Tests for primordial parity symmetry

Maresuke Shiraishi

Suwa University of Science, Japan

14. May. 2024 @Riken

In single-field slow-roll inflation based on GR

primordial scalar & tensor modes are monochromatic

nearly scale-invariant
symmetric (isotropic, parity-even, …)
almost Gaussian

parity violation is a fingerprint of the Chern-Simons couplings
$\mathscr{L} = f(\phi)F\tilde{F}, f(\phi)R\tilde{R}$

P-odd sector in 2,3,4-point correlator should be examined

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}F^2 - \frac{\alpha}{4f}\phi F\tilde{F}$$

inflaton = axion

e.g., Sorbo: 1101.1525, Barnaby +: 1210.3257

$$A_\lambda'' + k^2 A_\lambda = 0$$

unpolarized,
i.e., $A_+ = A_-$

$$+2\lambda\xi\frac{k}{\tau}A_\lambda$$

A_+ enhanced
as $e^{\pi\xi}$

$$\xi \equiv \frac{\alpha|\dot{\phi}|}{2fH} = \sqrt{\frac{\epsilon}{2}}\frac{\alpha M_p}{f}$$

★pseudoscalar

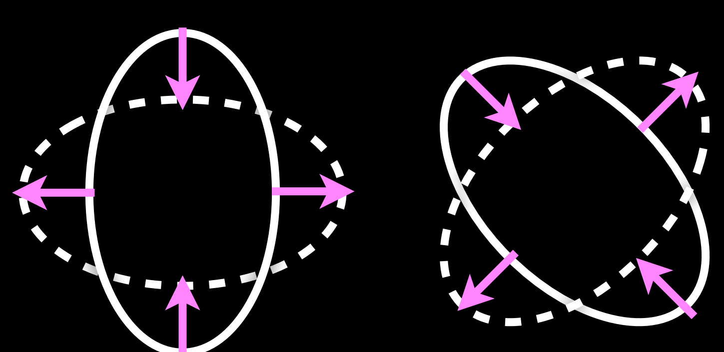
$$A + A \to \phi \to \zeta$$

$$\zeta_{\rm sou} \propto \delta\phi_{\rm sou} \propto \mathbf{E}\cdot\mathbf{B}$$

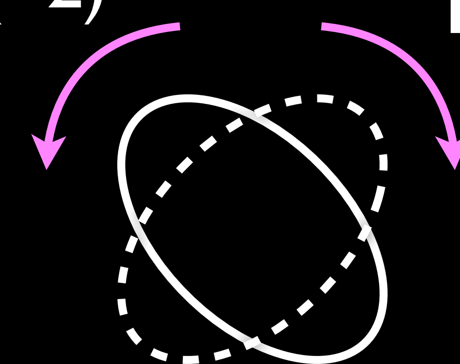
$$A + A \to h$$

★chiral GW

h+ h×



$h^{(+2)}$ >> $h^{(-2)}$



Characteristic shapes of primordial polyspectra

✤inflaton = axion $\mathscr{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}F^2 - \frac{\alpha}{4f}\phi F\tilde{F}$ [Barnaby +: 1102.4333]

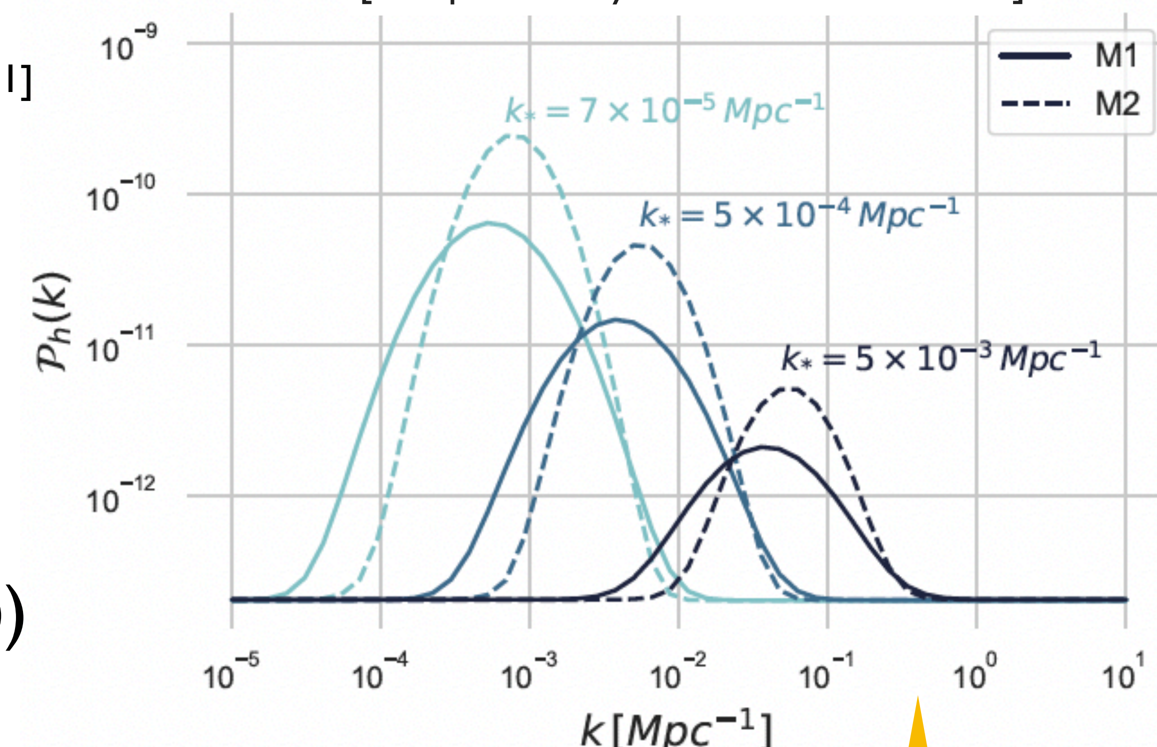
- scale-invariant spectra (since ϕ has to roll slowly)
- equilateral-type NG

✤inflaton ≠ axion [Namba, Peloso, MS, Sorbo, Unal: 1509.07521]

$$\mathscr{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi)$$
$$-\frac{1}{2}(\partial\sigma)^2 - U(\sigma) - \frac{1}{4}F^2 - \frac{\alpha}{4f}\sigma F\tilde{F}$$

- scale-dependent spectra (by tuning $U(\sigma)$)

[Campeti, Ozsoy, Obata, MS: 2203.03401]



if $U(\sigma) \propto \left(\cos\frac{\sigma}{f} + 1\right)$,
Gaussian-like bump is imprinted

✤arbitrary running coupling

$$\mathscr{L} \supset f(\phi)\left(-\frac{1}{4}F^2 + \frac{\gamma}{4}F\tilde{F}\right)$$

[Bartolo, Mattarese, Peloso, MS: 1505.02193]
[MS: 1608.00368]

- squeezed-type NG (by tuning $f(\phi)$)

✤more efficient GW production with SU(2)-gauge coupling

[Dimastrogiovanni +: 1608.04216]
[Agrawal +: 1707.03023]

Parity violation search

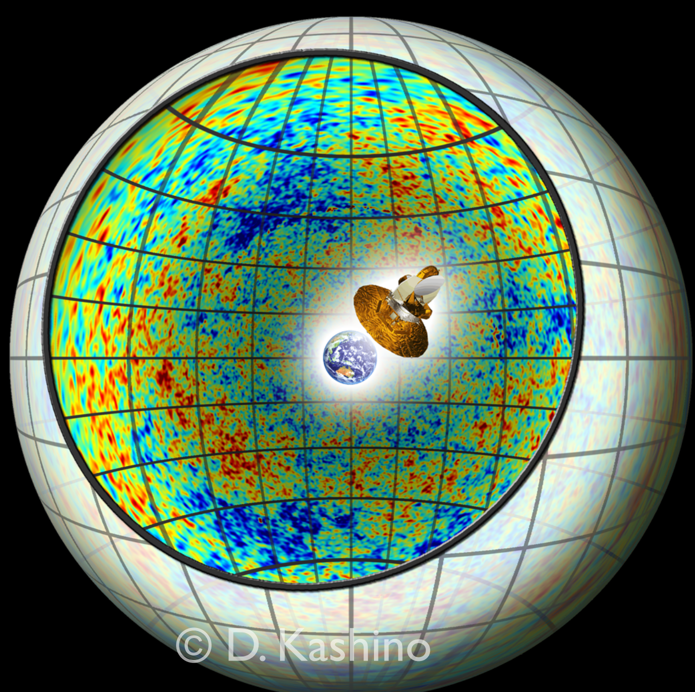
CMB

relic of the primordial fluctuations stretched by the inflationary expansion

$$T/E(n) \sim \zeta \, \Delta_{T/E}^{(s)}$$

$$T/E(n) \sim [h^{(+2)} + h^{(-2)}] \, \Delta_{T/E}^{(t)}$$

$$B(n) \sim [h^{(+2)} - h^{(-2)}] \, \Delta_{B}^{(t)}$$



$$a_{\ell m}^{X} = \int d^2n \, X(n) \, Y_{\ell m}^{*}(n)$$

$$a_{\ell m}^{T/E} \sim \zeta \, \Delta_{T/E}^{(s)}$$

$$a_{\ell m}^{T/E} \sim [h^{(+2)} + (-1)^{\ell} h^{(-2)}] \, \Delta_{T/E}^{(t)}$$

$$a_{\ell m}^{B} \sim [h^{(+2)} - (-1)^{\ell} h^{(-2)}] \, \Delta_{B}^{(t)}$$

「P-odd theorem in ℓ space」

CMB correlators including $\begin{matrix}\text{even}\\ \text{odd}\end{matrix}$ numbers of B modes from P-odd GW correlators are nonzero only for $\ell_1 + \ell_2 + \ell_3 = \begin{matrix}\text{odd}\\ \text{even}\end{matrix}$

Kamionkowski & Souradeep: 1010.4304
MS, Nitta, Yokoyama: 1107.0682

P-odd GW correlators obeys $\langle h^{(+2)} \cdots h^{(+2)} \rangle = - \langle h^{(-2)} \cdots h^{(-2)} \rangle$

Using $a_{\ell m}^T \sim h^{(+)} + (-1)^\ell h^{(-)}$, $a_{\ell m}^B \sim h^{(+)} - (-1)^\ell h^{(-)}$, induced CMB correlators are computed as

$$\langle a^T_{\ell_1 m_1} a^T_{\ell_2 m_2} a^T_{\ell_3 m_3} \rangle \sim \langle h^{(+)} h^{(+)} h^{(+)} \rangle + (-1)^{\ell_1+\ell_2+\ell_3} \langle h^{(-)} h^{(-)} h^{(-)} \rangle$$
$$\sim (1 - (-1)^{\ell_1+\ell_2+\ell_3}) \langle h^{(+)} h^{(+)} h^{(+)} \rangle$$
$$\neq 0 \text{ only if } \ell_1 + \ell_2 + \ell_3 = \text{odd}$$

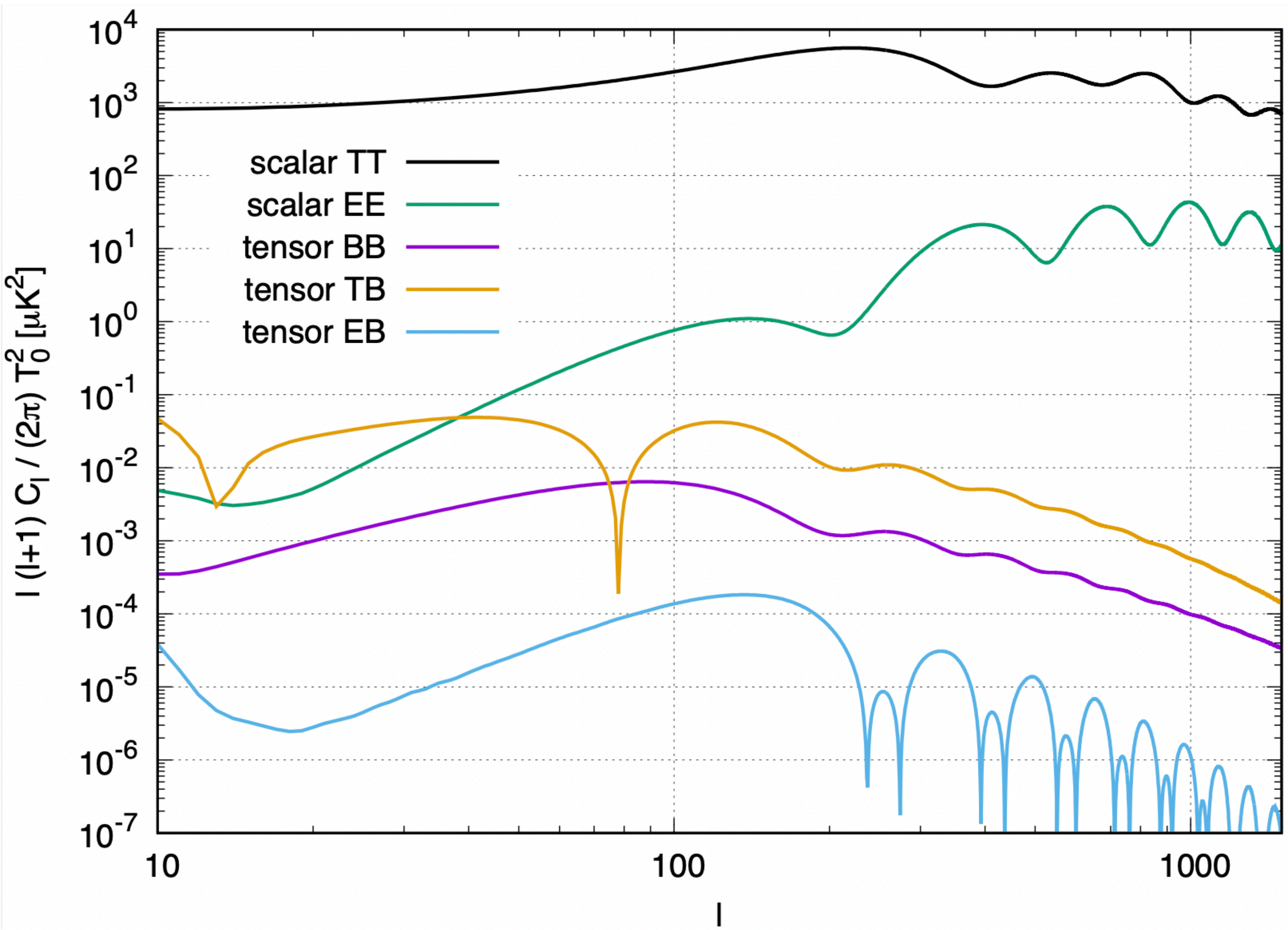
$$\langle a^B_{\ell_1 m_1} a^B_{\ell_2 m_2} a^B_{\ell_3 m_3} \rangle \sim \langle h^{(+)} h^{(+)} h^{(+)} \rangle - (-1)^{\ell_1+\ell_2+\ell_3} \langle h^{(-)} h^{(-)} h^{(-)} \rangle$$
$$\sim (1 + (-1)^{\ell_1+\ell_2+\ell_3}) \langle h^{(+)} h^{(+)} h^{(+)} \rangle$$
$$\neq 0 \text{ only if } \ell_1 + \ell_2 + \ell_3 = \text{even}$$

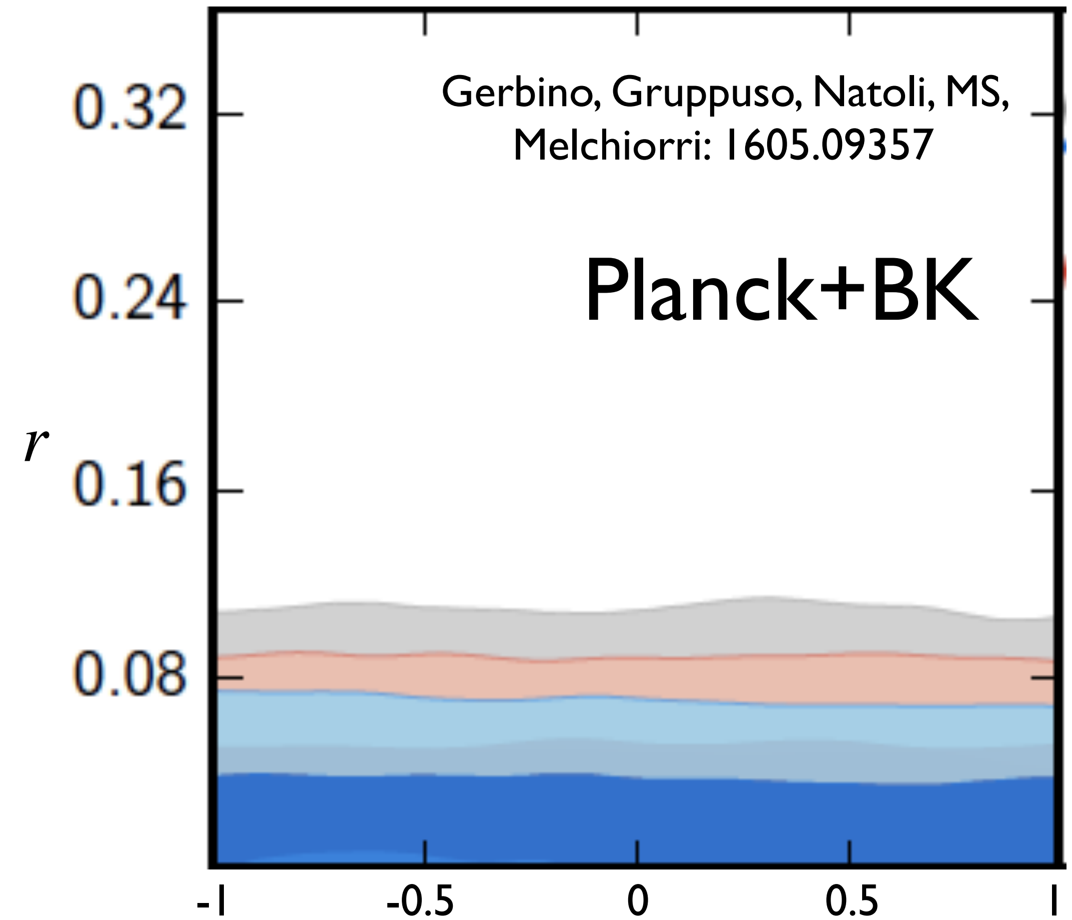
P-odd signal is extractable by this ℓ-space filtering !

Limits on scale-invariant chiral GW power

$$C_\ell^{BB} \propto P_h^{(+)} + P_h^{(-)} = r P_\zeta$$

$$C_\ell^{TB/EB} \propto P_h^{(+)} - P_h^{(-)} = r \chi P_\zeta$$



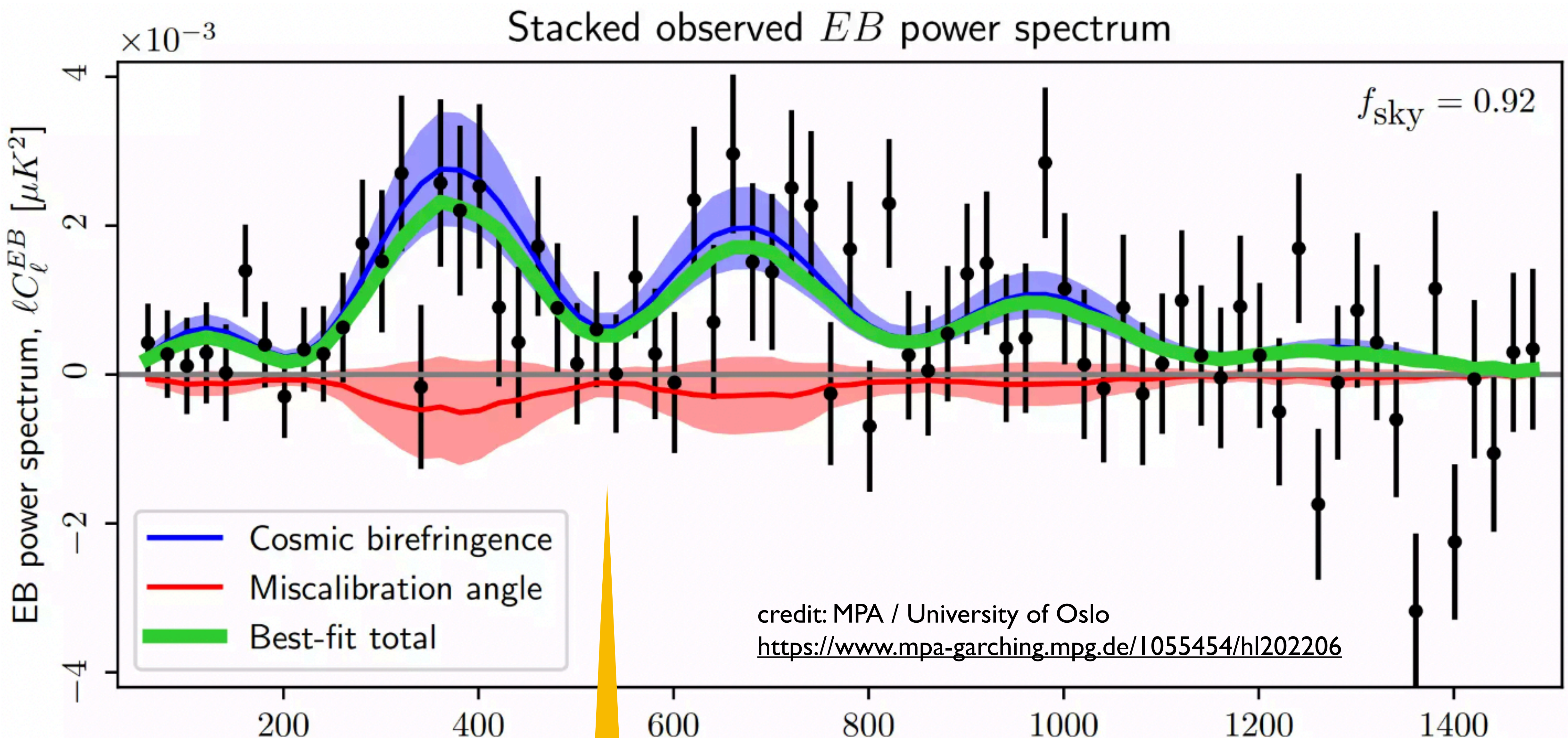


$$\chi \equiv \frac{P_h^{(+)} - P_h^{(-)}}{P_h^{(+)} + P_h^{(-)}}$$

$$\left(\frac{S}{N}\right)^2 = \sum_\ell (2\ell+1) \frac{\left(C_\ell^{TB/EB}\right)^2}{C_\ell^{TT/EE} C_\ell^{BB}}$$

unconstrained since $C_\ell^{TT/EE} \gg C_\ell^{BB}, C_\ell^{TB/EB}$

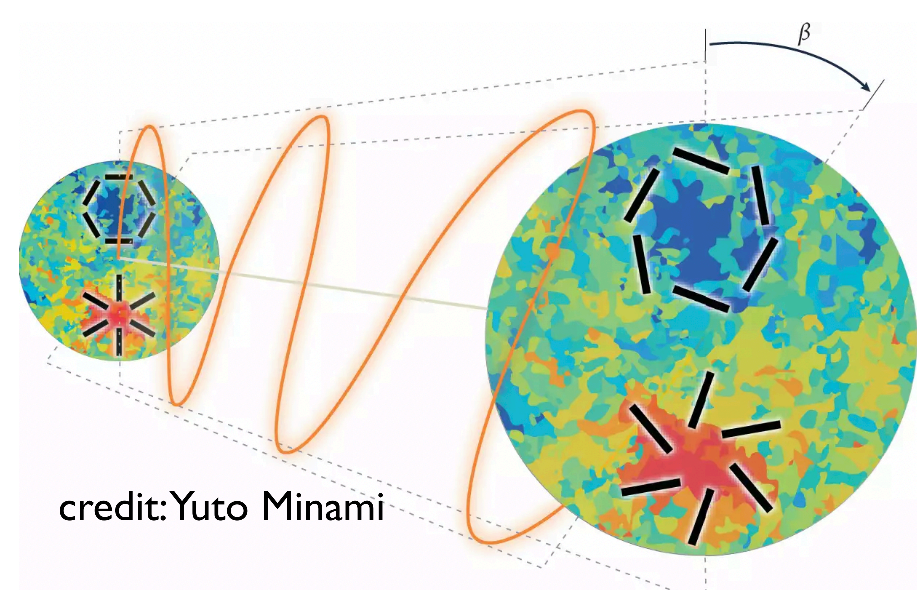
But 3σ signal in the EB power spectrum was found !



data is well fitted with the transformation of (scalar-induced) EE into EB by rotating the polarization plane (cosmic birefringence)

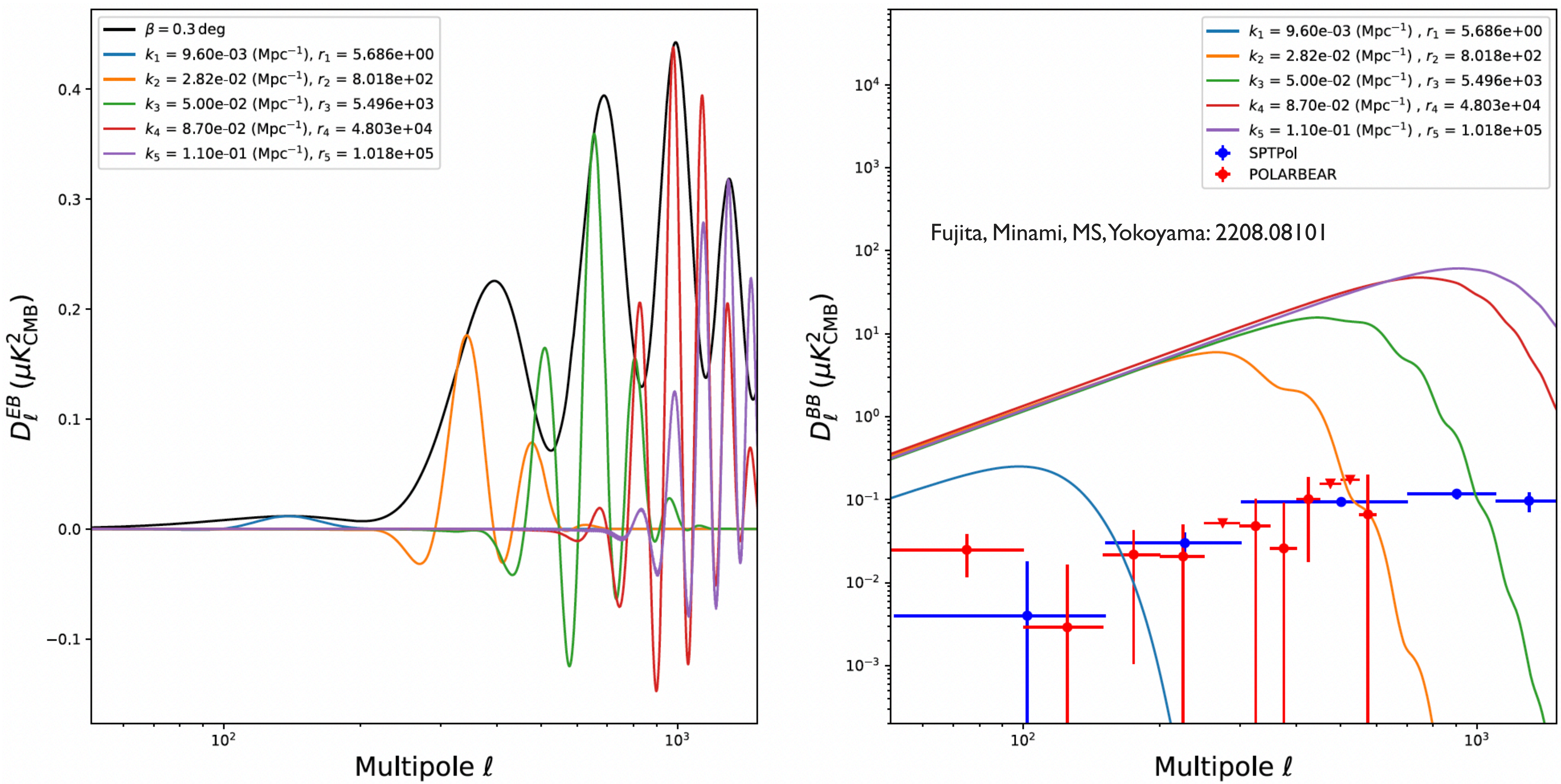
$$C^{EB}_{\ell,\rm obs} \approx \sin(4\beta) C^{EE}_{\ell,\rm scal}$$

scale-dependent chiral GW can fit ?



It seems hard to explain observed EB with chiral GWs

even if considering multiple bumps $P_h^{\lambda_1\lambda_2}(k) = P_\zeta(k) \sum_i r_i \exp\left[-\frac{1}{2\sigma^2}\ln^2\left(\frac{k}{k_i}\right)\right]\delta_{\lambda_1,+2}\delta_{\lambda_2,+2}$



When GWs with 5 bumps reproduce the shape of observed EB, simultaneously-made BB is too large to be consistent with the current data

CMB bispectra violate parity ?

Model-independent null tests using public Planck PR4 data show no significant signals

Analysis	Parity-even	Parity-odd
Fiducial $(T+E+B)$	0.8σ	1.1σ
T only	-0.4σ	-1.2σ
E only	-1.2σ	0.4σ
B only	1.7σ	1.5σ
$T+E$	-0.9σ	0.2σ
$T+B$	0.4σ	1.0σ
$E+B$	1.2σ	1.3σ
$\ell_{\rm min}=4$	1.8σ	2.3σ
$\ell_{\rm max}=375$	0.8σ	1.3σ

[Philcox & MS: 2312.12498]

Limits on scale-invariant equilateral-type chiral GW bispectrum

$$f_{\rm NL}^{ttt,\rm eq} = \lim_{k_i\to k} \frac{\langle h^{(+2)}_{\mathbf{k}_1} h^{(+2)}_{\mathbf{k}_2} h^{(+2)}_{\mathbf{k}_3}\rangle}{\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle_{f_{\rm NL}^{\rm equil}=1}}$$

$f_{\rm NL}^{ttt,\rm eq}$ / 100	P-even	P-odd	All
WMAP T [MS, Liguori, Fergusson: 1409.0265]	4 ± 15	90 ± 100	6 ± 15
Planck T + E [Planck Collaboration: 1905.05697]	11 ± 14	1 ± 18	8 ± 11
Planck T + E + B [Philcox & MS: 2312.12498]	11 ± 8	0 ± 14	9 ± 7

axion-U(1) model: $\xi = \dfrac{\alpha|\dot\phi|}{2fH} < 3.3$

axion-SU(2) model: $\dfrac{r^2}{\Omega_A} \lesssim 1000$

LiteBIRD BBB would capture $f_{\rm NL}^{ttt,\rm eq} \sim 1$

[MS: 1905.12485]

How about the (pseudo)scalar correlators?

[MS: 1608.00368]

parity transformation is $\zeta(\mathbf{k}) \to \zeta(-\mathbf{k})$, so parity-violating correlators should satisfy $\langle \zeta(\mathbf{k}_1)\cdots\zeta(\mathbf{k}_N)\rangle \neq \langle \zeta(-\mathbf{k}_1)\cdots\zeta(-\mathbf{k}_N)\rangle$

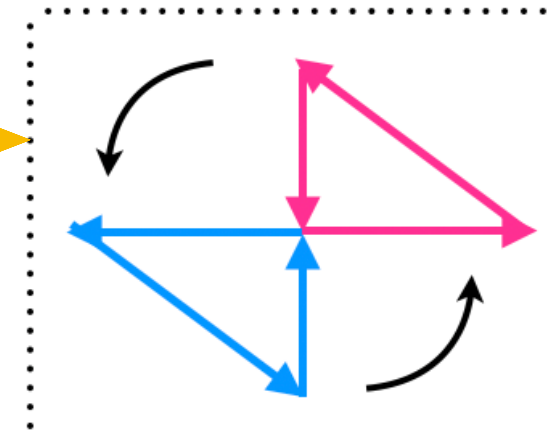
✤ In 2,3-pt correlators, isotropy (rotational symmetry) hides P-odd signal…

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = \langle\zeta(-\mathbf{k}_1)\zeta(-\mathbf{k}_2)\rangle$$

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = \langle\zeta(-\mathbf{k}_1)\zeta(-\mathbf{k}_2)\zeta(-\mathbf{k}_3)\rangle$$

☞ $\langle a^T_{\ell_1 m_1} a^T_{\ell_2 m_2} a^T_{\ell_3 m_3}\rangle = 0$ in $\ell_1 + \ell_2 + \ell_3 = \mathrm{odd}$

rotational & parity transforms are equivalent to each other



✤ In N(≧4)-pt ones, P-odd signal can be seen even if isotropic!

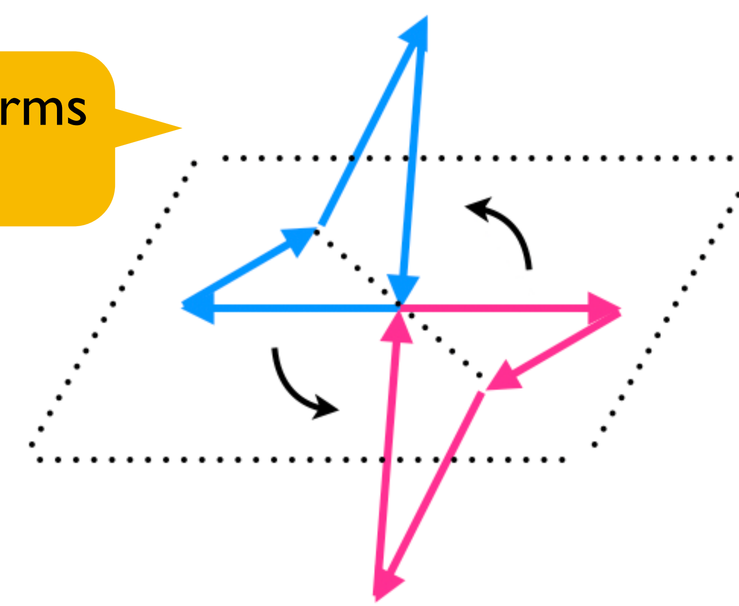
$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\zeta(\mathbf{k}_4)\rangle \quad \in \mathbb{C}$$

$$\neq \langle\zeta(-\mathbf{k}_1)\zeta(-\mathbf{k}_2)\zeta(-\mathbf{k}_3)\zeta(-\mathbf{k}_4)\rangle$$

$$=$$

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\zeta(\mathbf{k}_4)\rangle^* \text{ (reality condition } \zeta(\mathbf{x}) \in \mathbb{R})$$

rotational & parity transforms are distinguishable



☞ $\langle a^T_{\ell_1 m_1} a^T_{\ell_2 m_2} a^T_{\ell_3 m_3} a^T_{\ell_4 m_4}\rangle \neq 0$ in $\ell_1 + \ell_2 + \ell_3 + \ell_4 = \mathrm{odd}$

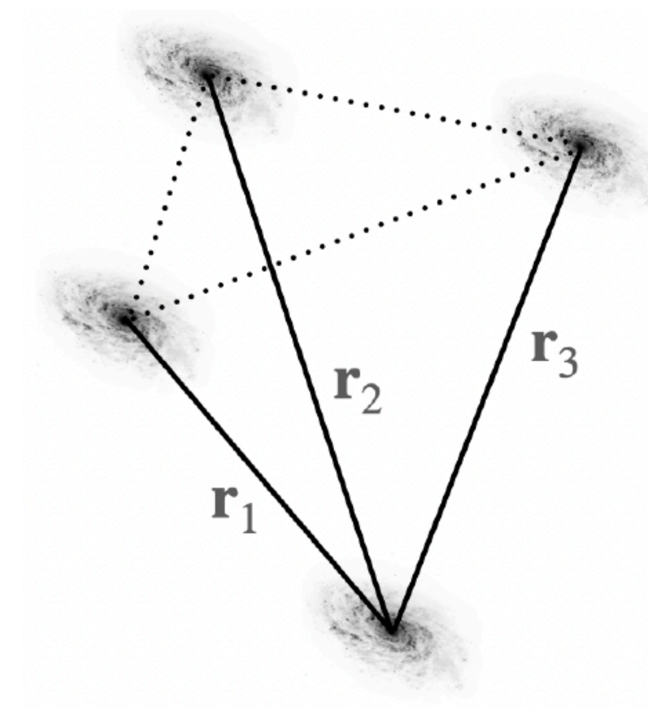
☞ $\mathrm{Im}\langle \delta_g(\mathbf{k}_1)\delta_g(\mathbf{k}_2)\delta_g(\mathbf{k}_3)\delta_g(\mathbf{k}_4)\rangle \neq 0$

CMB and galaxy trispectra violate parity ?

CMB (galaxy) trispectrum estimator becomes computable by a coarse-graining method: binning in harmonic (Fourier) space and replacing the sum over ℓ_n, m_n (k_n) with that over bin index b_n

$$\hat{A}_{\rm tris} = \frac{1}{N} \sum_{\substack{\ell_1\ell_2\ell_3\ell_4 \\ m_1m_2m_3m_4}} \frac{\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} a_{\ell_4 m_4} \rangle}{C_{\ell_1} C_{\ell_2} C_{\ell_3} C_{\ell_4}} \hat{a}_{\ell_1 m_1} \hat{a}_{\ell_2 m_2} \hat{a}_{\ell_3 m_3} \hat{a}_{\ell_4 m_4}$$

$$\simeq \frac{1}{N} \sum_{b_1 b_2 b_3 b_4} \frac{\langle a^{b_1} a^{b_2} a^{b_2} a^{b_4} \rangle}{C^{b_1} C^{b_2} C^{b_3} C^{b_4}} \hat{a}^{b_1} \hat{a}^{b_2} \hat{a}^{b_3} \hat{a}^{b_4}$$



Model-independent null tests show

$\frac{7}{3}\sigma$ signal of $\mathrm{Im}\langle \delta_g^4 \rangle$ in BOSS CMASS ($z = 0.57$) / LOWZ ($z = 0.32$) [Hou +: 2206.03625]

3σ signal of $\mathrm{Im}\langle \delta_g^4 \rangle$ in BOSS CMASS ($z = 0.57$) [Philcox: 2206.04227]

for $0.06\, h\mathrm{Mpc}^{-1} \lesssim k \lesssim 0.4\, h\mathrm{Mpc}^{-1}$

↘

very sensitive to covariance estimates

Reanalyses with new N-body-based mocks reduce significances to $\frac{1}{3}\sigma$ in CMASS ($z = 0.57$) / LOWZ ($z = 0.32$)

[Philcox & Ereza: 2401.09523]

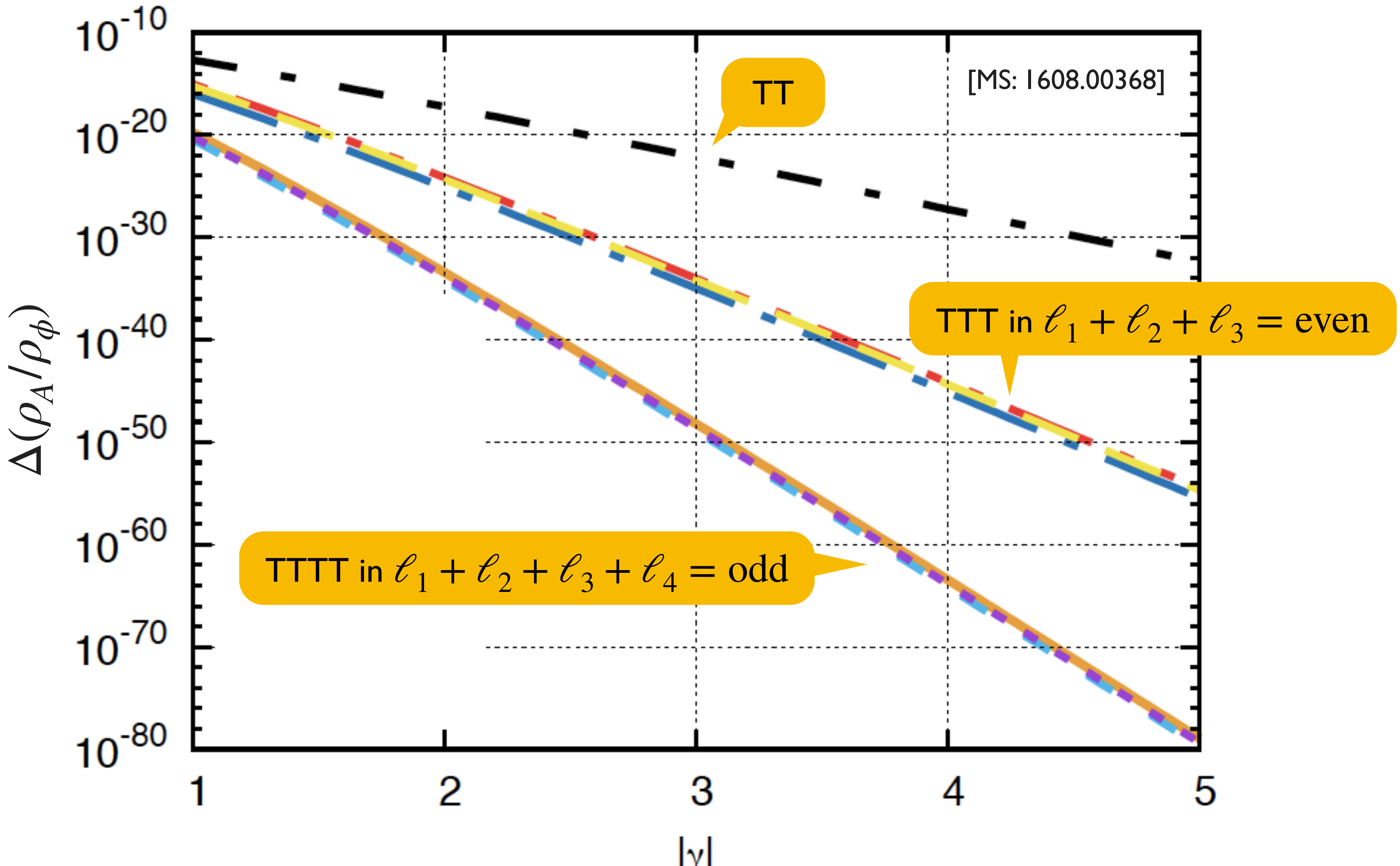
0.4σ signal of $\langle a_{\ell m}^4 \rangle$ in $\sum_{n=1}^{4} \ell_n = $ odd in Planck T or E

for $2 \leq \ell \leq 518$ ($10^{-4}\,\mathrm{Mpc}^{-1} \lesssim k \lesssim 0.04\,\mathrm{Mpc}^{-1}$)

[Philcox: 2303.12106 / Philcox & MS: 2308.03831]

Potential of scalar trispectrum

$$\mathcal{L} \supset f(\phi)\left(-\frac{1}{4}F^2 + \frac{\gamma}{4}F\tilde{F}\right) \quad ☞ \langle\zeta^2\rangle \propto \frac{e^{4\pi|\gamma|}}{|\gamma|^3}\frac{\rho_A}{\rho_\phi}, \quad \langle\zeta^3\rangle \propto \frac{e^{8\pi|\gamma|}}{|\gamma|^6}\frac{\rho_A}{\rho_\phi}, \quad \langle\zeta^4\rangle \propto \frac{e^{12\pi|\gamma|}}{|\gamma|^9}\frac{\rho_A}{\rho_\phi}$$



Depending on the model, the trispectrum will give the tightest limits!

Limits on scale-invariant collapsed-type P-odd scalar trispectrum

$$\left\langle \prod_{n=1}^{4} \zeta_{\mathbf{k}_n} \right\rangle = (2\pi)^3 \int d^3K \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{K}) \delta(\mathbf{k}_3 + \mathbf{k}_4 - \mathbf{K}) P_\zeta(k_1) P_\zeta(k_3) P_\zeta(K) T_{\hat{k}_3}^{\hat{k}_1}(\hat{K}) + 23 \text{ perm}$$

$$T_{\hat{k}_3}^{\hat{k}_1}(\hat{K}) = i \sum_n d_n^{\rm odd} \left[P_n(\hat{k}_1 \cdot \hat{k}_3) + P_n(\hat{k}_1 \cdot \hat{K}) + (-1)^n P_n(\hat{k}_3 \cdot \hat{K}) \right] \left[(\hat{k}_1 \times \hat{k}_3) \cdot \hat{K} \right]$$

[MS: 1608.00368]

- $\mathscr{L} \supset f(\phi) \left(-\frac{1}{4} F^2 + \frac{\gamma}{4} F \tilde{F} \right)$ predicts nonzero $d_{0,1}^{\rm odd}$
- $d_1^{\rm odd}$ is amplified at the collapsed (soft) limit $k_1 \sim k_2 \gg K$, but $d_0^{\rm odd}$ isn't

	$d_0^{\rm odd}$	$d_1^{\rm odd}$
BOSS δ_g [Philcox: 2206.04227]	-610 ± 800	-13000 ± 35000
Planck T (ℓ < 2000) [Philcox & MS: 2308.03831]	(-7.9 ± 3.1) × 10^9	45000 ± 33000
Planck T + E (ℓ < 2000) [Philcox & MS: 2308.03831]	(-0.9 ± 1.1) × 10^9	-3100 ± 9800

Uncertainty scales as $\Delta d_1^{\rm odd} \propto \ell_{\rm max}^{-2}$
(thanks to the collapsed-limit signal)
☞ maybe a big gain in future surveys

$$\frac{\rho_A}{\rho_\phi} \lesssim \begin{cases} 10^{-19} & (\gamma = 1) \\ 10^{-33} & (\gamma = 2) \end{cases}$$

Testing scale-dependent models

inflaton $\phi \neq$ axion χ $\quad \mathscr{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{2}(\partial\chi)^2 - U(\chi) - \frac{1}{4}F^2 - \frac{\lambda}{4f}\chi F\tilde{F}$

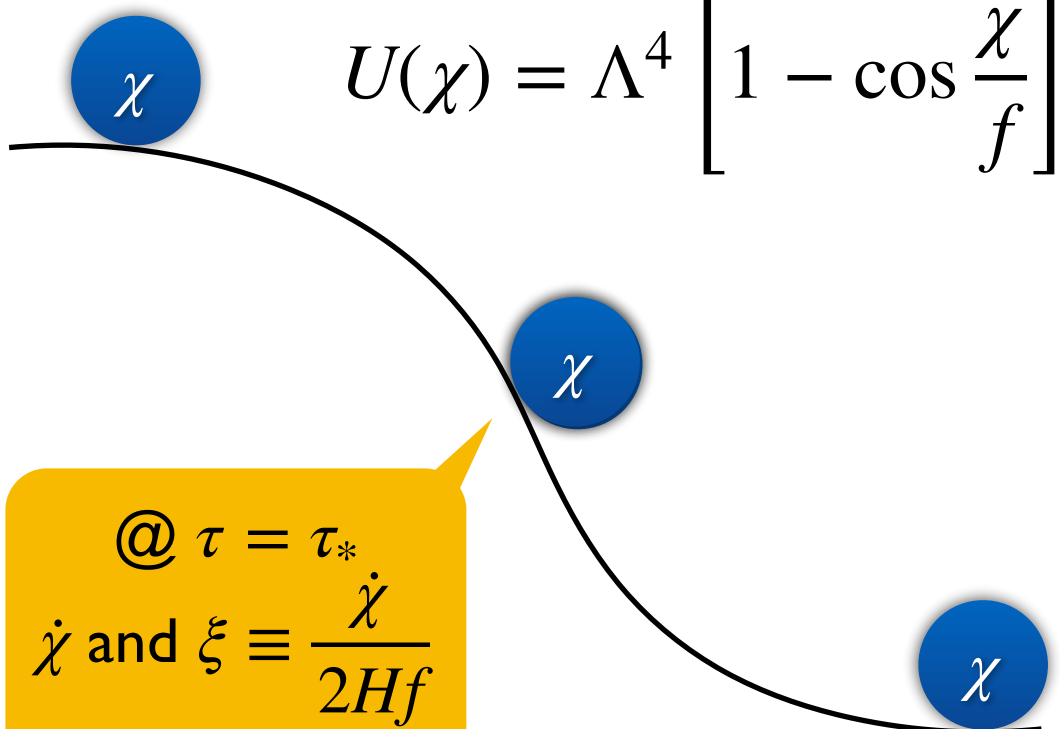
Namba, Peloso, MS, Sorbo, Unal: 1509.07521

The gauge field additionally produces metric perturbations on the usual vacuum contribution

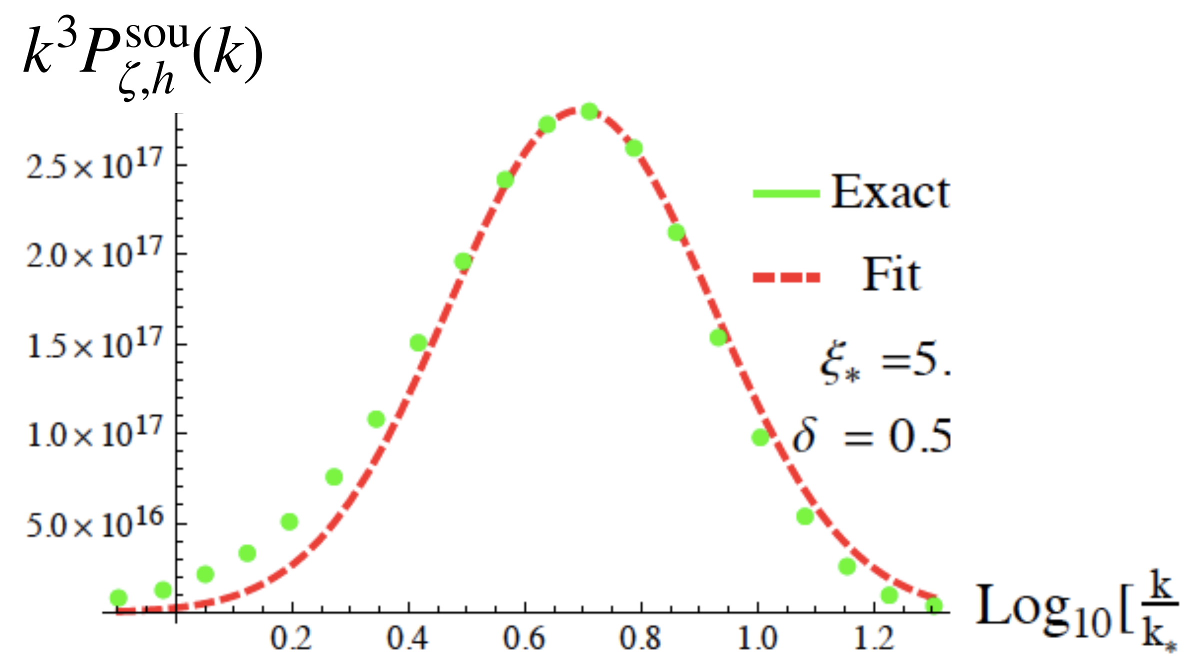
$$P_{\zeta,h}(k) = P^{\rm vac}_{\zeta,h}(k) + P^{\rm sou}_{\zeta,h}(k)$$

$\propto k^{-3}$ $\qquad \propto e^{\pi\xi}$

$$U(\chi) = \Lambda^4\left[1 - \cos\frac{\chi}{f}\right]$$



@ $\tau = \tau_*$
$\dot\chi$ and $\xi \equiv \frac{\dot\chi}{2Hf}$
are maximized



☞ $\zeta^{\rm sou}, h^{\rm sou}_{ij}$ have a bump

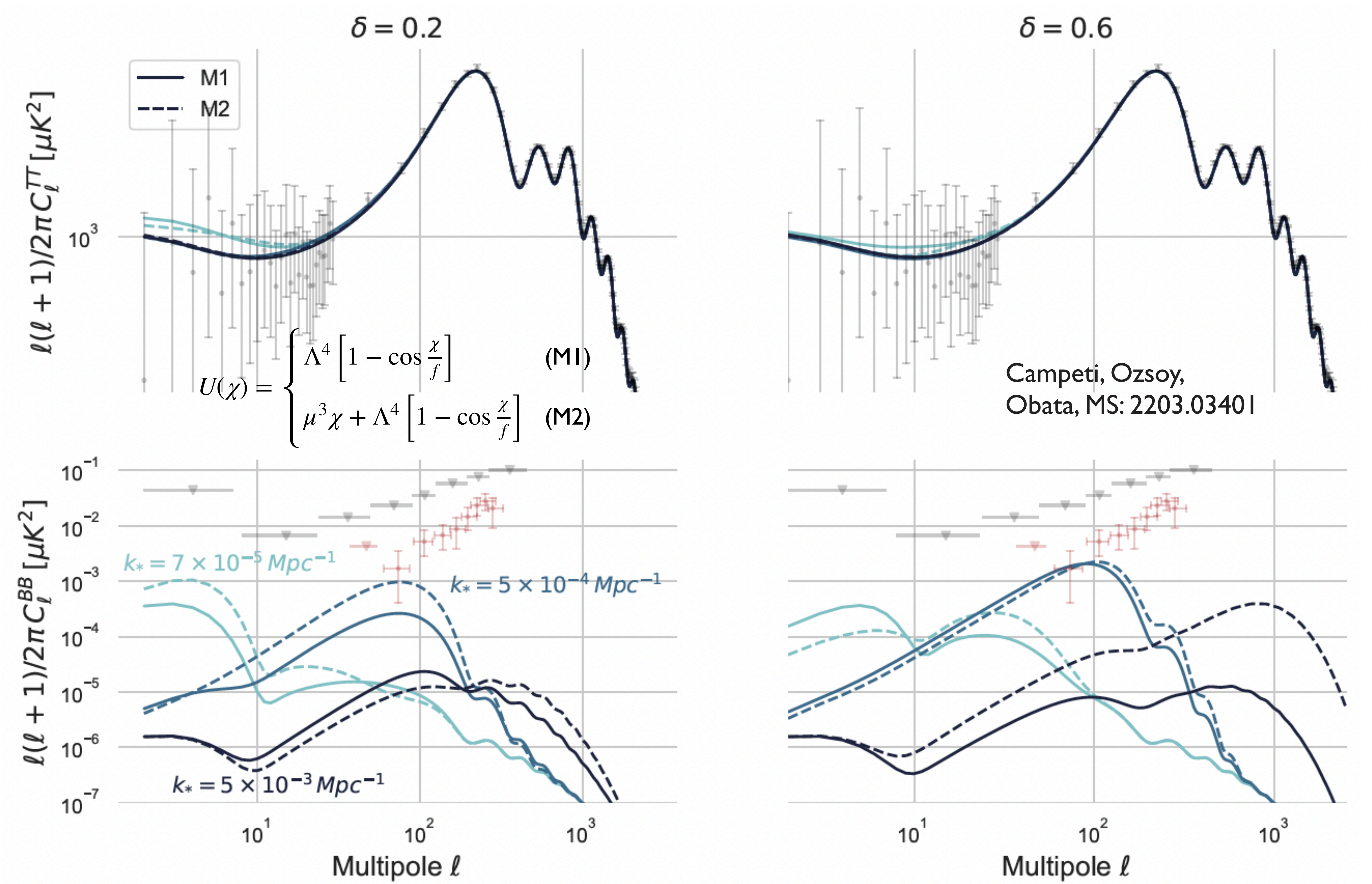
@ $k \sim k_* \equiv \tau_*^{-1}$

k_* ⋯ peak position ($\ell_{\rm peak} \sim k_*\eta_0$)

$\delta = \frac{\Lambda^4}{3H^2 f}$ ⋯ width (δ↑ ☞ width↓)

※ χ's rolls for a few e-folds $\Delta N_\chi \sim \delta^{-1}$

constraints on bumps mainly come from T+E

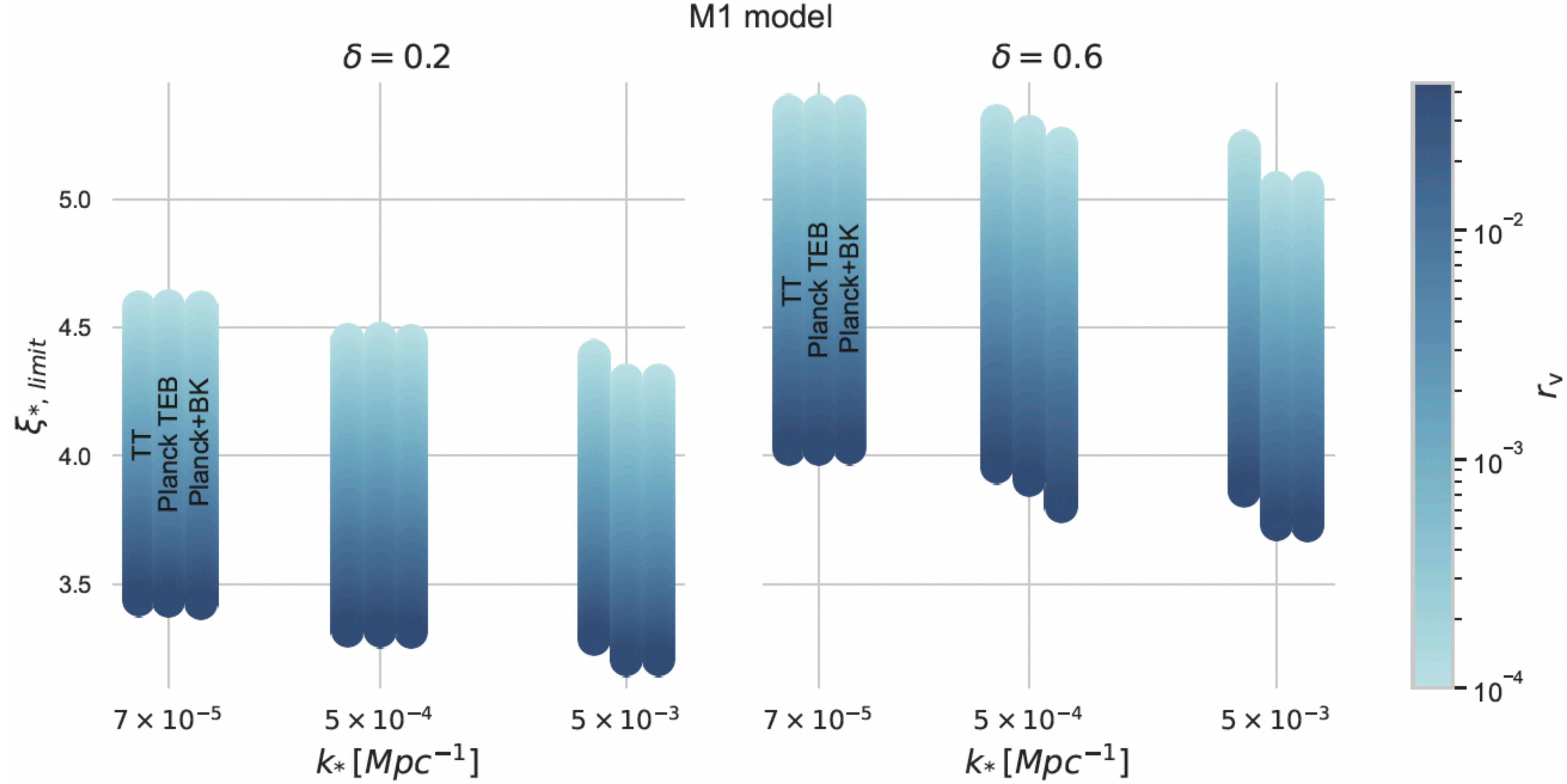


current BB data can constrain only the $k_* = 5\times10^{-4}\,\mathrm{Mpc}^{-1}$ case upper bounds on

$$\xi_* \equiv \xi(\tau_*) = \frac{\lambda\delta}{2}$$

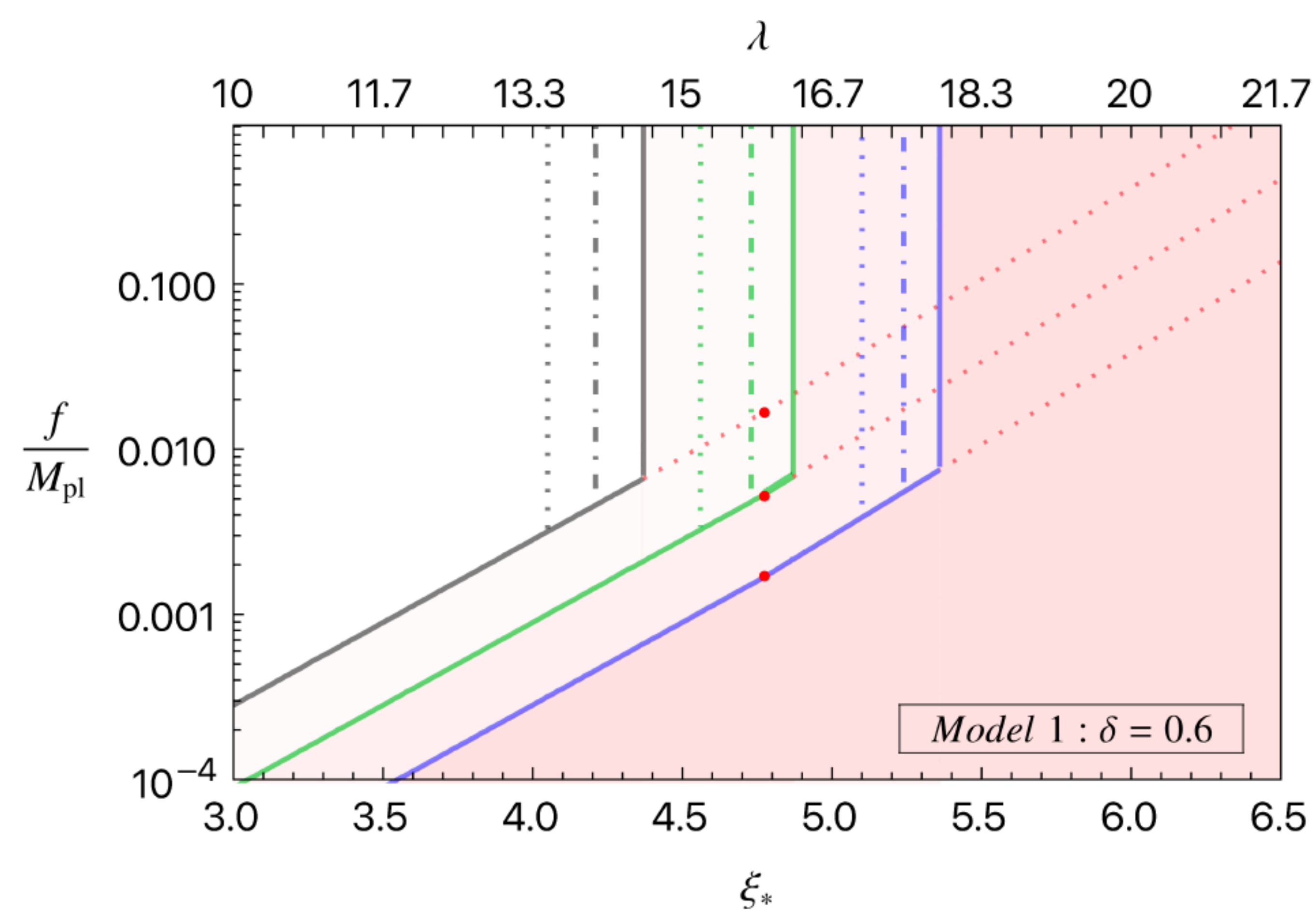
from Planck and/or BICEP/Keck data

Campeti, Ozsoy, Obata, MS: 2203.03401



allowed values on f, λ (white regions) from Planck+BK

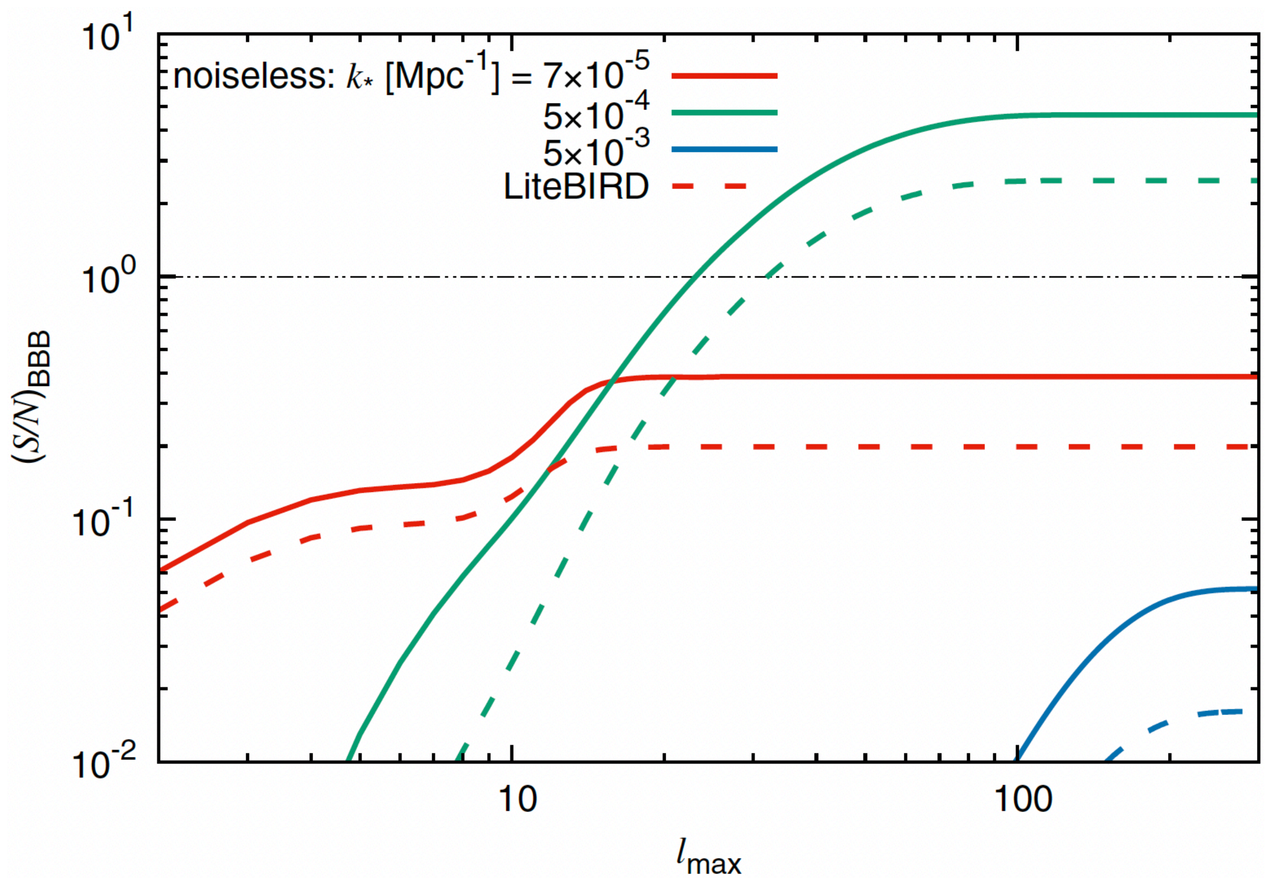
- red regions: prohibited by the backreaction & perturbativity constraints
- $k_* = \begin{cases} 7\times 10^{-5}\,\text{Mpc}^{-1} & \text{(solid)} \\ 5\times 10^{-4}\,\text{Mpc}^{-1} & \text{(dot-dashed)} \\ 5\times 10^{-3}\,\text{Mpc}^{-1} & \text{(dotted)} \end{cases}$
- $r_V = \begin{cases} 10^{-2} & \text{(black)} \\ 10^{-3} & \text{(green)} \\ 10^{-4} & \text{(blue)} \end{cases}$



Scale-dependent GW bispectrum & scalar trispectrum

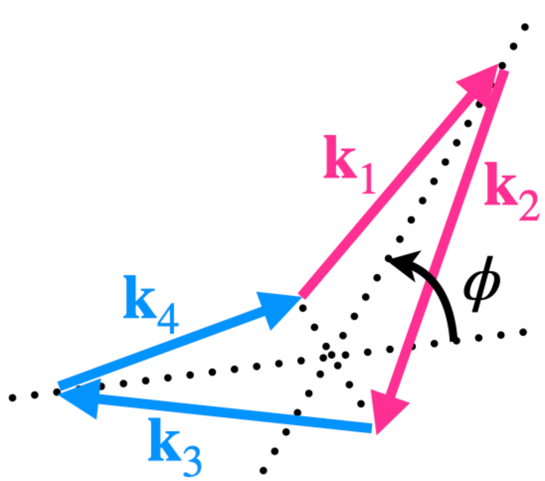
$\langle h^3_{(+2)} \rangle$ for $k_* = 5 \times 10^{-4}\,\mathrm{Mpc}^{-1}$ may be captured by LiteBIRD-level B-mode survey

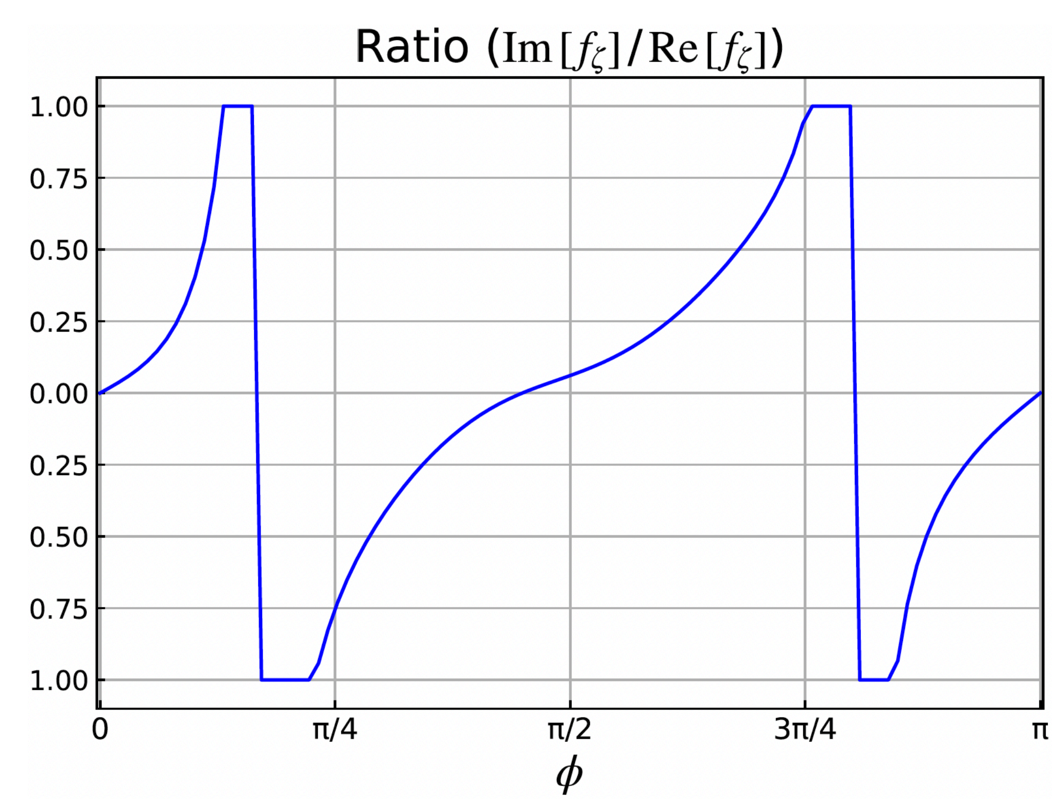
MS, Hikage, Namba, Namikawa, Hazumi: 1606.06082



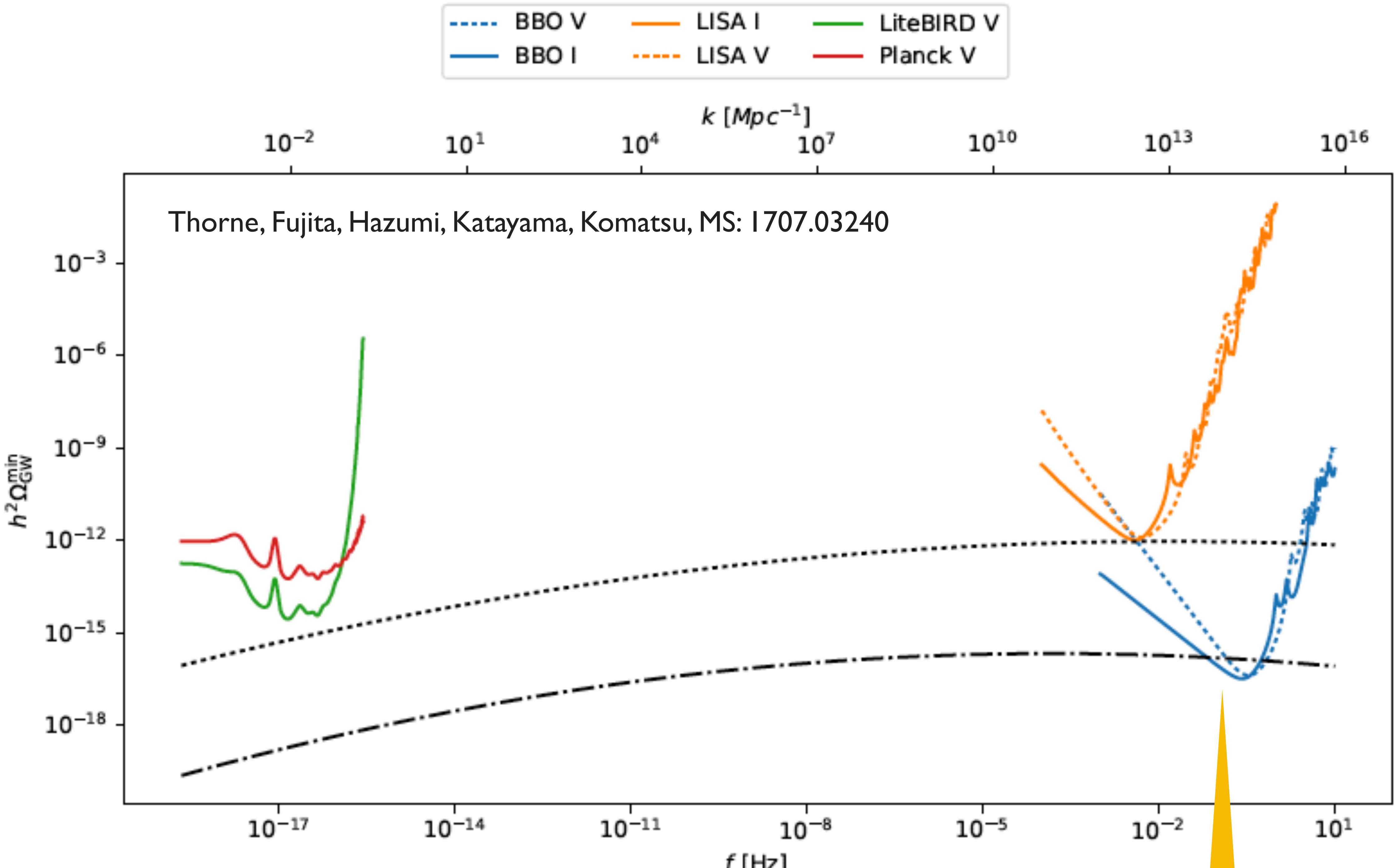
P-odd signal $\mathrm{Im}\langle\zeta^4\rangle$ can surpass P-even one $\mathrm{Re}\langle\zeta^4\rangle$ depending on ϕ

Fujita, Murata, Obata, MS: 2310.03551





$$\mathcal{L} = -\frac{1}{2}\left(\partial\phi\right)^2 - V(\phi) - \frac{1}{2}\left(\partial\chi\right)^2 - V(\chi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\lambda}{4f}\chi F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$$



V mode of GWB will be powerful indicator of very small-scale bumped features generated from axions

visible $g_{\rm NL}$-type GW trispectrum is also generated Fujita, Murai, Obata, MS: 2109.06457

Summary

❖clean parity-violating signal is extractable from

- $N(\geq 2)$-pt correlators of the tensor mode
- $N(\geq 4)$-pt correlators of the scalar mode

❖current observational results of primordial parity violation

- no evidence in Planck TB and EB
- no evidence in WMAP & Planck T+E+B bispectra
- no evidence in Planck T+E trispectra
- ~3σ signals in BOSS galaxy trispectrum

❖promising observables of primordial parity violation

- even $\ell_1 + \ell_2 + \ell_3$ BBB (LiteBIRD, CMBS4, $\ldots$)
- imaginary galaxy trispectrum (PFS, SPHEREx, Euclid, $\ldots$)