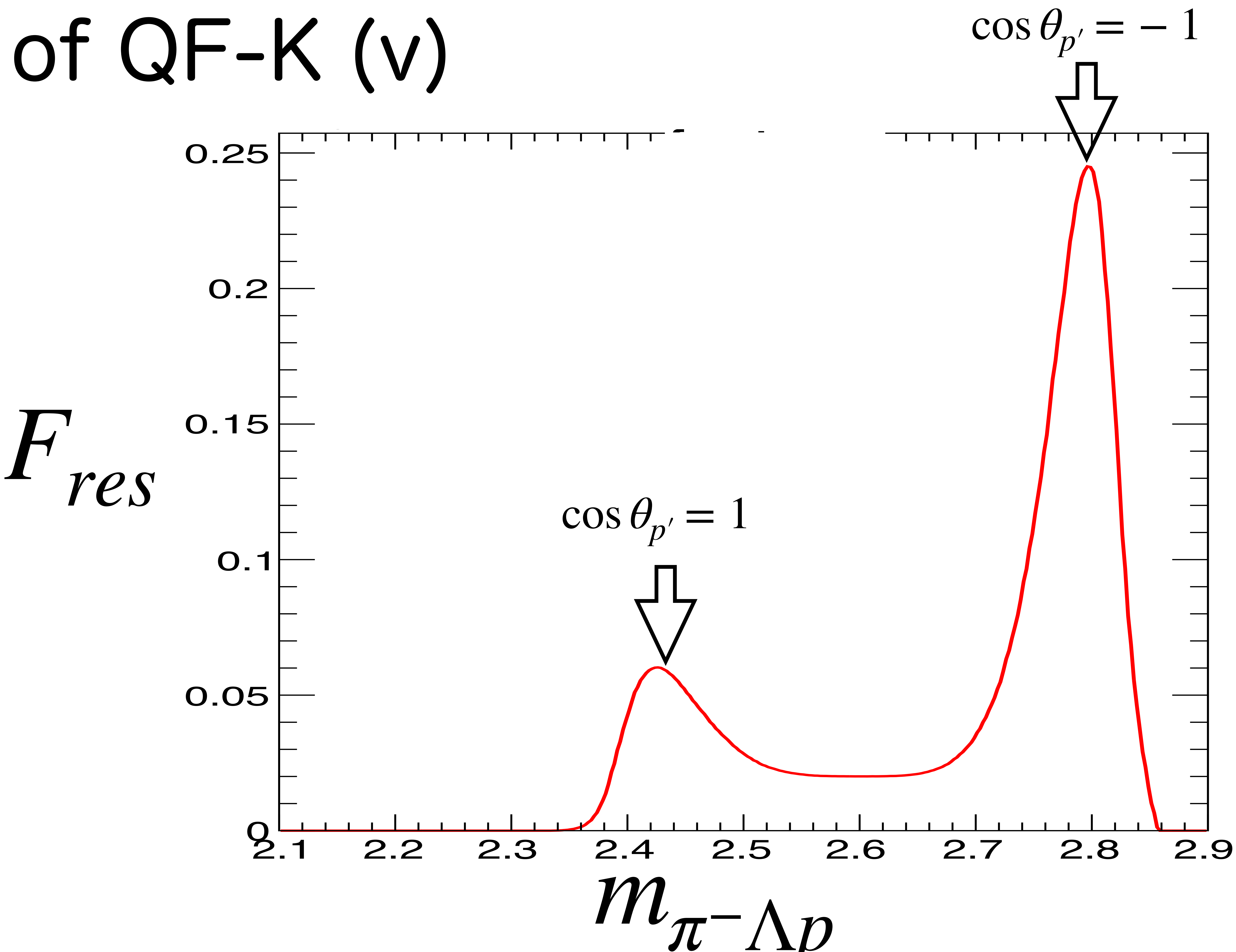
$$F_{res} \propto \int dw \int dT^*_{p'}\, p(w)\, p\left(T^*_{p'}\right) \delta\left(\left(w - m_p\right)^2 - w\,T^*_{p'} - m^2_{\pi^-\Lambda p}\right)$$

$$= \int dp_{K^-} \int d\boldsymbol{p}_f \int d\cos\theta_{K^-\tilde{p}\to\tilde{K}^-p'} \int dm_{\tilde{K}^-}\, p\left(p_{K^-}\right) p\left(\boldsymbol{p}_f\right) p\left(\cos\theta_{K^-\tilde{p}\to\tilde{K}^-p'}\right) p\left(m_{\tilde{K}^-}\right)\ \delta\left(\left(w - m_p\right)^2 - w\,T^*_{p'} - m^2_{\pi^-\Lambda p}\right)$$

$$= 2\pi \int dp_{K^-} \int dp_f \int d\cos\theta_f \int d\cos\theta_{K^-\tilde{p}\to\tilde{K}^-p'} \int dm_{\tilde{K}^-}\, p\left(p_{K^-}\right) p_f^2\, p\left(p_f\right) p\left(\cos\theta_{K^-\tilde{p}\to\tilde{K}^-p'}\right) p\left(m_{\tilde{K}^-}\right)\ \delta\left(\left(w - m_p\right)^2 - w\,T^*_{p'} - m^2_{\pi^-\Lambda p}\right)$$

Kinematics of QF-K (v)

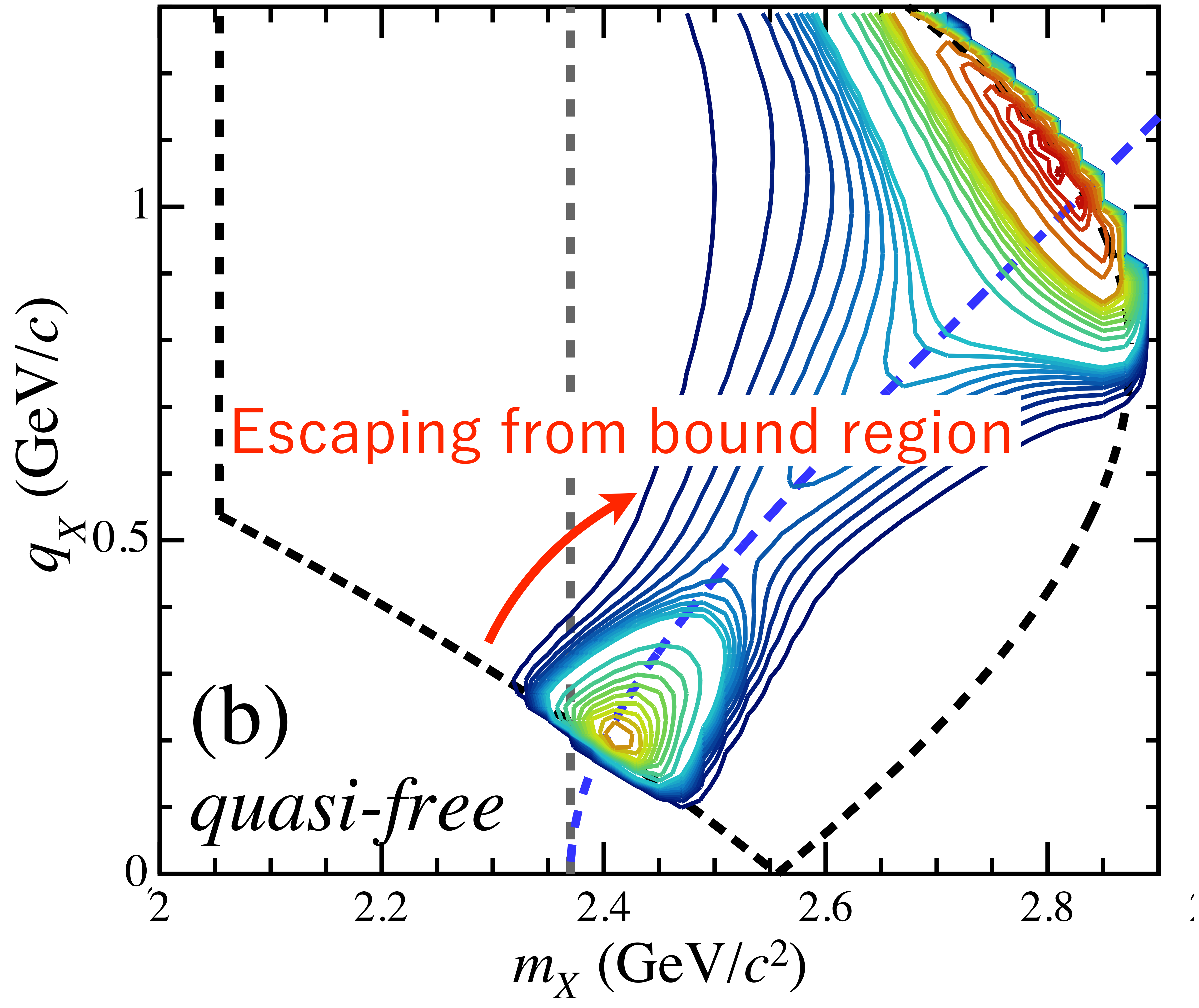




What we need to improve from E31 fit

- To extend angular region
  - We need to extend F_{res} to higher θ_N region. :: Possible
  - We need introduce additional factor (or reaction, such as point-like) to modify $\cos\theta_N$ dependence. Otherwise, $\cos\theta_N$ dependence will be fixed by the elementary process.

How to introduce q-dependence



⇨ Production CS will be rapidly decrease in higher-q.

⇩

We need something to enhance higher-q.

S-wave Gaussian form factor cannot do that.

What we need to improve from E31 fit

- To extend angular region
  - We need to extend F_{res} to higher θ_N region. :: Possible
  - We need introduce additional factor (or reaction, such as point-like) to modify $\cos\theta_N$ dependence. Otherwise, $\cos\theta_N$ dependence will be fixed by the elementary process. :: Need further consideration
- To properly treat three(or more)-body threshold effect
  - i.e. $\bar{K}NN$ threshold

$$T_2^{I'}(\bar{K}N, \bar{K}N) = \frac{A^{I'}}{1 - iA^{I'}k_2 + \frac{1}{2}A^{I'}R^{I'}k_2^2},$$

$$T_2^{I'}(\bar{K}N, \pi\Sigma) = \frac{e^{i\delta^{I'}}}{\sqrt{k_1}} \frac{\sqrt{\mathrm{Im}A^{I'} - \frac{1}{2}|A^{I'}|^2 \mathrm{Im}R^{I'}k_2^2}}{1 - iA^{I'}k_2 + \frac{1}{2}A^{I'}R^{I'}k_2^2},$$

⇦ These cannot be used in three-body coupled channel.

To take into account threshold effect

Let us consider relativistic BW with mass-dependent width

$$T_R = \frac{g}{M_R^2 - m^2 - iM_R\Gamma_{tot}^R}$$

- g : Coupling to the measured channel
- M_R : Resonance mass
- m : Measured mass
- Γ_{tot}^R : Mass-dependent width

Total decay width of $\bar{K}NN$

$\Gamma_{tot}^{\bar{K}NN}(m)$ is a sum of partial decay widths of all possible decay channels of the $\bar{K}NN$.

$$\Gamma_{tot}^{\bar{K}NN}(m) = \sum \Gamma_{YN}^{\bar{K}NN}(m) + \sum \Gamma_{\pi YN}^{\bar{K}NN}(m) + \sum \Gamma_{\bar{K}NN}^{\bar{K}NN}(m)$$

where $\Gamma_{YN}^{\bar{K}NN}(m)$ is non-mesonic two-body decay into YN channels, $\Gamma_{\pi YN}^{\bar{K}NN}(m)$ and $\Gamma_{\bar{K}NN}^{\bar{K}NN}(m)$ are mesonic three-body decay into πYN and $\bar{K}NN$ channels, respectively. All possible decay channels of the $\bar{K}NN_{I_3=+1/2}$ are,

- Λp
- $\Sigma^0 p$
- $\Sigma^+ n$

- $\pi^0 \Lambda p$
- $\pi^0 \Sigma^0 p$
- $\pi^+ \Lambda n$
- $\pi^- \Sigma^+ p$
- $\pi^0 \Sigma^+ n$
- $\pi^+ \Sigma^- p$
- $\pi^+ \Sigma^0 n$

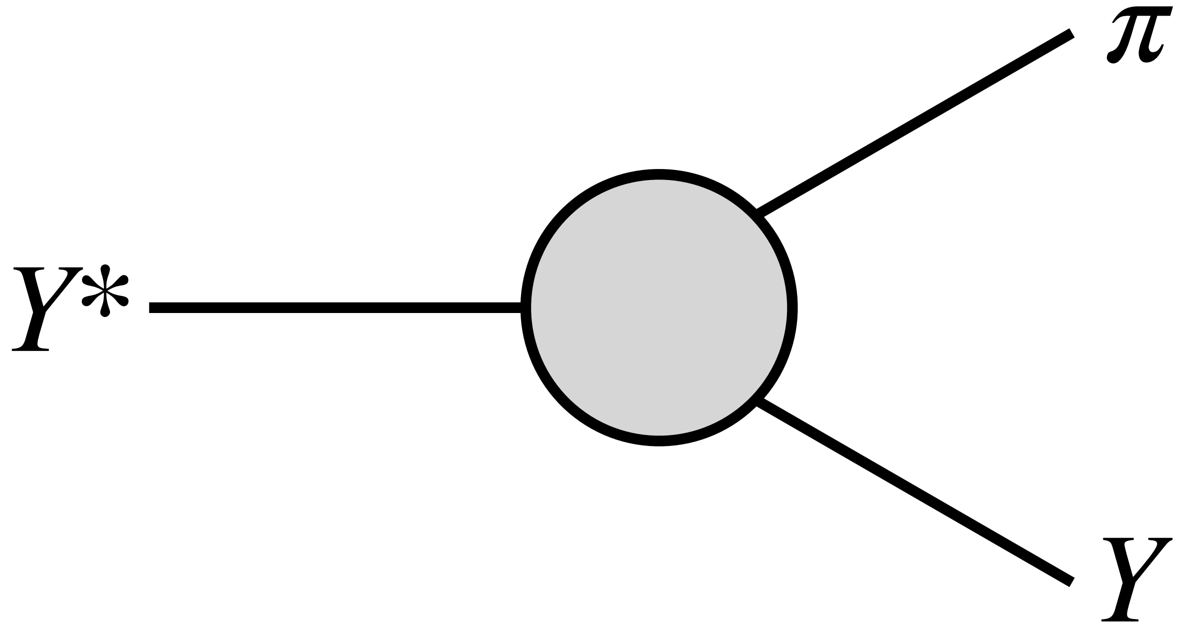
- $K^- pp$
- $\bar{K}^0 pn$

Partial decay widths can be obtained from the following equation,

Decay (taken from PDG "Kinematics")

$$d\Gamma = \frac{(2\pi)^4}{2M} |\mathscr{M}|^2 \, d\Phi_n \, (P;\ p_1, \ldots, p_n),$$

Cross section & Decay



Lorentz Invariant Phase Space

$$d\Gamma^{Y^*}_{\pi Y} = \frac{(2\pi)^4}{2m_{\pi Y}} \mathcal{M}^2 d\Phi_{\pi Y} = \frac{(2\pi)^4}{2m_{\pi Y}} \mathcal{M}^2 \frac{1}{4(2\pi)^6} \frac{\vec{p}^{(\pi Y)*}_Y}{m_{\pi Y}} d\Omega^{(\pi Y)*}_Y$$

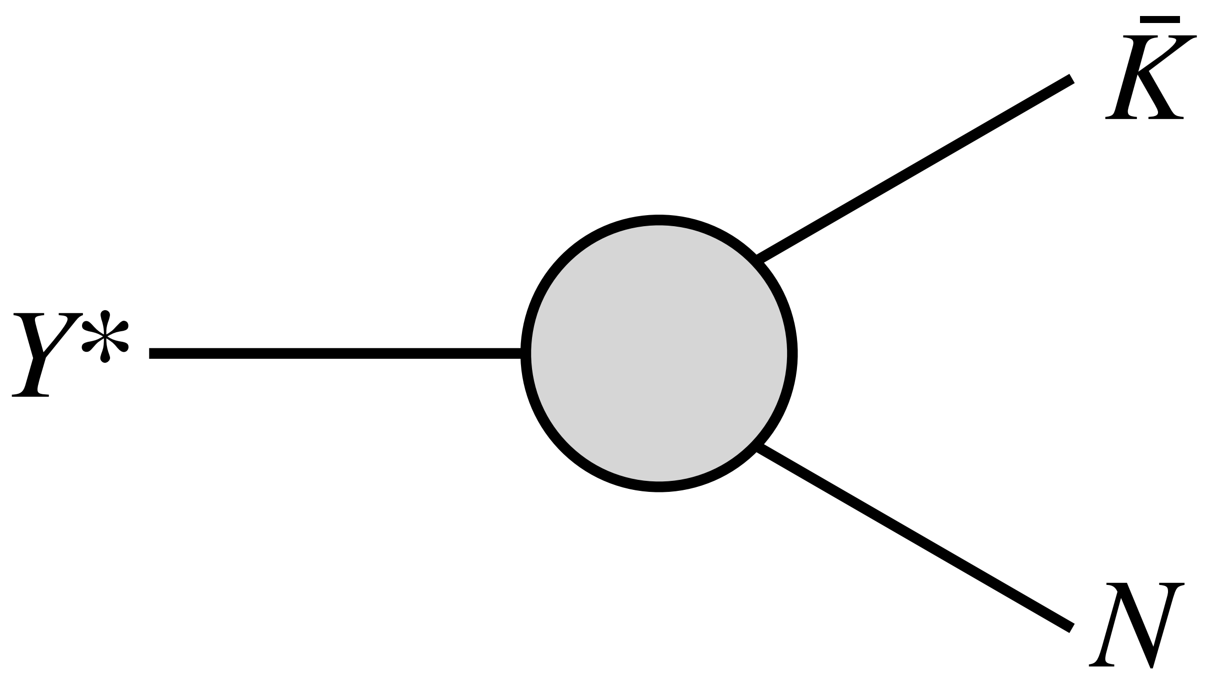
$$\mathcal{M} = g^{Y^*}_{\pi Y}$$

$$\sqrt{\left|\left(\vec{p}^{(\pi Y)*}_Y\right)^2\right|}$$

$$\Gamma_{\pi Y} = \frac{\left(g^{Y^*}_{\pi Y}\right)^2}{8\pi m^2_{\pi Y}} \frac{\sqrt{\left(m^2_{\pi Y} - (m_\pi + m_Y)^2\right)\left(m^2_{\pi Y} - (m_\pi - m_Y)^2\right)}}{2m_{\pi Y}}$$ (above the $m_\pi + m_Y$)

Imaginary ⇨ $$= i\frac{\left(g^{Y^*}_{\pi Y}\right)^2}{8\pi m^2_{\pi Y}} \frac{\sqrt{\left((m_\pi + m_Y)^2 - m^2_{\pi Y}\right)\left(m^2_{\pi Y} - (m_\pi - m_Y)^2\right)}}{2m_{\pi Y}}$$ (below the $m_\pi + m_Y$)

Cross section & Decay



$$d\Gamma^{Y^*}_{\bar{K}N} = \frac{(2\pi)^4}{2m_{\bar{K}N}} \mathscr{M}^2 \Phi_2 = \frac{(2\pi)^4}{2m_{\bar{K}N}} \mathscr{M}^2 \frac{1}{4\,(2\pi)^6} \frac{\vec{p}^{\,(\bar{K}N)*}_N}{m_{\bar{K}N}} d\Omega^{(\bar{K}N)*}_N$$

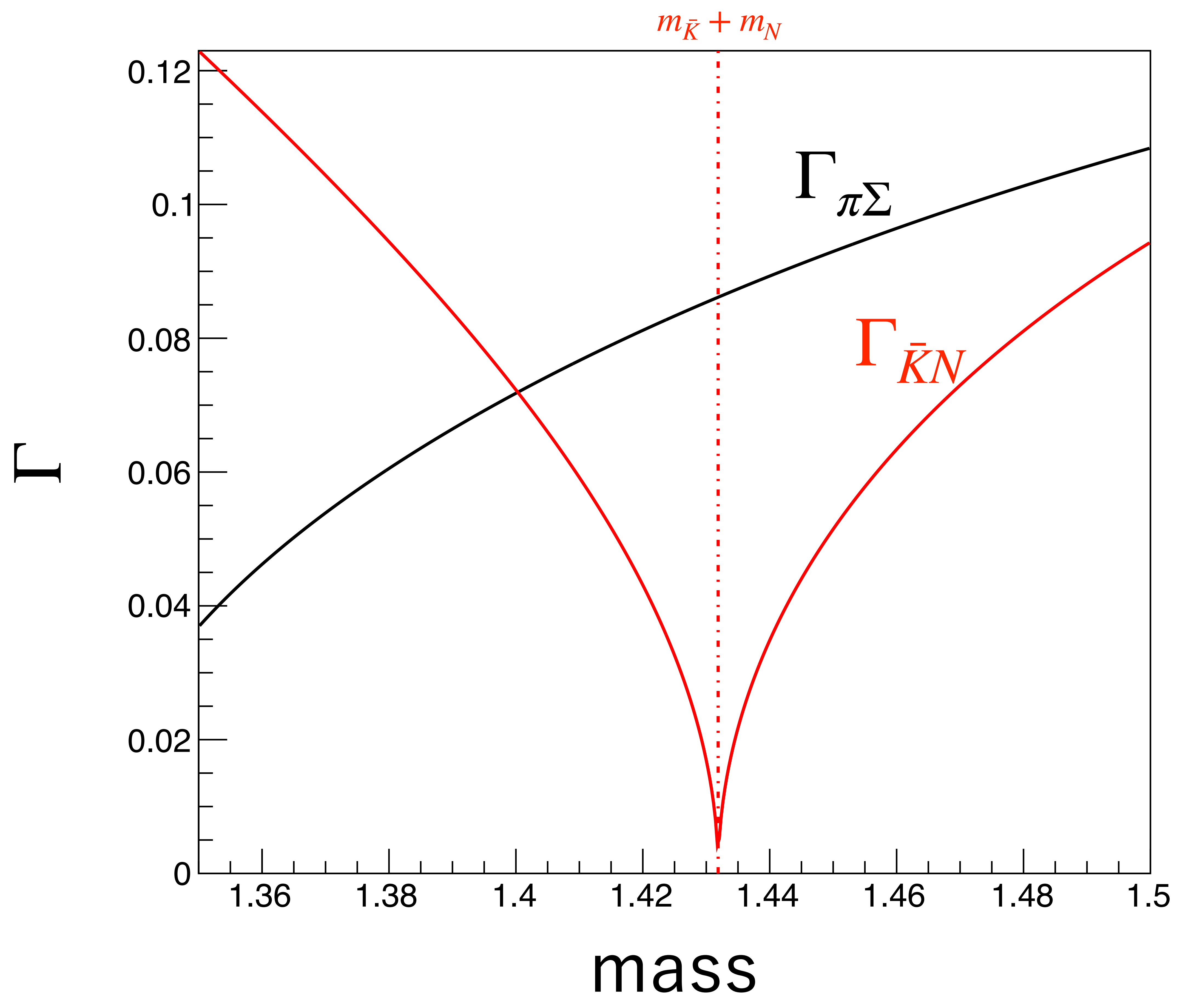
$$\boxed{\mathscr{M} = g^{Y^*}_{\bar{K}N}}$$

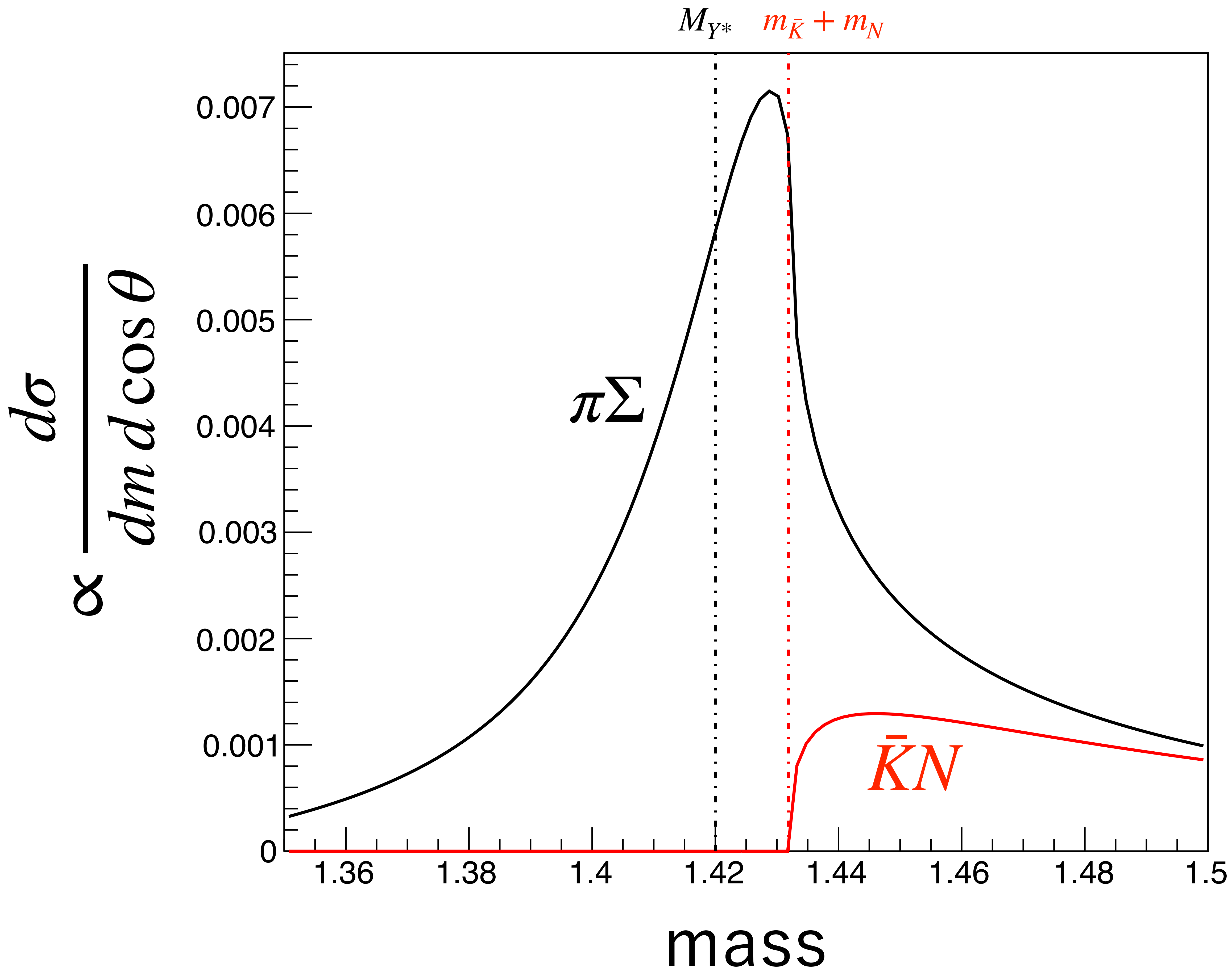
$$\Gamma_{\pi Y} = \frac{\left(g^{Y^*}_{\bar{K}N}\right)^2}{8\pi m^2_{\bar{K}N}} \frac{\sqrt{\left(m^2_{\bar{K}N} - (m_{\bar{K}} + m_N)^2\right)\left(m^2_{\bar{K}N} - (m_{\bar{K}} - m_N)^2\right)}}{2m_{\bar{K}N}} \text{ (above the } m_{\bar{K}} + m_N\text{)}$$

$$= i\frac{\left(g^{Y^*}_{\bar{K}N}\right)^2}{8\pi m^2_{\bar{K}N}} \frac{\sqrt{\left((m_{\bar{K}} + m_N)^2 - m^2_{\bar{K}N}\right)\left(m^2_{\bar{K}N} - (m_{\bar{K}} - m_N)^2\right)}}{2m_{\bar{K}N}} \text{ (below the } m_{\bar{K}} + m_N\text{)}$$

Example for $\Lambda(1405)$ case

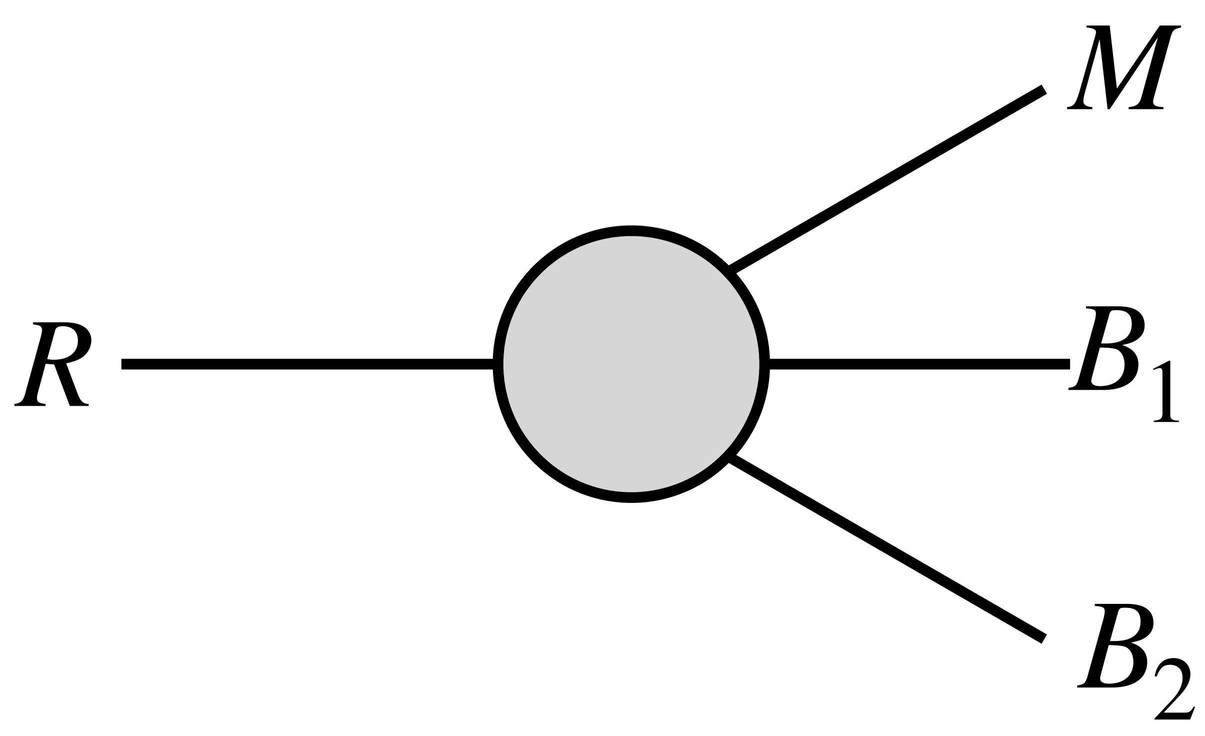
Parameters $\begin{cases} M_{Y*} = 1.42\ \mathrm{GeV}/c^2 \\ g^{Y*}_{\pi\Sigma} = g^{Y*}_{\bar{K}N} = 5 \end{cases}$





Threshold effect can be included in the line shape.

Three-body decay width



$$\begin{pmatrix} M \\ B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} \pi \\ Y \\ N \end{pmatrix}, \begin{pmatrix} \bar{K} \\ N \\ N \end{pmatrix}$$

The mesonic three-body decay widths $\Gamma_{MB_1B_2}$ can be expressed as,

$$d\Gamma_{MB_1B_2}(m_{MB_1B_2}) = \frac{(2\pi)^4}{2m_{MB_1B_2}} \mathscr{M}^{2} \Phi_{MB_1B_2}$$

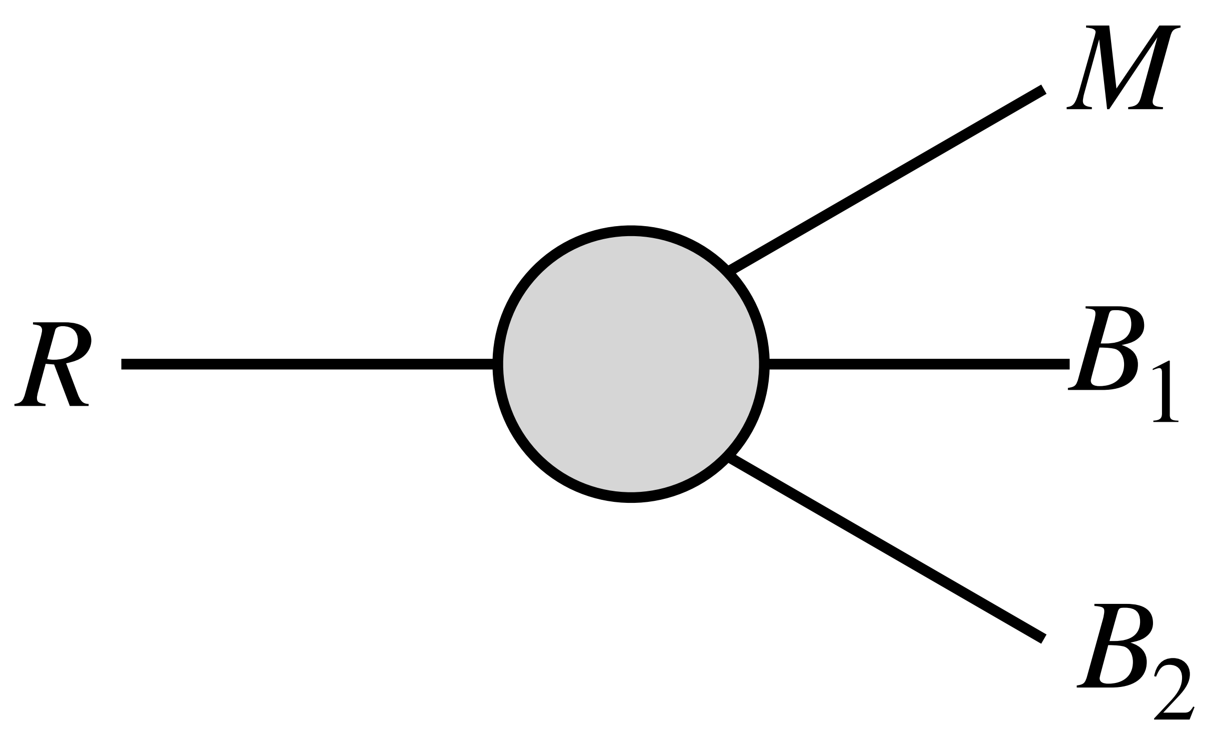
$$= \frac{(2\pi)^4}{2m_{MB_1B_2}} \mathscr{M}^{2} \left(\frac{1}{4(2\pi)^6} \frac{\vec{p}_{B_2}^{(MB_1B_2)*}}{m_{MB_1B_2}} d\Omega_{B_2}^{(MB_1B_2)*} \right) \left(\frac{1}{4(2\pi)^6} \frac{\vec{p}_{B_1}^{(MB_1)*}}{m_{MB_1}} d\Omega_{B_1}^{(MB_1)*} \right) \left((2\pi)^3 dm_{MB_1}^2 \right)$$

$(MB_1B_2) \to (MB_1) + B_2$ $\quad$ $(MB_1) \to M + B_1$

where

$$\left| \vec{p}_{B_2}^{(MB_1B_2)*} \right| = \frac{\sqrt{\left(m_{MB_1B_2}^2 - (m_{MB_1} + m_{B_2})^2 \right) \left(m_{MB_1B_2}^2 - (m_{MB_1} - m_{B_2})^2 \right)}}{2m_{MB_1B_2}} \qquad \left| \vec{p}_{B_1}^{(MB_1)*} \right| = \frac{\sqrt{\left(m_{MB_1}^2 - (m_M + m_{B_1})^2 \right) \left(m_{MB_1} - (m_M - m_{B_1})^2 \right)}}{2m_{MB_1}}$$

Three-body decay width



$$\begin{pmatrix} M \\ B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} \pi \\ Y \\ N \end{pmatrix}, \begin{pmatrix} \bar{K} \\ N \\ N \end{pmatrix}$$

If we simply consider $\mathscr{M}$ as a coupling constant,

$$\boxed{\mathscr{M} = g^{\bar{K}NN}_{MB_1B_2}}$$

the decay width can be expressed as,

We need to integral over m_{MB_1}

$$\Gamma_{MB_1B_2}(m_{MB_1B_2}) = \frac{\left(g^{\bar{K}NN}_{MB_1B_2}\right)^2}{32\pi^3 m^2_{MB_1B_2}} \int \left| \vec{p}^{\,(MB_1B_2)*}_{B_2} \right| \left| \vec{p}^{\,(MB_1)*}_{B_1} \right| dm_{MB_1}$$

$$\begin{cases} = \dfrac{\left(g^{\bar{K}NN}_{MB_1B_2}\right)^2}{32\pi^3 m^2_{MB_1B_2}} \displaystyle\int_{m_M+m_{B_1}}^{m_{MB_1B_2}-m_{B_2}} \frac{\sqrt{\left(m^2_{MB_1B_2}-(m_{MB_1}+m_{B_2})^2\right)\left(m^2_{MB_1B_2}-(m_{MB_1}-m_{B_2})^2\right)}}{2m_{MB_1B_2}} \frac{\sqrt{\left(m^2_{MB_1}-(m_M+m_{B_1})^2\right)\left(m^2_{MB_1}-(m_M-m_{B_1})^2\right)}}{2m_{MB_1}} dm_{MB_1} & \text{(for } m_{MB_1B_2} \geq m_M+m_{B_1}+m_{B_2}\text{)} \\ = -\dfrac{\left(g^{\bar{K}NN}_{MB_1B_2}\right)^2}{32\pi^3 m^2_{MB_1B_2}} \displaystyle\int_{m_M+m_{B_1}}^{m_{MB_1B_2}-m_{B_2}} \frac{\sqrt{\left((m_{MB_1}+m_{B_2})^2-m^2_{MB_1B_2}\right)\left(m^2_{MB_1B_2}-(m_{MB_1}-m_{B_2})^2\right)}}{2m_{MB_1B_2}} \frac{\sqrt{\left((m_M+m_{B_1})^2-m^2_{MB_1}\right)\left(m^2_{MB_1}-(m_M-m_{B_1})^2\right)}}{2m_{MB_1}} dm_{MB_1} & \text{(for } m_{MB_1B_2} < m_M+m_{B_1}+m_{B_2}\text{)} \end{cases}$$

Not an imaginary, but a negative real number

We can also include resonances coupled to MB_1 or MB_2 channel by using a proper $\mathscr{M}$.

Total width of $\bar{K}NN$

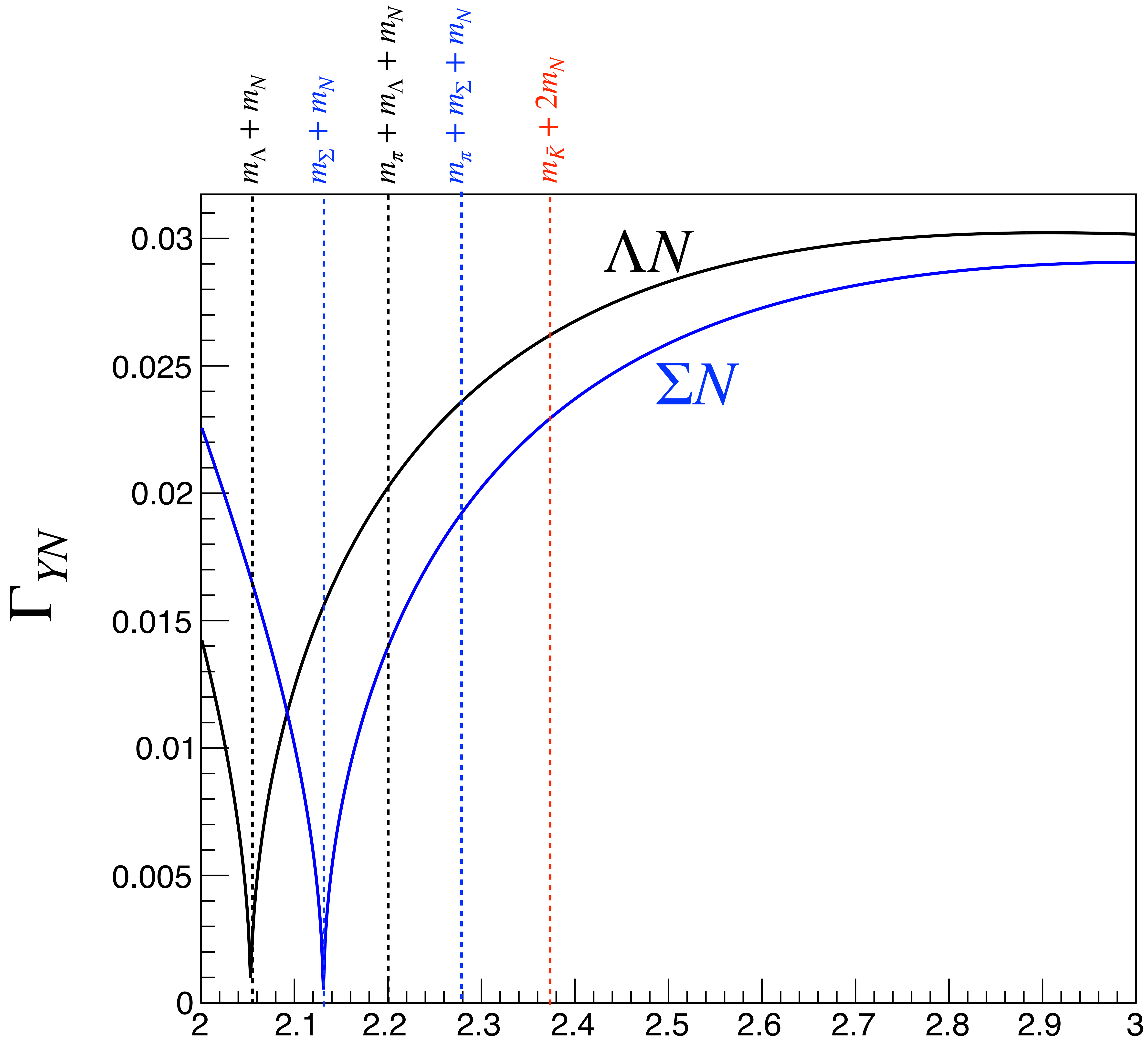
$\Gamma^{\bar{K}NN}_{tot}(m)$ is a sum of partial decay widths of all possible decay channels of the $\bar{K}NN$.

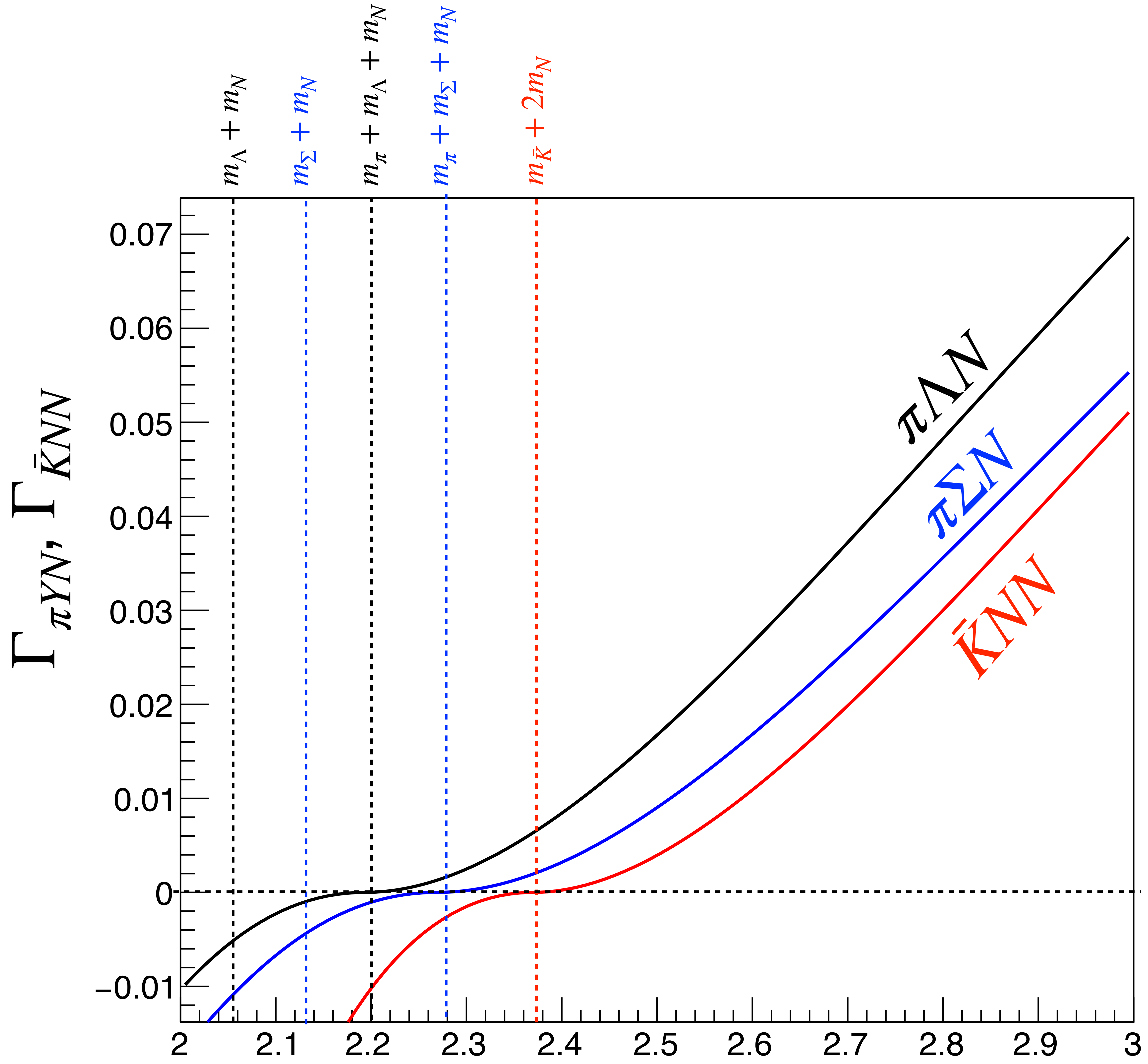
$$\Gamma^{\bar{K}NN}_{tot}(m) = \sum \Gamma^{\bar{K}NN}_{YN}(m) + \sum \Gamma^{\bar{K}NN}_{\pi YN}(m) + \sum \Gamma^{\bar{K}NN}_{\bar{K}NN}(m)$$

$$\Gamma_{YN}(m) = \begin{cases} \dfrac{\left(g^R_{YN}\right)^2}{8\pi m^2} \dfrac{\sqrt{\left(m^2 - (m_Y + m_N)^2\right)\left(m - (m_Y - m_N)^2\right)}}{2m} & \text{(for } m \geq m_Y + m_N) \\ i\dfrac{\left(g^R_{YN}\right)^2}{8\pi m^2} \dfrac{\sqrt{\left((m_Y + m_N)^2 - m^2\right)\left(m^2 - (m_Y - m_N)^2\right)}}{2m} & \text{(for } m < m_Y + m_N) \end{cases}$$

$$\Gamma_{MB_1B_2}(m) = \begin{cases} \dfrac{\left(g^{\bar{K}NN}_{MB_1B_2}\right)^2}{32\pi^3 m^2} \displaystyle\int_{m_M+m_{B_1}}^{m-m_{B_2}} \dfrac{\sqrt{\left(m^2 - (m_{MB_1} + m_{B_2})^2\right)\left(m^2 - (m_{MB_1} - m_{B_2})^2\right)}}{2m} \dfrac{\sqrt{\left(m^2_{MB_1} - (m_M + m_{B_1})^2\right)\left(m^2_{MB_1} - (m_M - m_{B_1})^2\right)}}{2m_{MB_1}} dm_{MB_1} & \text{(for } m \geq m_M + m_{B_1} + m_{B_2}) \\ \dfrac{\left(g^{\bar{K}NN}_{MB_1B_2}\right)^2}{32\pi^3 m^2} \displaystyle\int_{m_M+m_{B_1}}^{m-m_{B_2}} \dfrac{\sqrt{\left((m_{MB_1} + m_{B_2})^2 - m^2\right)\left(m^2 - (m_{MB_1} - m_{B_2})^2\right)}}{2m} \dfrac{\sqrt{\left((m_M + m_{B_1})^2 - m^2_{MB_1}\right)\left(m^2_{MB_1} - (m_M - m_{B_1})^2\right)}}{2m_{MB_1}} dm_{MB_1} & \text{(for } m < m_M + m_{B_1} + m_{B_2}) \end{cases}$$

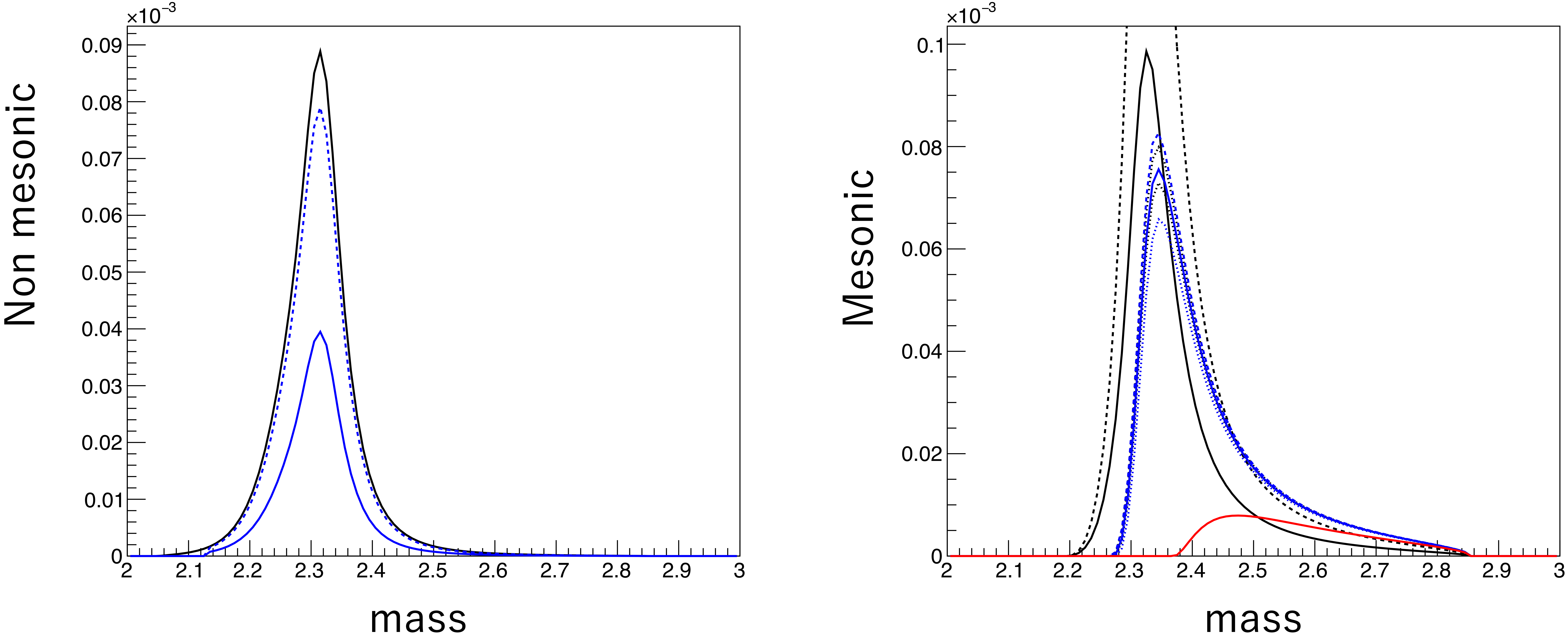
Partial widths of $\bar{K}NN$ vs. mass





Threshold effect (Phase space) of two- and three-body are much different!

Line shape for $\bar{K}NN$ including threshold effects



Three-body threshold effect seems to be smaller than that of two-body.

What we need to improve from E31 fit

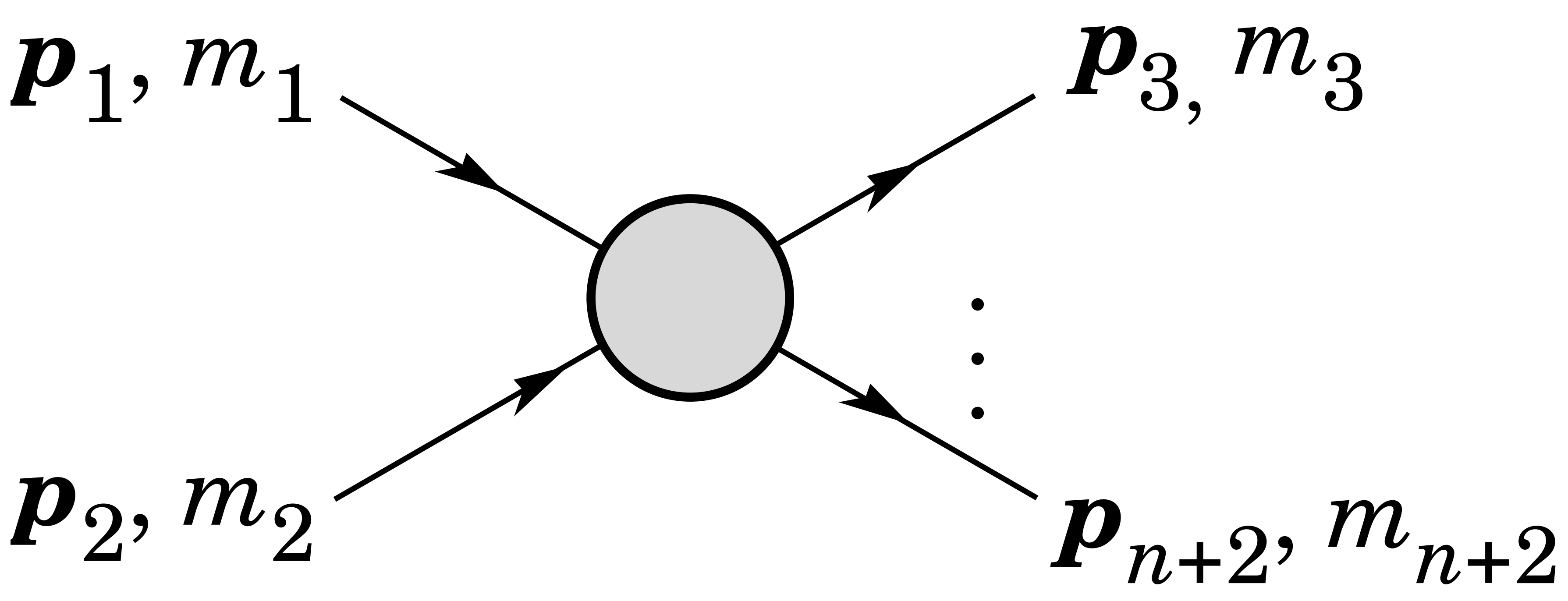
- To extend angular region
  - We need to extend F_{res} to higher θ_N region. :: Possible
  - We need introduce additional factor (or reaction, such as point-like) to modify $\cos\theta_N$ dependence. Otherwise, $\cos\theta_N$ dependence will be fixed by the elementary process. :: Need further consideration
- To properly treat three(or more)-body threshold effect
  - i.e. $\bar{K}NN$ threshold :: Possible

How to apply the model to mesonic decay?

Taken from PDG

$$\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2} = p_{1\mathrm{cm}}\sqrt{s}\ .$$

Cross sections (taken from PDG “Kinematics”)



$$d\sigma = \frac{(2\pi)^4 |\mathscr{M}|^2}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} \times d\Phi_n(p_1 + p_2;\ p_3, \ldots, p_{n+2})\ .$$

Amplitude

n-body phase space

It is very simple, just replacing $d\Phi_n$ properly.

Summary

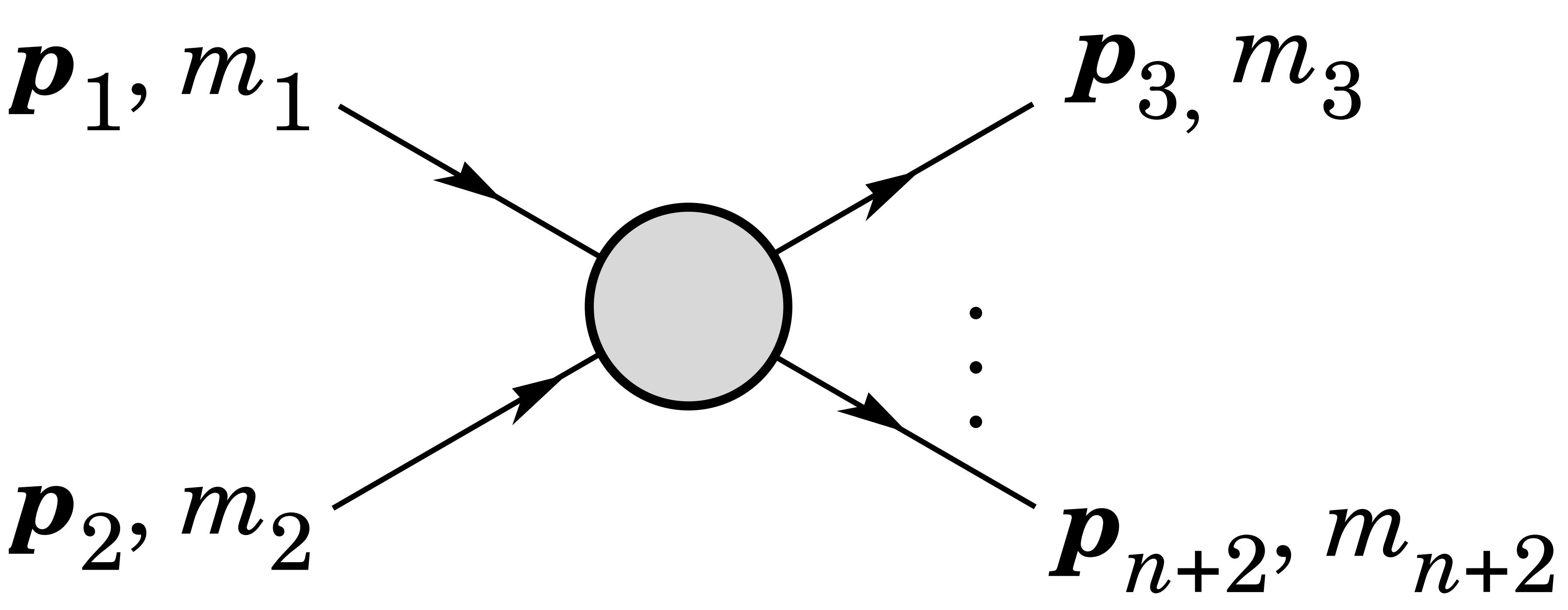
- We can introduce a new model functions including 2-step dynamics & proper threshold effects.
  - To apply it to four (or more)-body final state (i.e. mesonic decay channel) is simple.
- But, we need further consideration to apply it, particularly, how to treat q (or $\cos\theta_N$)-dependence.
  - This part is the most interesting and essential which would related to spatial structure & production mechanism!

Cross section & Decay

Starting from the cross section.

$$\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2} = p_{1\mathrm{cm}}\sqrt{s}\ .$$

Cross sections (taken from PDG "Kinematics")

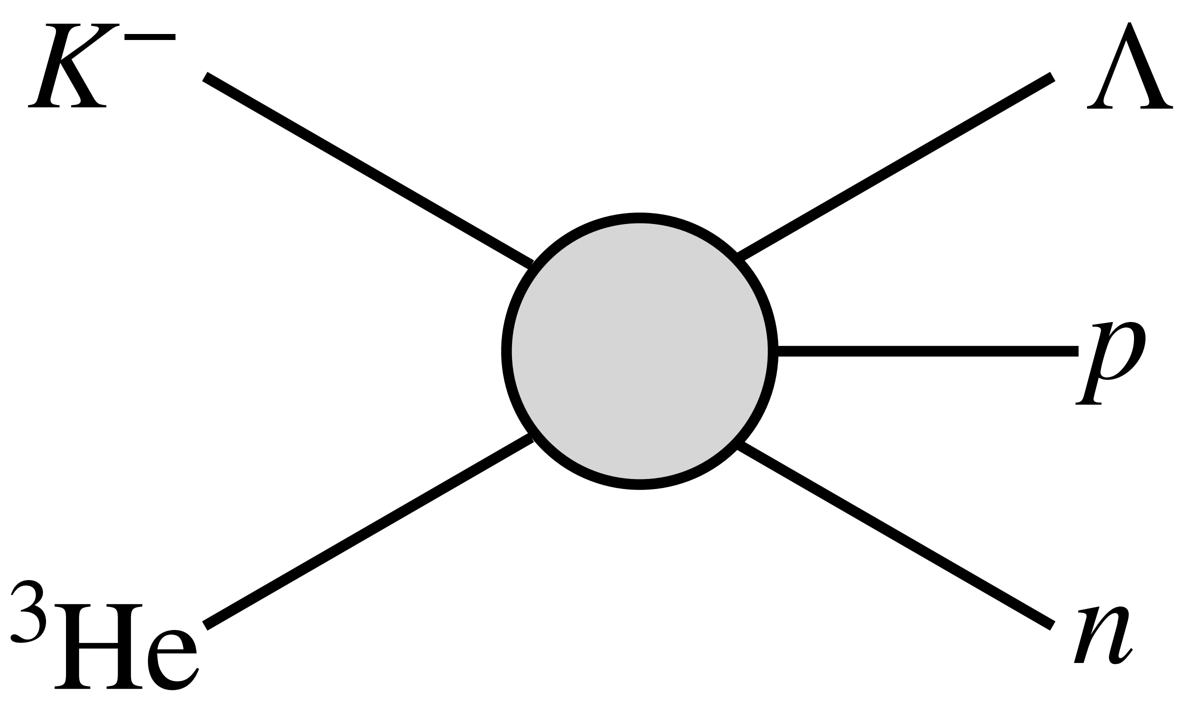


$$d\sigma = \frac{(2\pi)^4 |\mathcal{M}|^2}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} \times d\Phi_n(p_1 + p_2;\ p_3, \ldots, p_{n+2})\ .$$

Could be $\bar{K}NN$ amplitude

n-body phase space

Cross section & Decay



The cross section can be expressed as,

$$d\sigma_{\Lambda pn} = \frac{(2\pi)^4}{4p^*_{K^-}\sqrt{s}} \quad \mathscr{M}_{\Lambda pn} \quad ^2 \times d\Phi_{\Lambda pn}$$

where $d\Phi_{\Lambda pn}$ is the three-body phase space of the Λpn final state,

$$d\Phi_{\Lambda pn} = \left(\frac{1}{4\,(2\pi)^6} \frac{\vec{p}^*_n}{\sqrt{s}}\, d\Omega^*_n \right) \left(\frac{1}{4\,(2\pi)^6} \frac{\vec{p}^{(\Lambda p)*}_\Lambda}{m_{\Lambda p}}\, d\Omega^{(\Lambda p)*}_\Lambda \right) \left((2\pi)^3 dm^2_{\Lambda p} \right)$$

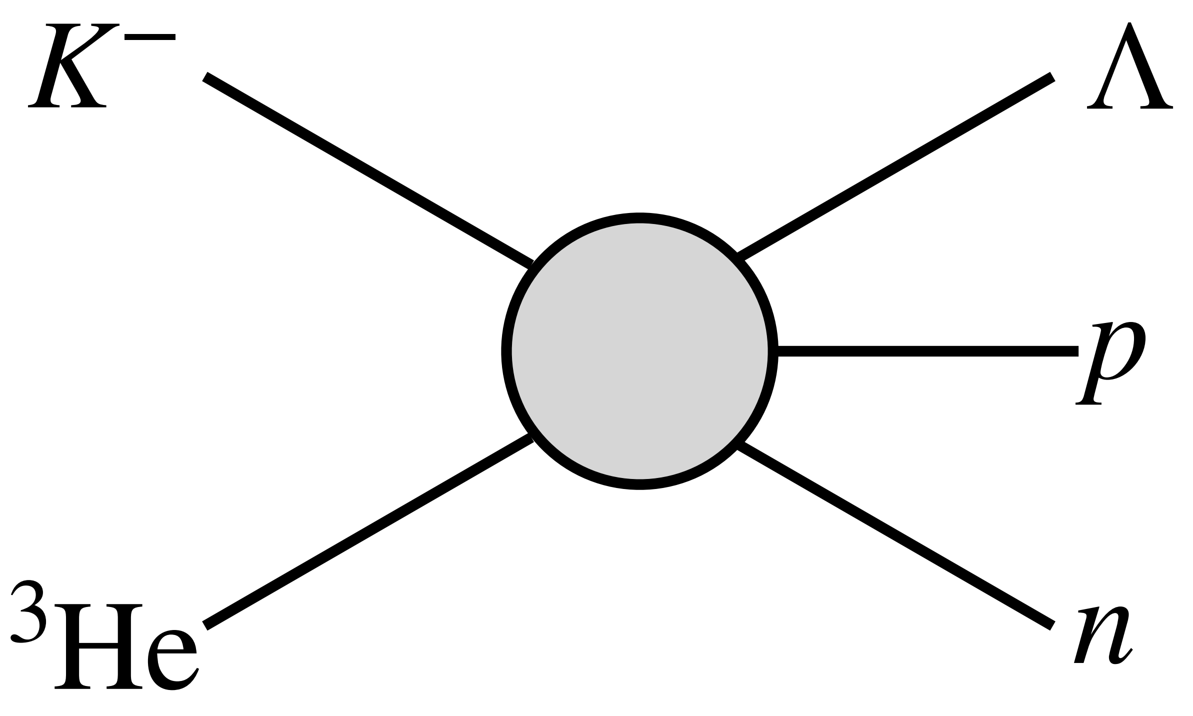
$p^*_n(\Omega^*_n)$ and $p^{(\Lambda p)*}_\Lambda(\Omega^{(\Lambda p)*}_\Lambda)$ are momenta (angles) of n and Λ in the $K^-{}^3\mathrm{He}$-c.m. and (Λp)-c.m. frame, respectively, as,

$$\left|\vec{p}^*_n\right| = \frac{\sqrt{\left(s-(m_{\Lambda p}+m_n)^2\right)\left(s-(m_{\Lambda p}-m_n)^2\right)}}{2\sqrt{s}} \qquad \left|\vec{p}^{(\Lambda p)*}_\Lambda\right| = \frac{\sqrt{\left(m^2_{\Lambda p}-(m_\Lambda+m_p)^2\right)\left(m_{\Lambda p}-(m_\Lambda-m_p)^2\right)}}{2m_{\Lambda p}}$$

We can integrate over $\Omega^{(\Lambda p)*}_\Lambda$ and ϕ^*_n by assuming uniform distribution. By using $dm^2_{\Lambda p} = 2m_{\Lambda p}dm_{\Lambda p}$, $d\Phi_{\Lambda pn}$ is as,

$$d\Phi_{\Lambda pn} = \frac{1}{4(2\pi)^7\sqrt{s}} \left|\vec{p}^*_n\right| \left|\vec{p}^{(\Lambda p)*}_\Lambda\right| dm_{\Lambda p}\, d\cos\theta^*_n$$

Cross section & Decay



By combining following two,

$$d\sigma_{\Lambda pn} = \frac{(2\pi)^4}{4p^*_{K^-}\sqrt{s}} \left|\mathscr{M}_{\Lambda pn}\right|^2 \times d\Phi_{\Lambda pn}$$

$$d\Phi_{\Lambda pn} = \frac{1}{4(2\pi)^7\sqrt{s}} \left|\vec{p}^*_n\right| \left|\vec{p}^{(\Lambda p)*}_\Lambda\right| dm_{\Lambda p}\, d\cos\theta^*_n$$

the double differential cross section of the $K^{-3}\mathrm{He} \to \Lambda pn$ reaction can be expressed as,

$$\frac{d\sigma_{\Lambda pn}}{dm_{\Lambda p}\, d\cos\theta^*_n} = \frac{1}{16(2\pi)^3}\frac{1}{p^*_{K^-}\, s} \left|\vec{p}^*_n\right| \left|\vec{p}^{(\Lambda p)*}_\Lambda\right| \left|\mathscr{M}_{\Lambda pn}\right|^2$$

If we consider the $\bar{K}NN_{I_3=+1/2}$ production decaying into Λp-pair with the Breit-Wigner parametrization, $\mathscr{M}_{\Lambda pn}$ can be expressed as,

$$\mathscr{M}_{\Lambda pn} = \frac{g^{\bar{K}NN}_{\Lambda p}}{M^2_{\bar{K}NN} - m^2_{\Lambda p} - iM_{\bar{K}NN}\Gamma^{\bar{K}NN}_{tot}} \cdot \mathscr{A}\left(\cos\theta^*_n\right)$$

$$\mathscr{M}_{\Lambda pn} = BW$$

where $g^{\bar{K}NN}_{\Lambda p}$ is a coupling constant of the $\bar{K}NN$ to Λp channel, $M_{\bar{K}NN}$ is the Breit-Wigner mass of the $\bar{K}NN$, $\Gamma^{\bar{K}NN}_{tot} = \Gamma^{\bar{K}NN}_{tot}(m)$ is the total decay width of the $\bar{K}NN$,

and$\mathscr{A}(\cos\theta^*_n)$ demonstrates an angular dependence of the $\bar{K}NN$ production.

$\Gamma_{tot}(m)$ is a sum of partial decay widths of all possible decay channels of the $\bar{K}NN$.

$$\Gamma^{\bar{K}NN}_{tot}(m) = \sum \Gamma_{YN}(m) + \sum \Gamma_{\pi YN}(m) + \sum \Gamma_{\bar{K}NN}(m)$$

Cross section & Decay

$$\mathscr{M}_{\Lambda pn} = \left\langle \Lambda pn \middle| T_{\Lambda pn} \middle| K^{-3}\mathrm{He} \right\rangle = \left\langle \Lambda pn \middle| \hat{T}^{(1/2)}_{\Lambda N} \middle| \Lambda NN' \right\rangle \left\langle \Lambda NN' \middle| \hat{T}^{(0)}_{\Lambda NN'} + \hat{T}^{(1)}_{\Lambda NN'} \middle| K^{-3}\mathrm{He} \right\rangle$$

- $T^{(I_{\Lambda NN'})}_{\Lambda NN'}$: Transition operator to the $\Lambda NN'$ final state in the isospin $I_{\Lambda NN'}$ channel
- $T^{(I_{\Lambda N})}_{\Lambda N}$: Transition operator to the ΛN channel in the isospin $I_{\Lambda N}$ channel

$$\left| K^{-3}\mathrm{He} \right\rangle = \sqrt{\frac{1}{2}} \underbrace{\left(\frac{\left| K^{-3}\mathrm{He} \right\rangle + \left| {}^{3}\mathrm{He}K^{-} \right\rangle}{\sqrt{2}} \right)}_{I=1} - \sqrt{\frac{1}{2}} \underbrace{\left(\frac{-\left| K^{-3}\mathrm{He} \right\rangle + \left| {}^{3}\mathrm{He}K^{-} \right\rangle}{\sqrt{2}} \right)}_{I=0}$$

$$\left| \Lambda pn \right\rangle = \sqrt{\frac{1}{2}} \underbrace{\left(\frac{\left| \Lambda pn \right\rangle + \left| \Lambda np \right\rangle}{\sqrt{2}} \right)}_{I=1} + \sqrt{\frac{1}{2}} \underbrace{\left(\frac{\left| \Lambda pn \right\rangle - \left| \Lambda np \right\rangle}{\sqrt{2}} \right)}_{I=0}$$

$$t^{(I)}_{\Lambda NN'} = \left\langle \Lambda NN' \middle| \hat{T}^{(I)}_{\Lambda NN'} \middle| K^{-3}\mathrm{He} \right\rangle \qquad t^{(I)}_{\Lambda N} = \left\langle \Lambda pn \middle| \hat{T}^{(I)}_{\Lambda N} \middle| \Lambda NN' \right\rangle$$

$$\Rightarrow \mathscr{M}_{\Lambda pn} = \frac{1}{2} t^{(1)}_{\Lambda NN'} \left(\frac{1}{2} t^{(1/2)}_{\Lambda p} + \frac{1}{2} t^{(1/2)}_{\Lambda n} \right) - \frac{1}{2} t^{(0)}_{\Lambda NN'} \left(\frac{1}{2} t^{(1/2)}_{\Lambda p} - \frac{1}{2} t^{(1/2)}_{\Lambda n} \right)$$

$$= \frac{1}{4} t^{(1/2)}_{\Lambda p} \left(t^{(1)}_{\Lambda NN'} - t^{(0)}_{\Lambda NN'} \right) + \frac{1}{4} t^{(1/2)}_{\Lambda n} \left(t^{(1)}_{\Lambda NN'} + t^{(0)}_{\Lambda NN'} \right)$$

“$K^- pp$” “$\bar{K}^0 pn$”

Cross section & Decay

$\Gamma_{tot}^{\bar{K}NN}(m)$ is a sum of partial decay widths of all possible decay channels of the $\bar{K}NN$.

$$\Gamma_{tot}^{\bar{K}NN}(m) = \sum \Gamma_{YN}^{\bar{K}NN}(m) + \sum \Gamma_{\pi YN}^{\bar{K}NN}(m) + \sum \Gamma_{\bar{K}NN}^{\bar{K}NN}(m)$$

where $\Gamma_{YN}^{\bar{K}NN}(m)$ is non-mesonic two-body decay into YN channels, $\Gamma_{\pi YN}^{\bar{K}NN}(m)$ and $\Gamma_{\bar{K}NN}^{\bar{K}NN}(m)$ are mesonic three-body decay into πYN and $\bar{K}NN$ channels, respectively. All possible decay channels of the $\bar{K}NN_{I_3=+1/2}$ are,

- Λp
- $\Sigma^0 p$
- $\Sigma^+ n$

- $\pi^0 \Lambda p$
- $\pi^0 \Sigma^0 p$
- $\pi^+ \Lambda n$
- $\pi^- \Sigma^+ p$
- $\pi^0 \Sigma^+ n$
- $\pi^+ \Sigma^- p$
- $\pi^+ \Sigma^0 n$

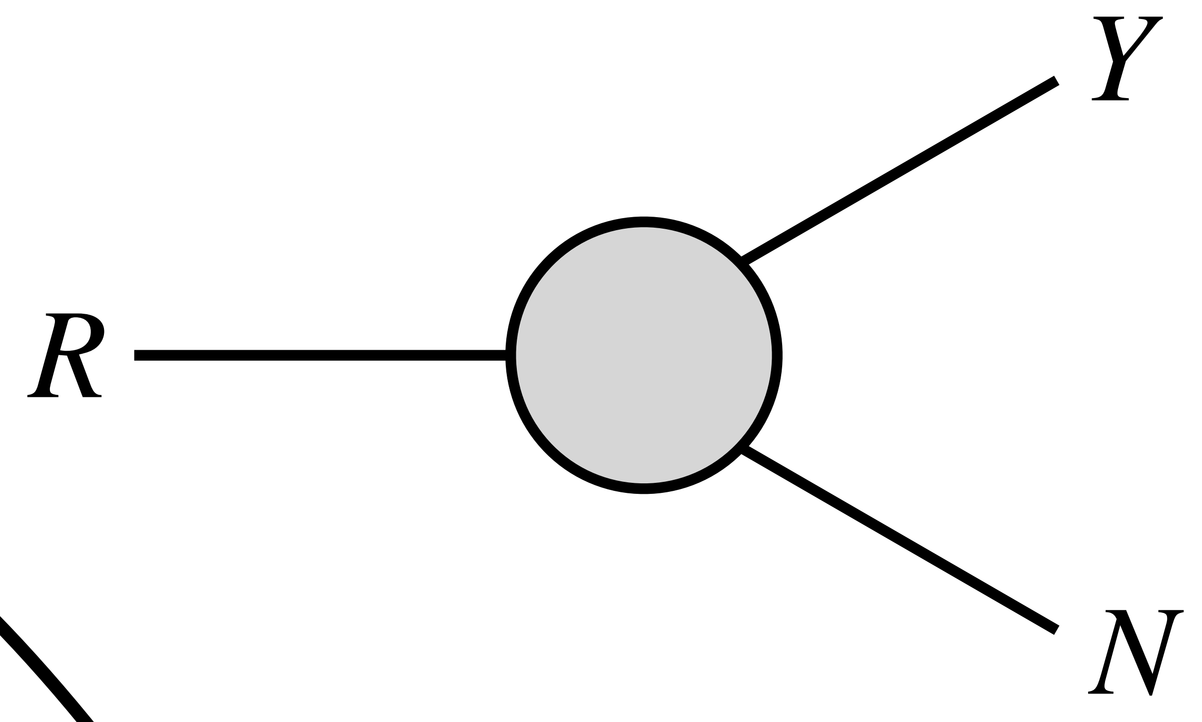
- $K^- pp$
- $\bar{K}^0 pn$

Partial decay widths can be obtained from the following equation,

Decay (taken from PDG "Kinematics")

$$d\Gamma = \frac{(2\pi)^4}{2M} |\mathscr{M}|^2 \, d\Phi_n \, (P; \; p_1, \, \ldots, \, p_n),$$

Cross section & Decay



The non-mesonic two-body decay widths Γ_{YN} can be expressed as,

$$d\Gamma_{YN}(m_{YN}) = \frac{(2\pi)^4}{2m_{YN}} \mathscr{M}^{\,2} \Phi_2 = \frac{(2\pi)^4}{2m_{YN}} \mathscr{M}^{\,2} \frac{1}{4(2\pi)^6} \frac{\vec{p}_Y^{\,(YN)*}}{m_{YN}} d\Omega_Y^{(YN)*}$$

$$\left|\vec{p}_Y^{\,(YN)*}\right| = \frac{\sqrt{\left(m_{YN}^2 - (m_Y + m_N)^2\right)\left(m_{YN} - (m_Y - m_N)^2\right)}}{2m_{YN}}$$

If we consider the amplitude $\mathscr{M}$ as a coupling constant to the YN channel,

$$\mathscr{M} = g_{YN}^{\bar{K}NN}$$

This $\mathscr{M}$ is a amplitude for the decay.
Not the same as the previous one!!

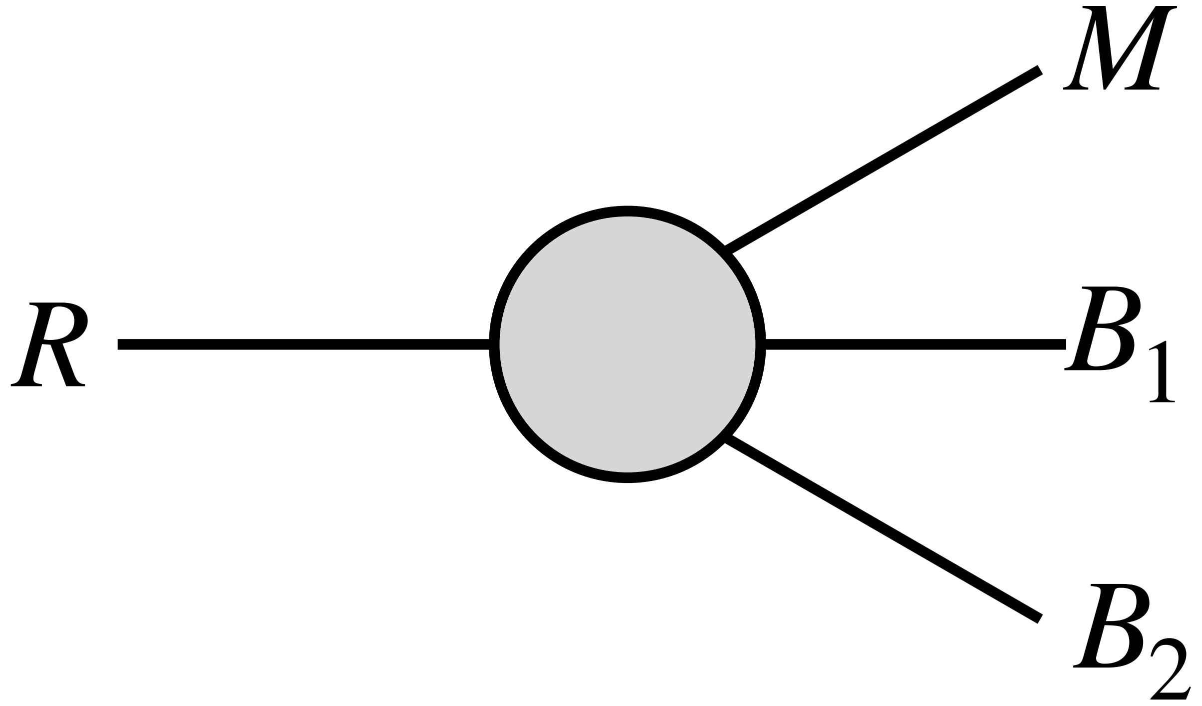
We can integrate over $\Omega_Y^{(YN)*}$, then,

$$\Gamma_{YN}(m_{YN}) = \frac{\left(g_{YN}^{\bar{K}NN}\right)^2}{8\pi m_{YN}^2} \vec{p}_Y^{\,(YN)*} = \frac{\left(g_{YN}^{\bar{K}NN}\right)^2}{8\pi m_{YN}^2} \frac{\sqrt{\left(m_{YN}^2 - (m_Y + m_N)^2\right)\left(m_{YN}^2 - (m_Y - m_N)^2\right)}}{2m_{YN}}$$

This expression is allowed only for above the $m_Y + m_N$ threshold, but we can expand it below the threshold by the Flatte parametrization as,

$$\Gamma_{YN}(m_{YN}) = \begin{cases} \frac{\left(g_{YN}^R\right)^2}{8\pi m_{YN}^2} \frac{\sqrt{\left(m_{YN}^2 - (m_Y + m_N)^2\right)\left(m_{YN}^2 - (m_Y - m_N)^2\right)}}{2m_{YN}} & (\text{for } m_{YN} \geq m_Y + m_N) \\ i\frac{\left(g_{YN}^R\right)^2}{8\pi m_{YN}^2} \frac{\sqrt{\left((m_Y + m_N)^2 - m_{YN}^2\right)\left(m_{YN}^2 - (m_Y - m_N)^2\right)}}{2m_{YN}} & (\text{for } m_{YN} < m_Y + m_N) \end{cases}$$

Cross section & Decay



$$\begin{pmatrix} M \\ B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} \pi \\ Y \\ N \end{pmatrix}, \begin{pmatrix} \bar{K} \\ N \\ N \end{pmatrix}$$

The mesonic three-body decay widths $\Gamma_{MB_1B_2}$ can be expressed as,

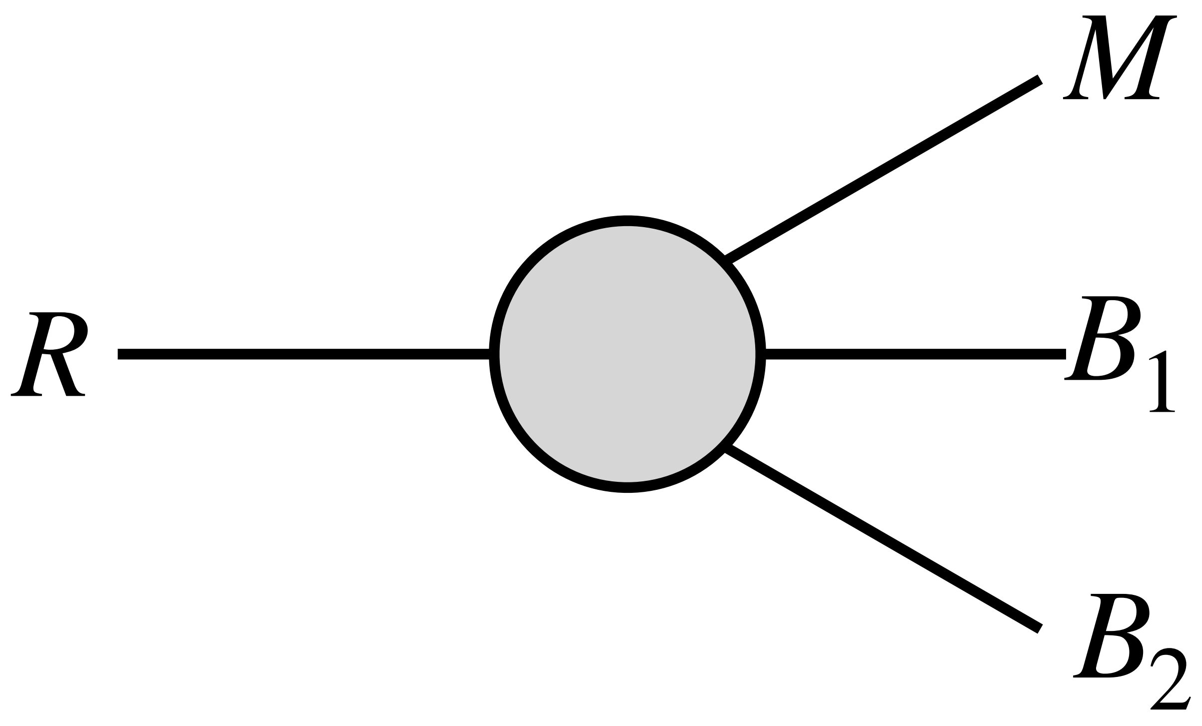
$$d\Gamma_{MB_1B_2}(m_{MB_1B_2}) = \frac{(2\pi)^4}{2m_{MB_1B_2}} \mathscr{M}^{2} \Phi_3$$

$$= \frac{(2\pi)^4}{2m_{MB_1B_2}} \mathscr{M}^{2} \left(\frac{1}{4(2\pi)^6} \frac{\vec{p}_{B_2}^{(MB_1B_2)*}}{m_{MB_1B_2}} d\Omega_{B_2}^{(MB_1B_2)*} \right) \left(\frac{1}{4(2\pi)^6} \frac{\vec{p}_{B_1}^{(MB_1)*}}{m_{MB_1}} d\Omega_{B_1}^{(MB_1)*} \right) \left((2\pi)^3 dm_{MB_1}^2 \right)$$

where

$$\left| \vec{p}_{B_2}^{(MB_1B_2)*} \right| = \frac{\sqrt{\left(m_{MB_1B_2}^2 - (m_{MB_1} + m_{B_2})^2\right)\left(m_{MB_1B_2}^2 - (m_{MB_1} - m_{B_2})^2\right)}}{2m_{MB_1B_2}} \qquad \left| \vec{p}_{B_1}^{(MB_1)*} \right| = \frac{\sqrt{\left(m_{MB_1}^2 - (m_M + m_{B_1})^2\right)\left(m_{MB_1} - (m_M - m_{B_1})^2\right)}}{2m_{MB_1}}$$

Cross section & Decay



$$\begin{pmatrix} M \\ B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} \pi \\ Y \\ N \end{pmatrix}, \begin{pmatrix} \bar{K} \\ N \\ N \end{pmatrix}$$

If we simply consider $\mathscr{M}$ as a coupling constant,

$$\boxed{\mathscr{M} = g^{\bar{K}NN}_{MB_1B_2}}$$

the decay width can be expressed as,

$$\Gamma_{MB_1B_2}(m_{MB_1B_2}) = \frac{\left(g^{\bar{K}NN}_{MB_1B_2}\right)^2}{32\pi^3 m^2_{MB_1B_2}} \int \left|\vec{p}^{\,(MB_1B_2)*}_{B_2}\right| \left|\vec{p}^{\,(MB_1)*}_{B_1}\right| dm_{MB_1}$$

$$\begin{cases} = \dfrac{\left(g^{\bar{K}NN}_{MB_1B_2}\right)^2}{32\pi^3 m^2_{MB_1B_2}} \displaystyle\int_{m_M+m_{B_1}}^{m_{MB_1B_2}-m_{B_2}} \frac{\sqrt{\left(m^2_{MB_1B_2}-(m_{MB_1}+m_{B_2})^2\right)\left(m^2_{MB_1B_2}-(m_{MB_1}-m_{B_2})^2\right)}}{2m_{MB_1B_2}} \frac{\sqrt{\left(m^2_{MB_1}-(m_M+m_{B_1})^2\right)\left(m^2_{MB_1}-(m_M-m_{B_1})^2\right)}}{2m_{MB_1}} dm_{MB_1} \quad \text{(for } m_{MB_1B_2} \geq m_M + m_{B_1} + m_{B_2}\text{)} \\ = -\dfrac{\left(g^{\bar{K}NN}_{MB_1B_2}\right)^2}{32\pi^3 m^2_{MB_1B_2}} \displaystyle\int_{m_M+m_{B_1}}^{m_{MB_1B_2}-m_{B_2}} \frac{\sqrt{\left((m_{MB_1}+m_{B_2})^2-m^2_{MB_1B_2}\right)\left(m^2_{MB_1B_2}-(m_{MB_1}-m_{B_2})^2\right)}}{2m_{MB_1B_2}} \frac{\sqrt{\left((m_M+m_{B_1})^2-m^2_{MB_1}\right)\left(m^2_{MB_1}-(m_M-m_{B_1})^2\right)}}{2m_{MB_1}} dm_{MB_1} \quad \text{(for } m_{MB_1B_2} < m_M + m_{B_1} + m_{B_2}\text{)} \end{cases}$$

We can also include resonances coupled to MB_1 or MB_2 channel by using a proper $\mathscr{M}$.

Cross section & Decay

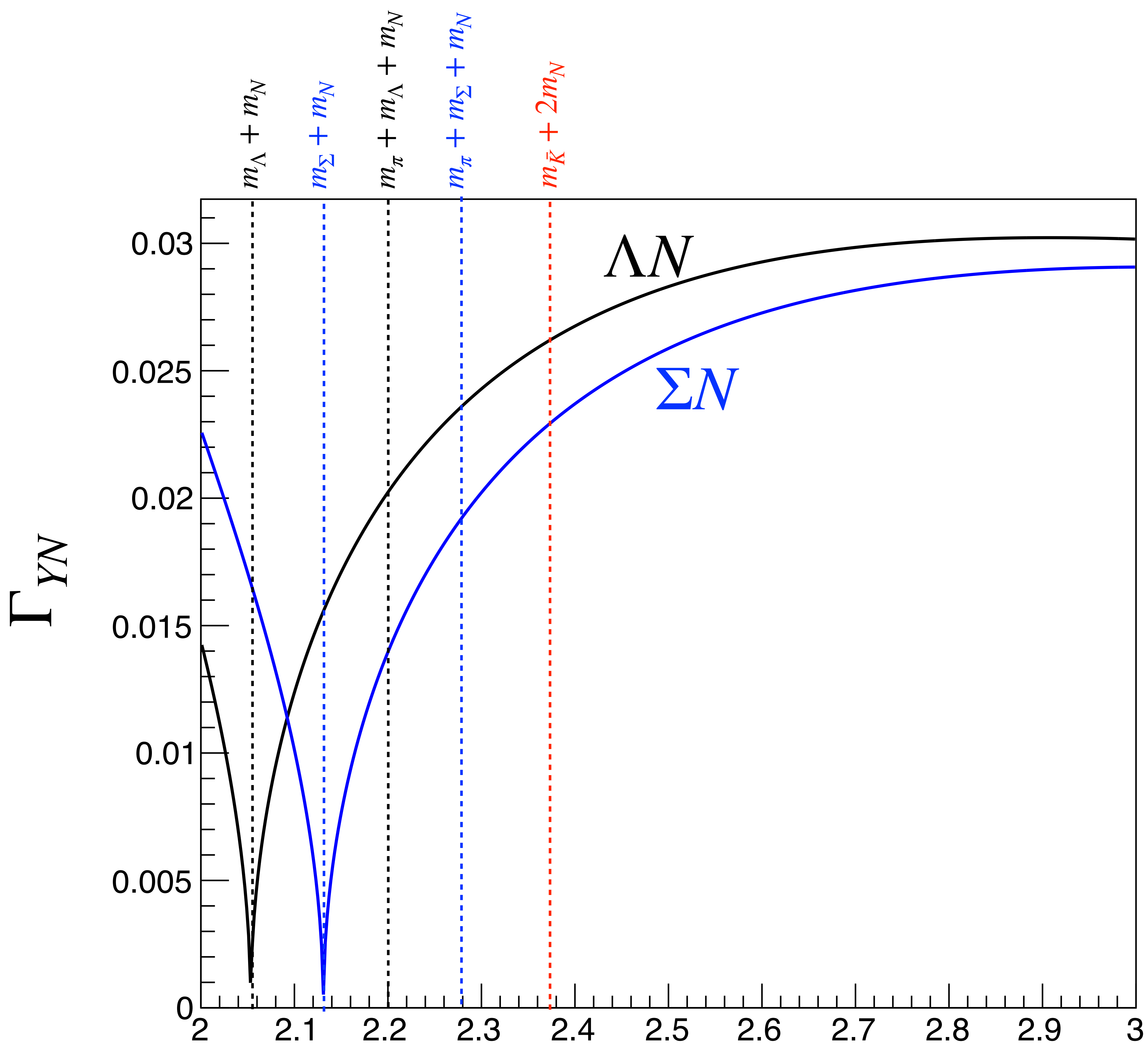
$\Gamma_{tot}^{\bar{K}NN}(m)$ is a sum of partial decay widths of all possible decay channels of the $\bar{K}NN$.

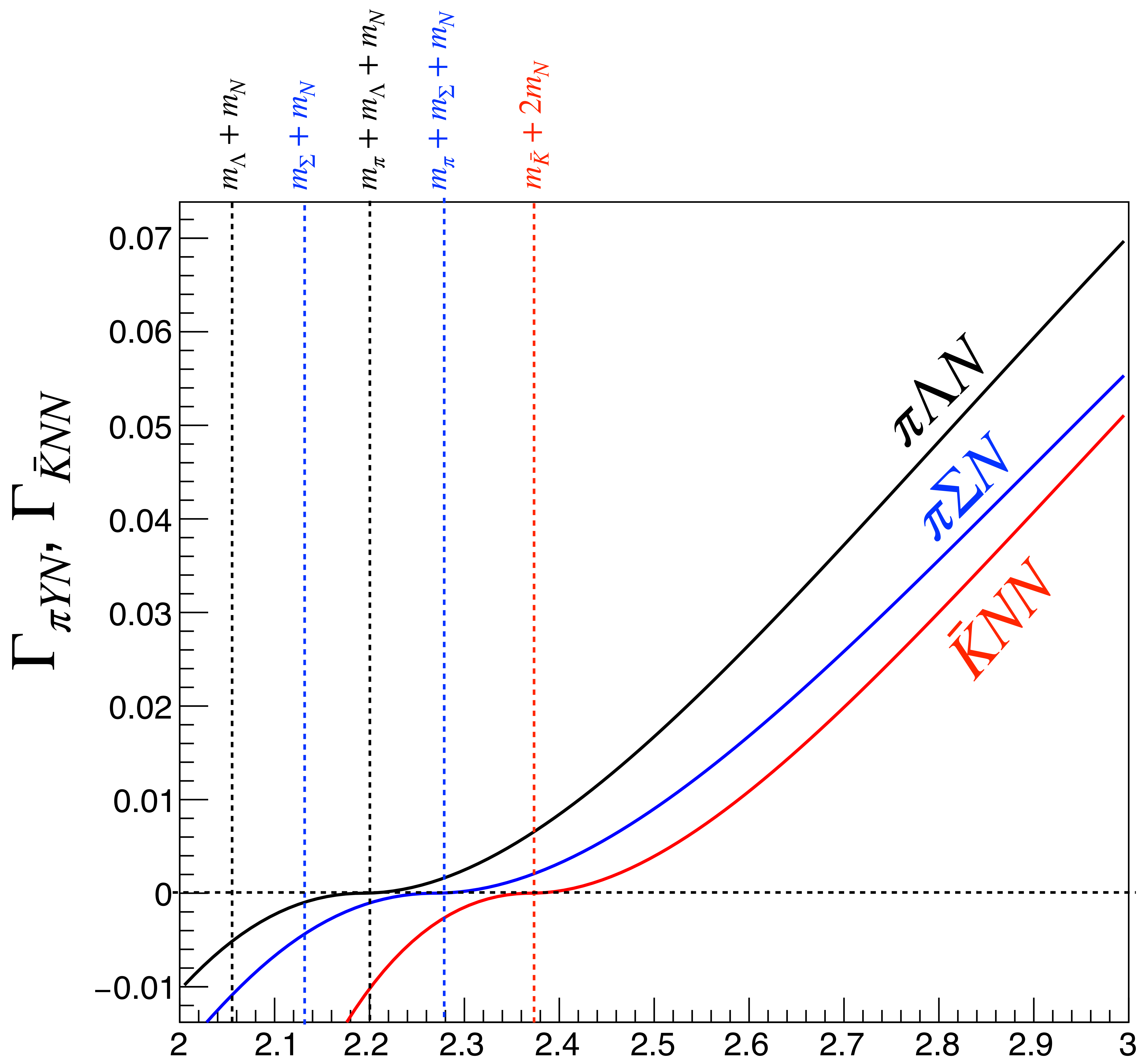
$$\Gamma_{tot}^{\bar{K}NN}(m) = \sum \Gamma_{YN}^{\bar{K}NN}(m) + \sum \Gamma_{\pi YN}^{\bar{K}NN}(m) + \sum \Gamma_{\bar{K}NN}^{\bar{K}NN}(m)$$

$$\Gamma_{YN}(m) = \begin{cases} \dfrac{\left(g_{YN}^{R}\right)^2}{8\pi m^2} \dfrac{\sqrt{\left(m^2-(m_Y+m_N)^2\right)\left(m-(m_Y-m_N)^2\right)}}{2m} & \text{(for } m \geq m_Y + m_N\text{)} \\ i\dfrac{\left(g_{YN}^{R}\right)^2}{8\pi m^2} \dfrac{\sqrt{\left((m_Y+m_N)^2-m^2\right)\left(m^2-(m_Y-m_N)^2\right)}}{2m} & \text{(for } m < m_Y + m_N\text{)} \end{cases}$$

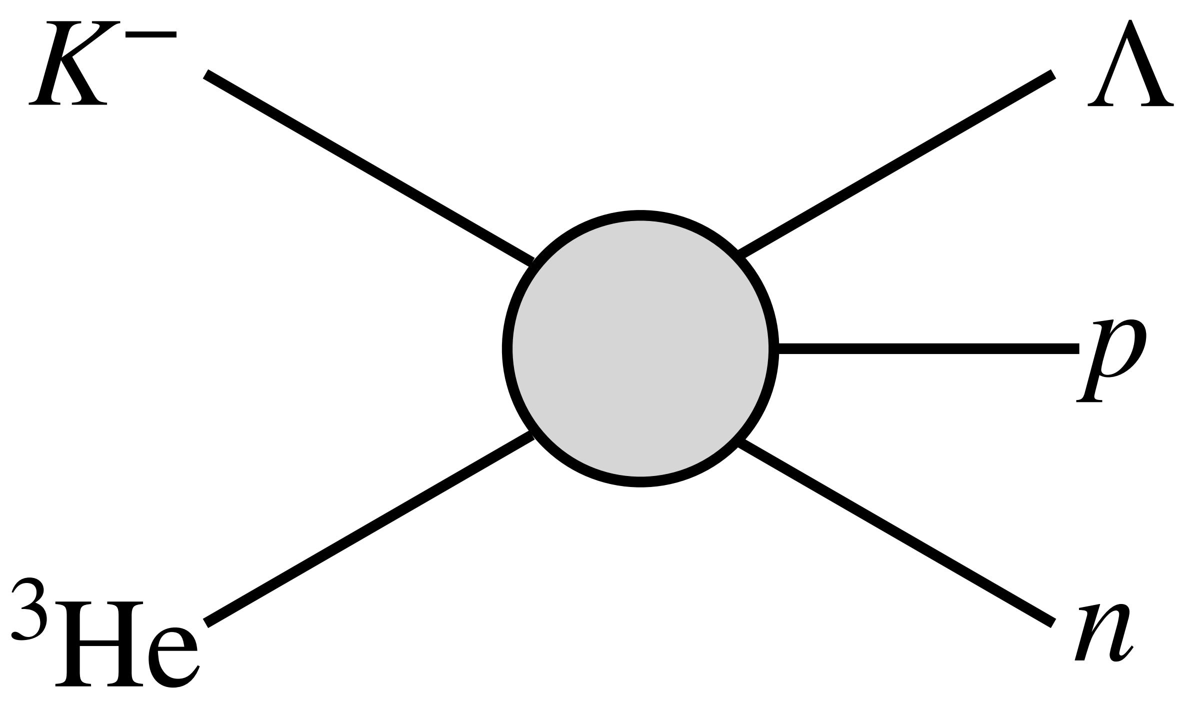
$$\Gamma_{MB_1B_2}(m) = \begin{cases} \dfrac{\left(g_{MB_1B_2}^{\bar{K}NN}\right)^2}{32\pi^3 m^2} \displaystyle\int_{m_M+m_{B_1}}^{m-m_{B_2}} \dfrac{\sqrt{\left(m^2-(m_{MB_1}+m_{B_2})^2\right)\left(m^2-(m_{MB_1}-m_{B_2})^2\right)}}{2m} \dfrac{\sqrt{\left(m_{MB_1}^2-(m_M+m_{B_1})^2\right)\left(m_{MB_1}^2-(m_M-m_{B_1})^2\right)}}{2m_{MB_1}} dm_{MB_1} & \text{(for } m \geq m_M + m_{B_1} + m_{B_2}\text{)} \\ \dfrac{\left(g_{MB_1B_2}^{\bar{K}NN}\right)^2}{32\pi^3 m^2} \displaystyle\int_{m_M+m_{B_1}}^{m-m_{B_2}} \dfrac{\sqrt{\left((m_{MB_1}+m_{B_2})^2-m^2\right)\left(m^2-(m_{MB_1}-m_{B_2})^2\right)}}{2m} \dfrac{\sqrt{\left((m_M+m_{B_1})^2-m_{MB_1}^2\right)\left(m_{MB_1}^2-(m_M-m_{B_1})^2\right)}}{2m_{MB_1}} dm_{MB_1} & \text{(for } m < m_M + m_{B_1} + m_{B_2}\text{)} \end{cases}$$

Cross section & Decay





Cross section & Decay



the double differential cross section of the $K^{-3}\mathrm{He} \to \Lambda pn$ reaction can be expressed as,

$$\frac{d\sigma_{\Lambda pn}}{dm_{\Lambda p}\, d\cos\theta_n^*} = \frac{1}{16(2\pi)^3}\frac{1}{p_{K^-}^* s}\left|\vec{p}_n^*\right|\left|\vec{p}_\Lambda^{(\Lambda p)*}\right|\left|\mathscr{M}_{\Lambda pn}\right|^2$$

$$\mathscr{M}_{\Lambda pn} = \frac{g_{\Lambda p}^{\bar{K}NN}}{M_{\bar{K}NN}^2 - m_{\Lambda p}^2 - iM_{\bar{K}NN}\Gamma_{tot}^{\bar{K}NN}} \cdot \mathscr{A}\left(\cos\theta_n^*\right)$$

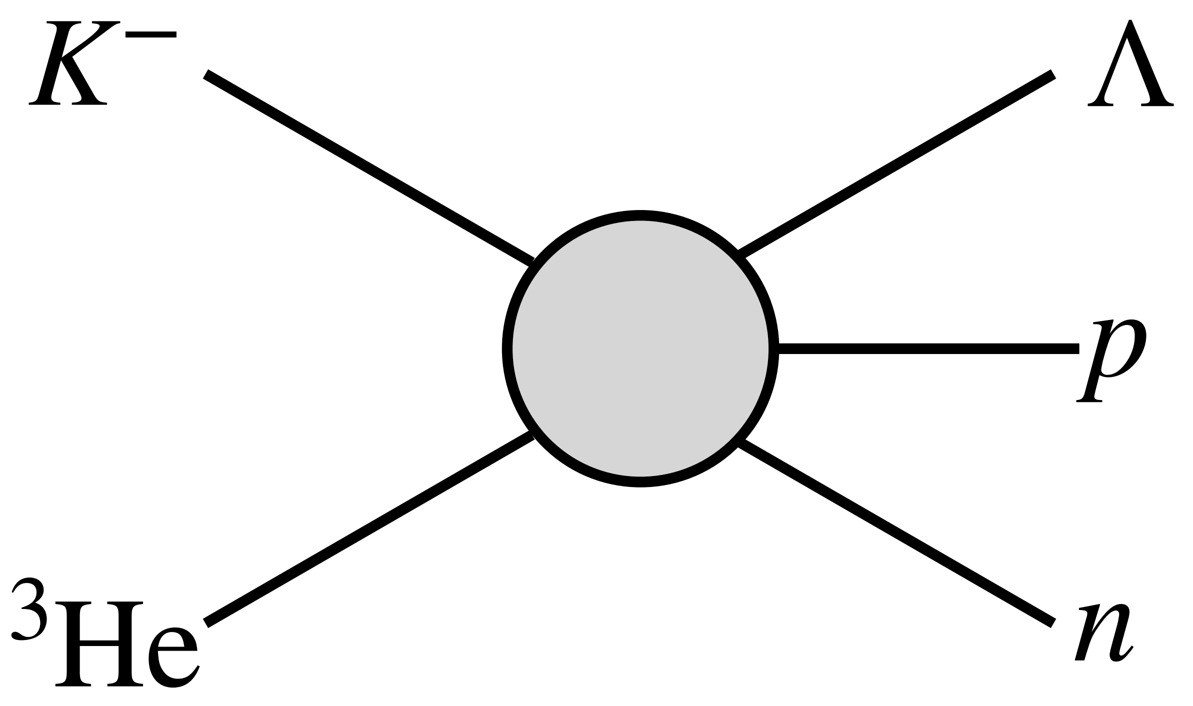
$$= \frac{1}{16(2\pi)^3}\frac{1}{p_{K^-}^* s}\left|\vec{p}_n^*\right|\left|\vec{p}_\Lambda^{(\Lambda p)*}\right|\left|\frac{g_{\Lambda p}^{\bar{K}NN}}{M_{\bar{K}NN}^2 - m_{\Lambda p}^2 - iM_{\bar{K}NN}\Gamma_{tot}^{\bar{K}NN}}\right|^2\left|\mathscr{A}\left(\cos\theta_n^*\right)\right|^2$$

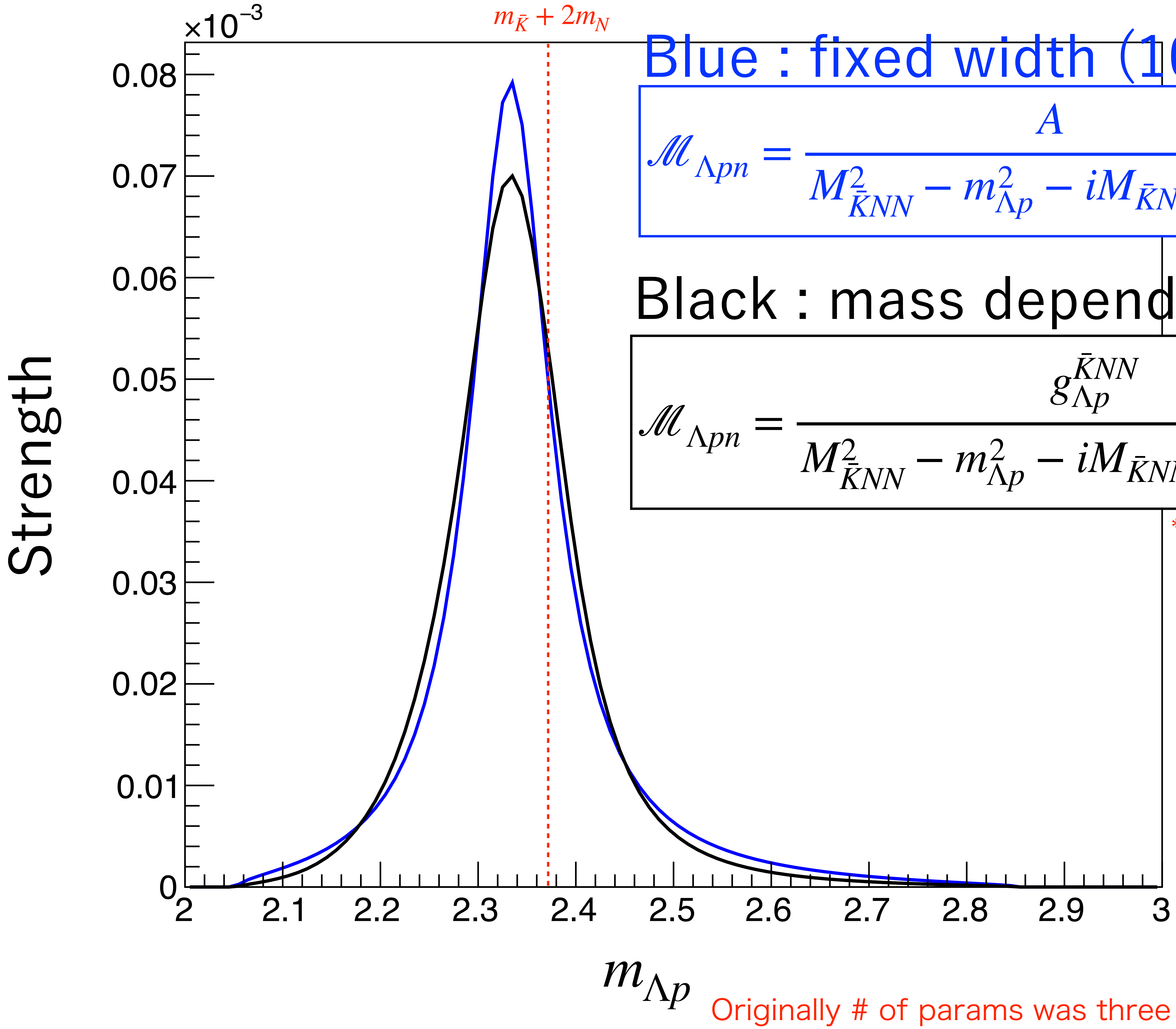
$$\Gamma_{tot}^{\bar{K}NN}(m) = \sum \Gamma_{YN}(m) + \sum \Gamma_{\pi YN}(m) + \sum \Gamma_{\bar{K}NN}(m)$$

If $m_{\bar{K}NN} = m_{\bar{K}} + 2m_N - 40\ \mathrm{MeV}$ and $\Gamma_{tot}^{\bar{K}NN} = 100\ \mathrm{MeV}$ (fixed), then line shape is (almost) the same as that PRC.

*In PRC, non-relativistic Breit-Wigner was used.

Cross section & Decay





Blue : fixed width (100 MeV)

$$\mathscr{M}_{\Lambda pn} = \frac{A}{M^2_{\bar{K}NN} - m^2_{\Lambda p} - iM_{\bar{K}NN}\Gamma_{\bar{K}NN}} \cdot \mathscr{A}\left(\cos\theta^*_n\right)$$

<-The same as PRC

Black : mass dependent width

$$\mathscr{M}_{\Lambda pn} = \frac{g^{\bar{K}NN}_{\Lambda p}}{M^2_{\bar{K}NN} - m^2_{\Lambda p} - iM_{\bar{K}NN}\Gamma^{\bar{K}NN}_{tot}(m_{\Lambda p})} \cdot \mathscr{A}\left(\cos\theta^*_n\right)$$

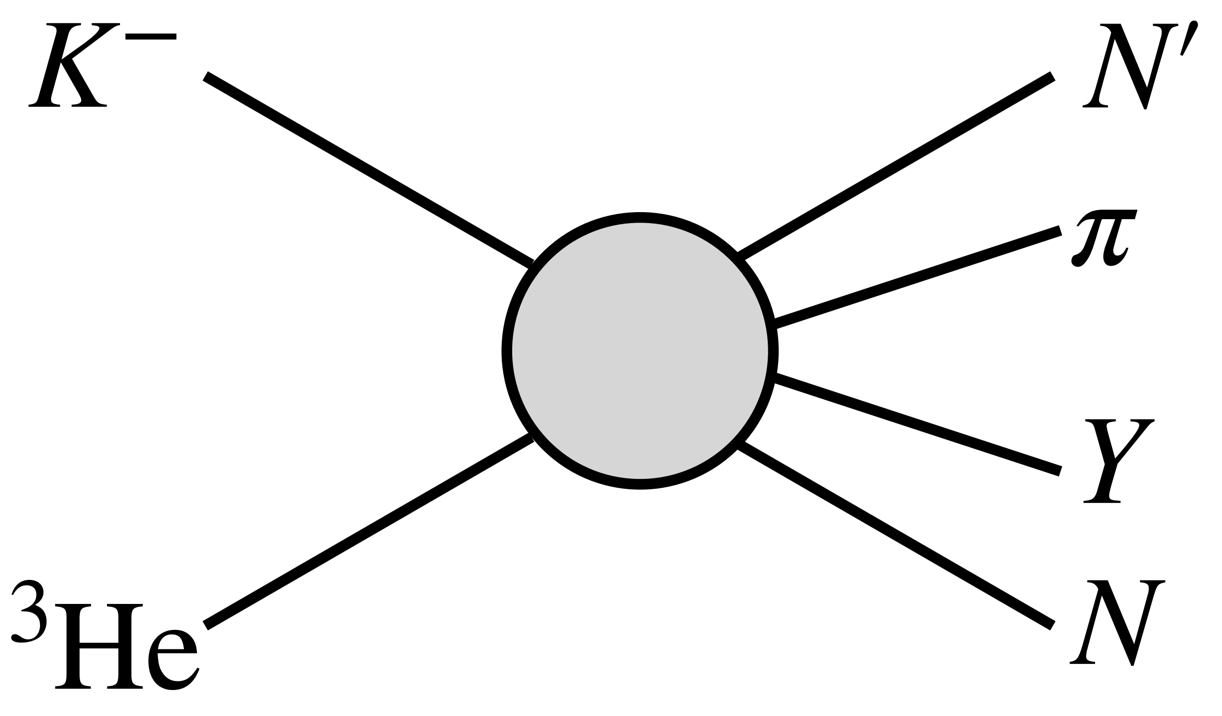
Error is so huge!!

*Parameters were set to fit the blue

NO.	NAME	VALUE	ERROR	
1	p0	2.41689e+00	2.62724e+00	$m_{\bar{K}NN}$
2	p1	1.69950e+00	1.15931e+02	$g_{\Lambda N}$
3	p2	6.98810e+01	6.81244e+01	$g_{\Sigma N}$
4	p3	6.81960e+02	6.82626e+04	$g_{\pi\Lambda N}$
5	p4	8.84178e+02	8.31748e+05	$g_{(\pi\Sigma)_{I=1}N}$
6	p5	6.36444e+01	1.15118e+06	$g_{\pi^0\Sigma^0 N}$
7	p6	1.44789e+02	1.19851e+06	$g_{\pi^-\Sigma^+ N}$
8	p7	4.11638e+02	1.16775e+06	$g_{\pi^+\Sigma^- N}$
9	p8	1.33888e+03	1.15010e+06	$g_{\bar{K}NN}$
10	p9	7.23045e-01	1.55019e+02	Scaling

Originally # of params was three

Cross section & Decay



$$d\sigma = \frac{(2\pi)^4}{4p^*_{K^-}\sqrt{s}} \; \mathscr{M}^{\;2} \times d\Phi_4$$

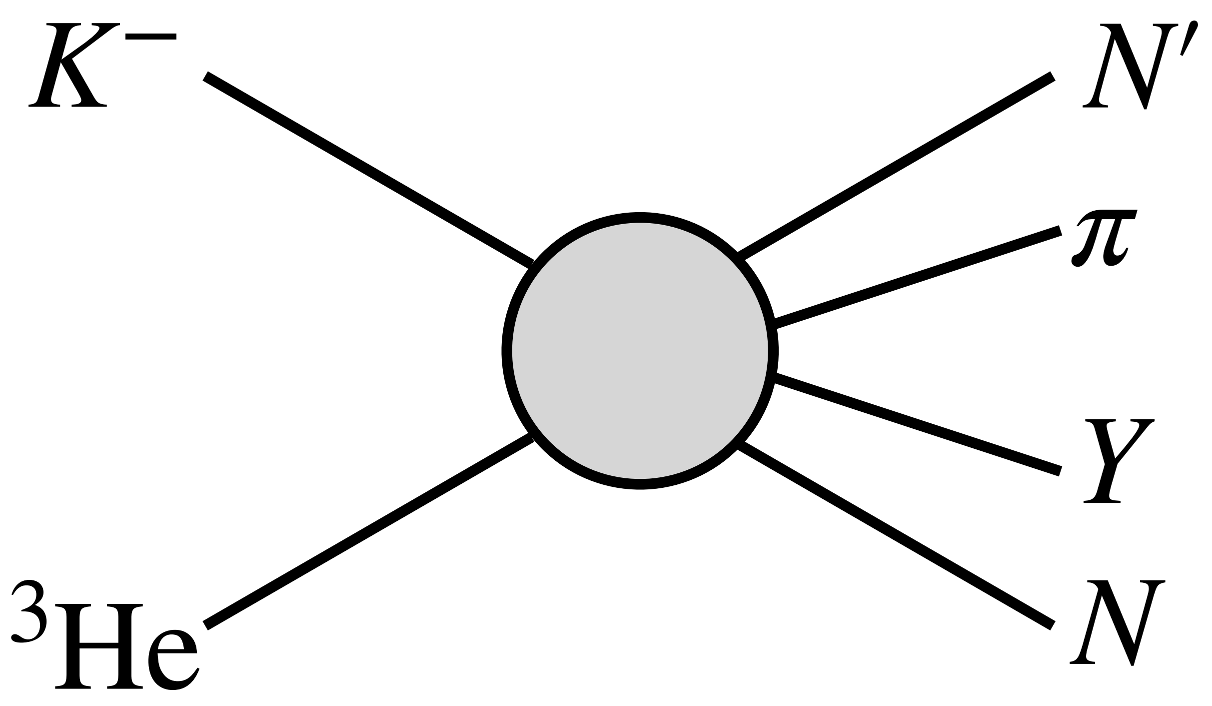
$$= \frac{1}{32(2\pi)^5}\frac{1}{p^*_{K^-}\,s}\left|p^*_{N'}\right|\left|p^{(\pi YN)*}_{N}\right|\left|p^{(\pi Y)*}_{Y}\right| \; \mathscr{M}^{\;2}\, dm_{\pi YN}\, dm_{\pi Y}\, d\cos\theta^*_{N'}\, d\cos\theta^{(\pi YN)*}_{N}$$

$$d\Phi_4 = \left(\frac{1}{4\,(2\pi)^6}\frac{\vec{p}^{\,*}_{N'}}{\sqrt{s}}\,d\Omega^*_{N'}\right)\left(\frac{1}{4\,(2\pi)^6}\frac{\vec{p}^{\,(\pi YN)*}_{N}}{m_{\pi YN}}\,d\Omega^{(\pi YN)*}_{N}\right)\left(\frac{1}{4\,(2\pi)^6}\frac{\vec{p}^{\,(\pi Y)*}_{Y}}{m_{\pi Y}}\,d\Omega^{(\pi Y)*}_{Y}\right)\left((2\pi)^3 dm^2_{\pi YN}\right)\left((2\pi)^3 dm^2_{\pi Y}\right)$$

$$\frac{d\sigma}{dm_{\pi YN}\, dm_{\pi Y}\, d\cos\theta^*_{N'}\, d\cos\theta^{(\pi YN)*}_{N}} = \frac{1}{32(2\pi)^5}\frac{1}{p^*_{K^-}\,s}\left|p^*_{N'}\right|\left|p^{(\pi YN)*}_{N}\right|\left|p^{(\pi Y)*}_{Y}\right| \; \mathscr{M}^{\;2}$$

$$\left|p^*_{N'}\right| = \frac{\sqrt{\left(s-(m_{\pi YN}+m_{N'})^2\right)\left(s-(m_{\pi YN}-m_{N'})^2\right)}}{2\sqrt{s}},\quad \left|p^{(\pi YN)*}_{N}\right| = \frac{\sqrt{\left(m^2_{\pi YN}-(m_{\pi Y}+m_N)^2\right)\left(m^2_{\pi YN}-(m_{\pi Y}-m_N)^2\right)}}{2m_{\pi YN}},\quad \left|p^{(\pi Y)*}_{Y}\right| = \frac{\sqrt{\left(m^2_{\pi Y}-(m_\pi+m_Y)^2\right)\left(m^2_{\pi Y}-(m_\pi-m_Y)^2\right)}}{2m_{\pi Y}}$$

Cross section & Decay



$$\frac{d\sigma}{dm_{\pi YN}\, dm_{\pi Y}\, d\cos\theta^*_{N'}\, d\cos\theta^{(\pi YN)*}_{N}} = \frac{1}{32(2\pi)^5}\frac{1}{p^*_{K^-}\, s}\left|p^*_{N'}\right|\left|p^{(\pi YN)*}_{N}\right|\left|p^{(\pi Y)*}_{Y}\right| \quad \mathscr{M}^{\;2}$$

$$\left|p^*_{N'}\right| = \frac{\sqrt{\left(s-(m_{\pi YN}+m_{N'})^2\right)\left(s-(m_{\pi YN}-m_{N'})^2\right)}}{2\sqrt{s}},\ \left|p^{(\pi YN)*}_{N}\right| = \frac{\sqrt{\left(m^2_{\pi YN}-(m_{\pi Y}+m_N)^2\right)\left(m^2_{\pi YN}-(m_{\pi Y}-m_N)^2\right)}}{2m_{\pi YN}},\ \left|p^{(\pi Y)*}_{Y}\right| = \frac{\sqrt{\left(m^2_{\pi Y}-(m_\pi+m_Y)^2\right)\left(m^2_{\pi Y}-(m_\pi-m_Y)^2\right)}}{2m_{\pi Y}}$$

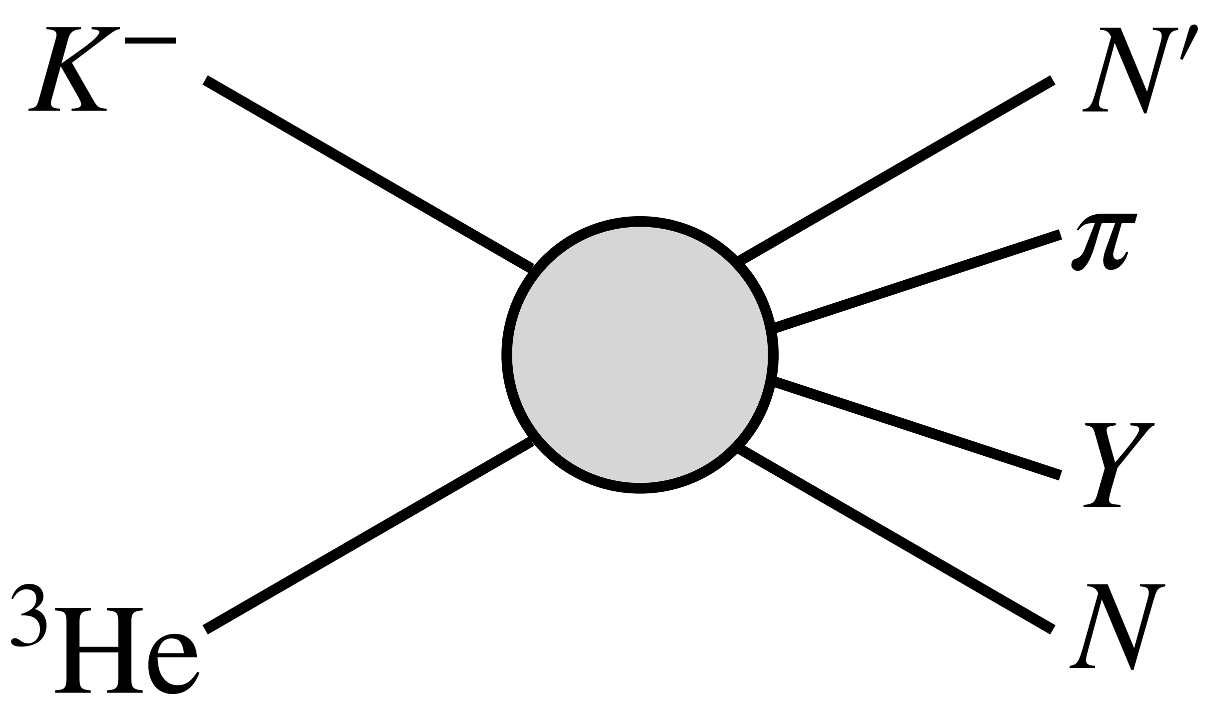
$$\mathscr{M} = \mathscr{M}_{\pi YN}\cdot\mathscr{M}_{\pi Y} = \frac{g^R_{\pi YN}}{M^2_R - m^2_{\pi YN} - iM_R\Gamma^R_{tot}}\cdot\left(g^{NR}_{\pi Y} + \frac{g^{Y*}_{\pi Y}}{M^2_{Y*} - m^2_{\pi Y} - iM_{Y*}\Gamma^{Y*}_{tot}}\right)\mathscr{A}\left(\cos\theta^*_{N'}\right)\mathscr{A}\left(\cos\theta^{(\pi YN)*}_{N}\right)$$

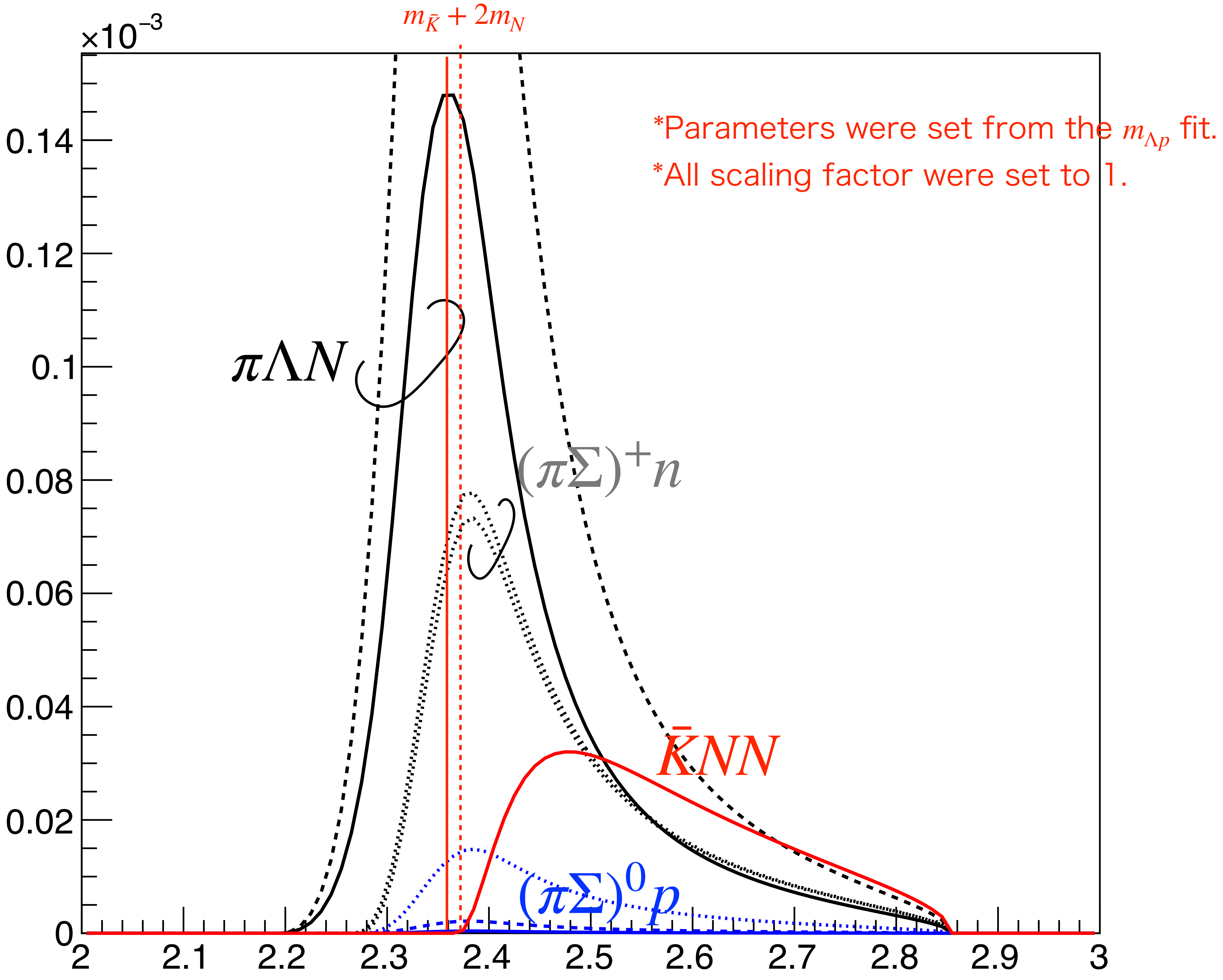
This term determines the $m_{\pi Y}$ distribution.

$$\frac{d\sigma}{dm_{\pi YN}\, dm_{\pi Y}\, d\cos\theta^*_{N'}\, d\cos\theta^{(\pi YN)*}_{N}} = \frac{1}{32(2\pi)^5}\frac{1}{p^*_{K^-}\, s}\left|p^*_{N'}\right|\left|p^{(\pi YN)*}_{N}\right|\left|p^{(\pi Y)*}_{Y}\right|\left|\frac{g^R_{\pi YN}}{M^2_R - m^2_{\pi YN} - iM_R\Gamma^R_{tot}}\right|^2\left|g^{NR}_{\pi Y} + \frac{g^{Y*}_{\pi Y}}{M^2_{Y*} - m^2_{\pi Y} - iM_{Y*}\Gamma^{Y*}_{tot}}\right|^2\left|\mathscr{A}\left(\cos\theta^*_{N'}\right)\right|^2\left|\mathscr{A}\left(\cos\theta^{(\pi YN)*}_{N}\right)\right|^2$$

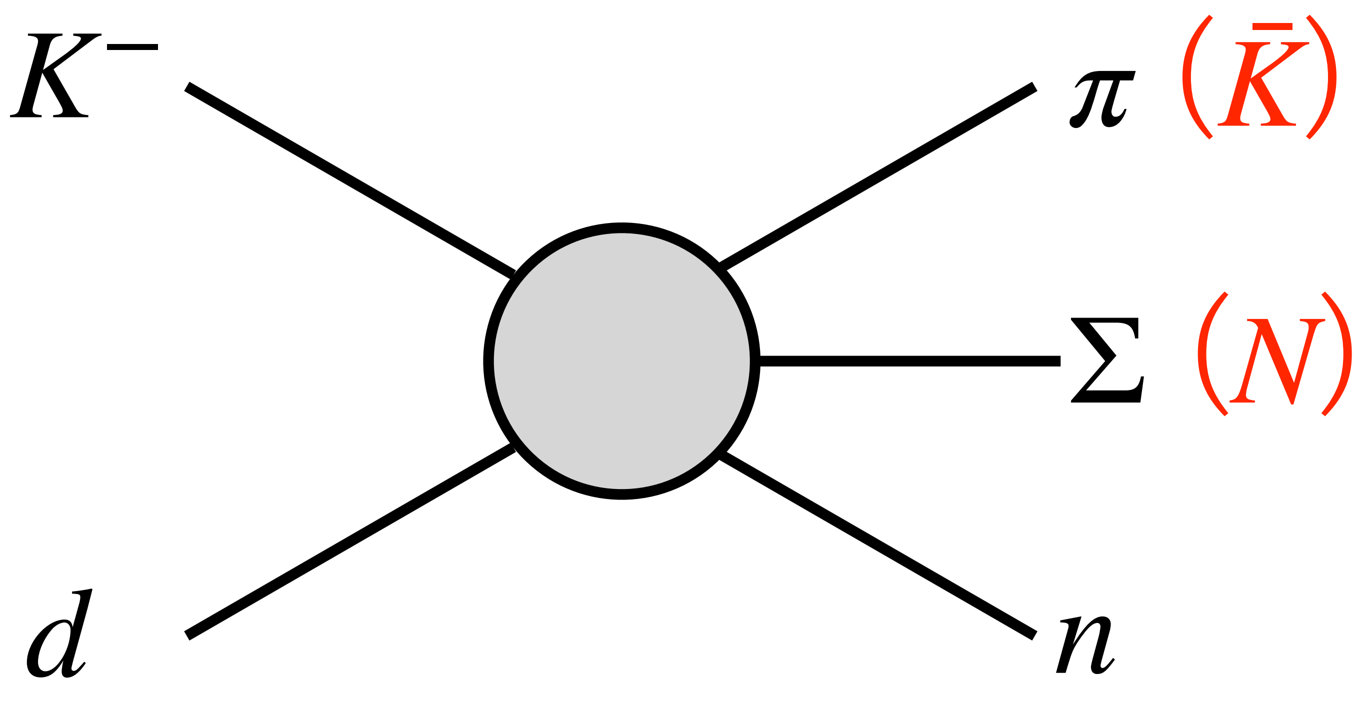
As the first step, let us ignore Y^* contribution.

Cross section & Decay





Cross section & Decay



$$\frac{d\sigma_{\pi\Sigma n}}{dm_{\pi\Sigma}\, d\cos\theta_n^*} = \frac{1}{16(2\pi)^3} \frac{1}{p_{K^-}^* s} \left| p_n^* \right| \left| p_\Sigma^{(\pi\Sigma)*} \right| \quad \mathscr{M}^{\;2}$$

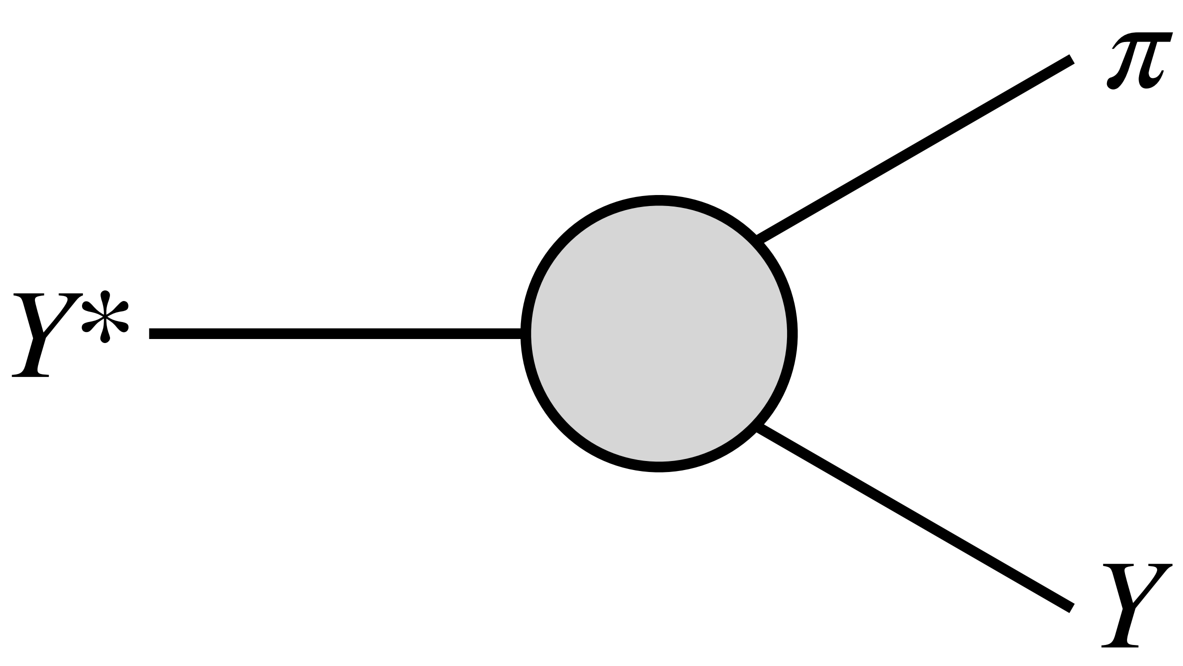
$$\mathscr{M} = \frac{g_{\pi Y}^{Y*}}{M_{Y*}^2 - m_{\pi\Sigma}^2 - iM_{Y*}\Gamma_{tot}^{Y*}} \cdot \mathscr{A}\left(\cos\theta_n^*\right)$$

$$\frac{d\sigma_{\pi\Sigma n}}{dm_{\pi\Sigma}\, d\cos\theta_n^*} = \frac{1}{16(2\pi)^3} \frac{1}{p_{K^-}^* s} \left| p_n^* \right| \left| p_\Sigma^{(\pi\Sigma)*} \right| \left| \frac{g_{\pi\Sigma}^{Y*}}{M_{Y*}^2 - m_{\pi\Sigma}^2 - iM_{Y*}\Gamma_{tot}^{Y*}} \right|^2 \left| \mathscr{A}\left(\cos\theta_n^*\right) \right|^2$$

$\dfrac{d\sigma_{\bar{K}Nn}}{dm_{\bar{K}N}\, d\cos\theta_n^*}$ is as the same.

$$\begin{cases} \dfrac{\left(g_{\pi\Sigma}^{Y*}\right)^2}{\left(M_{Y*}^2 - m_{\pi\Sigma}^2 + M_{Y*}\Gamma_{KN}\right)^2 + M_{Y*}^2\Gamma_{\pi\Sigma}^2}, & \text{(below the } m_{\bar{K}} + m_N \text{ threshold)} \\[2ex] \dfrac{\left(g_{\pi\Sigma}^{Y*}\right)^2}{\left(M_{Y*}^2 - m_{\pi\Sigma}^2\right)^2 + M_{Y*}^2\left(\Gamma_{\pi\Sigma} + \Gamma_{\bar{K}N}\right)^2}, & \text{(above the } m_{\bar{K}} + m_N \text{ threshold)} \end{cases}$$

Cross section & Decay



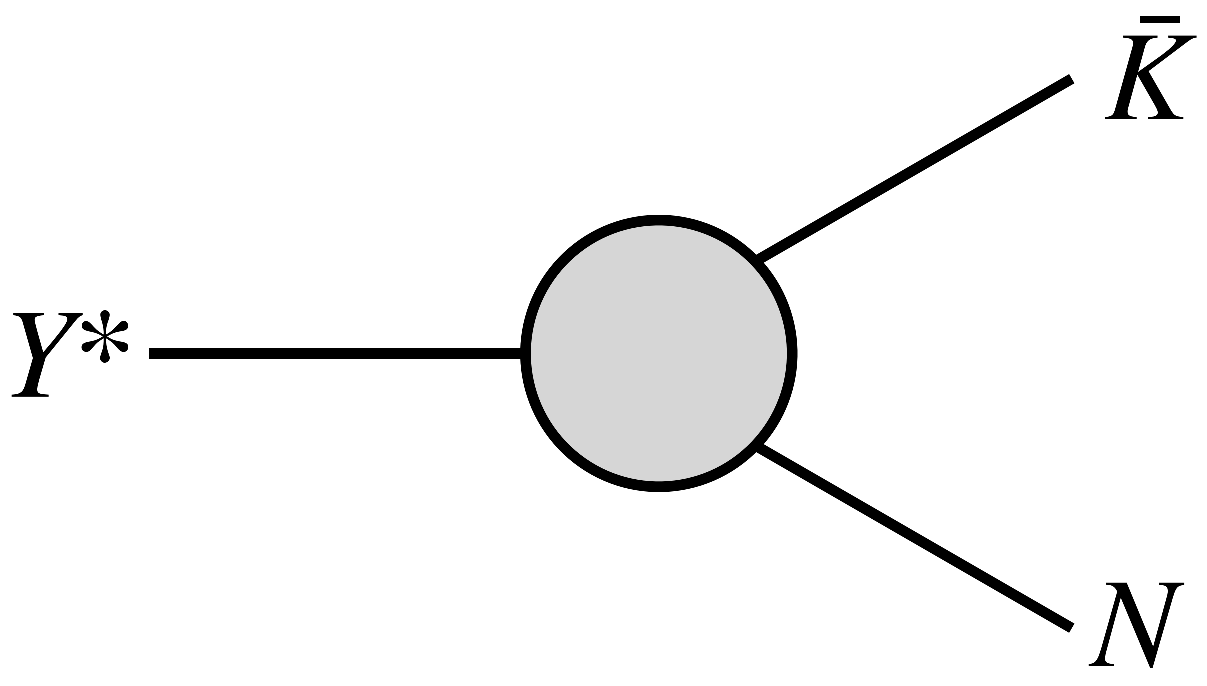
$$d\Gamma_{\pi Y}^{Y*} = \frac{(2\pi)^4}{2m_{\pi Y}} \mathcal{M}^{2} \Phi_2 \quad = \frac{(2\pi)^4}{2m_{\pi Y}} \mathcal{M}^{2} \frac{1}{4\,(2\pi)^6} \frac{\vec{p}_Y^{(\pi Y)*}}{m_{\pi Y}} d\Omega_Y^{(\pi Y)*}$$

$$\boxed{\mathcal{M} = g_{\pi Y}^{Y*}}$$

$$\Gamma_{\pi Y} = \frac{\left(g_{\pi Y}^{Y*}\right)^2}{8\pi m_{\pi Y}^2} \frac{\sqrt{\left(m_{\pi Y}^2 - (m_\pi + m_Y)^2\right)\left(m_{\pi Y}^2 - (m_\pi - m_Y)^2\right)}}{2m_{\pi Y}} \text{ (above the } m_\pi + m_Y\text{)}$$

$$= i\frac{\left(g_{\pi Y}^{Y*}\right)^2}{8\pi m_{\pi Y}^2} \frac{\sqrt{\left((m_\pi + m_Y)^2 - m_{\pi Y}^2\right)\left(m_{\pi Y}^2 - (m_\pi - m_Y)^2\right)}}{2m_{\pi Y}} \text{ (below the } m_\pi + m_Y\text{)}$$

Cross section & Decay



$$d\Gamma_{\bar{K}N}^{Y^*} = \frac{(2\pi)^4}{2m_{\bar{K}N}} \;\mathscr{M}\;^2 \Phi_2 \quad = \frac{(2\pi)^4}{2m_{\bar{K}N}} \;\mathscr{M}\;^2 \frac{1}{4\,(2\pi)^6} \frac{\vec{p}_N^{\,(\bar{K}N)*}}{m_{\bar{K}N}} d\Omega_N^{(\bar{K}N)*}$$

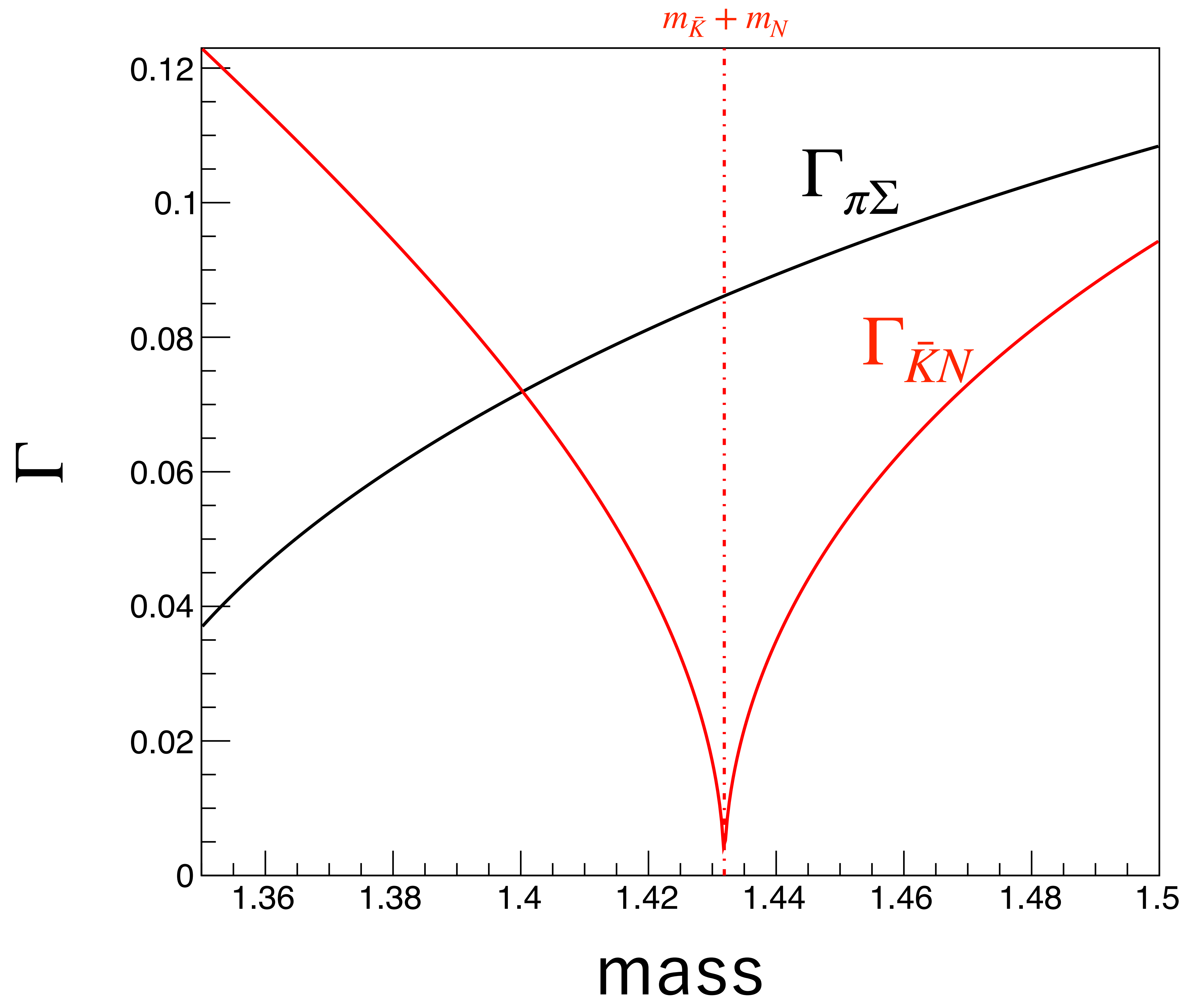
$$\boxed{\mathscr{M} = g_{\bar{K}N}^{Y^*}}$$

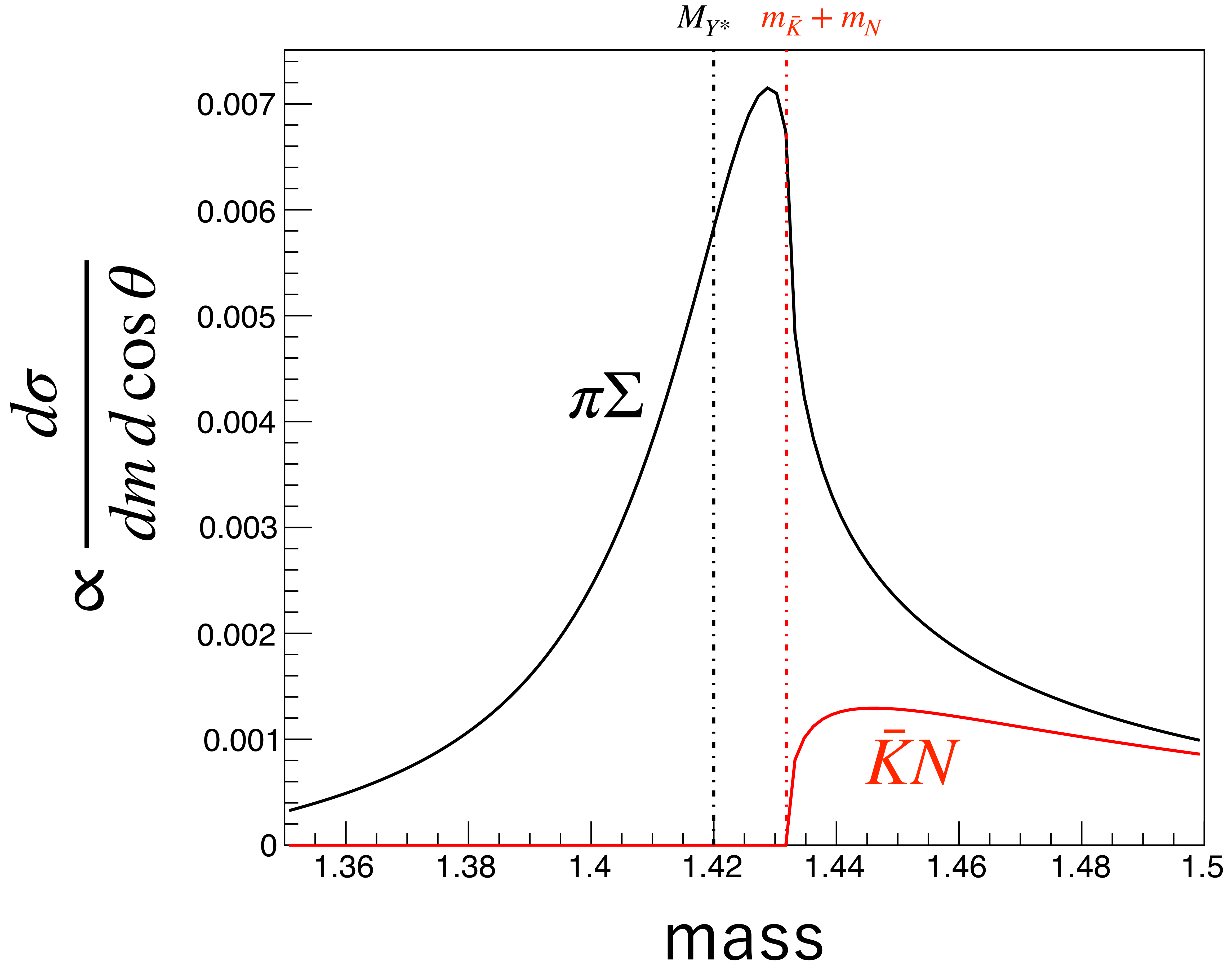
$$\Gamma_{\pi Y} = \frac{\left(g_{\bar{K}N}^{Y^*}\right)^2}{8\pi m_{\bar{K}N}^2} \frac{\sqrt{\left(m_{\bar{K}N}^2 - (m_{\bar{K}} + m_N)^2\right)\left(m_{\bar{K}N}^2 - (m_{\bar{K}} - m_N)^2\right)}}{2m_{\bar{K}N}} \text{ (above the } m_{\bar{K}} + m_N\text{)}$$

$$= i\frac{\left(g_{\bar{K}N}^{Y^*}\right)^2}{8\pi m_{\bar{K}N}^2} \frac{\sqrt{\left((m_{\bar{K}} + m_N)^2 - m_{\bar{K}N}^2\right)\left(m_{\bar{K}N}^2 - (m_{\bar{K}} - m_N)^2\right)}}{2m_{\bar{K}N}} \text{ (below the } m_{\bar{K}} + m_N\text{)}$$

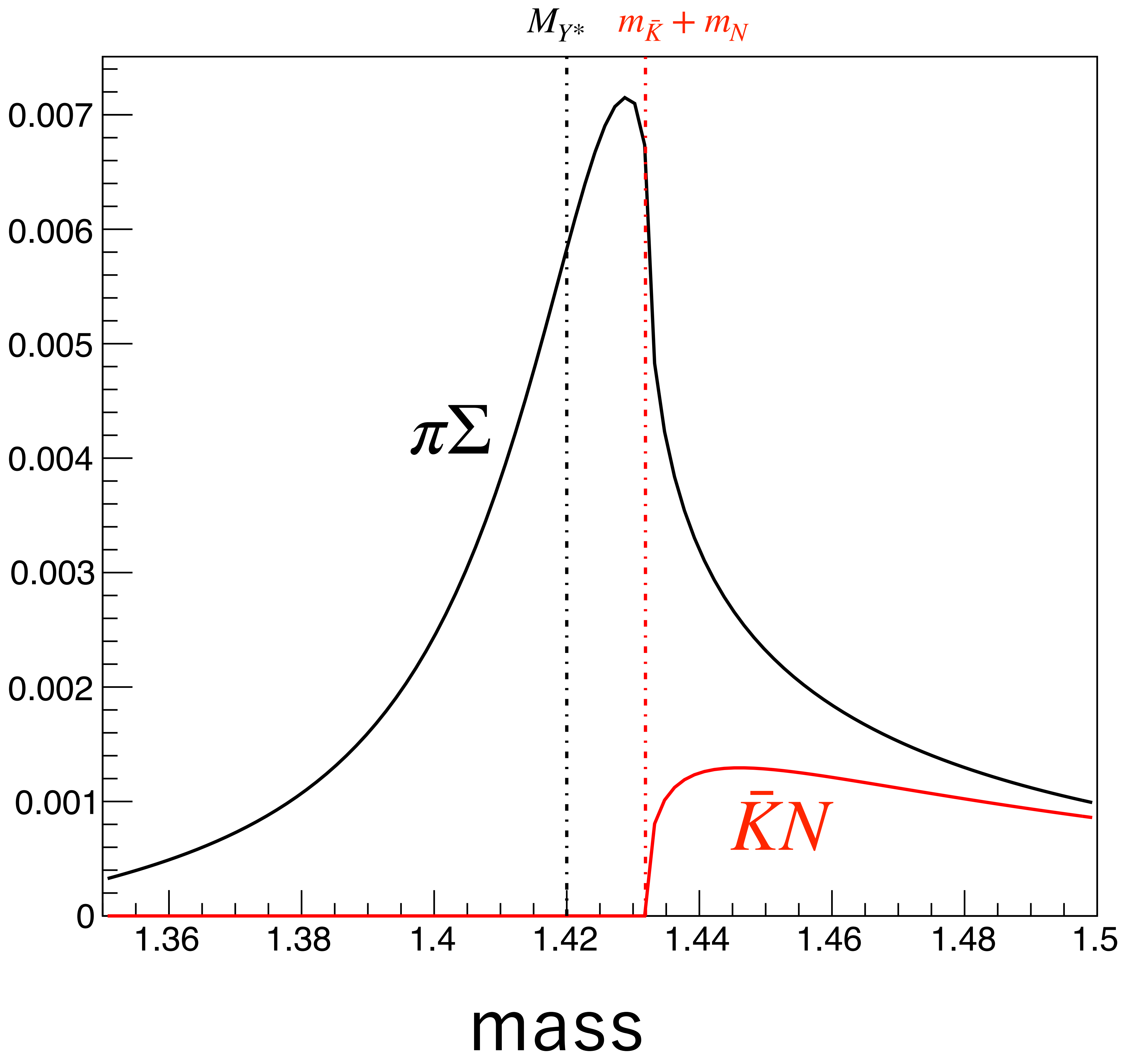
Cross section & Decay

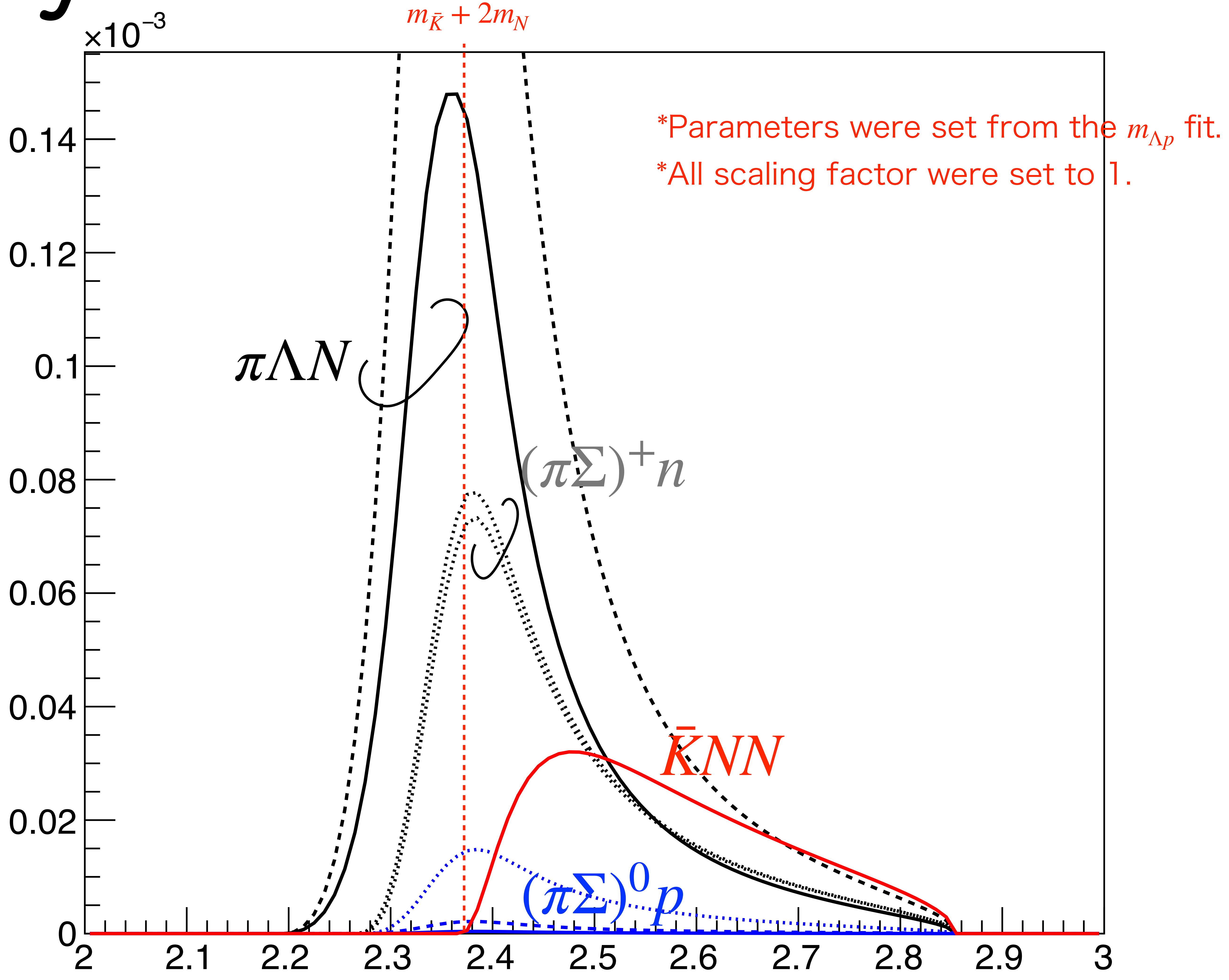
Parameters $\begin{cases} M_{Y*} = 1.42 \text{ GeV}/c^2 \\ g^{Y*}_{\pi\Sigma} = g^{Y*}_{\bar{K}N} = 5 \end{cases}$





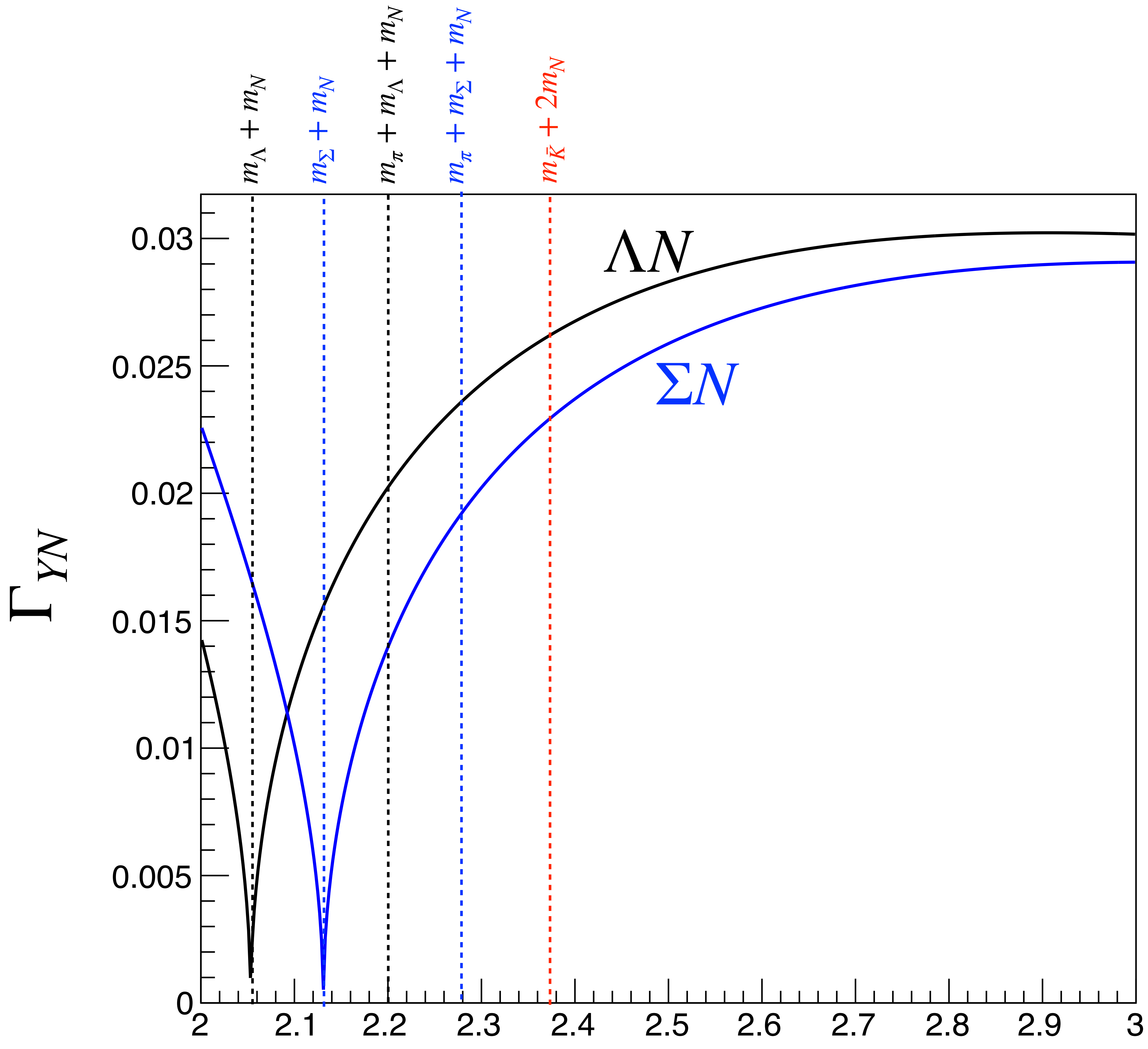
Cross section & Decay

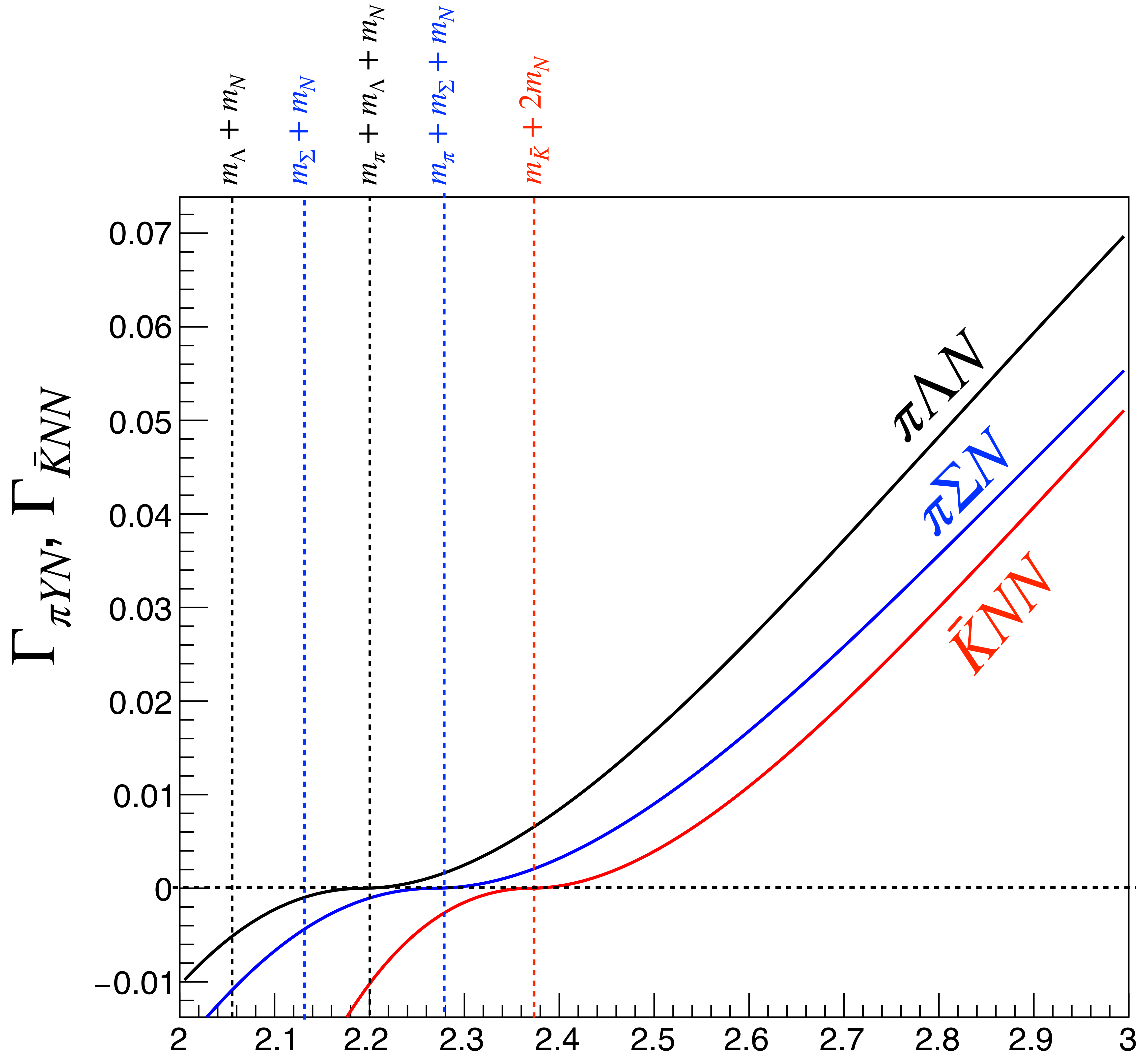




Spectral distortion seems to be much smaller than $\Lambda(1405)$ case.
It may due to the difference between two-body and three-body LIPS.

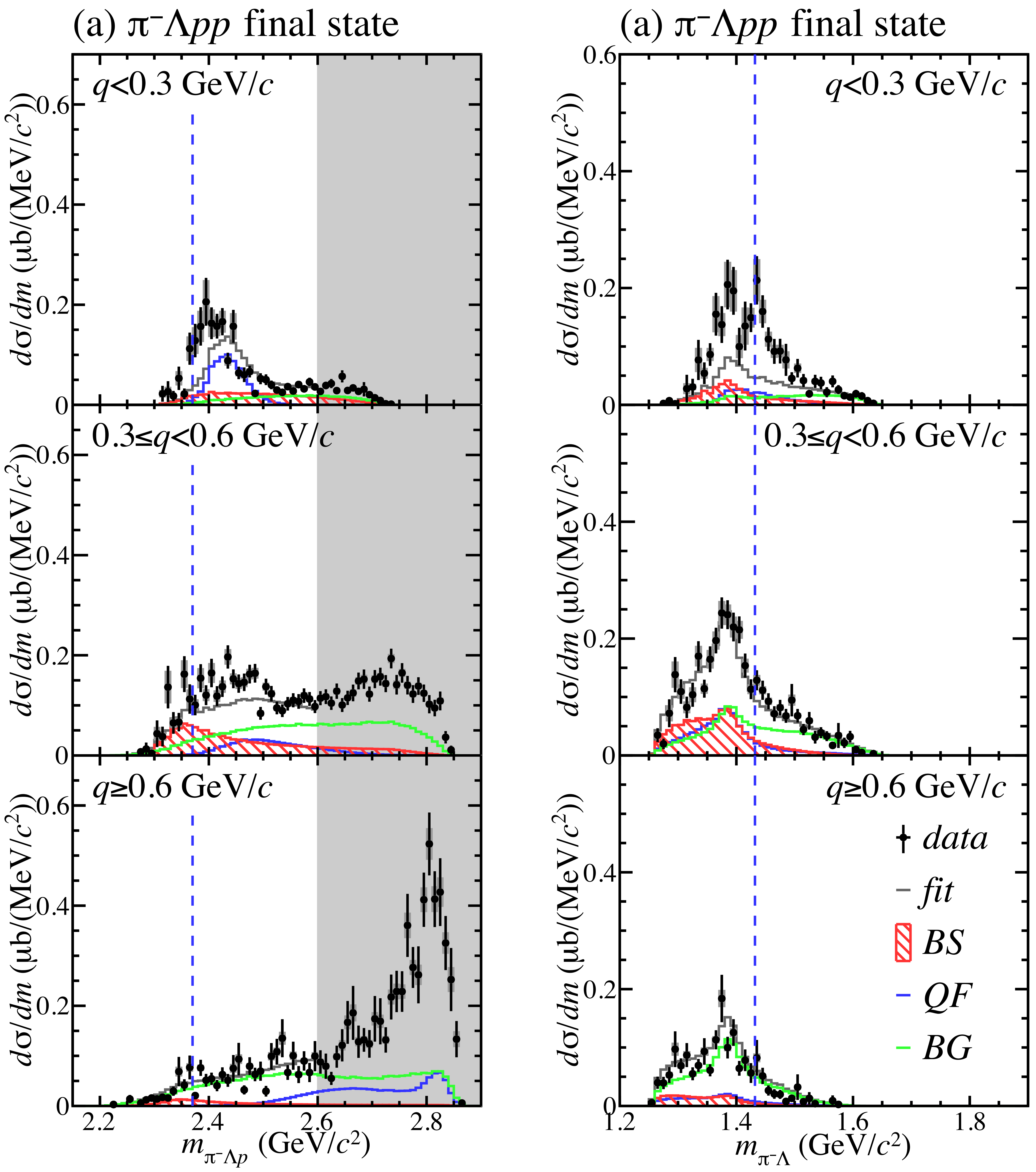
Cross section & Decay



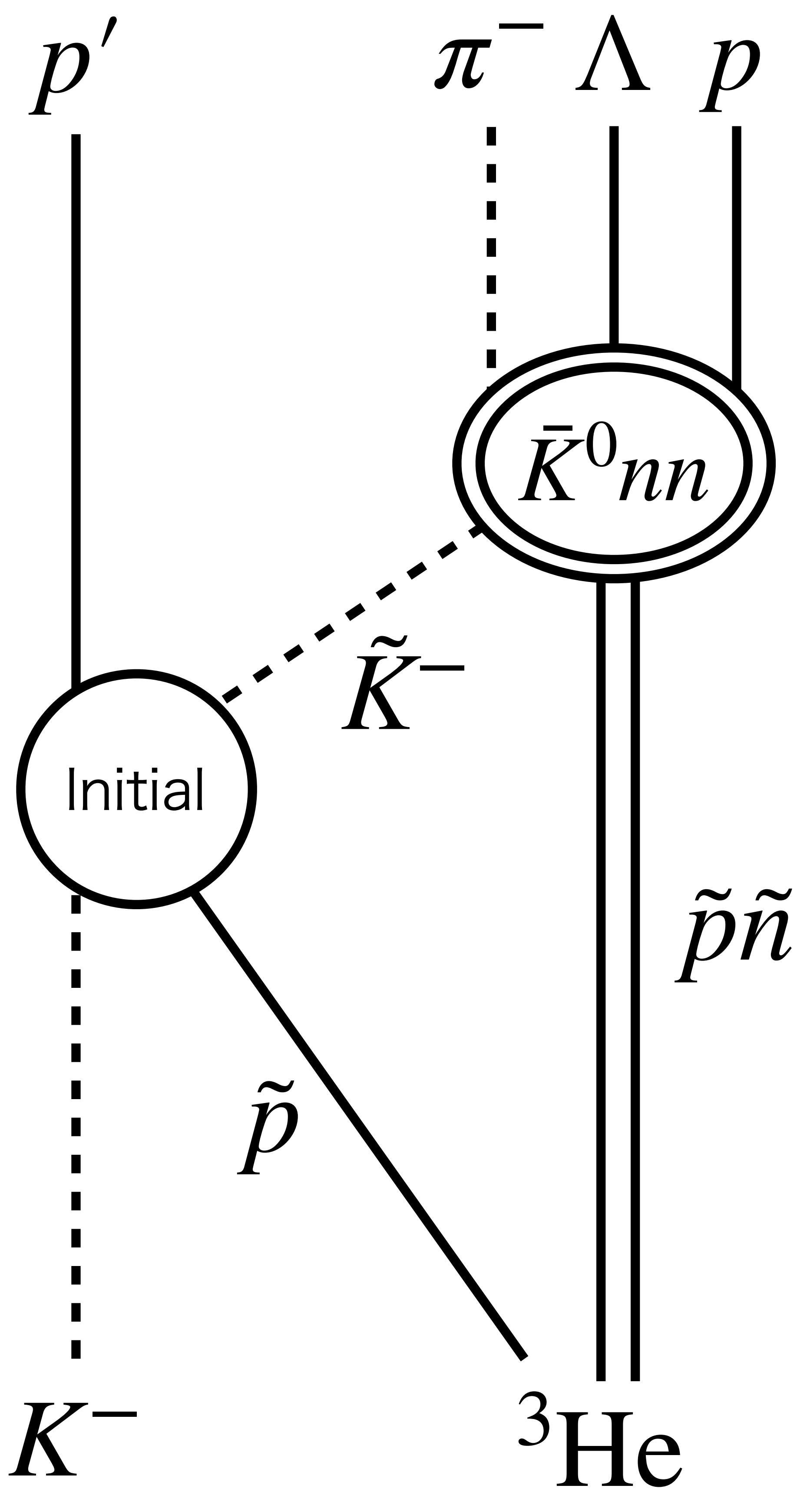


Overview

- More precise analysis of the $K^{-3}\mathrm{He} \to \pi^- \Lambda p p'$ reaction
- The same event selection with the distributed draft (Not submitted yet, but will be soon with small modification…)
- To search for $\bar{K}^0 nn$ with more precise fitting
  - $\bar{K}^0 nn$, QF-K, and QF-Y*



Analysis to select the $\pi^-\Lambda pp'$ final state



$\pi^-\Lambda p \rightarrow \pi^- \left(\pi^- p\right) p$: Measured by CDS

p' : Identified by the missing-mass method

→ Requiring four tracks in CDC

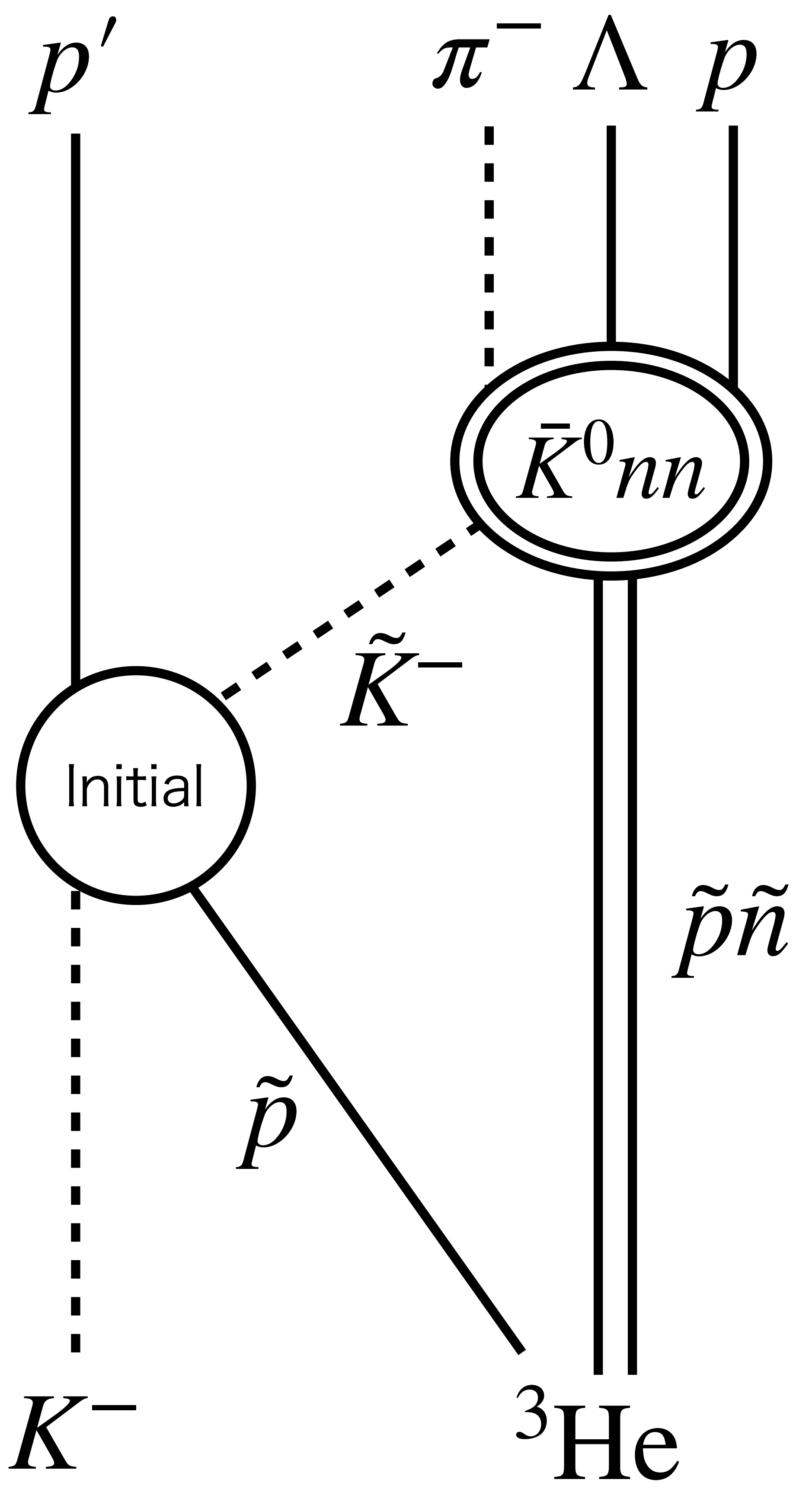
→ Event selection with log-likelihood

$$L = -\ln\left(p\left(\chi_{kin}\right) + \sum_{i}^{N_{\mathrm{DCA}}} p\left(\mathrm{DCA}_i\right) \right)$$

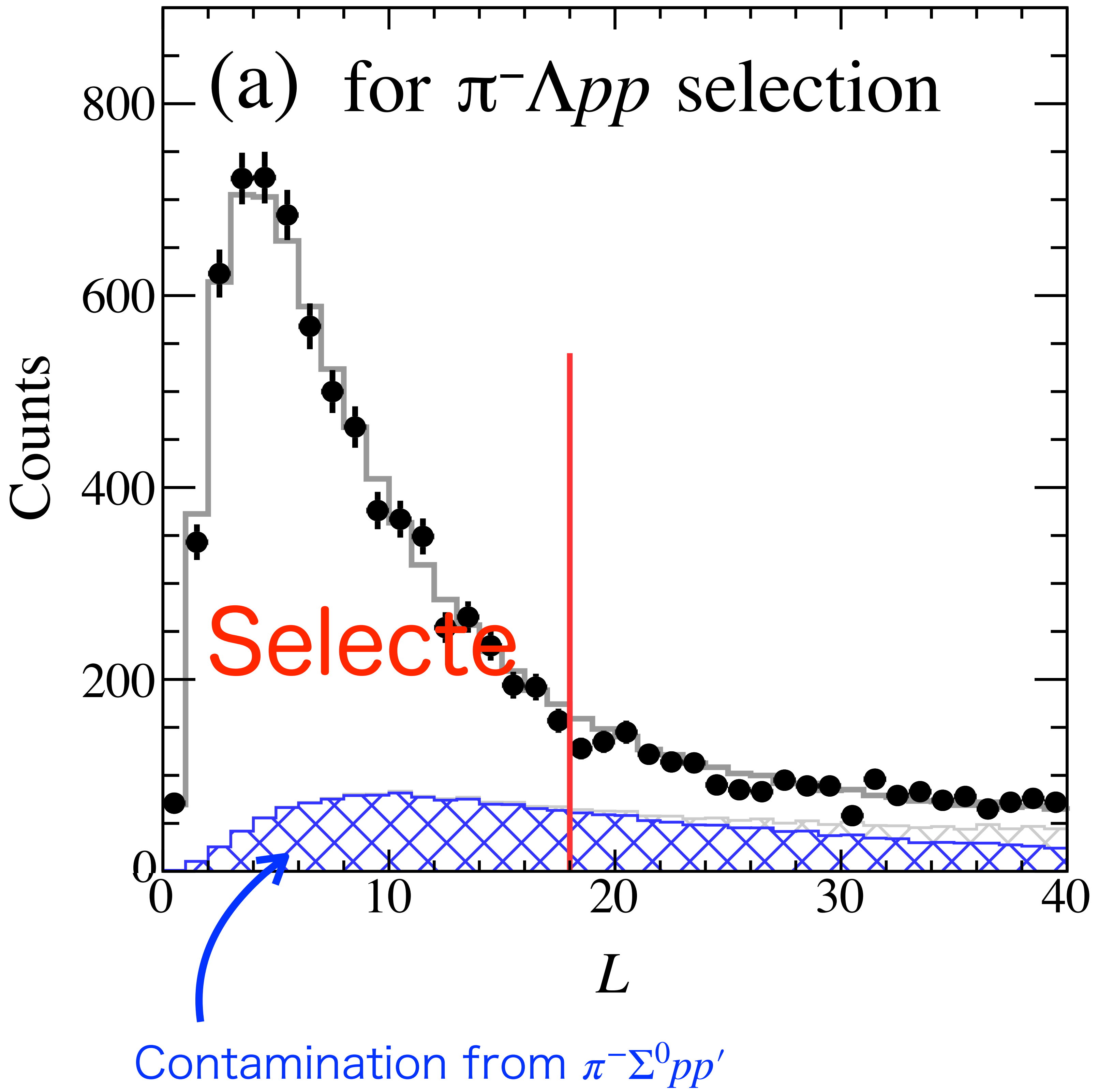
seven DCAs considered:

- π^--K^-
- π^--p
- p_Λ-p_{π^-}
- p-K^-
- p-Λ
- Λ-K^-
- Λ-π^-

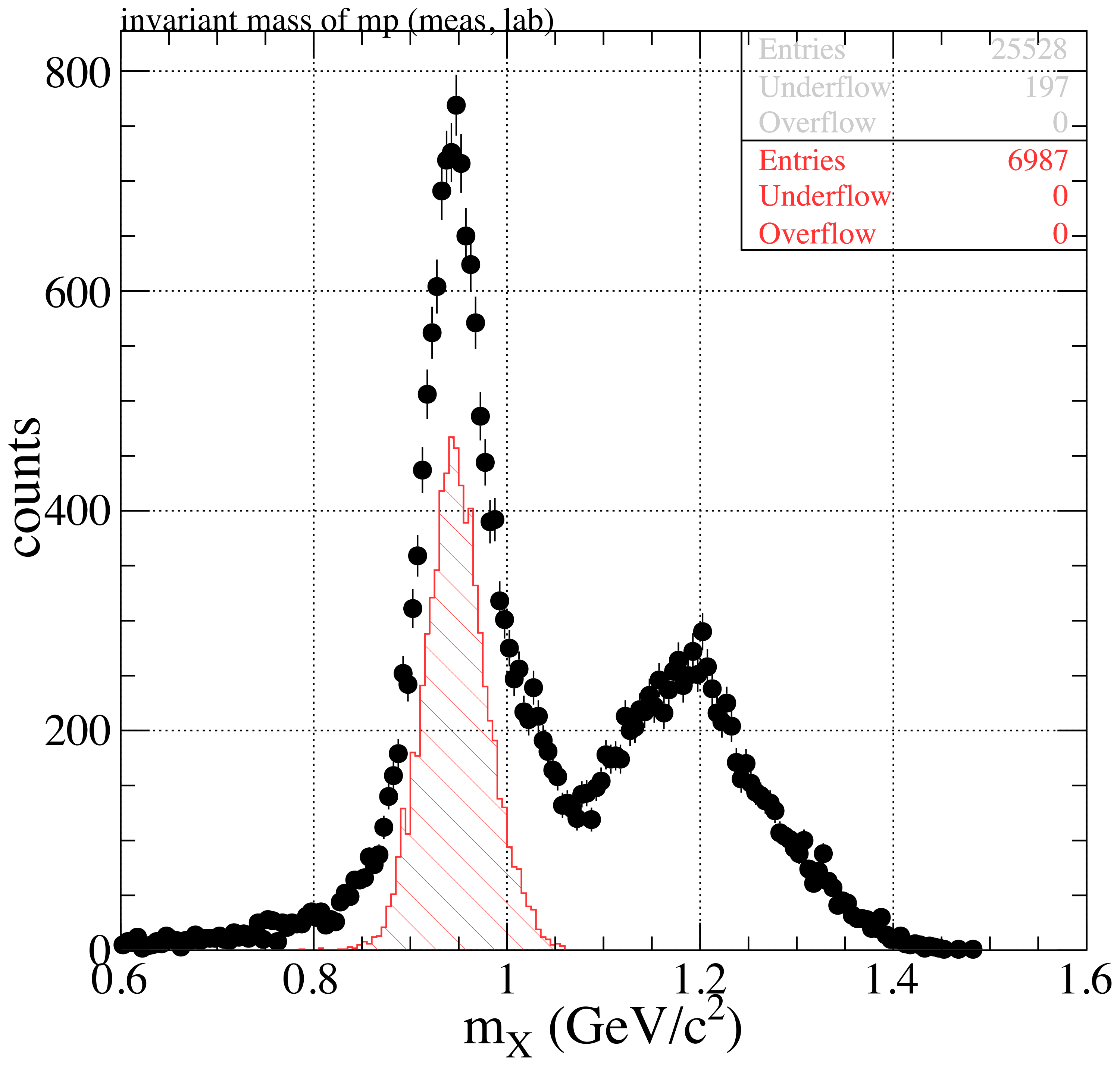
Analysis to select the $\pi^-\Lambda p p'$ final state (ii)



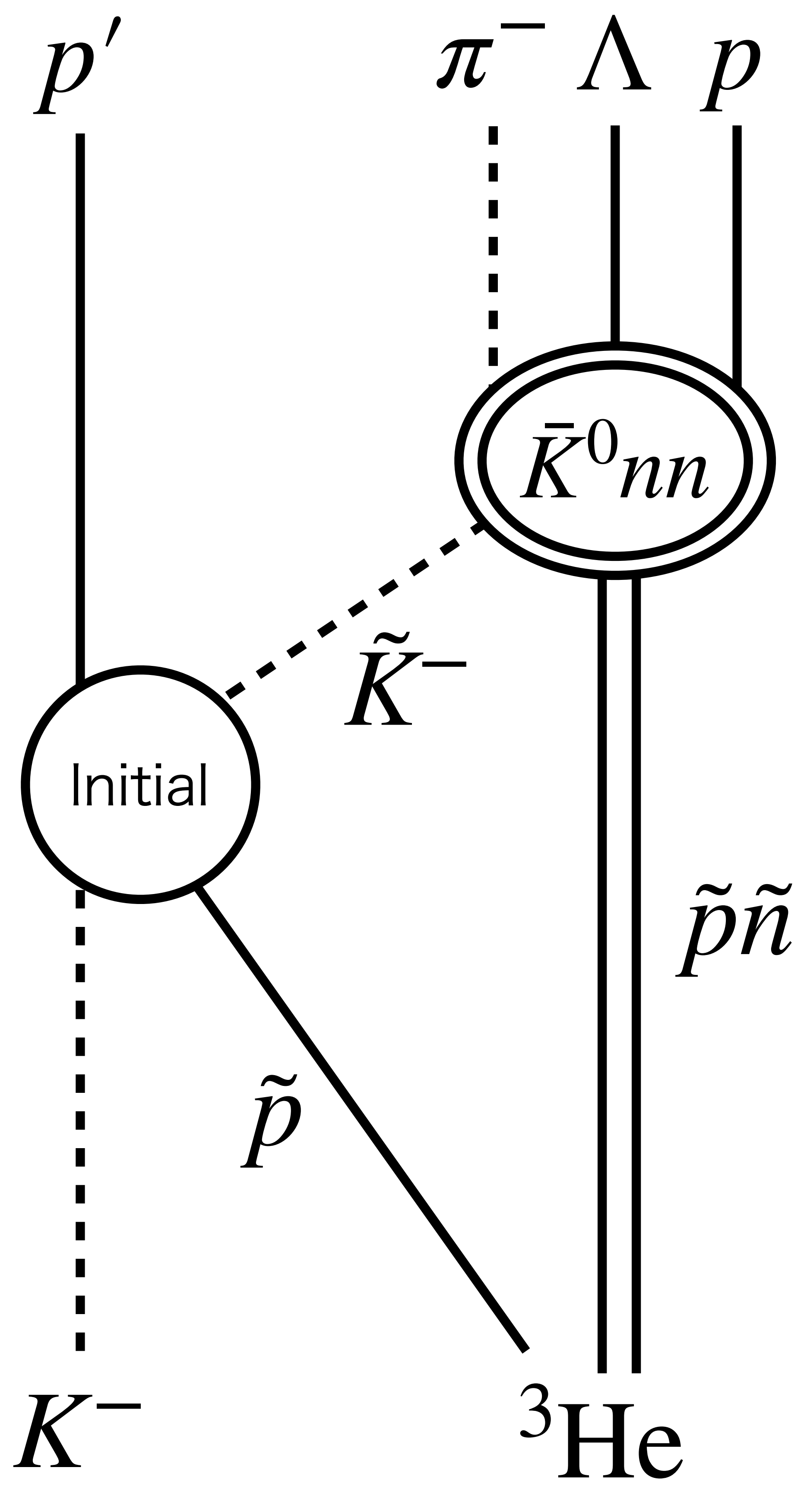
log-likelihood



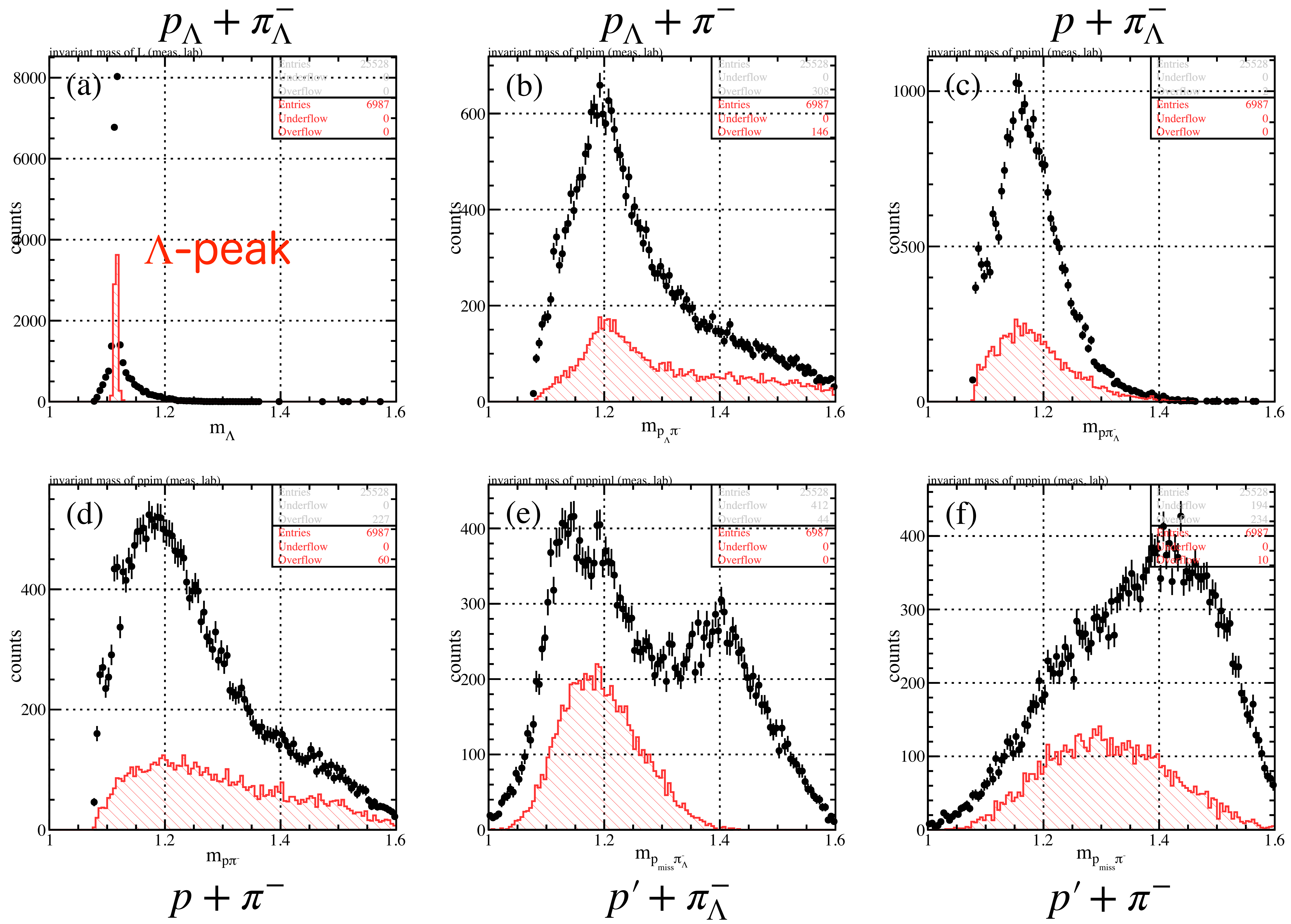
Missing-mass



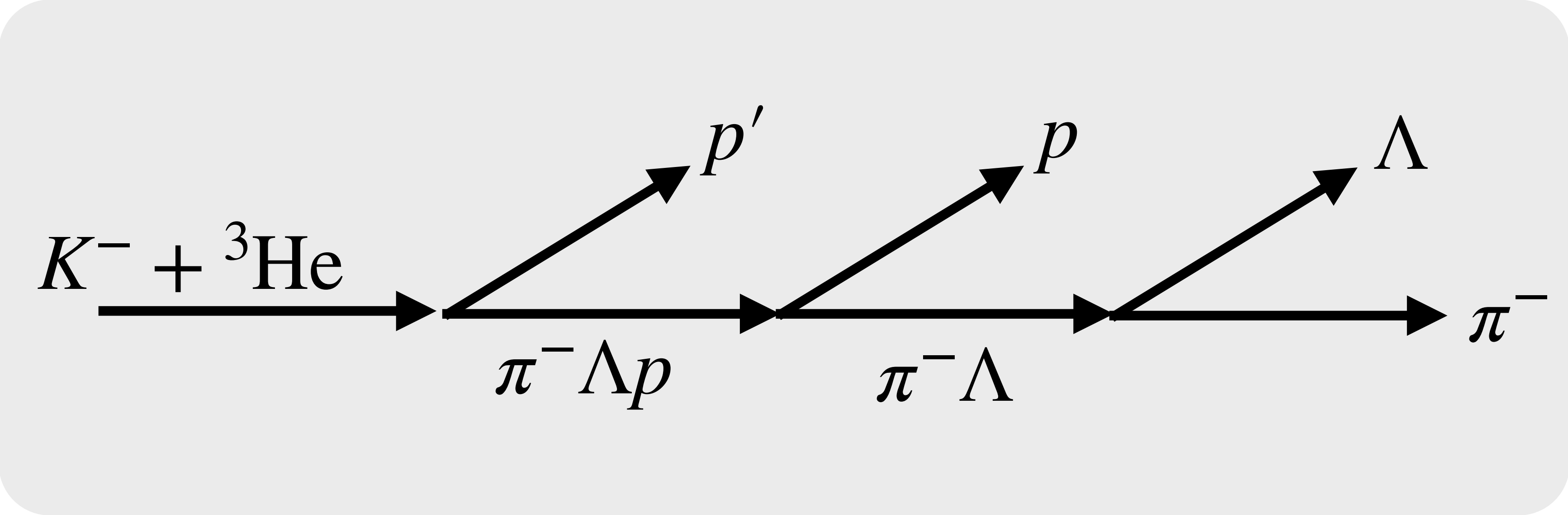
Analysis to select the $\pi^- \Lambda p p'$ final state (iii)



Invariant-masses of $p\pi^-$ pairs



Degrees of freedom in the reaction

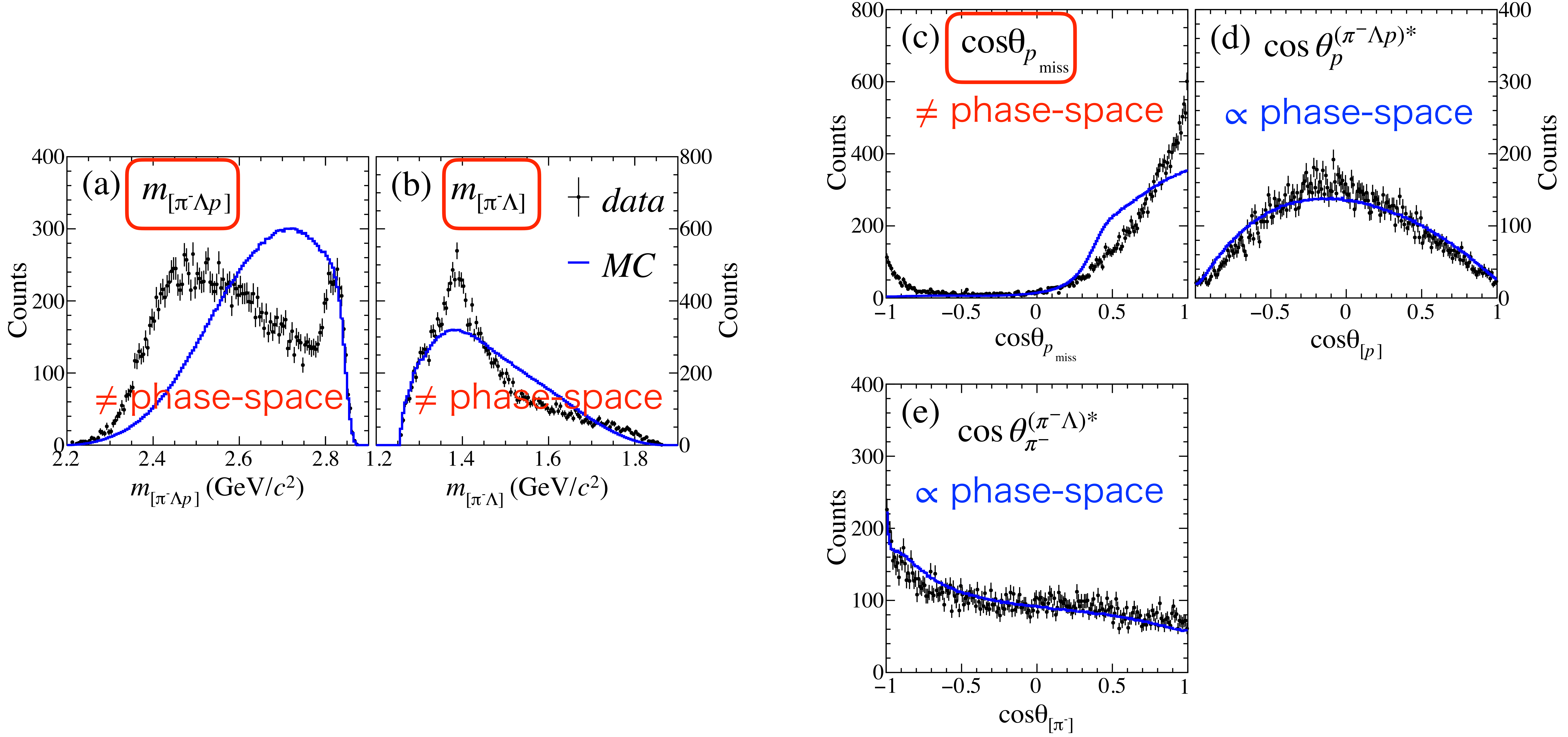


Eight degrees of freedom:

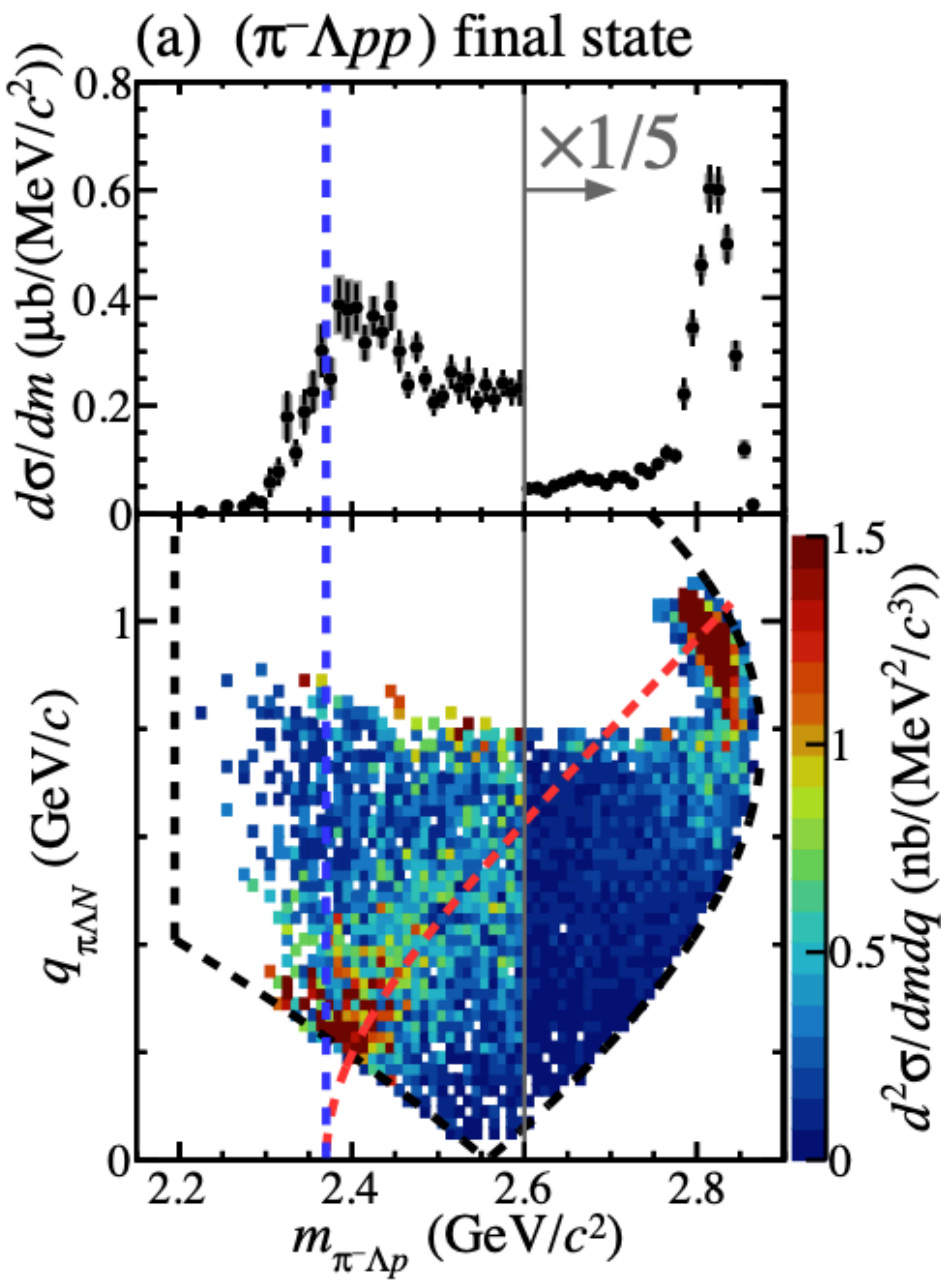
- $m_{\pi^-\Lambda p}$: invariant-mass of $\pi^-\Lambda p$
- $m_{\pi^-\Lambda}$: invariant-mass of $\pi^-\Lambda$
- $\theta^*_{p'}$: polar angle between $\boldsymbol{p}_{K^-}$ and $\boldsymbol{p}_{p'}$ in the c.m. frame
- $\phi^*_{p'}$: azimuthal angle between $\boldsymbol{p}_{K^-}$ and $\boldsymbol{p}_{p'}$ in the c.m. frame
- $\theta_p^{(\pi^-\Lambda p)*}$: polar angle between $\boldsymbol{p}_p$ and $-\boldsymbol{p}_{p'}$ in the $(\pi^-\Lambda p)$-c.m. frame
- $\phi_p^{(\pi^-\Lambda p)*}$: azimuthal angle between $\boldsymbol{p}_p$ and $-\boldsymbol{p}_{p'}$ in the $(\pi^-\Lambda p)$-c.m. frame
- $\theta_\Lambda^{(\pi^-\Lambda)*}$: polar angle between $\boldsymbol{p}_\Lambda$ and $-\boldsymbol{p}_p$ in the $(\pi^-\Lambda)$-c.m. frame
- $\phi_\Lambda^{(\pi^-\Lambda)*}$: azimutal angle between $\boldsymbol{p}_\Lambda$ and $-\boldsymbol{p}_p$ in the $(\pi^-\Lambda)$-c.m. frame

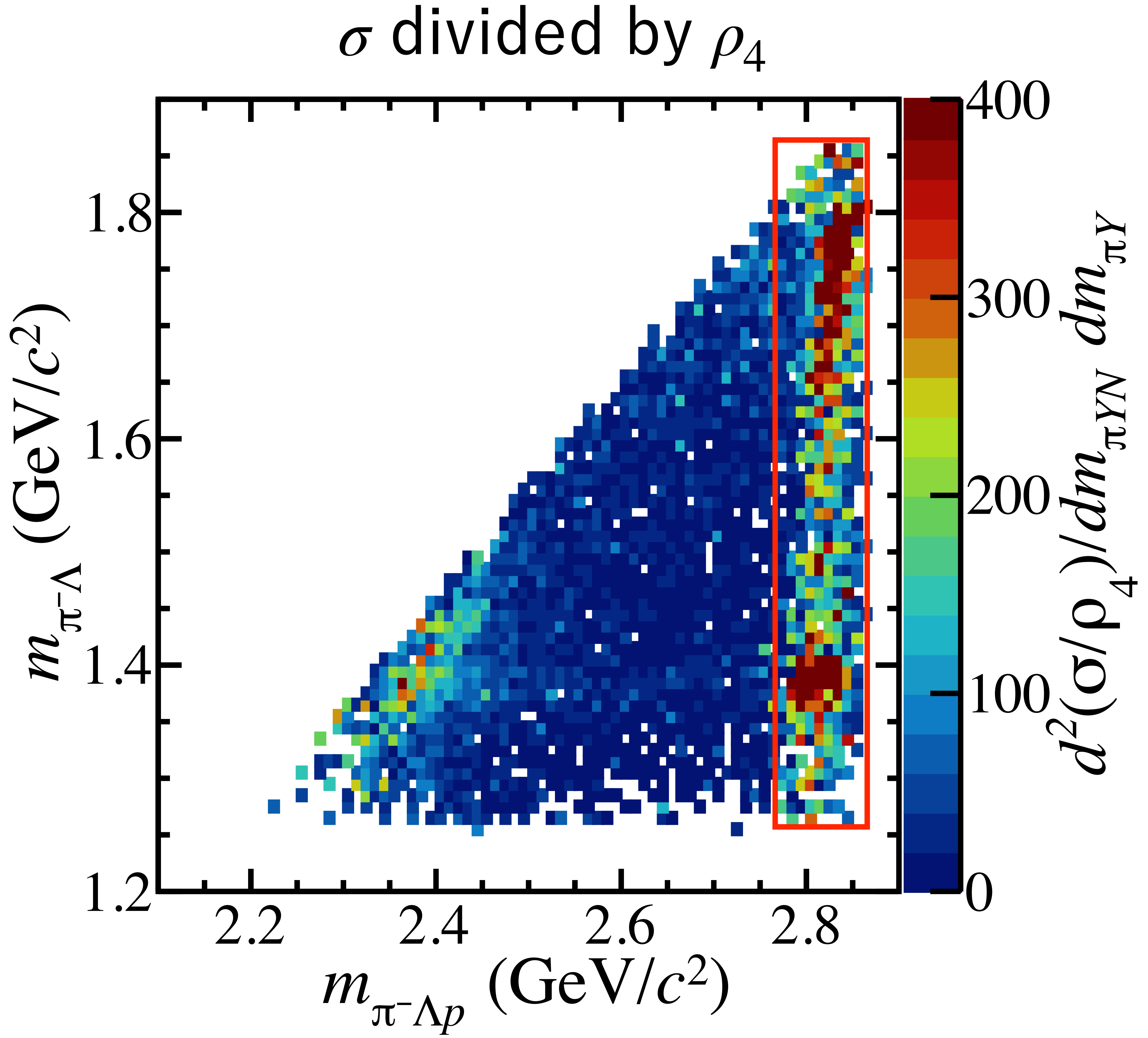
$\rightarrow$ $m_{\pi^-\Lambda p}$, $m_{\pi^-\Lambda}$, and $\theta^*_{p'}$ will be discussed.

Comparison
between data & sim (four-body phase-space)

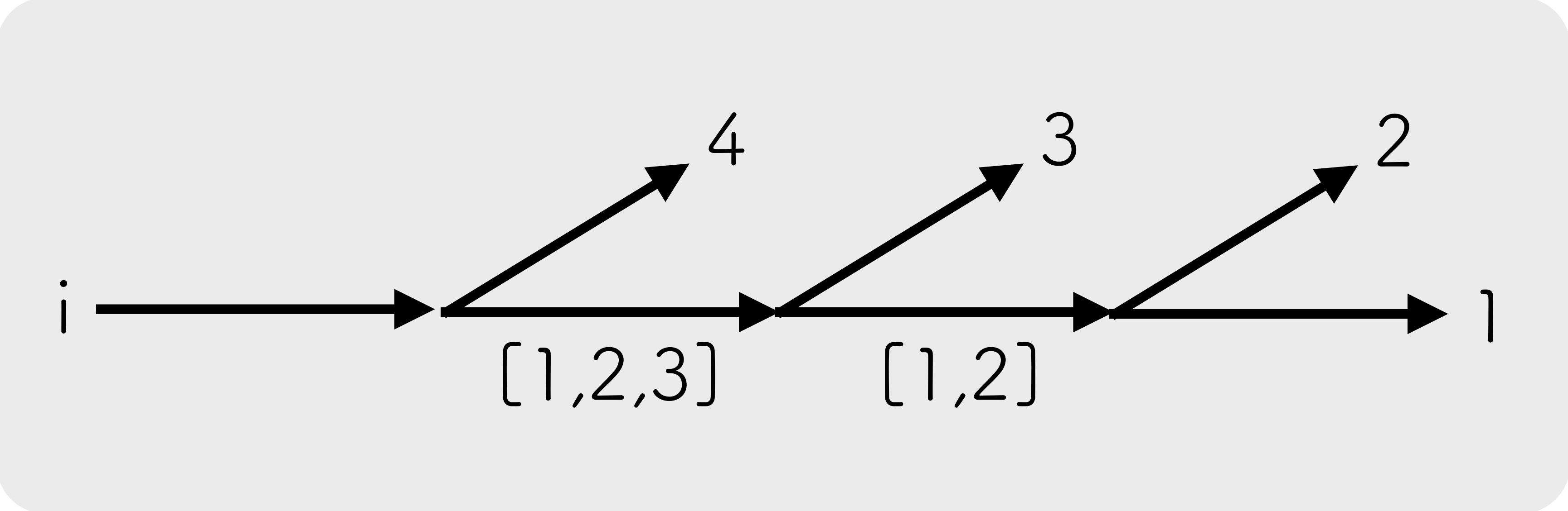


Obtained distributions





Four-body LIPS



Two-body LIPS

$$\rho_2 = \frac{\pi}{(2\pi)^6}\frac{\vec{p}^*}{m_i}$$

$$d\rho_4 = d\rho_2(i;\ [1,2,3],4)d\rho_2([1,2,3];\ [1,2],3)d\rho_2([1,2];\ 1,2)(2\pi)^3\, dm^2_{[1,2]}$$

$$= \frac{\pi}{(2\pi)^6}\frac{\vec{p_4}^*}{m_i}\frac{\pi}{(2\pi)^6}\frac{\vec{p_3}^*}{m_{[1,2,3]}}\frac{\pi}{(2\pi)^6}\frac{\vec{p_2}^*}{m_{[1,2]}}(2\pi)^6\, dm^2_{[1,2,3]}\, dm^2_{[1,2]} = \frac{\pi^3}{(2\pi)^{12}}\frac{\vec{p_4}^*}{m_i}\frac{\vec{p_3}^*}{m_{[1,2,3]}}\frac{\vec{p_2}^*}{m_{[1,2]}}\, dm^2_{[1,2,3]}\, dm^2_{[1,2]}$$

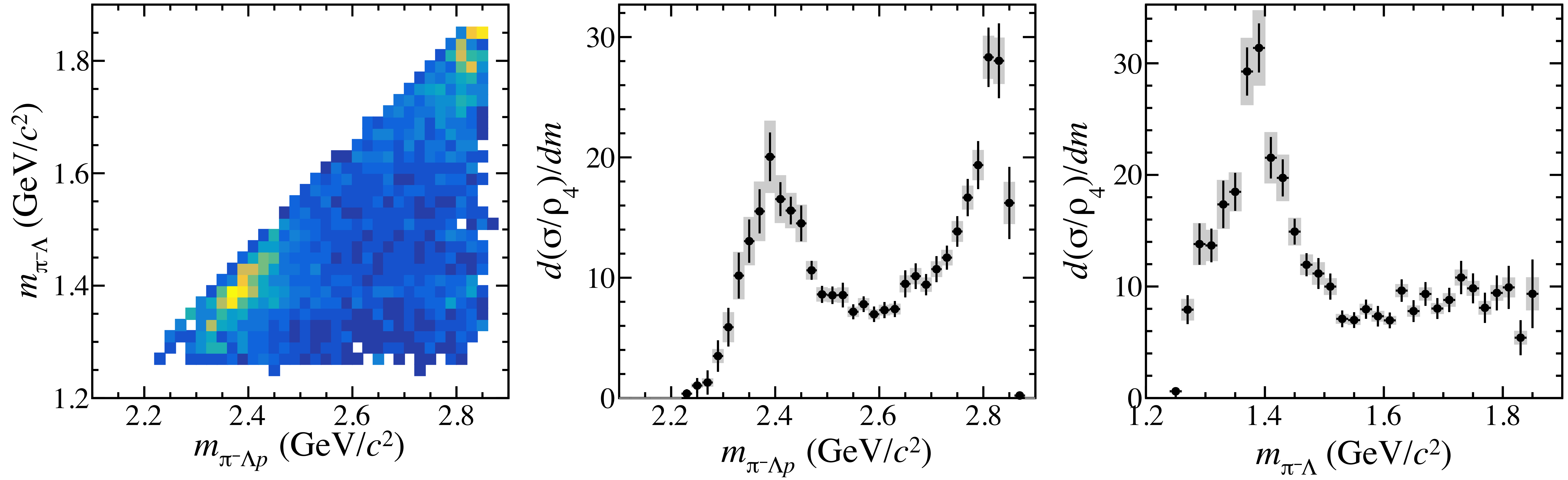
$dm^2_{[1,2,3]} = 2m_{[1,2,3]}\, dm_{[1,2,3]},\ dm^2_{[1,2]} = 2m_{[1,2]}\, dm_{[1,2]}$

$$\frac{d\rho_4}{dm_{[1,2,3]}\, dm_{[1,2]}} = \frac{4\pi^3}{(2\pi)^{12}}\frac{\vec{p_4}^*}{m_i}\ \vec{p_3}^*\ \vec{p_2}^*$$

$$\vec{p_4}^* = \frac{\sqrt{\left(m_i^2 - (m_{[1,2,3]} + m_4)^2\right)\left(m_i^2 - (m_{[1,2,3]} - m_4)^2\right)}}{2m_i},\quad \vec{p_3}^* = \frac{\sqrt{\left(m_{[1,2,3]}^2 - (m_{[1,2]} + m_3)^2\right)\left(m_{[1,2,3]}^2 - (m_{[1,2]} - m_3)^2\right)}}{2m_{[1,2,3]}},\quad \vec{p_2}^* = \frac{\sqrt{\left(m_{[1,2]}^2 - (m_1 + m_2)^2\right)\left(m_{[1,2]}^2 - (m_1 - m_2)^2\right)}}{2m_{[1,2]}}$$

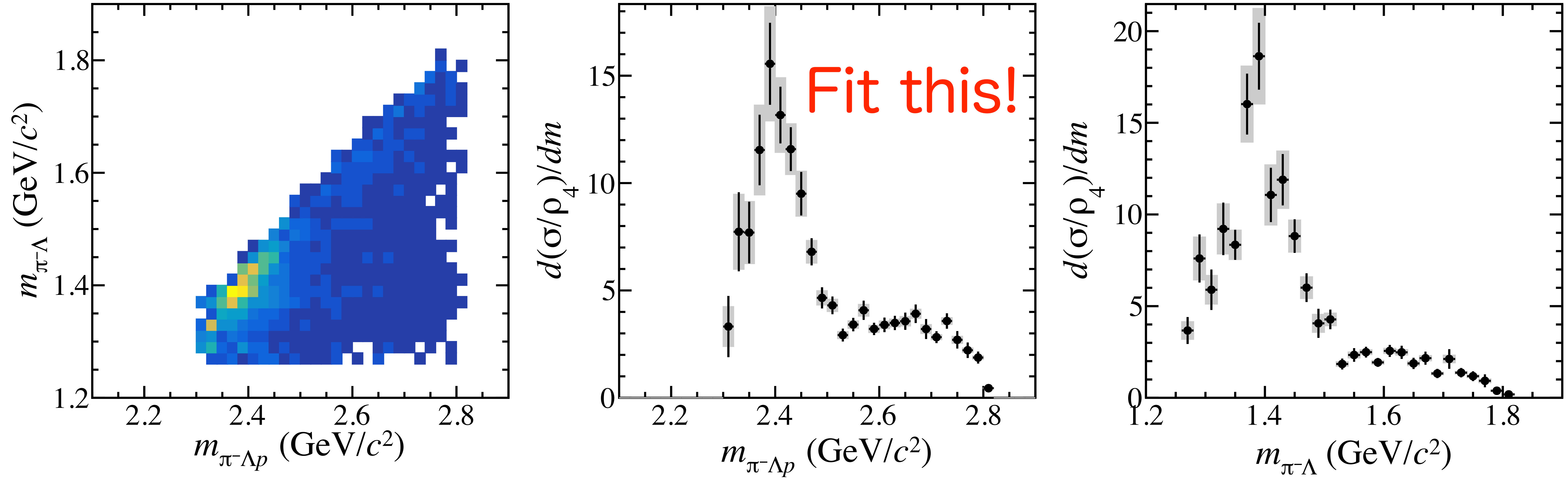
Obtained distributions (ii)

all events

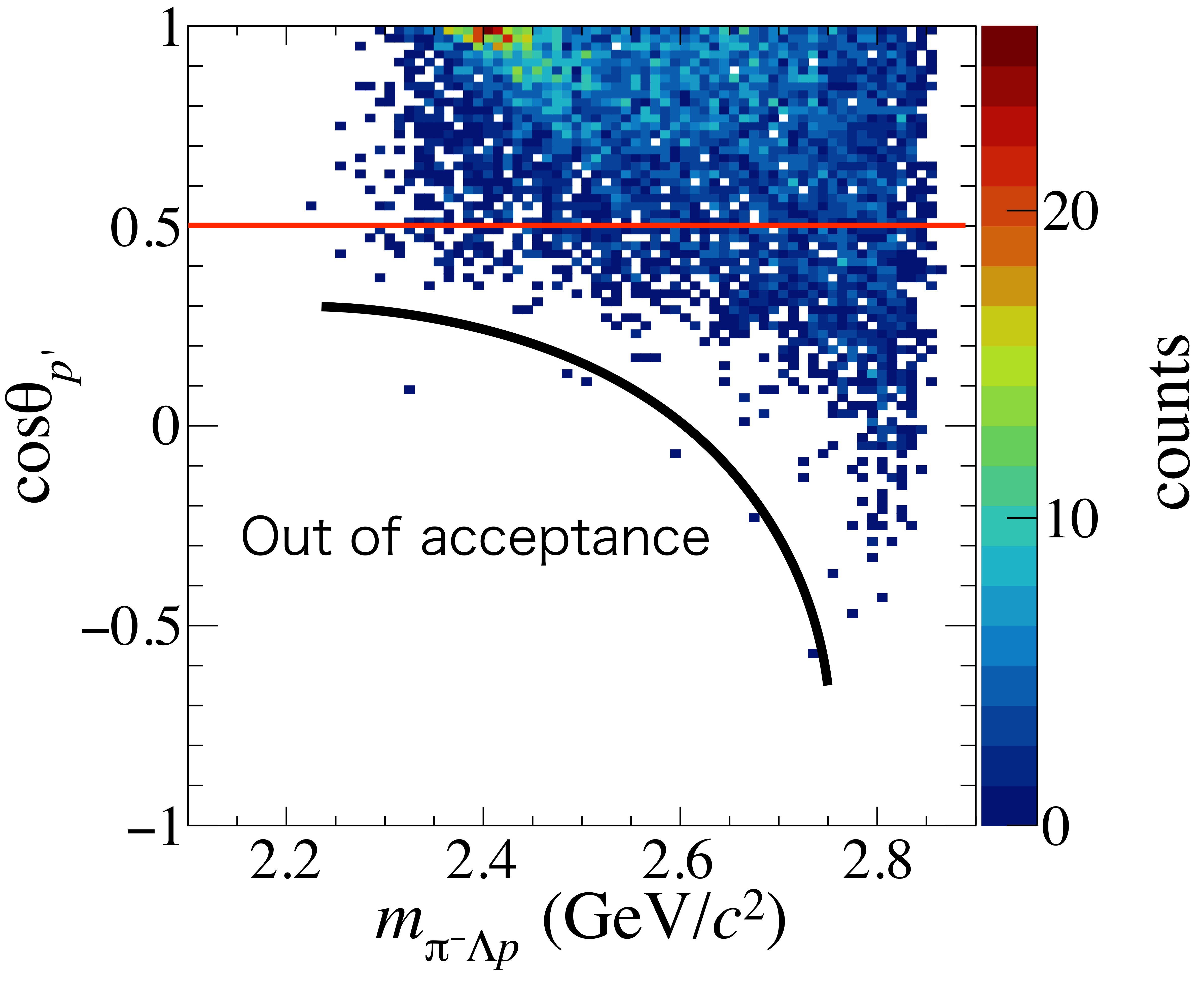


$\cos\theta^*_{p'} > 0.5$

selected

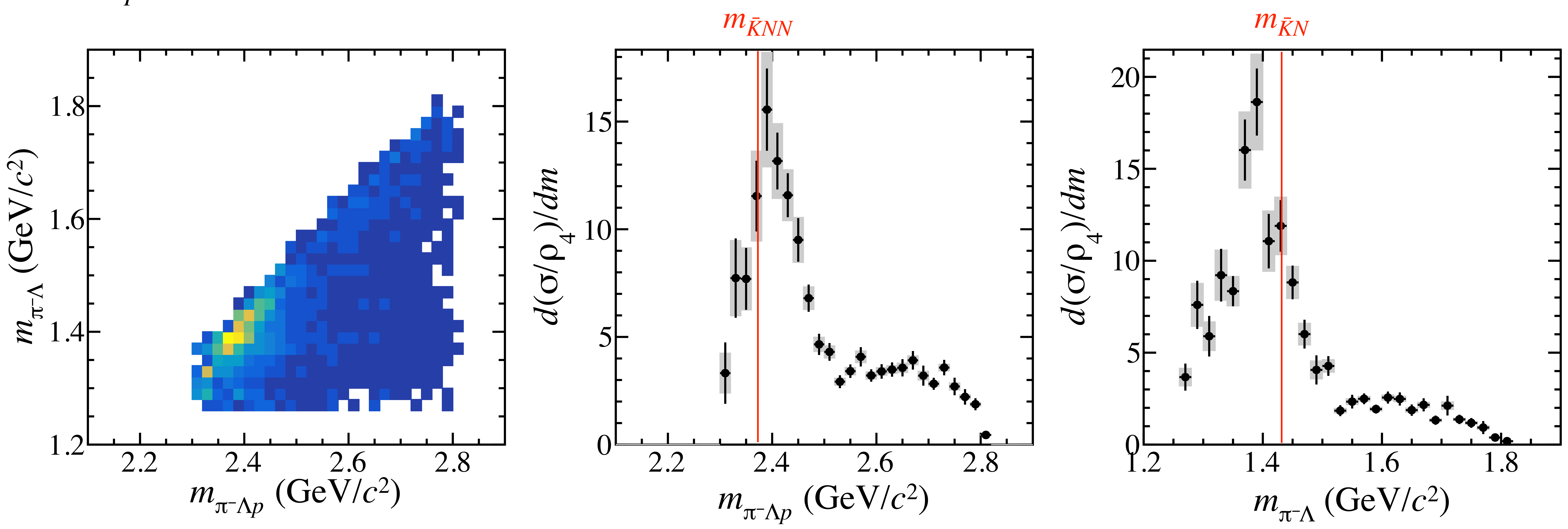


Obtained distributions (iii)

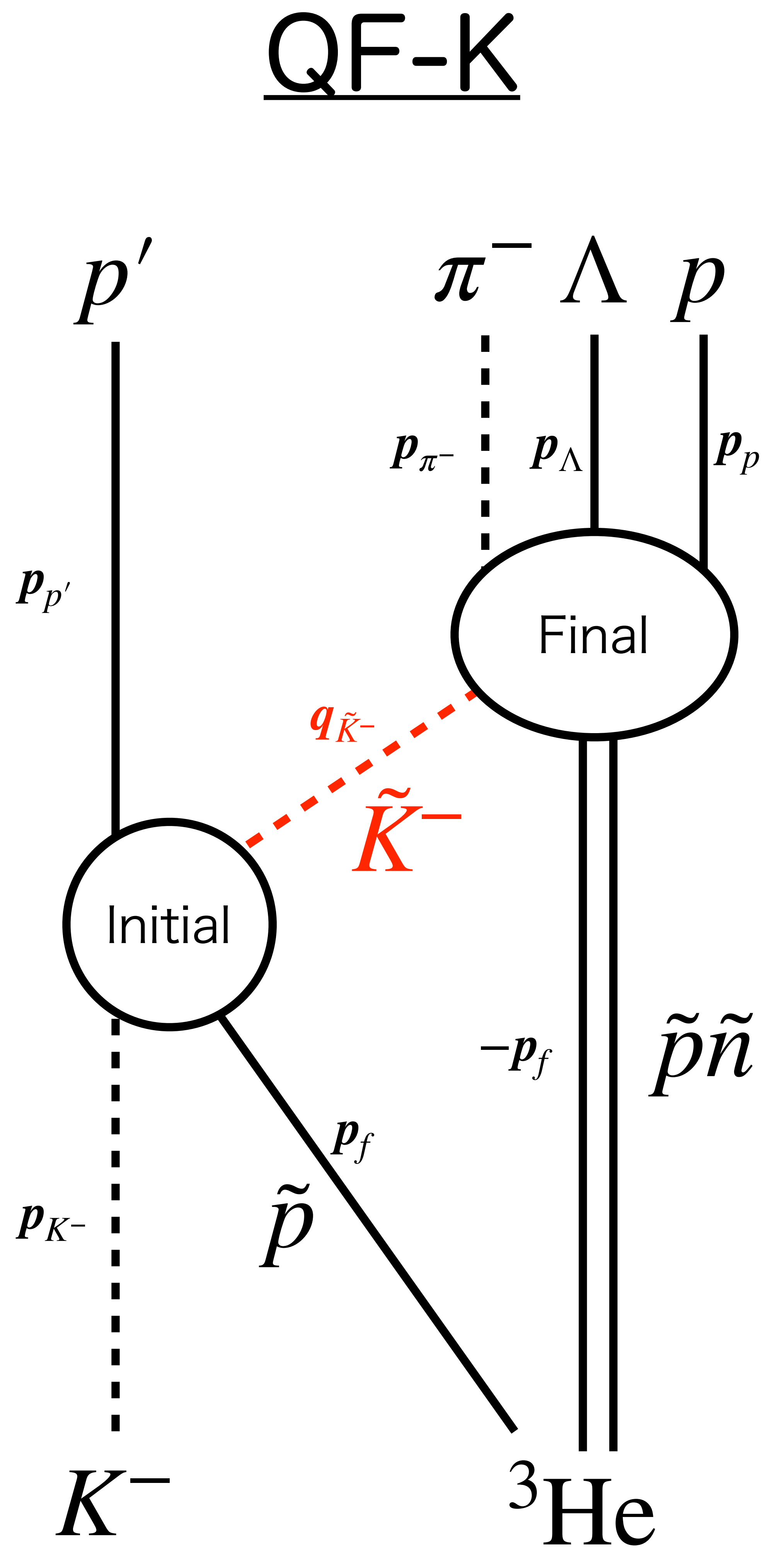


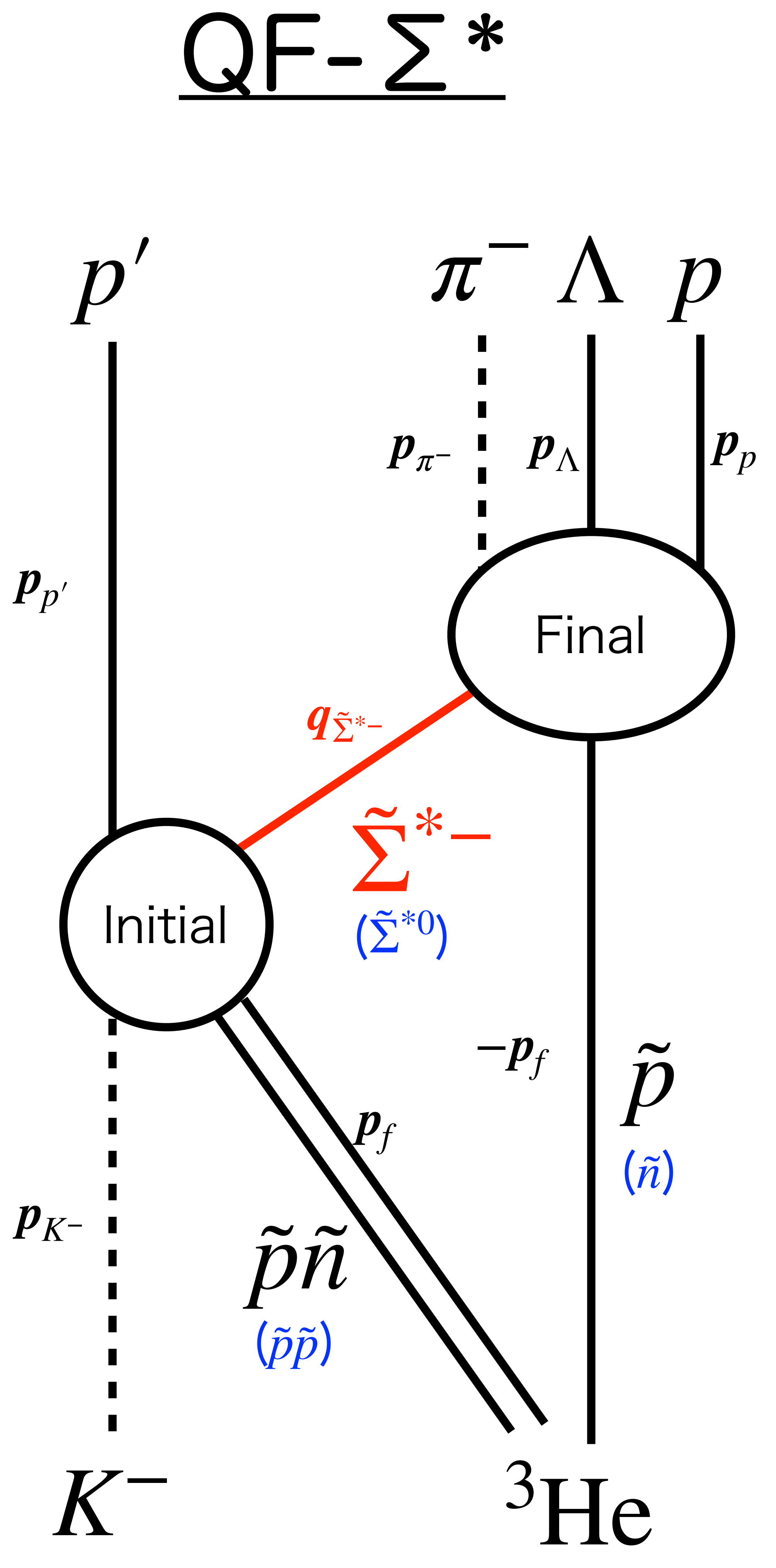
Obtained distributions (iv)

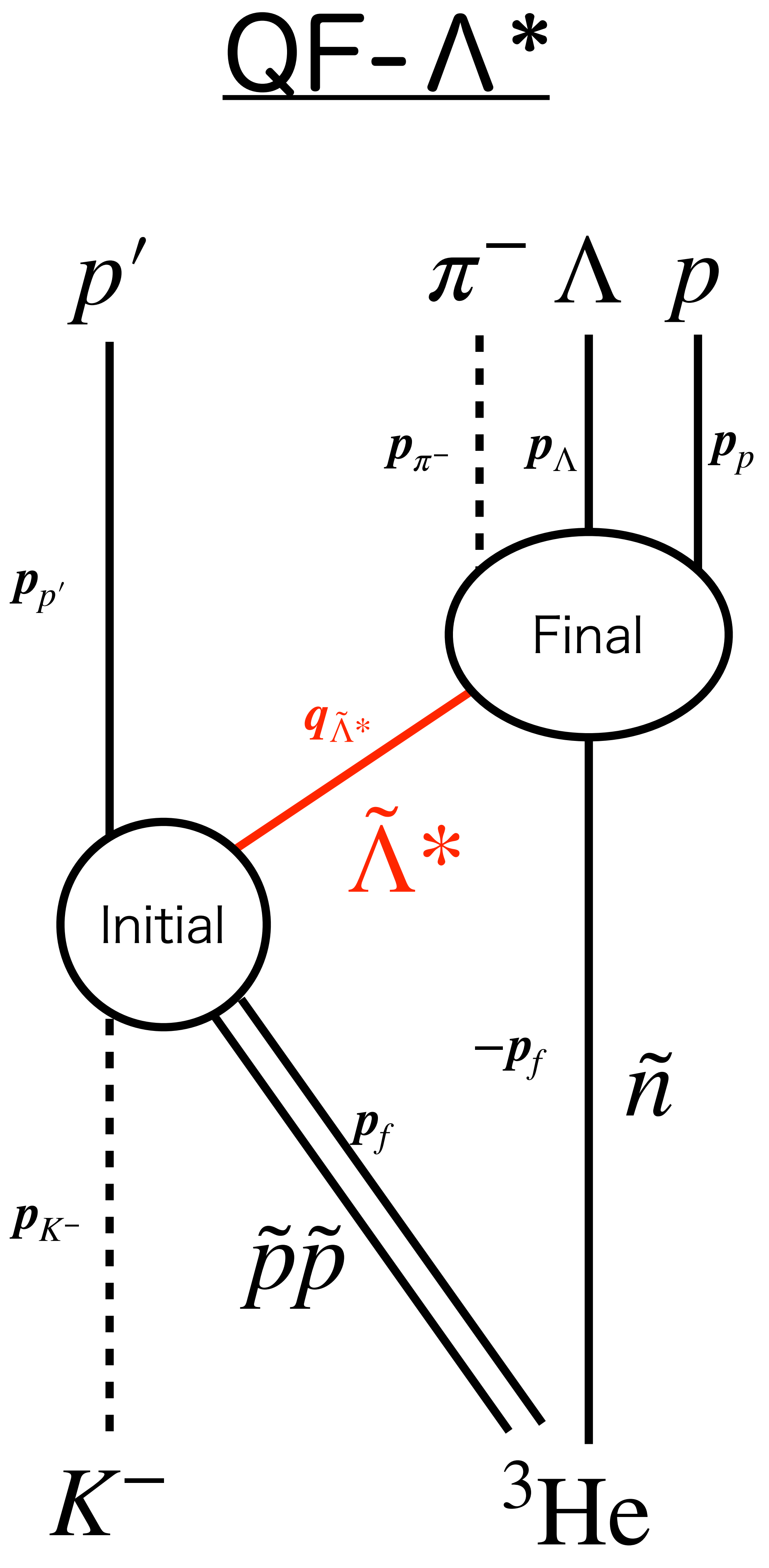
$\cos\theta^*_{p'} > 0.5$ selected



QF processes

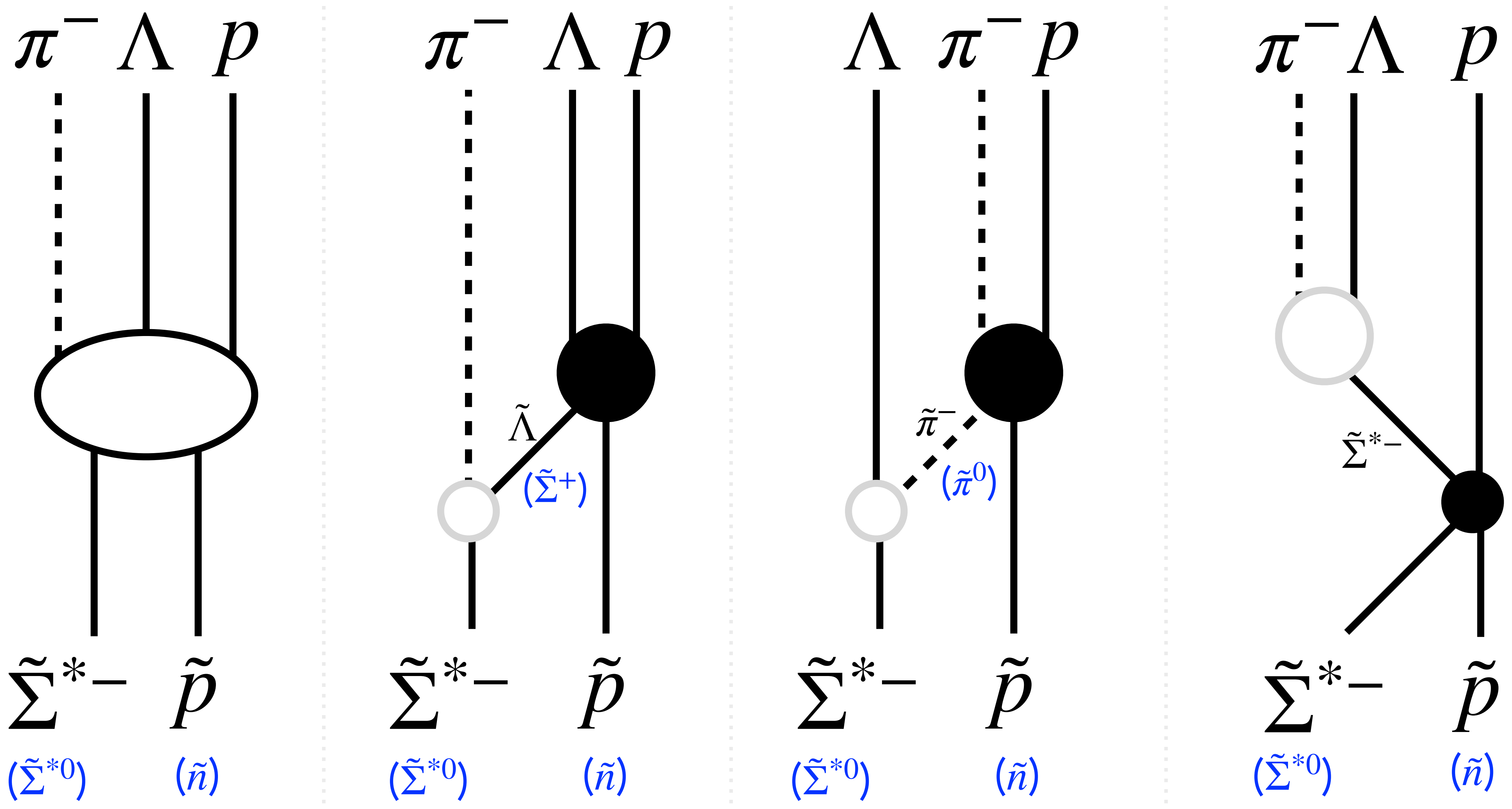






QF processes : Focusing on the final step

QF-Σ*



$m_{\pi^-\Lambda}$ dist.

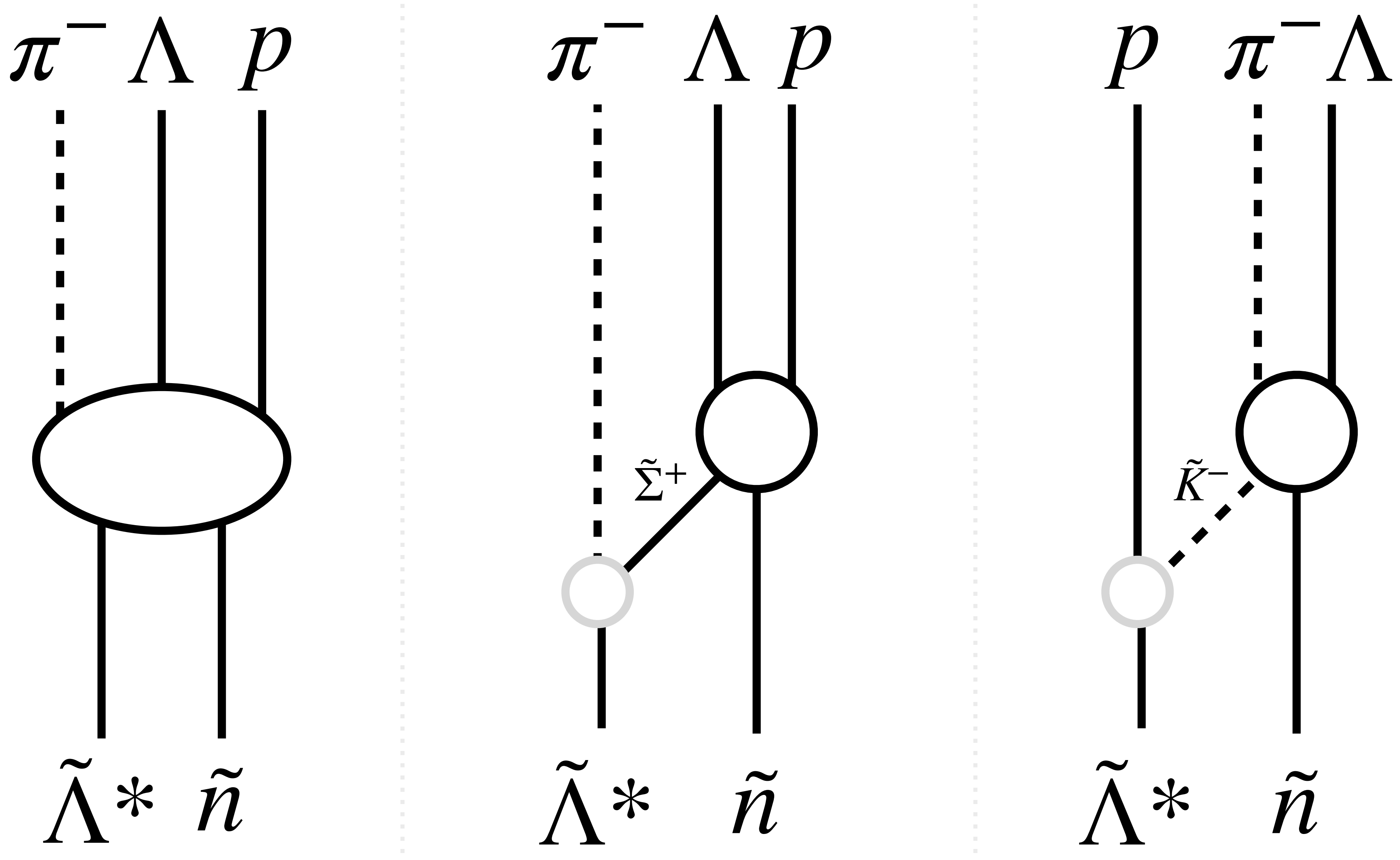
Phase space

Σ*-peak with tail

Σ*-peak with tail

Σ*-peak

QF-Λ*



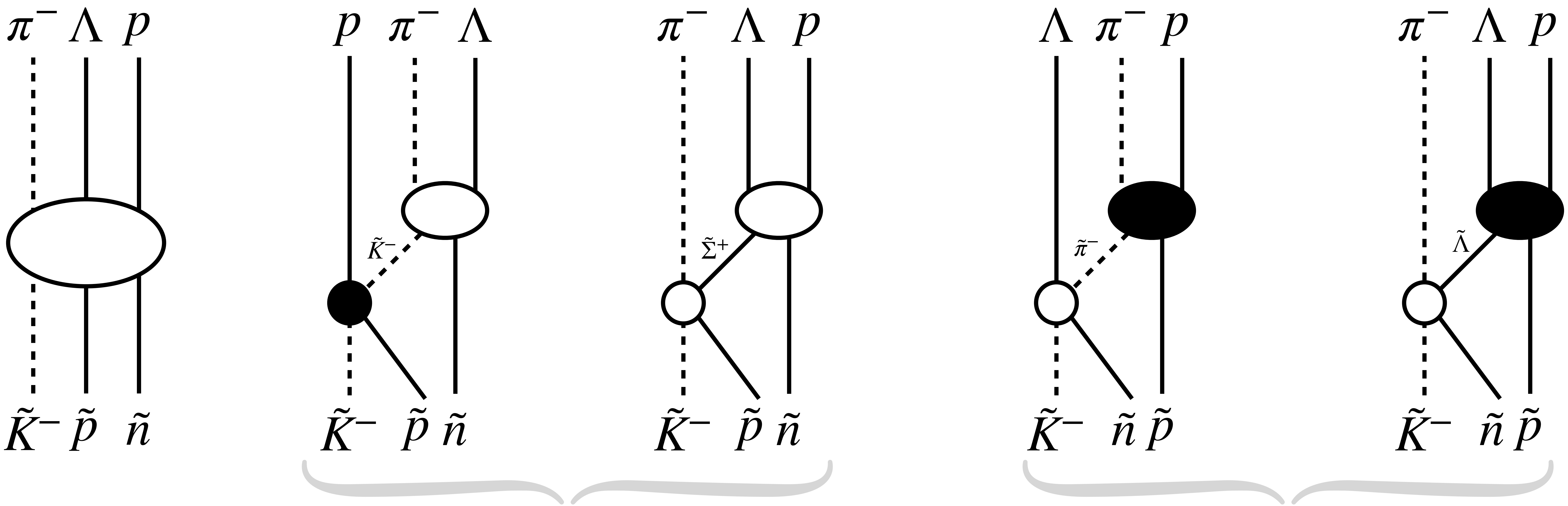
Phase space

Λ*-peak with tail

Λ*-peak with tail
Σ*-peak

QF processes : Focusing on the final step (ii)

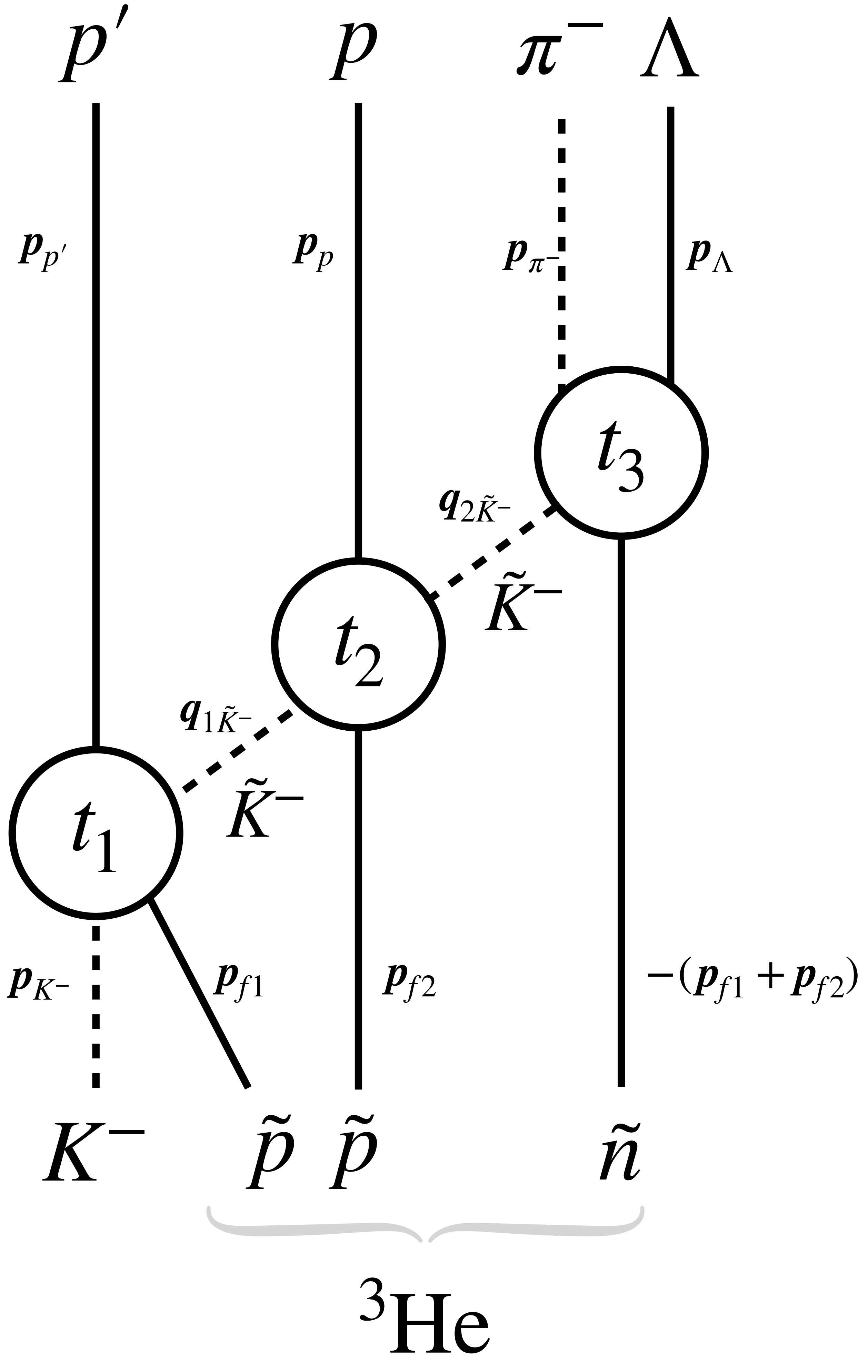
QF-K



similar to QF-Λ^* if $\tilde{K}^-\tilde{p}$ strongly couples to Λ^*

similar to QF-Σ^* if $\tilde{K}^-\tilde{n}$ strongly couples to Σ^*

QF processes : a realistic(?) expression of QF-K



$$d\sigma \propto \left|\left\langle \pi^-\Lambda p p' \left| T \right| K^-\Phi_{^3\mathrm{He}}\right\rangle\right|^2 \delta^4\left(p_{\pi^-}+p_\Lambda+p_p+p_{p'}-p_{K^-}-p_{^3\mathrm{He}}\right)\frac{d^3\boldsymbol{p}_{\pi^-}}{(2\pi)^3\,2E_{\pi^-}}\frac{d^3\boldsymbol{p}_{\Lambda}}{(2\pi)^3\,2E_{\Lambda}}\frac{d^3\boldsymbol{p}_{p}}{(2\pi)^3\,2E_{p}}\frac{d^3\boldsymbol{p}_{p'}}{(2\pi)^3\,2E_{p'}}$$

$$\left\langle \pi^-\Lambda p p' \left| T \right| K^-\Phi_{^3\mathrm{He}}\right\rangle = \left\langle \pi^-\Lambda \left| t_3\, G_{\tilde{K}^-} \right| \tilde{K}^-\tilde{n}\right\rangle \left\langle \tilde{K}^- p \left| t_2\, G_{\tilde{K}^-} \right| \tilde{K}^-\tilde{p}\right\rangle \left\langle \tilde{K}^- p' \left| t_1 \right| K^-\tilde{p}\right\rangle \left|\Phi_{^3\mathrm{He}}\right\rangle$$

$$= \int\frac{d^3\boldsymbol{p}_{f1}}{(2\pi)^3}\int\frac{d^3\boldsymbol{p}_{f2}}{(2\pi)^3}\; t_3\left(\boldsymbol{p}_{\pi^-}, \boldsymbol{p}_\Lambda, \boldsymbol{q}_{2\tilde{K}^-}, -(\boldsymbol{p}_{f1}+\boldsymbol{p}_{f2})\right)\; G_{\tilde{K}^-}\left(\boldsymbol{q}_{2\tilde{K}^-}, -(\boldsymbol{p}_{f1}+\boldsymbol{p}_{f2})\right)\; t_2\left(\boldsymbol{q}_{2\tilde{K}^-}, \boldsymbol{p}_p, \boldsymbol{q}_{1\tilde{K}^-}, \boldsymbol{p}_{f2}\right)\; G_{\tilde{K}^-}\left(\boldsymbol{q}_{1\tilde{K}^-}, \boldsymbol{p}_{f2}\right)\; t_1\left(\boldsymbol{q}_{1\tilde{K}^-}, \boldsymbol{p}_{p'}, \boldsymbol{p}_{K^-}, \boldsymbol{p}_{f1}\right)\; \Phi_{^3\mathrm{He}}\left(\boldsymbol{p}_{f1}, \boldsymbol{p}_{f2}\right)$$

× form factors?

$t_3 : K^-n \to \pi^-\Lambda$ amplitude $\to \Sigma(1385)^-$-pole

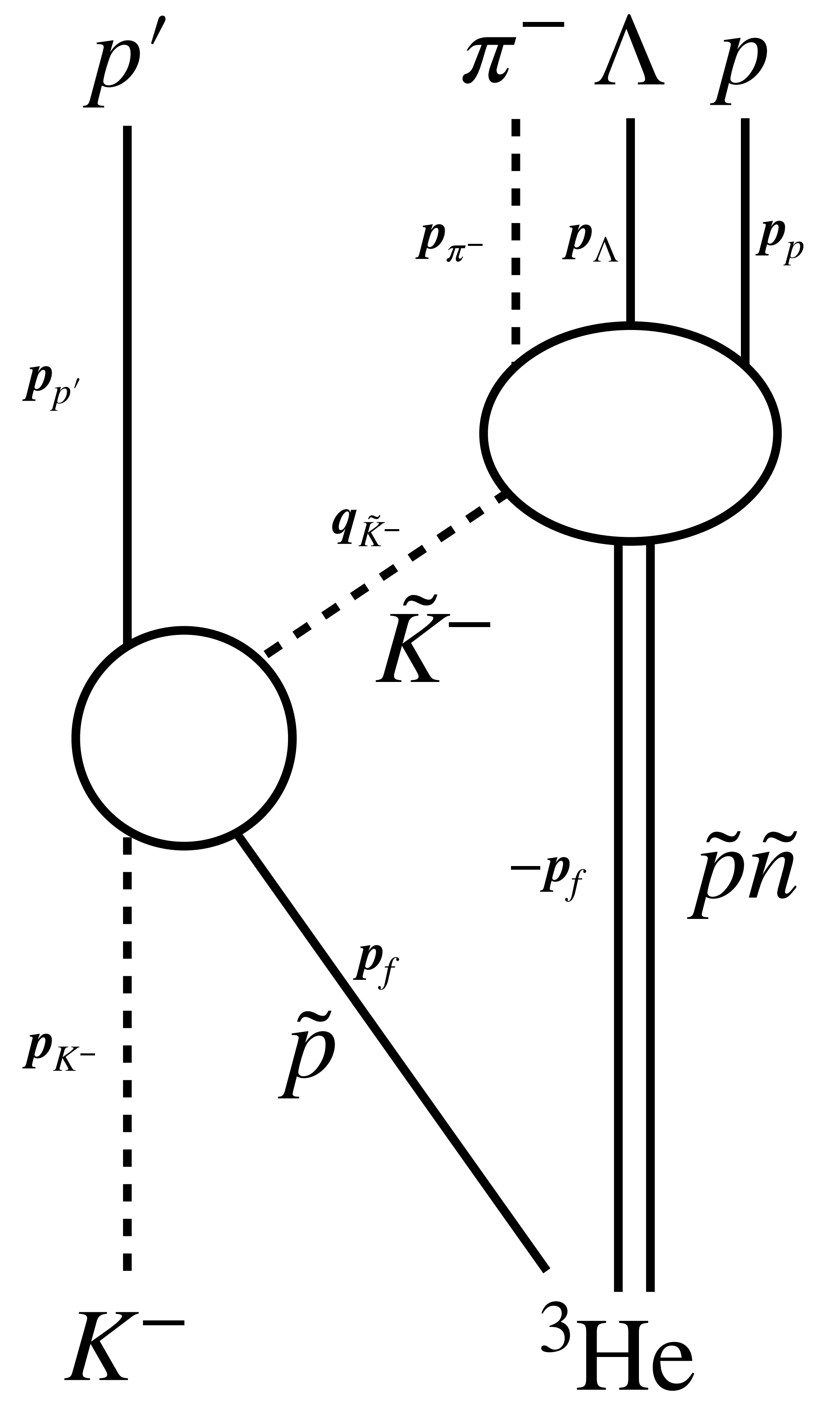
$t_2 : K^-p \to K^-p$ amplitude $\to \Lambda(1405)$-pole

$t_1 : K^-p \to K^-p$ amplitude $\to Y^*(\sim 1800)$-poles

Phenomenological shape

This is future work. Today's work is much more simple.

Kinematics of QF-K



Center of mass energy of the initial step

$$w_I = \left(m_{K^-}^2 + m_{\tilde{p}}^2 + 2E_{K^-}E_{\tilde{p}} - 2\boldsymbol{p}_{K^-}\cdot\boldsymbol{p}_f\right)^{\frac{1}{2}}$$

Momentum of p' in the $(K^-\tilde{p})$-cm frame

$$p_{p'}^{(K^-\tilde{p})*} = \frac{\sqrt{\left(w_I^2 - \left(m_{p'} + m_{\tilde{K}^-}\right)^2\right)\left(w_I^2 - \left(m_{p'} - m_{\tilde{K}^-}\right)^2\right)}}{2w_I}$$

↓ boost to the $(K^{-3}\mathrm{He})$-cm frame

$p_{p'}^*$: Momentum of p' in the $(K^{-3}\mathrm{He})$-cm frame

Kinetic energy of p' in the $(K^{-3}\mathrm{He})$-cm frame

$$T_{p'}^* = \sqrt{m_p^2 + p_{p'}^{*2}} - m_p$$

Mass of off-shell proton/neutron

$$m_{\tilde{p}}^2 = m_{\tilde{n}}^2 = \frac{5m_{^3\mathrm{He}}^2 - 4m_{^3\mathrm{He}}\sqrt{m_{^3\mathrm{He}}^2 + 9p_f^2}}{9}$$

(To satisfy $(p_{\tilde{p}} + p_{\tilde{p}\tilde{n}})^\mu (p_{\tilde{p}} + p_{\tilde{p}\tilde{n}})_\mu = m_{^3\mathrm{He}}$)

Mass distribution of off-shell antikaon

$$p(m_{\tilde{K}^-}) = \frac{1}{2\pi}\frac{\Gamma_{\tilde{K}^-}}{\left(m_{\tilde{K}^-} - m_{K^-}\right)^2 + \left(\Gamma_{\tilde{K}^-}/2\right)^2}$$

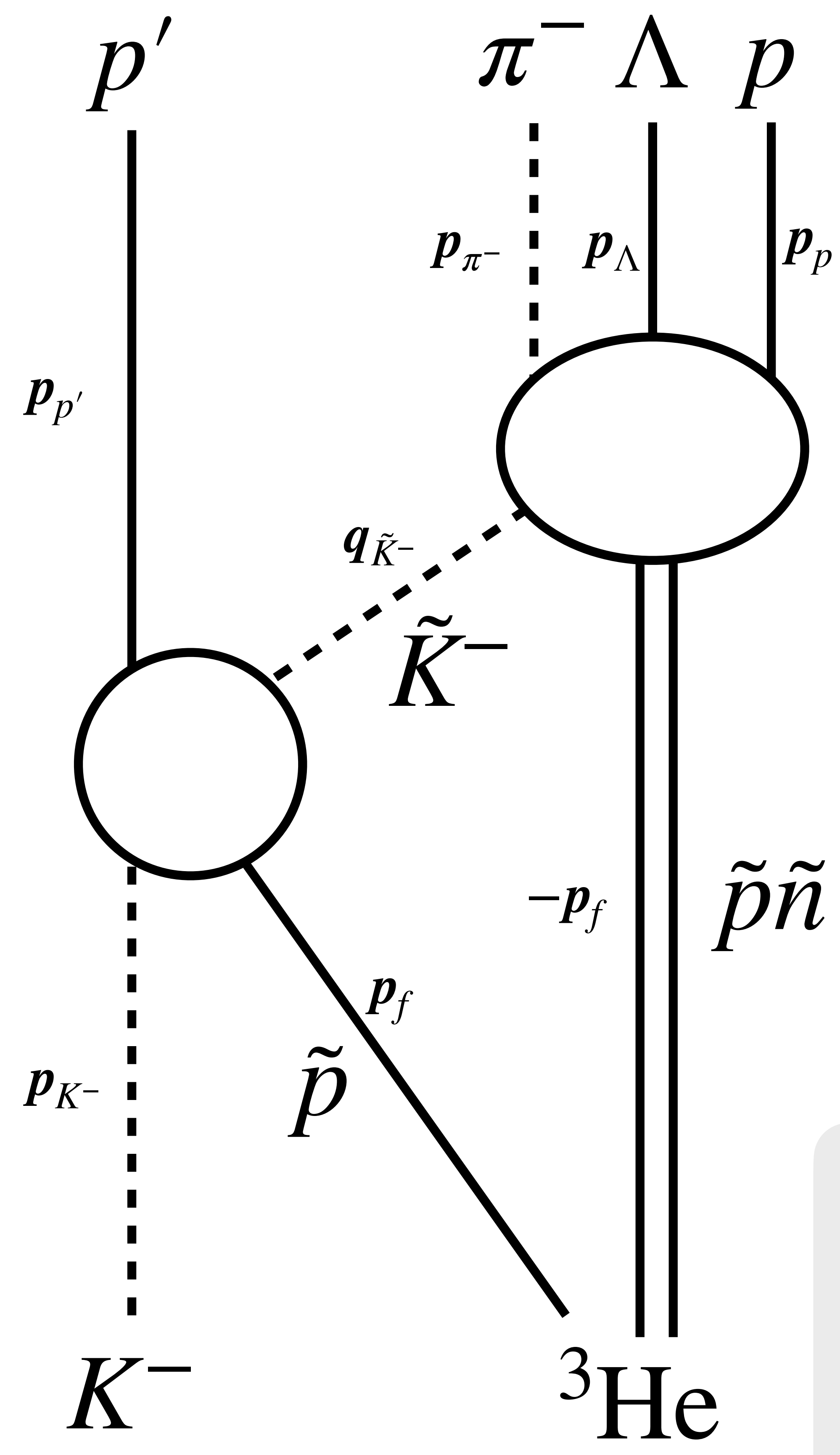
Center of mass energy of the final step

$$w_F = \left(\left(w - m_p\right)^2 - w\,T_{p'}^*\right)^{\frac{1}{2}} = m_{\pi^-\Lambda p}$$

Center of mass energy of the $K^- + {}^3\mathrm{He}$ reaction

$$w = \left(m_{K^-}^2 + m_{^3\mathrm{He}}^2 + 2m_{^3\mathrm{He}}E_{K^-}\right)^{\frac{1}{2}}$$

Kinematics of QF-K (ii)



Four-momentum of K^- in the $(K^-\tilde{p})$-cm frame

$$\begin{pmatrix} E_{K^-}^{(K^-p)*} \\ p_{K^-}^{(K^-p)*}\cos\theta_{K^-}^{(K^-p)*} \\ p_{K^-}^{(K^-p)*}\sin\theta_{K^-}^{(K^-p)*} \end{pmatrix} = \frac{1}{w_I} \begin{pmatrix} E_{K^-}+E_p & -(p_{K^-}+p_f\cos\theta_f) & -p_f\sin\theta_f \\ -(p_{K^-}+p_f\cos\theta_f) & w_I + \frac{\left(p_{K^-}+p_f\cos\theta_f\right)^2}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-w_I\right) & \frac{\left(p_{K^-}+p_f\cos\theta\right)p_f\sin\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-w_I\right) \\ -p_f\sin\theta_f & \frac{\left(p_{K^-}+p_f\cos\theta\right)p_f\sin\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-w_I\right) & w_I + \frac{p_f^2\sin^2\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-w_I\right) \end{pmatrix} \begin{pmatrix} \sqrt{m_{K^-}^2+p_{K^-}^2} \\ p_{K^-} \\ 0 \end{pmatrix}$$

$$\cos\theta_f = \frac{\boldsymbol{p}_{K^-}\cdot\boldsymbol{p}_f}{\boldsymbol{p}_{K^-}\quad\boldsymbol{p}_f} \text{ (in the lab frame)}$$

Four-momentum of p' in the $(K^-\tilde{p})$-cm frame

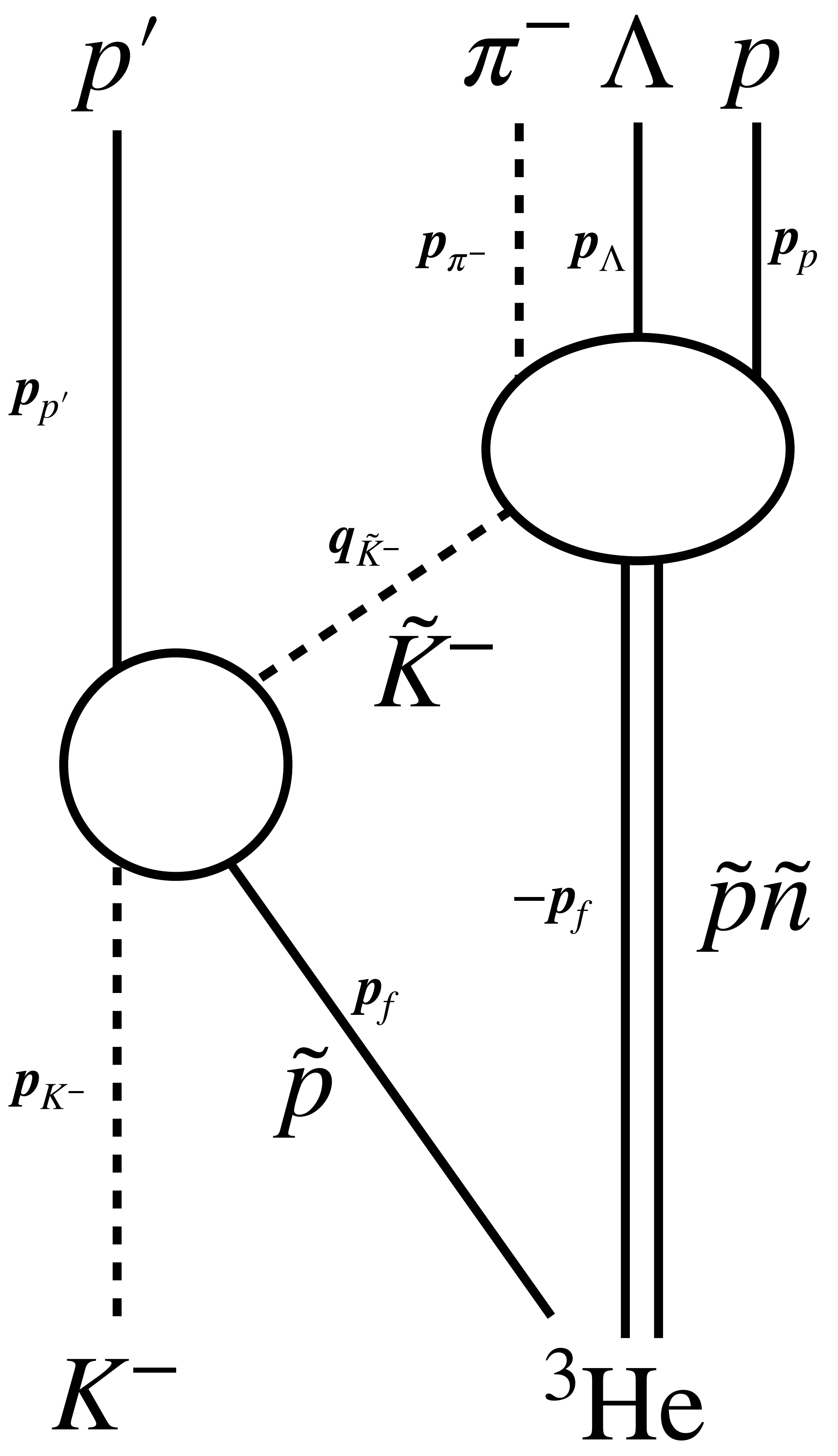
$$\begin{pmatrix} E_{p'}^{(K^-p)*} \\ p_{p'}^{(K^-p)*}\cos\theta_{p'}^{(K^-p)*} \\ p_{p'}^{(K^-p)*}\sin\theta_{p'}^{(K^-p)*} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{K^-}^{(K^-p)*} & -\sin\theta_{K^-}^{(K^-p)*} \\ 0 & \sin\theta_{K^-}^{(K^-p)*} & \cos\theta_{K^-}^{(K^-p)*} \end{pmatrix} \begin{pmatrix} \sqrt{m_p^2+p_{p'}^{(K^-\tilde{p})*2}} \\ p_{p'}^{(K^-\tilde{p})*}\cos\theta_{K^-\tilde{p}\to\tilde{K}^-p'} \\ p_{p'}^{(K^-\tilde{p})*}\sin\theta_{K^-\tilde{p}\to\tilde{K}^-p'} \end{pmatrix}$$

$$\cos\theta_{K^-\tilde{p}\to\tilde{K}^-p'} = \frac{\boldsymbol{p}_{K^-}^{(K^-\tilde{p})*}\cdot\boldsymbol{p}_{p'}^{(K^-\tilde{p})*}}{\boldsymbol{p}_{K^-}^{(K^-\tilde{p})*}\quad\boldsymbol{p}_{p'}^{(K^-\tilde{p})*}} \text{ (in the } (K^-\tilde{p})\text{-cm frame)}$$

Four-momentum of p' in the $(K^{-3}\mathrm{He})$-cm frame

$$\begin{pmatrix} E_{p'}^* \\ p_{p'}^*\cos\theta_{p'}^* \\ p_{p'}^*\sin\theta_{p'}^* \end{pmatrix} = \frac{1}{w}\frac{1}{w_I} \begin{pmatrix} E_{K^-}+m_{^3\mathrm{He}} & -p_{K^-} & 0 \\ -p_{K^-} & E_{K^-}+m_{^3\mathrm{He}} & 0 \\ 0 & 0 & w \end{pmatrix} \begin{pmatrix} E_{K^-}+E_p & p_{K^-}+p_f\cos\theta_f & p_f\sin\theta_f \\ p_{K^-}+p_f\cos\theta_f & w_I + \frac{\left(p_{K^-}+p_f\cos\theta_f\right)^2}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-w_I\right) & \frac{\left(p_{K^-}+p_f\cos\theta\right)p_f\sin\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-w_I\right) \\ p_f\sin\theta_f & \frac{\left(p_{K^-}+p_f\cos\theta\right)p_f\sin\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-w_I\right) & w_I + \frac{p_f^2\sin^2\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-w_I\right) \end{pmatrix} \begin{pmatrix} E_{p'}^{(K^-p)*} \\ p_{p'}^{(K^-p)*}\cos\theta_{p'}^{(K^-p)*} \\ p_{p'}^{(K^-p)*}\sin\theta_{p'}^{(K^-p)*} \end{pmatrix}$$

Kinematics of QF-K (iii)



Center of mass energy of the final step

$$w_F = \left(\left(w - m_p\right)^2 - w\,T^*_{p'}\right)^{\frac{1}{2}} = m_{\pi^-\Lambda p}$$

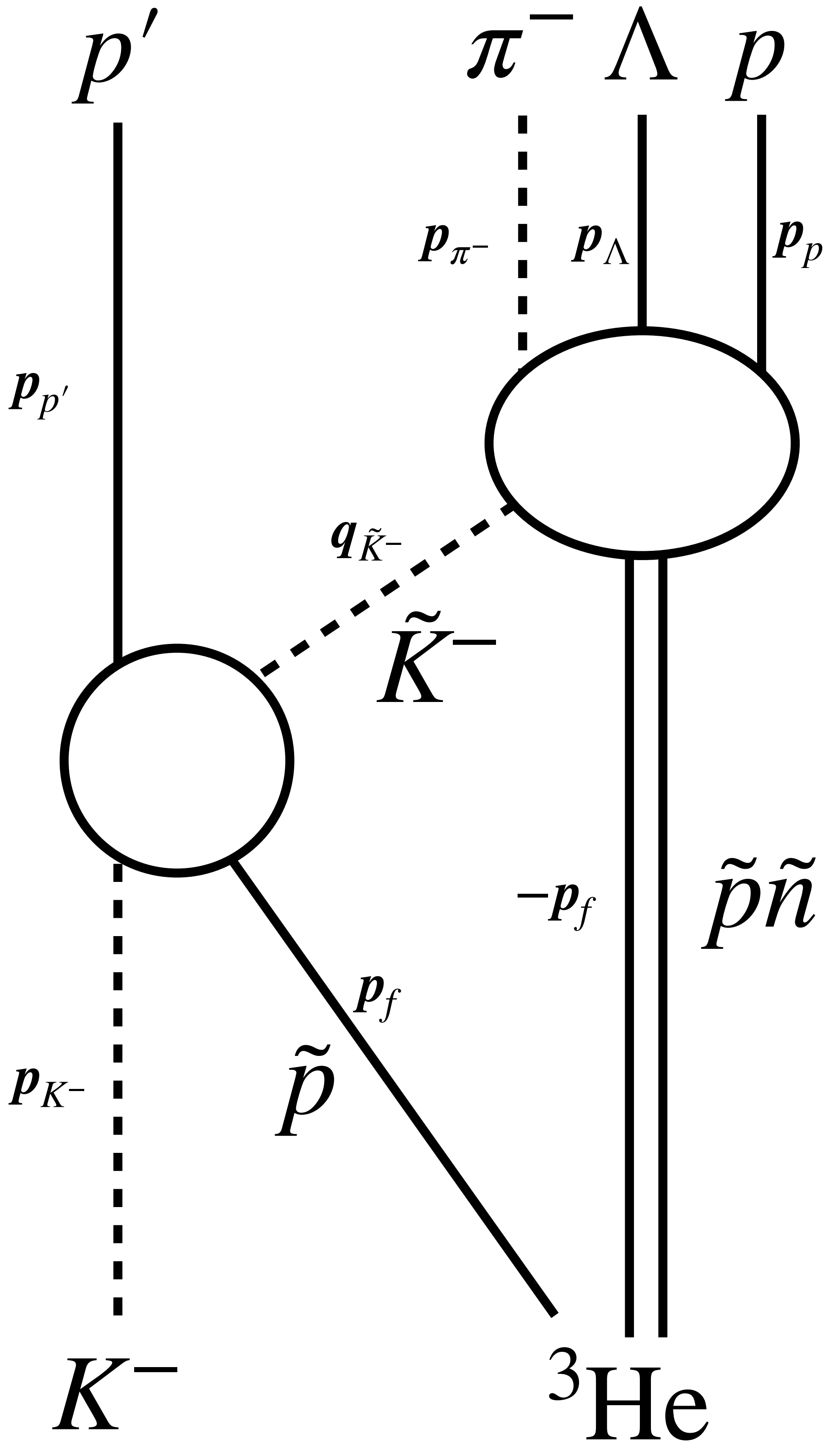
Distribution of $m_{\pi^-\Lambda p}$

$$\frac{d\sigma}{dm_{\pi^-\Lambda p}} \propto \int dw \int dT^*_{p'}\, p\,(w)\, p\left(T^*_{p'}\right) \delta\left(\left(w - m_p\right)^2 - w\,T^*_{p'} - m^2_{\pi^-\Lambda p}\right)$$

$$= \int dp_{K^-} \int d\boldsymbol{p}_f \int d\cos\theta_{K^-\tilde{p}\to\tilde{K}^-p'} \int dm_{\tilde{K}^-}\, p\left(p_{K^-}\right) p\left(\boldsymbol{p}_f\right) p\left(\cos\theta_{K^-\tilde{p}\to\tilde{K}^-p'}\right) p\left(m_{\tilde{K}^-}\right)\ \delta\left(\left(w - m_p\right)^2 - w\,T^*_{p'} - m^2_{\pi^-\Lambda p}\right)$$

$$= 2\pi \int dp_{K^-} \int dp_f \int d\cos\theta_f \int d\cos\theta_{K^-\tilde{p}\to\tilde{K}^-p'} \int dm_{\tilde{K}^-}\, p\left(p_{K^-}\right) p_f^2\, p\left(p_f\right) p\left(\cos\theta_{K^-\tilde{p}\to\tilde{K}^-p'}\right) p\left(m_{\tilde{K}^-}\right)\ \delta\left(\left(w - m_p\right)^2 - w\,T^*_{p'} - m^2_{\pi^-\Lambda p}\right)$$

Kinematics of QF-K (iv)



Distribution of $m_{\pi^-\Lambda p}$

$$\frac{d\sigma}{dm_{\pi^-\Lambda p}} \propto 2\pi \int dp_{K^-} \int dp_f \int d\cos\theta_f \int d\cos\theta_{K^-\tilde{p}\to\tilde{K}^-p'} \int dm_{\tilde{K}^-}\, p\left(p_{K^-}\right) p_f^2\, p\left(p_f\right) p\left(\cos\theta_{K^-\tilde{p}\to\tilde{K}^-p'}\right) p\left(m_{\tilde{K}^-}\right)\, \delta\left(\left(w - m_p\right)^2 - w\,T^*_{p'} - m^2_{\pi^-\Lambda p}\right)$$

$$p\left(p_{K^-}\right) = \delta\left(p_{K^-} - 1\ \mathrm{GeV}/c\right)$$

$$p(m_{\tilde{K}^-}) = \frac{1}{2\pi}\frac{\Gamma_{\tilde{K}^-}}{\left(m_{\tilde{K}^-} - m_{K^-}\right)^2 + \left(\Gamma_{\tilde{K}^-}/2\right)^2}$$

$m_{K^-} = 0.4936\ \mathrm{GeV}/c^2$, $\Gamma_{\tilde{K}^-} = 0.02\ \mathrm{GeV}/c^2$,

Not any specific evidence

$$p\left(p_f\right) = a_0 \exp\left(-\frac{p_f^2}{p_0^2}\right) + a_1 \exp\left(-\frac{p_f^2}{p_1^2}\right) + a_2 \exp\left(-\frac{p_f^2}{p_2^2}\right)$$

$a_0 = 0.406113$, $p_0 = 0.0440577\ \mathrm{GeV}/c$,

$a_1 = 0.244472$, $p_1 = 0.0791843\ \mathrm{GeV}/c$,

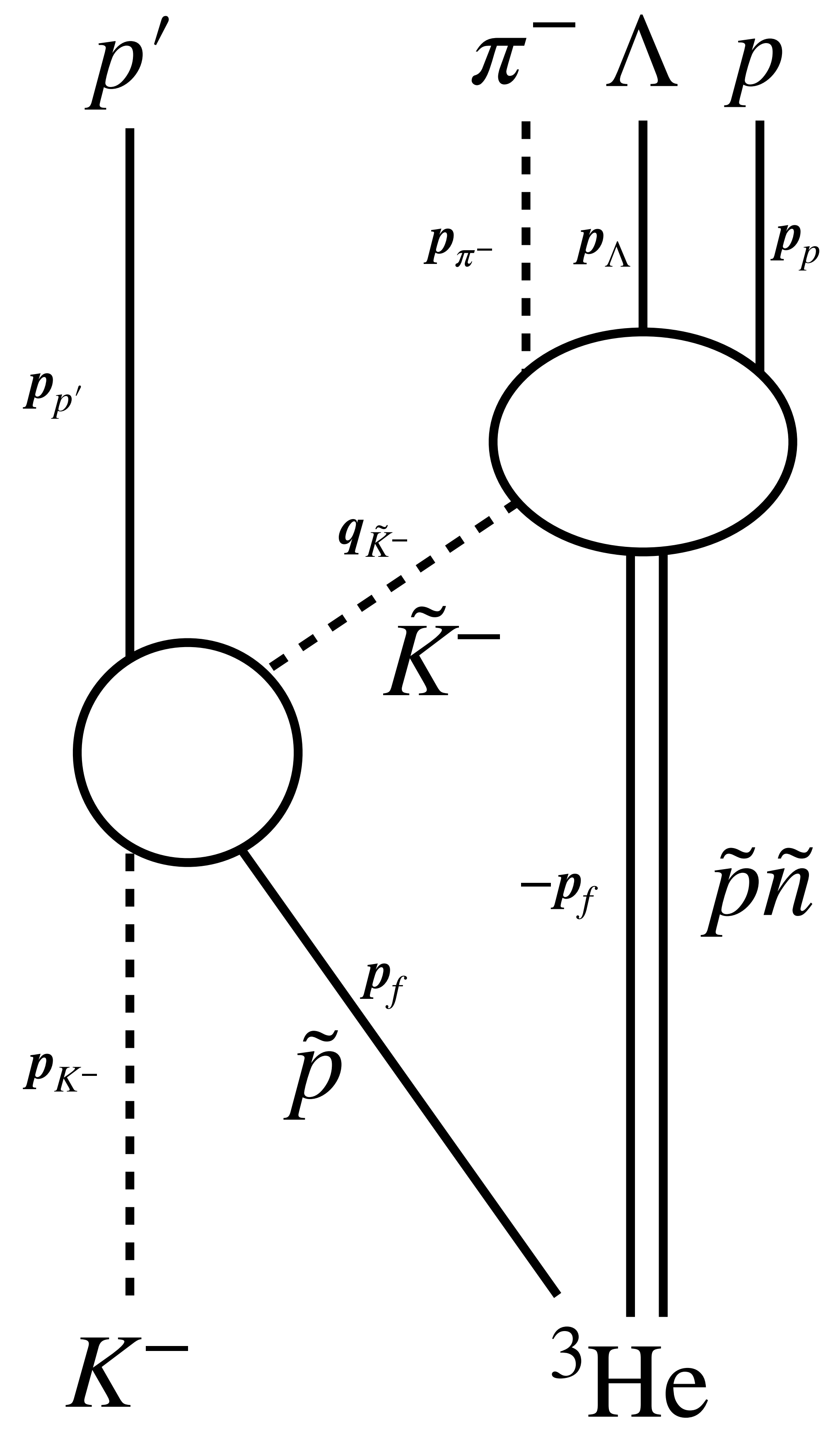
$a_2 = 0.0169276$, $p_2 = 0.125564\ \mathrm{GeV}/c$

E. Jans et al., Physical Review Letters 49, 974 (1982).

$$p\left(\cos\theta_{K^-\tilde{p}\to\tilde{K}^-p'}\right) = \sum_{i=0}^{6} b_i P_i\left(\cos\theta_{K^-\tilde{p}\to\tilde{K}^-p'}\right)$$

b_i : free parameters

Kinematics of QF-K (v)



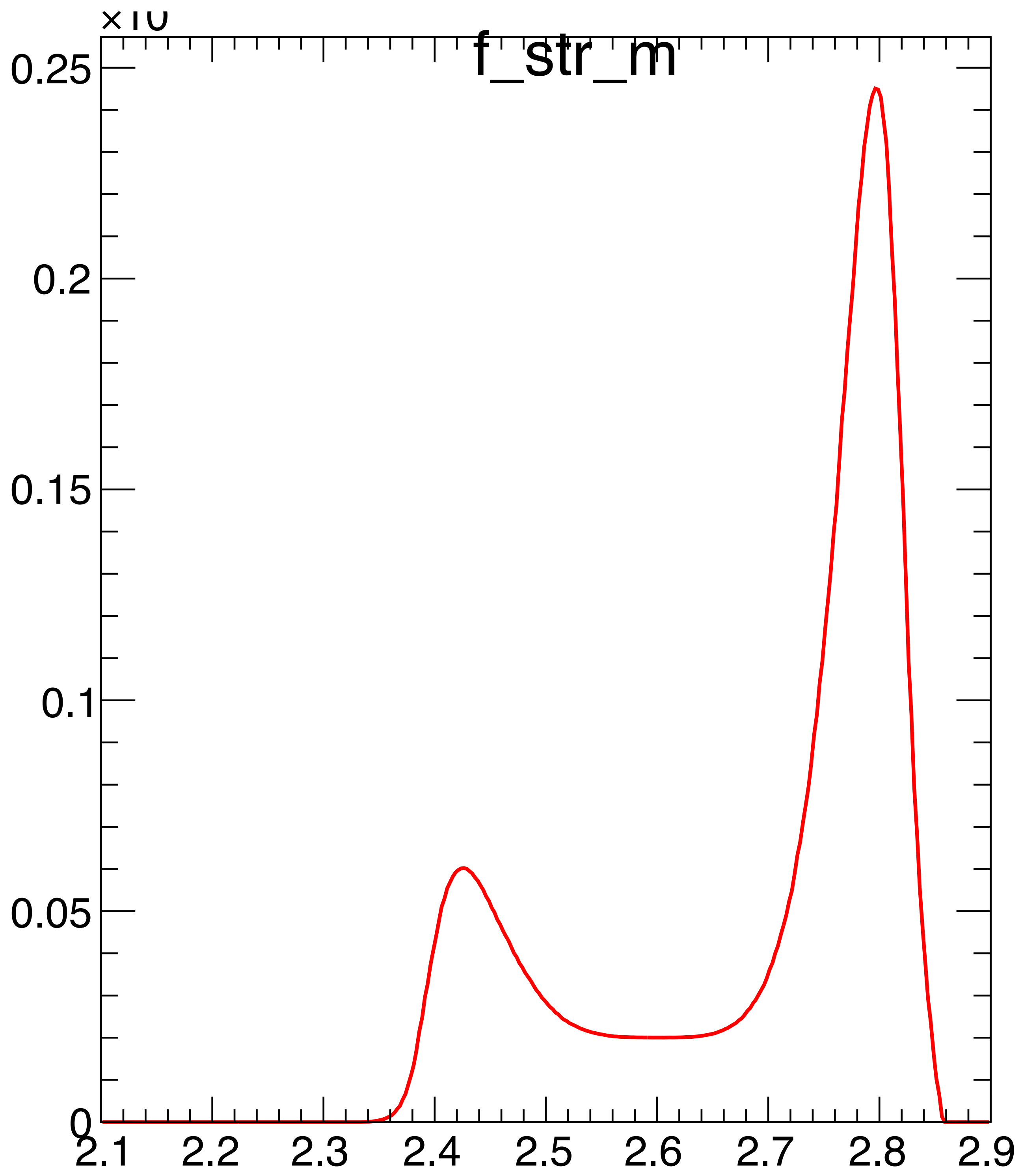
Distribution of $m_{\pi^-\Lambda p}$

$$\frac{d\sigma}{dm_{\pi^-\Lambda p}} \propto 2\pi \int dp_{K^-} \int dp_f \int d\cos\theta_f \int d\cos\theta_{K^-\tilde{p}\to\tilde{K}^-p'} \int dm_{\tilde{K}^-}\, p\left(p_{K^-}\right) p_f^2\, p\left(p_f\right) p\left(\cos\theta_{K^-\tilde{p}\to\tilde{K}^-p'}\right) p\left(m_{\tilde{K}^-}\right)\, \delta\left(\left(w - m_p\right)^2 - w\,T^*_{p'} - m^2_{\pi^-\Lambda p}\right)$$

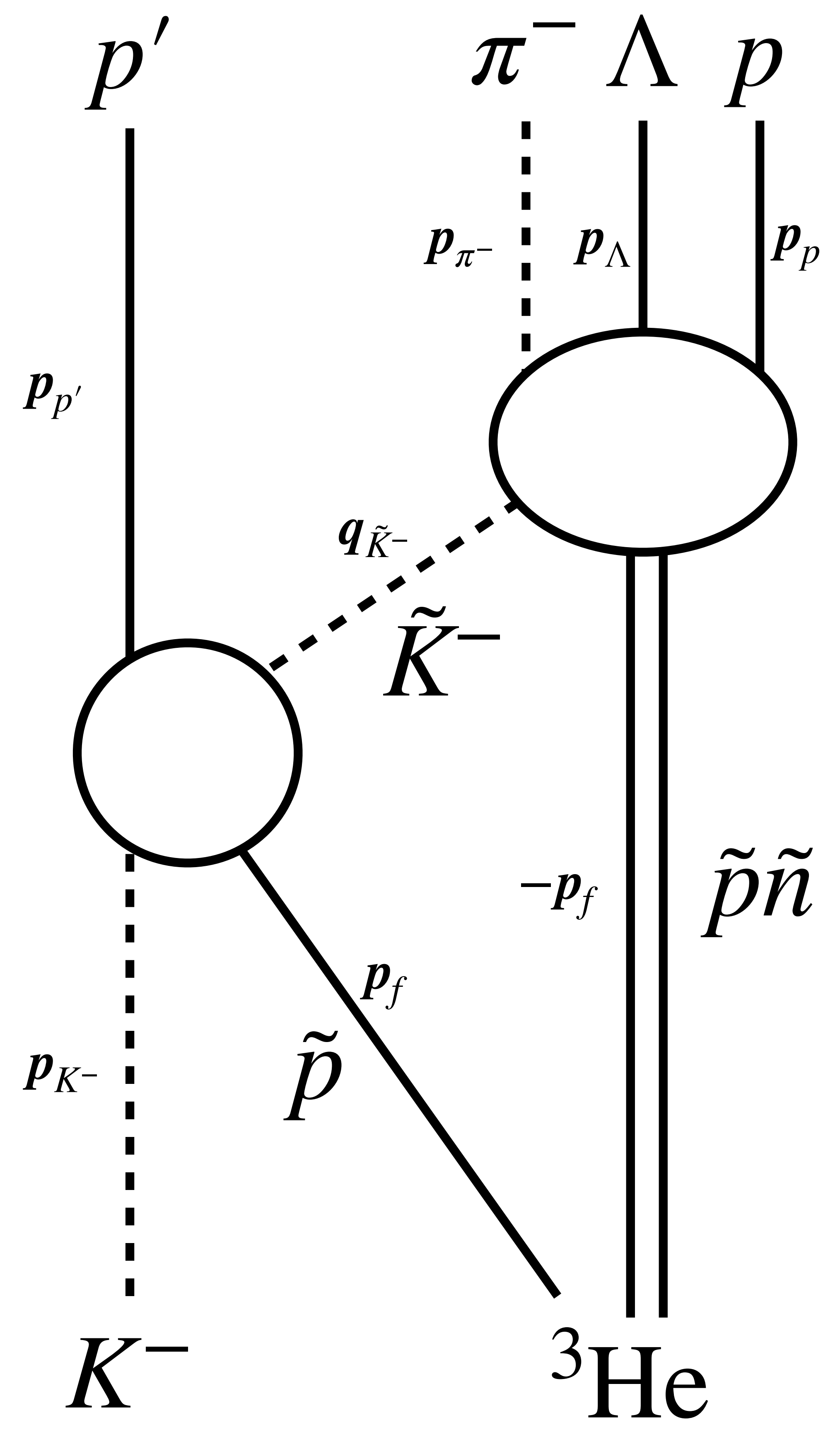
$$p\left(\cos\theta_{K^-\tilde{p}\to\tilde{K}^-p'}\right) = \sum_{i=0}^{6} b_i P_i\left(\cos\theta_{K^-\tilde{p}\to\tilde{K}^-p'}\right)$$

→ If it is elementary $K^-p \to K^-p$

@ $p_{K^-} = 1\ \mathrm{GeV}/c$



Re: Kinematics of QF-K



Center of mass energy of the initial step

$$w_I = \left(m_{K^-}^2 + m_{\tilde{p}}^2 + 2E_{K^-}E_{\tilde{p}} - 2\boldsymbol{p}_{K^-}\cdot\boldsymbol{p}_f\right)^{\frac{1}{2}}$$

Momentum of p' in the $(K^-\tilde{p})$-cm frame

$$p_{p'}^{(K^-\tilde{p})*} = \frac{\sqrt{\left(w_I^2 - \left(m_{p'} + m_{\tilde{K}^-}\right)^2\right)\left(w_I^2 - \left(m_{p'} - m_{\tilde{K}^-}\right)^2\right)}}{2w_I}$$

↓ boost to the $(K^-{}^3\mathrm{He})$-cm frame

$p_{p'}^*$: Momentum of p' in the $(K^-{}^3\mathrm{He})$-cm frame

Kinetic energy of p' in the $(K^-{}^3\mathrm{He})$-cm frame

$$T_{p'}^* = \sqrt{m_p^2 + p_{p'}^{*2}} - m_p$$

Mass of off-shell proton/neutron

$$m_{\tilde{p}}^2 = m_{\tilde{n}}^2 = \frac{5m_{^3\mathrm{He}}^2 - 4m_{^3\mathrm{He}}\sqrt{m_{^3\mathrm{He}}^2 + 9p_f^2}}{9}$$

(To satisfy $(p_{\tilde{p}} + p_{\tilde{p}\tilde{n}})^\mu (p_{\tilde{p}} + p_{\tilde{p}\tilde{n}})_\mu = m_{^3\mathrm{He}}$)

Mass distribution of off-shell antikaon

$$p(m_{\tilde{K}^-}) = \frac{1}{2\pi}\frac{\Gamma_{\tilde{K}^-}}{\left(m_{\tilde{K}^-} - m_{K^-}\right)^2 + \left(\Gamma_{\tilde{K}^-}/2\right)^2}$$

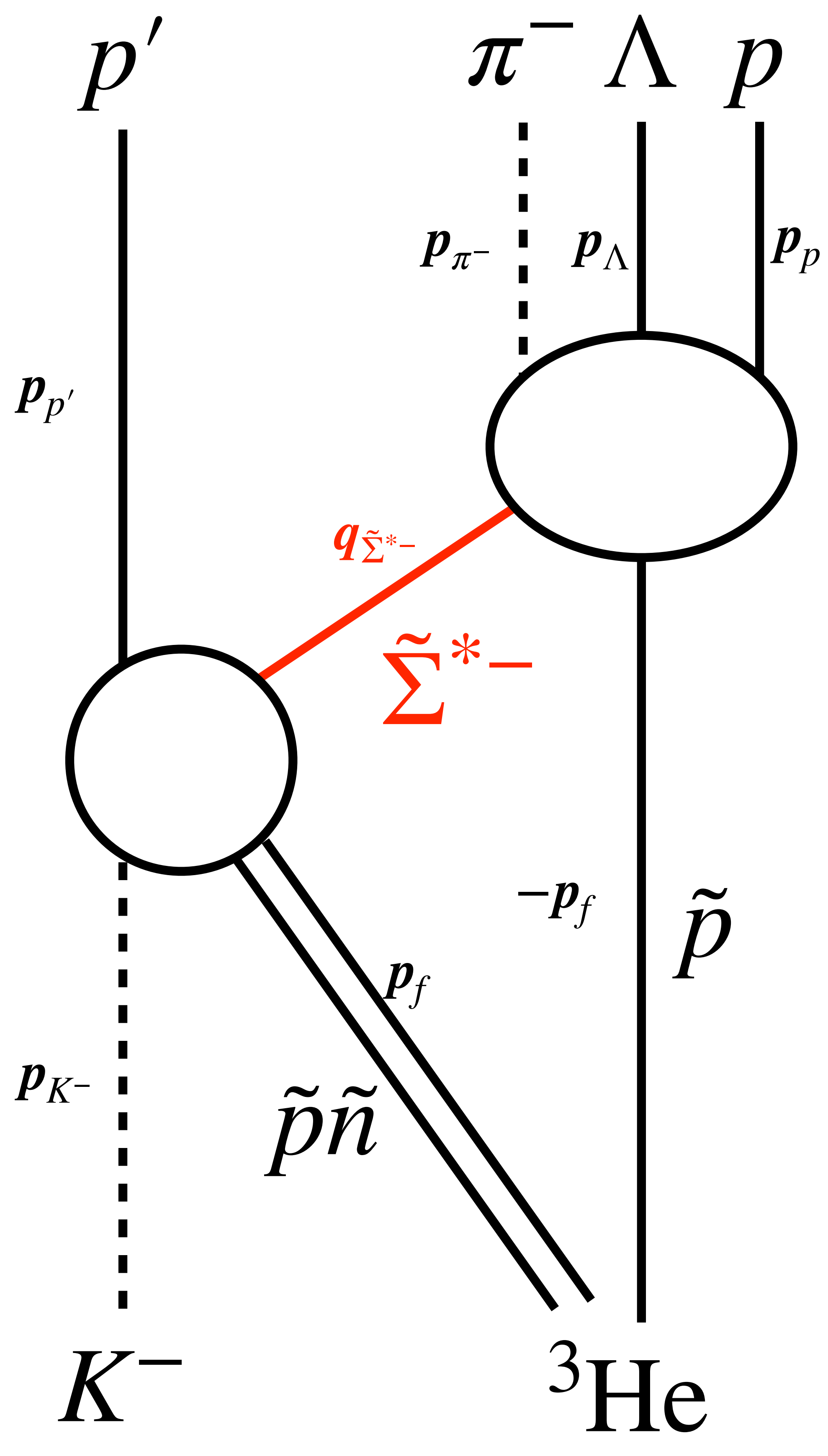
Center of mass energy of the final step

$$w_F = \left(\left(w - m_p\right)^2 - w\,T_{p'}^*\right)^{\frac{1}{2}} = m_{\pi^-\Lambda p}$$

Center of mass energy of the $K^- + {}^3\mathrm{He}$ reaction

$$w = \left(m_{K^-}^2 + m_{^3\mathrm{He}}^2 + 2m_{^3\mathrm{He}}E_{K^-}\right)^{\frac{1}{2}}$$

Kinematics of QF-Y*



Center of mass energy of the initial step

$$w_I = \left(m_{K^-}^2 + m_{\tilde p\tilde n}^2 + 2E_{K^-}E_{\tilde p\tilde n} - 2\mathbf{p}_{K^-}\cdot\mathbf{p}_f\right)^{\frac{1}{2}}$$

Momentum of p' in the $(K^-\tilde p\tilde n)$-cm frame

$$p_{p'}^{(K^-\tilde p\tilde n)*} = \frac{\sqrt{\left(w_I^2 - \left(m_{p'} + m_{\tilde Y^*}\right)^2\right)\left(w_I^2 - \left(m_{p'} - m_{\tilde Y^*}\right)^2\right)}}{2w_I}$$

boost to the $(K^{-3}\mathrm{He})$-cm frame

$p_{p'}^*$: Momentum of p' in the $(K^{-3}\mathrm{He})$-cm frame

Kinetic energy of p' in the $(K^{-3}\mathrm{He})$-cm frame

$$T_{p'}^* = \sqrt{m_p^2 + p_{p'}^{*2}} - m_p$$

Mass of off-shell proton/neutron

$$m_{\tilde p}^2 = m_{\tilde n}^2 = \frac{5m_{^3\mathrm{He}}^2 - 4m_{^3\mathrm{He}}\sqrt{m_{^3\mathrm{He}}^2 + 9p_f^2}}{9}$$

(To satisfy $(p_{\tilde p} + p_{\tilde p\tilde n})^\mu (p_{\tilde p} + p_{\tilde p\tilde n})_\mu = m_{^3\mathrm{He}}$)

Mass distribution of off-shell Y*

$$p(m_{\tilde Y^*}) = \frac{1}{2\pi}\frac{\Gamma_{\tilde Y^*}}{\left(m_{\tilde Y^*} - m_{Y^*}\right)^2 + \left(\Gamma_{\tilde Y^*}/2\right)^2}$$

Center of mass energy of the final step

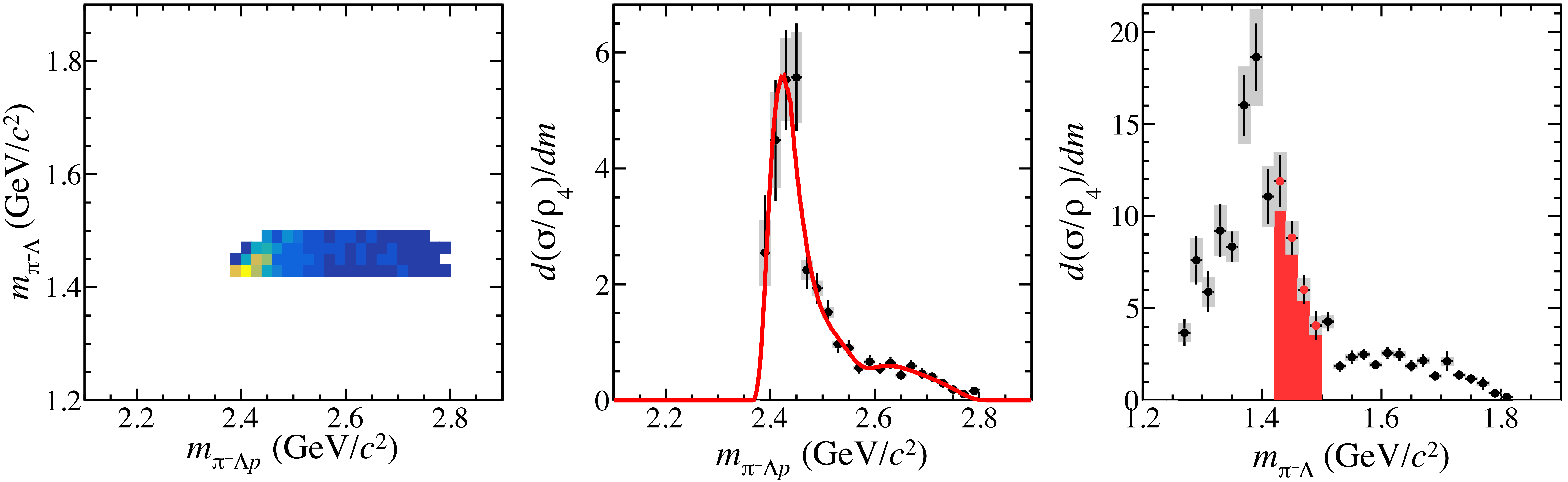
$$w_F = \left(\left(w - m_p\right)^2 - w\,T_{p'}^*\right)^{\frac{1}{2}} = m_{\pi^-\Lambda p}$$

Center of mass energy of the $K^- + {}^3\mathrm{He}$ reaction

$$w = \left(m_{K^-}^2 + m_{^3\mathrm{He}}^2 + 2m_{^3\mathrm{He}}E_{K^-}\right)^{\frac{1}{2}}$$

Fit results (very preliminary)

- Fit $m_{\pi^-\Lambda p}$ with selecting $1.42 < m_{\pi^-\Lambda} < 1.5\ \mathrm{GeV}/c^2$
- Using only QF-K
- Not considering resolution

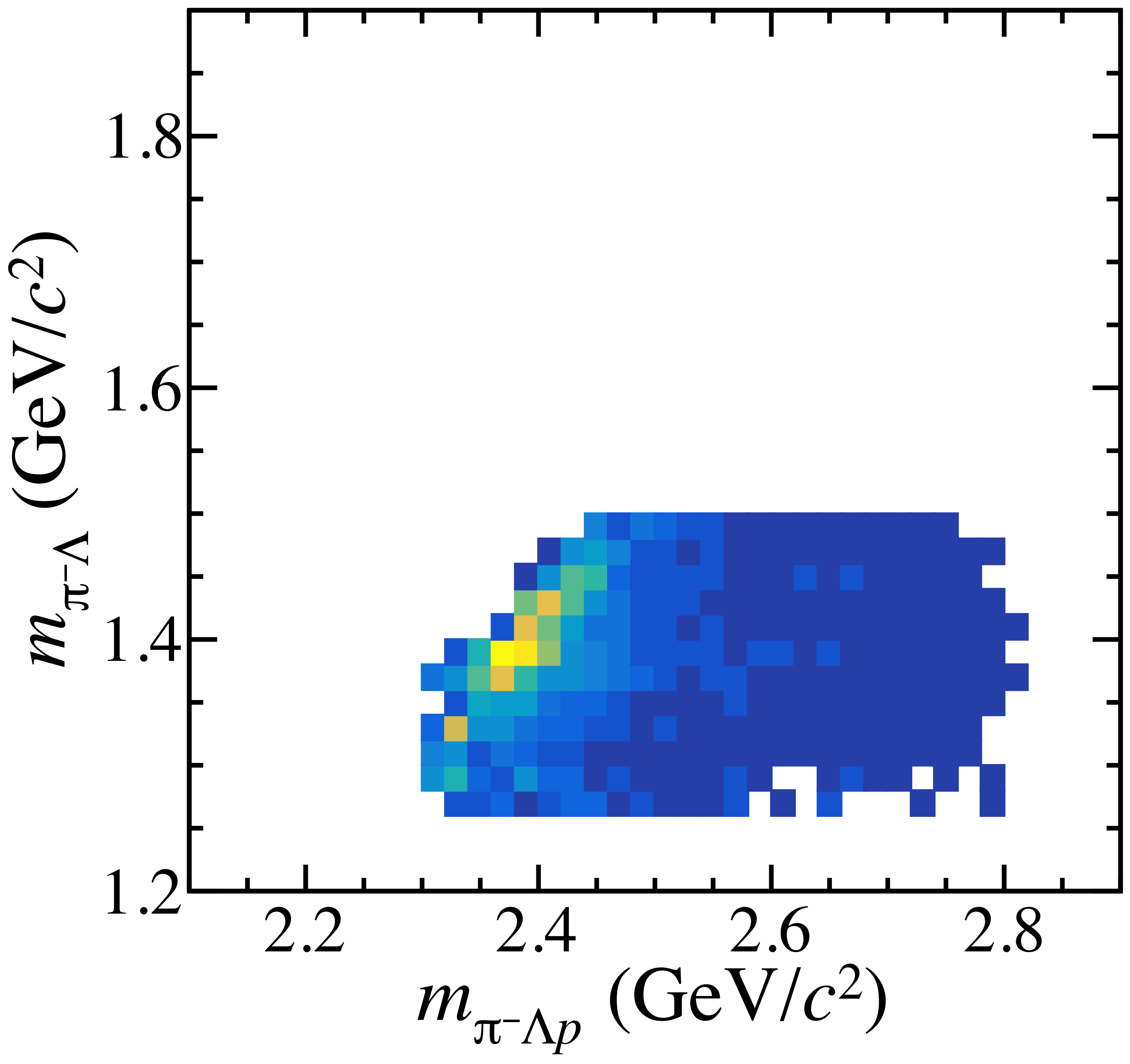


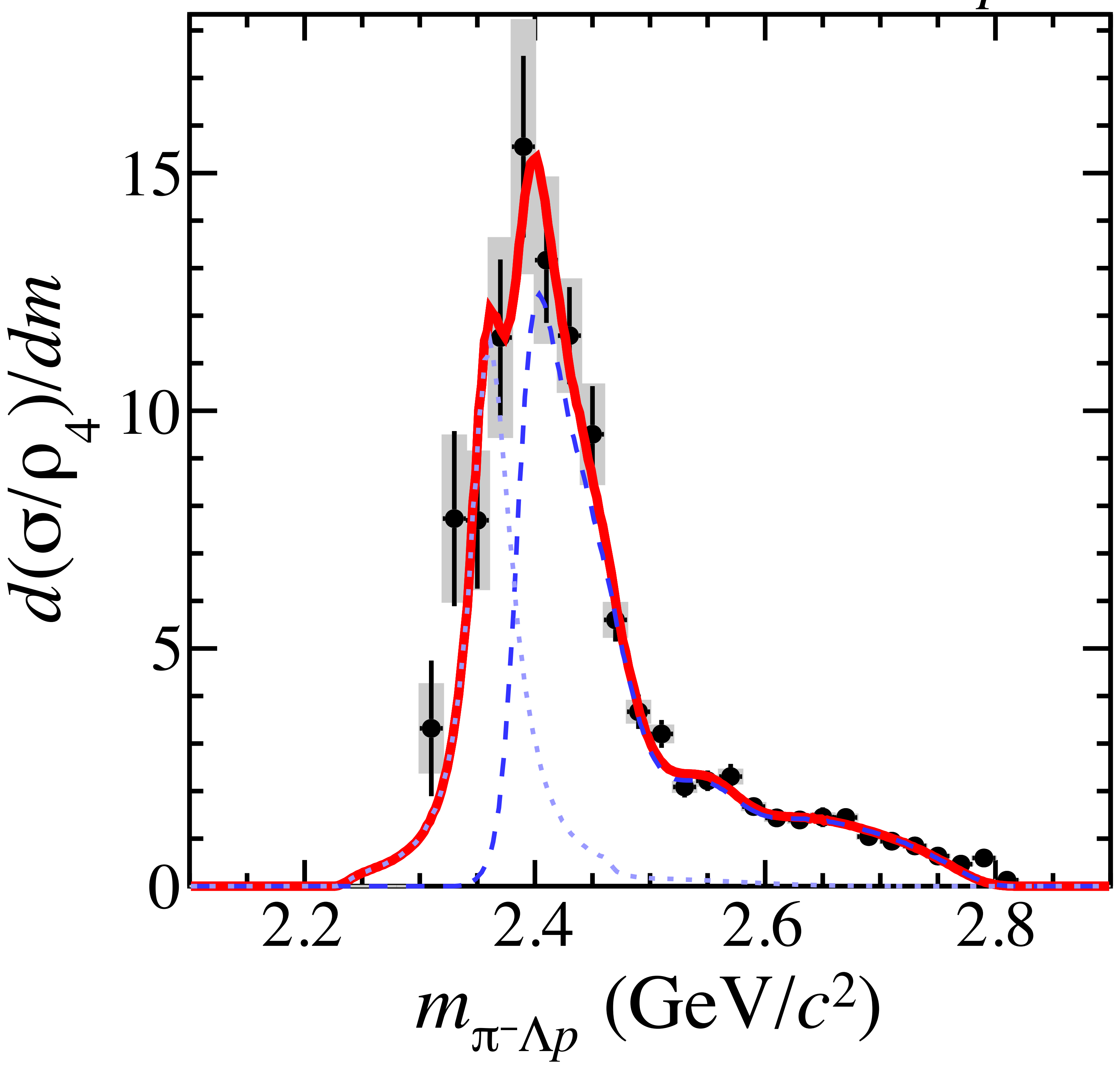
Fit results (very preliminary)

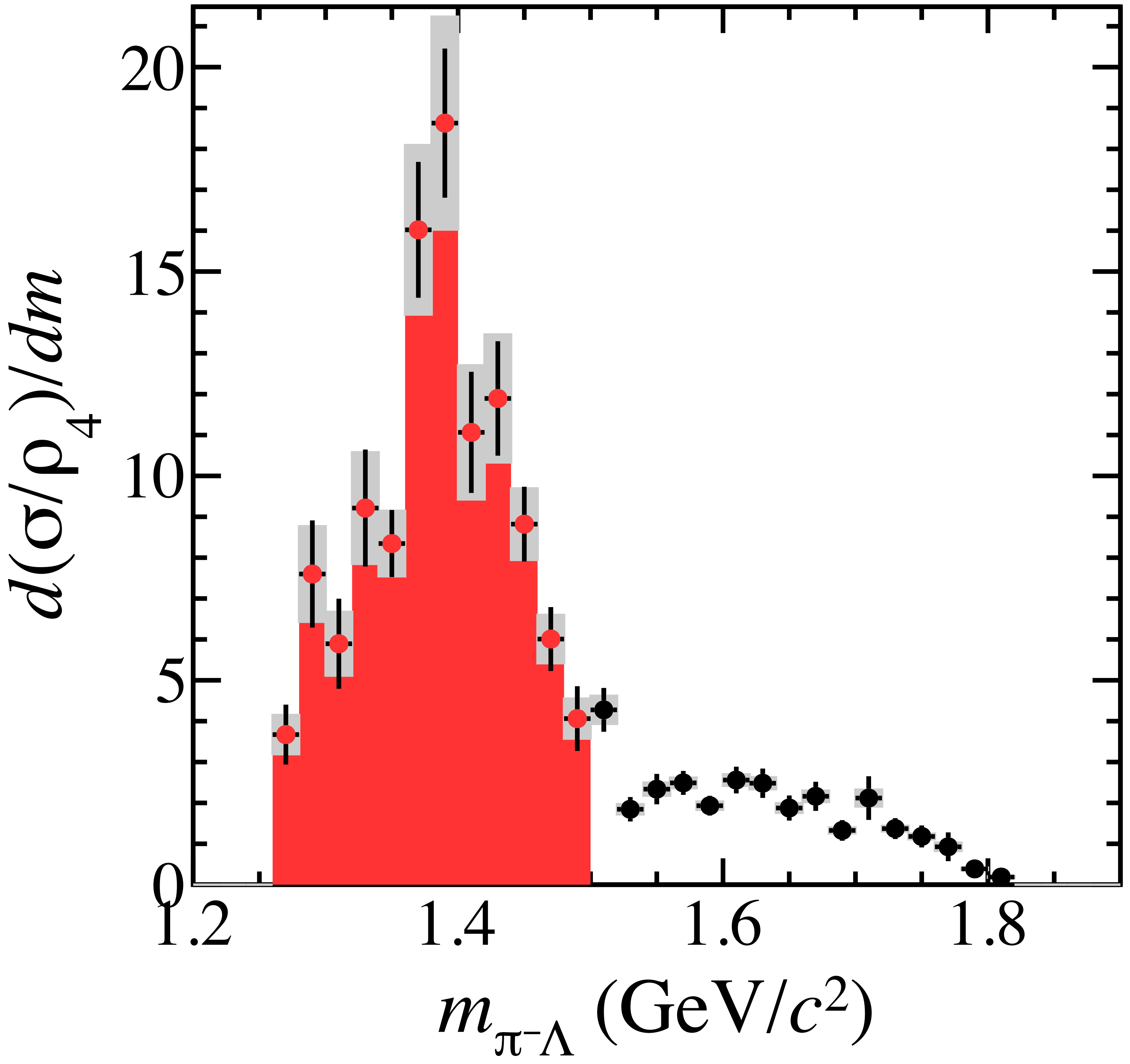
- Fit $m_{\pi^- \Lambda p}$ with selecting $1.26 < m_{\pi^- \Lambda} < 1.5$ GeV/c^2
- Using only QF-K + QF-Σ*
- Not considering resolution

No $\bar{K}^0 nn$?

→ $\cos\theta^*_{p'}$ distribution should be checked.





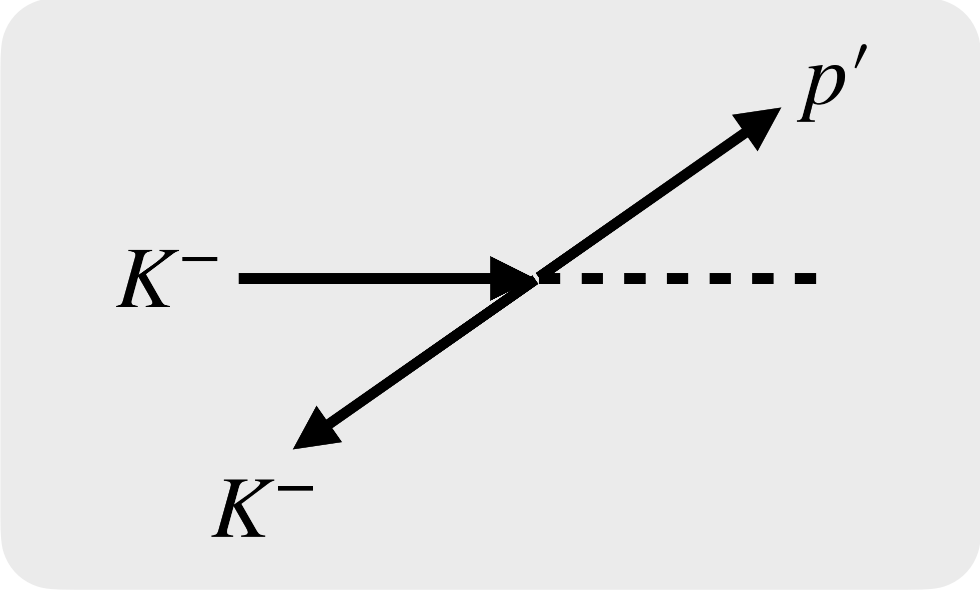


Summary & to do

- More precise fit for the $\pi^- \Lambda p p'$ reaction to search for $\bar{K}^0 nn$ has been done.
  - QF-K & QF-Σ* reactions with a simple model function were introduced.
  - Very preliminary fit results were shown in which spectra can be reproduced only with QF-K & QF-Σ*.
  - $\cos\theta^*_{p'}$ dependence should be checked.
- To do:
  - Need more time for fitting

$\cos\theta_{p'}^{(\pi^-\Lambda pp')-cm}$とQF質量の関係

$$E_{p'}^{(K^-p)*} = \sqrt{m_p^2 + \left(p_{p'}^{(K^-p)*}\right)^2}$$

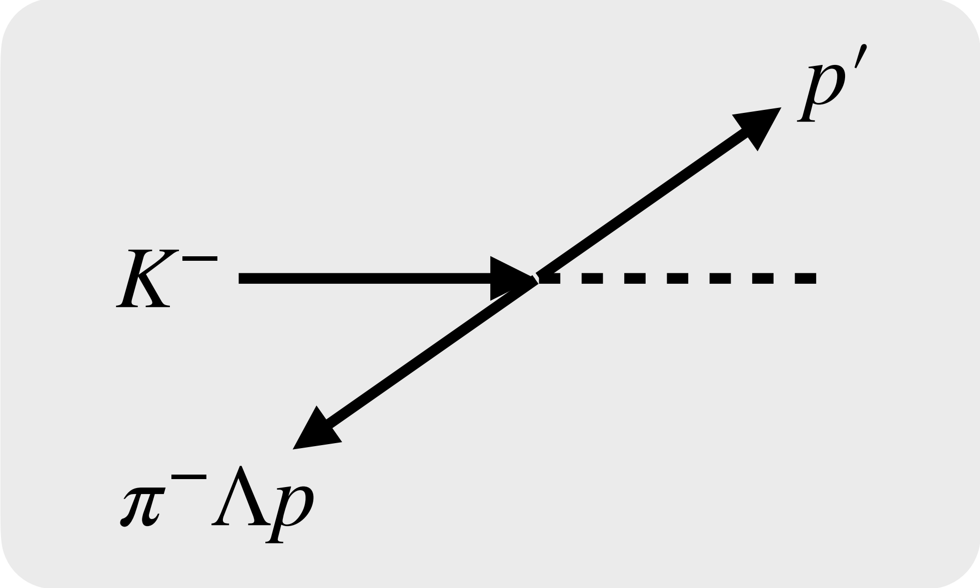


$$p_{p'}^{(K^-p)*} = \begin{pmatrix} E_{p'}^{(K^-p)*} \\ p_{p'}^{(K^-p)*}\cos\theta \\ p_{p'}^{(K^-p)*}\sin\theta \end{pmatrix}$$

$$p_{p'}^{(K^-p)*} = \frac{\sqrt{\left(m_{K^-p}^2 - (m_{K^-} + m_p)^2\right)\left(m_{K^-p}^2 - (m_{K^-} - m_p)^2\right)}}{2m_{K^-p}}$$

$$m_{K^-p} = \sqrt{m_{K^-}^2 + m_p^2 + 2E_{K^-}E_p \mp 2p_{K^-}p_f}$$

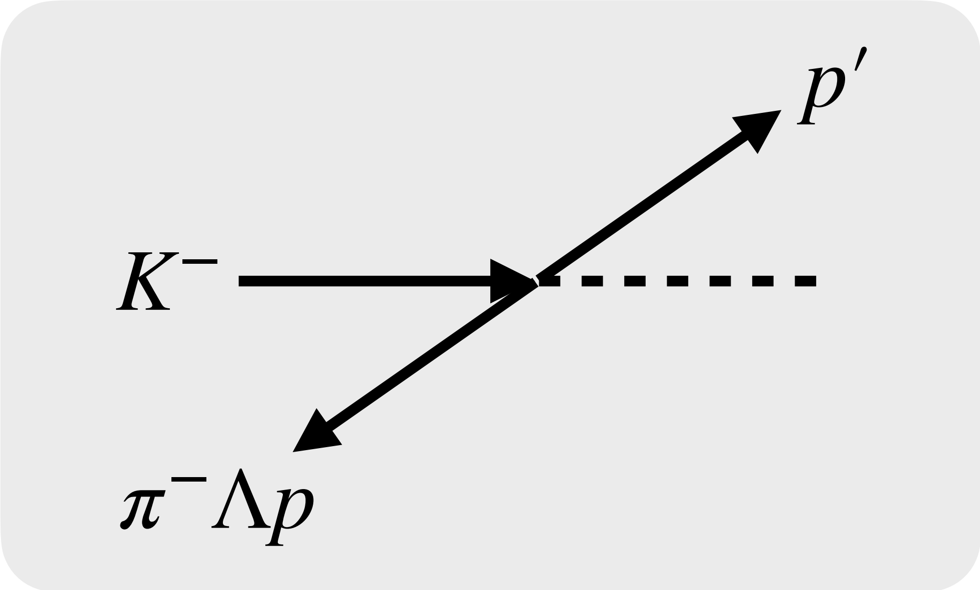
$$\frac{1}{m_{K^-p}}\begin{pmatrix} E_{K^-} + E_p & p_{K^-} + p_f & 0 \\ p_{K^-} + p_f & E_{K^-} + E_p & 0 \\ 0 & 0 & m_{K^-p} \end{pmatrix}$$



$$p_{p'}^{lab} = \frac{1}{m_{K^-p}}\begin{pmatrix} (E_{K^-} + E_p)\,E_{p'}^{(K^-p)*} + (p_{K^-} \pm p_f)\,p_{p'}^{(K^-p)*}\cos\theta \\ (p_{K^-} \pm p_f)\,E_{p'}^{(K^-p)*} + (E_{K^-} + E_p)\,p_{p'}^{(K^-p)*}\cos\theta \\ m_{K^-p}\,p_{p'}^{(K^-p)*}\sin\theta \end{pmatrix}$$

$$\frac{1}{m_{K^-{}^3\mathrm{He}}}\begin{pmatrix} E_{K^-} + m_{^3\mathrm{He}} & -p_{K^-} & 0 \\ -p_{K^-} & E_{K^-} + m_{^3\mathrm{He}} & 0 \\ 0 & 0 & m_{K^-{}^3\mathrm{He}} \end{pmatrix}$$

$$m_{K^-{}^3\mathrm{He}} = \sqrt{m_{K^-}^2 + m_{^3\mathrm{He}}^2 + 2m_{^3\mathrm{He}}E_{K^-}}$$



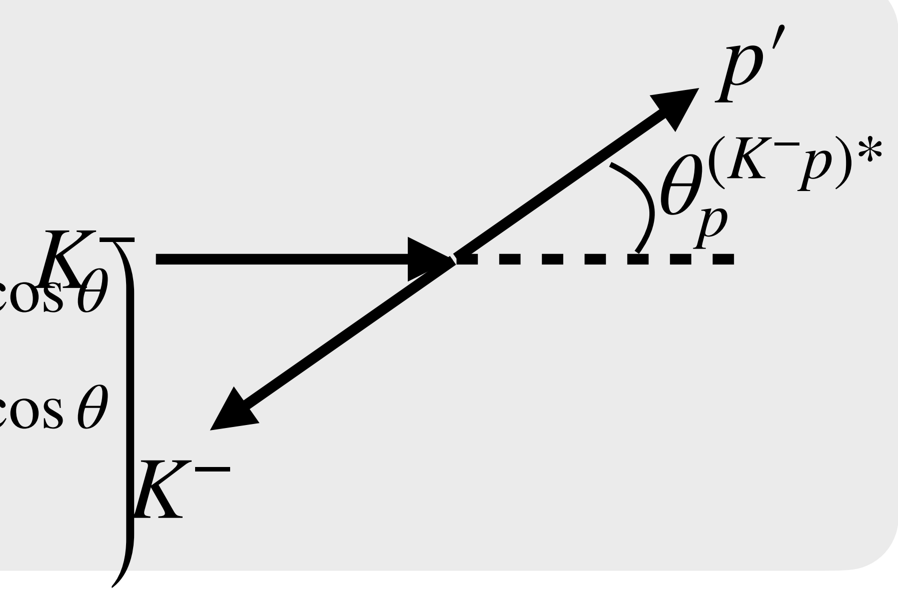
$$p_{p'}^{(\pi^-\Lambda pp')*} = \frac{1}{m_{K^-{}^3\mathrm{He}}}\frac{1}{m_{K^-p}}\begin{pmatrix} \left(E_{K^-{}^3\mathrm{He}}\,E_{K^-p} - p_{K^-}\,p_{K^-p}\right)E_{p'}^{(K^-p)*} + \left(E_{K^-{}^3\mathrm{He}}\,p_{K^-p} - p_{K^-}\,E_{K^-p}\right)p_{p'}^{(K^-p)*}\cos\theta \\ \left(E_{K^-{}^3\mathrm{He}}\,p_{K^-p} - p_{K^-}\,E_{K^-p}\right)E_{p'}^{(K^-p)*} + \left(E_{K^-{}^3\mathrm{He}}\,E_{K^-p} - p_{K^-}\,p_{K^-p}\right)p_{p'}^{(K^-p)*}\cos\theta \\ m_{K^-{}^3\mathrm{He}}\,m_{K^-p}\,p_{p'}^{(K^-p)*}\sin\theta \end{pmatrix}$$

$$E_{K^-{}^3\mathrm{He}} = E_{K^-} + m_{^3\mathrm{He}} \qquad E_{K^-p} = E_{K^-} + E_p \qquad p_{K^-p} = p_{K^-} \pm p_f$$

$\cos\theta_{p'}^{(\pi^-\Lambda pp')-cm}$とQF質量の関係

$$E_{p'}^{(K^-p)*} = \sqrt{m_p^2 + \left(p_{p'}^{(K^-p)*}\right)^2}$$

$$p_{p'}^{(K^-p)*} = \frac{\sqrt{\left(m_{K^-p}^2 - (m_{K^-}+m_p)^2\right)\left(m_{K^-p}^2 - (m_{K^-}-m_p)^2\right)}}{2m_{K^-p}}$$

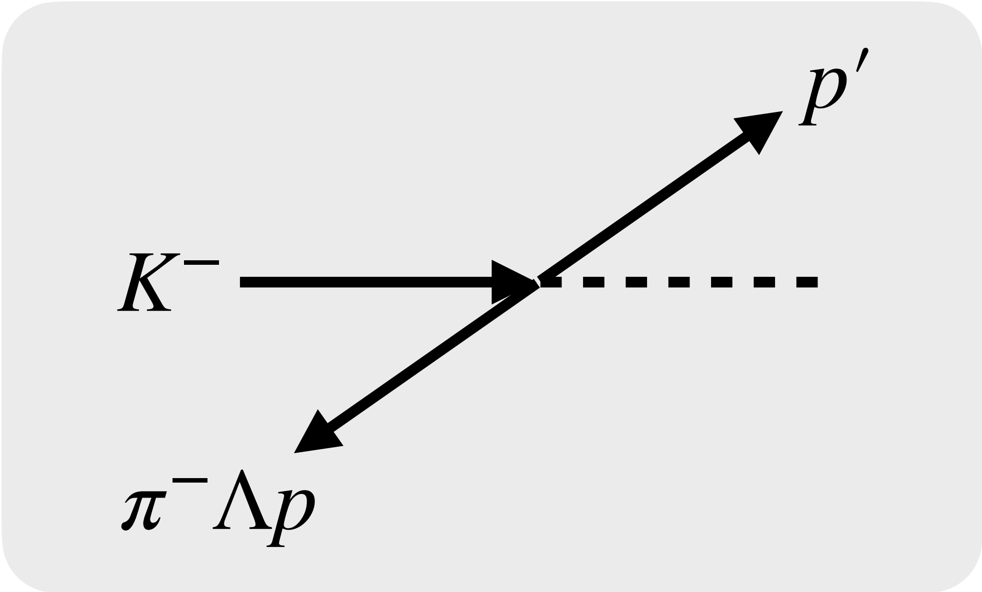


$$p_{p'}^{lab} = \frac{1}{m_{K^-p}}\begin{pmatrix} (E_{K^-}+E_p)\,E_{p'}^{(K^-p)*} + (p_{K^-}\pm p_f)\,p_{p'}^{(K^-p)*}\cos\theta \\ (p_{K^-}\pm p_f)\,E_{p'}^{(K^-p)*} + (E_{K^-}+E_p)\,p_{p'}^{(K^-p)*}\cos\theta \\ m_{K^-p}\,p_{p'}^{(K^-p)*}\sin\theta \end{pmatrix}$$

$$p_{p'}^{(K^-p)*} = \begin{pmatrix} E_{p'}^{(K^-p)*} \\ p_{p'}^{(K^-p)*}\cos\theta \\ p_{p'}^{(K^-p)*}\sin\theta \end{pmatrix}$$

$$m_{K^-p} = \sqrt{m_{K^-}^2 + m_p^2 + 2E_{K^-}E_p \mp 2p_{K^-}\,p_f\cos\theta_f}$$

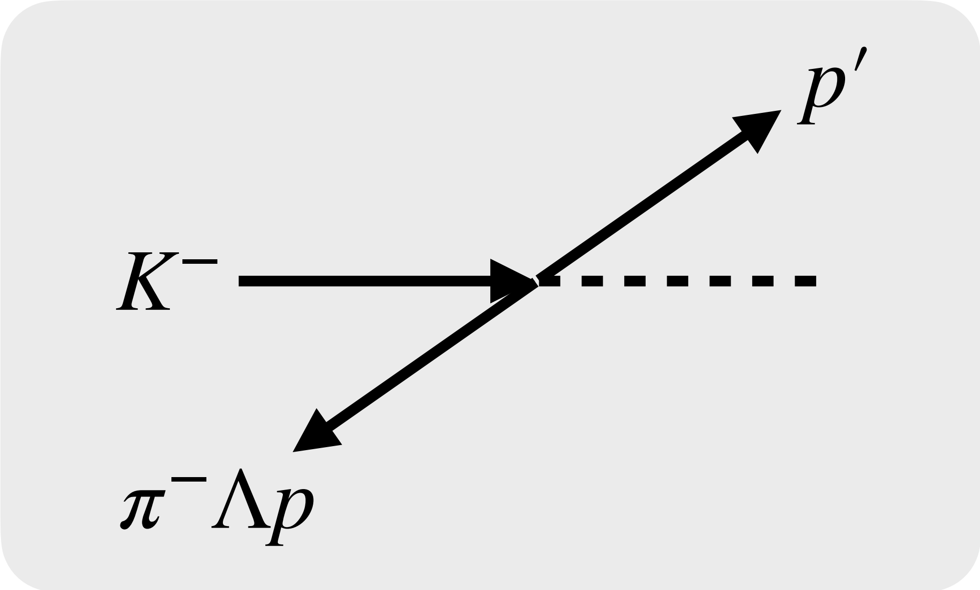
$\cos\theta_f$は実験室系でのフェルミモーションのビームに対する角度。



$$\frac{1}{m_{K^-p}}\begin{pmatrix} E_{K^-}+E_p & p_{K^-}+p_f\cos\theta_f & p_f\sin\theta_f \\ p_{K^-}+p_f\cos\theta_f & m_{K^-p} + \frac{\left(p_{K^-}+p_f\cos\theta_f\right)^2}{\left(p_{K^-}+p_f\cos\theta_f\right)^2 + p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-m_{K^-p}\right) & m_{K^-p} + \frac{\left(p_{K^-}+p_f\cos\theta\right)p_f\sin\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2 + p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-m_{K^-p}\right) \\ p_f\sin\theta_f & m_{K^-p} + \frac{\left(p_{K^-}+p_f\cos\theta\right)p_f\sin\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2 + p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-m_{K^-p}\right) & m_{K^-p} + \frac{p_f^2\sin^2\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2 + p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-m_{K^-p}\right) \end{pmatrix}$$

$$\frac{1}{m_{K^{-3}\mathrm{He}}}\begin{pmatrix} E_{K^-}+m_{^3\mathrm{He}} & -p_{K^-} & 0 \\ -p_{K^-} & E_{K^-}+m_{^3\mathrm{He}} & 0 \\ 0 & 0 & m_{K^{-3}\mathrm{He}} \end{pmatrix}$$

$$m_{K^{-3}\mathrm{He}} = \sqrt{m_{K^-}^2 + m_{^3\mathrm{He}}^2 + 2m_{^3\mathrm{He}}E_{K^-}}$$



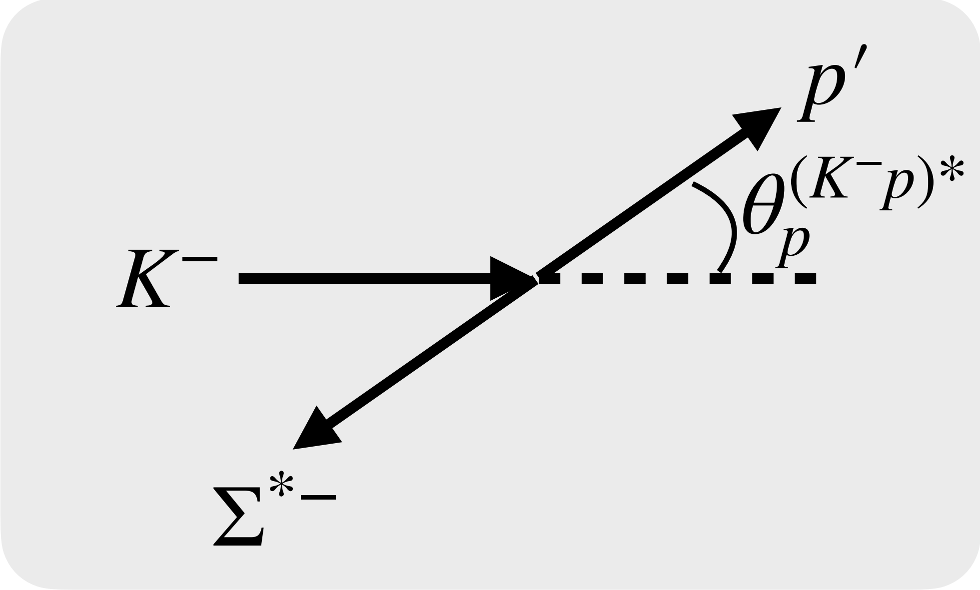
$$p_{p'}^{(\pi^-\Lambda pp')*} = \frac{1}{m_{K^{-3}\mathrm{He}}}\frac{1}{m_{K^-p}}\begin{pmatrix} \left(E_{K^{-3}\mathrm{He}}\ E_{K^-p} - p_{K^-}\ p_{K^-p}\right)E_{p'}^{(K^-p)*} + \left(E_{K^{-3}\mathrm{He}}\ p_{K^-p} - p_{K^-}\ E_{K^-p}\right)p_{p'}^{(K^-p)*}\cos\theta \\ \left(E_{K^{-3}\mathrm{He}}\ p_{K^-p} - p_{K^-}\ E_{K^-p}\right)E_{p'}^{(K^-p)*} + \left(E_{K^{-3}\mathrm{He}}\ E_{K^-p} - p_{K^-}\ p_{K^-p}\right)p_{p'}^{(K^-p)*}\cos\theta \\ m_{K^{-3}\mathrm{He}}\ m_{K^-p}\ p_{p'}^{(K^-p)*}\sin\theta \end{pmatrix}$$

$$E_{K^{-3}\mathrm{He}} = E_{K^-} + m_{^3\mathrm{He}} \qquad E_{K^-p} = E_{K^-} + E_p \qquad p_{K^-p} = p_{K^-} \pm p_f$$

$\cos\theta_{p'}^{(\pi^-\Lambda pp')-cm}$ とQF質量の関係

$$E_{p'}^{(K^-p)*} = \sqrt{m_p^2 + \left(p_{p'}^{(K^-p)*}\right)^2}$$

$$\begin{pmatrix} (E_{K^-} + E_p)\,E_{p'}^{(K^-p)*} + (p_{K^-} \pm p_f)\,p_{p'}^{(K^-p)*}\cos\theta \\ (p_{K^-} \pm p_f)\,E_{p'}^{(K^-p)*} + (E_{K^-} + E_p)\,p_{p'}^{(K^-p)*}\cos\theta \\ m_{K^-p}\,p_{p'}^{(K^-p)*}\sin\theta \end{pmatrix}$$

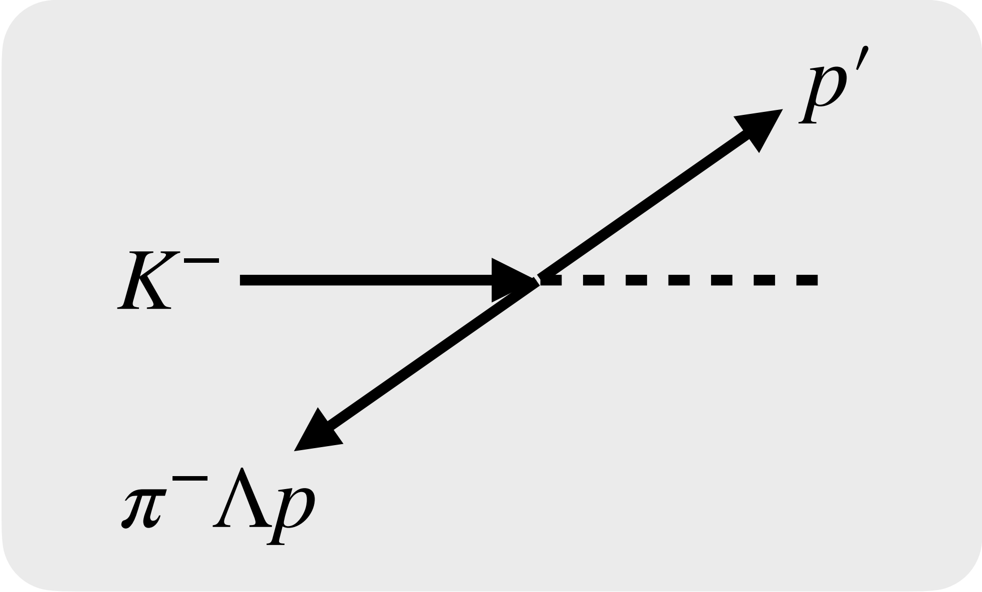


$$p_{p'}^{(K^-p)*} = \begin{pmatrix} E_{p'}^{(K^-p)*} \\ p_{p'}^{(K^-p)*}\cos\theta \\ p_{p'}^{(K^-p)*}\sin\theta \end{pmatrix}$$

$$p_{p'}^{(K^-p)*} = \frac{\sqrt{\left(m_{K^-p}^2 - (m_{K^-} + m_p)^2\right)\left(m_{K^-p}^2 - (m_{K^-} - m_p)^2\right)}}{2m_{K^-p}}$$

$$m_{K^-pn} = \sqrt{m_{K^-}^2 + 4m_N^2 + 2E_{K^-}E_{pn} \mp 2p_{K^-}\,p_f\cos\theta_f}$$

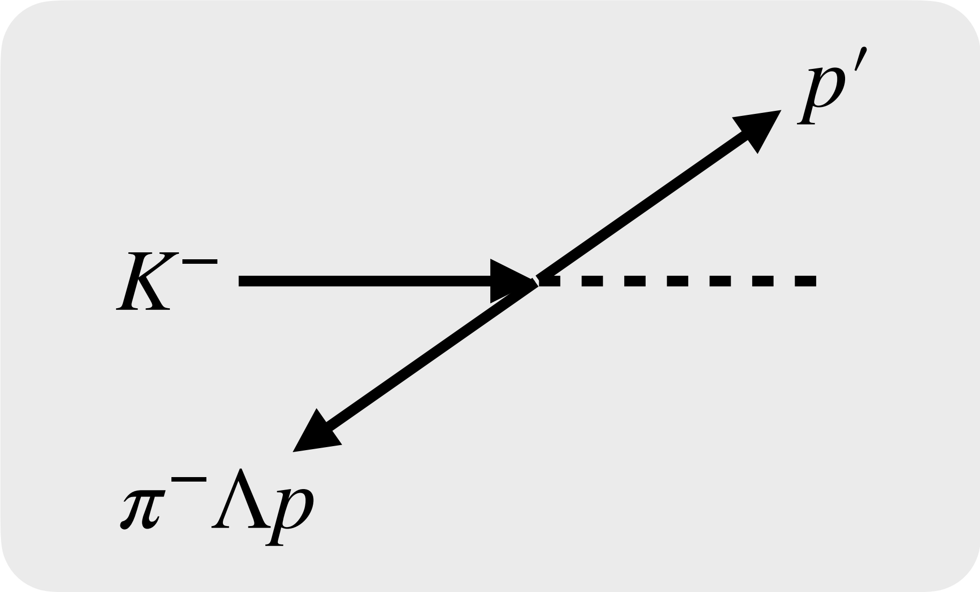
$\cos\theta_f$は実験室系でのフェルミモーションのビームに対する角度。



$$\frac{1}{m_{K^-p}}\begin{pmatrix} E_{K^-} + E_p & p_{K^-} + p_f\cos\theta_f & p_f\sin\theta_f \\ p_{K^-} + p_f\cos\theta_f & m_{K^-p} + \frac{\left(p_{K^-} + p_f\cos\theta_f\right)^2}{\left(p_{K^-} + p_f\cos\theta_f\right)^2 + p_f^2\sin^2\theta_f}\left(E_{K^-} + E_p - m_{K^-p}\right) & m_{K^-p} + \frac{\left(p_{K^-} + p_f\cos\theta\right)p_f\sin\theta_f}{\left(p_{K^-} + p_f\cos\theta_f\right)^2 + p_f^2\sin^2\theta_f}\left(E_{K^-} + E_p - m_{K^-p}\right) \\ p_f\sin\theta_f & m_{K^-p} + \frac{\left(p_{K^-} + p_f\cos\theta\right)p_f\sin\theta_f}{\left(p_{K^-} + p_f\cos\theta_f\right)^2 + p_f^2\sin^2\theta_f}\left(E_{K^-} + E_p - m_{K^-p}\right) & m_{K^-p} + \frac{p_f^2\sin^2\theta_f}{\left(p_{K^-} + p_f\cos\theta_f\right)^2 + p_f^2\sin^2\theta_f}\left(E_{K^-} + E_p - m_{K^-p}\right) \end{pmatrix}$$

$$\frac{1}{m_{K^-{}^3\mathrm{He}}}\begin{pmatrix} E_{K^-} + m_{{}^3\mathrm{He}} & -p_{K^-} & 0 \\ -p_{K^-} & E_{K^-} + m_{{}^3\mathrm{He}} & 0 \\ 0 & 0 & m_{K^-{}^3\mathrm{He}} \end{pmatrix}$$

$$m_{K^-{}^3\mathrm{He}} = \sqrt{m_{K^-}^2 + m_{{}^3\mathrm{He}}^2 + 2m_{{}^3\mathrm{He}}E_{K^-}}$$

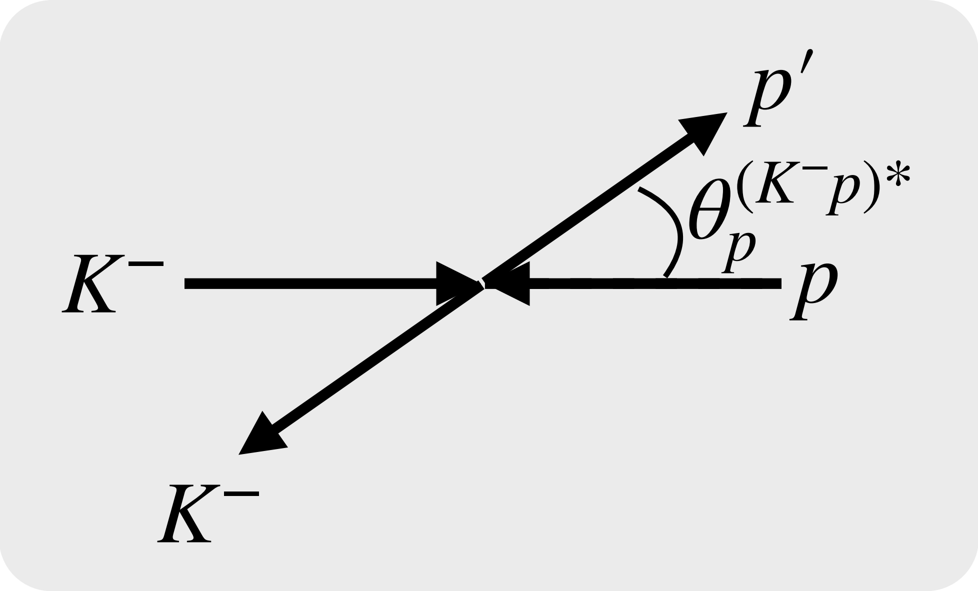


$$p_{p'}^{(\pi^-\Lambda pp')*} = \frac{1}{m_{K^-{}^3\mathrm{He}}}\frac{1}{m_{K^-p}}\begin{pmatrix} \left(E_{K^-{}^3\mathrm{He}}\ E_{K^-p} - p_{K^-}\ p_{K^-p}\right)E_{p'}^{(K^-p)*} + \left(E_{K^-{}^3\mathrm{He}}\ p_{K^-p} - p_{K^-}\ E_{K^-p}\right)p_{p'}^{(K^-p)*}\cos\theta \\ \left(E_{K^-{}^3\mathrm{He}}\ p_{K^-p} - p_{K^-}\ E_{K^-p}\right)E_{p'}^{(K^-p)*} + \left(E_{K^-{}^3\mathrm{He}}\ E_{K^-p} - p_{K^-}\ p_{K^-p}\right)p_{p'}^{(K^-p)*}\cos\theta \\ m_{K^-{}^3\mathrm{He}}\ m_{K^-p}\,p_{p'}^{(K^-p)*}\sin\theta \end{pmatrix}$$

$E_{K^-{}^3\mathrm{He}} = E_{K^-} + m_{{}^3\mathrm{He}}$ $\quad E_{K^-p} = E_{K^-} + E_p$ $\quad p_{K^-p} = p_{K^-} \pm p_f$

$\cos\theta_{p'}^{(\pi^-\Lambda pp')-cm}$とQF質量の関係

(K^-p)cm



$$m_{K^-p} = \sqrt{m_{K^-}^2 + m_p^2 + 2E_{K^-}E_p \mp 2p_{K^-}p_f\cos\theta_f}$$

$$p_{p'}^{(K^-p)*} = \frac{\sqrt{\left(m_{K^-p}^2 - (m_{K^-}+m_p)^2\right)\left(m_{K^-p}^2 - (m_{K^-}-m_p)^2\right)}}{2m_{K^-p}}$$

$$E_{p'}^{(K^-p)*} = \sqrt{m_p^2 + \left(p_{p'}^{(K^-p)*}\right)^2}$$

$$p_{p'}^{(K^-p)*} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{K^-}^{(K^-p)*} & -\sin\theta_{K^-}^{(K^-p)*} \\ 0 & \sin\theta_{K^-}^{(K^-p)*} & \cos\theta_{K^-}^{(K^-p)*} \end{pmatrix} \begin{pmatrix} E_{p'}^{(K^-p)*} \\ p_{p'}^{(K^-p)*}\cos\theta \\ p_{p'}^{(K^-p)*}\sin\theta \end{pmatrix}$$

$\cos\theta_f$は実験室系でのフェルミモーションのビームに対する角度。

$$\begin{pmatrix} E_{K^-}^{(K^-p)*} \\ p_{K^-}^{(K^-p)*}\cos\theta_{K^-}^{(K^-p)*} \\ p_{K^-}^{(K^-p)*}\sin\theta_{K^-}^{(K^-p)*} \end{pmatrix} = \frac{1}{m_{K^-p}} \begin{pmatrix} E_{K^-}\left(E_{K^-}+E_p\right) - p_{K^-}\left(p_{K^-}+p_f\cos\theta_f\right) \\ -E_{K^-}\left(p_{K^-}+p_f\cos\theta_f\right) + p_{K^-}\left(m_{K^-p} + \frac{\left(p_{K^-}+p_f\cos\theta_f\right)^2}{\left(p_{K^-}+p_f\cos\theta_f\right)^2 + p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-m_{K^-p}\right)\right) \\ -E_{K^-}p_f\sin\theta_f + p_{K^-}\left(\frac{\left(p_{K^-}+p_f\cos\theta\right)p_f\sin\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2 + p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-m_{K^-p}\right)\right) \end{pmatrix}$$

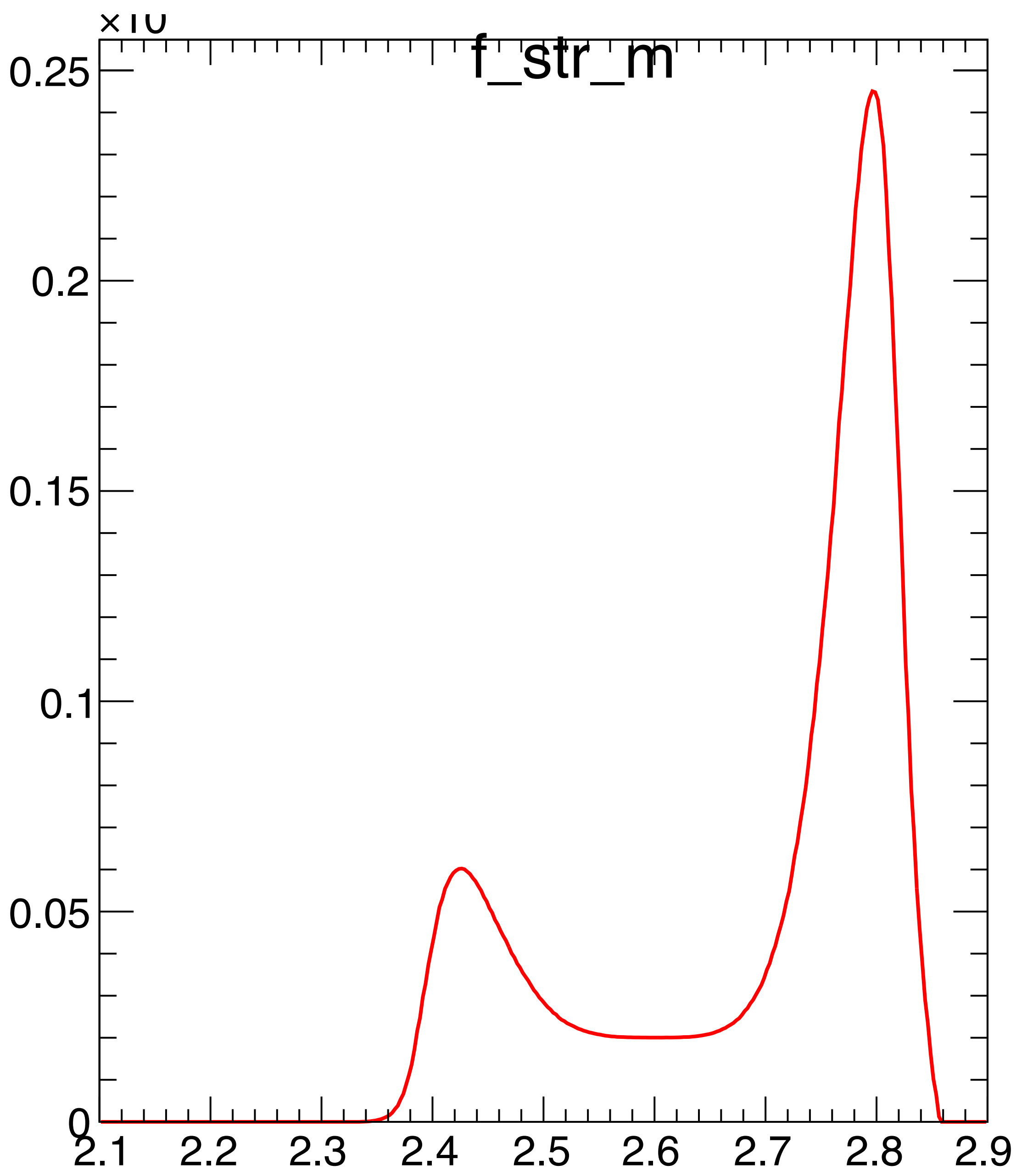
(K^-p)cm→LabのLorentz変換

$$\frac{1}{m_{K^-p}} \begin{pmatrix} E_{K^-}+E_p & p_{K^-}+p_f\cos\theta_f & p_f\sin\theta_f \\ p_{K^-}+p_f\cos\theta_f & m_{K^-p} + \frac{\left(p_{K^-}+p_f\cos\theta_f\right)^2}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-m_{K^-p}\right) & \frac{\left(p_{K^-}+p_f\cos\theta\right)p_f\sin\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-m_{K^-p}\right) \\ p_f\sin\theta_f & \frac{\left(p_{K^-}+p_f\cos\theta\right)p_f\sin\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-m_{K^-p}\right) & m_{K^-p} + \frac{p_f^2\sin^2\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-m_{K^-p}\right) \end{pmatrix}$$

結局、

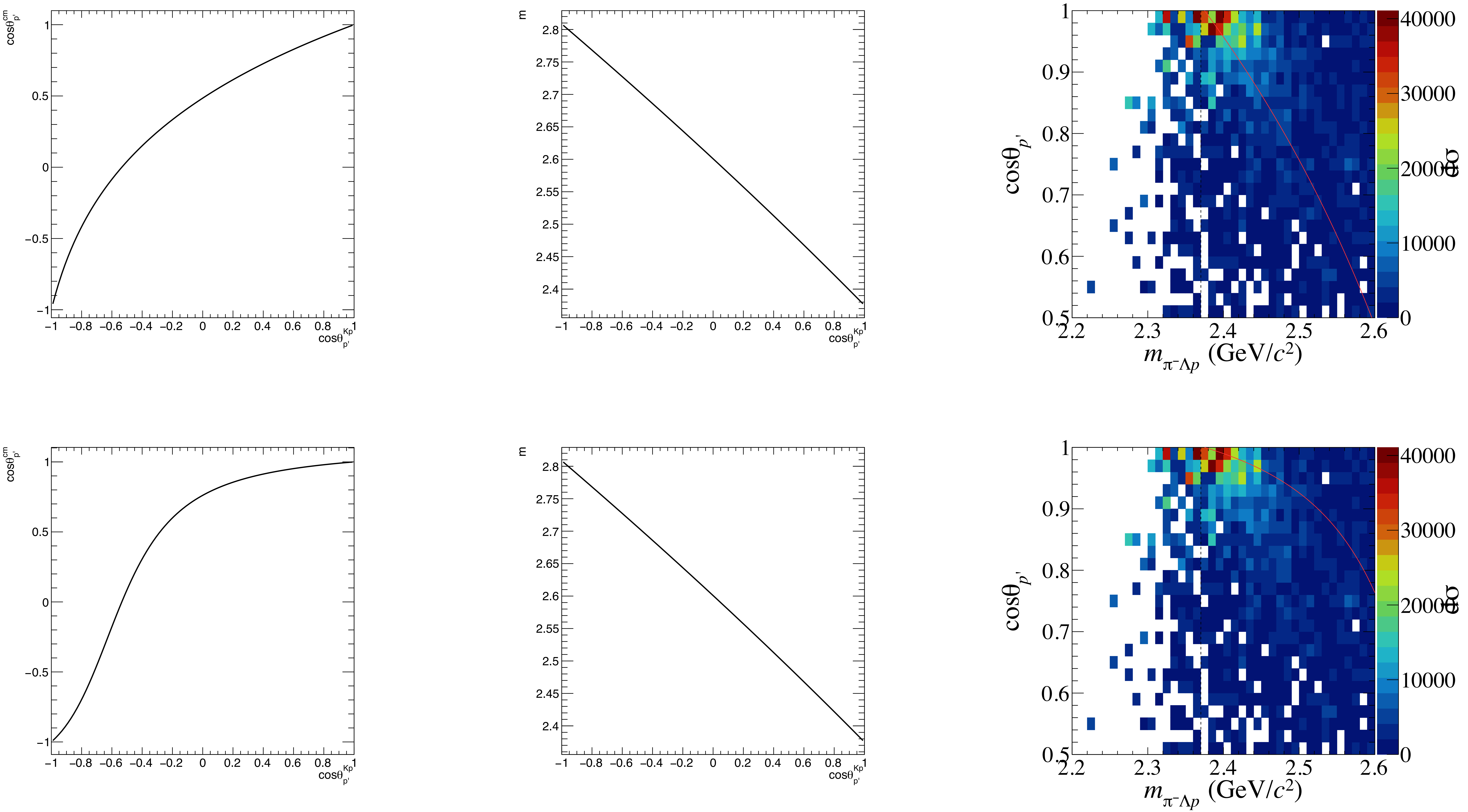
$$\begin{pmatrix} E^*_{p'} \\ p^*_{p'\,x} \\ p^*_{p'\,y}\end{pmatrix} = \frac{1}{m_{K^-{}^3\mathrm{He}}}\frac{1}{m_{K^-p}}\begin{pmatrix} E_{K^-}+m_{^3\mathrm{He}} & -p_{K^-} & 0 \\ -p_{K^-} & E_{K^-}+m_{^3\mathrm{He}} & 0 \\ 0 & 0 & m_{K^-{}^3\mathrm{He}}\end{pmatrix}\begin{pmatrix} E_{K^-}+E_p & p_{K^-}+p_f\cos\theta_f & p_f\sin\theta_f \\ p_{K^-}+p_f\cos\theta_f & m_{K^-p}+\frac{\left(p_{K^-}+p_f\cos\theta_f\right)^2}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-m_{K^-p}\right) & \frac{\left(p_{K^-}+p_f\cos\theta\right)p_f\sin\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-m_{K^-p}\right) \\ p_f\sin\theta_f & \frac{\left(p_{K^-}+p_f\cos\theta\right)p_f\sin\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-m_{K^-p}\right) & m_{K^-p}+\frac{p_f^2\sin^2\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-m_{K^-p}\right)\end{pmatrix}\begin{pmatrix} E^{(K^-p)*}_{p'} \\ p^{(K^-p)*}_{p'}\cos\theta^{(K^-p)*}_{p'} \\ p^{(K^-p)*}_{p'}\sin\theta^{(K^-p)*}_{p'}\end{pmatrix}$$

$$\begin{pmatrix} E^*_{p'} \\ p^*_{p'\,x} \\ p^*_{p'\,y}\end{pmatrix} = \frac{1}{m_{K^-{}^3\mathrm{He}}}\frac{1}{m_{K^-p}}\begin{pmatrix} E_{K^-}+m_{^3\mathrm{He}} & -p_{K^-} & 0 \\ -p_{K^-} & E_{K^-}+m_{^3\mathrm{He}} & 0 \\ 0 & 0 & m_{K^-{}^3\mathrm{He}}\end{pmatrix}\begin{pmatrix} E_{K^-}+E_p & p_{K^-}+p_f\cos\theta_f & p_f\sin\theta_f \\ p_{K^-}+p_f\cos\theta_f & m_{K^-p}+\frac{\left(p_{K^-}+p_f\cos\theta_f\right)^2}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-m_{K^-p}\right) & \frac{\left(p_{K^-}+p_f\cos\theta\right)p_f\sin\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-m_{K^-p}\right) \\ p_f\sin\theta_f & \frac{\left(p_{K^-}+p_f\cos\theta\right)p_f\sin\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-m_{K^-p}\right) & m_{K^-p}+\frac{p_f^2\sin^2\theta_f}{\left(p_{K^-}+p_f\cos\theta_f\right)^2+p_f^2\sin^2\theta_f}\left(E_{K^-}+E_p-m_{K^-p}\right)\end{pmatrix}\begin{pmatrix} E^{(K^-p)*}_{p'} \\ p^{(K^-p)*}_{p'}\cos\theta^{(K^-p)*}_{p'} \\ p^{(K^-p)*}_{p'}\sin\theta^{(K^-p)*}_{p'}\end{pmatrix}$$


×10
f_str_m
0.25
0.2
0.15
0.1
0.05
0
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9

```
---- matrix ----
cos_p : 1
lv_p_prime_cm = (1.26966, 0.85539, 0)
---- matrix ----
cos_p : 0.998
lv_p_prime_cm = (1.26937, 0.854275, 0.0160382)
---- matrix ----
cos_p : 0.996
lv_p_prime_cm = (1.26907, 0.85316, 0.0226701)
---- matrix ----
cos_p : 0.994
lv_p_prime_cm = (1.26878, 0.852045, 0.0277512)
---- matrix ----
cos_p : 0.992
lv_p_prime_cm = (1.26848, 0.85093, 0.0320283)
---- matrix ----
cos_p : 0.99
lv_p_prime_cm = (1.26818, 0.849815, 0.0357907)
---- matrix ----
cos_p : 0.988
lv_p_prime_cm = (1.26789, 0.8487, 0.0391871)
---- matrix ----
cos_p : 0.986
lv_p_prime_cm = (1.26759, 0.847585, 0.0423056)
---- matrix ----
cos_p : 0.984
lv_p_prime_cm = (1.26729, 0.84647, 0.0452038)
---- matrix ----
cos_p : 0.982
lv_p_prime_cm = (1.267, 0.845355, 0.0479217)
---- matrix ----
cos_p : 0.98
lv_p_prime_cm = (1.2667, 0.84424, 0.0504884)
---- matrix ----
cos_p : 0.978
lv_p_prime_cm = (1.26641, 0.843125, 0.0529259)
---- matrix ----
cos_p : 0.976
lv_p_prime_cm = (1.26611, 0.84201, 0.0552514)
```

```
---- direct ----
cos_p : 1
lv_p_prime_cm = (1.26966, 0.85539, 0)
---- direct ----
cos_p : 0.998
lv_p_prime_cm = (1.26937, 0.854275, 0.0339738)
---- direct ----
cos_p : 0.996
lv_p_prime_cm = (1.26907, 0.85316, 0.0480221)
---- direct ----
cos_p : 0.994
lv_p_prime_cm = (1.26878, 0.852045, 0.0587854)
---- direct ----
cos_p : 0.992
lv_p_prime_cm = (1.26848, 0.85093, 0.0678455)
---- direct ----
cos_p : 0.99
lv_p_prime_cm = (1.26818, 0.849815, 0.0758155)
---- direct ----
cos_p : 0.988
lv_p_prime_cm = (1.26789, 0.8487, 0.0830099)
---- direct ----
cos_p : 0.986
lv_p_prime_cm = (1.26759, 0.847585, 0.0896159)
---- direct ----
cos_p : 0.984
lv_p_prime_cm = (1.26729, 0.84647, 0.0957551)
---- direct ----
cos_p : 0.982
lv_p_prime_cm = (1.267, 0.845355, 0.101512)
---- direct ----
cos_p : 0.98
lv_p_prime_cm = (1.2667, 0.84424, 0.10695)
---- direct ----
cos_p : 0.978
lv_p_prime_cm = (1.26641, 0.843125, 0.112113)
---- direct ----
cos_p : 0.976
lv_p_prime_cm = (1.26611, 0.84201, 0.117039)
```



$\cos\theta_{p'}^{cm}$
$\cos\theta_{p'}^{Kp}$
m
$\cos\theta_{p'}$
$m_{\pi^-\Lambda p}$ (GeV/c^2)
dσ

^{3}Heの中のNの質量

$$m_{^3\mathrm{He}} = \sqrt{4m_N^2 + m_N^2 + 2\sqrt{4m_N^2 + p_f^2}\,\sqrt{m_N^2 + p_f^2} + 2p_f^2}$$

$$(m_{^3\mathrm{He}}^2 - 5m_N^2 - 2p_f^2)^2 = 4(4m_N^2 + p_f^2)(m_N^2 + p_f^2)$$

$$m_{^3\mathrm{He}}^4 + 25m_N^4 + 4p_f^4 - 10m_{^3\mathrm{He}}^2 m_N^2 - 4m_{^3\mathrm{He}}^2 p_f^2 + 20m_N^2 p_f^2 = 16m_N^4 + 20m_N^2 p_f^2 + 4p_f^4$$

$$9m_N^4 - 10m_{^3\mathrm{He}}^2 m_N^2 + m_{^3\mathrm{He}}^4 - 4m_{^3\mathrm{He}}^2 p_f^2 = 0$$

$$9m_N^4 - 10m_{^3\mathrm{He}}^2 m_N^2 + (m_{^3\mathrm{He}}^2 + 2m_{^3\mathrm{He}} p_f)(m_{^3\mathrm{He}}^2 - 2m_{^3\mathrm{He}} p_f) = 0$$

$$m_N^2 = \frac{10m_{^3\mathrm{He}}^2 \pm \sqrt{100m_{^3\mathrm{He}}^4 - 36m_{^3\mathrm{He}}^2(m_{^3\mathrm{He}} + 2p_f)(m_{^3\mathrm{He}} - 2p_f)}}{18}$$

$$= \frac{10m_{^3\mathrm{He}}^2 \pm m_{^3\mathrm{He}}\sqrt{100m_{^3\mathrm{He}}^2 - 36(m_{^3\mathrm{He}} + 2p_f)(m_{^3\mathrm{He}} - 2p_f)}}{18}$$

$$= \frac{10m_{^3\mathrm{He}}^2 \pm m_{^3\mathrm{He}}\sqrt{64m_{^3\mathrm{He}}^2 + 144p_f^2}}{18} \quad = \frac{5m_{^3\mathrm{He}}^2 \pm 4m_{^3\mathrm{He}}\sqrt{m_{^3\mathrm{He}}^2 + 9p_f^2}}{9}$$

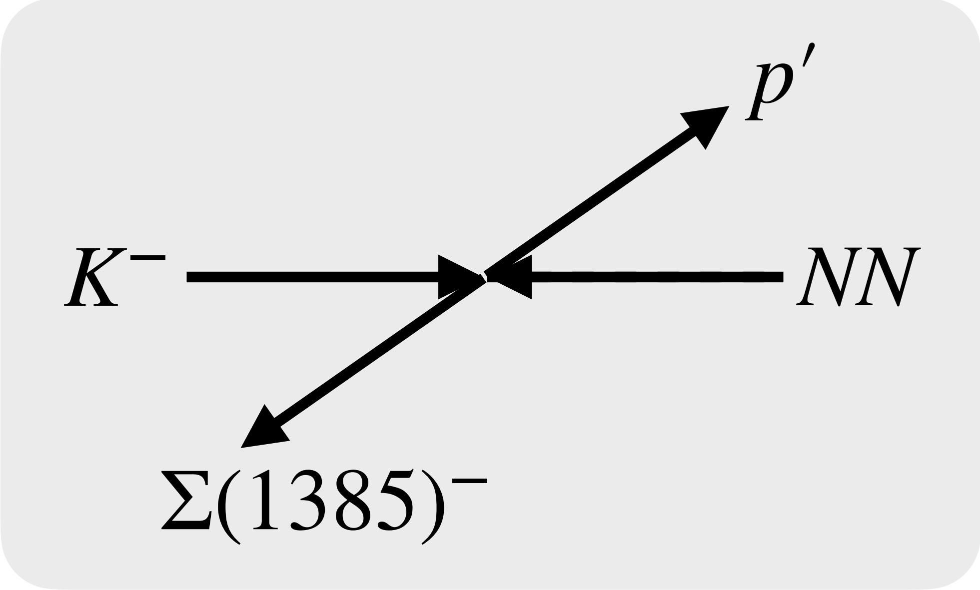
$m_N < m_{^3\mathrm{He}}/3$なので、負号側のみ解

$$m_N = \frac{\sqrt{5m_{^3\mathrm{He}}^2 - 4m_{^3\mathrm{He}}\sqrt{m_{^3\mathrm{He}}^2 + 9p_f^2}}}{3}$$

$\cos\theta_{p'}^{(\pi^-\Lambda pp')-cm}$ とQF質量の関係

$$E_{NN}=\sqrt{4m_N^2+p_f^2}$$

$$m_{K^-+NN}=\sqrt{m_{K^-}^2+4m_N^2+2E_{K^-}E_{NN}\mp 2p_{K^-}\,p_f}$$

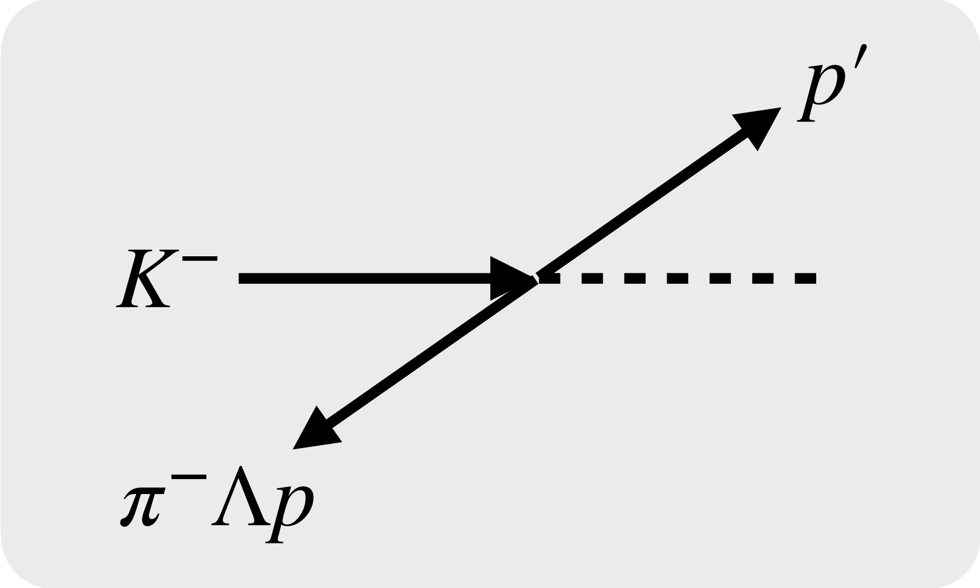


$$p_{p'}^{(K^-p)*}=\begin{pmatrix}E_{p'}^{(K^-p)*}\\ p_{p'}^{(K^-p)*}\cos\theta\\ p_{p'}^{(K^-p)*}\sin\theta\end{pmatrix}$$

$$p_{p'}^{(K^-p)*}=\frac{\sqrt{\left(m_{K^-p}^2-(m_{K^-}+m_p)^2\right)\left(m_{K^-p}^2-(m_{K^-}-m_p)^2\right)}}{2m_{K^-p}}$$

$$E_{p'}^{(K^-pn)*}=\sqrt{m_p^2+\left(p_{p'}^{(K^-p)*}\right)^2}$$

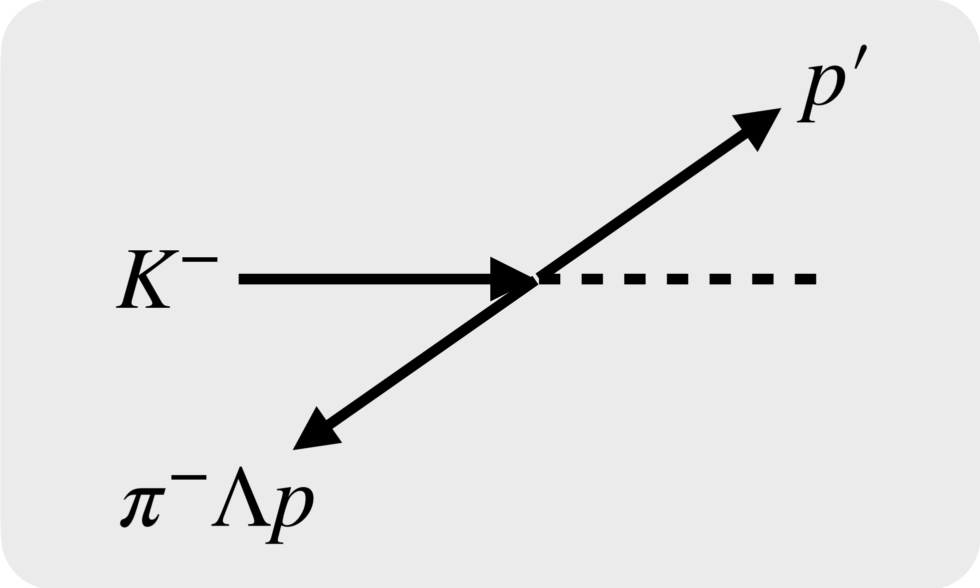
$$\frac{1}{m_{K^-p}}\begin{pmatrix}E_{K^-}+E_p & p_{K^-}+p_f & 0\\ p_{K^-}+p_f & E_{K^-}+E_p & 0\\ 0 & 0 & m_{K^-p}\end{pmatrix}$$



$$p_{p'}^{lab}=\frac{1}{m_{K^-p}}\begin{pmatrix}(E_{K^-}+E_p)\,E_{p'}^{(K^-p)*}+(p_{K^-}\pm p_f)\,p_{p'}^{(K^-p)*}\cos\theta\\ (p_{K^-}\pm p_f)\,E_{p'}^{(K^-p)*}+(E_{K^-}+E_p)\,p_{p'}^{(K^-p)*}\cos\theta\\ m_{K^-p}\,p_{p'}^{(K^-p)*}\sin\theta\end{pmatrix}$$

$$\frac{1}{m_{K^-{}^3\mathrm{He}}}\begin{pmatrix}E_{K^-}+m_{^3\mathrm{He}} & -p_{K^-} & 0\\ -p_{K^-} & E_{K^-}+m_{^3\mathrm{He}} & 0\\ 0 & 0 & m_{K^-{}^3\mathrm{He}}\end{pmatrix}$$

$$m_{K^-{}^3\mathrm{He}}=\sqrt{m_{K^-}^2+m_{^3\mathrm{He}}^2+2m_{^3\mathrm{He}}E_{K^-}}$$

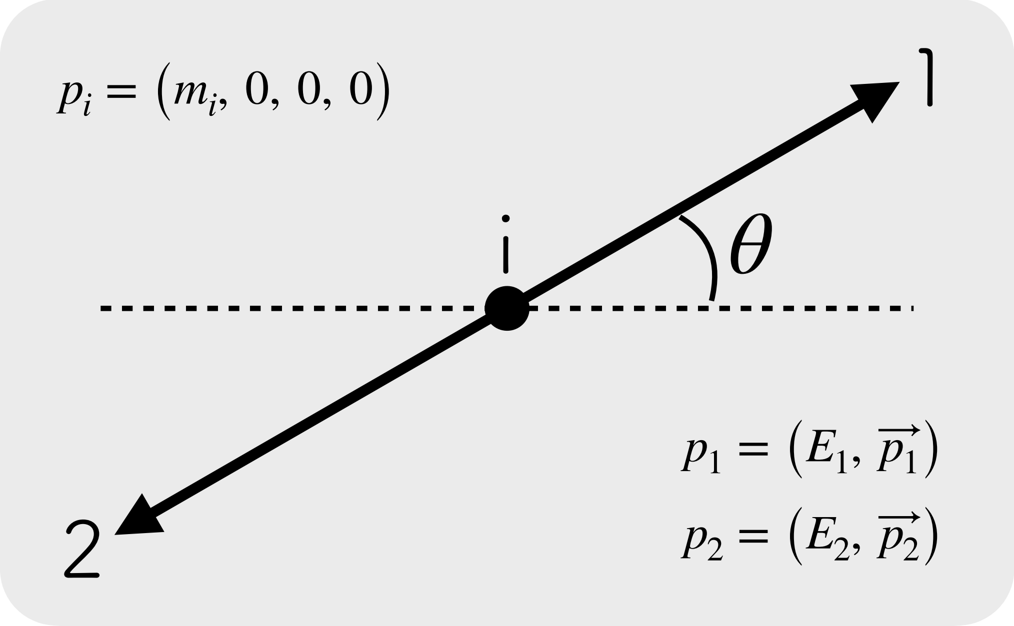


$$p_{p'}^{(\pi^-\Lambda pp')*}=\frac{1}{m_{K^-{}^3\mathrm{He}}}\frac{1}{m_{K^-p}}\begin{pmatrix}\left(E_{K^-{}^3\mathrm{He}}\,E_{K^-p}-p_{K^-}\,p_{K^-p}\right)E_{p'}^{(K^-p)*}+\left(E_{K^-{}^3\mathrm{He}}\,p_{K^-p}-p_{K^-}\,E_{K^-p}\right)p_{p'}^{(K^-p)*}\cos\theta\\ \left(E_{K^-{}^3\mathrm{He}}\,p_{K^-p}-p_{K^-}\,E_{K^-p}\right)E_{p'}^{(K^-p)*}+\left(E_{K^-{}^3\mathrm{He}}\,E_{K^-p}-p_{K^-}\,p_{K^-p}\right)p_{p'}^{(K^-p)*}\cos\theta\\ m_{K^-{}^3\mathrm{He}}\,m_{K^-p}\,p_{p'}^{(K^-p)*}\sin\theta\end{pmatrix}$$

$$E_{K^-{}^3\mathrm{He}}=E_{K^-}+m_{^3\mathrm{He}}\qquad E_{K^-p}=E_{K^-}+E_p\qquad p_{K^-p}=p_{K^-}\pm p_f$$

$m_{\pi^-\Lambda p}$ vs. $m_{\pi^-\Lambda}$

２体の場合のLIPS



$$d^6\rho_2 = \delta\left(m_i - E_1 - E_2\right)\ \delta^3\left(\vec{p_1} + \vec{p_2}\right)\ \frac{d^3\vec{p_1}}{(2\pi)^3\, 2E_1} \cdot \frac{d^3\vec{p_2}}{(2\pi)^3\, 2E_2}$$

$\delta^3\left(\vec{p_1} + \vec{p_2}\right)$を使って$p_2$について積分すると、

$$d^3\rho_2 = \delta\left(m_i - E_1 - E_2\right)\ \frac{d^3\vec{p_1}}{(2\pi)^3\, 2E_1} \cdot \frac{1}{(2\pi)^3\, 2E_2}$$

*ただし、$E_2^2 = m_2^2 + |\vec{p_1}|^2$ $(\vec{p_2} = -\vec{p_1})$

$d^3\vec{p_1} = |\vec{p_1}|^2 \sin\theta\, d|\vec{p_1}|\, d\theta\, d\phi = |\vec{p_1}|^2 d|\vec{p_1}|\, d\Omega$なので、

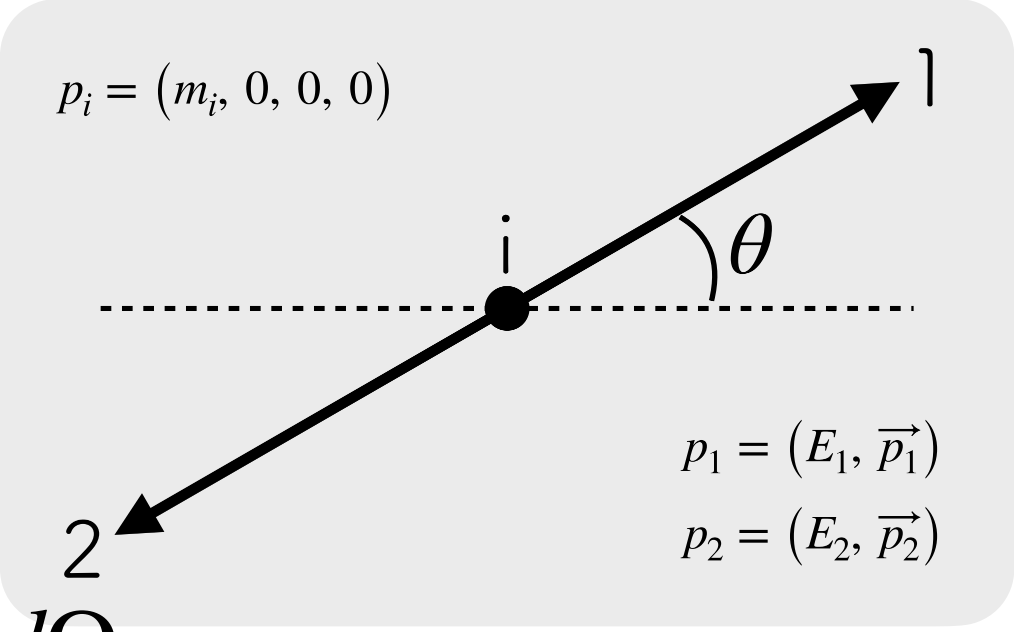
$$d^3\rho_2 = \frac{1}{(2\pi)^6}\delta\left(m_i - E_1 - E_2\right)\ \frac{|\vec{p_1}|^2}{4E_1E_2} d|\vec{p_1}|\ d\Omega$$

$$= \frac{1}{4\,(2\pi)^6}\delta\left(m_i - \sqrt{m_1^2 + |\vec{p_1}|^2} - \sqrt{m_2^2 + |\vec{p_1}|^2}\right)\ \frac{|\vec{p_1}|^2}{\sqrt{m_1^2 + |\vec{p_1}|^2}\sqrt{m_2^2 + |\vec{p_1}|^2}} d|\vec{p_1}|\ d\Omega$$

これは、「$\delta\left(f(x)\right) g(x)$」という形（$f(|\vec{p_1}|) = m_i - \sqrt{m_1^2 + |\vec{p_1}|^2} - \sqrt{m_2^2 + |\vec{p_1}|^2}$、$g(|\vec{p_1}|) = \frac{|\vec{p_1}|^2}{\sqrt{m_1^2 + |\vec{p_1}|^2}\sqrt{m_2^2 + |\vec{p_1}|^2}}$）

なので、$|\vec{p_1}|$についての積分は、

２体の場合のLIPS（続）



$$d^3\rho_2 = \frac{1}{4(2\pi)^6}\delta\left(m_i - \sqrt{m_1^2 + \vec{p_1}^{\,2}} - \sqrt{m_2^2 + \vec{p_1}^{\,2}}\right)\frac{\vec{p_1}^{\,2}}{\sqrt{m_1^2 + \vec{p_1}^{\,2}}\sqrt{m_2^2 + \vec{p_1}^{\,2}}}d\,\vec{p_1}\,d\Omega$$

これは、「$\delta\left(f(x)\right)g(x)$」という形（$f(\vec{p_1}) = m_i - \sqrt{m_1^2 + \vec{p_1}^{\,2}} - \sqrt{m_2^2 + \vec{p_1}^{\,2}}$、$g(\vec{p_1}) = \dfrac{\vec{p_1}^{\,2}}{\sqrt{m_1^2 + \vec{p_1}^{\,2}}\sqrt{m_2^2 + \vec{p_1}^{\,2}}}$）

なので、$\vec{p_1}$ についての積分は、$\displaystyle\int \delta\left(f(x)\right)g(x)\,dx = \frac{1}{df/dx\,\Big|_{x=a}}\cdot g(a)$（ただし、$f(a) = 0$）となる。

$f = 0$となる $\vec{p_1}$ を $\vec{p}*$ とする。

$$\frac{df}{d\,\vec{p_1}} = -\frac{\vec{p_1}}{\sqrt{m_1^2 + \vec{p_1}^{\,2}}} - \frac{\vec{p_1}}{\sqrt{m_2^2 + \vec{p_1}^{\,2}}} = -\frac{\vec{p_1}}{E_1} - \frac{\vec{p_1}}{E_2} = -\frac{E_1 + E_2}{E_1E_2}\vec{p_1}$$ なので、

$$d^2\rho_2 = \frac{1}{4(2\pi)^6}d\Omega\left|\frac{E_1E_2}{(E_1 + E_2)\,\vec{p_1}}\frac{\vec{p_1}^{\,2}}{E_1E_2}\right|_{\vec{p_1} = \vec{p}*} = \frac{1}{4(2\pi)^6}d\Omega\left|\frac{\vec{p_1}}{E_1 + E_2}\right|_{\vec{p_1} = \vec{p}*} = \frac{1}{4(2\pi)^6}\frac{\vec{p}*}{m_i}d\Omega$$

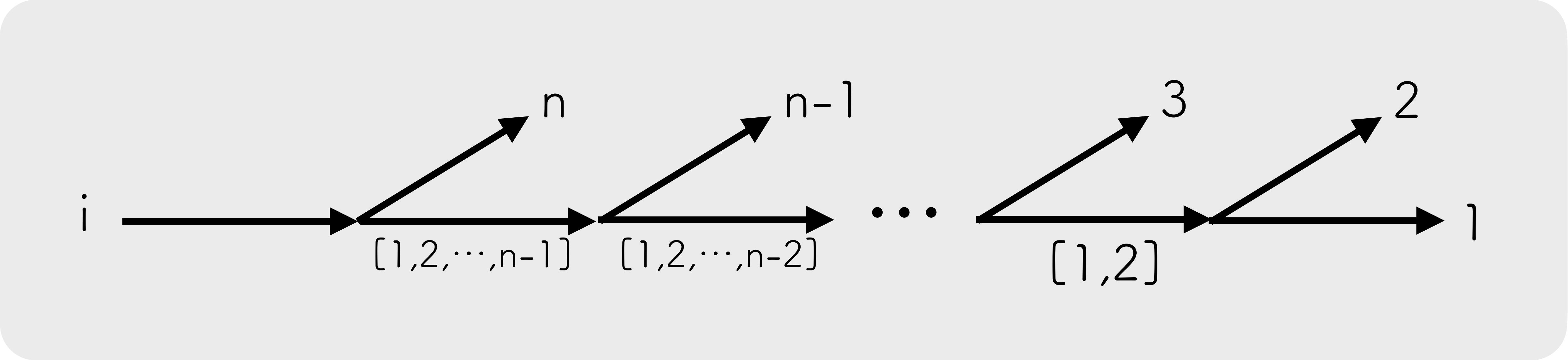
$d\Omega$についても積分すると、（$\int d\Omega = 4\pi$）

$$\rho_2 = \frac{1}{64\pi^5}\frac{\vec{p}*}{m_i}$$

ただし、$\vec{p}* = \dfrac{\sqrt{\left(m_i^2 - (m_1 + m_2)^2\right)\left(m_i^2 - (m_1 - m_2)^2\right)}}{2m_i}$

３体以上の場合のLIPS

３体以上について、２体の場合のように計算していくのは大変。（３体までならできそうだけど。。。
しかし、以下のように２対崩壊が連続して起こると考えると、n体のLIPSを逐次的に計算できる。



つまり、

$$d\rho_n = d\rho_2\,(i;\ \ [1,2,\cdots,n-1],n)\ d\rho_{n-1}\,([1,2,\cdots,n-1];\ \ 1,2,\cdots,n-1)\ (2\pi)^3\,dm^2_{[1,2,\cdots,n-1]}$$

ただし、$[1,2,\cdots,n-1]$は1~n-1番目の粒子を一塊と思ったもの。

例えば、３体のLIPSは、

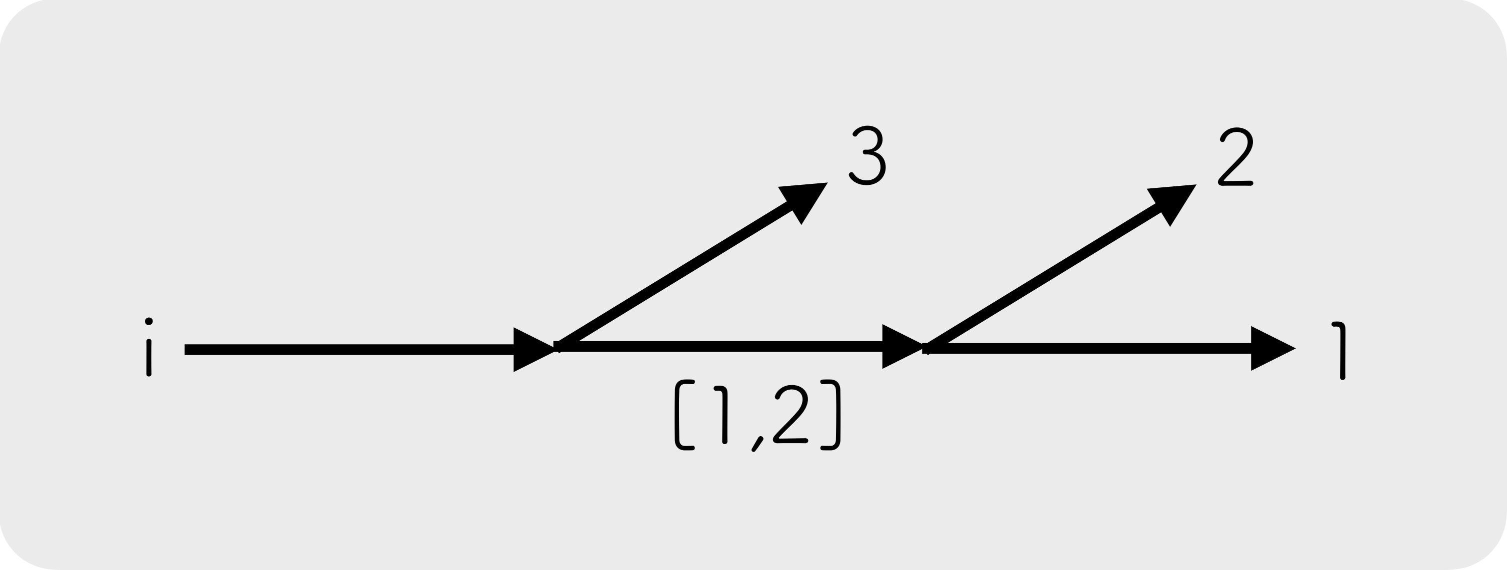
$$d\rho_3 = d\rho_2(i;\ \ [1,2],3)\,d\rho_2([1,2];\ \ 1,2)\,(2\pi)^3\,dm^2_{[1,2]}$$

4体のLIPSは、

$$d\rho_4 = d\rho_2(i;\ \ [1,2,3],4)\,d\rho_3([1,2,3];\ \ 1,2,3)\,(2\pi)^3\,dm^2_{[1,2,3]}$$

$$= d\rho_2(i;\ \ [1,2,3],4)\,d\rho_2([1,2,3];\ \ [1,2],3)\,d\rho_2([1,2];\ \ 1,2)\,(2\pi)^6\,dm^2_{[1,2,3]}dm^2_{[1,2]}$$

3体のLIPS



2体のLIPS

$$\rho_2 = \frac{\pi}{(2\pi)^6}\frac{\vec{p}^*}{m_i}$$

$$d\rho_3 = d\rho_2(i;\ [1{,}2],3)\,d\rho_2([1{,}2];\ 1{,}2)(2\pi)^3\,dm^2_{[1,2]}$$

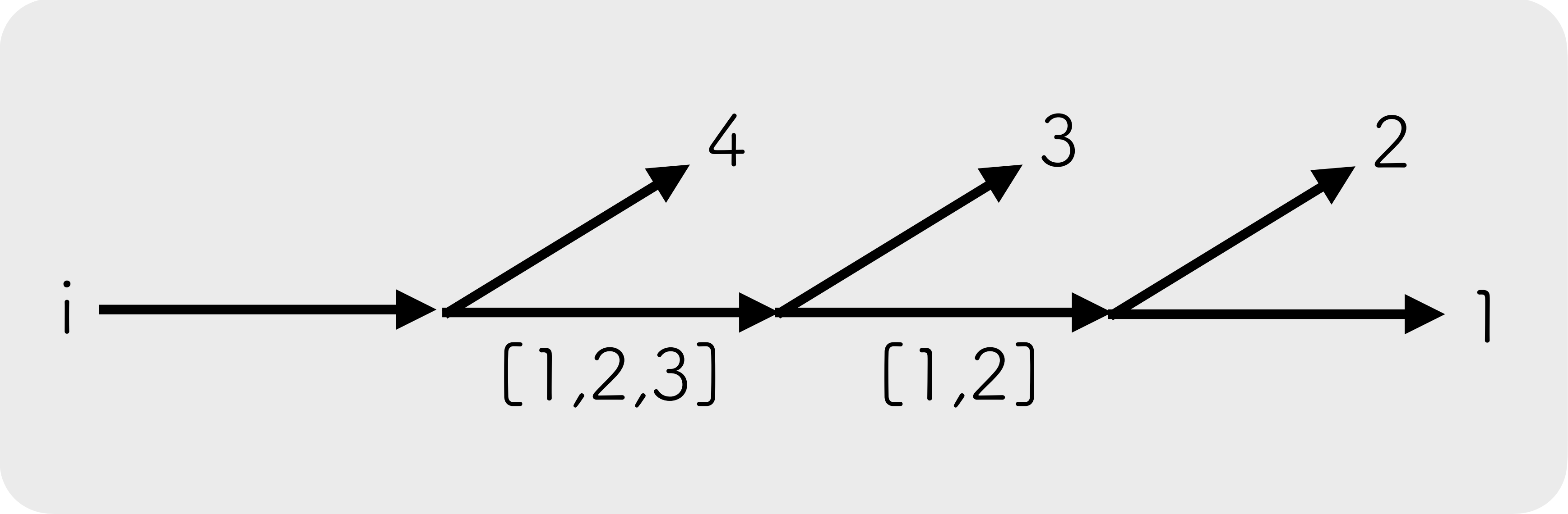
$$= \frac{\pi}{(2\pi)^6}\frac{\vec{p_3}^*}{m_i}\frac{\pi}{(2\pi)^6}\frac{\vec{p_2}^*}{m_{[1,2]}}(2\pi)^3\,dm^2_{[1,2]} = \frac{\pi^2}{(2\pi)^9}\frac{\vec{p_3}^*}{m_i}\frac{\vec{p_2}^*}{m_{[1,2]}}\,dm^2_{[1,2]}$$

$dm^2_{[1,2]} = 2m_{[1,2]}\,dm_{[1,2]}$なので、

$$\frac{d\rho_3}{dm_{[1,2]}} = \frac{2\pi^2}{(2\pi)^9}\frac{\vec{p_3}^*}{m_i}\vec{p_2}^*$$

ただし、 $\vec{p_3}^* = \dfrac{\sqrt{\left(m_i^2-(m_{[1,2]}+m_3)^2\right)\left(m_i^2-(m_{[1,2]}-m_3)^2\right)}}{2m_i}$, $\vec{p_2}^* = \dfrac{\sqrt{\left(m_{[1,2]}^2-(m_1+m_2)^2\right)\left(m_{[1,2]}^2-(m_1-m_2)^2\right)}}{2m_{[1,2]}}$

Four-body LIPS



Two-body LIPS

$$\rho_2 = \frac{\pi}{(2\pi)^6}\frac{\vec{p}^*}{m_i}$$

$$d\rho_4 = d\rho_2(i;\ [1,2,3],4)\,d\rho_2([1,2,3];\ [1,2],3)\,d\rho_2([1,2];\ 1,2)(2\pi)^3\, dm^2_{[1,2]}$$

$$= \frac{\pi}{(2\pi)^6}\frac{\vec{p_4}^*}{m_i}\frac{\pi}{(2\pi)^6}\frac{\vec{p_3}^*}{m_{[1,2,3]}}\frac{\pi}{(2\pi)^6}\frac{\vec{p_2}^*}{m_{[1,2]}}(2\pi)^6\, dm^2_{[1,2,3]}\, dm^2_{[1,2]} = \frac{\pi^3}{(2\pi)^{12}}\frac{\vec{p_4}^*}{m_i}\frac{\vec{p_3}^*}{m_{[1,2,3]}}\frac{\vec{p_2}^*}{m_{[1,2]}}\, dm^2_{[1,2,3]}\, dm^2_{[1,2]}$$

$dm^2_{[1,2,3]} = 2m_{[1,2,3]}\, dm_{[1,2,3]}$, $dm^2_{[1,2]} = 2m_{[1,2]}\, dm_{[1,2]}$

$$\frac{d\rho_4}{dm_{[1,2,3]}\, dm_{[1,2]}} = \frac{4\pi^3}{(2\pi)^{12}}\frac{\vec{p_4}^*}{m_i}\,\vec{p_3}^*\,\vec{p_2}^*$$

$$\vec{p_4}^* = \frac{\sqrt{\left(m_i^2 - (m_{[1,2,3]} + m_4)^2\right)\left(m_i^2 - (m_{[1,2,3]} - m_4)^2\right)}}{2m_i},\quad \vec{p_3}^* = \frac{\sqrt{\left(m_{[1,2,3]}^2 - (m_{[1,2]} + m_3)^2\right)\left(m_{[1,2,3]}^2 - (m_{[1,2]} - m_3)^2\right)}}{2m_{[1,2,3]}},\quad \vec{p_2}^* = \frac{\sqrt{\left(m_{[1,2]}^2 - (m_1 + m_2)^2\right)\left(m_{[1,2]}^2 - (m_1 - m_2)^2\right)}}{2m_{[1,2]}}$$

デルタ関数の公式

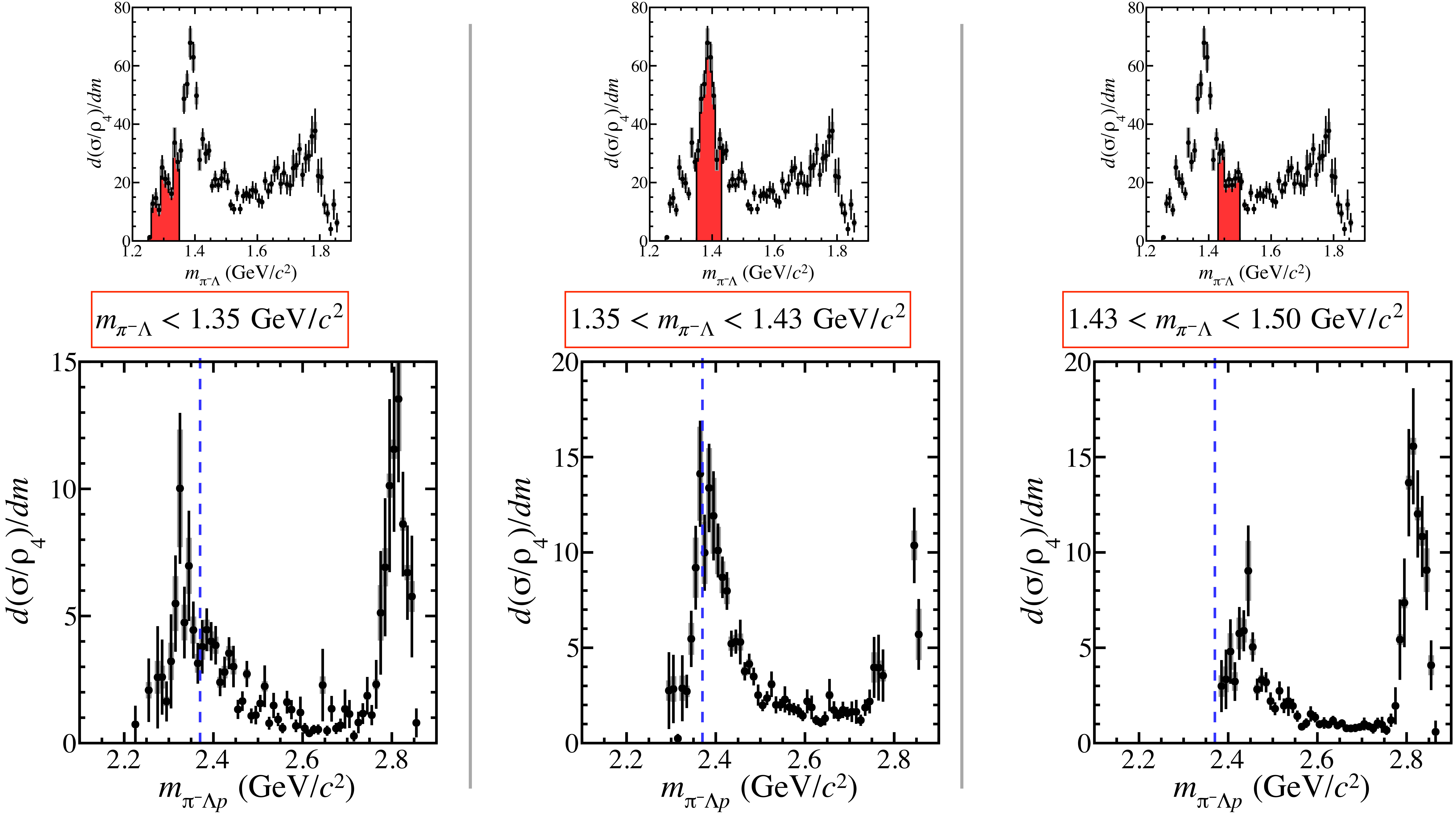
$$\int f(x)\,\delta(x)\,dx = f(0) \quad ・・・(1)$$

$$\int f(x)\,\delta(x-a)\,dx = f(a) \quad ・・・(2)$$

$$\delta(x) = \delta(-x) \quad ・・・(3)$$

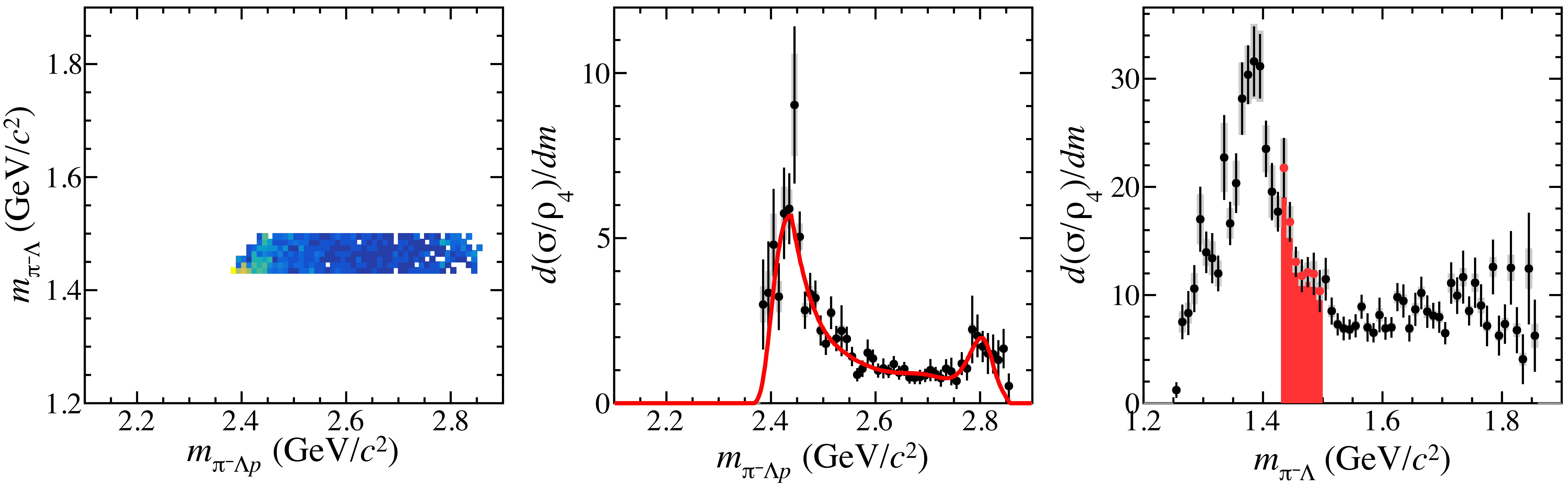
$$\delta(ax) = \frac{1}{a}\delta(x) \quad ・・・(4)$$

$$\delta\left(f(x)\right) = \sum_i \frac{1}{df/dx}\bigg|_{x=a_i} \delta(x-a_i) \quad ・・・(5)$$



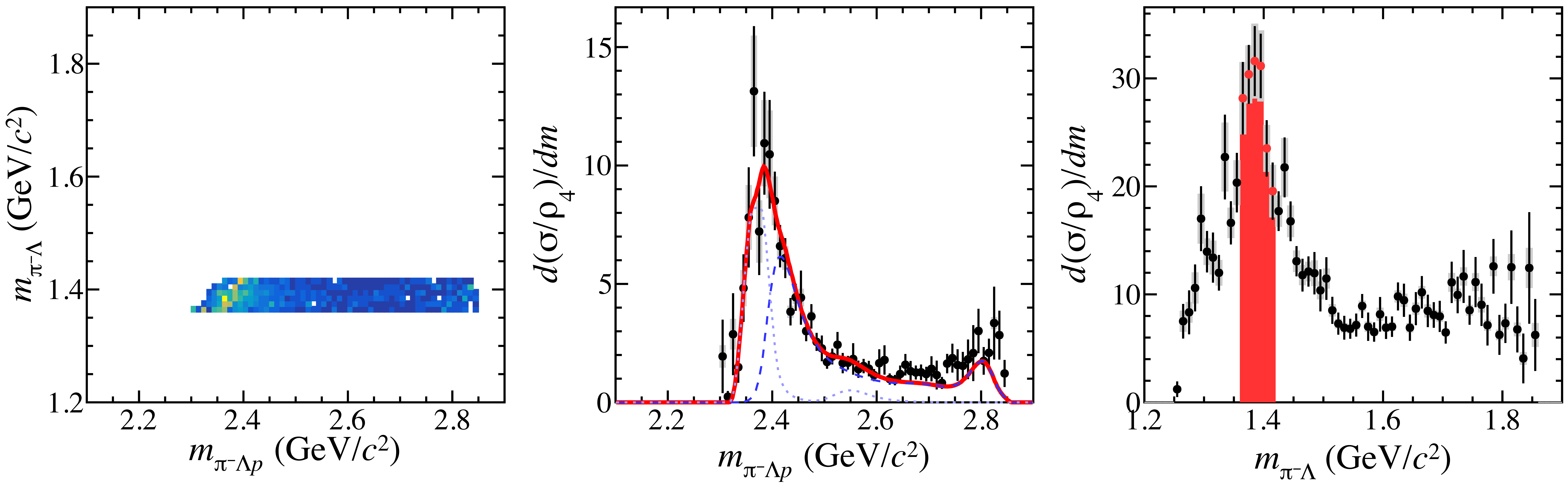
$m_{\pi^-\Lambda} < 1.35$ GeV/c^2
$1.35 < m_{\pi^-\Lambda} < 1.43$ GeV/c^2
$1.43 < m_{\pi^-\Lambda} < 1.50$ GeV/c^2
$d(\sigma/\rho_4)/dm$
$m_{\pi^-\Lambda}$ (GeV/c^2)
$m_{\pi^-\Lambda p}$ (GeV/c^2)

QF-K region



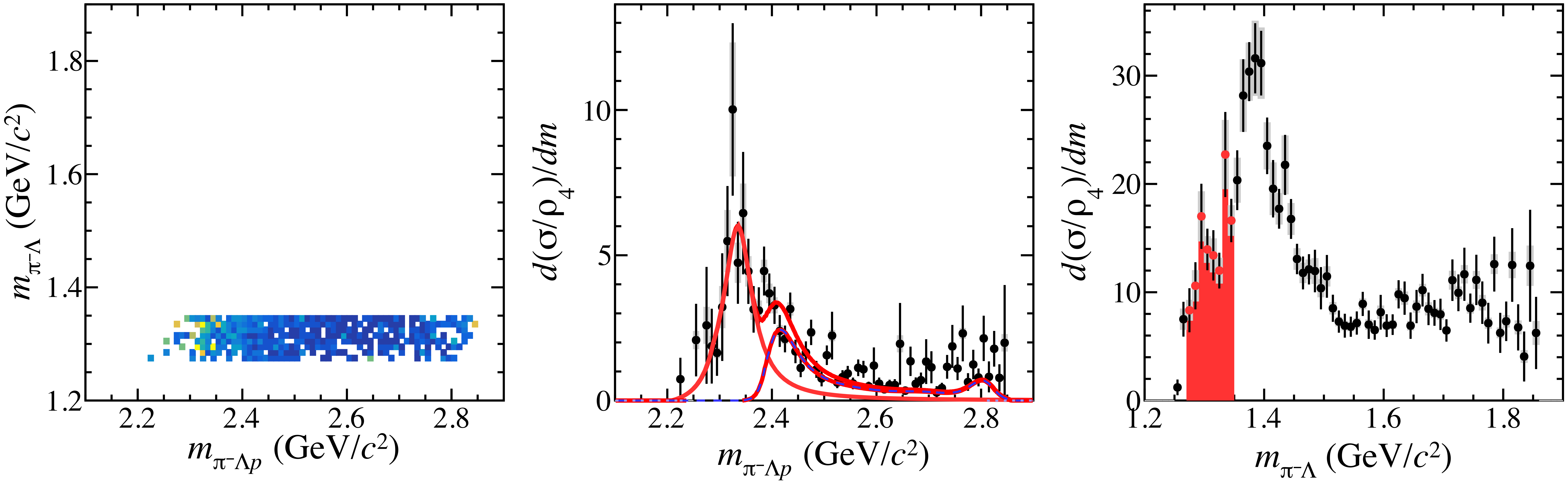
1.8
1.6
1.4
1.2
$m_{\pi^-\Lambda}$ (GeV/c^2)
2.2
2.4
2.6
2.8
$m_{\pi^-\Lambda p}$ (GeV/c^2)
10
5
0
$d(\sigma/\rho_4)/dm$
2.2
2.4
2.6
2.8
$m_{\pi^-\Lambda p}$ (GeV/c^2)
30
20
10
0
$d(\sigma/\rho_4)/dm$
1.2
1.4
1.6
1.8
$m_{\pi^-\Lambda}$ (GeV/c^2)

Σ(1385) region



$\bar{K}NN$ region

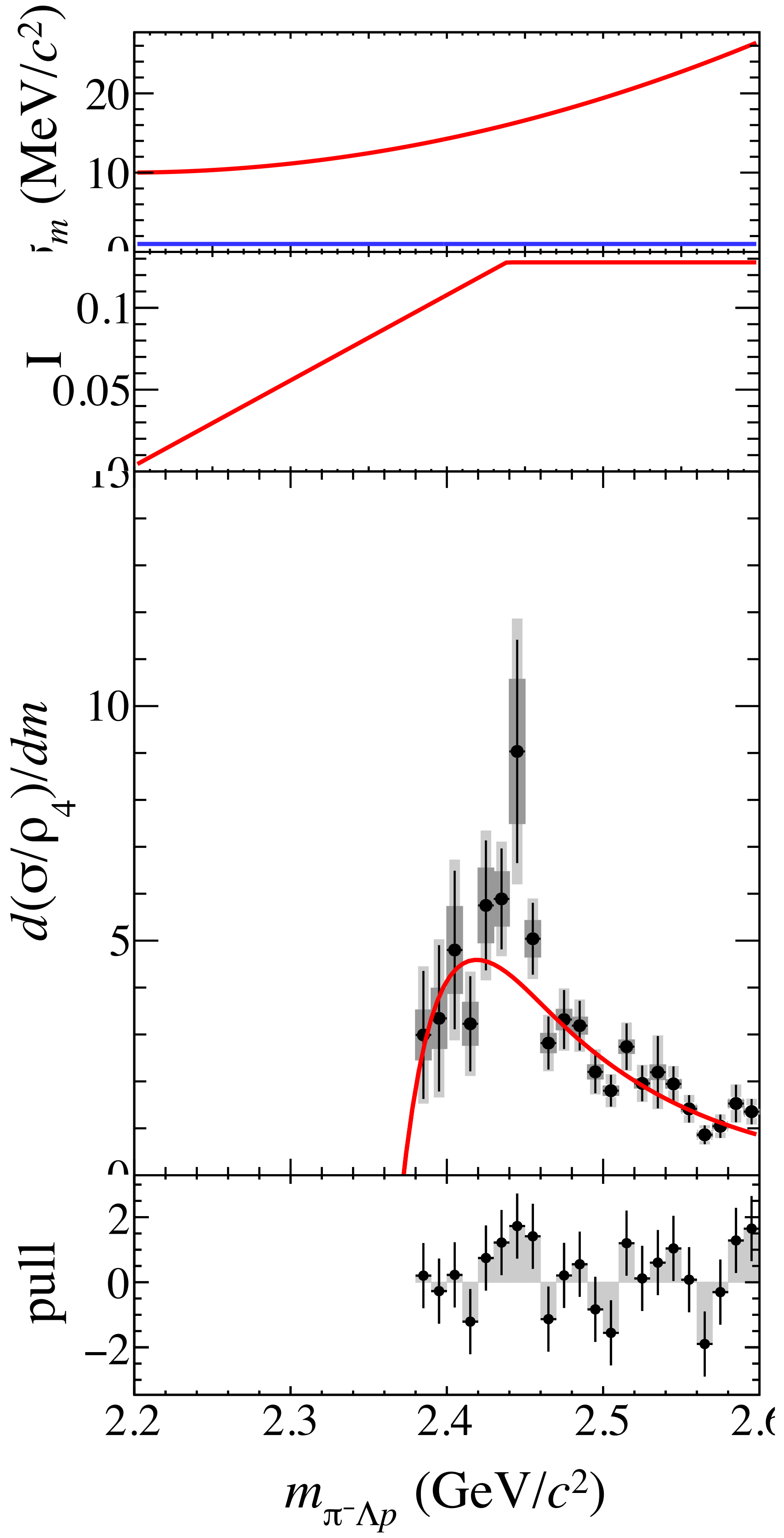
```
21  p20            2.33495e+00   4.41851e-03
  22  p21            6.62616e-02   1.38292e-02
```



QFのフィット（Double-Gaussian）

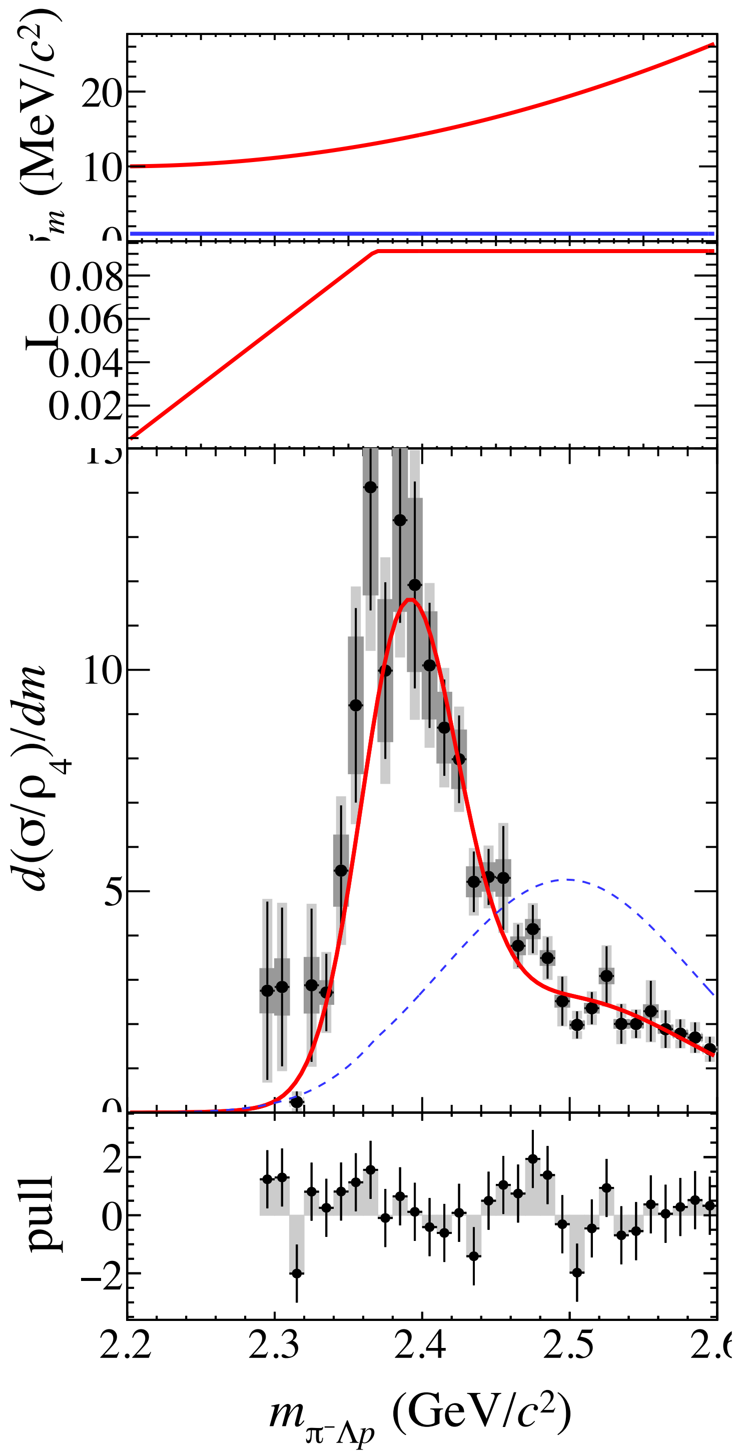
$1.43 < m_{\pi^-\Lambda} < 1.50\ \mathrm{GeV}/c^2$

3	p2	-1.75961e+04	3.50759e+04	-1.08055e+03	-3.00089e-06
4	p3	2.17574e+00	1.04407e-01	-3.05840e-03	2.58661e+00
5	p4	5.80218e-02	2.61597e-02	5.57939e-04	9.78496e+00
6	p5	6.54721e+03	1.26143e+04	3.90315e+02	-1.31474e-05
7	p6	1.36319e+00	4.51704e-01	-1.54501e-02	4.08812e-01
8	p7	3.33083e-01	7.80956e-02	2.82174e-03	3.75086e+00

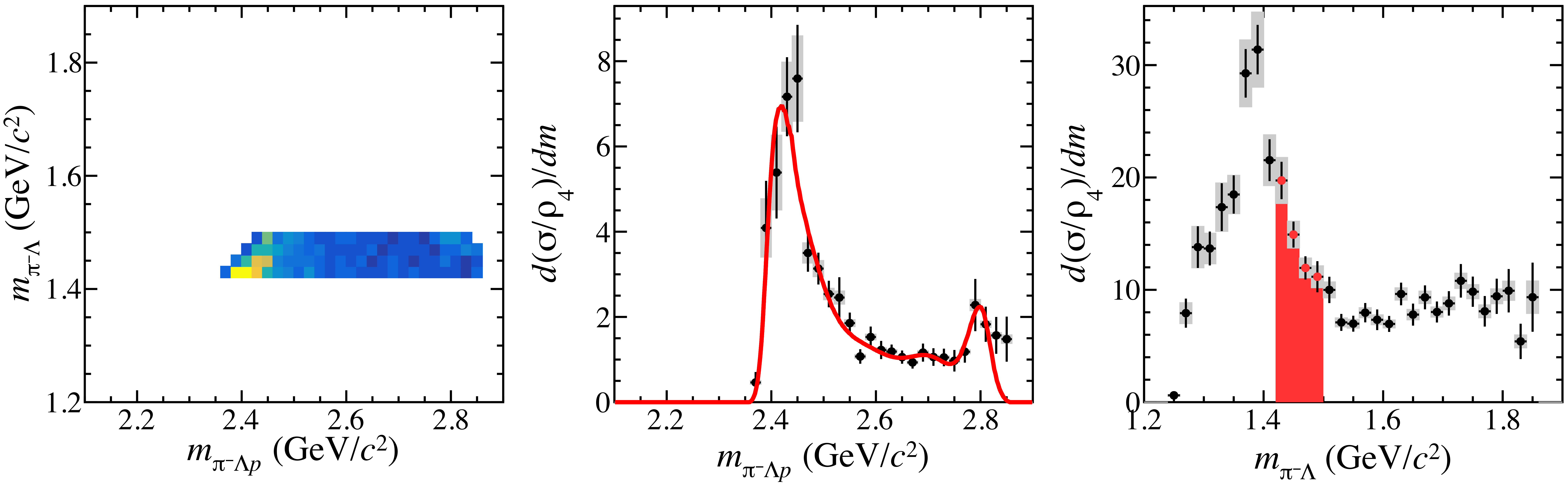


$1.35 < m_{\pi^-\Lambda} < 1.43\ \mathrm{GeV}/c^2$

3	p2	2.88240e+01	3.06421e+00	-9.70414e-03	-5.36149e-04
4	p3	2.49781e+00	1.81420e-02	7.73131e-05	-9.78580e-02
5	p4	-8.53636e-02	1.17035e-02	-2.53362e-07	1.17945e-01
6	p5	1.24821e+02	1.63160e+01	2.65250e-02	6.10031e-05
7	p6	2.39138e+00	3.25751e-03	4.52180e-06	2.63293e-01
8	p7	-3.02081e-02	3.51902e-03	-5.10249e-06	8.46429e-02



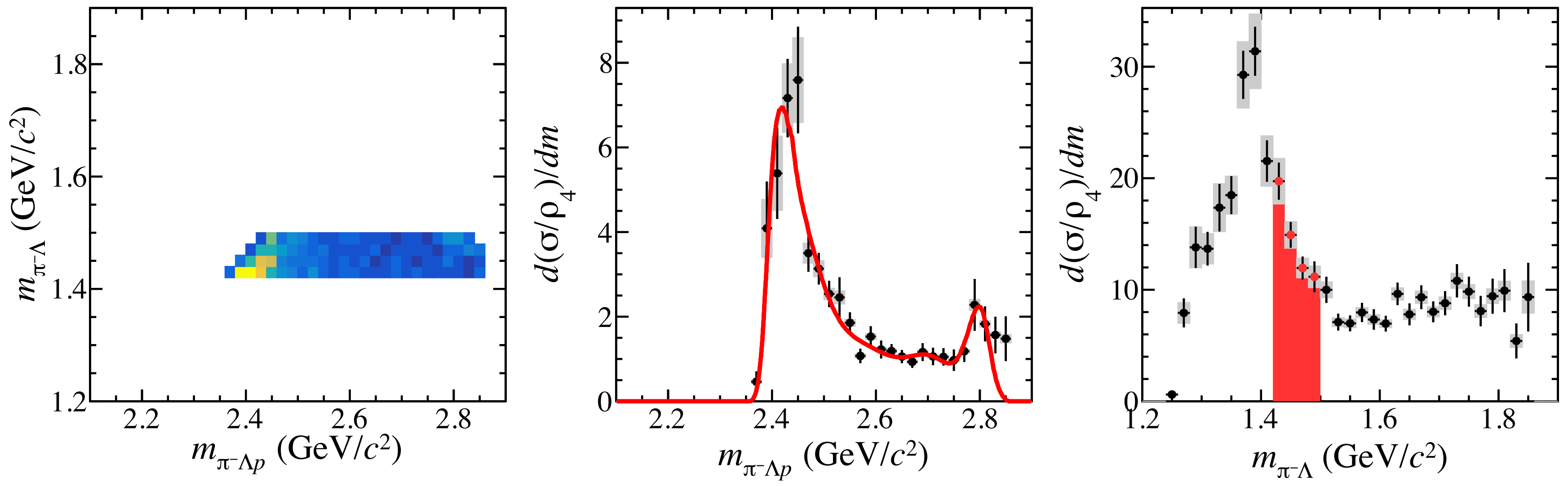
Fit (all $\cos\theta^*_{p'}$ events, QF-K region)

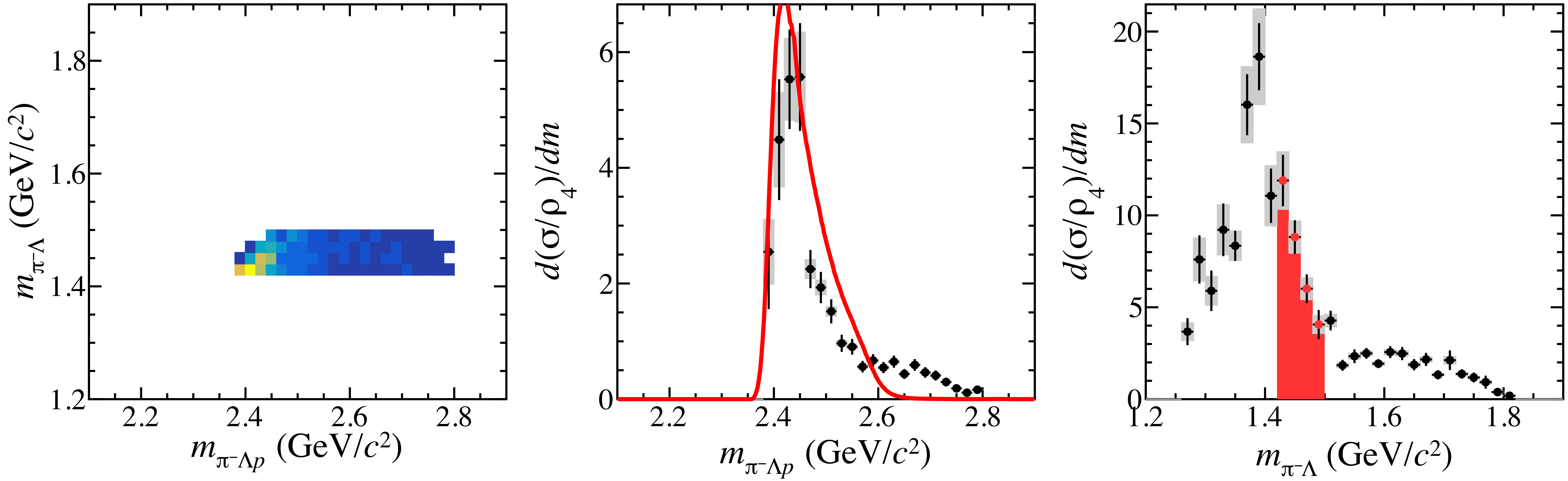


これでQF-Kのパラメータを決める。

Fit ($\cos\theta^*_{p'} > 0.5$ events, QF-K region)

前ページで決めたQF-Kのパラメータをそのまま使う。

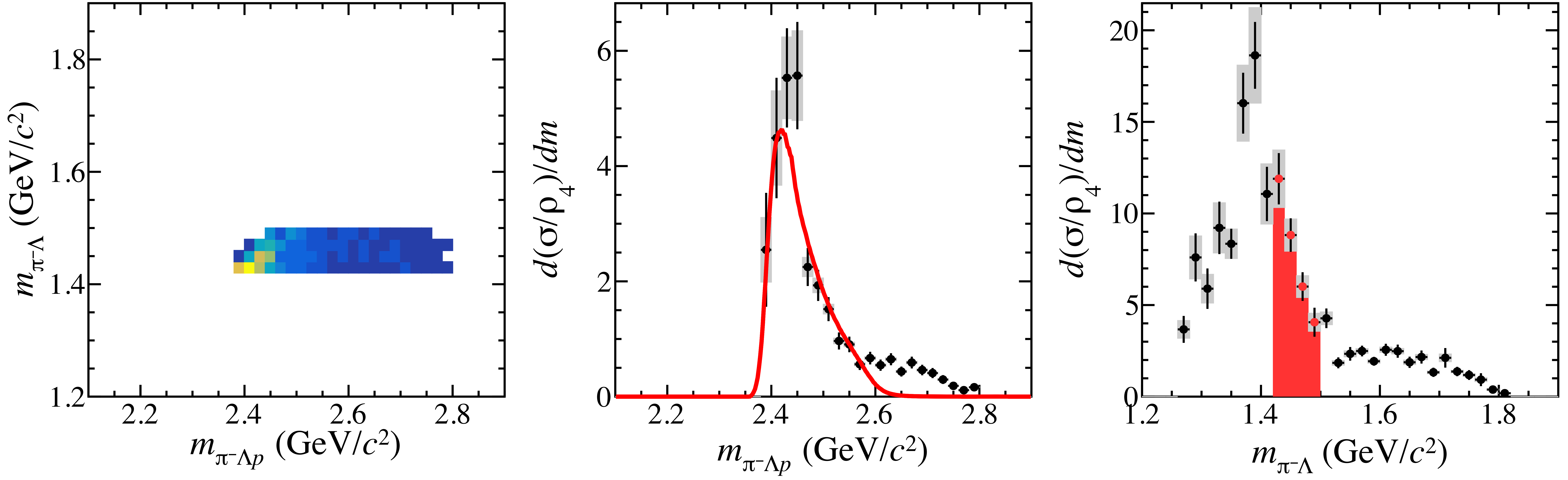




パラメータそのままだと、$\cos\theta^*_{p'} > 0.5$のイベントは合いが悪い。

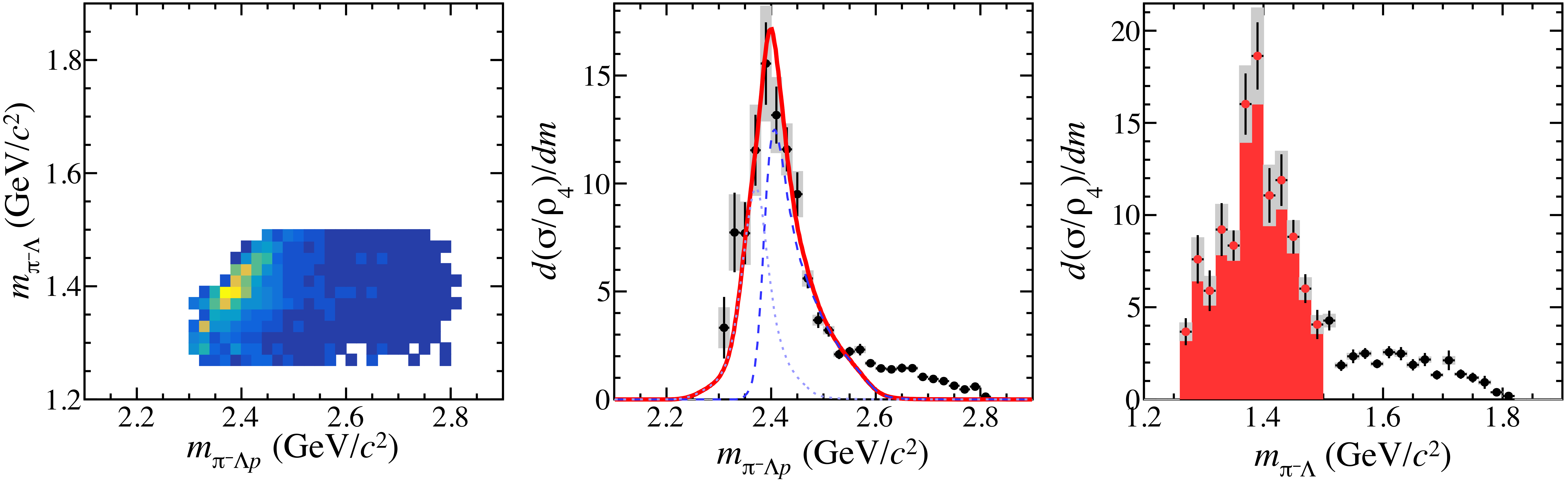
Fit ($\cos\theta^*_{p'} > 0.5$ events, QF-K region)

全ページから、データを再現するように全体をスケール。

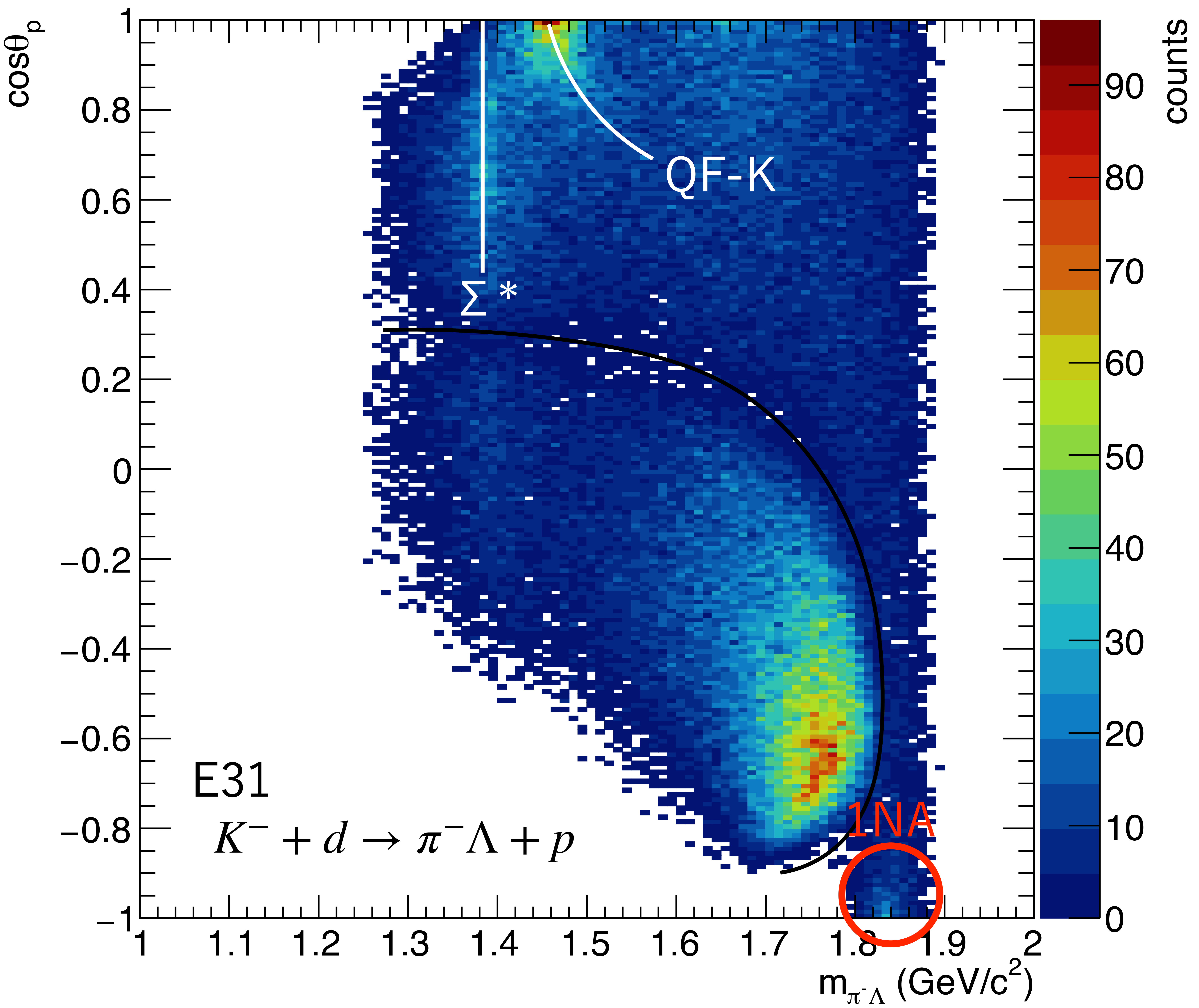


$m_{\pi^-\Lambda p} < 2.6$ GeV/c^2でデータをよく再現するのでよしとする。

パラメータをこれで固定。



1.8
1.6
1.4
1.2
2.2
2.4
2.6
2.8
$m_{\pi^-\Lambda}$ (GeV/c^2)
$m_{\pi^-\Lambda p}$ (GeV/c^2)
15
10
5
0
$d(\sigma/\rho_4)/dm$
$m_{\pi^-\Lambda p}$ (GeV/c^2)
20
15
10
5
0
1.2
1.4
1.6
1.8
$d(\sigma/\rho_4)/dm$
$m_{\pi^-\Lambda}$ (GeV/c^2)

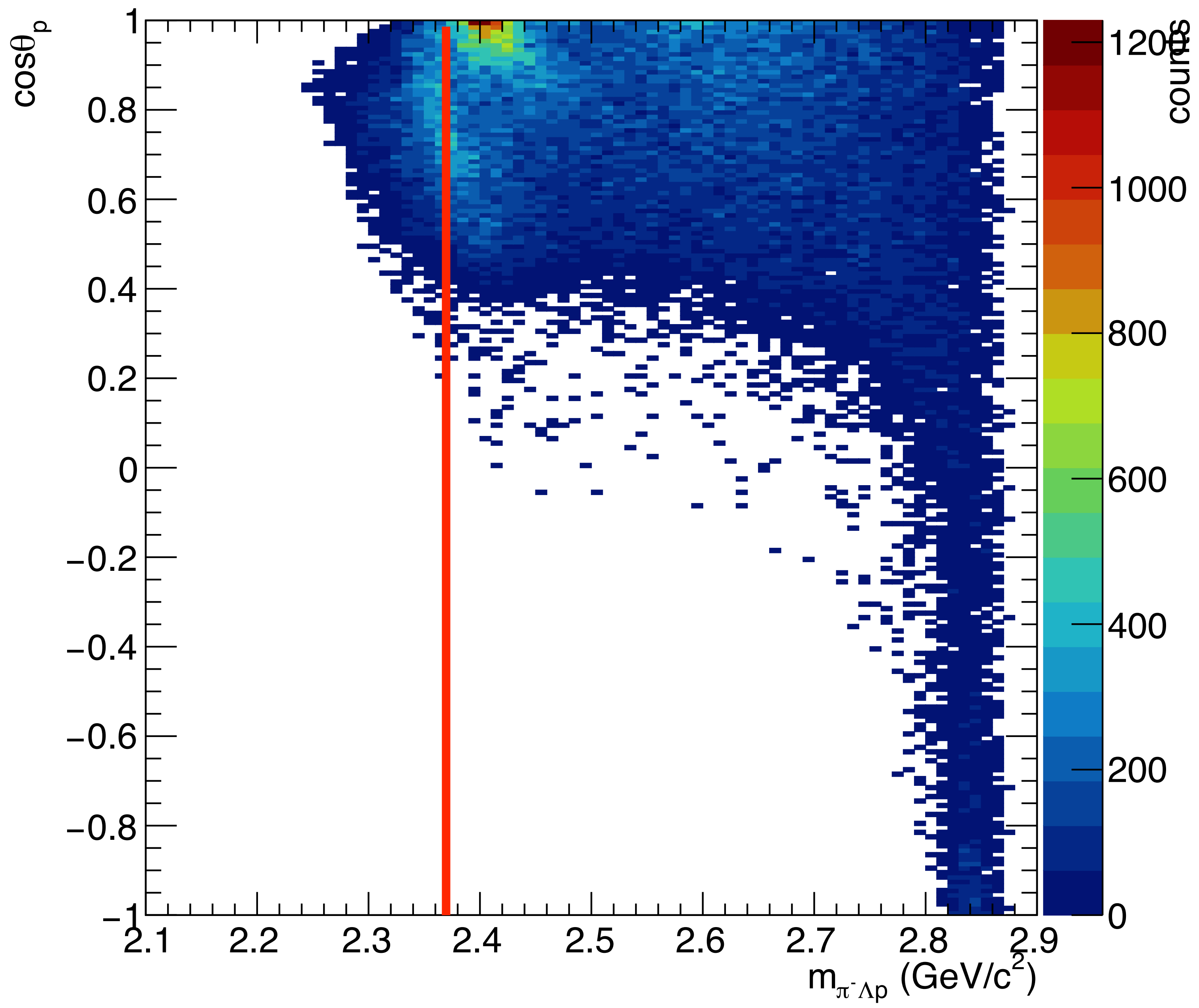


cosθp
mπ-Λ (GeV/c2)
counts
QF-K
Σ*
1NA
E31
K⁻ + d → π⁻Λ + p

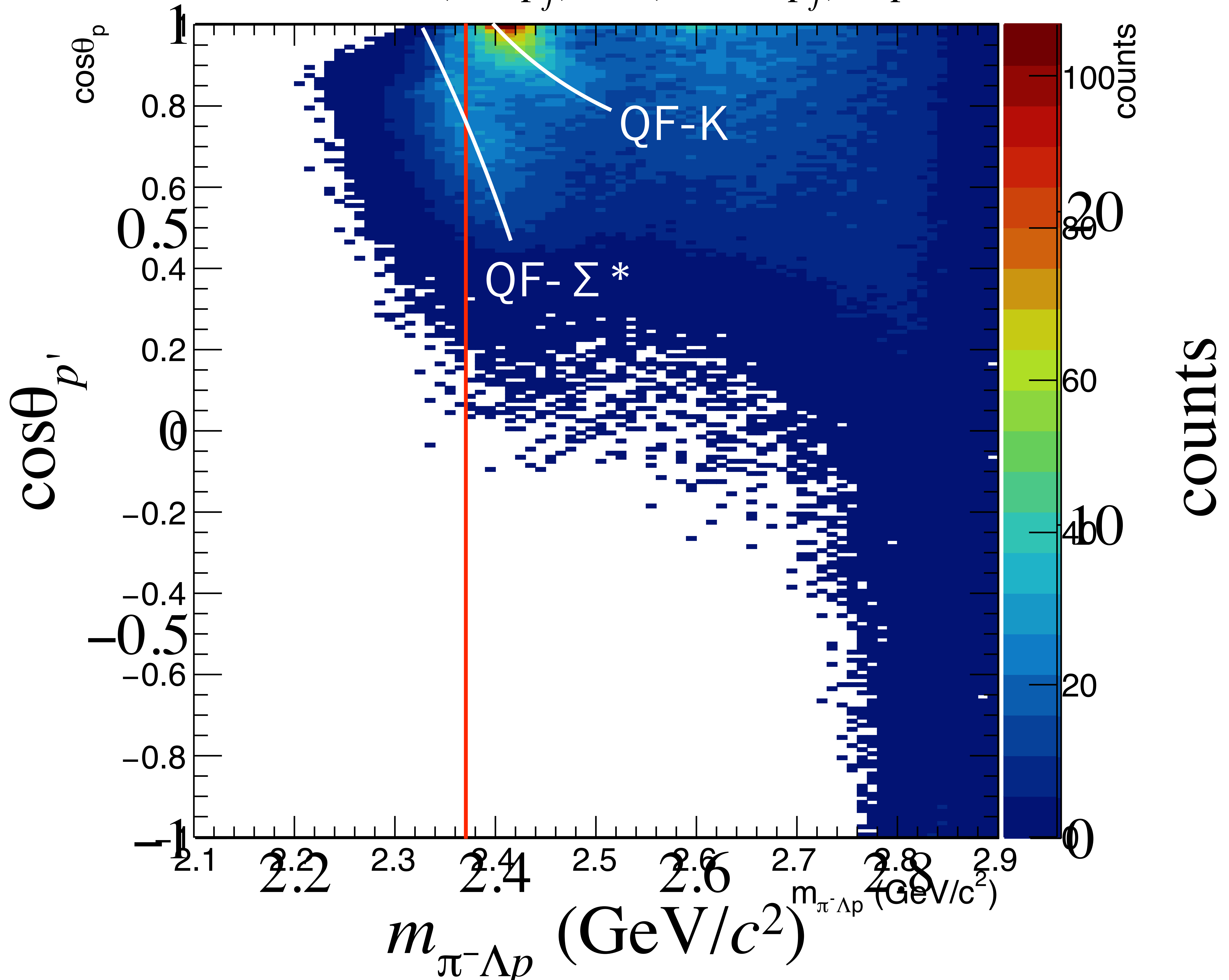
Estimation with deuteron data

Σ* events seem to be weaker in he3 than that in d. (Σ*/QF-K~1/3 in d, ~1/10 in he3)

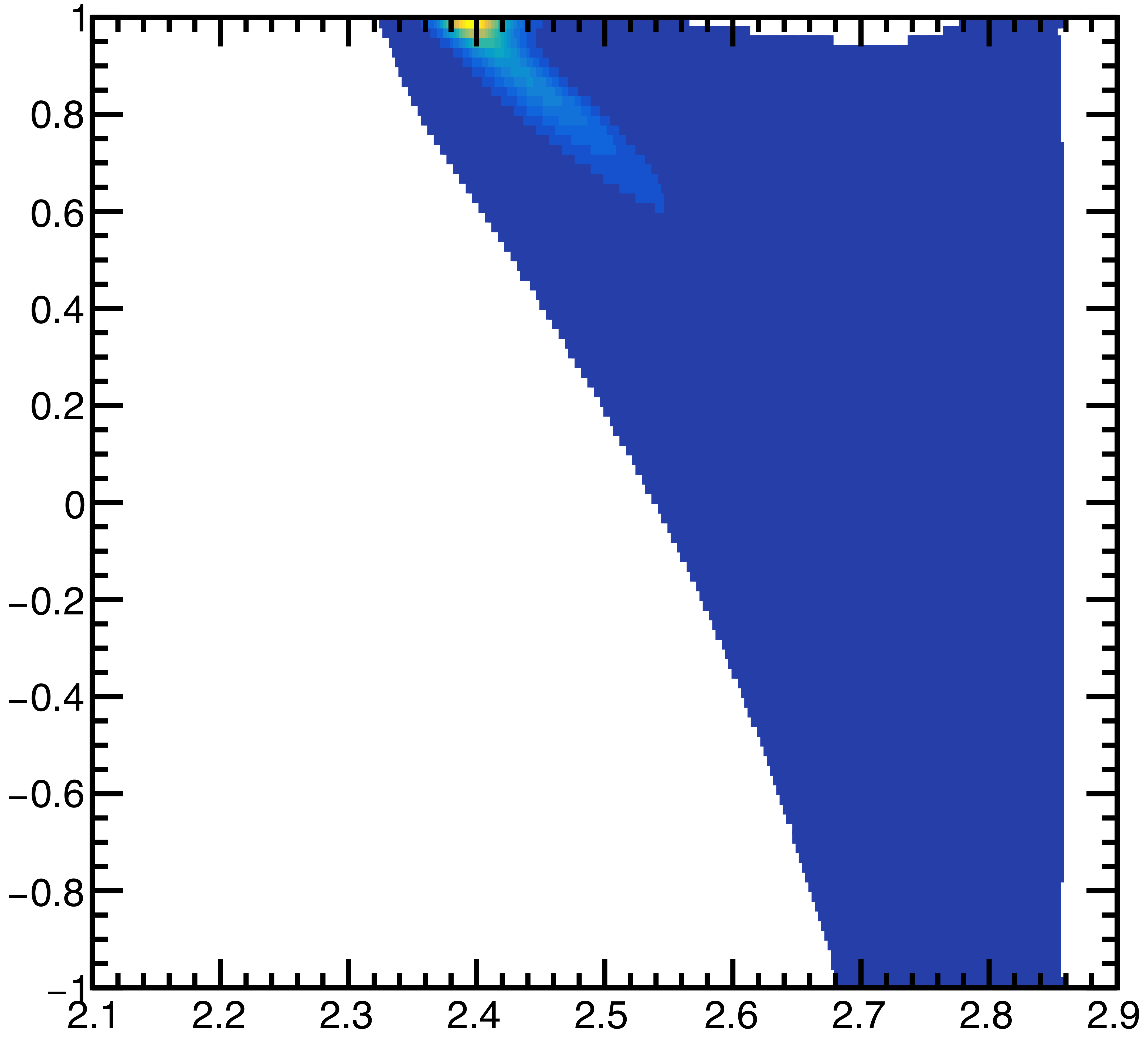
$p_f = 0$ (fixed) case



$K^- + (d + p_f) \to (\pi^-\Lambda + p_f) + p'$



$\cos\theta_{p'}^{(K^-\tilde{p}\tilde{n})^*}$ dependence for QF-K

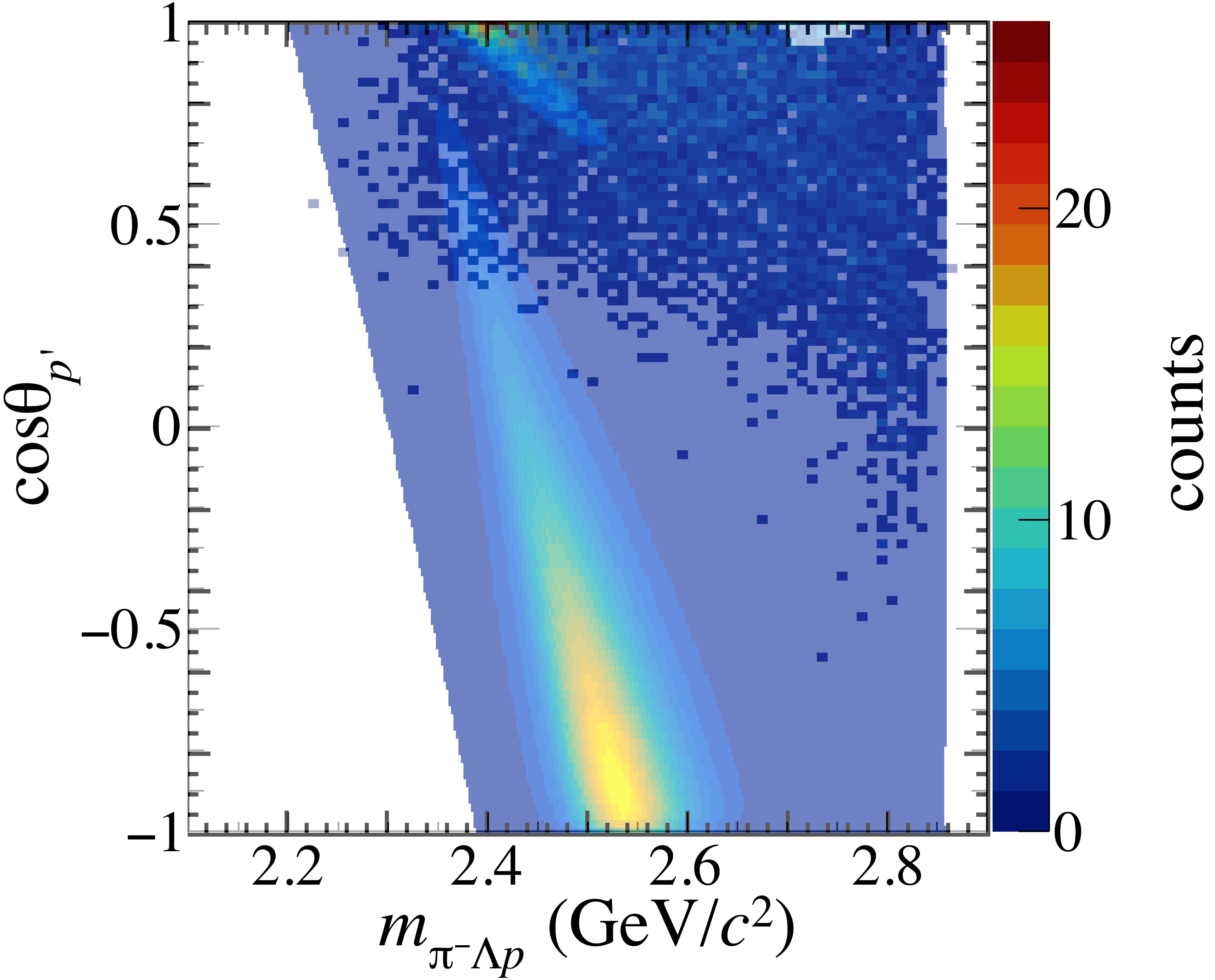


$\cos\theta_{p'}^{(K^-\tilde{p}\tilde{n})^*}$ dependence for QF-Σ*

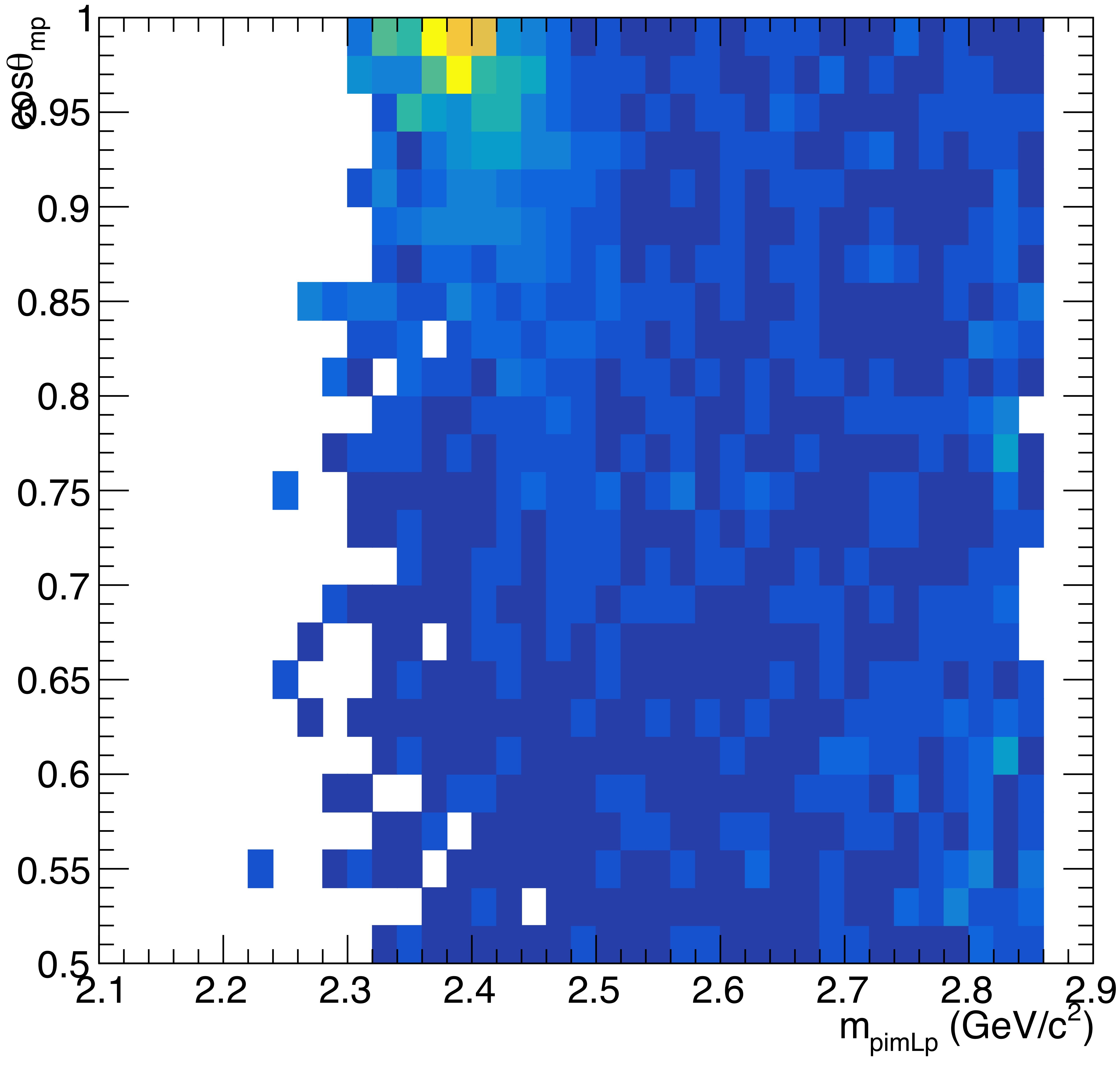
Legendre with the same parameters as the $K^-p \to K^-p$ reaction

Exponential : $\exp\left(a_0(x-1)\right)$

Gaussian



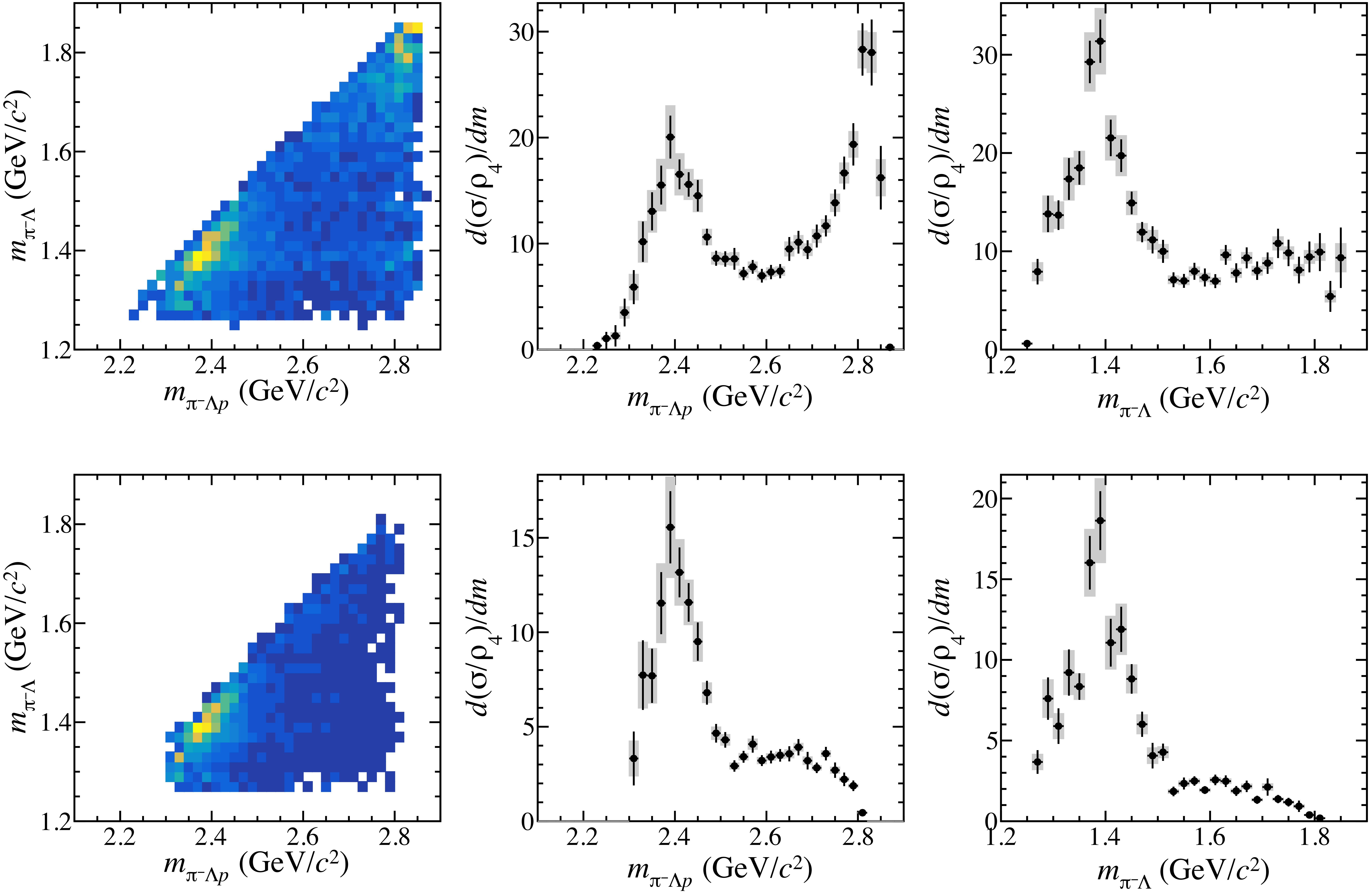
1
0.5
0
−0.5
−1
cosθ$_{p'}$
2.2
2.4
2.6
2.8
$m_{\pi^-\Lambda p}$ (GeV/c^2)
20
10
0
counts



$\cos\theta_{mp}$
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
m_{pimLp} (GeV/c^2)

all events

$\cos\theta_{p'} > 0.5$

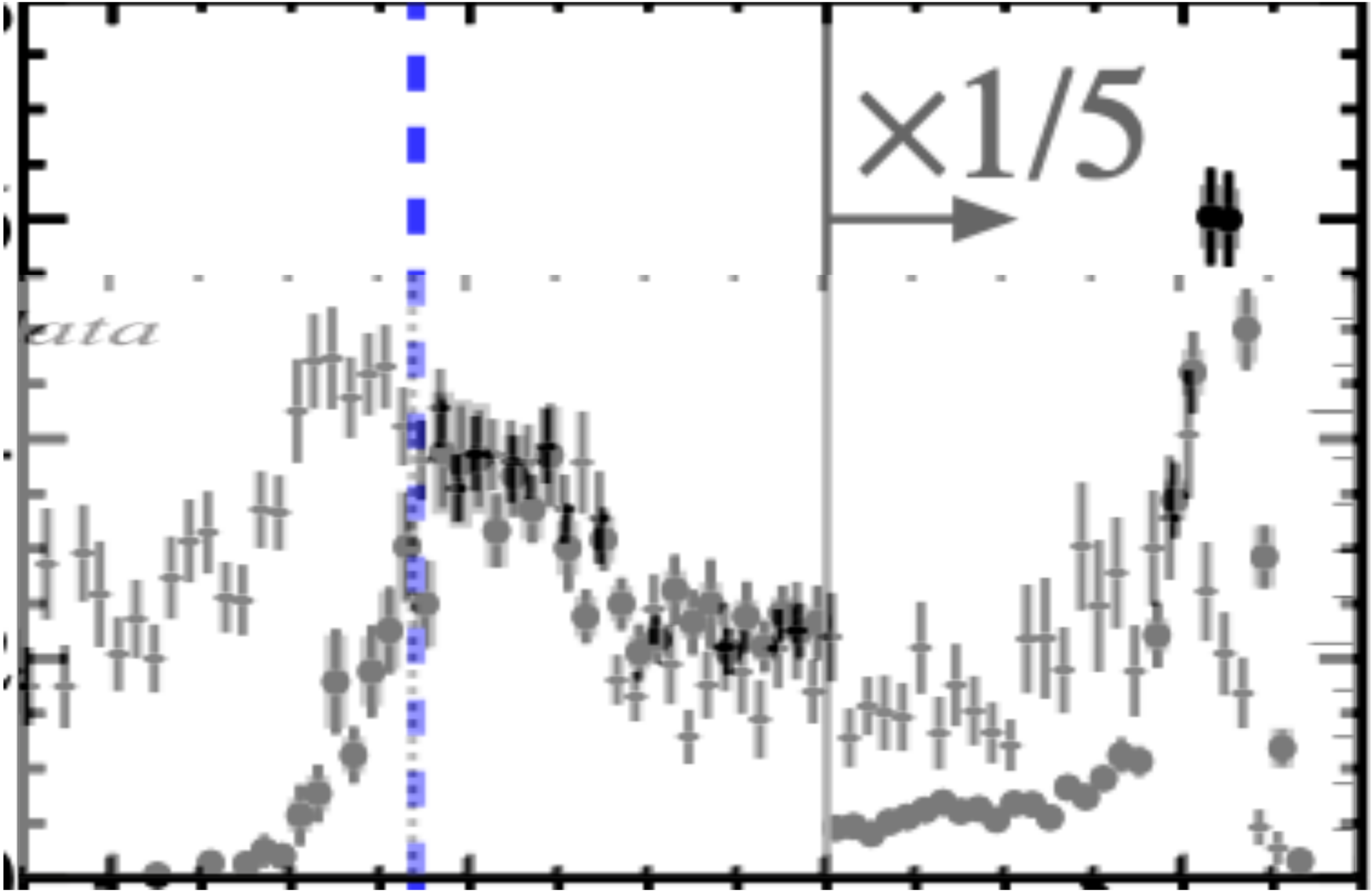


data

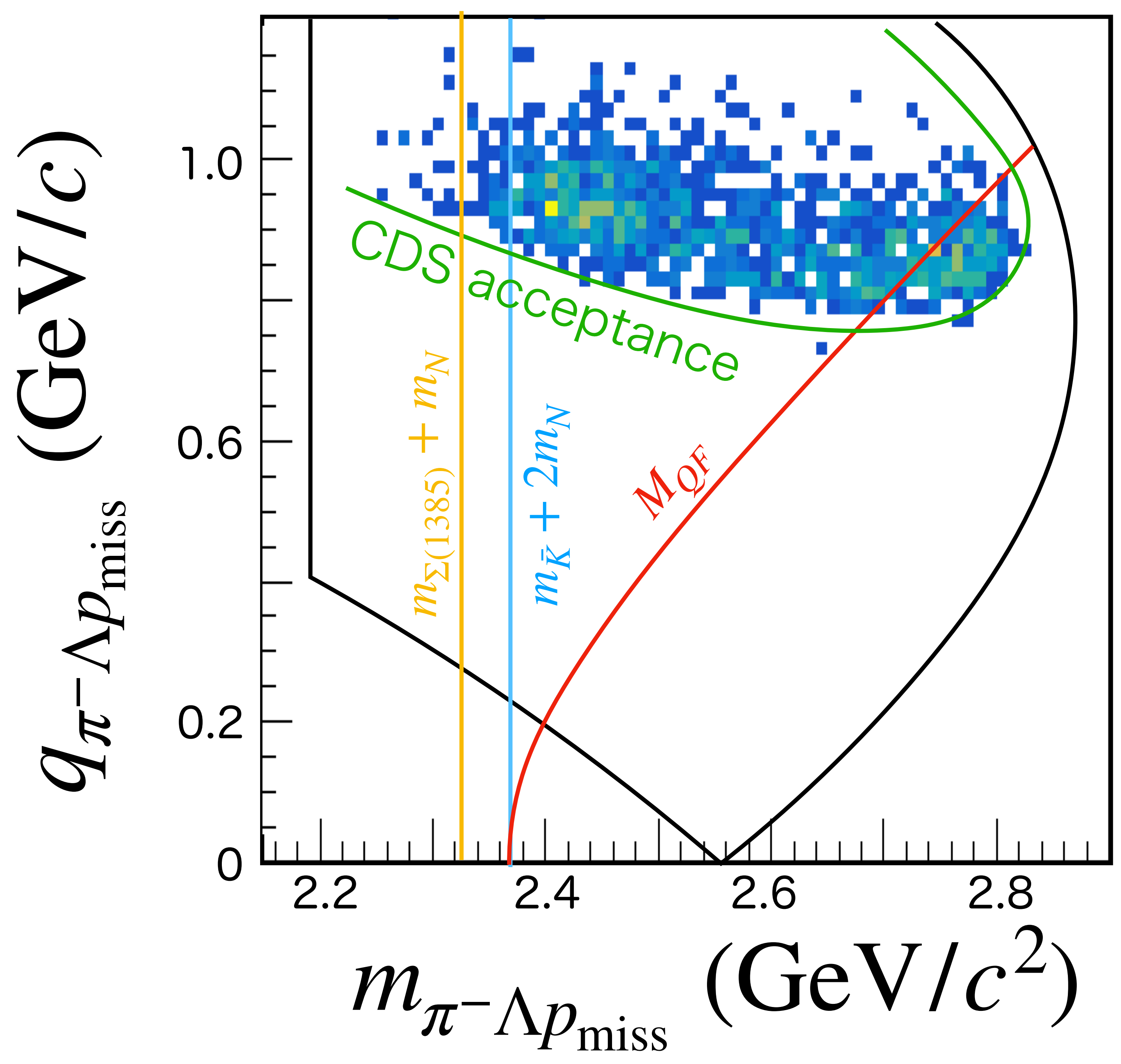
ver	
70	all
71	$\cos\theta > 0.5$

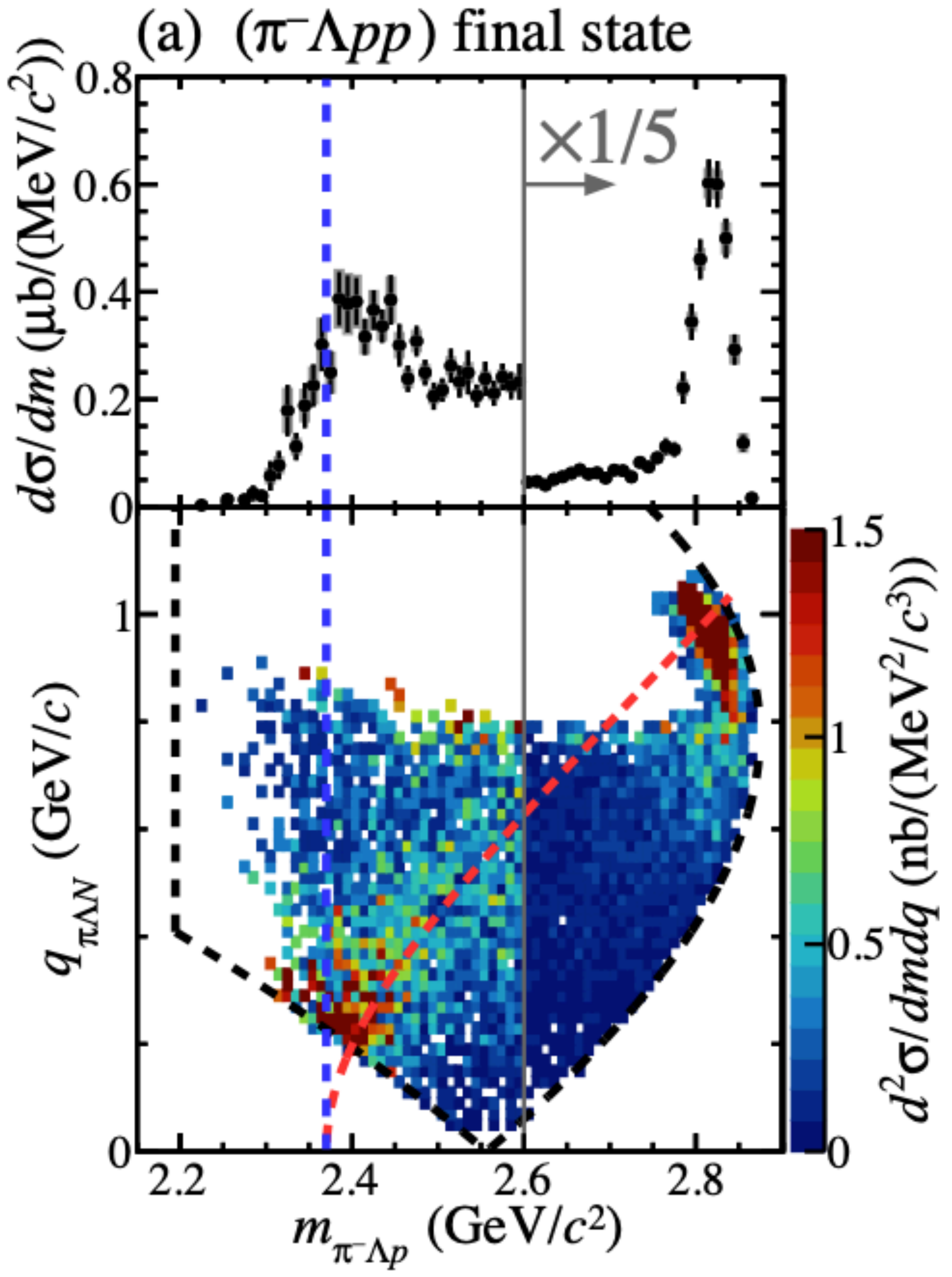
To do

- Draw event map & acceptance map
  - $m_{\pi^- \Lambda p}$ vs. $m_{\pi^- \Lambda}$ by selecting $\cos\theta^*_{p'}$ regions
-



×1/5


CDS acceptance
$m_{\Sigma(1385)} + m_N$
$m_{\bar{K}} + 2m_N$
M_{QF}
$q_{\pi^-\Lambda p_{\rm miss}}$ (GeV/c)
$m_{\pi^-\Lambda p_{\rm miss}}$ (GeV/c^2)

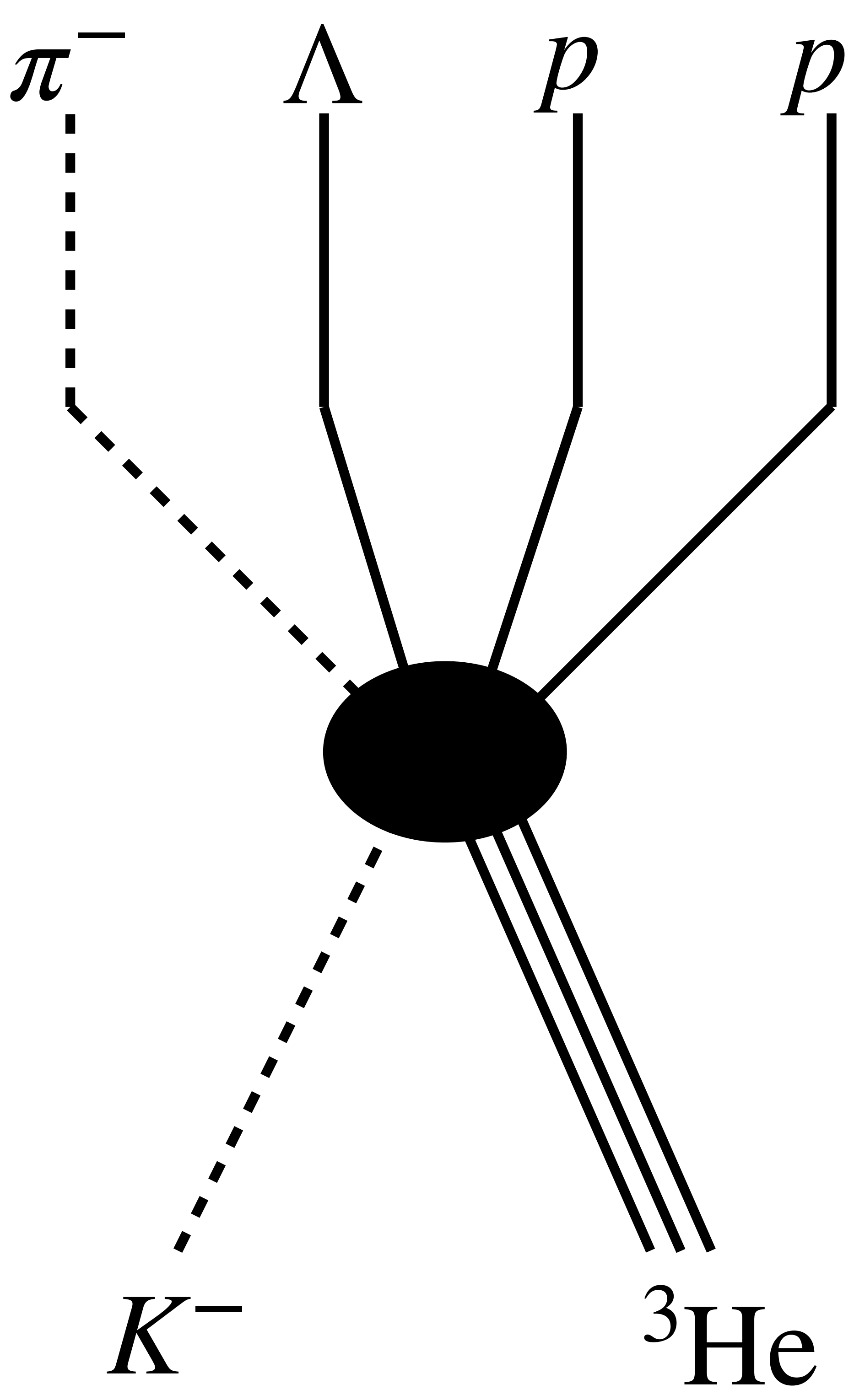

(a) ($\pi^-\Lambda pp$) final state
×1/5
$d\sigma/dm$ (μb/(MeV/c^2))
$q_{\pi\Lambda N}$ (GeV/c)
$m_{\pi^-\Lambda p}$ (GeV/c^2)
$d^2\sigma/dmdq$ (nb/(MeV^2/c^3))

メモ

- フィットは、phase space, QF-K($p_{\mathrm{miss}} = p'$), QF-K($p_{\mathrm{miss}} \neq p'$)かな。最初は。

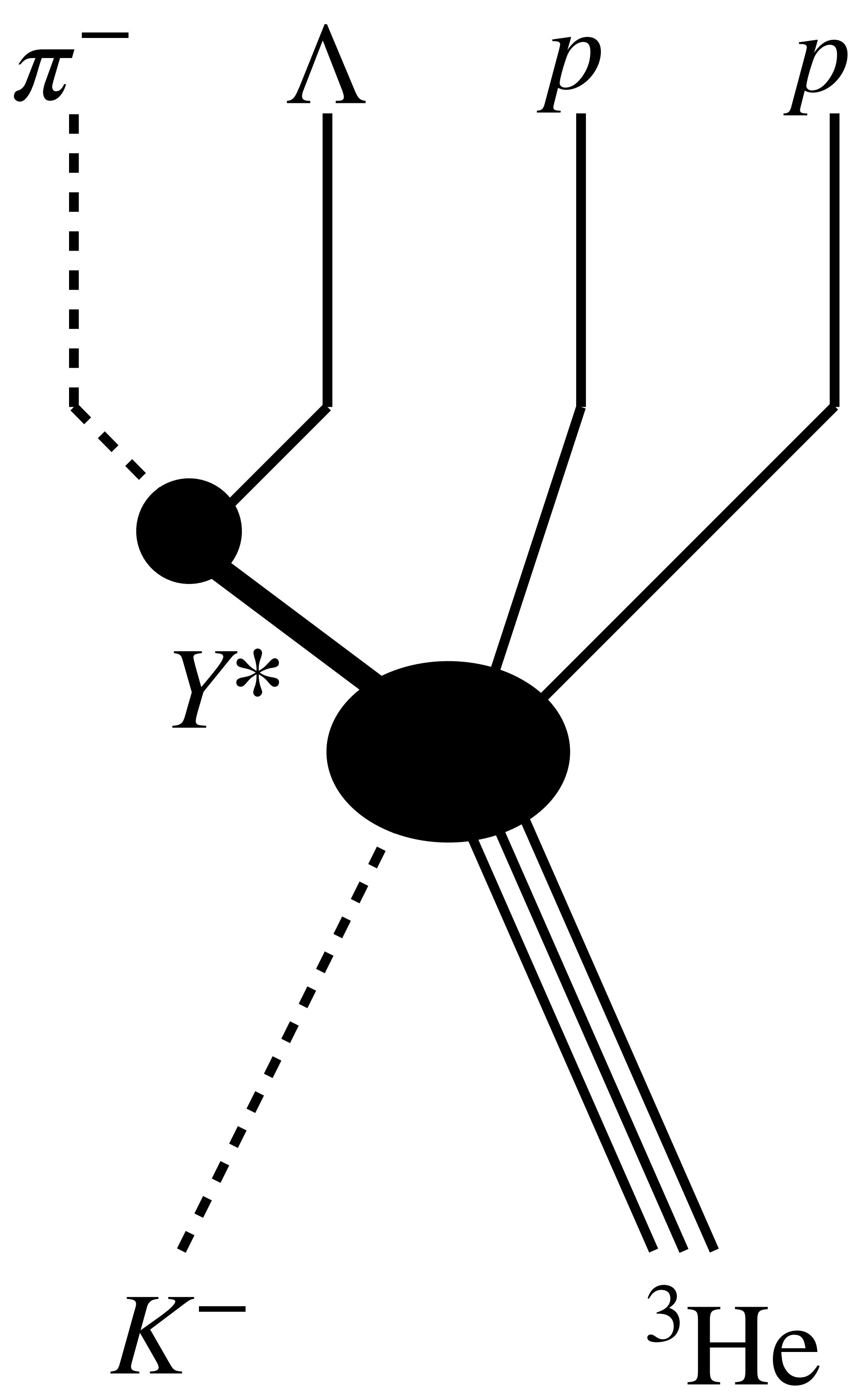
Reactions to be considered

point-like



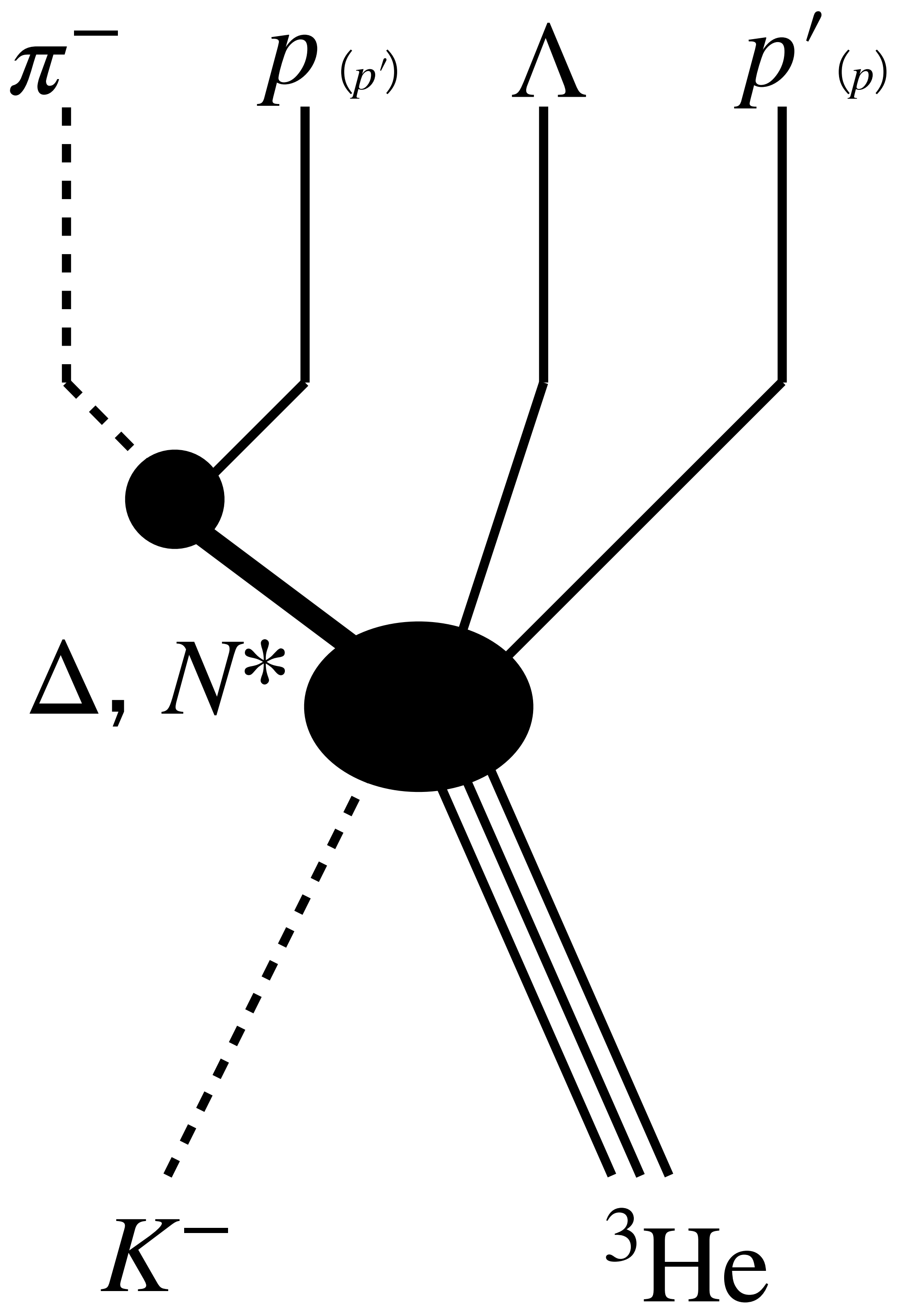
$N(m_{\pi^-\Lambda p}, m_{\pi^-\Lambda}, \cos\theta_{p'}) \propto \rho(m_{\pi^-\Lambda p}, m_{\pi^-\Lambda}, \cos\theta_{p'})$

point-like w/ Y^*



$N(m_{\pi^-\Lambda p}, m_{\pi^-\Lambda}, \cos\theta_{p'}) \propto f_{Y^*}(m_{\pi^-\Lambda})\ \rho(m_{\pi^-\Lambda p}, m_{\pi^-\Lambda}, \cos\theta_{p'})$

point-like w/ Δ, N^*



$N(m_{\pi^-\Lambda p}, m_{\pi^- p}, \cos\theta_{p'}) \propto f_{\Delta,N^*}(m_{\pi^- p})\ \rho(m_{\pi^-\Lambda p}, m_{\pi^- p}, \cos\theta_{p'})$

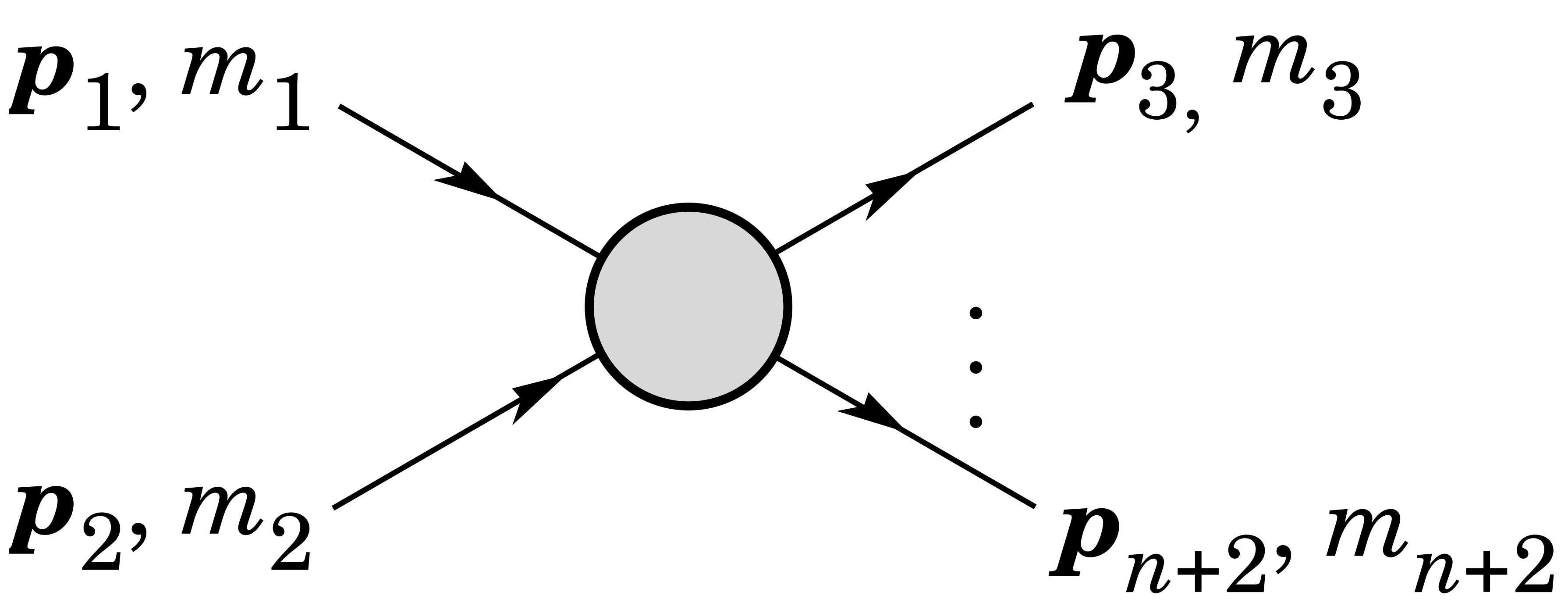
Cross section & Decay

Cross section & Decay

Starting from the cross section.

$$\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2} = p_{1\mathrm{cm}}\sqrt{s}\ .$$

Cross sections (taken from PDG "Kinematics")

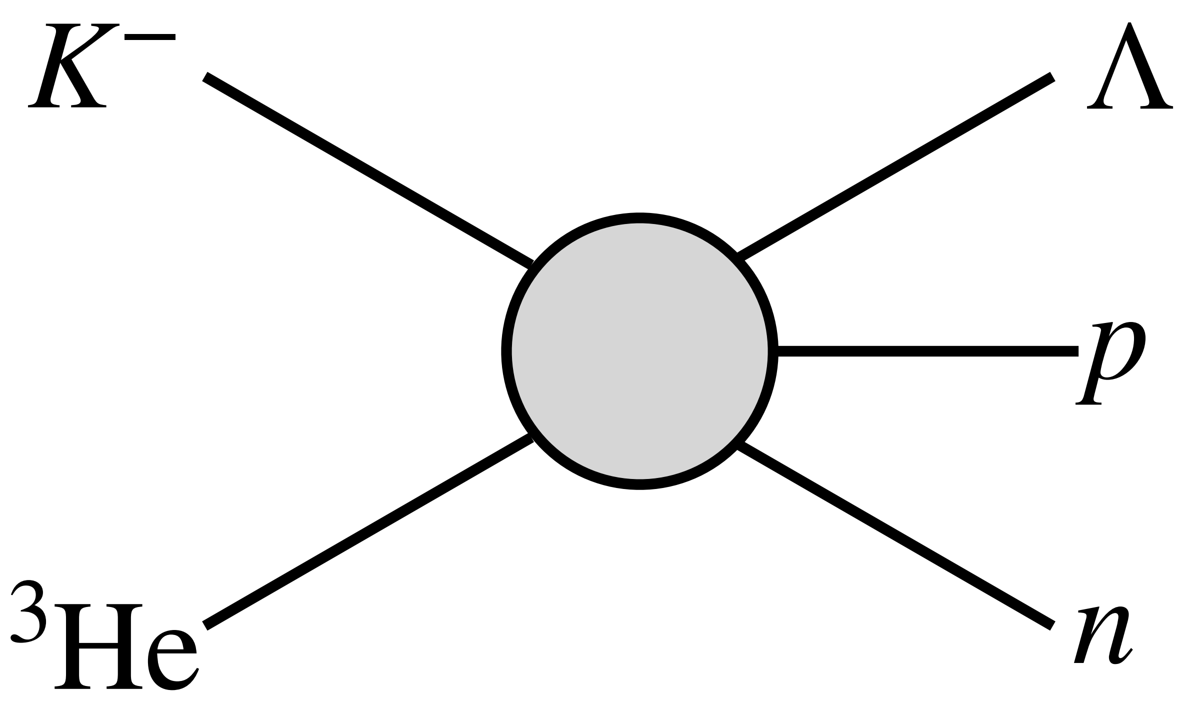


$$d\sigma = \frac{(2\pi)^4 |\mathscr{M}|^2}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} \times d\Phi_n(p_1 + p_2;\ p_3, \ldots, p_{n+2})\ .$$

Could be $\bar{K}NN$ amplitude

n-body phase space

Cross section & Decay



The cross section can be expressed as,

$$d\sigma_{\Lambda pn} = \frac{(2\pi)^4}{4p^*_{K^-}\sqrt{s}} \quad \mathscr{M}_{\Lambda pn} \quad ^2 \times d\Phi_{\Lambda pn}$$

where $d\Phi_{\Lambda pn}$ is the three-body phase space of the Λpn final state,

$$d\Phi_{\Lambda pn} = \left(\frac{1}{4\,(2\pi)^6} \frac{\vec{p}^{\,*}_n}{\sqrt{s}}\, d\Omega^*_n \right) \left(\frac{1}{4\,(2\pi)^6} \frac{\vec{p}^{\,(\Lambda p)*}_\Lambda}{m_{\Lambda p}}\, d\Omega^{(\Lambda p)*}_\Lambda \right) \left((2\pi)^3 dm^2_{\Lambda p} \right)$$

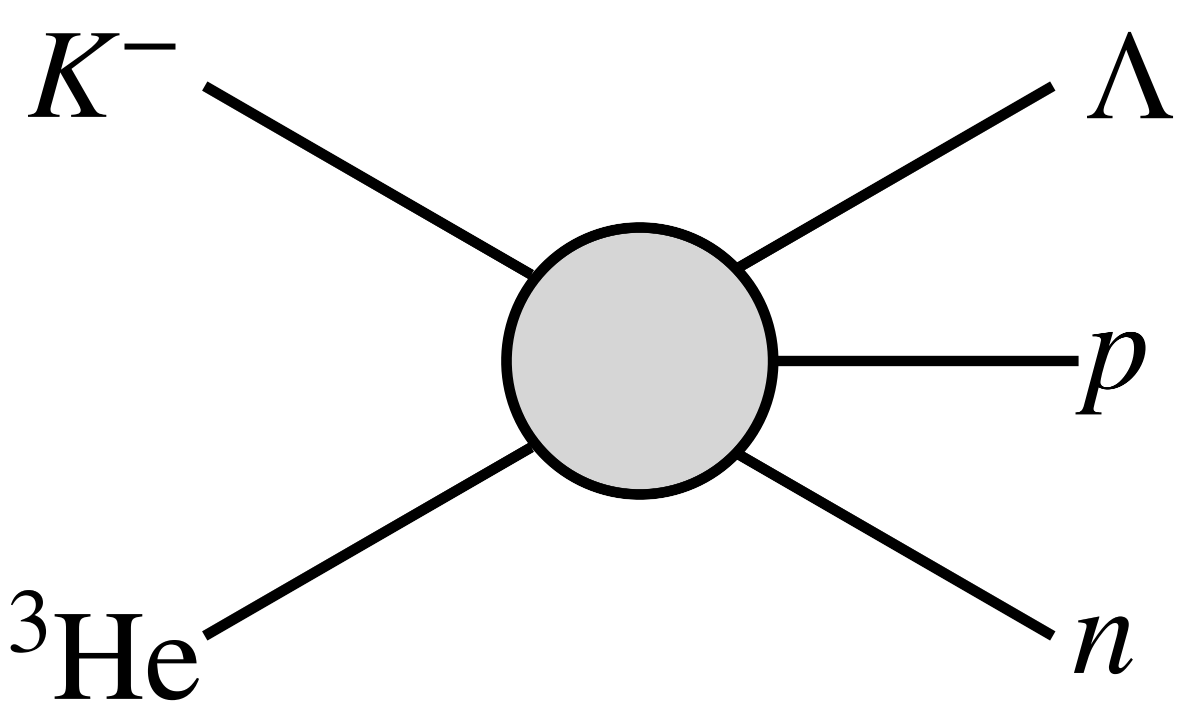
p^*_n(Ω^*_n) and $p^{(\Lambda p)*}_\Lambda$($\Omega^{(\Lambda p)*}_\Lambda$) are momenta (angles) of n and Λ in the $K^{-3}\mathrm{He}$-c.m. and (Λp)-c.m. frame, respectively, as,

$$\left|\vec{p}^{\,*}_n\right| = \frac{\sqrt{\left(s-(m_{\Lambda p}+m_n)^2\right)\left(s-(m_{\Lambda p}-m_n)^2\right)}}{2\sqrt{s}} \qquad \left|\vec{p}^{\,(\Lambda p)*}_\Lambda\right| = \frac{\sqrt{\left(m^2_{\Lambda p}-(m_\Lambda+m_p)^2\right)\left(m_{\Lambda p}-(m_\Lambda-m_p)^2\right)}}{2m_{\Lambda p}}$$

We can integrate over $\Omega^{(\Lambda p)*}_\Lambda$ and ϕ^*_n by assuming uniform distribution. By using $dm^2_{\Lambda p} = 2m_{\Lambda p}dm_{\Lambda p}$, $d\Phi_{\Lambda pn}$ is as,

$$d\Phi_{\Lambda pn} = \frac{1}{4(2\pi)^7\sqrt{s}} \left|\vec{p}^{\,*}_n\right| \left|\vec{p}^{\,(\Lambda p)*}_\Lambda\right| dm_{\Lambda p}\, d\cos\theta^*_n$$

Cross section & Decay



By combining following two,

$$d\sigma_{\Lambda pn} = \frac{(2\pi)^4}{4p^*_{K^-}\sqrt{s}} \left|\mathscr{M}_{\Lambda pn}\right|^2 \times d\Phi_{\Lambda pn}$$

$$d\Phi_{\Lambda pn} = \frac{1}{4(2\pi)^7\sqrt{s}} \left|\vec{p}^{\,*}_n\right| \left|\vec{p}^{\,(\Lambda p)*}_\Lambda\right| dm_{\Lambda p}\, d\cos\theta^*_n$$

the double differential cross section of the $K^{-3}\mathrm{He} \rightarrow \Lambda pn$ reaction can be expressed as,

$$\frac{d\sigma_{\Lambda pn}}{dm_{\Lambda p}\, d\cos\theta^*_n} = \frac{1}{16(2\pi)^3}\frac{1}{p^*_{K^-}\, s} \left|\vec{p}^{\,*}_n\right| \left|\vec{p}^{\,(\Lambda p)*}_\Lambda\right| \left|\mathscr{M}_{\Lambda pn}\right|^2$$

If we consider the $\bar{K}NN_{I_3=+1/2}$ production decaying into Λp-pair with the Breit-Wigner parametrization, $\mathscr{M}_{\Lambda pn}$ can be expressed as,

$$\mathscr{M}_{\Lambda pn} = \frac{g^{\bar{K}NN}_{\Lambda p}}{M^2_{\bar{K}NN} - m^2_{\Lambda p} - iM_{\bar{K}NN}\Gamma^{\bar{K}NN}_{tot}} \cdot \mathscr{A}\left(\cos\theta^*_n\right)$$

$\Leftarrow$ $\mathscr{M}_{\Lambda pn} = BW$

where $g^{\bar{K}NN}_{\Lambda p}$ is a coupling constant of the $\bar{K}NN$ to Λp channel, $M_{\bar{K}NN}$ is the Breit-Wigner mass of the $\bar{K}NN$, $\Gamma^{\bar{K}NN}_{tot} = \Gamma^{\bar{K}NN}_{tot}(m)$ is the total decay width of the $\bar{K}NN$, and $\mathscr{A}(\cos\theta^*_n)$ demonstrates an angular dependence of the $\bar{K}NN$ production.

$\Gamma_{tot}(m)$ is a sum of partial decay widths of all possible decay channels of the $\bar{K}NN$.

$$\Gamma^{\bar{K}NN}_{tot}(m) = \sum \Gamma_{YN}(m) + \sum \Gamma_{\pi YN}(m) + \sum \Gamma_{\bar{K}NN}(m)$$

Cross section & Decay

$$\mathscr{M}_{\Lambda pn} = \left\langle \Lambda pn \middle| T_{\Lambda pn} \middle| K^{-3}\mathrm{He} \right\rangle = \left\langle \Lambda pn \middle| \hat{T}^{(1/2)}_{\Lambda N} \middle| \Lambda NN' \right\rangle \left\langle \Lambda NN' \middle| \hat{T}^{(0)}_{\Lambda NN'} + \hat{T}^{(1)}_{\Lambda NN'} \middle| K^{-3}\mathrm{He} \right\rangle$$

- $T^{(I_{\Lambda NN'})}_{\Lambda NN'}$: Transition operator to the $\Lambda NN'$ final state in the isospin $I_{\Lambda NN'}$ channel
- $T^{(I_{\Lambda N})}_{\Lambda N}$: Transition operator to the ΛN channel in the isospin $I_{\Lambda N}$ channel

$$\left| K^{-3}\mathrm{He} \right\rangle = \sqrt{\frac{1}{2}} \underbrace{\left(\frac{\left| K^{-3}\mathrm{He} \right\rangle + \left| {}^{3}\mathrm{He}K^{-} \right\rangle}{\sqrt{2}} \right)}_{I=1} - \sqrt{\frac{1}{2}} \underbrace{\left(\frac{-\left| K^{-3}\mathrm{He} \right\rangle + \left| {}^{3}\mathrm{He}K^{-} \right\rangle}{\sqrt{2}} \right)}_{I=0}$$

$$\left| \Lambda pn \right\rangle = \sqrt{\frac{1}{2}} \underbrace{\left(\frac{\left| \Lambda pn \right\rangle + \left| \Lambda np \right\rangle}{\sqrt{2}} \right)}_{I=1} + \sqrt{\frac{1}{2}} \underbrace{\left(\frac{\left| \Lambda pn \right\rangle - \left| \Lambda np \right\rangle}{\sqrt{2}} \right)}_{I=0}$$

$$t^{(I)}_{\Lambda NN'} = \left\langle \Lambda NN' \middle| \hat{T}^{(I)}_{\Lambda NN'} \middle| K^{-3}\mathrm{He} \right\rangle \qquad t^{(I)}_{\Lambda N} = \left\langle \Lambda pn \middle| \hat{T}^{(I)}_{\Lambda N} \middle| \Lambda NN' \right\rangle$$

$$\Rightarrow \mathscr{M}_{\Lambda pn} = \frac{1}{2} t^{(1)}_{\Lambda NN'} \left(\frac{1}{2} t^{(1/2)}_{\Lambda p} + \frac{1}{2} t^{(1/2)}_{\Lambda n} \right) - \frac{1}{2} t^{(0)}_{\Lambda NN'} \left(\frac{1}{2} t^{(1/2)}_{\Lambda p} - \frac{1}{2} t^{(1/2)}_{\Lambda n} \right)$$

$$= \frac{1}{4} t^{(1/2)}_{\Lambda p} \left(t^{(1)}_{\Lambda NN'} - t^{(0)}_{\Lambda NN'} \right) + \frac{1}{4} t^{(1/2)}_{\Lambda n} \left(t^{(1)}_{\Lambda NN'} + t^{(0)}_{\Lambda NN'} \right)$$

“$K^{-}pp$” “$\bar{K}^{0}pn$”

Cross section & Decay

$\Gamma^{\bar{K}NN}_{tot}(m)$ is a sum of partial decay widths of all possible decay channels of the $\bar{K}NN$.

$$\Gamma^{\bar{K}NN}_{tot}(m) = \sum \Gamma^{\bar{K}NN}_{YN}(m) + \sum \Gamma^{\bar{K}NN}_{\pi YN}(m) + \sum \Gamma^{\bar{K}NN}_{\bar{K}NN}(m)$$

where $\Gamma^{\bar{K}NN}_{YN}(m)$ is non-mesonic two-body decay into YN channels, $\Gamma^{\bar{K}NN}_{\pi YN}(m)$ and $\Gamma^{\bar{K}NN}_{\bar{K}NN}(m)$ are mesonic three-body decay into πYN and $\bar{K}NN$ channels, respectively. All possible decay channels of the $\bar{K}NN_{I_3=+1/2}$ are,

- Λp
- $\Sigma^0 p$
- $\Sigma^+ n$

- $\pi^0 \Lambda p$
- $\pi^0 \Sigma^0 p$
- $\pi^+ \Lambda n$
- $\pi^- \Sigma^+ p$
- $\pi^0 \Sigma^+ n$
- $\pi^+ \Sigma^- p$
- $\pi^+ \Sigma^0 n$

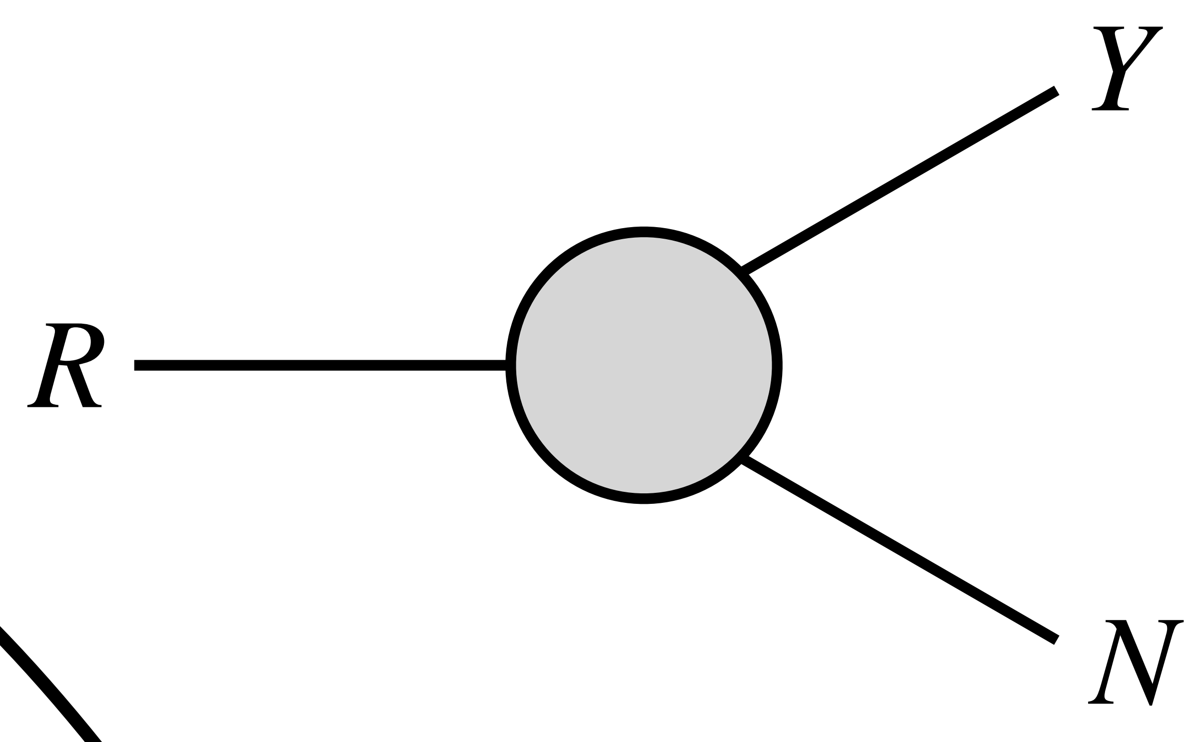
- $K^- pp$
- $\bar{K}^0 pn$

Partial decay widths can be obtained from the following equation,

Decay (taken from PDG "Kinematics")

$$d\Gamma = \frac{(2\pi)^4}{2M} |\mathscr{M}|^2 \, d\Phi_n \, (P;\; p_1, \ldots, p_n),$$

Cross section & Decay



The non-mesonic two-body decay widths Γ_{YN} can be expressed as,

$$d\Gamma_{YN}(m_{YN}) = \frac{(2\pi)^4}{2m_{YN}} |\mathscr{M}|^2 \Phi_2 = \frac{(2\pi)^4}{2m_{YN}} |\mathscr{M}|^2 \frac{1}{4(2\pi)^6} \frac{\vec{p}_Y^{\,(YN)*}}{m_{YN}} d\Omega_Y^{(YN)*}$$

$$\left|\vec{p}_Y^{\,(YN)*}\right| = \frac{\sqrt{\left(m_{YN}^2 - (m_Y + m_N)^2\right)\left(m_{YN} - (m_Y - m_N)^2\right)}}{2m_{YN}}$$

If we consider the amplitude $\mathscr{M}$ as a coupling constant to the YN channel,

$$\mathscr{M} = g_{YN}^{\bar{K}NN}$$

This $\mathscr{M}$ is a amplitude for the decay.
Not the same as the previous one!!

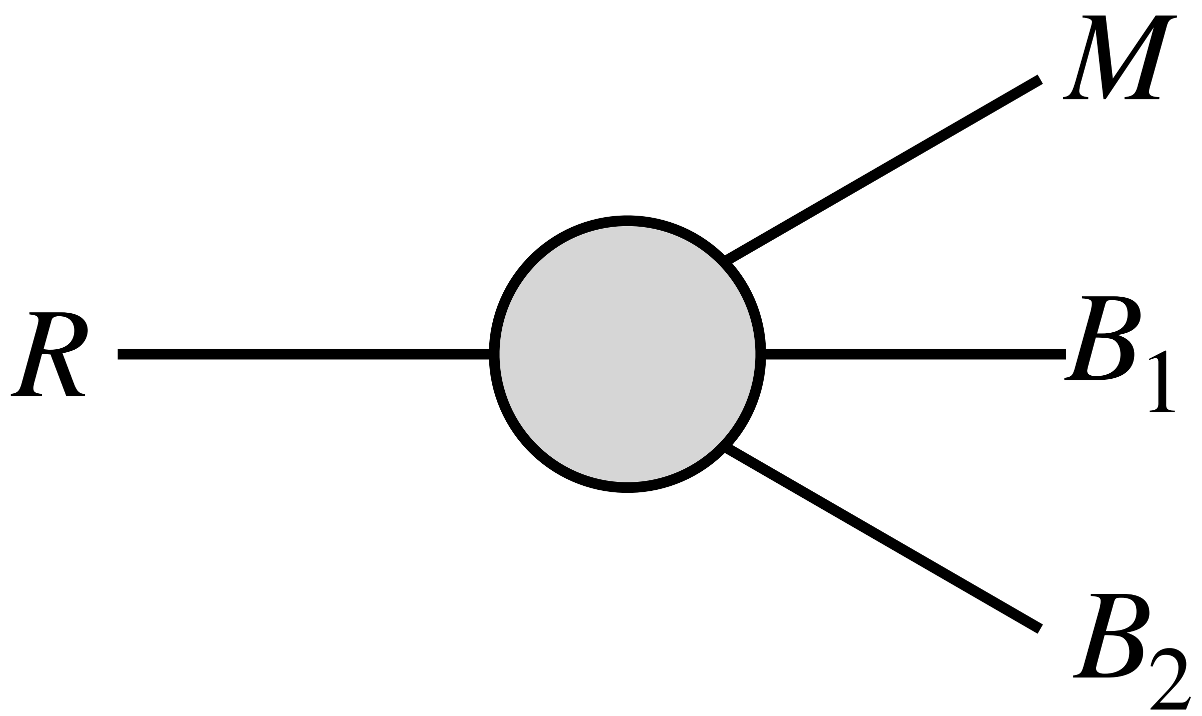
We can integrate over $\Omega_Y^{(YN)*}$, then,

$$\Gamma_{YN}(m_{YN}) = \frac{\left(g_{YN}^{\bar{K}NN}\right)^2}{8\pi m_{YN}^2} \vec{p}_Y^{\,(YN)*} = \frac{\left(g_{YN}^{\bar{K}NN}\right)^2}{8\pi m_{YN}^2} \frac{\sqrt{\left(m_{YN}^2 - (m_Y + m_N)^2\right)\left(m_{YN}^2 - (m_Y - m_N)^2\right)}}{2m_{YN}}$$

This expression is allowed only for above the $m_Y + m_N$ threshold, but we can expand it below the threshold by the Flatte parametrization as,

$$\Gamma_{YN}(m_{YN}) = \begin{cases} \frac{\left(g_{YN}^R\right)^2}{8\pi m_{YN}^2} \frac{\sqrt{\left(m_{YN}^2 - (m_Y + m_N)^2\right)\left(m_{YN}^2 - (m_Y - m_N)^2\right)}}{2m_{YN}} & \text{(for } m_{YN} \geq m_Y + m_N) \\ i\frac{\left(g_{YN}^R\right)^2}{8\pi m_{YN}^2} \frac{\sqrt{\left((m_Y + m_N)^2 - m_{YN}^2\right)\left(m_{YN}^2 - (m_Y - m_N)^2\right)}}{2m_{YN}} & \text{(for } m_{YN} < m_Y + m_N) \end{cases}$$

Cross section & Decay



$$\begin{pmatrix} M \\ B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} \pi \\ Y \\ N \end{pmatrix}, \begin{pmatrix} \bar{K} \\ N \\ N \end{pmatrix}$$

The mesonic three-body decay widths $\Gamma_{MB_1B_2}$ can be expressed as,

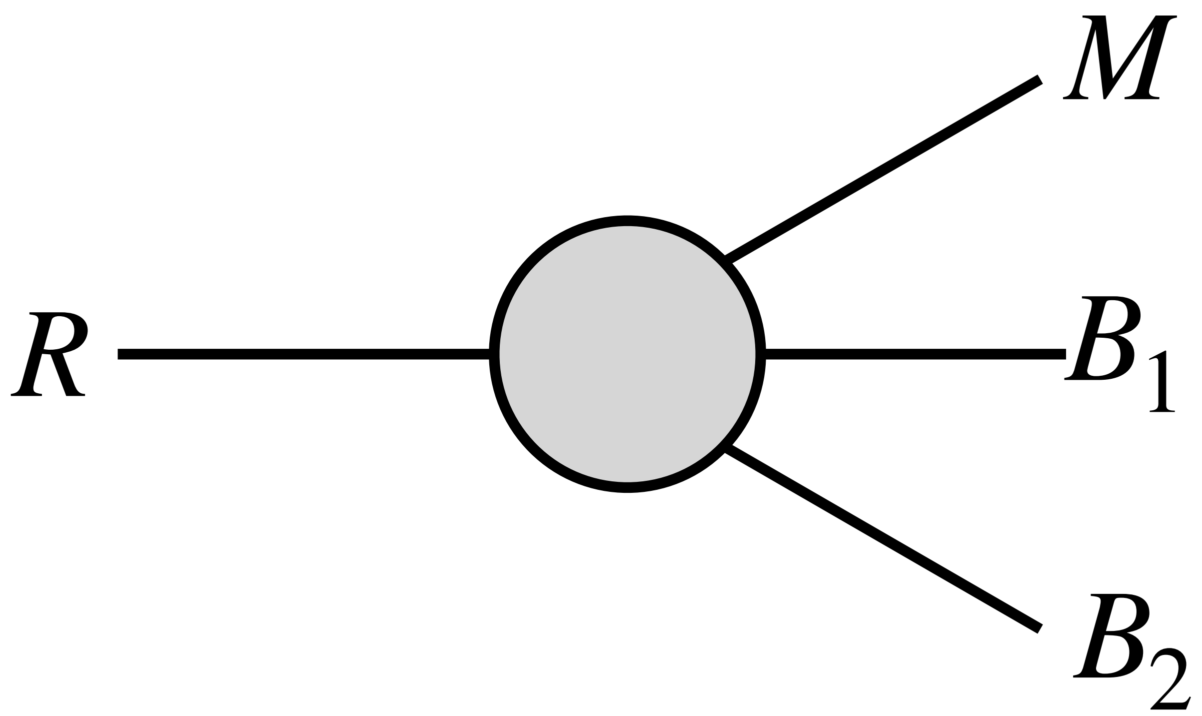
$$d\Gamma_{MB_1B_2}(m_{MB_1B_2}) = \frac{(2\pi)^4}{2m_{MB_1B_2}} \mathscr{M}^{2} \Phi_3$$

$$= \frac{(2\pi)^4}{2m_{MB_1B_2}} \mathscr{M}^{2} \left(\frac{1}{4(2\pi)^6} \frac{\vec{p}_{B_2}^{(MB_1B_2)*}}{m_{MB_1B_2}} d\Omega_{B_2}^{(MB_1B_2)*} \right) \left(\frac{1}{4(2\pi)^6} \frac{\vec{p}_{B_1}^{(MB_1)*}}{m_{MB_1}} d\Omega_{B_1}^{(MB_1)*} \right) \left((2\pi)^3 dm_{MB_1}^2 \right)$$

where

$$\left| \vec{p}_{B_2}^{(MB_1B_2)*} \right| = \frac{\sqrt{\left(m_{MB_1B_2}^2 - (m_{MB_1} + m_{B_2})^2 \right) \left(m_{MB_1B_2}^2 - (m_{MB_1} - m_{B_2})^2 \right)}}{2m_{MB_1B_2}} \qquad \left| \vec{p}_{B_1}^{(MB_1)*} \right| = \frac{\sqrt{\left(m_{MB_1}^2 - (m_M + m_{B_1})^2 \right) \left(m_{MB_1} - (m_M - m_{B_1})^2 \right)}}{2m_{MB_1}}$$

Cross section & Decay



$$\begin{pmatrix} M \\ B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} \pi \\ Y \\ N \end{pmatrix}, \begin{pmatrix} \bar{K} \\ N \\ N \end{pmatrix}$$

If we simply consider $\mathscr{M}$ as a coupling constant,

$$\boxed{\mathscr{M} = g^{\bar{K}NN}_{MB_1B_2}}$$

the decay width can be expressed as,

$$\Gamma_{MB_1B_2}(m_{MB_1B_2}) = \frac{\left(g^{\bar{K}NN}_{MB_1B_2}\right)^2}{32\pi^3 m^2_{MB_1B_2}} \int \left| \vec{p}^{\,(MB_1B_2)*}_{B_2} \right| \left| \vec{p}^{\,(MB_1)*}_{B_1} \right| dm_{MB_1}$$

$$\begin{cases} = \dfrac{\left(g^{\bar{K}NN}_{MB_1B_2}\right)^2}{32\pi^3 m^2_{MB_1B_2}} \displaystyle\int_{m_M+m_{B_1}}^{m_{MB_1B_2}-m_{B_2}} \frac{\sqrt{\left(m^2_{MB_1B_2} - (m_{MB_1} + m_{B_2})^2\right)\left(m^2_{MB_1B_2} - (m_{MB_1} - m_{B_2})^2\right)}}{2m_{MB_1B_2}} \frac{\sqrt{\left(m^2_{MB_1} - (m_M + m_{B_1})^2\right)\left(m^2_{MB_1} - (m_M - m_{B_1})^2\right)}}{2m_{MB_1}} dm_{MB_1} \ \text{(for } m_{MB_1B_2} \geq m_M + m_{B_1} + m_{B_2}) \\ = -\dfrac{\left(g^{\bar{K}NN}_{MB_1B_2}\right)^2}{32\pi^3 m^2_{MB_1B_2}} \displaystyle\int_{m_M+m_{B_1}}^{m_{MB_1B_2}-m_{B_2}} \frac{\sqrt{\left((m_{MB_1} + m_{B_2})^2 - m^2_{MB_1B_2}\right)\left(m^2_{MB_1B_2} - (m_{MB_1} - m_{B_2})^2\right)}}{2m_{MB_1B_2}} \frac{\sqrt{\left((m_M + m_{B_1})^2 - m^2_{MB_1}\right)\left(m^2_{MB_1} - (m_M - m_{B_1})^2\right)}}{2m_{MB_1}} dm_{MB_1} \ \text{(for } m_{MB_1B_2} < m_M + m_{B_1} + m_{B_2}) \end{cases}$$

We can also include resonances coupled to MB_1 or MB_2 channel by using a proper $\mathscr{M}$.

Cross section & Decay

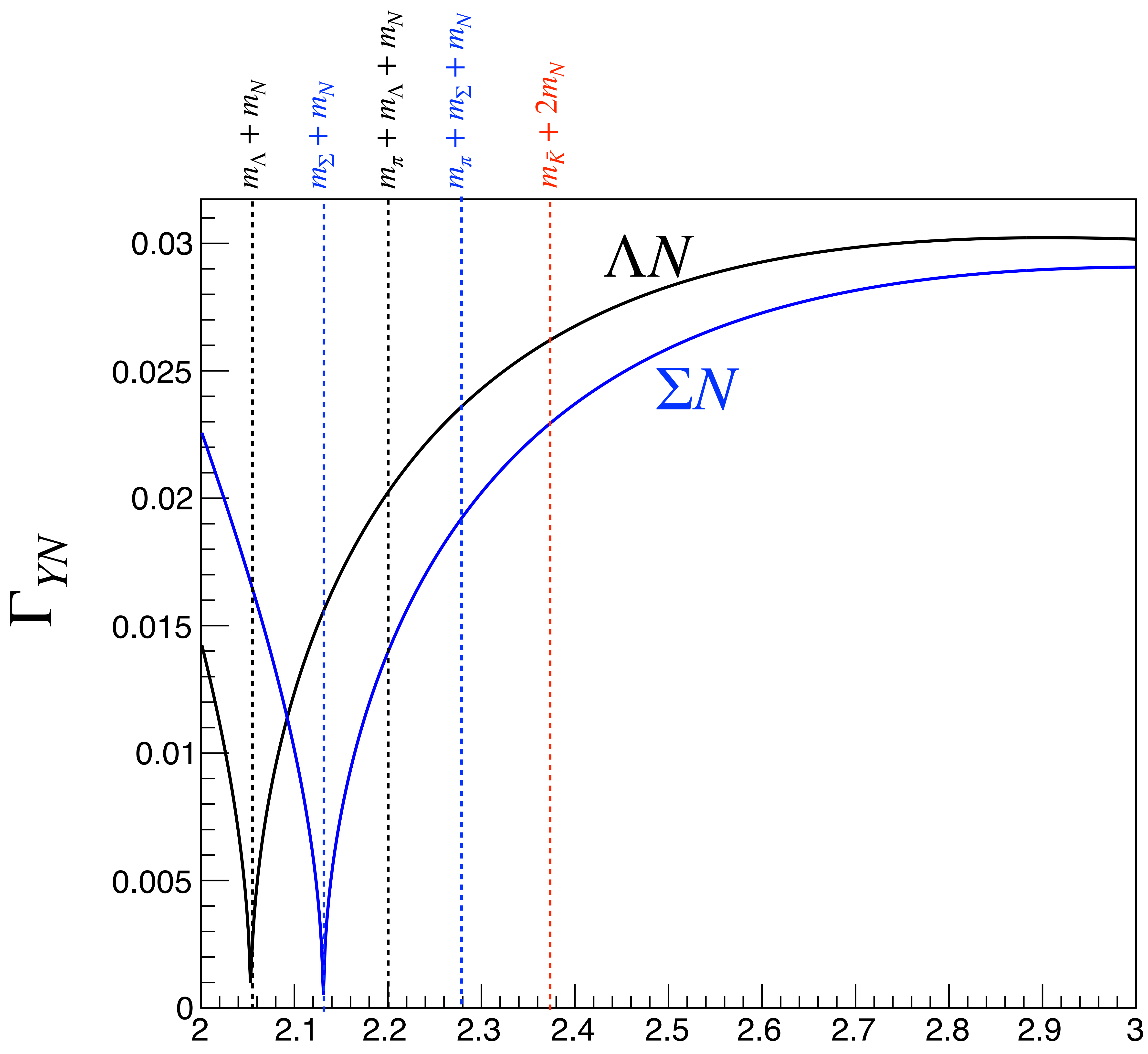
$\Gamma_{tot}^{\bar{K}NN}(m)$ is a sum of partial decay widths of all possible decay channels of the $\bar{K}NN$.

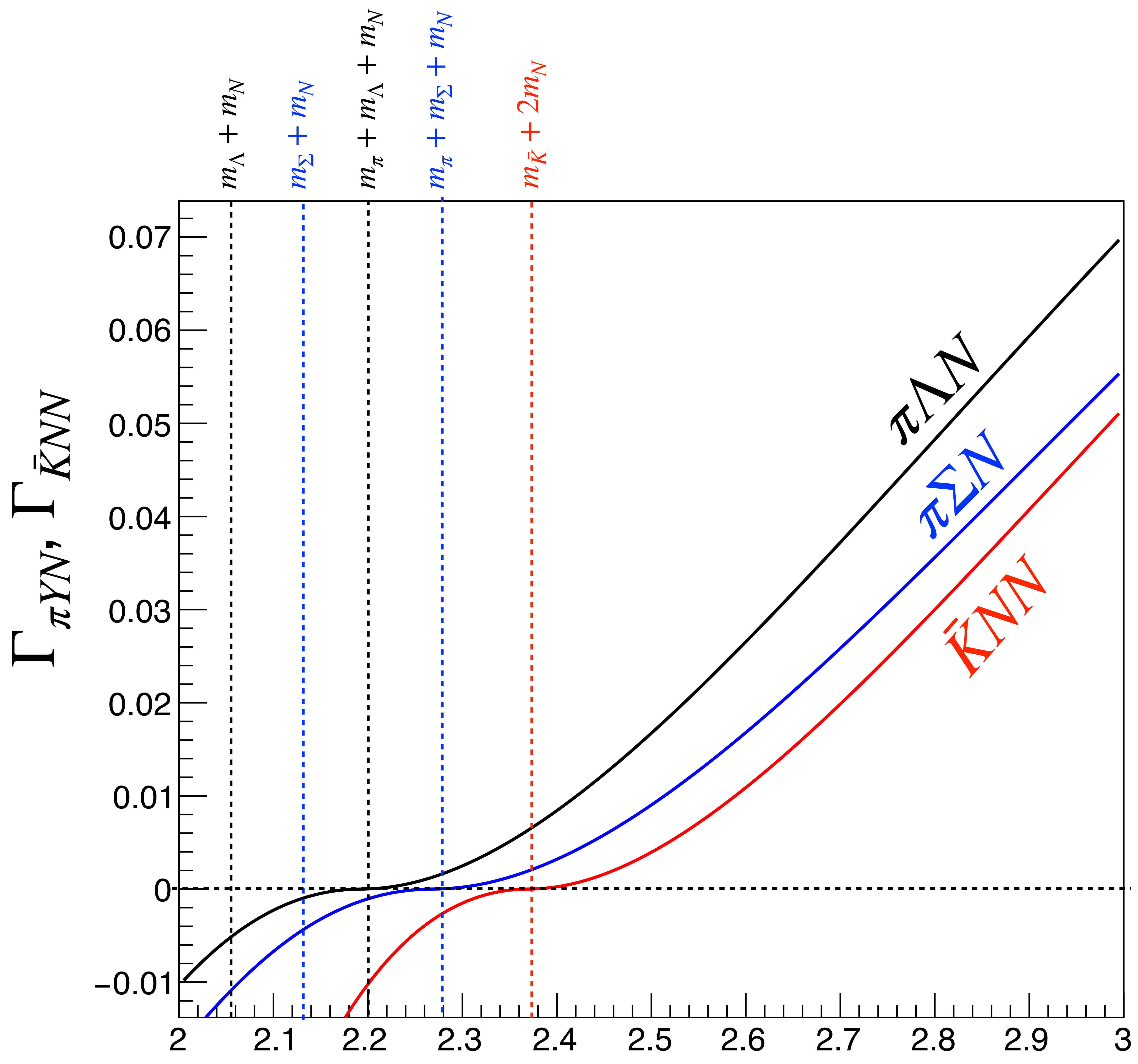
$$\Gamma_{tot}^{\bar{K}NN}(m) = \sum \Gamma_{YN}^{\bar{K}NN}(m) + \sum \Gamma_{\pi YN}^{\bar{K}NN}(m) + \sum \Gamma_{\bar{K}NN}^{\bar{K}NN}(m)$$

$$\Gamma_{YN}(m) = \begin{cases} \frac{\left(g_{YN}^{R}\right)^2}{8\pi m^2} \frac{\sqrt{\left(m^2 - (m_Y + m_N)^2\right)\left(m - (m_Y - m_N)^2\right)}}{2m} & \text{(for } m \geq m_Y + m_N\text{)} \\ i\frac{\left(g_{YN}^{R}\right)^2}{8\pi m^2} \frac{\sqrt{\left((m_Y + m_N)^2 - m^2\right)\left(m^2 - (m_Y - m_N)^2\right)}}{2m} & \text{(for } m < m_Y + m_N\text{)} \end{cases}$$

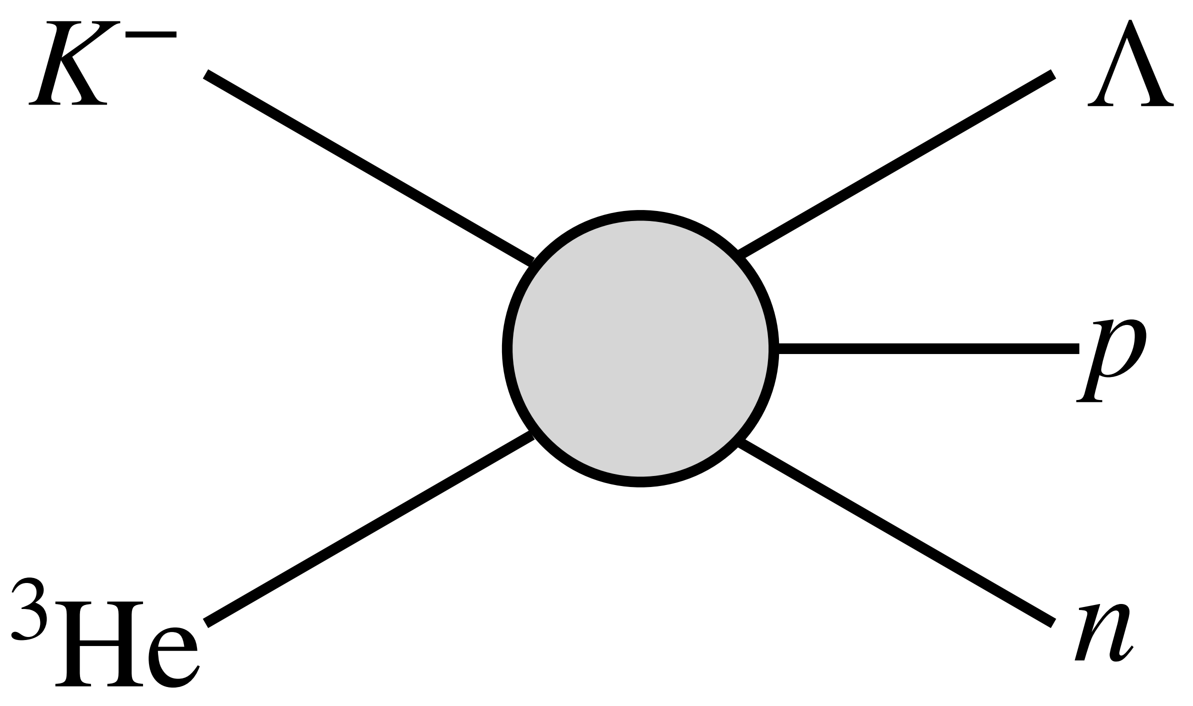
$$\Gamma_{MB_1B_2}(m) = \begin{cases} \frac{\left(g_{MB_1B_2}^{\bar{K}NN}\right)^2}{32\pi^3 m^2} \int_{m_M+m_{B_1}}^{m-m_{B_2}} \frac{\sqrt{\left(m^2 - (m_{MB_1} + m_{B_2})^2\right)\left(m^2 - (m_{MB_1} - m_{B_2})^2\right)}}{2m} \frac{\sqrt{\left(m_{MB_1}^2 - (m_M + m_{B_1})^2\right)\left(m_{MB_1}^2 - (m_M - m_{B_1})^2\right)}}{2m_{MB_1}} dm_{MB_1} & \text{(for } m \geq m_M + m_{B_1} + m_{B_2}\text{)} \\ -\frac{\left(g_{MB_1B_2}^{\bar{K}NN}\right)^2}{32\pi^3 m^2} \int_{m_M+m_{B_1}}^{m-m_{B_2}} \frac{\sqrt{\left((m_{MB_1} + m_{B_2})^2 - m^2\right)\left(m^2 - (m_{MB_1} - m_{B_2})^2\right)}}{2m} \frac{\sqrt{\left((m_M + m_{B_1})^2 - m_{MB_1}^2\right)\left(m_{MB_1}^2 - (m_M - m_{B_1})^2\right)}}{2m_{MB_1}} dm_{MB_1} & \text{(for } m < m_M + m_{B_1} + m_{B_2}\text{)} \end{cases}$$

Cross section & Decay





Cross section & Decay



the double differential cross section of the $K^- {}^3\mathrm{He} \to \Lambda p n$ reaction can be expressed as,

$$\frac{d\sigma_{\Lambda pn}}{dm_{\Lambda p}\, d\cos\theta_n^*} = \frac{1}{16(2\pi)^3}\frac{1}{p_{K^-}^* s}\left|\vec{p}_n^{\,*}\right|\left|\vec{p}_\Lambda^{\,(\Lambda p)*}\right|\left|\mathcal{M}_{\Lambda pn}\right|^2$$

$$\mathcal{M}_{\Lambda pn} = \frac{g_{\Lambda p}^{\bar{K}NN}}{M_{\bar{K}NN}^2 - m_{\Lambda p}^2 - iM_{\bar{K}NN}\Gamma_{tot}^{\bar{K}NN}} \cdot \mathcal{A}\left(\cos\theta_n^*\right)$$

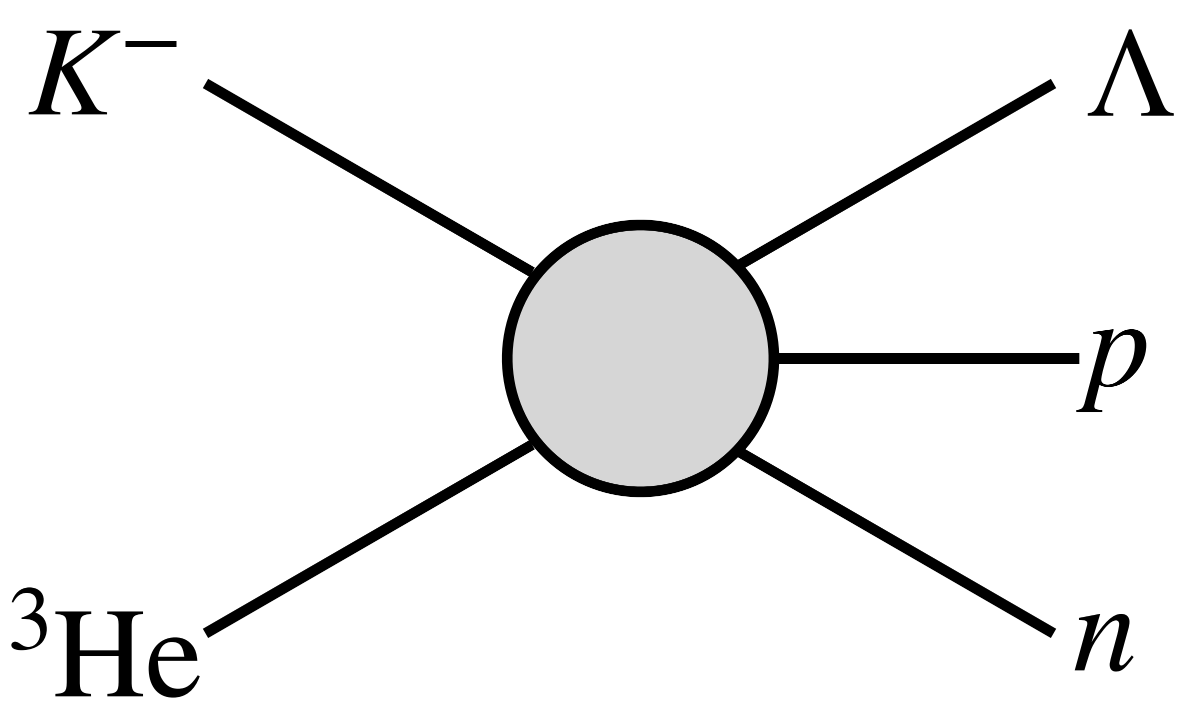
$$= \frac{1}{16(2\pi)^3}\frac{1}{p_{K^-}^* s}\left|\vec{p}_n^{\,*}\right|\left|\vec{p}_\Lambda^{\,(\Lambda p)*}\right|\left|\frac{g_{\Lambda p}^{\bar{K}NN}}{M_{\bar{K}NN}^2 - m_{\Lambda p}^2 - iM_{\bar{K}NN}\Gamma_{tot}^{\bar{K}NN}}\right|^2\left|\mathcal{A}\left(\cos\theta_n^*\right)\right|^2$$

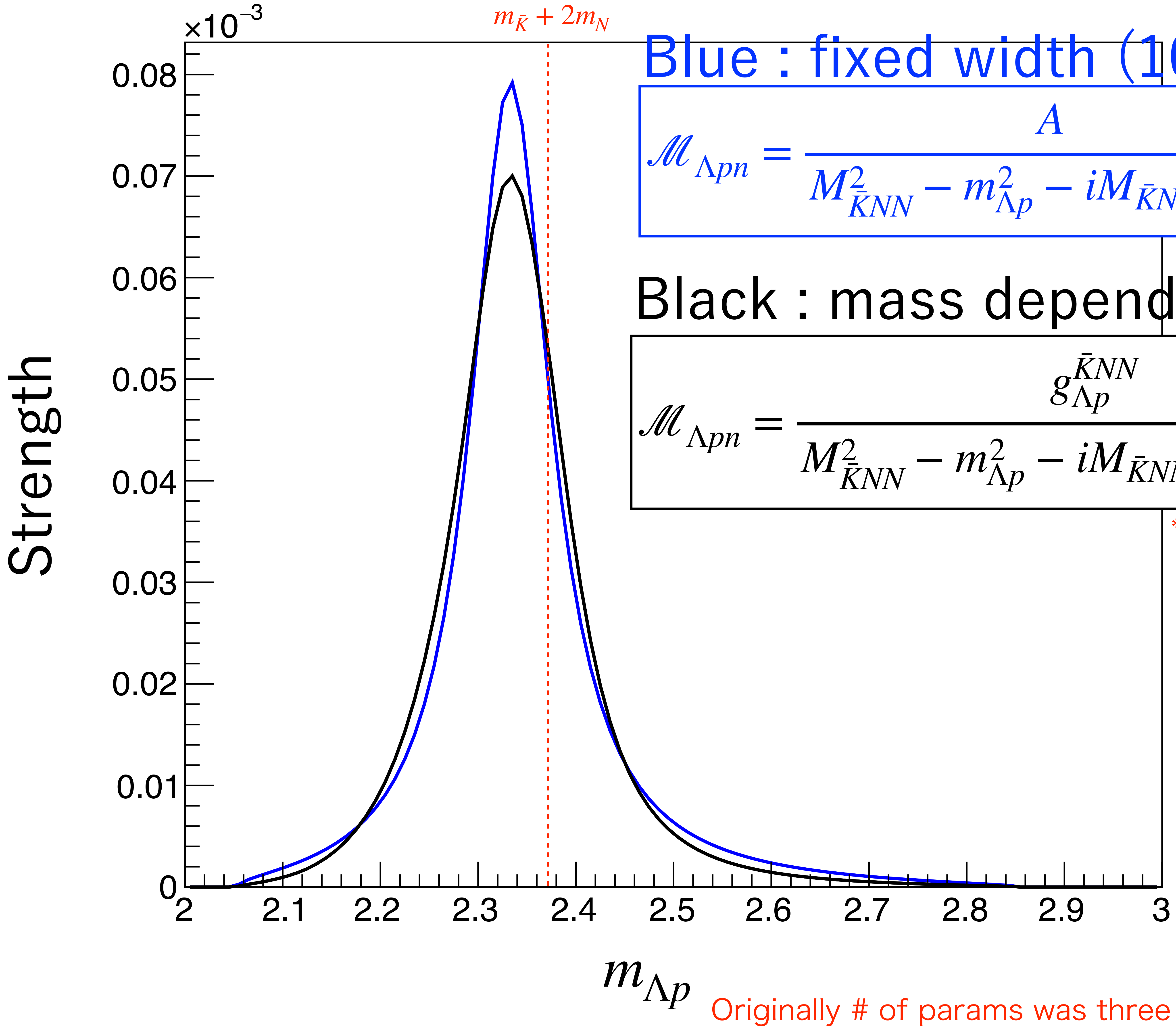
$$\Gamma_{tot}^{\bar{K}NN}(m) = \sum \Gamma_{YN}(m) + \sum \Gamma_{\pi YN}(m) + \sum \Gamma_{\bar{K}NN}(m)$$

If $m_{\bar{K}NN} = m_{\bar{K}} + 2m_N - 40\ \mathrm{MeV}$ and $\Gamma_{tot}^{\bar{K}NN} = 100\ \mathrm{MeV}$ (fixed), then line shape is (almost) the same as that PRC.

*In PRC, non-relativistic Breit-Wigner was used.

Cross section & Decay





Blue : fixed width (100 MeV)

$$\mathscr{M}_{\Lambda pn} = \frac{A}{M_{\bar{K}NN}^2 - m_{\Lambda p}^2 - iM_{\bar{K}NN}\Gamma_{\bar{K}NN}} \cdot \mathscr{A}\left(\cos\theta_n^*\right)$$

<-The same as PRC

Black : mass dependent width

$$\mathscr{M}_{\Lambda pn} = \frac{g_{\Lambda p}^{\bar{K}NN}}{M_{\bar{K}NN}^2 - m_{\Lambda p}^2 - iM_{\bar{K}NN}\Gamma_{tot}^{\bar{K}NN}(m_{\Lambda p})} \cdot \mathscr{A}\left(\cos\theta_n^*\right)$$

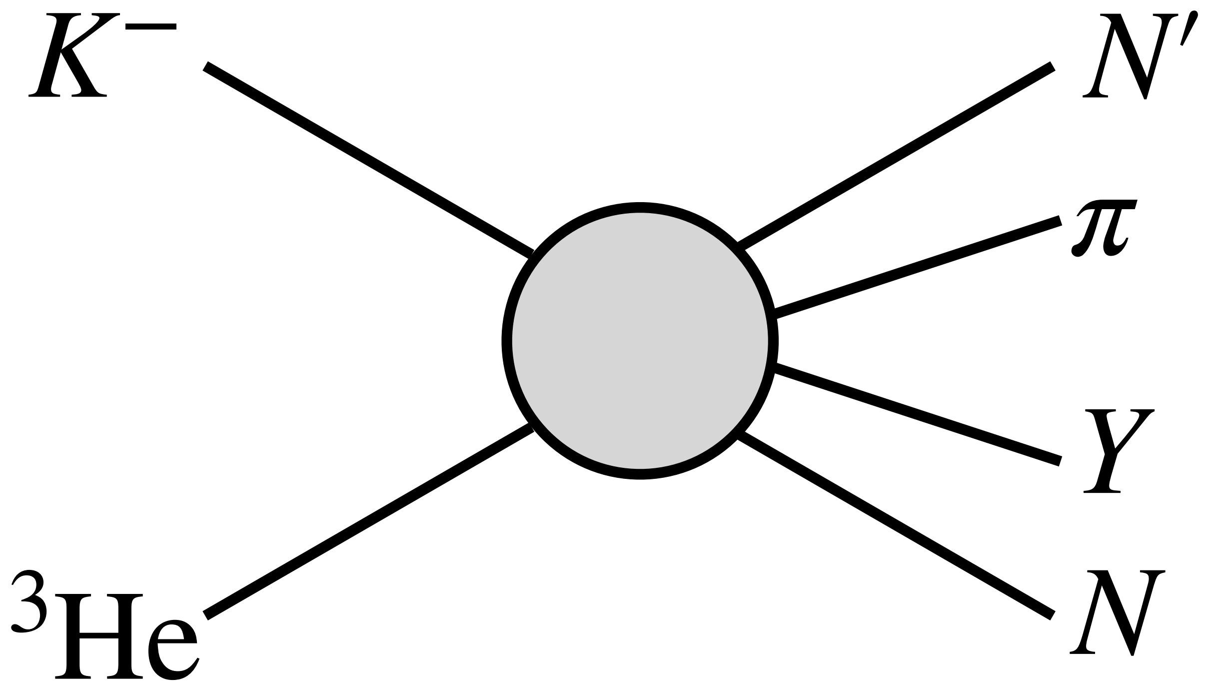
Originally # of params was three

*Parameters were set to fit the blue

Error is so huge!!

NO.	NAME	VALUE	ERROR	
1	p0	2.41689e+00	2.62724e+00	$m_{\bar{K}NN}$
2	p1	1.69950e+00	1.15931e+02	$g_{\Lambda N}$
3	p2	6.98810e+01	6.81244e+01	$g_{\Sigma N}$
4	p3	6.81960e+02	6.82626e+04	$g_{\pi\Lambda N}$
5	p4	8.84178e+02	8.31748e+05	$g_{(\pi\Sigma)_{I=1}N}$
6	p5	6.36444e+01	1.15118e+06	$g_{\pi^0\Sigma^0 N}$
7	p6	1.44789e+02	1.19851e+06	$g_{\pi^-\Sigma^+ N}$
8	p7	4.11638e+02	1.16775e+06	$g_{\pi^+\Sigma^- N}$
9	p8	1.33888e+03	1.15010e+06	$g_{\bar{K}NN}$
10	p9	7.23045e-01	1.55019e+02	Scaling

Cross section & Decay



$$d\sigma = \frac{(2\pi)^4}{4p^*_{K^-}\sqrt{s}} \; \mathscr{M}^{\,2} \times d\Phi_4$$

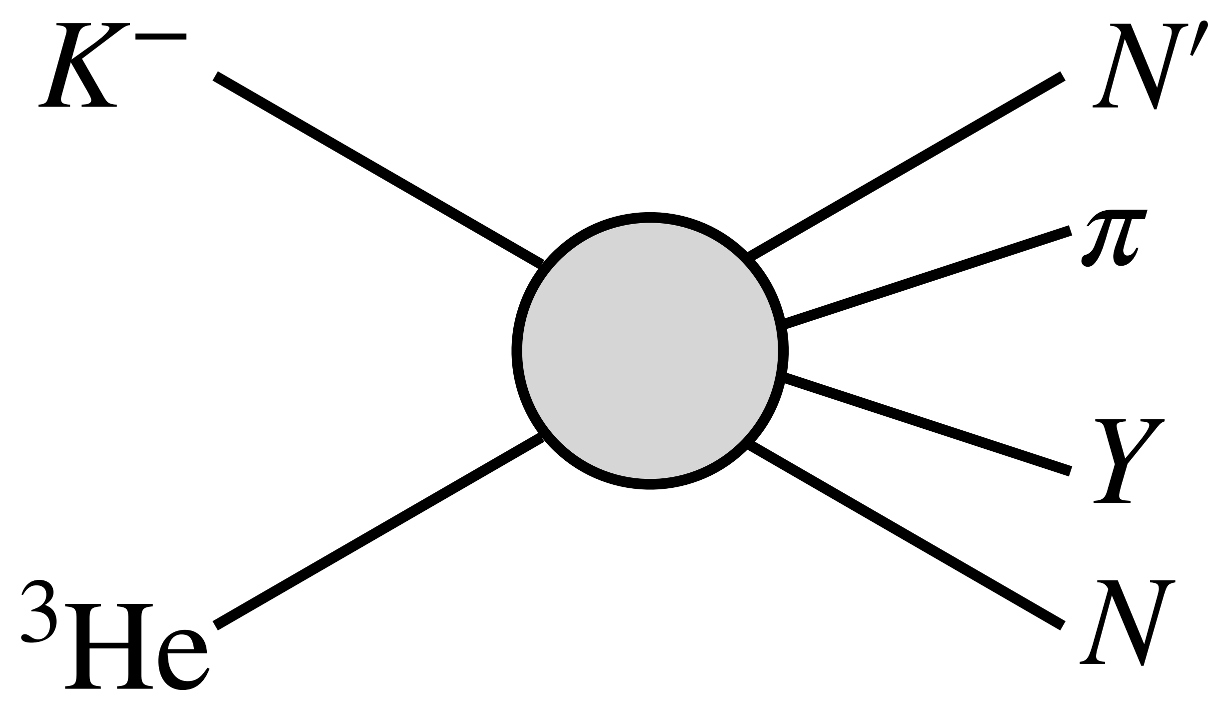
$$= \frac{1}{32(2\pi)^5}\frac{1}{p^*_{K^-}\,s} \left|p^*_{N'}\right| \left|p_N^{(\pi YN)*}\right| \left|p_Y^{(\pi Y)*}\right| \; \mathscr{M}^{\,2}\, dm_{\pi YN}\, dm_{\pi Y}\, d\cos\theta^*_{N'}\, d\cos\theta_N^{(\pi YN)*}$$

$$d\Phi_4 = \left(\frac{1}{4\,(2\pi)^6}\frac{\vec{p}^{\,*}_{N'}}{\sqrt{s}}\, d\Omega^*_{N'}\right)\left(\frac{1}{4\,(2\pi)^6}\frac{\vec{p}_N^{\,(\pi YN)*}}{m_{\pi YN}}\, d\Omega_N^{(\pi YN)*}\right)\left(\frac{1}{4\,(2\pi)^6}\frac{\vec{p}_Y^{\,(\pi Y)*}}{m_{\pi Y}}\, d\Omega_Y^{(\pi Y)*}\right)\left((2\pi)^3 dm^2_{\pi YN}\right)\left((2\pi)^3 dm^2_{\pi Y}\right)$$

$$\frac{d\sigma}{dm_{\pi YN}\, dm_{\pi Y}\, d\cos\theta^*_{N'}\, d\cos\theta_N^{(\pi YN)*}} = \frac{1}{32(2\pi)^5}\frac{1}{p^*_{K^-}\,s} \left|p^*_{N'}\right| \left|p_N^{(\pi YN)*}\right| \left|p_Y^{(\pi Y)*}\right| \; \mathscr{M}^{\,2}$$

$$\left|p^*_{N'}\right| = \frac{\sqrt{\left(s-(m_{\pi YN}+m_{N'})^2\right)\left(s-(m_{\pi YN}-m_{N'})^2\right)}}{2\sqrt{s}},\quad \left|p_N^{(\pi YN)*}\right| = \frac{\sqrt{\left(m^2_{\pi YN}-(m_{\pi Y}+m_N)^2\right)\left(m^2_{\pi YN}-(m_{\pi Y}-m_N)^2\right)}}{2m_{\pi YN}},\quad \left|p_Y^{(\pi Y)*}\right| = \frac{\sqrt{\left(m^2_{\pi Y}-(m_\pi+m_Y)^2\right)\left(m^2_{\pi Y}-(m_\pi-m_Y)^2\right)}}{2m_{\pi Y}}$$

Cross section & Decay



$$\frac{d\sigma}{dm_{\pi YN}\, dm_{\pi Y}\, d\cos\theta^*_{N'}\, d\cos\theta^{(\pi YN)*}_N} = \frac{1}{32(2\pi)^5}\frac{1}{p^*_{K^-} s}\left|p^*_{N'}\right|\left|p^{(\pi YN)*}_N\right|\left|p^{(\pi Y)*}_Y\right| \quad \mathscr{M}\ ^2$$

$$\left|p^*_{N'}\right| = \frac{\sqrt{\left(s-(m_{\pi YN}+m_{N'})^2\right)\left(s-(m_{\pi YN}-m_{N'})^2\right)}}{2\sqrt{s}},\ \left|p^{(\pi YN)*}_N\right| = \frac{\sqrt{\left(m^2_{\pi YN}-(m_{\pi Y}+m_N)^2\right)\left(m^2_{\pi YN}-(m_{\pi Y}-m_N)^2\right)}}{2m_{\pi YN}},\ \left|p^{(\pi Y)*}_Y\right| = \frac{\sqrt{\left(m^2_{\pi Y}-(m_\pi+m_Y)^2\right)\left(m^2_{\pi Y}-(m_\pi-m_Y)^2\right)}}{2m_{\pi Y}}$$

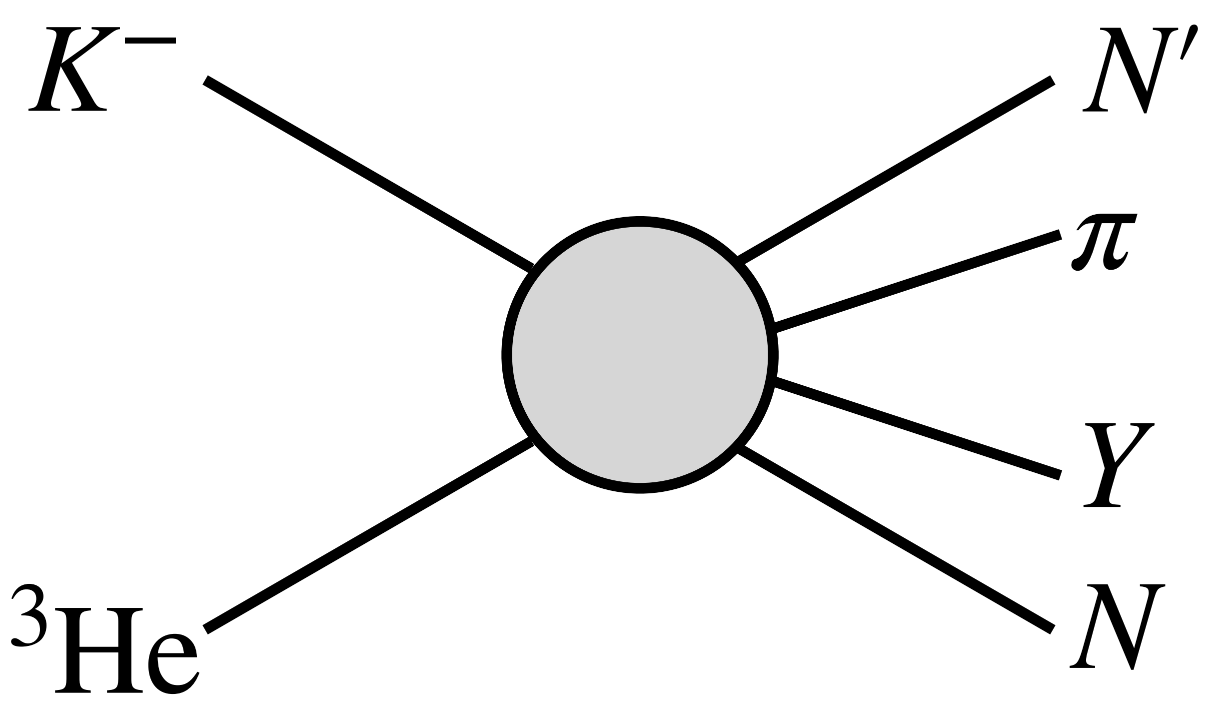
$$\mathscr{M} = \mathscr{M}_{\pi YN}\cdot\mathscr{M}_{\pi Y} = \frac{g^R_{\pi YN}}{M^2_R - m^2_{\pi YN} - iM_R\Gamma^R_{tot}}\cdot\left(g^{NR}_{\pi Y} + \frac{g^{Y*}_{\pi Y}}{M^2_{Y*} - m^2_{\pi Y} - iM_{Y*}\Gamma^{Y*}_{tot}}\right)\mathscr{A}\left(\cos\theta^*_{N'}\right)\mathscr{A}\left(\cos\theta^{(\pi YN)*}_N\right)$$

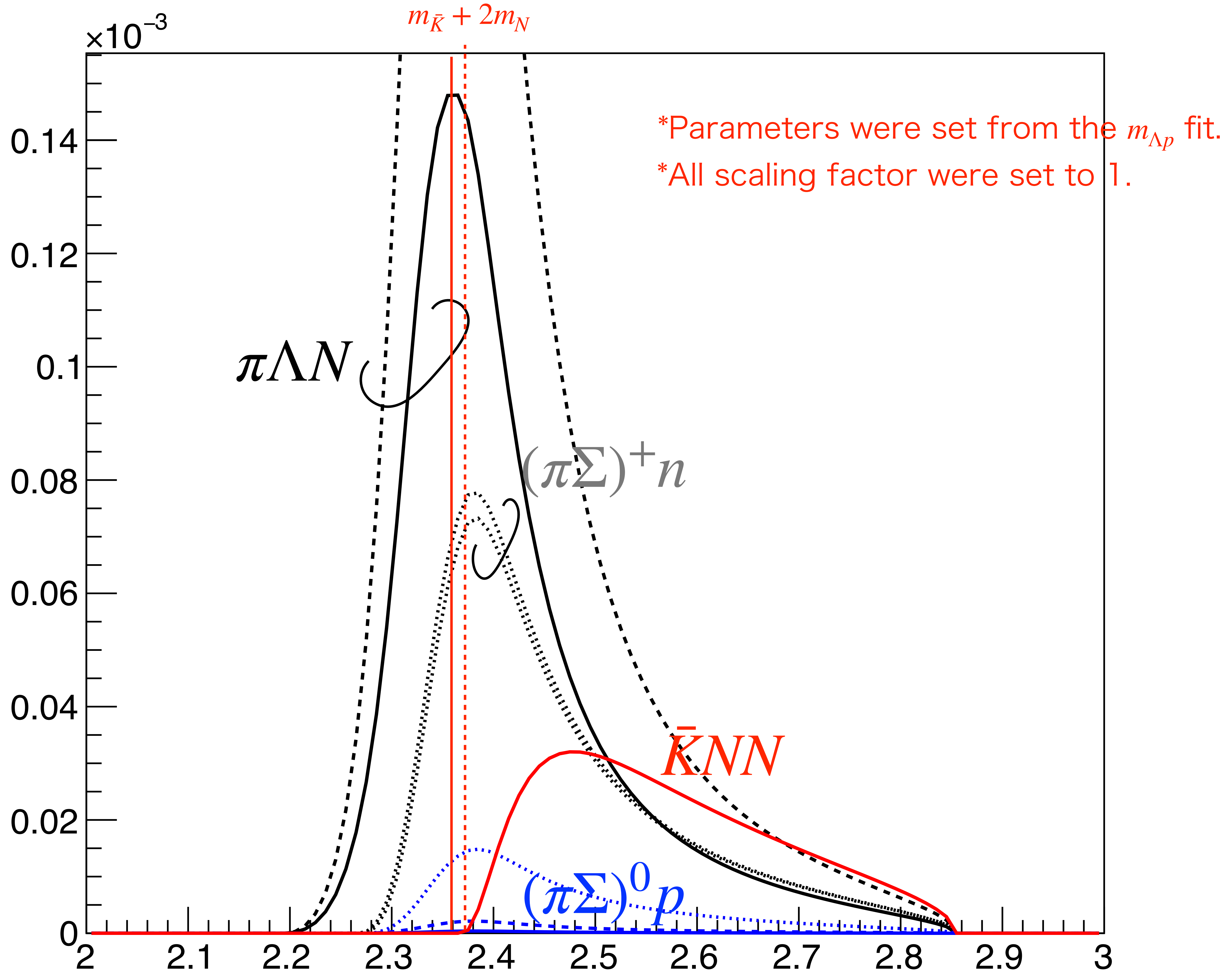
This term determines the $m_{\pi Y}$ distribution.

$$\frac{d\sigma}{dm_{\pi YN}\, dm_{\pi Y}\, d\cos\theta^*_{N'}\, d\cos\theta^{(\pi YN)*}_N} = \frac{1}{32(2\pi)^5}\frac{1}{p^*_{K^-} s}\left|p^*_{N'}\right|\left|p^{(\pi YN)*}_N\right|\left|p^{(\pi Y)*}_Y\right|\left|\frac{g^R_{\pi YN}}{M^2_R - m^2_{\pi YN} - iM_R\Gamma^R_{tot}}\right|^2\left|g^{NR}_{\pi Y} + \frac{g^{Y*}_{\pi Y}}{M^2_{Y*} - m^2_{\pi Y} - iM_{Y*}\Gamma^{Y*}_{tot}}\right|^2\left|\mathscr{A}\left(\cos\theta^*_{N'}\right)\right|^2\left|\mathscr{A}\left(\cos\theta^{(\pi YN)*}_N\right)\right|^2$$

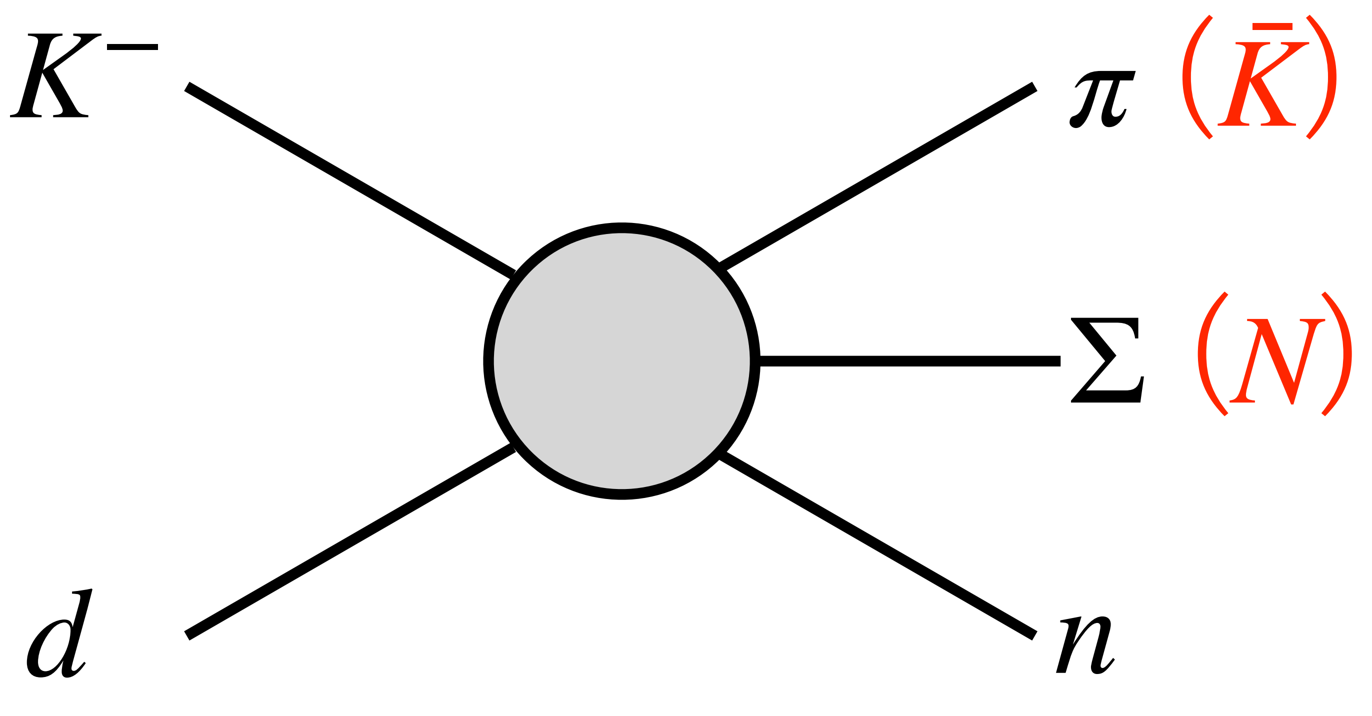
As the first step, let us ignore Y^* contribution.

Cross section & Decay





Cross section & Decay



$$\frac{d\sigma_{\pi\Sigma n}}{dm_{\pi\Sigma}\,d\cos\theta_n^*} = \frac{1}{16(2\pi)^3}\frac{1}{p_{K^-}^* s}\left|p_n^*\right|\left|p_\Sigma^{(\pi\Sigma)*}\right| \mathscr{M}^{2}$$

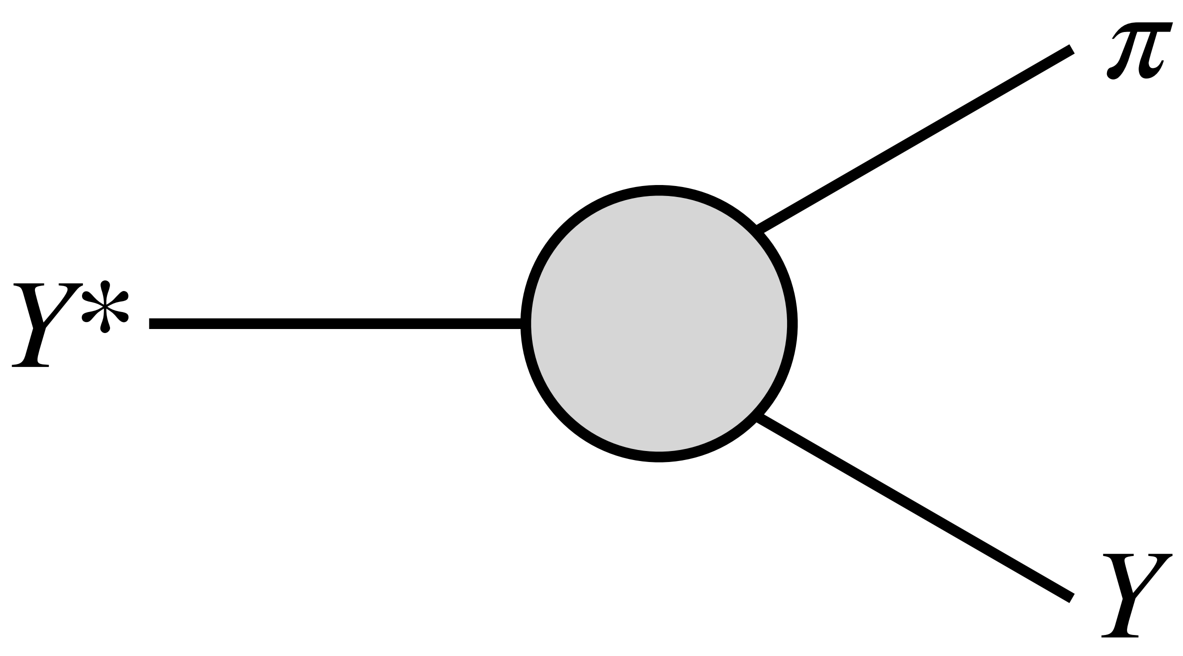
$$\mathscr{M} = \frac{g_{\pi Y}^{Y*}}{M_{Y*}^2 - m_{\pi\Sigma}^2 - iM_{Y*}\Gamma_{tot}^{Y*}} \cdot \mathscr{A}\left(\cos\theta_n^*\right)$$

$$\frac{d\sigma_{\pi\Sigma n}}{dm_{\pi\Sigma}\,d\cos\theta_n^*} = \frac{1}{16(2\pi)^3}\frac{1}{p_{K^-}^* s}\left|p_n^*\right|\left|p_\Sigma^{(\pi\Sigma)*}\right|\left|\frac{g_{\pi\Sigma}^{Y*}}{M_{Y*}^2 - m_{\pi\Sigma}^2 - iM_{Y*}\Gamma_{tot}^{Y*}}\right|^2\left|\mathscr{A}\left(\cos\theta_n^*\right)\right|^2$$

$$\begin{cases} \dfrac{\left(g_{\pi\Sigma}^{Y*}\right)^2}{\left(M_{Y*}^2 - m_{\pi\Sigma}^2 + M_{Y*}\Gamma_{KN}\right)^2 + M_{Y*}^2\Gamma_{\pi\Sigma}^2}, & \text{(below the } m_{\bar{K}} + m_N \text{ threshold)} \\ \dfrac{\left(g_{\pi\Sigma}^{Y*}\right)^2}{\left(M_{Y*}^2 - m_{\pi\Sigma}^2\right)^2 + M_{Y*}^2\left(\Gamma_{\pi\Sigma} + \Gamma_{\bar{K}N}\right)^2}, & \text{(above the } m_{\bar{K}} + m_N \text{ threshold)} \end{cases}$$

$\dfrac{d\sigma_{\bar{K}Nn}}{dm_{\bar{K}N}\,d\cos\theta_n^*}$ is as the same.

Cross section & Decay



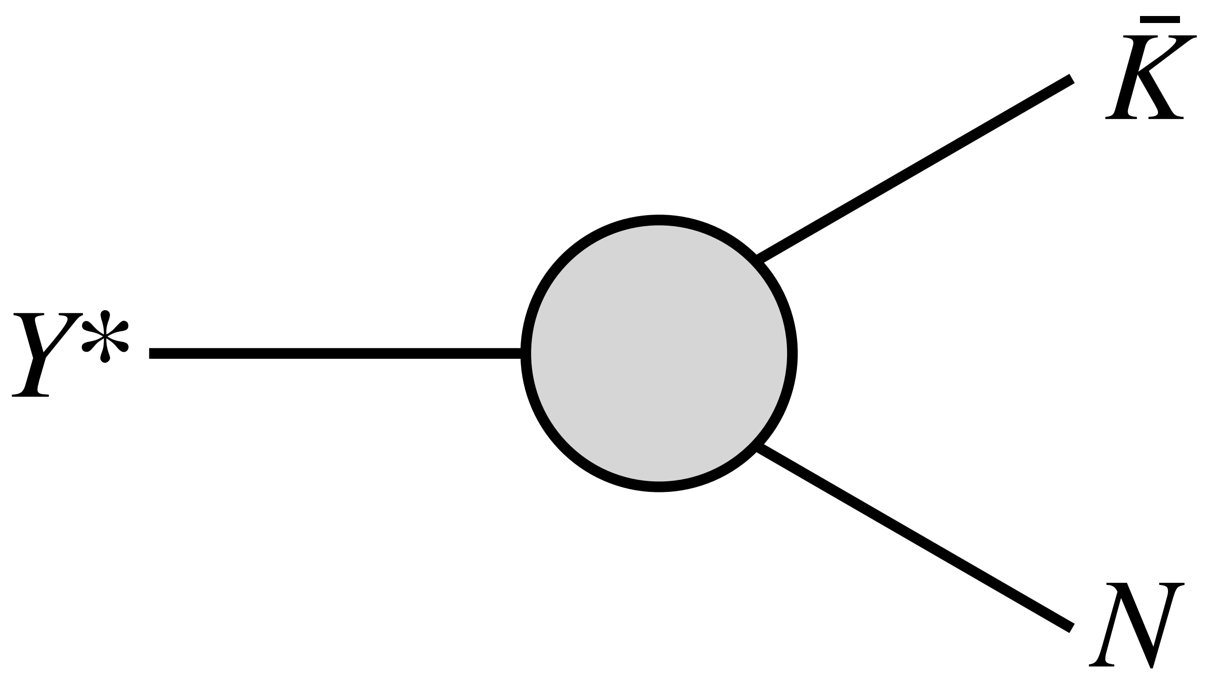
$$d\Gamma_{\pi Y}^{Y*} = \frac{(2\pi)^4}{2m_{\pi Y}} \mathscr{M}\;\;^2 \Phi_2 \quad = \frac{(2\pi)^4}{2m_{\pi Y}} \mathscr{M}\;\;^2 \frac{1}{4\,(2\pi)^6} \frac{\vec{p}_Y^{(\pi Y)*}}{m_{\pi Y}} d\Omega_Y^{(\pi Y)*}$$

$$\boxed{\mathscr{M} = g_{\pi Y}^{Y*}}$$

$$\Gamma_{\pi Y} = \frac{\left(g_{\pi Y}^{Y*}\right)^2}{8\pi m_{\pi Y}^2} \frac{\sqrt{\left(m_{\pi Y}^2 - (m_\pi + m_Y)^2\right)\left(m_{\pi Y}^2 - (m_\pi - m_Y)^2\right)}}{2m_{\pi Y}} \text{ (above the } m_\pi + m_Y\text{)}$$

$$= i\frac{\left(g_{\pi Y}^{Y*}\right)^2}{8\pi m_{\pi Y}^2} \frac{\sqrt{\left((m_\pi + m_Y)^2 - m_{\pi Y}^2\right)\left(m_{\pi Y}^2 - (m_\pi - m_Y)^2\right)}}{2m_{\pi Y}} \text{ (below the } m_\pi + m_Y\text{)}$$

Cross section & Decay



$$d\Gamma^{Y^*}_{\bar{K}N} = \frac{(2\pi)^4}{2m_{\bar{K}N}} \left|\mathcal{M}\right|^2 \Phi_2 = \frac{(2\pi)^4}{2m_{\bar{K}N}} \left|\mathcal{M}\right|^2 \frac{1}{4\,(2\pi)^6} \frac{\left|\vec{p}^{\,(\bar{K}N)*}_N\right|}{m_{\bar{K}N}} d\Omega^{(\bar{K}N)*}_N$$

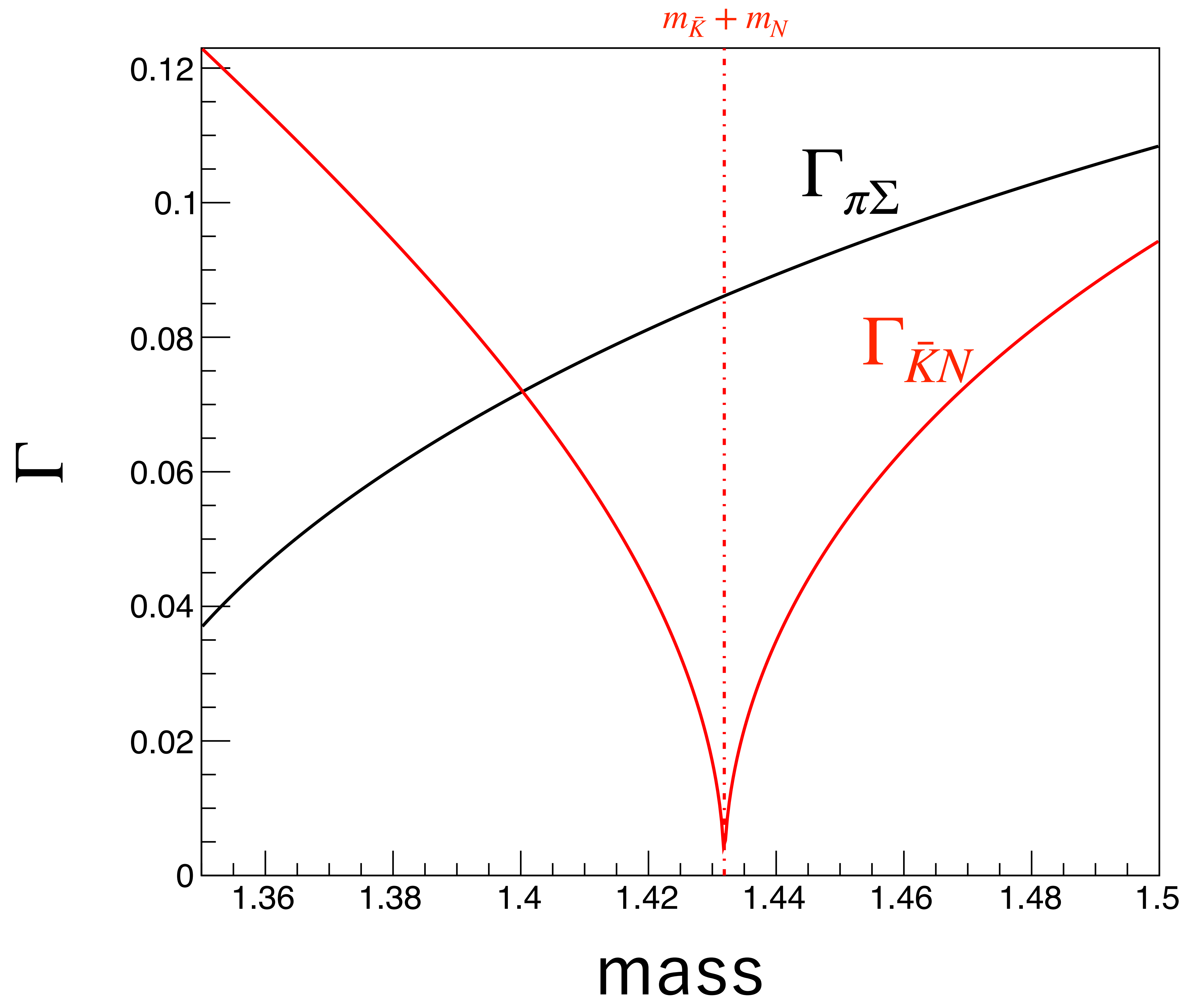
$$\boxed{\mathcal{M} = g^{Y^*}_{\bar{K}N}}$$

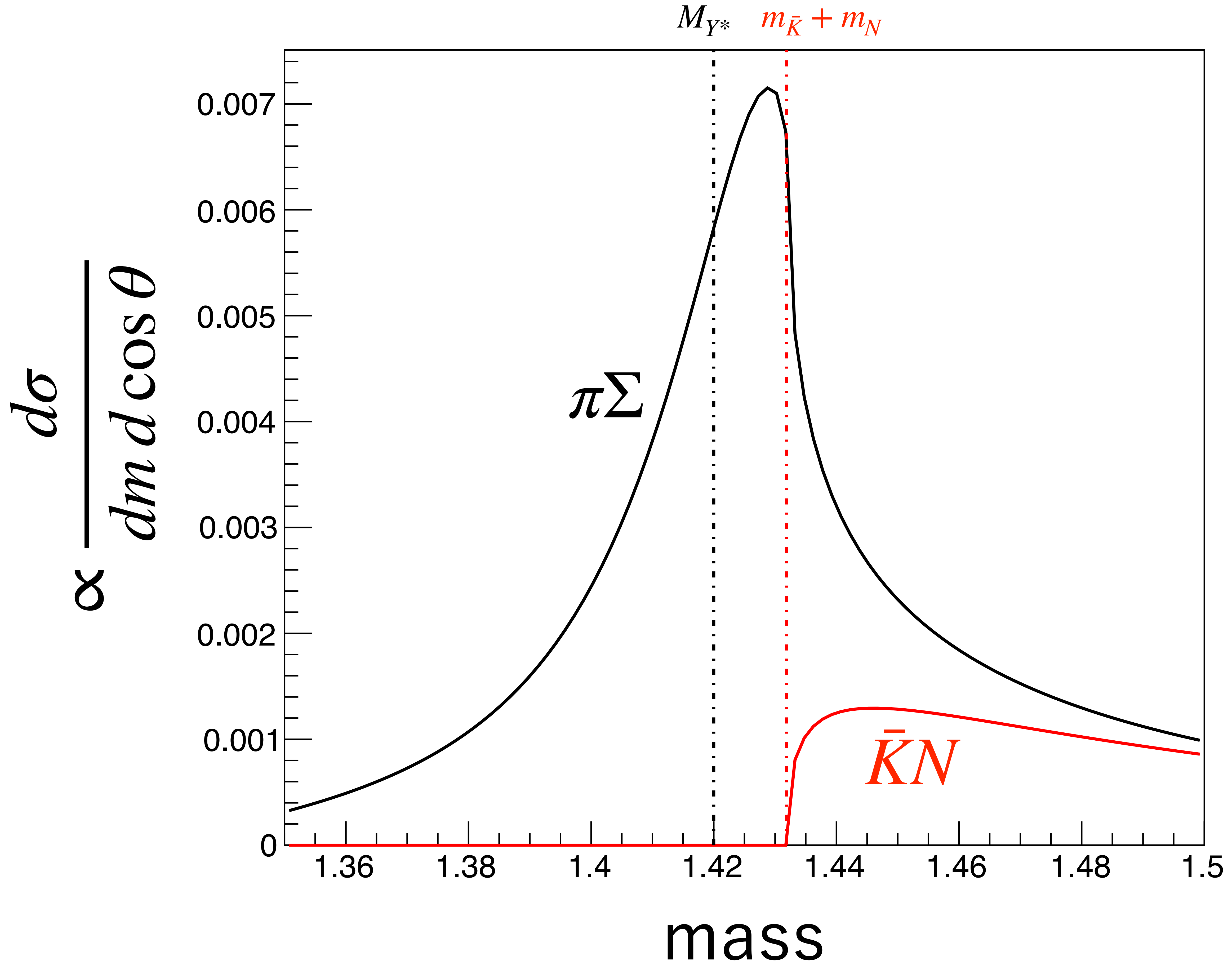
$$\Gamma_{\pi Y} = \frac{\left(g^{Y^*}_{\bar{K}N}\right)^2}{8\pi m^2_{\bar{K}N}} \frac{\sqrt{\left(m^2_{\bar{K}N} - (m_{\bar{K}} + m_N)^2\right)\left(m^2_{\bar{K}N} - (m_{\bar{K}} - m_N)^2\right)}}{2m_{\bar{K}N}} \text{ (above the } m_{\bar{K}} + m_N)$$

$$= i\frac{\left(g^{Y^*}_{\bar{K}N}\right)^2}{8\pi m^2_{\bar{K}N}} \frac{\sqrt{\left((m_{\bar{K}} + m_N)^2 - m^2_{\bar{K}N}\right)\left(m^2_{\bar{K}N} - (m_{\bar{K}} - m_N)^2\right)}}{2m_{\bar{K}N}} \text{ (below the } m_{\bar{K}} + m_N)$$

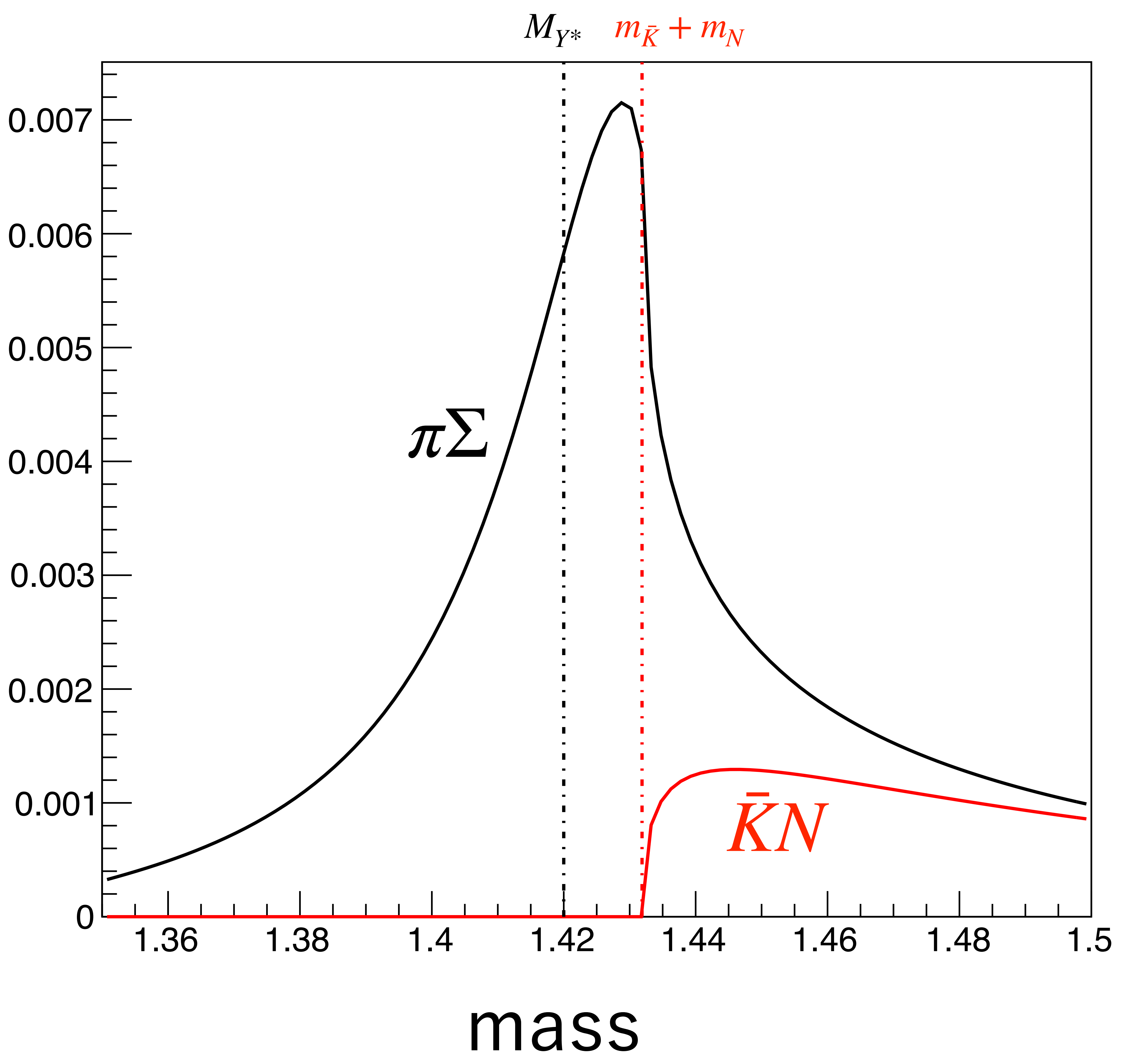
Cross section & Decay

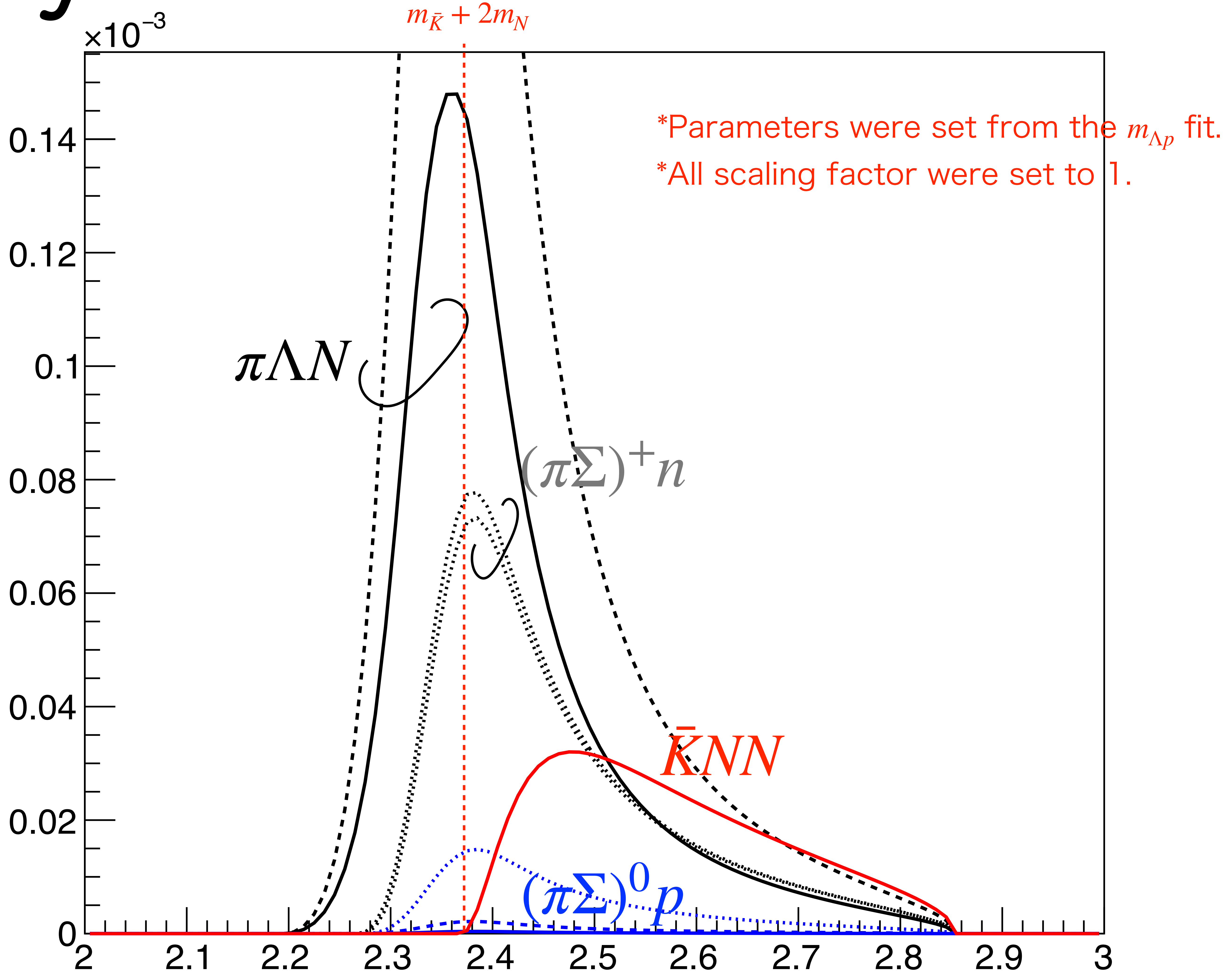
Parameters
$$\begin{cases} M_{Y^*} = 1.42\ \mathrm{GeV}/c^2 \\ g^{Y^*}_{\pi\Sigma} = g^{Y^*}_{\bar{K}N} = 5 \end{cases}$$





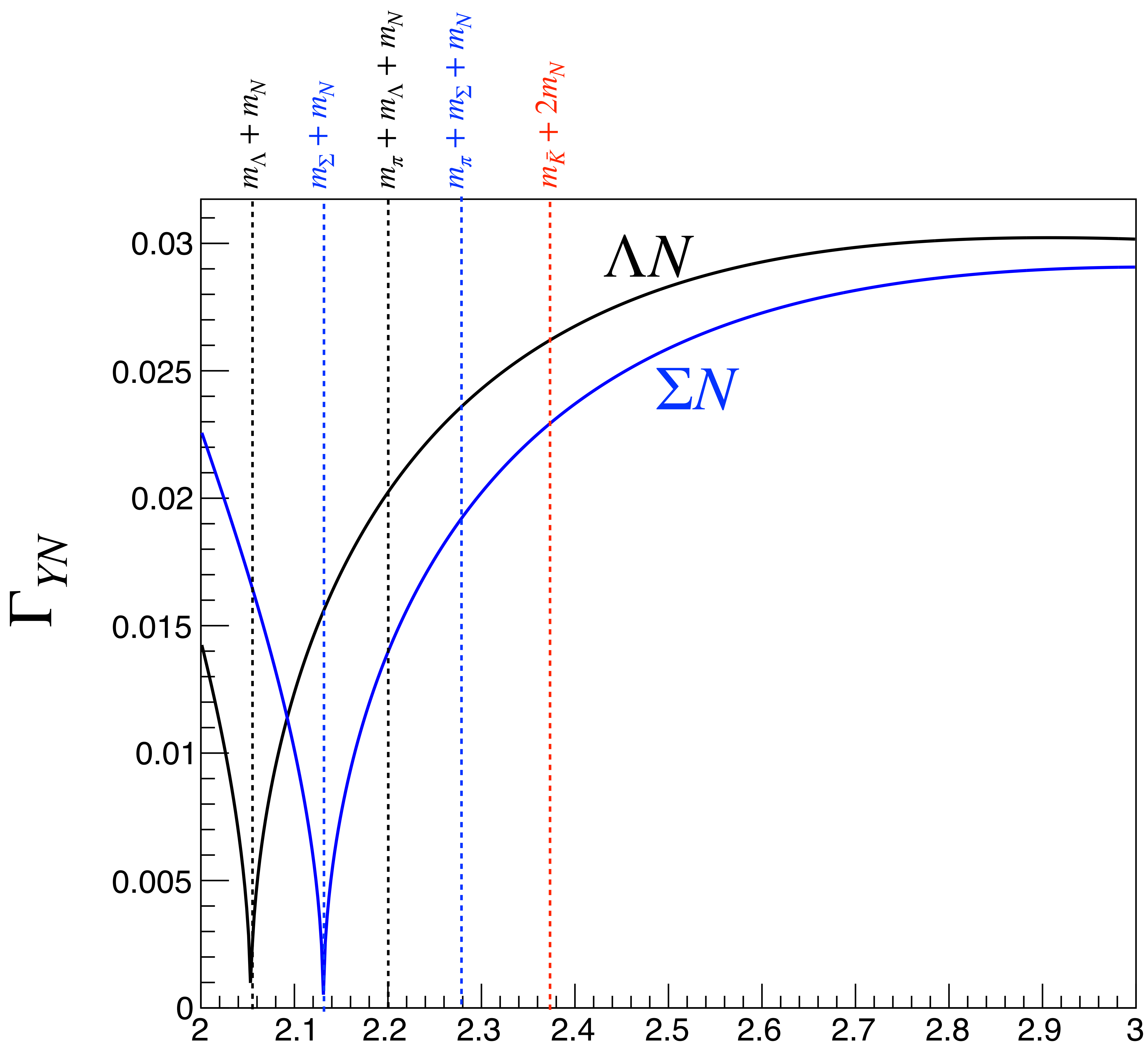
Cross section & Decay

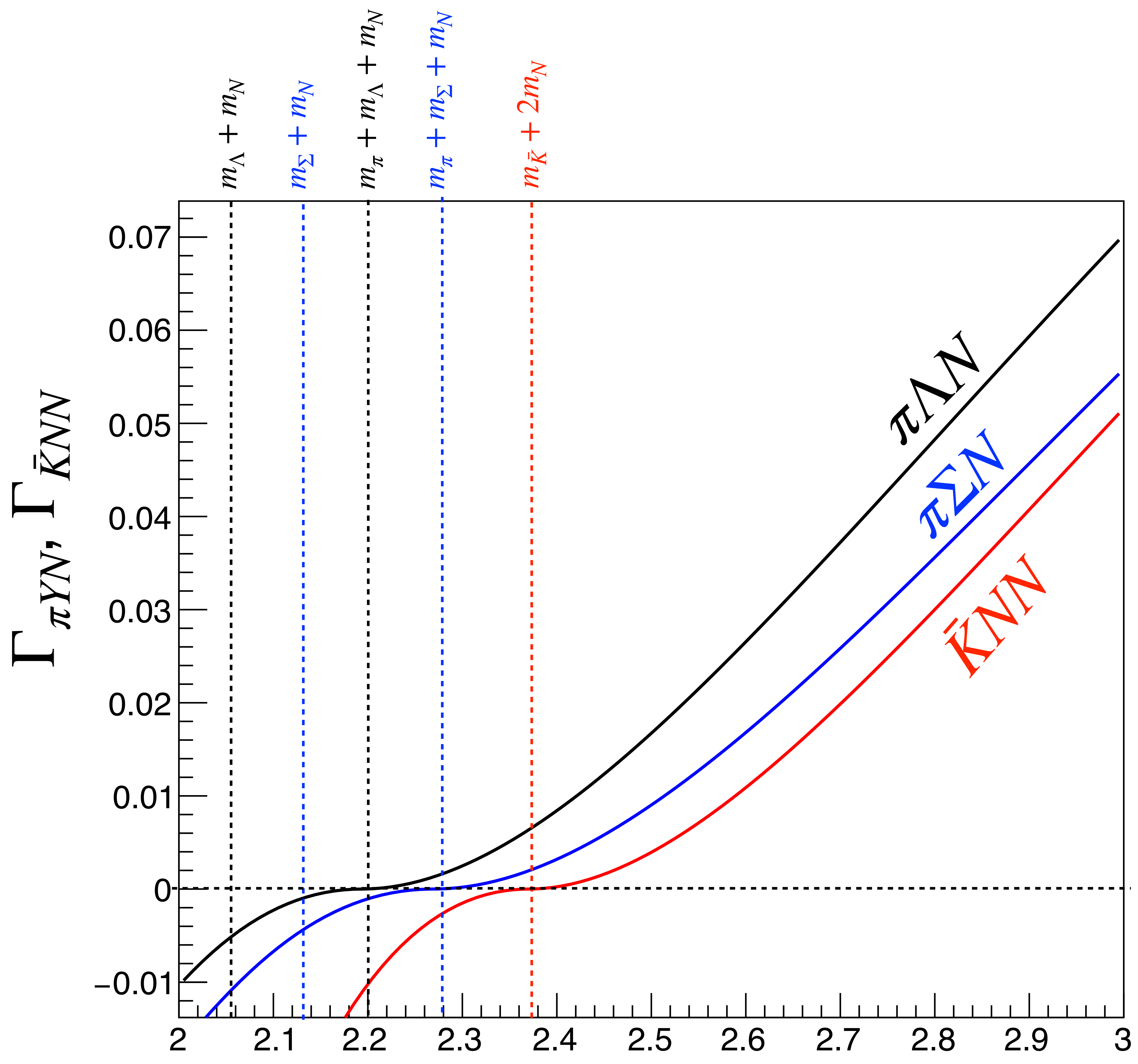




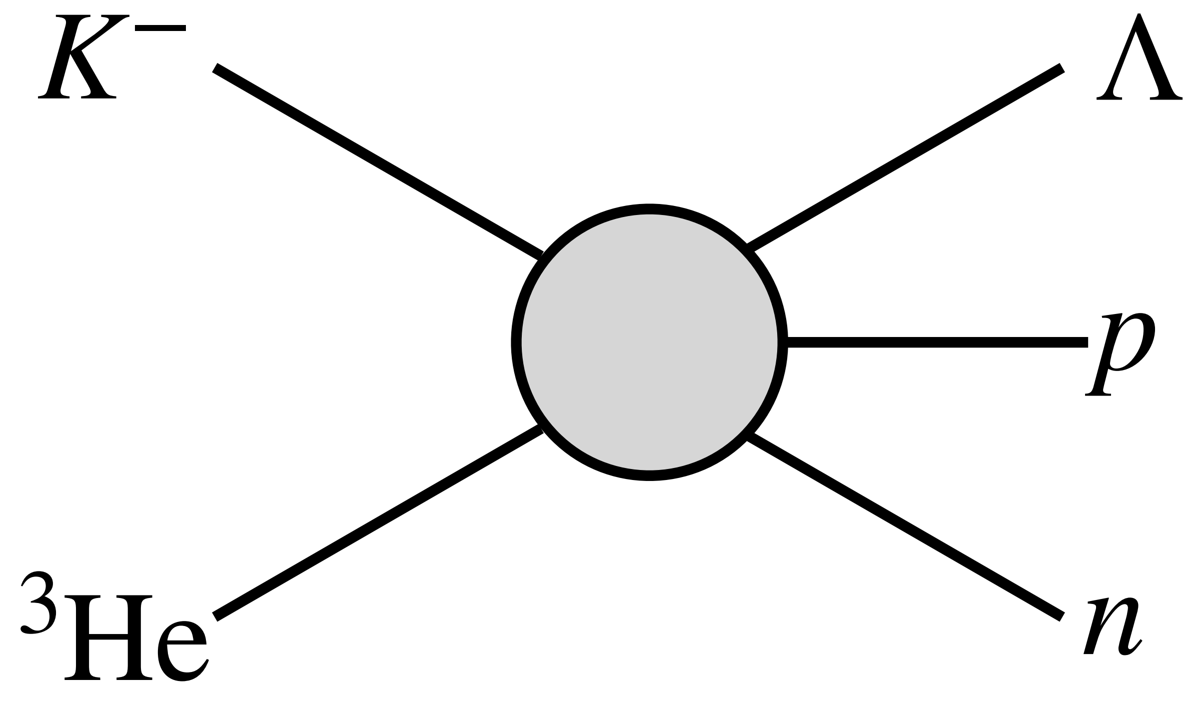
Spectral distortion seems to be much smaller than $\Lambda(1405)$ case.
It may due to the difference between two-body and three-body LIPS.

Cross section & Decay





Cross section & Decay



The cross section can be expressed as,

$$d\sigma_{\Lambda pn} = \frac{(2\pi)^4}{4p^*_{K^-}\sqrt{s}}\ \mathscr{M}_{\Lambda pn}\ ^{2} \times d\Phi_{\Lambda pn}$$

where $d\Phi_{\Lambda pn}$ is the three-body phase space of the Λpn final state,

$$d\Phi_{\Lambda pn} = \left(\frac{1}{4\,(2\pi)^6}\frac{\vec{p}^{\,*}_n}{\sqrt{s}}\,d\Omega^*_n\right)\left(\frac{1}{4\,(2\pi)^6}\frac{\vec{p}^{\,(\Lambda p)*}_{\Lambda}}{m_{\Lambda p}}\,d\Omega^{(\Lambda p)*}_{\Lambda}\right)\left((2\pi)^3 dm^2_{\Lambda p}\right)$$

$p^*_n(\Omega^*_n)$ and $p^{(\Lambda p)*}_{\Lambda}(\Omega^{(\Lambda p)*}_{\Lambda})$ are momenta (angles) of n and Λ in the K^{-3}He-c.m. and (Λp)-c.m. frame, respectively, as,

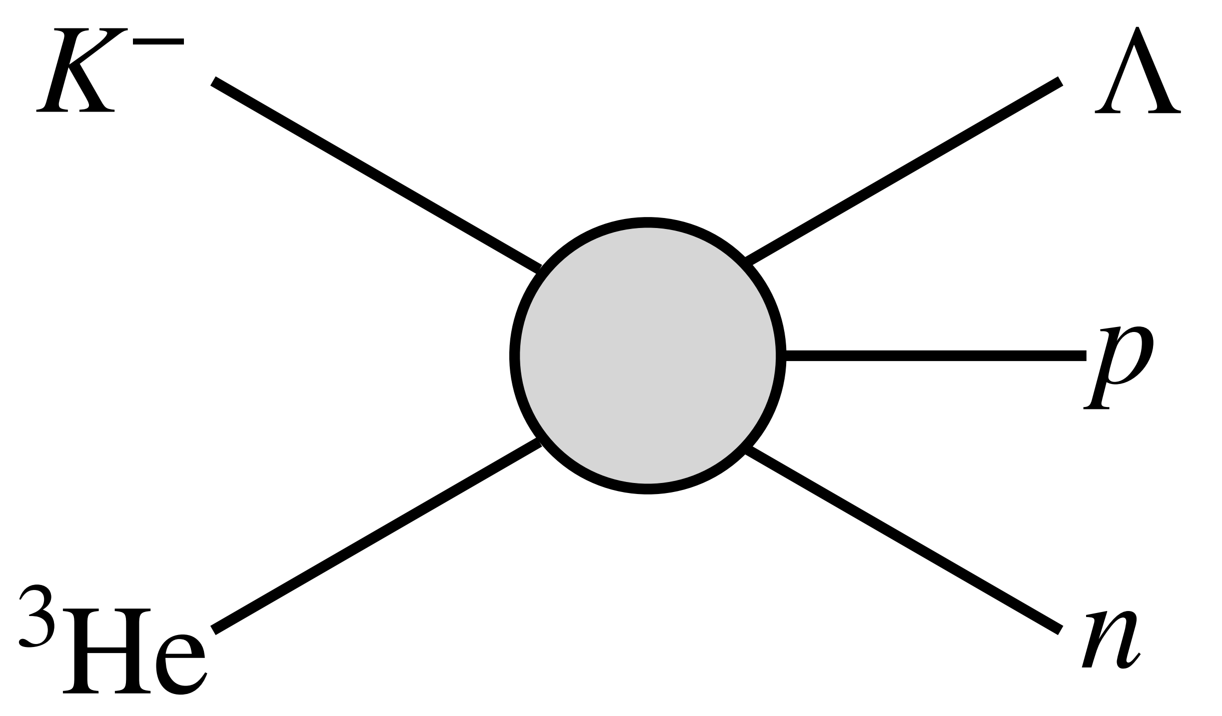
$$\left|p^*_n\right| = \frac{\sqrt{\left(s-(m_{\Lambda p}+m_n)^2\right)\left(s-(m_{\Lambda p}-m_n)^2\right)}}{2\sqrt{s}} \qquad \left|p^{(\Lambda p)*}_p\right| = \frac{\sqrt{\left(m^2_{\Lambda p}-(m_\Lambda+m_p)^2\right))\left(m_{\Lambda p}-(m_\Lambda-m_p)^2\right)}}{2m_{\Lambda p}}$$

$$\mathscr{M} = \frac{g^R_{\Lambda p}}{M^2_R - m^2_{\Lambda p} - iM_R\Gamma^R_{tot}}\cdot\mathscr{A}\left(\cos\theta^*_n\right)$$

$$\frac{d\sigma}{dm_{\Lambda p}\,d\cos\theta^*_n} = \frac{1}{16(2\pi)^3}\frac{1}{p^*_{K^-}\,s}\left|p^*_n\right|\left|p^{(\Lambda p)*}_p\right|\left|\frac{g^R_{\Lambda p}}{M^2_R - m^2_{\Lambda p} - iM_R\Gamma^R_{tot}}\right|^2\left|\mathscr{A}\left(\cos\theta^*_n\right)\right|^2$$

Cross section & Decay

簡単のため、$K^{-3}\mathrm{He} \to \Lambda pn$反応からはじめる。というか終状態3体で、アイソスピンの組み合わせが少ないこのチャンネルしかできなさそうなのだが…



例の微分断面積の表式に粒子を当てはめる。

$$d\sigma_{\Lambda pn} = \frac{(2\pi)^4}{4p^*_{K^-}\sqrt{s}} \mathscr{M}_{\Lambda pn}{}^2 \times d\Phi_3 = \frac{1}{16(2\pi)^3}\frac{1}{p^*_{K^-}s}\left|p^*_n\right|\left|p^{(\Lambda p)*}_{\Lambda}\right| \mathscr{M}_{\Lambda pn}{}^2\, dm_{\Lambda p}\, d\cos\theta^*_n$$

$d\Phi_3$の書き方には何通りかあるが、Λpの不変質量分布$d\sigma/dm_{\Lambda p}$に興味があると思い、

$$d\Phi_3 = \left(\frac{1}{4(2\pi)^6}\frac{\vec{p}^*_n}{\sqrt{s}}\, d\Omega^*_n\right)\left(\frac{1}{4(2\pi)^6}\frac{\vec{p}^{(\Lambda p)*}_{\Lambda}}{m_{\Lambda p}}\, d\Omega^{(\Lambda p)*}_{\Lambda}\right)\left((2\pi)^3 dm^2_{\Lambda p}\right)$$

の形とした。ここで、p^*_n、$p^{(\Lambda p)*}_{\Lambda}$はそれぞれ、$\Lambda pn$重心系での$n$の運動量、$\Lambda p$重心系での$\Lambda$の運動量で、

$$\left|p^*_n\right| = \frac{\sqrt{\left(s-(m_{\Lambda p}+m_n)^2\right)\left(s-(m_{\Lambda p}-m_n)^2\right)}}{2\sqrt{s}}、\quad \left|p^{(\Lambda p)*}_{\Lambda}\right| = \frac{\sqrt{\left(m^2_{\Lambda p}-(m_\Lambda+m_p)^2\right)\Big)\left(m_{\Lambda p}-(m_\Lambda-m_p)^2\right)}}{2m_{\Lambda p}}$$

$K^{-3}\mathrm{He} \to \Lambda pn$反応を記述する

Λpn終状態は3体で、かつΛのアイソスピンがゼロで、アイソスピンの組み合わせの数が少ないので、数ある終状態の中で記述するのが一番簡単（なはず）である。ということで、ここでは最も簡単な$K^{-3}\mathrm{He} \to \Lambda pn$反応を記述することを目指す。というか、現実的には、この反応しか記述できなさそう…
例の反応断面積の表式から出発して、

$$d\sigma_{\Lambda pn} = \frac{(2\pi)^4}{4p^*_{K^-}\sqrt{s}}\ \mathscr{M}_{\Lambda pn}\ ^{2} \times d\Phi_3 = \frac{1}{16(2\pi)^3}\frac{1}{p^*_{K^-}\,s}\left|p^*_n\right|\left|p^{(\Lambda p)*}_{\Lambda}\right|\ \mathscr{M}_{\Lambda pn}\ ^{2}\, dm_{\Lambda p} d\cos\theta^*_n$$

ここで、$d\Phi_3$の書き方には何通りかあるが、Λpの不変質量分布$d\sigma/dm_{\Lambda p}$に興味があるので、

$$d\Phi_3 = \left(\frac{1}{4\,(2\pi)^6}\frac{\vec{p}^{\,*}_n}{\sqrt{s}}\,d\Omega^*_n\right)\left(\frac{1}{4\,(2\pi)^6}\frac{\vec{p}^{\,(\Lambda p)*}_{\Lambda}}{m_{\Lambda p}}\,d\Omega^{(\Lambda p)*}_{\Lambda}\right)\left((2\pi)^3 dm^2_{\Lambda p}\right)$$

とした。ここで、p^*_n、$p^{(\Lambda p)*}_{\Lambda}$はそれぞれ、$\Lambda pn$重心系での$n$の運動量、$\Lambda p$重心系での$\Lambda$の運動量で、

$$\left|p^*_n\right| = \frac{\sqrt{\left(s-(m_{\Lambda p}+m_n)^2\right)\left(s-(m_{\Lambda p}-m_n)^2\right)}}{2\sqrt{s}}、$$

$$\left|p^{(\Lambda p)*}_{\Lambda}\right| = \frac{\sqrt{\left(m^2_{\Lambda p}-(m_\Lambda+m_p)^2\right))\left(m_{\Lambda p}-(m_\Lambda-m_p)^2\right)}}{2m_{\Lambda p}}$$

である。ということで、興味のある$m_{\Lambda p}$と$\bar{K}NN_{I_3=+1/2}(K^-pp)$が生成したときに前方に飛ぶ$n$の角度、$\cos\theta^*_n$についての２重微分断面積が、

$$\frac{d\sigma_{\Lambda pn}}{dm_{\Lambda p}\,d\cos\theta^*_n} = \frac{1}{16(2\pi)^3}\frac{1}{p^*_{K^-}\,s}\left|p^*_n\right|\left|p^{(\Lambda p)*}_{p}\right|\ \mathscr{M}_{\Lambda pn}\ ^{2}$$

の形で得られる。

Cross section & Decay

$$|0,0\rangle_{YNN} = \sqrt{\frac{1}{2}}\left|\frac{1}{2}, +\frac{1}{2}\right\rangle_{YN}|n\rangle - \sqrt{\frac{1}{2}}\left|\frac{1}{2}, -\frac{1}{2}\right\rangle_{YN}|p\rangle$$

$$|1,0\rangle_{YNN} = \left(\sqrt{\frac{1}{2}}\left|\frac{1}{2}, +\frac{1}{2}\right\rangle_{YN}|n\rangle + \sqrt{\frac{1}{2}}\left|\frac{1}{2}, -\frac{1}{2}\right\rangle_{YN}|p\rangle\right)$$

$$\left|\frac{1}{2}, +\frac{1}{2}\right\rangle_{YN} = g_{\Lambda N}|\Lambda p\rangle + g_{\Sigma N}\left(\sqrt{\frac{2}{3}}|\Sigma^+ n\rangle - \sqrt{\frac{1}{3}}|\Sigma^0 p\rangle\right)$$

$$\mathscr{M}_{\Lambda pn} = \left\langle \Lambda pn \middle| T_{\Lambda pn} \middle| K^{-3}\mathrm{He}\right\rangle = \left\langle \Lambda pn \middle| \hat{T}^{(1/2)}_{\Lambda N} \middle| \Lambda NN' \right\rangle \left\langle \Lambda NN' \middle| \hat{T}^{(0)}_{\Lambda NN'} + \hat{T}^{(1)}_{\Lambda NN'} \middle| K^{-3}\mathrm{He}\right\rangle$$

- $T^{(I_{\Lambda NN'})}_{\Lambda NN'}$: Transition operator to the $\Lambda NN'$ final state in the isospin $I_{\Lambda NN'}$ channel
- $T^{(I_{\Lambda N})}_{\Lambda N}$: Transition operator to the ΛN channel in the isospin $I_{\Lambda N}$ channel

$$|K^{-3}\mathrm{He}\rangle = \sqrt{\frac{1}{2}}\underbrace{\left(\frac{|K^{-3}\mathrm{He}\rangle + |{}^3\mathrm{He}K^-\rangle}{\sqrt{2}}\right)}_{I=1} - \sqrt{\frac{1}{2}}\underbrace{\left(\frac{-|K^{-3}\mathrm{He}\rangle + |{}^3\mathrm{He}K^-\rangle}{\sqrt{2}}\right)}_{I=0}$$

$$|\Lambda pn\rangle = \sqrt{\frac{1}{2}}\underbrace{\left(\frac{|\Lambda pn\rangle + |\Lambda np\rangle}{\sqrt{2}}\right)}_{I=1} + \sqrt{\frac{1}{2}}\underbrace{\left(\frac{|\Lambda pn\rangle - |\Lambda np\rangle}{\sqrt{2}}\right)}_{I=0}$$

$$t^{(I)}_{\Lambda NN'} = \left\langle \Lambda NN' \middle| \hat{T}^{(I)}_{\Lambda NN'} \middle| K^{-3}\mathrm{He}\right\rangle \qquad t^{(I)}_{\Lambda N} = \left\langle \Lambda pn \middle| \hat{T}^{(I)}_{\Lambda N} \middle| \Lambda NN' \right\rangle$$

$$\Rightarrow \mathscr{M}_{\Lambda pn} = \frac{1}{2}t^{(1)}_{\Lambda NN'}\left(\frac{1}{2}t^{(1/2)}_{\Lambda p} + \frac{1}{2}t^{(1/2)}_{\Lambda n}\right) - \frac{1}{2}t^{(0)}_{\Lambda NN'}\left(\frac{1}{2}t^{(1/2)}_{\Lambda p} - \frac{1}{2}t^{(1/2)}_{\Lambda n}\right)$$

$$= \underbrace{\frac{1}{4}t^{(1/2)}_{\Lambda p}\left(t^{(1)}_{\Lambda NN'} - t^{(0)}_{\Lambda NN'}\right)}_{\text{“}K^-pp\text{”}} + \underbrace{\frac{1}{4}t^{(1/2)}_{\Lambda n}\left(t^{(1)}_{\Lambda NN'} + t^{(0)}_{\Lambda NN'}\right)}_{\text{“}\bar{K}^0pn\text{”}}$$

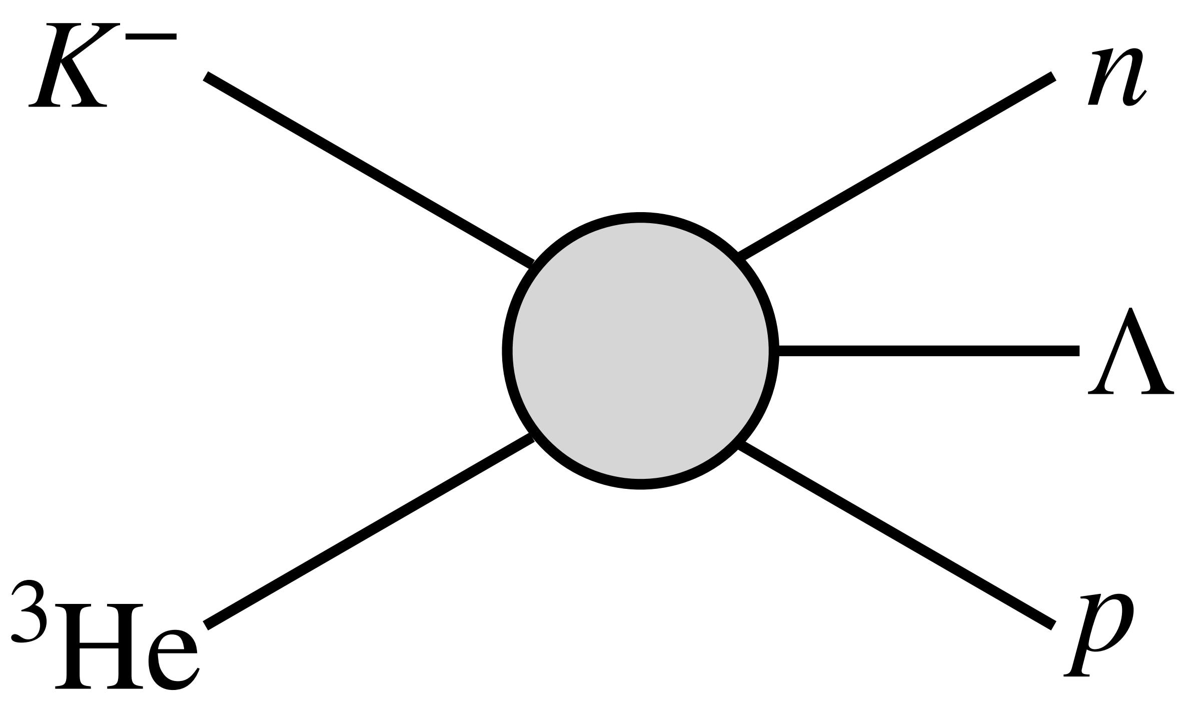
Cross section & Decay

$$\mathscr{M}_{\Lambda pn} = \frac{1}{4} t^{(1/2)}_{\Lambda p} \left(t^{(1)}_{\Lambda NN'} - t^{(0)}_{\Lambda NN'} \right) + \frac{1}{4} t^{(1/2)}_{\Lambda n} \left(t^{(1)}_{\Lambda NN'} + t^{(0)}_{\Lambda NN'} \right) \equiv A(s)\, t^{(1/2)}_{\Lambda p} + B(s)\, t^{(1/2)}_{\Lambda n}$$

$$t^{(1/2)}_{\Lambda N} = \frac{g^{X}_{\Lambda N}}{M_X^2 - m_{\Lambda N}^2 - i M_X \Gamma^R_{tot}} \cdot \mathscr{A}\left(\Omega_{\Lambda N}\right)$$

$\dfrac{d\sigma_{\Lambda pn}}{dm_{\Lambda p}\, d\cos\theta^*_n} = \dfrac{1}{16(2\pi)^3} \dfrac{1}{p^*_{K^-}\, s} \left| p^*_n \right| \left| p_p^{(\Lambda p)*} \right| \ \mathscr{M}_{\Lambda pn} \ {}^2$ の式にいれても、よくわからない式になる…

Cross section & Decay



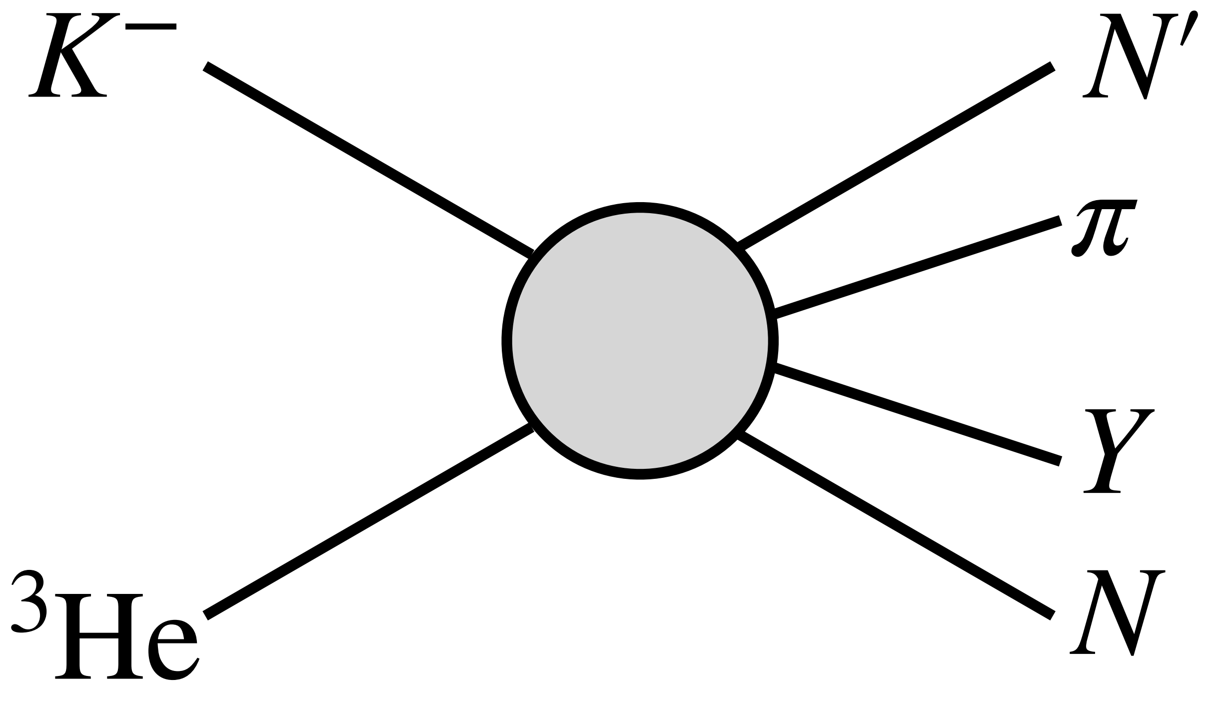
$$\frac{d\sigma_{\Lambda pn}}{dm_{\Lambda p}\, d\cos\theta_n^*} = \frac{1}{16(2\pi)^3}\frac{1}{p_{K^-}^* \, s} \left|p_n^*\right| \left|p_p^{(\Lambda p)*}\right| \;\; \mathscr{M}_{\Lambda pn}{}^{2}$$

$$\left|p_n^*\right| = \frac{\sqrt{\left(s-(m_{\Lambda p}+m_n)^2\right)\left(s-(m_{\Lambda p}-m_n)^2\right)}}{2\sqrt{s}} \qquad \left|p_p^{(\Lambda p)*}\right| = \frac{\sqrt{\left(m_{\Lambda p}^2-(m_\Lambda+m_p)^2\right)\!)\left(m_{\Lambda p}-(m_\Lambda-m_p)^2\right)}}{2m_{\Lambda p}}$$

$$\mathscr{M} = \frac{g_{\Lambda p}^R}{M_R^2 - m_{\Lambda p}^2 - iM_R\Gamma_{tot}^R} \cdot \mathscr{A}\left(\cos\theta_n^*\right)$$

$$\frac{d\sigma}{dm_{\Lambda p}\, d\cos\theta_n^*} = \frac{1}{16(2\pi)^3}\frac{1}{p_{K^-}^* \, s} \left|p_n^*\right| \left|p_p^{(\Lambda p)*}\right| \left|\frac{g_{\Lambda p}^R}{M_R^2 - m_{\Lambda p}^2 - iM_R\Gamma_{tot}^R}\right|^2 \left|\mathscr{A}\left(\cos\theta_n^*\right)\right|^2$$

Cross section & Decay



$$d\sigma = \frac{(2\pi)^4}{4p^*_{K^-}\sqrt{s}}\ \mathscr{M}\ ^2 \times d\Phi_4$$

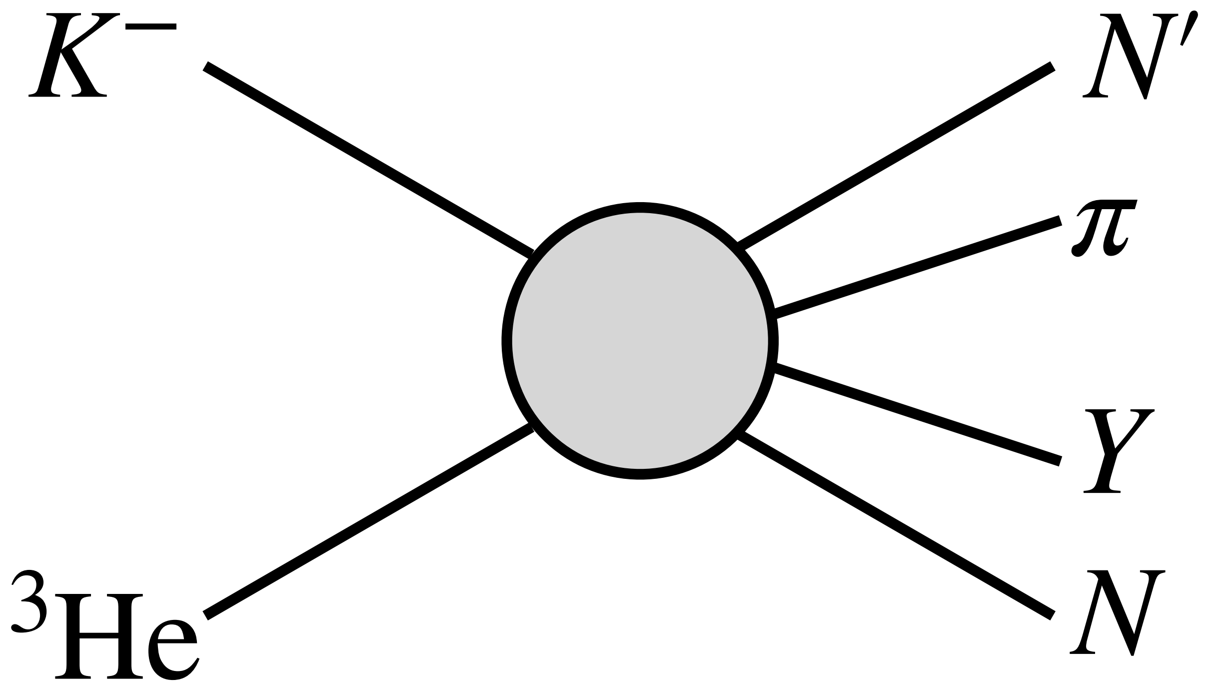
$$= \frac{1}{32(2\pi)^5}\frac{1}{p^*_{K^-}\, s}\left|p^*_{N'}\right|\left|p_N^{(\pi YN)*}\right|\left|p_Y^{(\pi Y)*}\right|\ \mathscr{M}\ ^2 dm_{\pi YN}\, dm_{\pi Y}\, d\cos\theta^*_{N'}\, d\cos\theta_N^{(\pi YN)*}$$

$$d\Phi_4 = \left(\frac{1}{4\,(2\pi)^6}\frac{\vec{p}^{\,*}_{N'}}{\sqrt{s}}\,d\Omega^*_{N'}\right)\left(\frac{1}{4\,(2\pi)^6}\frac{\vec{p}_N^{\,(\pi YN)*}}{m_{\pi YN}}\,d\Omega_N^{(\pi YN)*}\right)\left(\frac{1}{4\,(2\pi)^6}\frac{\vec{p}_Y^{\,(\pi Y)*}}{m_{\pi Y}}\,d\Omega_Y^{(\pi Y)*}\right)\left((2\pi)^3 dm^2_{\pi YN}\right)\left((2\pi)^3 dm^2_{\pi Y}\right)$$

$$\frac{d\sigma}{dm_{\pi YN}\, dm_{\pi Y}\, d\cos\theta^*_{N'}\, d\cos\theta_N^{(\pi YN)*}} = \frac{1}{32(2\pi)^5}\frac{1}{p^*_{K^-}\, s}\left|p^*_{N'}\right|\left|p_N^{(\pi YN)*}\right|\left|p_Y^{(\pi Y)*}\right|\ \mathscr{M}\ ^2$$

$$\left|p^*_{N'}\right| = \frac{\sqrt{\left(s-(m_{\pi YN}+m_{N'})^2\right)\left(s-(m_{\pi YN}-m_{N'})^2\right)}}{2\sqrt{s}},\quad \left|p_N^{(\pi YN)*}\right| = \frac{\sqrt{\left(m^2_{\pi YN}-(m_{\pi Y}+m_N)^2\right)\left(m^2_{\pi YN}-(m_{\pi Y}-m_N)^2\right)}}{2m_{\pi YN}},\quad \left|p_Y^{(\pi Y)*}\right| = \frac{\sqrt{\left(m^2_{\pi Y}-(m_\pi+m_Y)^2\right)\left(m^2_{\pi Y}-(m_\pi-m_Y)^2\right)}}{2m_{\pi Y}}$$

Cross section & Decay



$$\frac{d\sigma}{dm_{\pi YN}\, dm_{\pi Y}\, d\cos\theta^*_{N'}\, d\cos\theta^{(\pi YN)*}_{N}} = \frac{1}{32(2\pi)^5}\frac{1}{p^*_{K^-}\, s}\left|p^*_{N'}\right|\left|p^{(\pi YN)*}_{N}\right|\left|p^{(\pi Y)*}_{Y}\right| \mathscr{M}^{2}$$

$$\left|p^*_{N'}\right| = \frac{\sqrt{\left(s-(m_{\pi YN}+m_{N'})^2\right)\left(s-(m_{\pi YN}-m_{N'})^2\right)}}{2\sqrt{s}},\quad \left|p^{(\pi YN)*}_{N}\right| = \frac{\sqrt{\left(m^2_{\pi YN}-(m_{\pi Y}+m_N)^2\right)\left(m^2_{\pi YN}-(m_{\pi Y}-m_N)^2\right)}}{2m_{\pi YN}},\quad \left|p^{(\pi Y)*}_{Y}\right| = \frac{\sqrt{\left(m^2_{\pi Y}-(m_\pi+m_Y)^2\right)\left(m^2_{\pi Y}-(m_\pi-m_Y)^2\right)}}{2m_{\pi Y}}$$

$$\mathscr{M} = \mathscr{M}_{\pi YN}\cdot\mathscr{M}_{\pi Y} = \frac{g^R_{\pi YN}}{M_R^2 - m^2_{\pi YN} - iM_R\Gamma^R_{tot}}\cdot\left(g^{NR}_{\pi Y} + \frac{g^{Y*}_{\pi Y}}{M^2_{Y*} - m^2_{\pi Y} - iM_{Y*}\Gamma^{Y*}_{tot}}\right)\mathscr{A}\left(\cos\theta^*_{N'}\right)\mathscr{A}\left(\cos\theta^{(\pi YN)*}_{N}\right)$$

$$\frac{d\sigma}{dm_{\pi YN}\, dm_{\pi Y}\, d\cos\theta^*_{N'}\, d\cos\theta^{(\pi YN)*}_{N}} = \frac{1}{32(2\pi)^5}\frac{1}{p^*_{K^-}\, s}\left|p^*_{N'}\right|\left|p^{(\pi YN)*}_{N}\right|\left|p^{(\pi Y)*}_{Y}\right|\left|\frac{g^R_{\pi YN}}{M_R^2 - m^2_{\pi YN} - iM_R\Gamma^R_{tot}}\right|^2\left|g^{NR}_{\pi Y} + \frac{g^{Y*}_{\pi Y}}{M^2_{Y*} - m^2_{\pi Y} - iM_{Y*}\Gamma^{Y*}_{tot}}\right|^2\left|\mathscr{A}\left(\cos\theta^*_{N'}\right)\right|^2\left|\mathscr{A}\left(\cos\theta^{(\pi YN)*}_{N}\right)\right|^2$$

Cross section & Decay

50.3.1 *The Breit–Wigner parametrization*

The relativistic Breit–Wigner parametrization provides a propagator for a single, isolated resonance,

$$\mathcal{A}(s) = \frac{N_a(s)}{M_{\mathrm{BW}}^2 - s - iM_{\mathrm{BW}}\Gamma(s)} \tag{50.23}$$

where M_{BW} is the Breit–Wigner mass, and $\Gamma_{\mathrm{BW}} = \Gamma(M_{\mathrm{BW}}^2)$ is the Breit–Wigner width. The function $\Gamma(s)$ is determined by the channels that the resonance can decay to. The numerator function $N_a(s)$ is specific to the production process. It includes kinematic factors and couplings related to the production process and the decay. Breit–Wigner functions with a s-independent width are justified only, if there is no relevant threshold in the vicinity of the resonance.

To give a concrete example, we consider a resonance observed in the channel a, that is also coupled to a set of channels labeled by index $b = 1, 2, \ldots$, with the orbital angular momentum l_b. Couplings to the channels are denoted, g_b.

$$N_a(s) = \alpha\, g_a\, n_a(s) \tag{50.24}$$

$$\Gamma(s) = \frac{1}{M_{\mathrm{BW}}} \sum_b g_b^2 \rho_b(s) n_b^2(s) \tag{50.25}$$

where the factor $n_a(s)$ includes the kinematic threshold factor q^{l_a}, and the barrier factor $F_{l_a}(q_a/q_0)$ that regularize the high–energy behaviour:

$$n_a = (q_a/q_0)^{l_a} F_{l_a}(q_a/q_0)\,, \tag{50.26}$$

with l_a being the orbital angular momentum in channel a, $q_a(s)$ is defined in Eq. (50.5), and q_0 denotes some conveniently chosen momentum scale. The factor $(q_a)^l$ guarantees the correct threshold behavior. The rapid growth of this factor for angular momenta $l > 0$ is commonly compensated at higher energies by a phenomenological form factor, here denoted by $F_{l_a}(q_a, q_0)$. Often, the Blatt-Weisskopf form factors, $F_j(q/q_0)$, are used [50–52]:

$$\begin{aligned} F_0^2(z) &= 1\,, \\ F_1^2(z) &= 1/(1+z^2)\,, \\ F_2^2(z) &= 1/(9+3z^2+z^4)\,, \end{aligned} \tag{50.27}$$

with the scale parameter $R = 1/q_0$ in the range from $1\,\mathrm{GeV}^{-1}$ to $5\,\mathrm{GeV}^{-1}$. Instead of using coupling constant in Eq. (50.25), one can define the energy-dependent partial width:

$$\Gamma_b(s) = \Gamma_{\mathrm{BW},b} \frac{\rho_b(s)}{\rho_b(M_{\mathrm{BW}}^2)} \left(\frac{q_b}{q_{b\,\mathrm{R}}}\right)^{2l_b} \frac{F_{l_b}^2(q_b, q_0)}{F_{l_b}^2(q_{b\,\mathrm{R}}, q_0)}\,. \tag{50.28}$$

Here $q_{a\,\mathrm{R}}$ are the values of the break-up momentum evaluated at $s = M_{\mathrm{BW}}^2$. The substitution is possible only for those channels where the threshold of the decay channel is located below the nominal resonance mass, otherwise, Eq. (50.25) should be used.

The Breit–Wigner parametrization provides an effective description of resonance phenomena. However, the parameters agree with the pole parameters only if the resonance is narrow, isolated (no nearby resonances in the same partial wave) and the background is smooth. Otherwise, the Breit–Wigner parameters deviate from the pole parameters and are reaction-dependent. If there is more than one resonance in one partial wave that significantly couples to the same channel, it is in general incorrect to use a sum of Breit–Wigner functions, for this usually leads to a violation of unitarity constraints, and hence, a non-quantifiable bias to resonance properties which are inferred from the reaction amplitude. In case of overlapping resonances in the same partial wave more refined methods should be used, like the K-matrix approach described in the next section.

Cross section & Decay

$K^- pp$-$\bar{K}^0 pn$

- Λp
- $\Sigma^0 p$
- $\Sigma^+ n$

- $\pi^0 \Lambda p$
- $\pi^+ \Lambda n$
- $\pi^0 \Sigma^0 p$
- $\pi^- \Sigma^+ p$
- $\pi^+ \Sigma^- p$
- $\pi^0 \Sigma^+ n$
- $\pi^+ \Sigma^0 n$

$$\left|\frac{1}{2},+\frac{1}{2}\right\rangle_{YN} = g_{\Lambda N}\left|\Lambda p\right\rangle + g_{\Sigma N}\left(\sqrt{\frac{2}{3}}\left|\Sigma^+ n\right\rangle - \sqrt{\frac{1}{3}}\left|\Sigma^0 p\right\rangle\right)$$

$$\left|\frac{1}{2},+\frac{1}{2}\right\rangle_{\pi\Lambda N} = g_{\pi\Lambda N}\left(\sqrt{\frac{2}{3}}\left|\pi^+ \Lambda n\right\rangle - \sqrt{\frac{1}{3}}\left|\pi^0 \Lambda p\right\rangle\right)$$

$$\left|\frac{1}{2},+\frac{1}{2}\right\rangle_{\pi\Sigma N} = g_{\pi\Sigma N}\left(g^0_{\pi\Sigma}\left|\pi\Sigma\right\rangle_{I=0}\left|p\right\rangle + g^1_{\pi\Sigma}\left(\sqrt{\frac{2}{3}}\left|\pi\Sigma\right\rangle_{I=\left|1,+1\right\rangle}\left|n\right\rangle - \sqrt{\frac{1}{3}}\left|\pi\Sigma\right\rangle_{I=\left|1,0\right\rangle}\left|p\right\rangle\right)\right)$$

$$= g_{\pi\Sigma N}\, g^0_{\pi\Sigma}\left(\sqrt{\frac{1}{3}}\left|\pi^+\Sigma^- p\right\rangle_{I_{\pi\Sigma}=0} - \sqrt{\frac{1}{3}}\left|\pi^0\Sigma^0 p\right\rangle_{I_{\pi\Sigma}=0} + \sqrt{\frac{1}{3}}\left|\pi^-\Sigma^+ p\right\rangle_{I_{\pi\Sigma}=0}\right)$$

$$+ g_{\pi\Sigma N}\, g^1_{\pi\Sigma}\left(\sqrt{\frac{2}{3}}\left(\sqrt{\frac{1}{2}}\left|\pi^+\Sigma^0 n\right\rangle - \sqrt{\frac{1}{2}}\left|\pi^0\Sigma^+ n\right\rangle\right) - \sqrt{\frac{1}{3}}\left(\sqrt{\frac{1}{2}}\left|\pi^+\Sigma^- p\right\rangle_{I_{\pi\Sigma}=1} - \sqrt{\frac{1}{2}}\left|\pi^-\Sigma^+ p\right\rangle_{I_{\pi\Sigma}=1}\right)\right)$$

$$= g_{\pi\Sigma N}\, g^0_{\pi\Sigma}\left(-\sqrt{\frac{1}{3}}\left|\pi^0\Sigma^0 p\right\rangle_{I_{\pi\Sigma}=0}\right) + g_{\pi\Sigma N}\, g^1_{\pi\Sigma}\left(\sqrt{\frac{1}{3}}\left|\pi^+\Sigma^0 n\right\rangle - \sqrt{\frac{1}{3}}\left|\pi^0\Sigma^+ n\right\rangle\right)$$

$$+ g_{\pi\Sigma N}\left(g^0_{\pi\Sigma}\sqrt{\frac{2}{6}}\left|\pi^+\Sigma^- p\right\rangle_{I_{\pi\Sigma}=0} - g^1_{\pi\Sigma}\sqrt{\frac{1}{6}}\left|\pi^+\Sigma^- p\right\rangle_{I_{\pi\Sigma}=1}\right) + g_{\pi\Sigma N}\left(g^0_{\pi\Sigma}\sqrt{\frac{2}{6}}\left|\pi^-\Sigma^+ p\right\rangle_{I_{\pi\Sigma}=0} + g^1_{\pi\Sigma}\sqrt{\frac{1}{6}}\left|\pi^-\Sigma^+ p\right\rangle_{I_{\pi\Sigma}=1}\right)$$

Cross section & Decay

$K^{-}pp\text{-}\bar{K}^0pn$

$$\Gamma_{tot} = \sum \Gamma_{YN} + \sum \Gamma_{\pi YN} + \sum \Gamma_{\bar{K}NN}$$

- Λp : $g_{\Lambda N}$
- $\Sigma^0 p$: $g_{\Sigma N}$
- $\Sigma^+ n$: $\sqrt{2} g_{\Sigma N}$

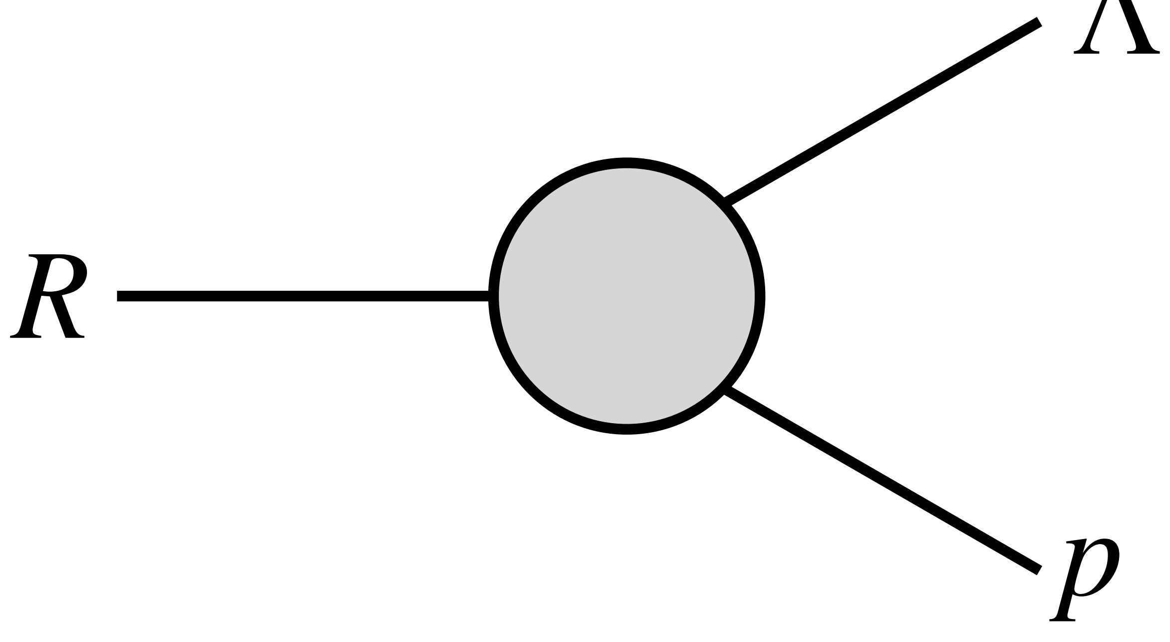
- $\pi^0 \Lambda p$: $g_{\pi\Lambda N}$
- $\pi^+ \Lambda n$: $\sqrt{2} g_{\pi\Lambda N}$
- $\pi^0 \Sigma^0 p$: $g_{\pi^0\Sigma^0 N}$
- $\pi^- \Sigma^+ p$: $g_{\pi^-\Sigma^+ N}$
- $\pi^+ \Sigma^- p$: $g_{\pi^+\Sigma^- N}$
- $\pi^0 \Sigma^+ n$: $g_{(\pi\Sigma)^\pm N}$
- $\pi^+ \Sigma^0 n$: $g_{(\pi\Sigma)^\pm N}$

- $K^- pp$: $g_{\bar{K}NN}$
- $\bar{K}^0 pn$: $g_{\bar{K}NN}$

Cross section & Decay

Decay (taken from PDG “Kinematics”)

$$d\Gamma = \frac{(2\pi)^4}{2M} \, |\mathscr{M}|^2 \, d\Phi_n \, (P;\ p_1, \ldots, p_n),$$

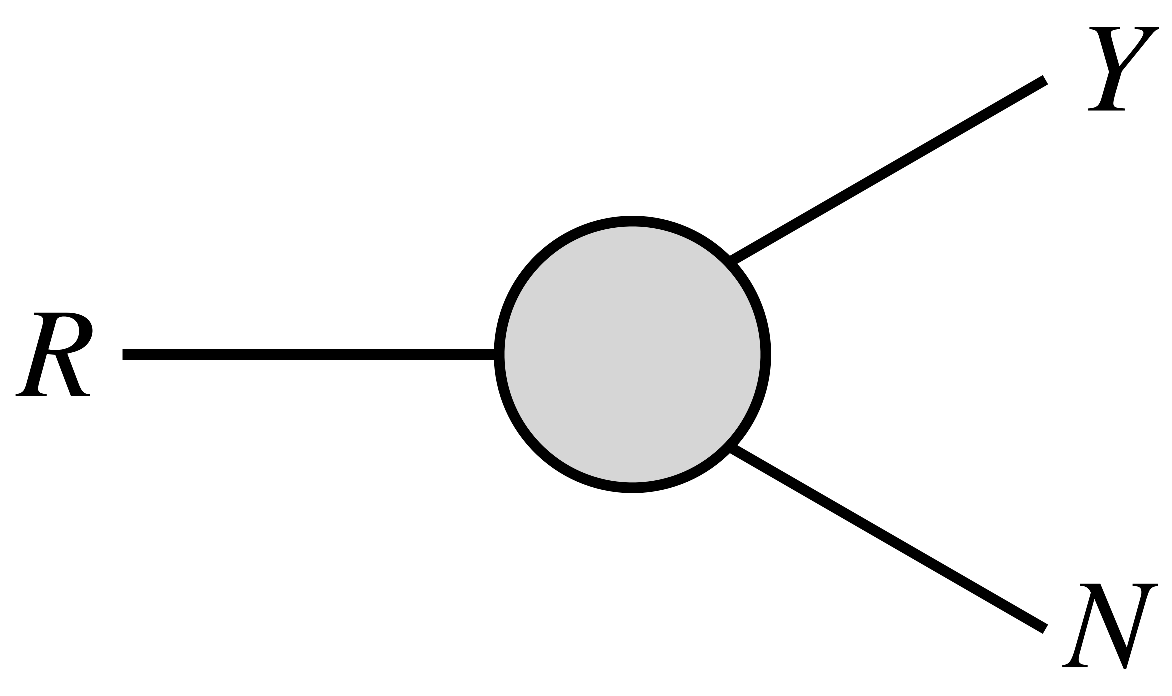


$$d\Gamma_{\Lambda p} = \frac{(2\pi)^4}{2m_{\Lambda p}} \ \mathscr{M}\ ^2 \Phi_2 \quad = \frac{(2\pi)^4}{2m_{\Lambda p}} \ \mathscr{M}\ ^2 \frac{1}{4\,(2\pi)^6} \frac{\vec{p}_p^{(\Lambda p)*}}{m_{\Lambda p}} \, d\Omega_p^{(\Lambda p)*}$$

$$\boxed{\mathscr{M} = g_{\Lambda p}^{R}}$$

$$\Gamma_{\Lambda p} = \frac{\left(g_{\Lambda p}^{R}\right)^2}{8\pi m_{\Lambda p}^2} \ \vec{p}_p^{(\Lambda p)*} \quad = \frac{\left(g_{\Lambda p}^{R}\right)^2}{8\pi m_{\Lambda p}^2} \frac{\sqrt{\left(m_{\Lambda p}^2 - (m_\Lambda + m_p)^2\right)\left(m_{\Lambda p}^2 - (m_\Lambda - m_p)^2\right)}}{2m_{\Lambda p}}$$

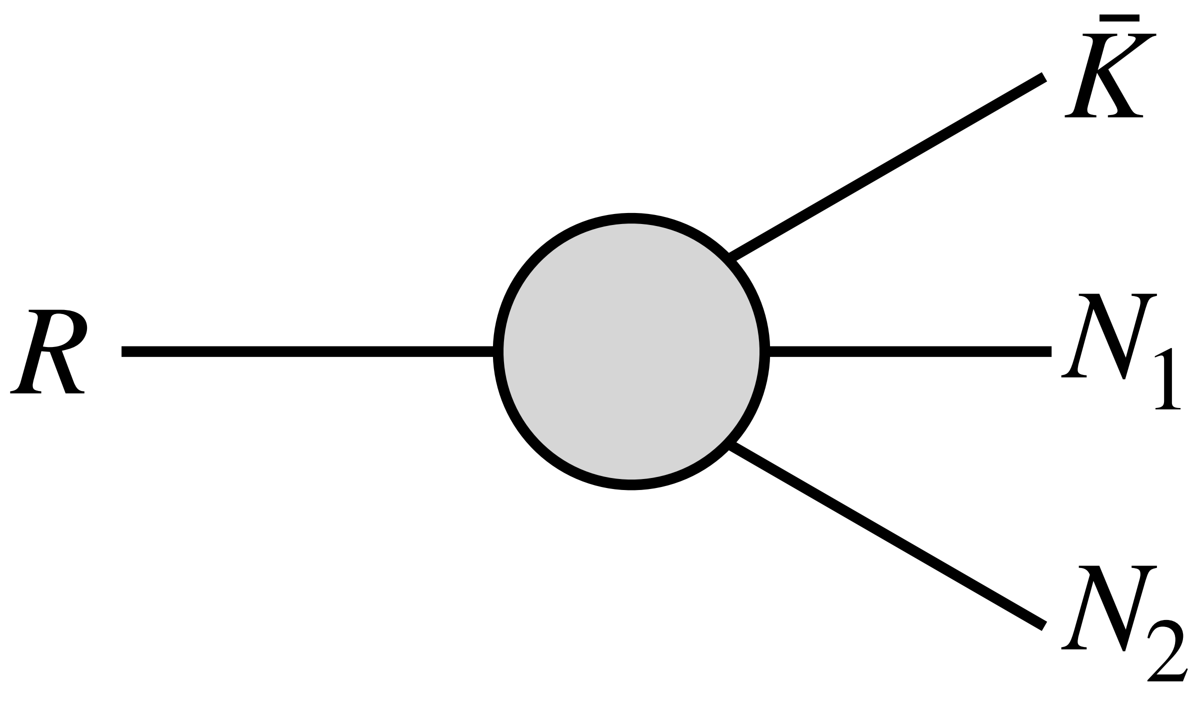
Cross section & Decay



$$\Gamma_{YN} = \frac{\left(g^R_{YN}\right)^2}{8\pi m^2_{YN}} \frac{\sqrt{\left(m^2_{YN} - (m_Y + m_N)^2\right)\left(m^2_{YN} - (m_Y - m_N)^2\right)}}{2m_{YN}}$$

$$\begin{cases} = \frac{\left(g^R_{YN}\right)^2}{8\pi m^2_{YN}} \frac{\sqrt{\left(m^2_{YN} - (m_Y + m_N)^2\right)\left(m^2_{YN} - (m_Y - m_N)^2\right)}}{2m_{YN}} \\ = i\frac{\left(g^R_{YN}\right)^2}{8\pi m^2_{YN}} \frac{\sqrt{\left((m_Y + m_N)^2 - m^2_{YN}\right)\left(m^2_{YN} - (m_Y - m_N)^2\right)}}{2m_{YN}} \end{cases}$$

Cross section & Decay



$$d\Gamma_{\bar{K}NN} = \frac{(2\pi)^4}{2m_{\bar{K}NN}} \mathscr{M}^{\ 2} \Phi_3$$

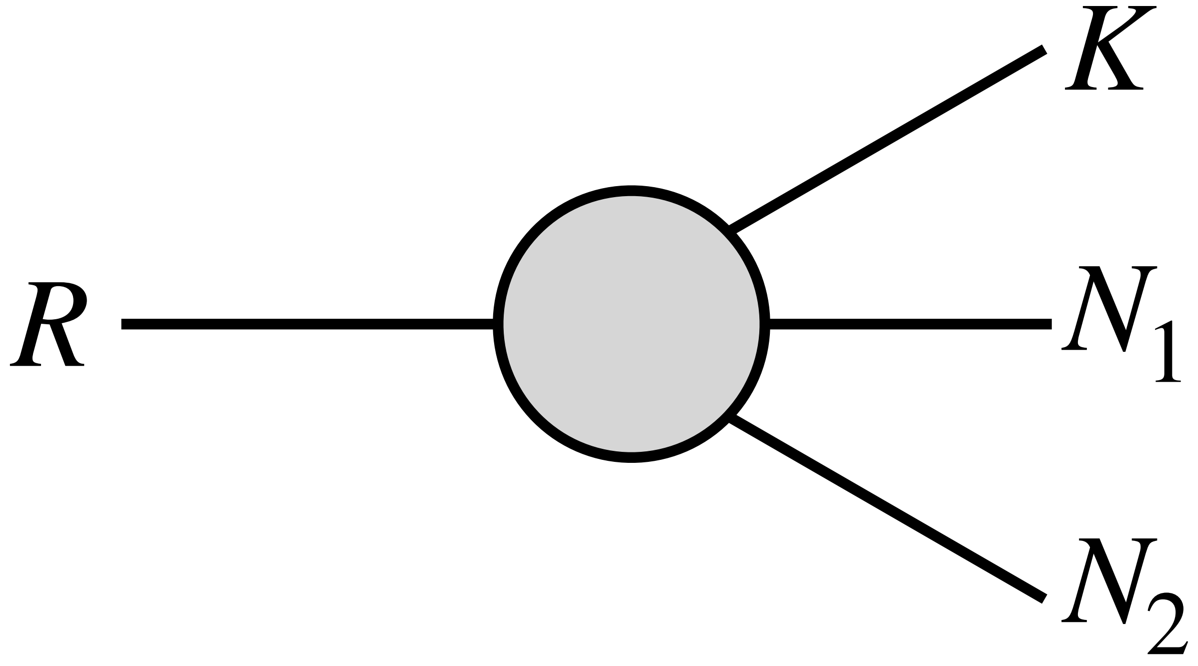
$$= \frac{(2\pi)^4}{2m_{\bar{K}NN}} \mathscr{M}^{\ 2} \left(\frac{1}{4\,(2\pi)^6} \frac{\vec{p}_{N_2}^{(\bar{K}NN)*}}{m_{\bar{K}NN}} d\Omega_{N_2}^{(\bar{K}NN)*} \right) \left(\frac{1}{4\,(2\pi)^6} \frac{\vec{p}_{N_1}^{(\bar{K}N_1)*}}{m_{\bar{K}N_1}} d\Omega_{N_1}^{(\bar{K}N_1)*} \right) \left((2\pi)^3 dm_{\bar{K}N_1}^2 \right)$$

$$\boxed{\mathscr{M} = g_{\bar{K}NN}^R}$$

$$\Gamma_{\bar{K}NN} = \frac{\left(g_{\bar{K}NN}^R\right)^2}{32\pi^3 m_{\bar{K}NN}^2} \int \left| \vec{p}_{N_2}^{(\bar{K}NN)*} \right| \left| \vec{p}_{N_1}^{(\bar{K}N_1)*} \right| dm_{\bar{K}N}$$

$$\begin{cases} = \dfrac{\left(g_{\bar{K}NN}^R\right)^2}{32\pi^3 m_{\bar{K}NN}^2} \displaystyle\int_{m_{\bar{K}}+m_N}^{m_{\bar{K}NN}-m_N} \frac{\sqrt{\left(m_{\bar{K}NN}^2 - (m_{\bar{K}N}+m_N)^2\right)\left(m_{\bar{K}NN}^2 - (m_{\bar{K}N}-m_N)^2\right)}}{2m_{\bar{K}NN}} \frac{\sqrt{\left(m_{\bar{K}N}^2 - (m_{\bar{K}}+m_N)^2\right)\left(m_{\bar{K}N}^2 - (m_{\bar{K}}-m_N)^2\right)}}{2m_{\bar{K}N_1}} dm_{\bar{K}N} \text{ (above the } m_{\bar{K}} + 2m_N \text{ threshold)} \\ = -\dfrac{\left(g_{\bar{K}NN}^R\right)^2}{32\pi^3 m_{\bar{K}NN}^2} \displaystyle\int_{m_{\bar{K}NN}-m_N}^{m_{\bar{K}}+m_N} \frac{\sqrt{\left((m_{\bar{K}N}+m_N)^2 - m_{\bar{K}NN}^2\right)\left(m_{\bar{K}NN}^2 - (m_{\bar{K}N}-m_N)^2\right)}}{2m_{\bar{K}NN}} \frac{\sqrt{\left((m_{\bar{K}}+m_N)^2 - m_{\bar{K}N}^2\right)\left(m_{\bar{K}N}^2 - (m_{\bar{K}}-m_N)^2\right)}}{2m_{\bar{K}N_1}} dm_{\bar{K}N} \text{ (below the } m_{\bar{K}} + 2m_N \text{ threshold)} \end{cases}$$

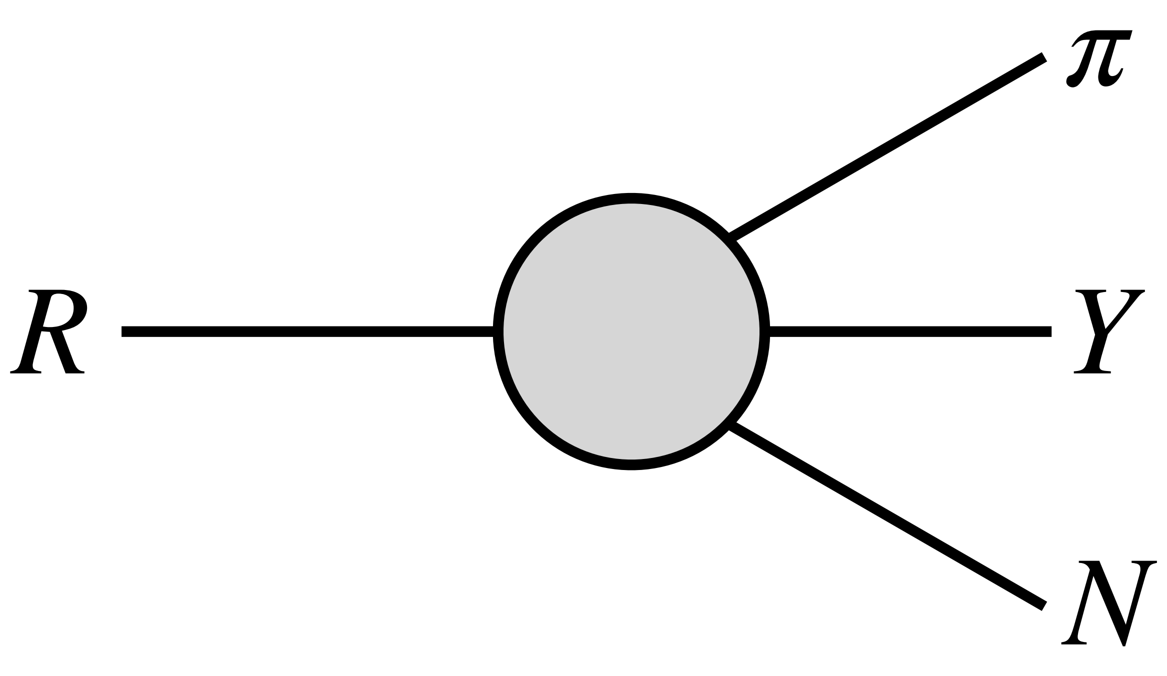
Cross section & Decay



$$\mathscr{M} = g^{R}_{\bar{K}NN}\left(g^{NR}_{\bar{K}N} + \frac{g^{Y^*}_{\bar{K}N}}{M^2_{Y^*} - m^2_{\bar{K}N} - iM_{Y^*}\Gamma^{Y^*}_{tot}} \right)$$

$$\Gamma_{\bar{K}NN} = \frac{\left(g^{R}_{\bar{K}NN}\right)^2}{32\pi^3 m^2_{\bar{K}NN}} \int \left| \vec{p}^{\,(\bar{K}NN)^*}_{N_2} \right| \left| \vec{p}^{\,(\bar{K}N_1)^*}_{N_1} \right| \left(\left| g^{NR}_{\bar{K}N} \right|^2 + \left| \frac{g^{Y^*}_{\bar{K}N}}{M^2_{Y^*} - m^2_{\bar{K}N} - iM_{Y^*}\Gamma^{Y^*}_{tot}} \right|^2 + 2\mathrm{Re}\left(\frac{g^{NR}_{\bar{K}N} \cdot g^{Y^*}_{\bar{K}N}}{M^2_{Y^*} - m^2_{\bar{K}N} - iM_{Y^*}\Gamma^{Y^*}_{tot}} \right) \right) dm_{\bar{K}N}$$

Cross section & Decay



$$d\Gamma_{\pi YN} = \frac{(2\pi)^4}{2m_{\pi YN}} \; \mathscr{M}^{\;2}\,\Phi_3$$

$$= \frac{(2\pi)^4}{2m_{\pi YN}} \; \mathscr{M}^{\;2}\left(\frac{1}{4\,(2\pi)^6}\frac{\vec{p}_N^{(\pi YN)*}}{m_{\pi YN}}\,d\Omega_N^{(\pi YN)*}\right)\left(\frac{1}{4\,(2\pi)^6}\frac{\vec{p}_Y^{(\pi Y)*}}{m_{\pi Y}}\,d\Omega_Y^{(\pi Y)*}\right)\left((2\pi)^3 dm_{\pi Y}^2\right)$$

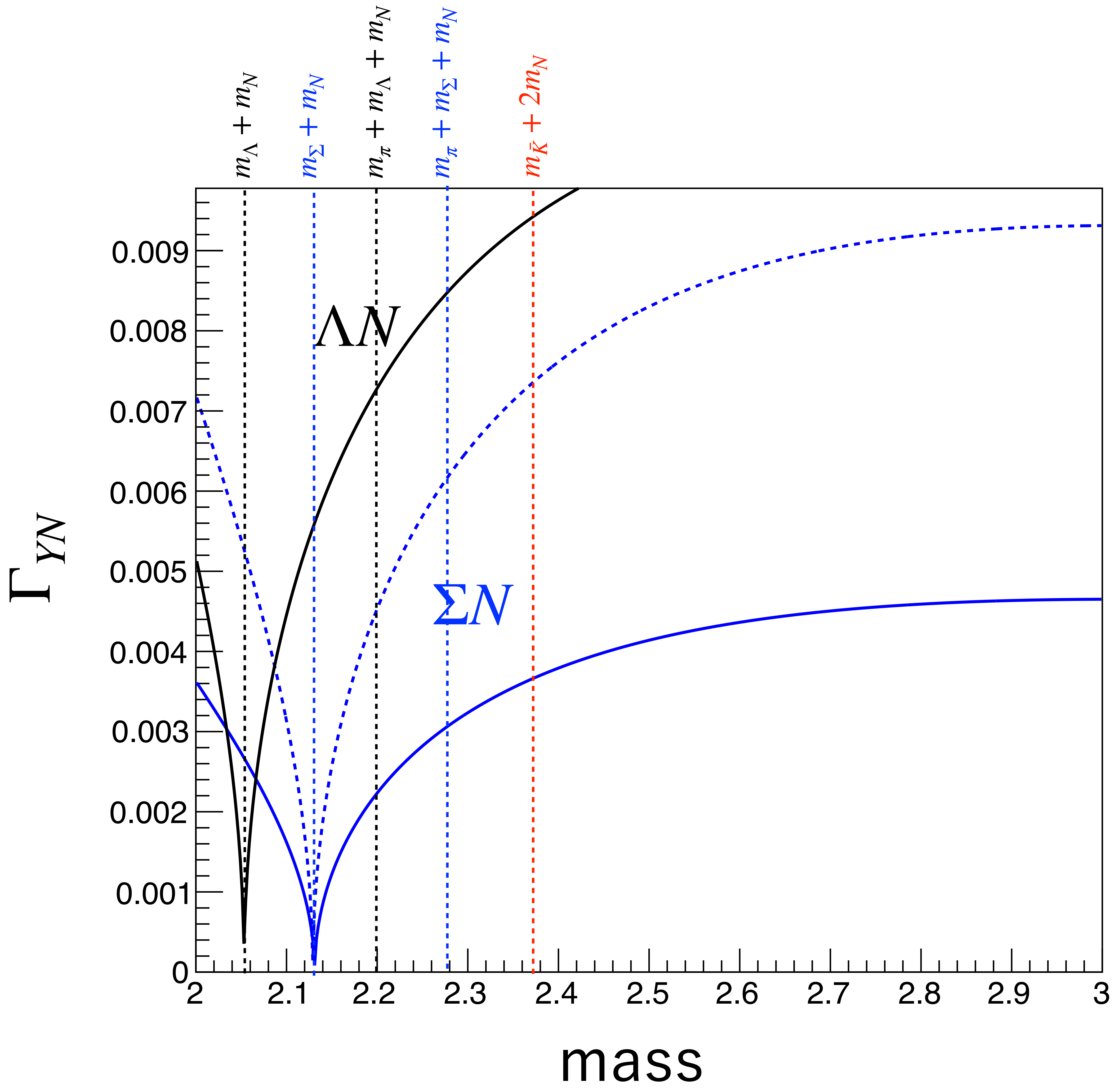
$$\boxed{\mathscr{M} = g_{\pi YN}\left(g_{\pi Y}^{NR} + \frac{g_{\pi Y}^{Y*}}{M_{Y*}^2 - m_{\pi Y}^2 - iM_{Y*}\Gamma_{tot}^{Y*}}\right)}$$

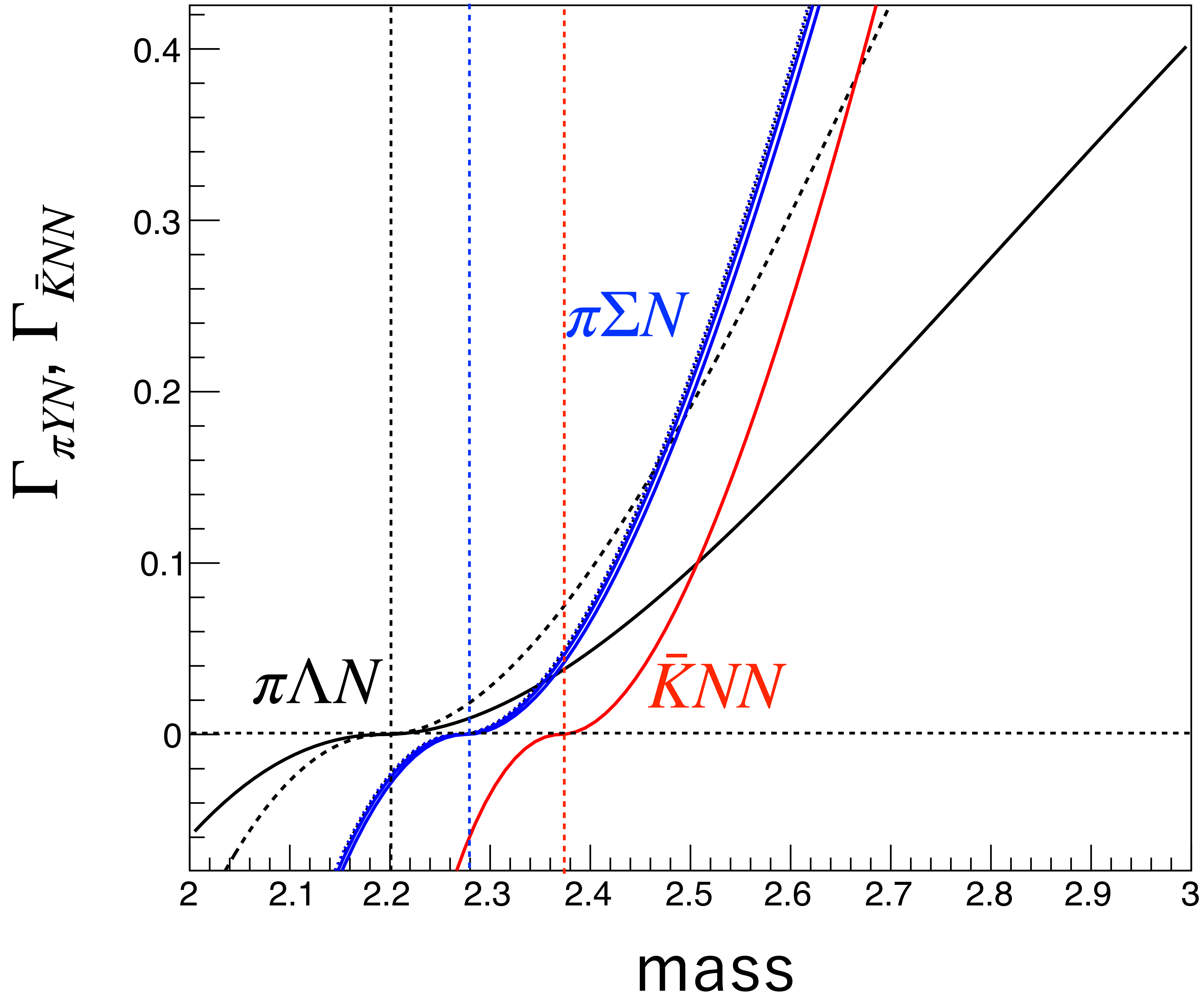
$$\Gamma_{\bar{K}NN} = \frac{g_{\bar{K}NN}^2}{32\pi^3 m_{\bar{K}NN}^2}\int \left|\vec{p}_{N_2}^{(\bar{K}NN)*}\right| \left|\vec{p}_{N_1}^{(\bar{K}N_1)*}\right| dm_{\bar{K}N}$$

$$= \frac{g_{\bar{K}NN}^2}{32\pi^3 m_{\bar{K}NN}^2}\int_{m_{\bar{K}}+m_N}^{m_{\bar{K}NN}-m_N} \frac{\sqrt{\left(m_{\bar{K}NN}^2-(m_{\bar{K}N}+m_N)^2\right)\left(m_{\bar{K}NN}^2-(m_{\bar{K}N}-m_N)^2\right)}}{2m_{\bar{K}NN}} \frac{\sqrt{\left(m_{\bar{K}N}^2-(m_{\bar{K}}+m_N)^2\right)\left(m_{\bar{K}N}^2-(m_{\bar{K}}-m_N)^2\right)}}{2m_{\bar{K}N_1}} dm_{\bar{K}N}$$
(above the $m_{\bar{K}} + 2m_N$ threshold)

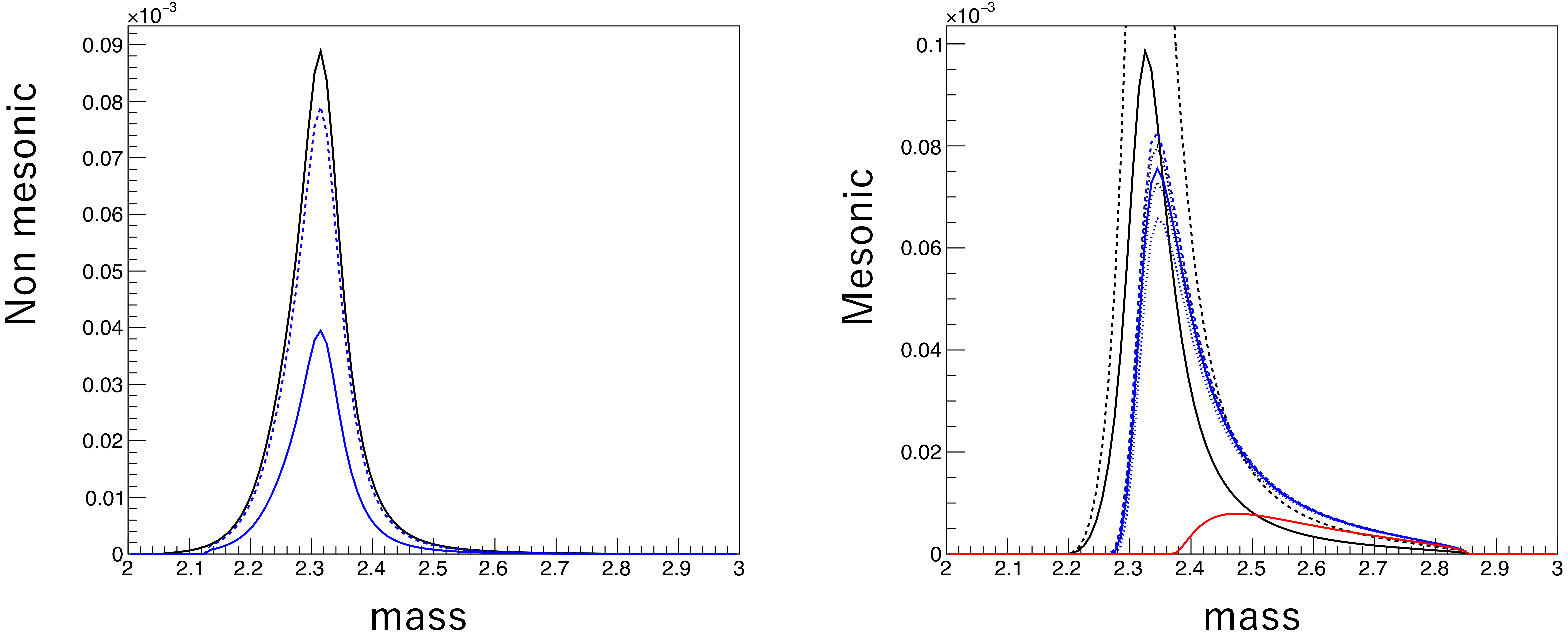
$$= -\frac{g_{\bar{K}NN}^2}{32\pi^3 m_{\bar{K}NN}^2}\int_{m_{\bar{K}NN}-m_N}^{m_{\bar{K}}+m_N} \frac{\sqrt{\left((m_{\bar{K}N}+m_N)^2-m_{\bar{K}NN}^2\right)\left(m_{\bar{K}NN}^2-(m_{\bar{K}N}-m_N)^2\right)}}{2m_{\bar{K}NN}} \frac{\sqrt{\left((m_{\bar{K}}+m_N)^2-m_{\bar{K}N}^2\right)\left(m_{\bar{K}N}^2-(m_{\bar{K}}-m_N)^2\right)}}{2m_{\bar{K}N_1}} dm_{\bar{K}N}$$
(below the $m_{\bar{K}} + 2m_N$ threshold)

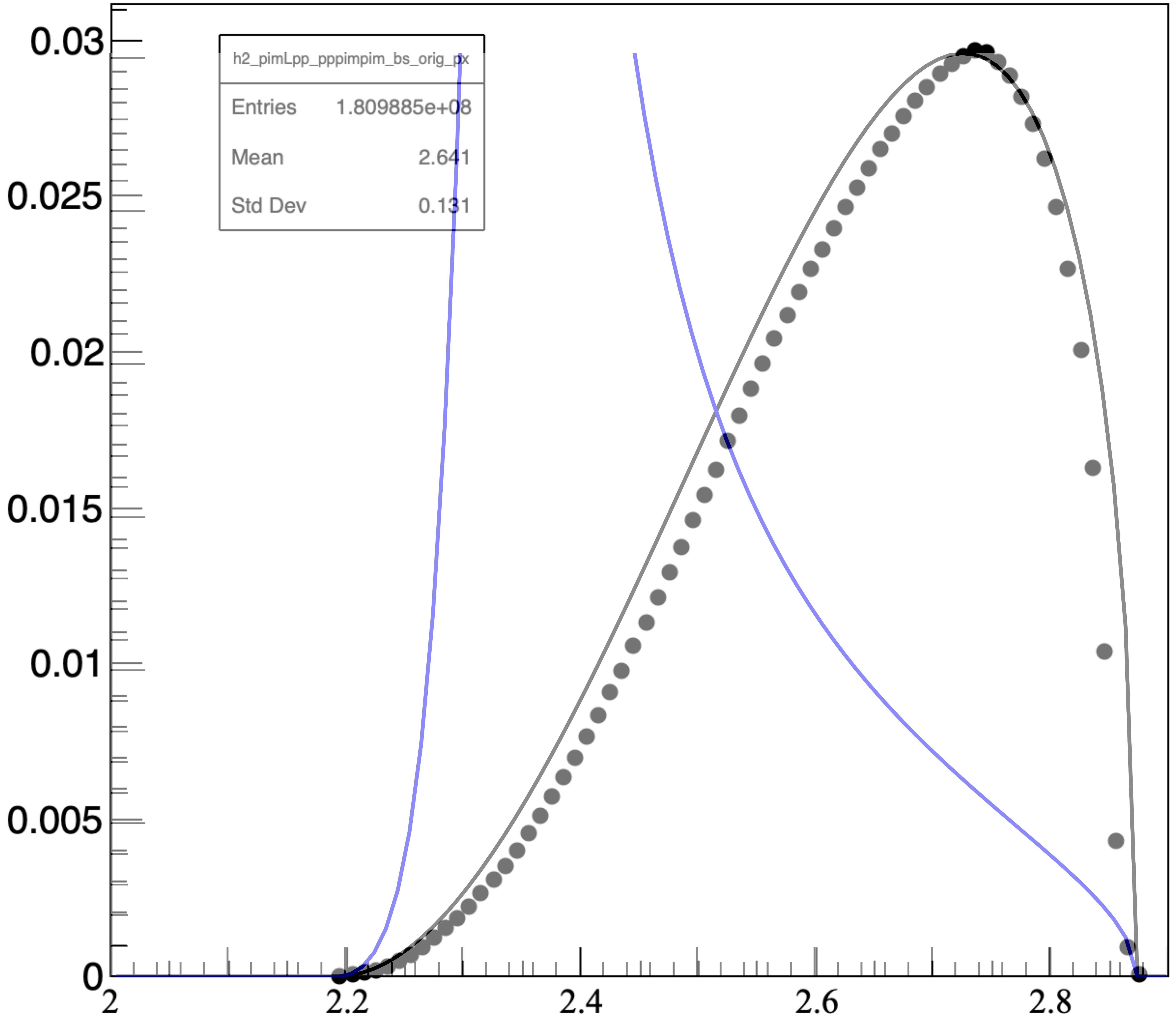
Cross section & Decay



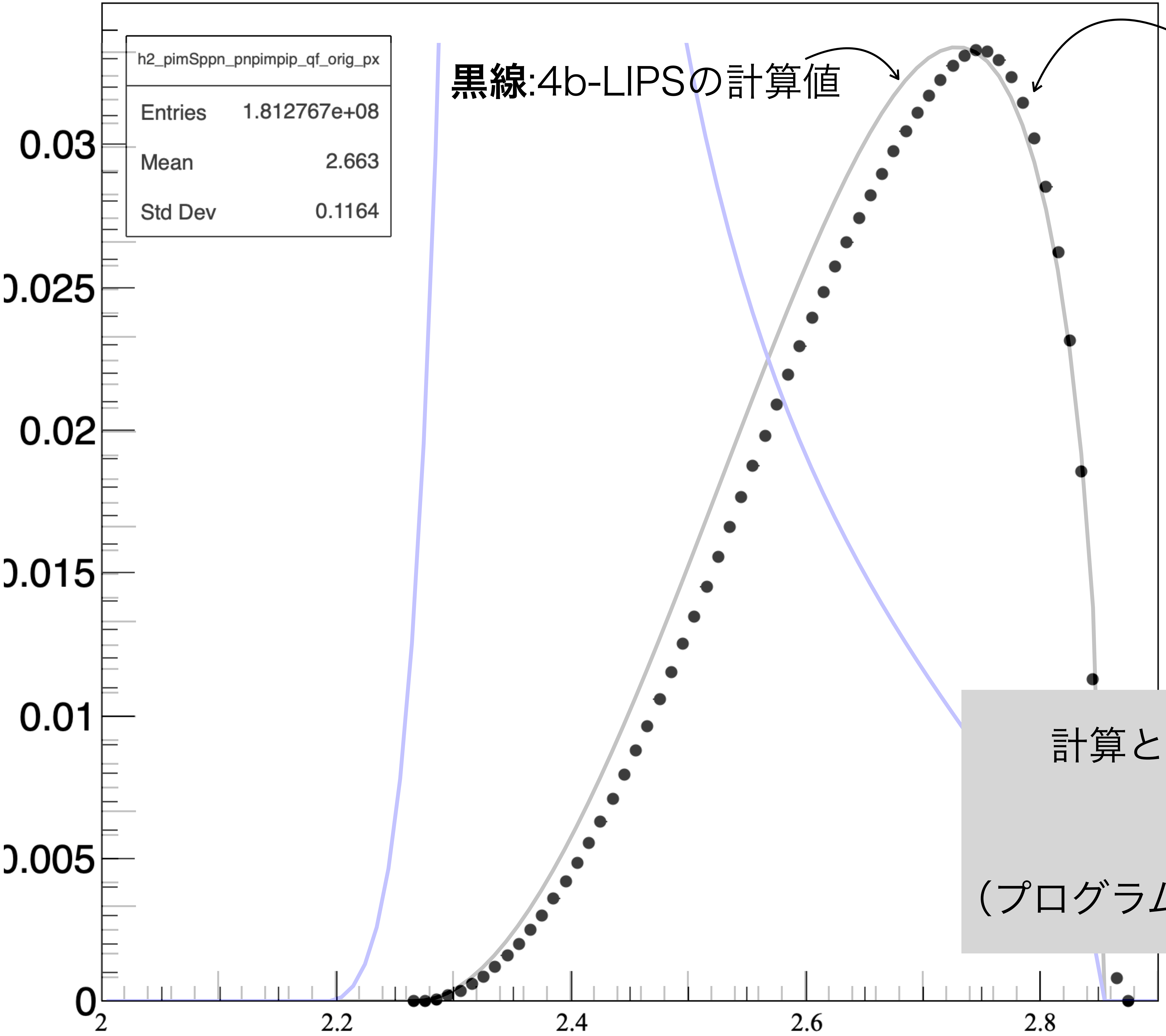


Cross section & Decay





h2_pimLpp_pppimpim_bs_orig_px
Entries 1.809885e+08
Mean 2.641
Std Dev 0.131
0.03
0.025
0.02
0.015
0.01
0.005
0
2
2.2
2.4
2.6
2.8



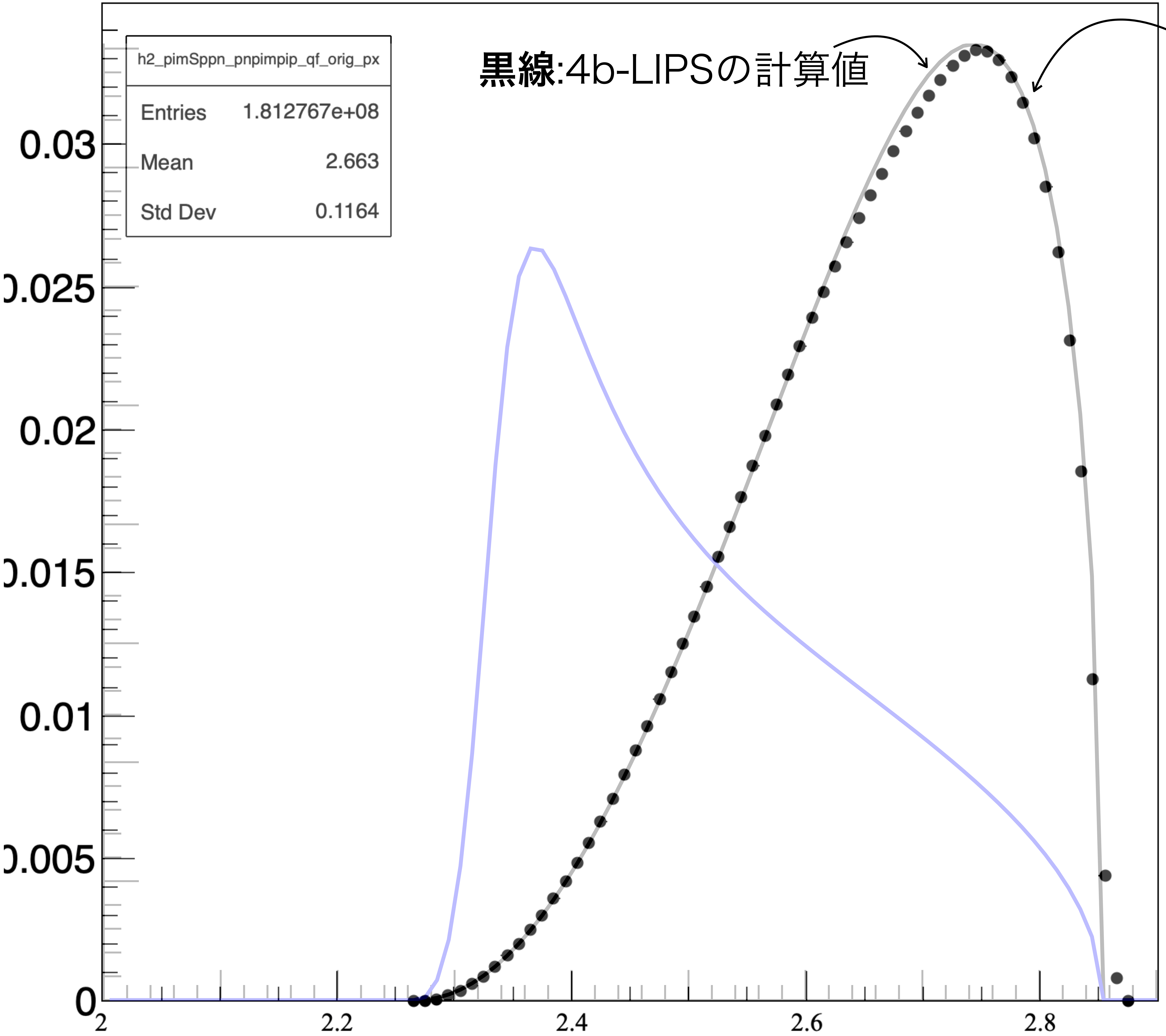
黒線:4b-LIPSの計算値

黒点:knuclで作った4b-LIPS

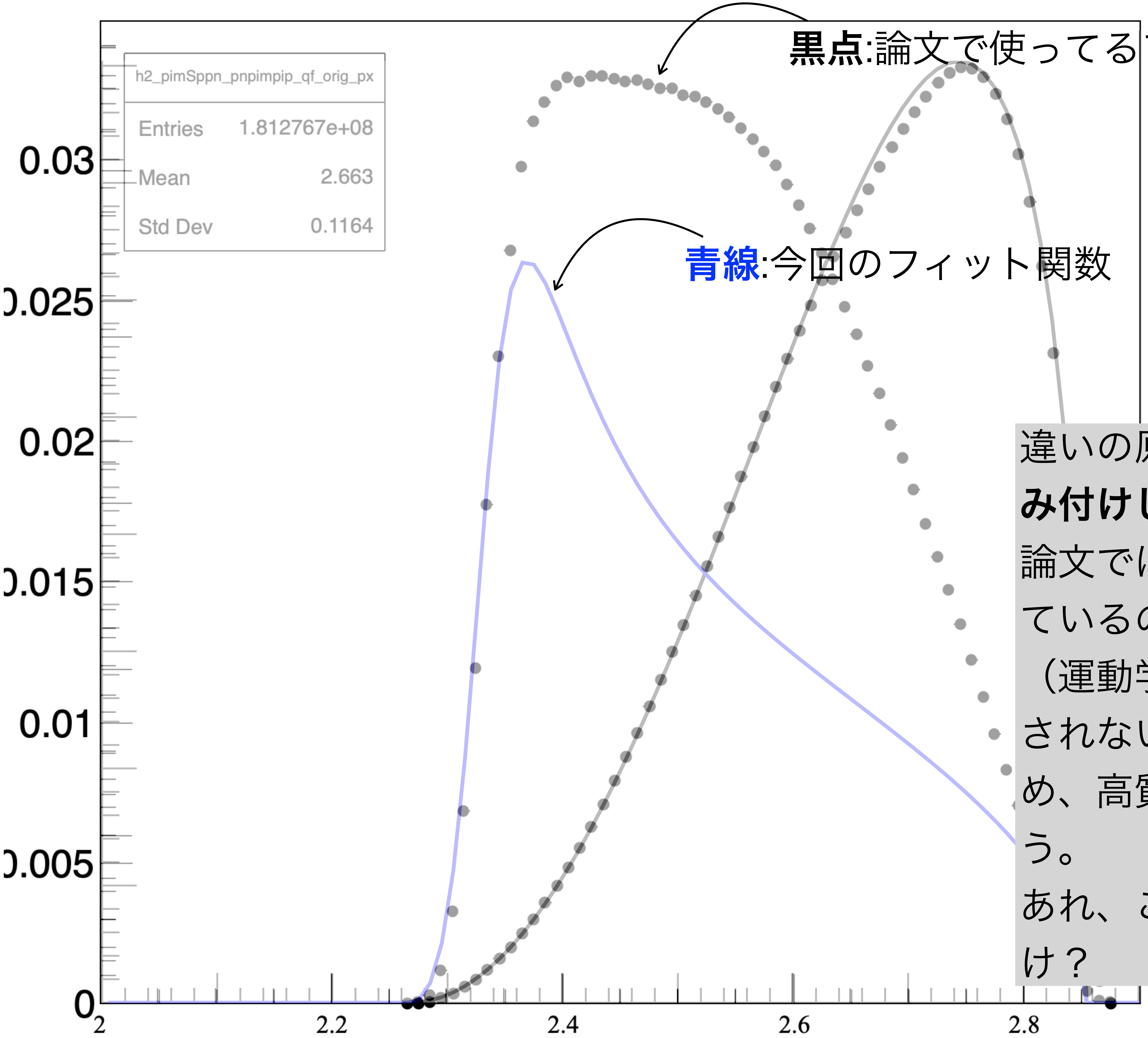
計算とknuclのLIPSがズレてた。

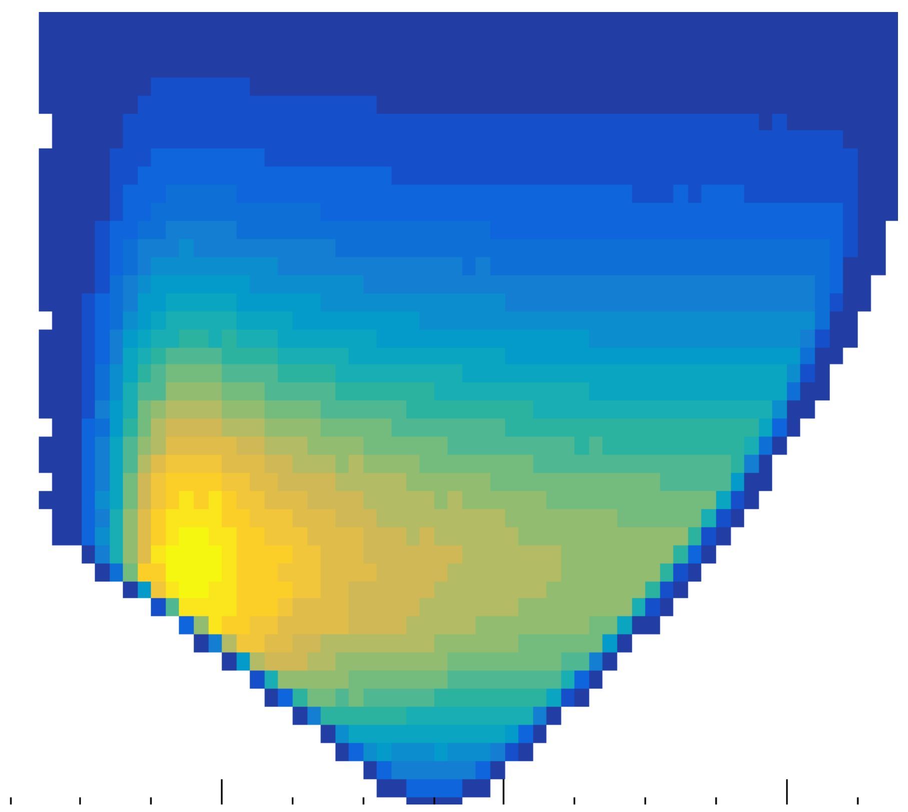
↓

（プログラム上の）計算ミスに気がつく。



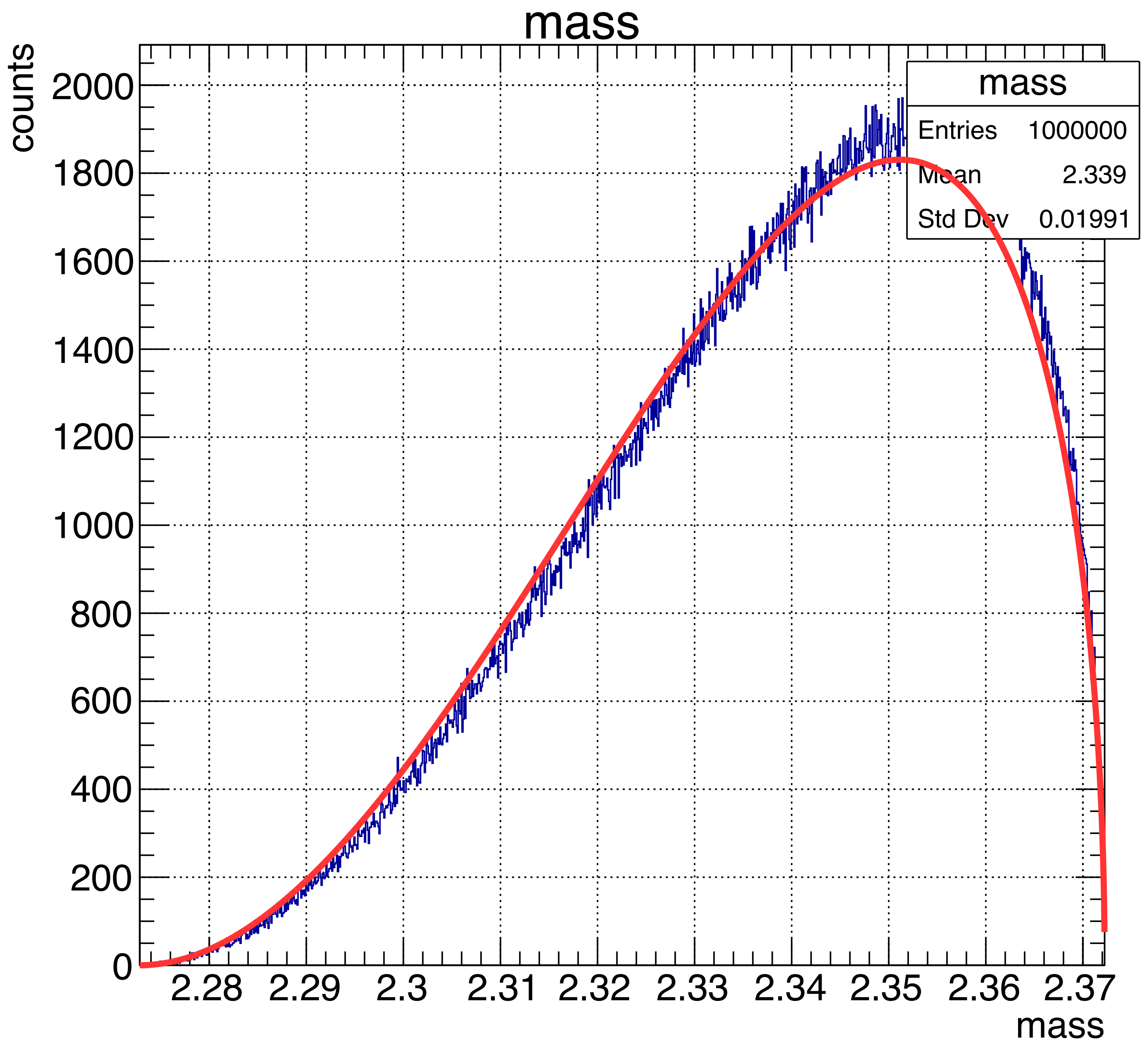
わずかな違いは、knucl
で入れてるビーム運動量
広がりによるもの



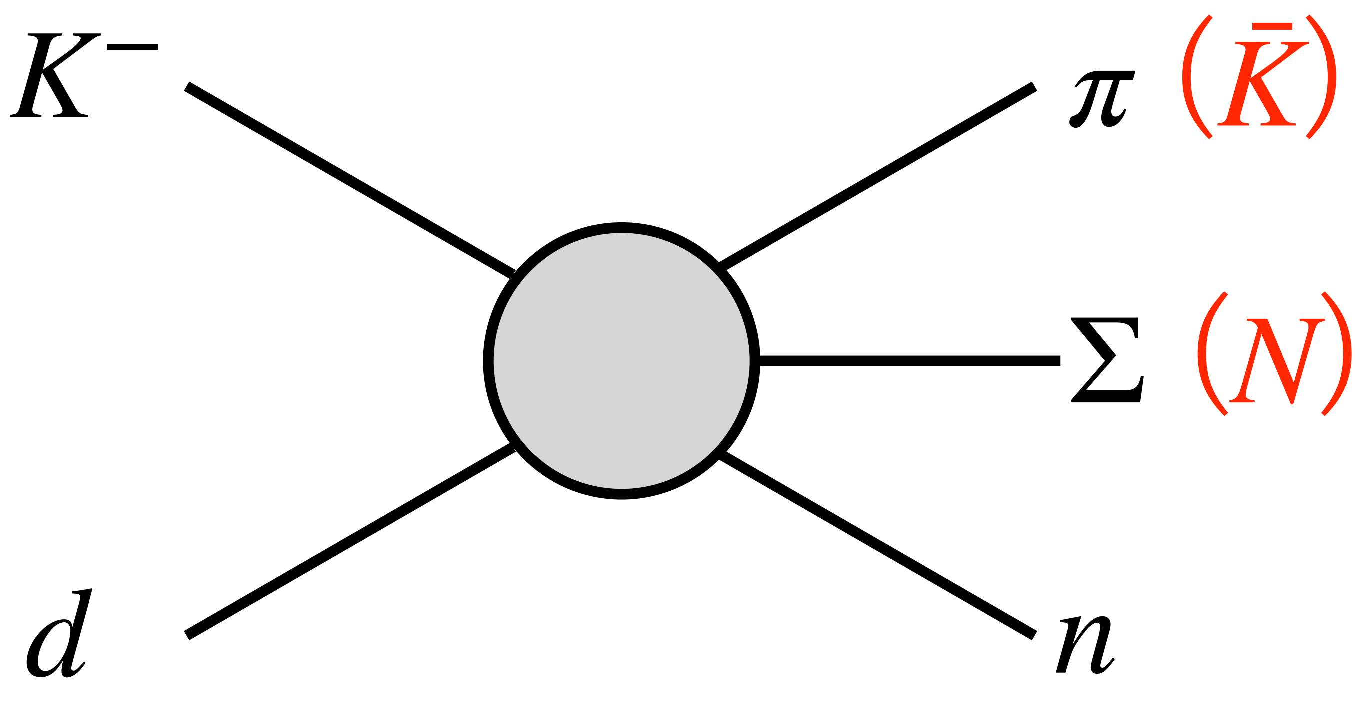


違いの原因は、「**運動量移行で重み付けしている**」こと。
論文では運動量移行で重みをつけているので、低い運動量移行領域（運動学的に高い質量領域しか許されない領域）の強度が強いため、高質量側が強調されてしまう。
あれ、この重み付け正しいんだっけ？

$p_K = 0.1$ GeV/c


mass
counts
mass
Entries 1000000
Mean 2.339
Std Dev 0.01991
2000
1800
1600
1400
1200
1000
800
600
400
200
0
2.28 2.29 2.3 2.31 2.32 2.33 2.34 2.35 2.36 2.37
mass

Cross section & Decay



$$\frac{d\sigma_{\pi\Sigma n}}{dm_{\pi\Sigma}\, d\cos\theta_n^*} = \frac{1}{16(2\pi)^3}\frac{1}{p_{K^-}^* s}\left|p_n^*\right|\left|p_\Sigma^{(\pi\Sigma)*}\right| \mathscr{M}^{2}$$

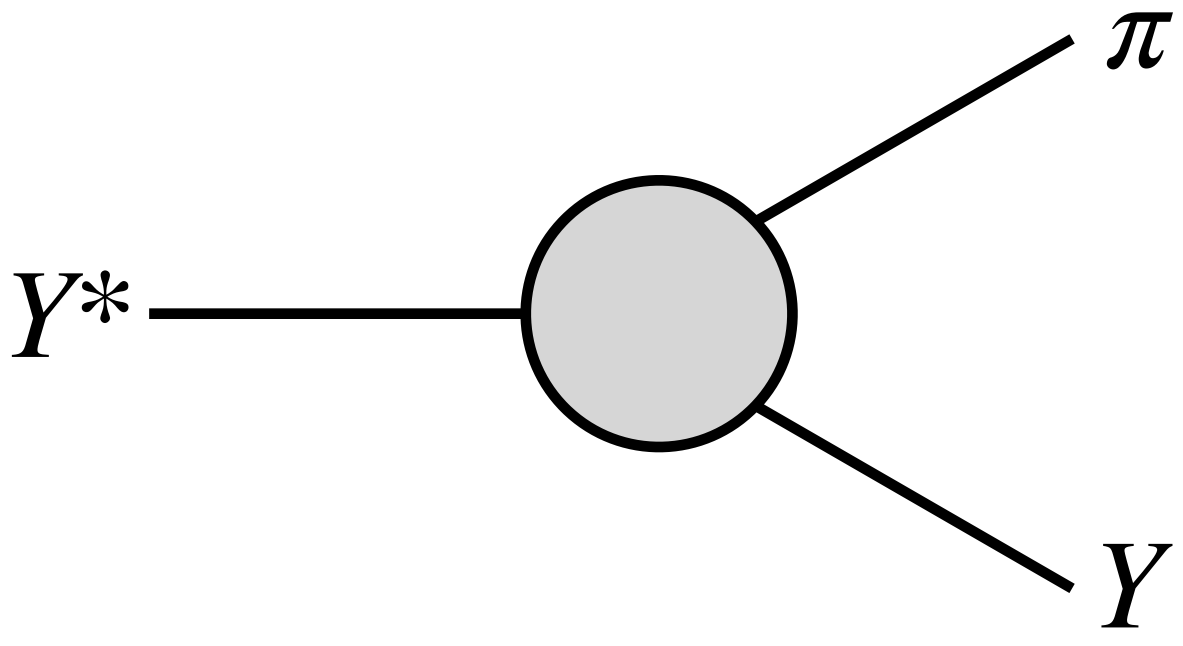
$$\mathscr{M} = \frac{g_{\pi Y}^{Y*}}{M_{Y*}^2 - m_{\pi\Sigma}^2 - iM_{Y*}\Gamma_{tot}^{Y*}} \cdot \mathscr{A}\left(\cos\theta_n^*\right)$$

$$\frac{d\sigma_{\pi\Sigma n}}{dm_{\pi\Sigma}\, d\cos\theta_n^*} = \frac{1}{16(2\pi)^3}\frac{1}{p_{K^-}^* s}\left|p_n^*\right|\left|p_\Sigma^{(\pi\Sigma)*}\right|\left|\frac{g_{\pi\Sigma}^{Y*}}{M_{Y*}^2 - m_{\pi\Sigma}^2 - iM_{Y*}\Gamma_{tot}^{Y*}}\right|^2\left|\mathscr{A}\left(\cos\theta_n^*\right)\right|^2$$

$$\begin{cases} \dfrac{\left(g_{\pi\Sigma}^{Y*}\right)^2}{\left(M_{Y*}^2 - m_{\pi\Sigma}^2 + M_{Y*}\Gamma_{KN}\right)^2 + M_{Y*}^2\Gamma_{\pi\Sigma}^2}, & \text{(below the } m_{\bar{K}} + m_N \text{ threshold)} \\ \dfrac{\left(g_{\pi\Sigma}^{Y*}\right)^2}{\left(M_{Y*}^2 - m_{\pi\Sigma}^2\right)^2 + M_{Y*}^2\left(\Gamma_{\pi\Sigma} + \Gamma_{\bar{K}N}\right)^2}, & \text{(above the } m_{\bar{K}} + m_N \text{ threshold)} \end{cases}$$

$\dfrac{d\sigma_{\bar{K}Nn}}{dm_{\bar{K}N}\, d\cos\theta_n^*}$ is as the same.

Cross section & Decay



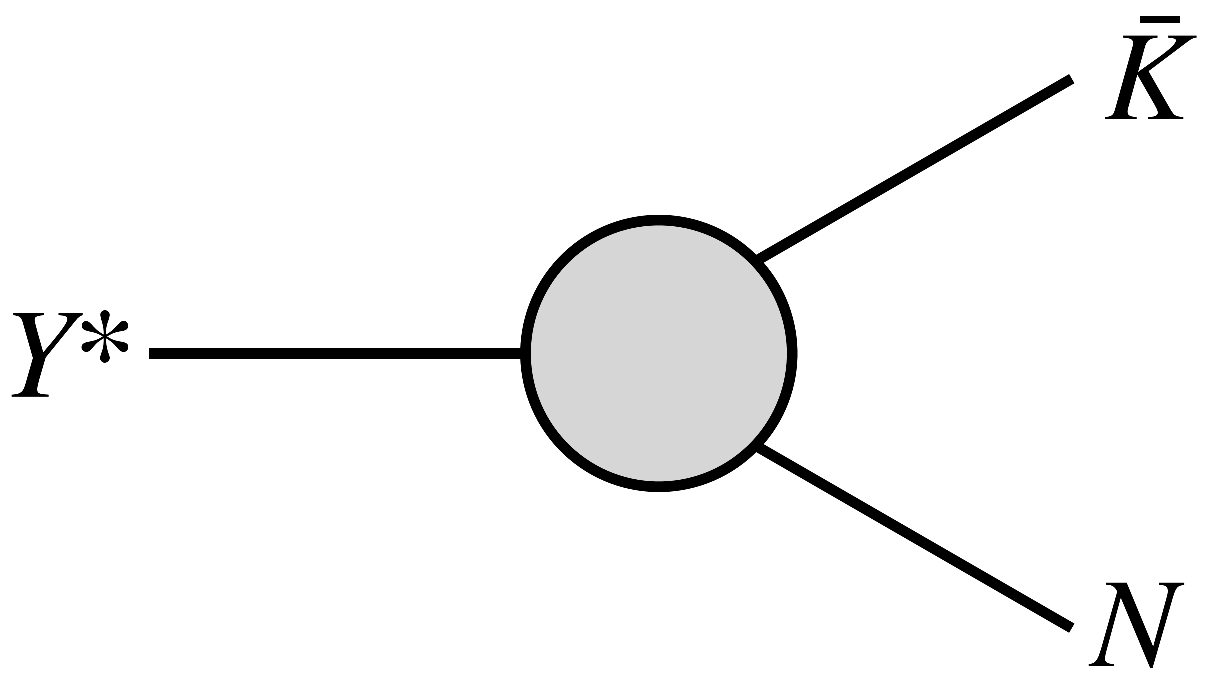
$$d\Gamma_{\pi Y}^{Y^*} = \frac{(2\pi)^4}{2m_{\pi Y}} \left|\mathscr{M}\right|^2 \Phi_2 \quad = \frac{(2\pi)^4}{2m_{\pi Y}} \left|\mathscr{M}\right|^2 \frac{1}{4\,(2\pi)^6} \frac{\left|\vec{p}_Y^{(\pi Y)^*}\right|}{m_{\pi Y}} d\Omega_Y^{(\pi Y)^*}$$

$$\boxed{\mathscr{M} = g_{\pi Y}^{Y^*}}$$

$$\Gamma_{\pi Y} = \frac{\left(g_{\pi Y}^{Y^*}\right)^2}{8\pi m_{\pi Y}^2} \frac{\sqrt{\left(m_{\pi Y}^2 - (m_\pi + m_Y)^2\right)\left(m_{\pi Y}^2 - (m_\pi - m_Y)^2\right)}}{2m_{\pi Y}} \text{ (above the } m_\pi + m_Y\text{)}$$

$$= i\frac{\left(g_{\pi Y}^{Y^*}\right)^2}{8\pi m_{\pi Y}^2} \frac{\sqrt{\left((m_\pi + m_Y)^2 - m_{\pi Y}^2\right)\left(m_{\pi Y}^2 - (m_\pi - m_Y)^2\right)}}{2m_{\pi Y}} \text{ (below the } m_\pi + m_Y\text{)}$$

Cross section & Decay



$$d\Gamma^{Y^*}_{\bar{K}N} = \frac{(2\pi)^4}{2m_{\bar{K}N}} |\mathcal{M}|^2 \Phi_2 = \frac{(2\pi)^4}{2m_{\bar{K}N}} |\mathcal{M}|^2 \frac{1}{4\,(2\pi)^6} \frac{\vec{p}_N^{\,(\bar{K}N)*}}{m_{\bar{K}N}} d\Omega_N^{(\bar{K}N)*}$$

$$\boxed{\mathcal{M} = g^{Y^*}_{\bar{K}N}}$$

$$\Gamma_{\pi Y} = \frac{\left(g^{Y^*}_{\bar{K}N}\right)^2}{8\pi m^2_{\bar{K}N}} \frac{\sqrt{\left(m^2_{\bar{K}N} - (m_{\bar{K}} + m_N)^2\right)\left(m^2_{\bar{K}N} - (m_{\bar{K}} - m_N)^2\right)}}{2m_{\bar{K}N}} \text{ (above the } m_{\bar{K}} + m_N\text{)}$$

$$= i\frac{\left(g^{Y^*}_{\bar{K}N}\right)^2}{8\pi m^2_{\bar{K}N}} \frac{\sqrt{\left((m_{\bar{K}} + m_N)^2 - m^2_{\bar{K}N}\right)\left(m^2_{\bar{K}N} - (m_{\bar{K}} - m_N)^2\right)}}{2m_{\bar{K}N}} \text{ (below the } m_{\bar{K}} + m_N\text{)}$$

Cross section & Decay

Parameters
$$\begin{cases} M_{Y*} = 1.42\ \mathrm{GeV}/c^2 \\ g_{\pi\Sigma}^{Y*} = g_{\bar{K}N}^{Y*} = 5 \end{cases}$$