Nonlinear dynamics of relativistic magnetized jet with field reversals

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What is a relativistic jet?

collimated bipolar outflow from gravitationally bounded object

- active galactic nuclei (AGN) jet: γ ~ 30
- microquasar jet: v ~ 0.9c
- Gamma-ray burst: γ > 100

Lorentz factor

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

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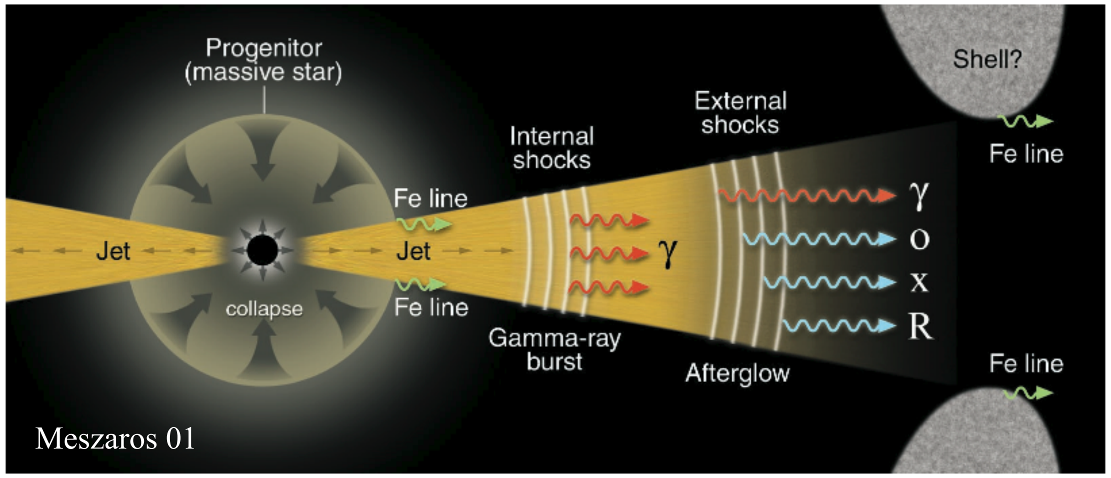
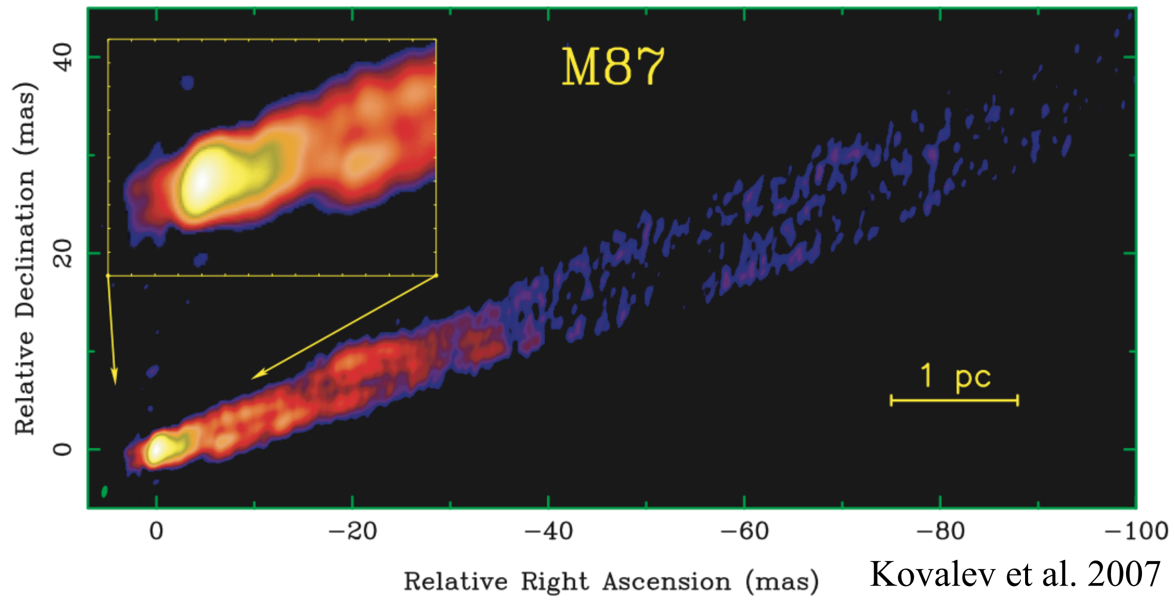
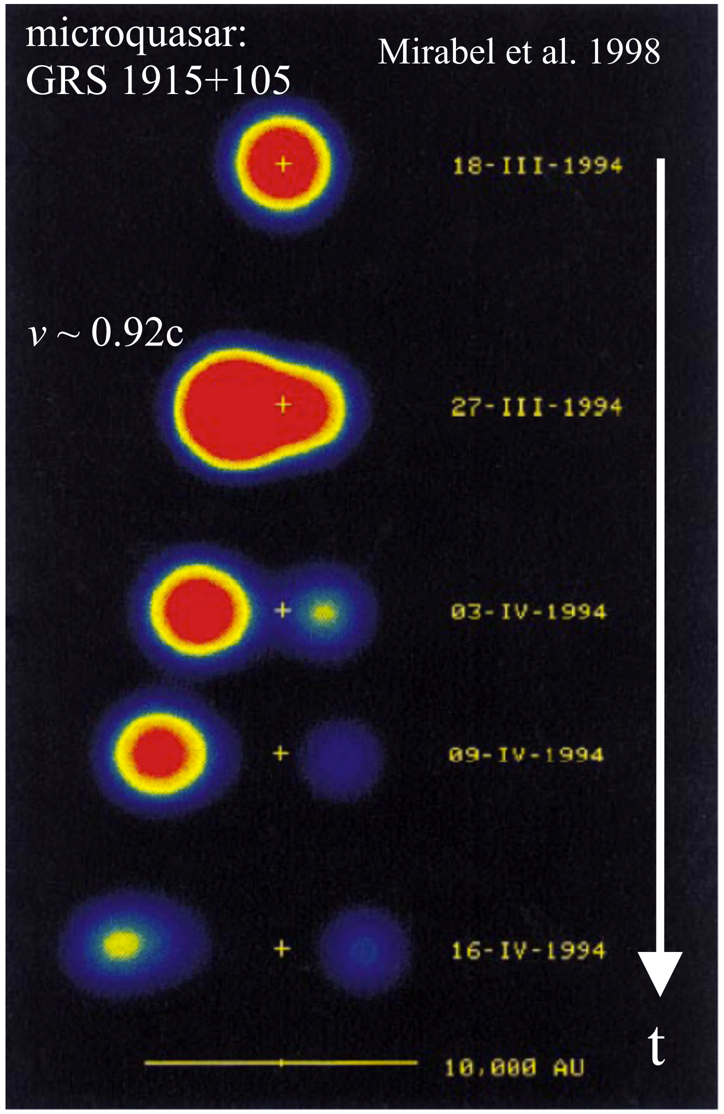
schematic picture of the GRB jet





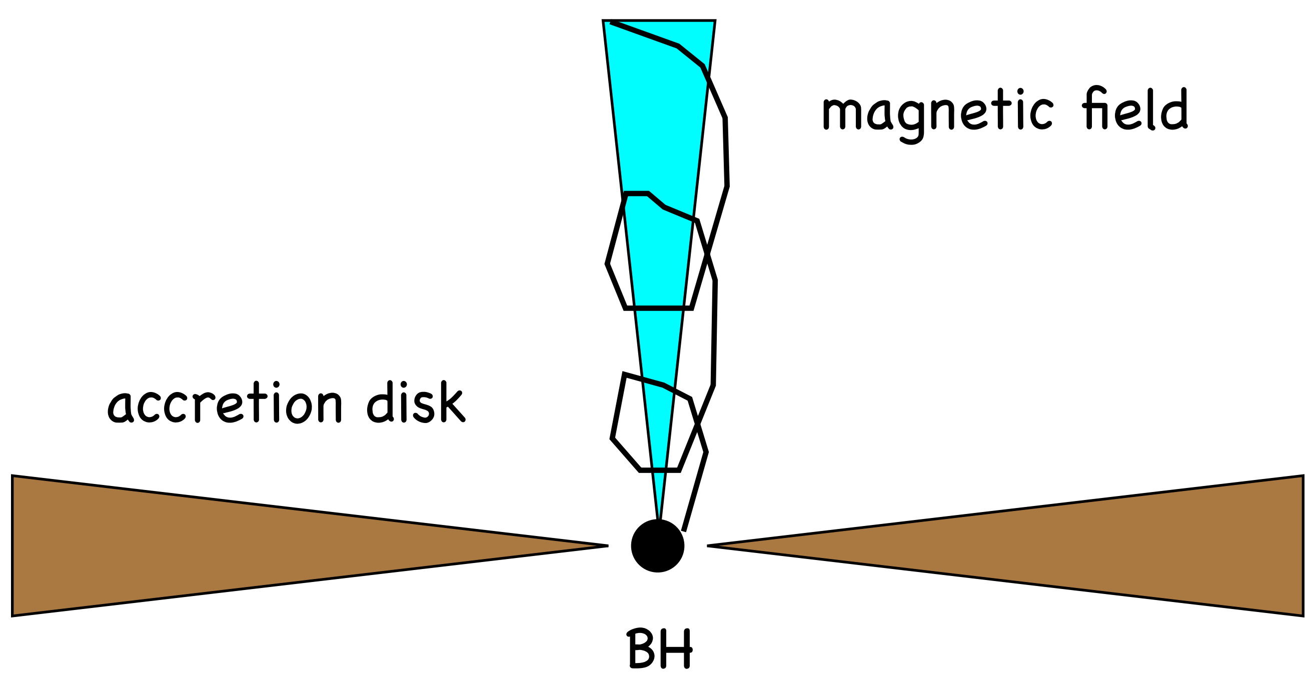


Open questions

Launching mechanism of the jet

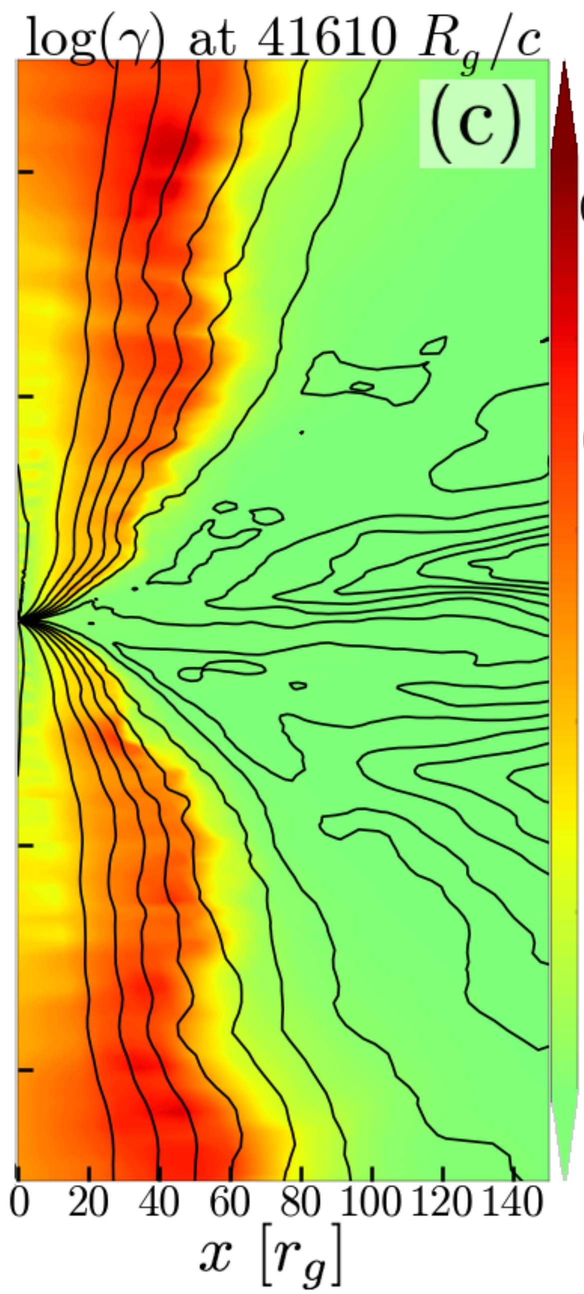
Acceleration of the jet: $\gamma \sim$ 30 (AGN) - 100 (GRB)

Collimation of the jet: related to the stability

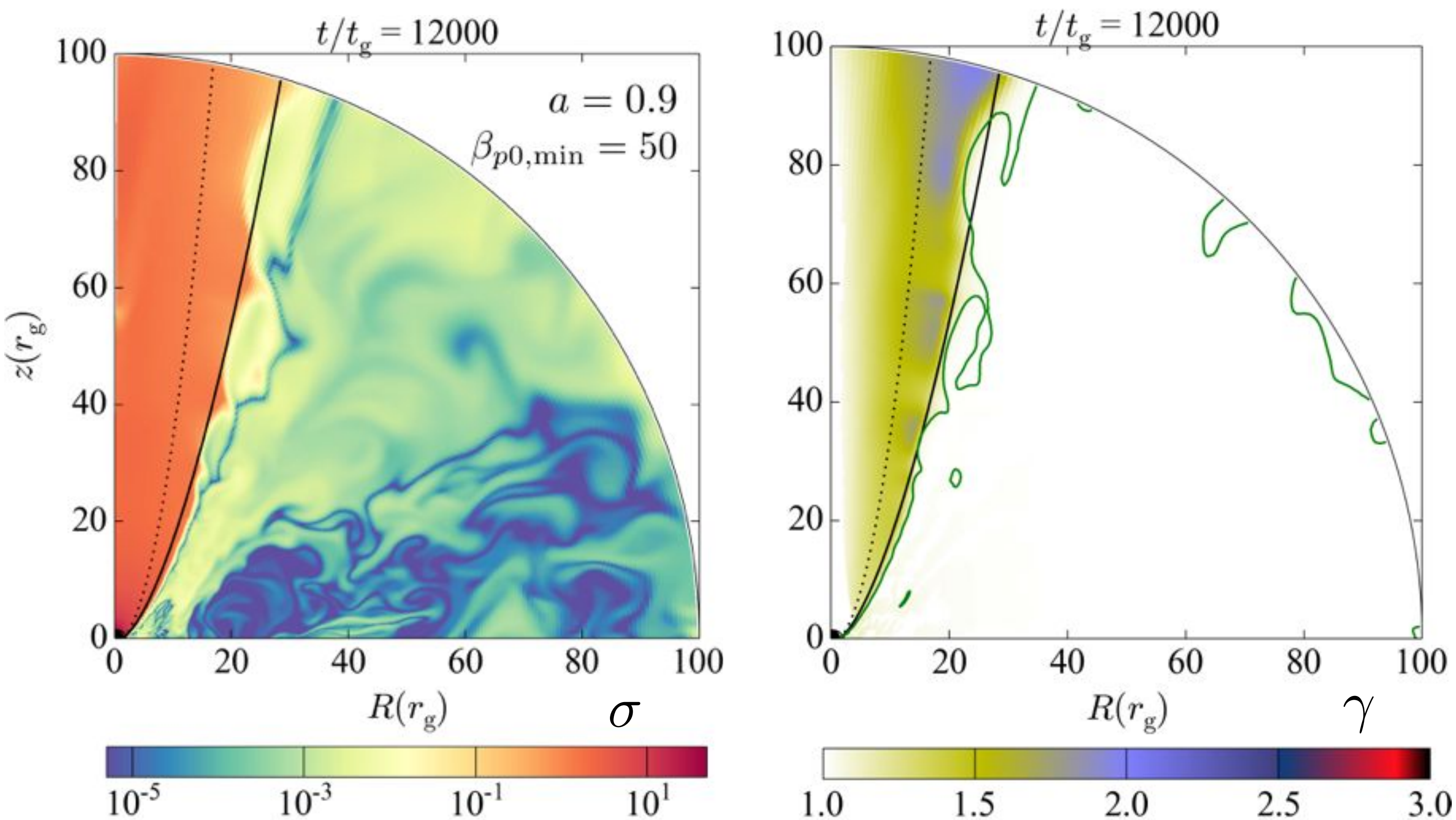


GRMHD simulations for launching jet

Liska+ 20



Nakamura+ 18



$\gamma \sim 7$ at z = 100 r_g

$\gamma \sim 2$ at z = 100 r_g

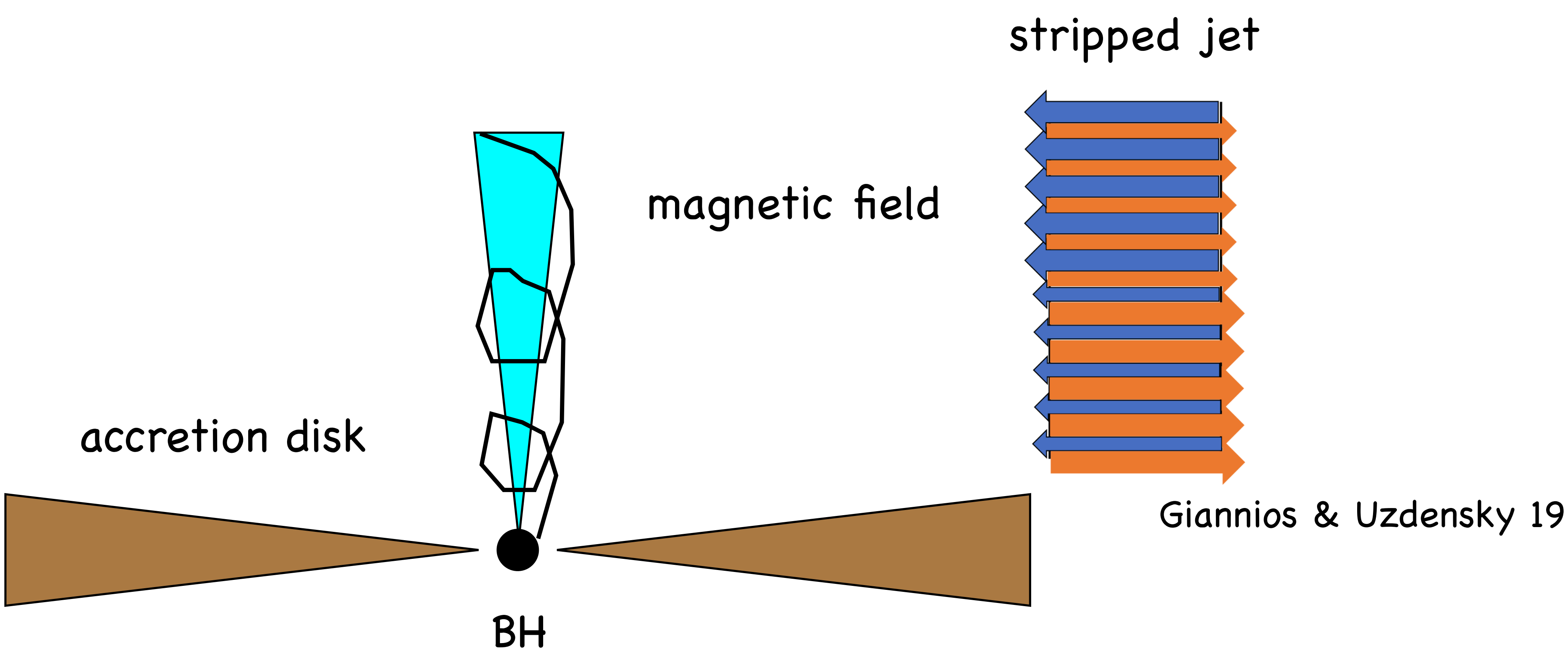
Poynting-flux-dominated jets with mildly relativistic velocity are launched.

Further acceleration of the jet is necessary during the propagation of jet.

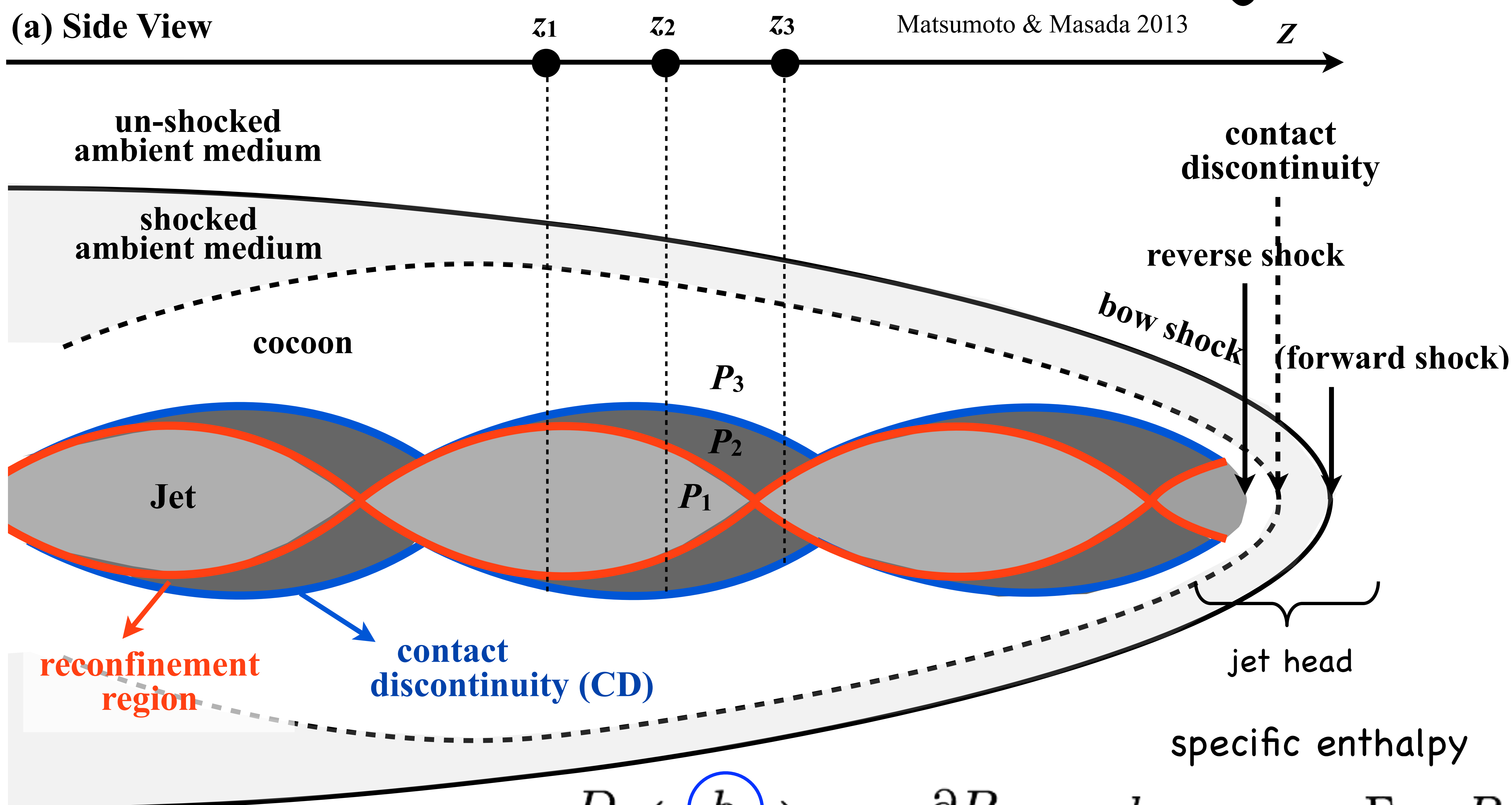
Promising scenario for acceleration

- magnetic dissipation
- in situ energy conversion
- energy conversion from magnetic energy into thermal energy

Idea: striped jet (Drenkhahn+ 02, Giannios & Uzdensky 19)



Rarefaction acceleration of jet



specific enthalpy

equation of motion: $\gamma\rho\frac{D}{Dt}\left(\gamma\frac{h}{c^2}v\right) = -\frac{\partial P}{\partial x}$ $\quad \frac{h}{c^2} = 1 + \frac{\Gamma}{\Gamma - 1}\frac{P}{\rho c^2}$

Bernoulli equation: $\gamma h \sim$ const.

When the jet radius expands, h decreases.
Then the Lorentz factor is accelerated.

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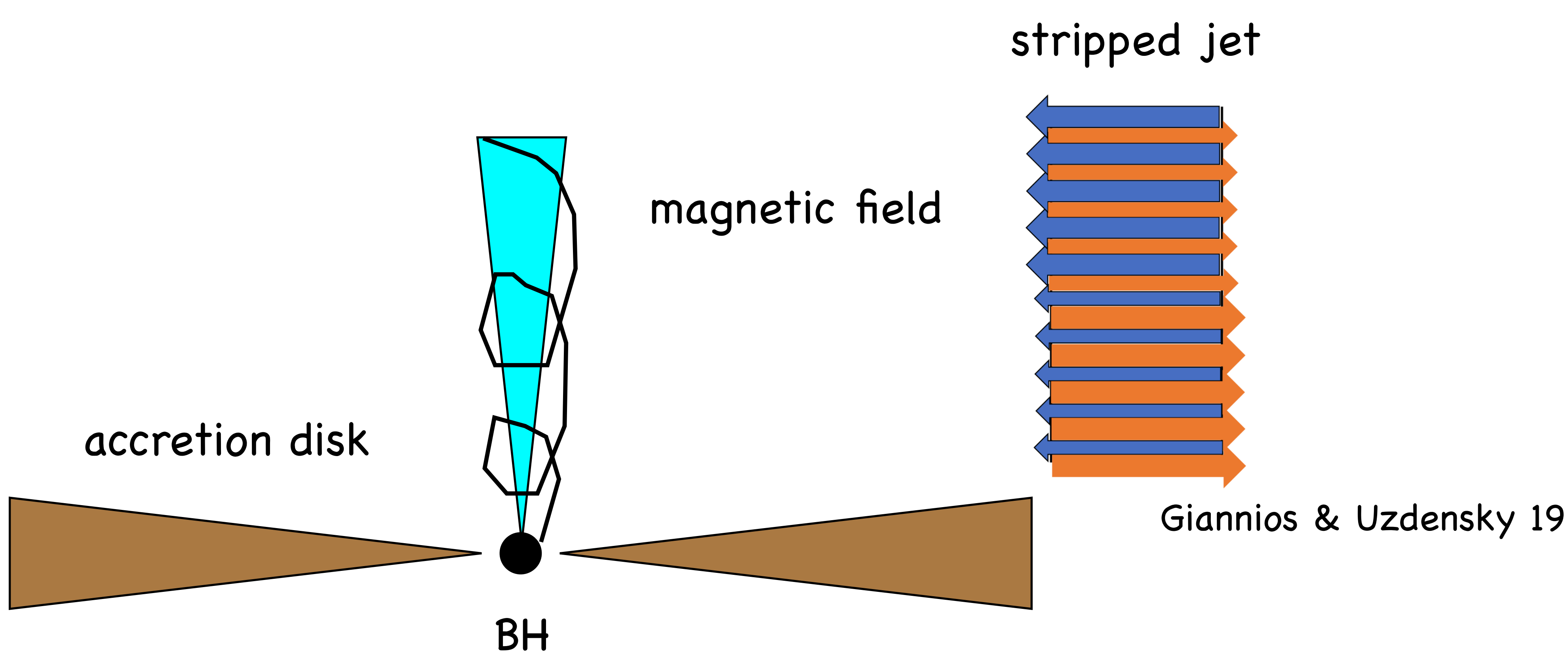
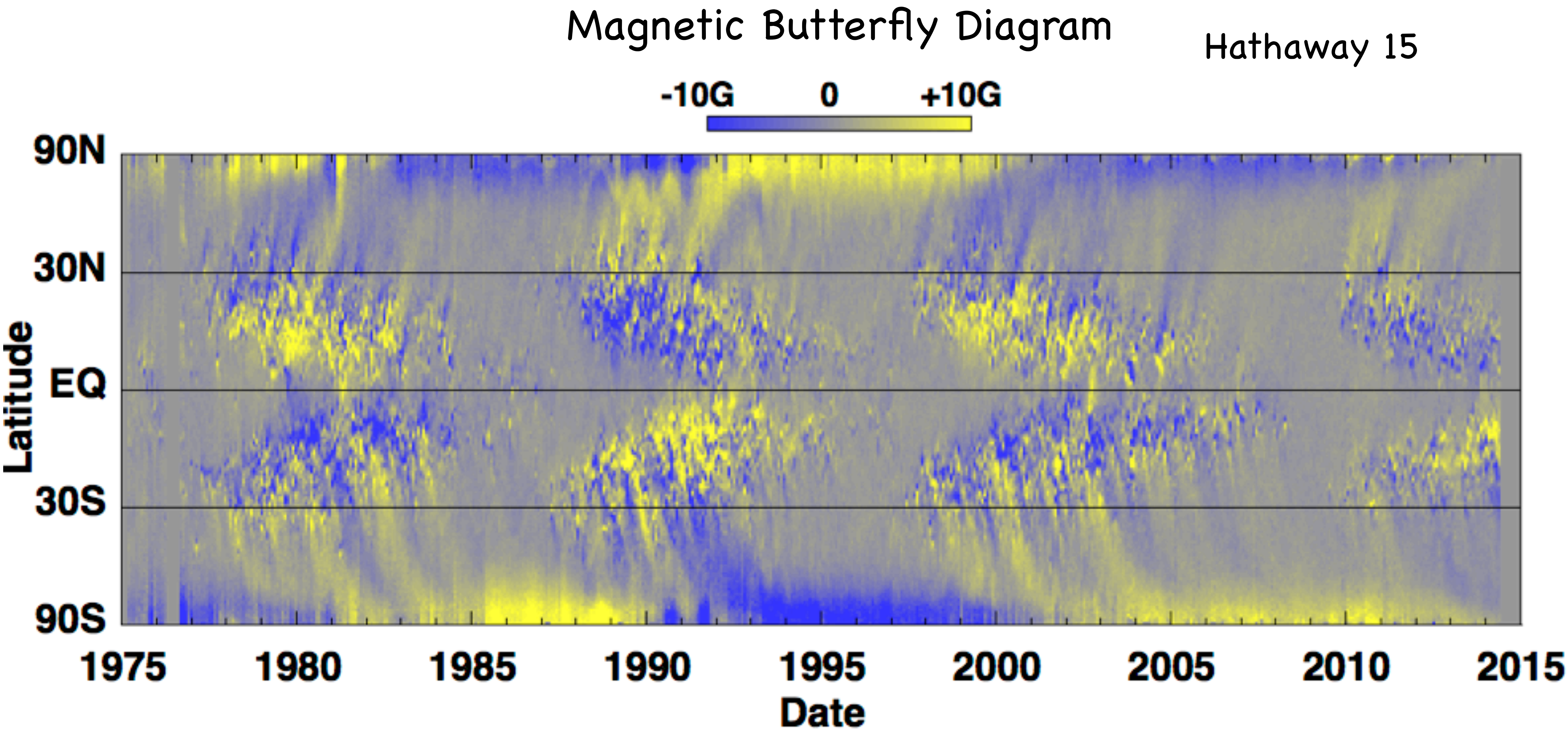


Polar field reversals in the Sun



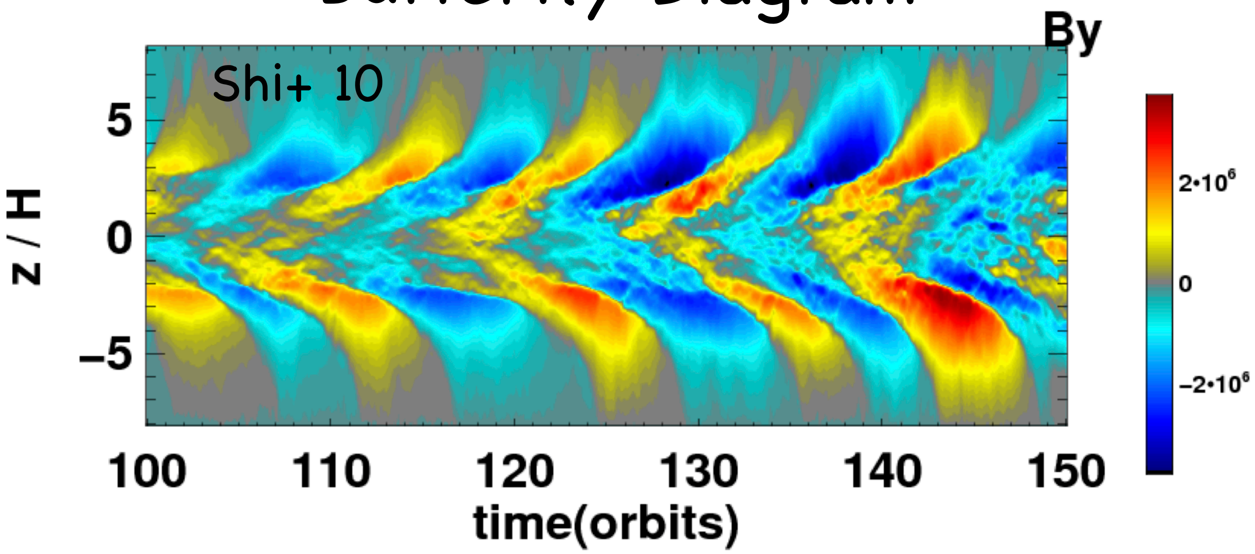
longitudinally averaged radial magnetic field obtained from instruments on Kitt Peak and SOHO

11-year cycle for the polarity of the magnetic field in the sun

Field reversals in accretion disks

(Brandenburg+ 95)

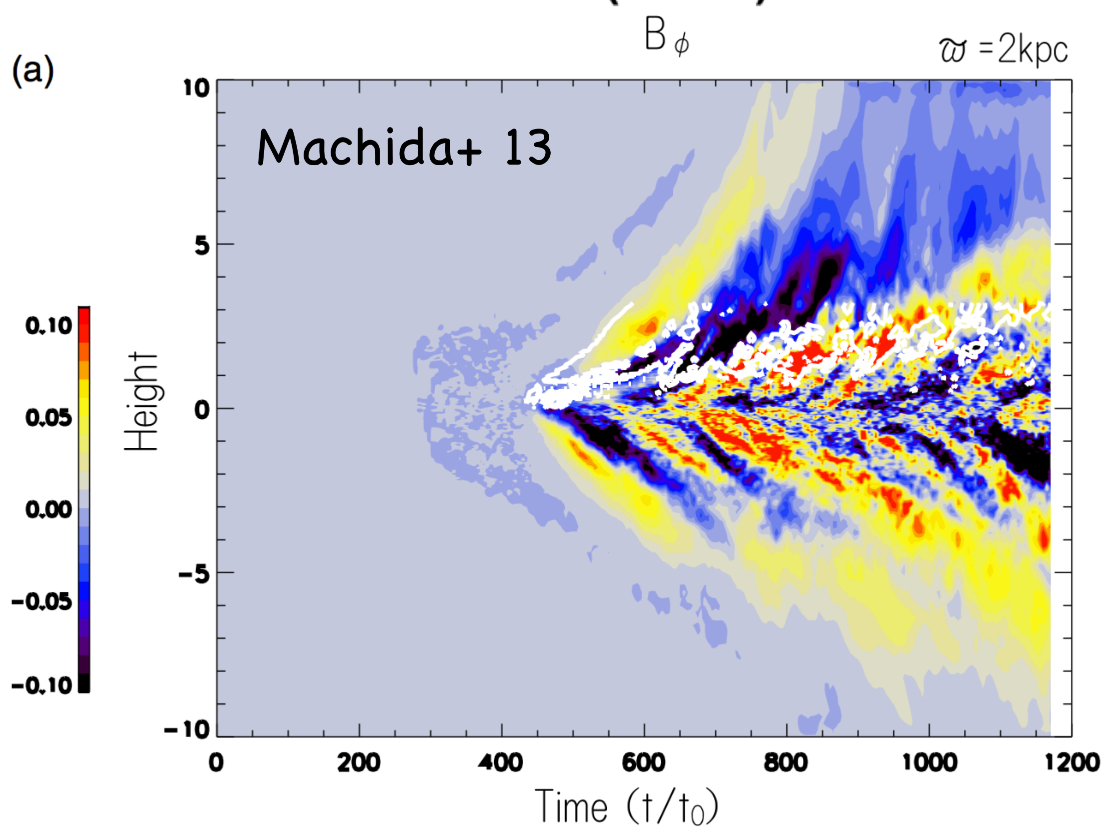
Butterfly Diagram



proto-stellar disk

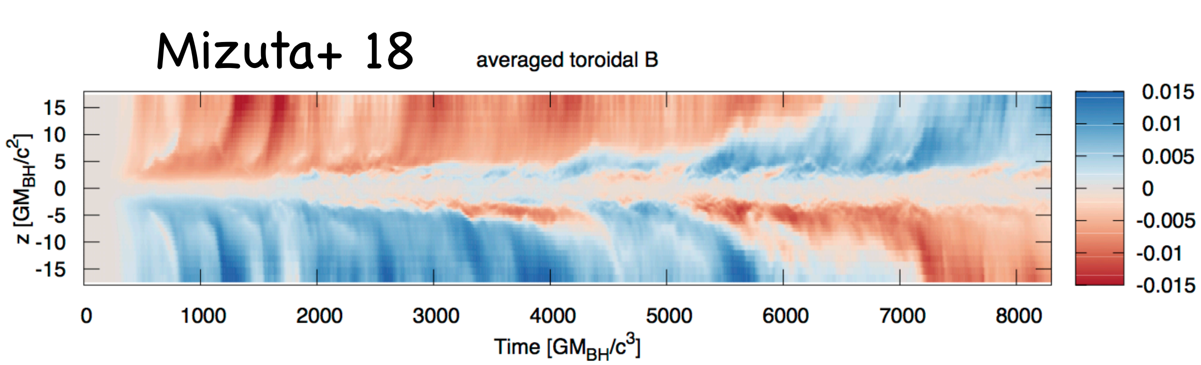
local simulation: shearing box

10 orbital periods



3D global simulation

(e.g., Nishikori+ 06, Machida+ 13)

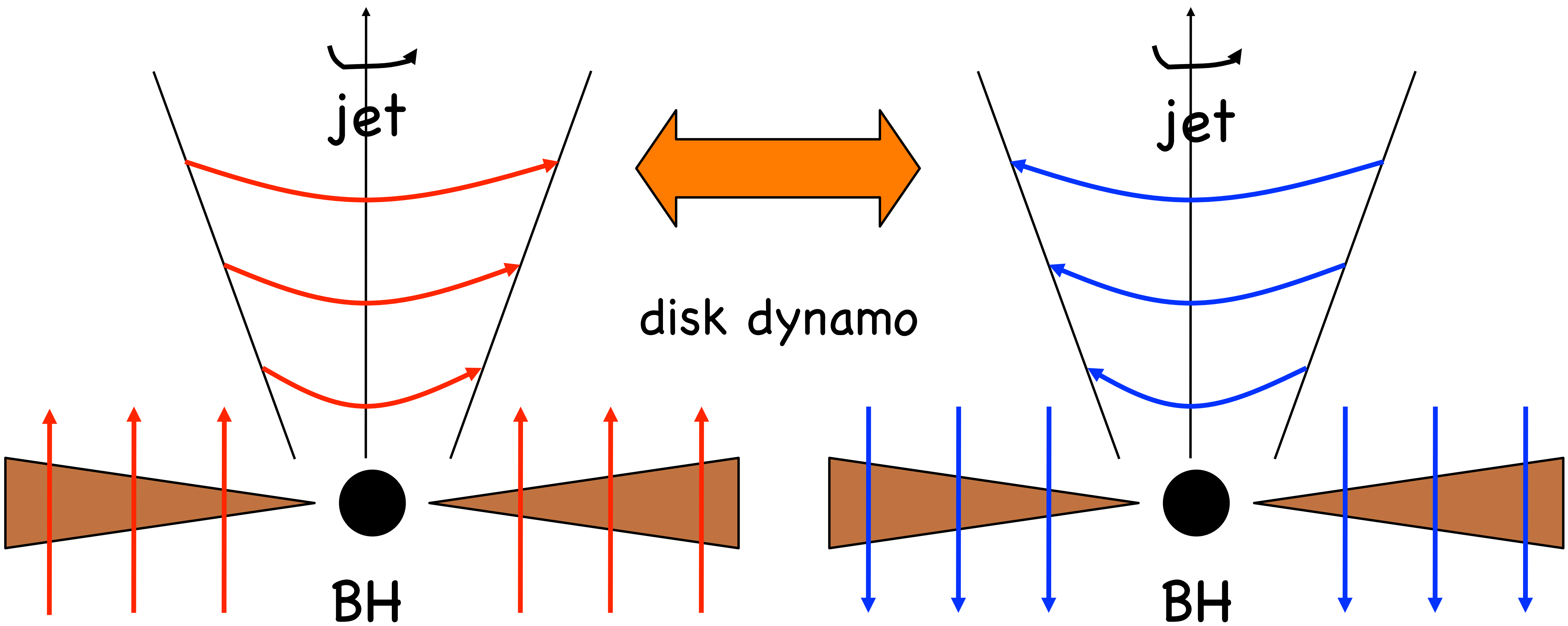


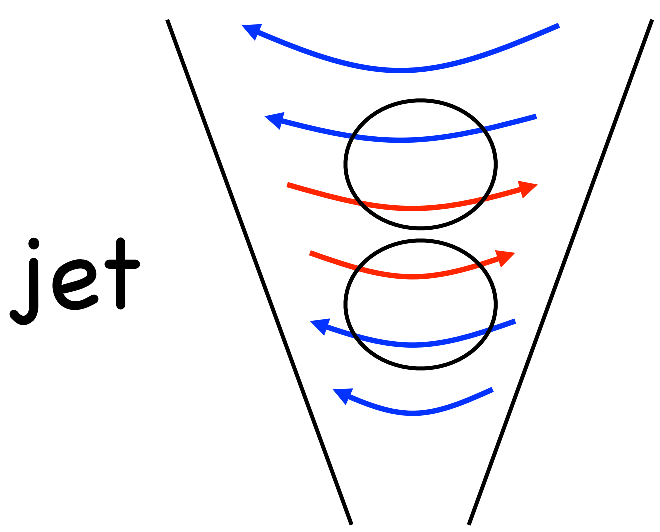
global GRMHD simulation

(e.g., Siegel+ 18, Mizuta+ 18, Jacquemin-Ide+ 24)

Alternating polarity are generated by MHD turbulence.

Field reversals in jet



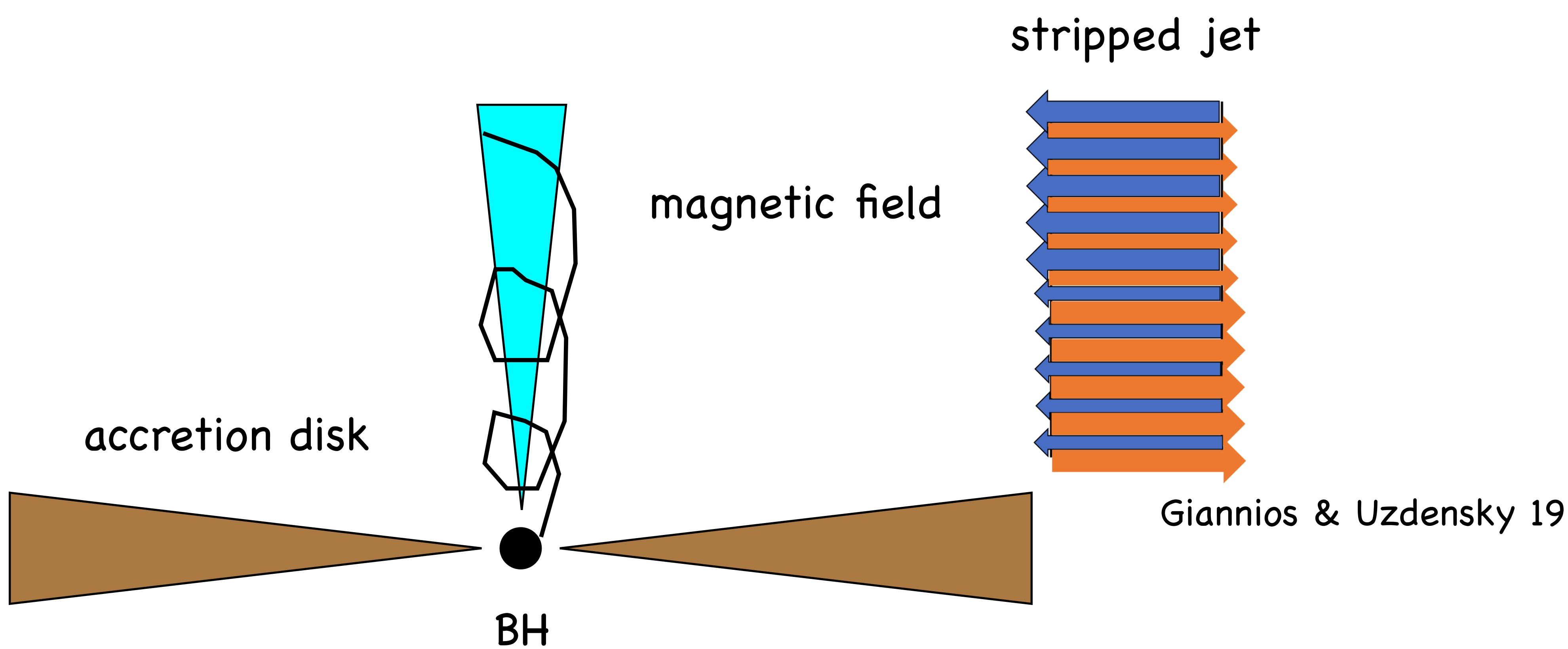


Magnetic field is expected to be dissipated due to the magnetic reconnection.

Promising scenario for acceleration

- magnetic dissipation
- magnetic reconnection in jet
- energy conversion from magnetic energy into thermal energy

Idea: striped jet (Drenkhahn+ 02, Giannios & Uzdensky 19)



We addressed the dynamics of jets with toroidal magnetic field reversals through axisymmetric special relativistic magnetohydrodynamic simulations.

Basic equations

mass conservation

$$\partial_\alpha(\rho u^\alpha) = 0$$

energy-momentum conservation

$$\partial_\alpha\left[(\rho h + b^2)u^\alpha u^\beta - b^\alpha b^\beta + \left(P + \frac{1}{2}b^2\right)\eta^{\alpha\beta}\right] = 0$$

Maxwell's equations

$$\partial_\alpha(u^\alpha b^\beta - u^\beta b^\alpha) = 0$$

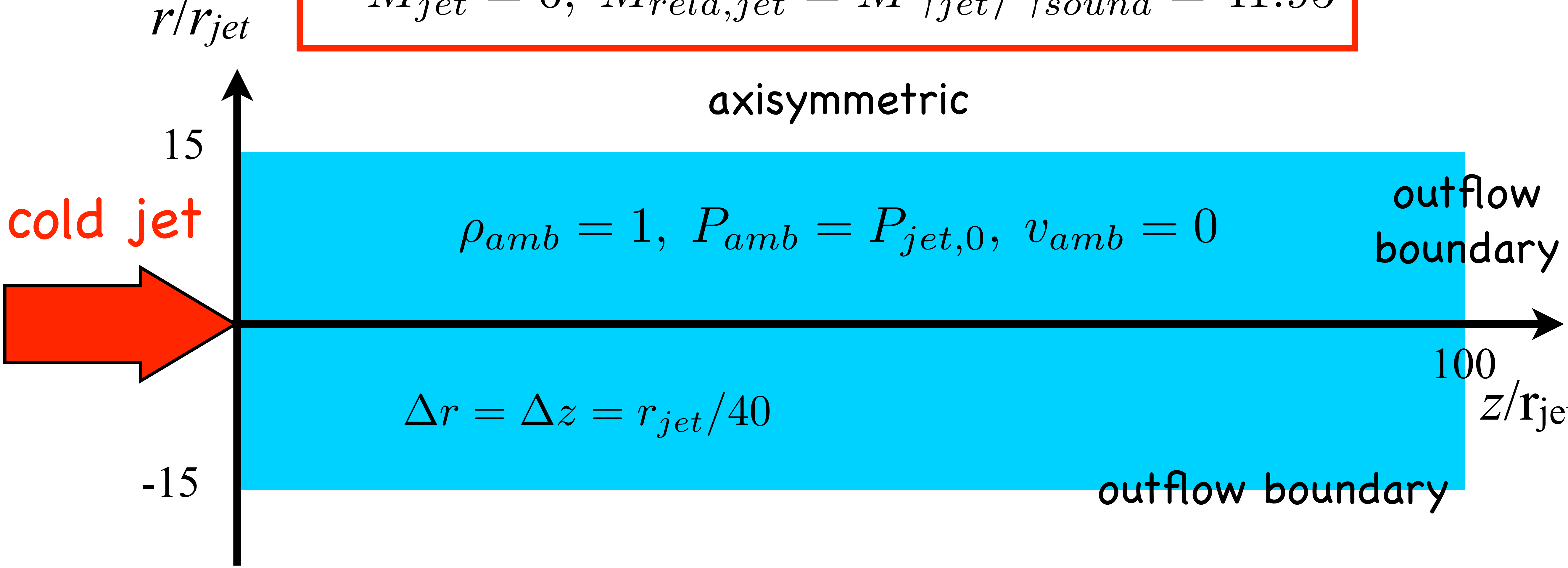
- cylindrical coordinate system
- axisymmetric: $\partial_\phi = 0$
- pure toroidal field: only jet
- ideal MHD
- relativistic HLLD (Mignone+ 09)

Numerical settings

$$\rho_{jet} = 0.01, \; v_z = v_{jet} = 0.99c, \; \gamma_{jet} \sim 7.09$$

$$P_{jet,0} = 2.2 \times 10^{-4}$$

$$M_{jet} = 6, \; M_{rela,jet} = M\gamma_{jet}/\gamma_{sound} = 41.95$$



$$\sigma_{jet} = {b_m}^2/(\rho h)_{jet} = 0.41$$

$T_{rev}=\infty$, 10, 1, 0.1

$$\beta_{jet} = 2P_{jet,0}/{b_m}^2 = 0.1$$

Distribution of Magnetic Field/Pressure

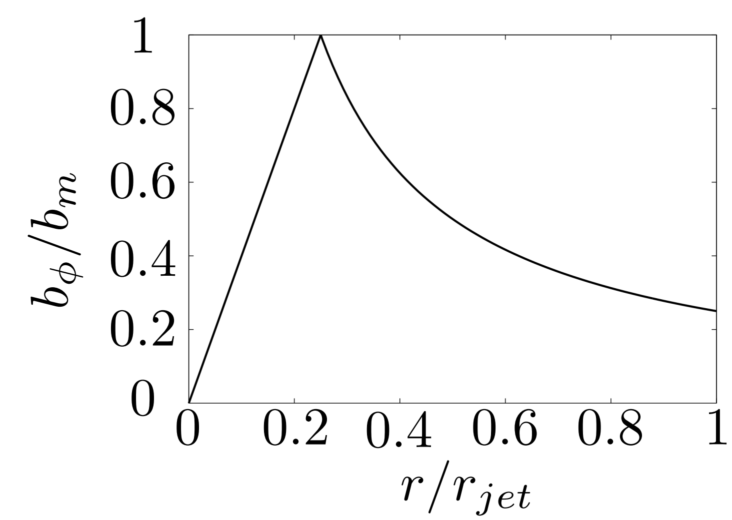
- Komissarov 1999

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{b^\phi}{r}\frac{\mathrm{d}(rb^\phi)}{\mathrm{d}r}.$$

balancing equation of the momentum in the radial direction

Hence, we have

configuration of B-field



$$b^\phi = \begin{cases} b_\mathrm{m}^\phi (r/r_\mathrm{m}) & \text{if} \quad r < r_\mathrm{m}, \\ b_\mathrm{m}^\phi (r_\mathrm{m}/r) & \text{if} \quad r_\mathrm{m} < r < r_\mathrm{j}, \\ 0 & \text{if} \quad r > r_\mathrm{j}, \end{cases}$$

$$p = \begin{cases} p_\mathrm{e}\left[\alpha + \frac{2}{\beta}\left(1 - (r/r_\mathrm{m})^2\right)\right] & \text{if} \quad r < r_\mathrm{m}, \\ \alpha p_\mathrm{e} & \text{if} \quad r_\mathrm{m} < r < r_\mathrm{j}, \\ p_\mathrm{e} & \text{if} \quad r > r_\mathrm{j}, \end{cases}$$

where r_m is the radius of the jet core $p_\mathrm{e} = \beta (b_\mathrm{m}^\phi)^2/2$, $\alpha = 1 - (1/\beta)(r_\mathrm{j}/r_\mathrm{m})^2$.

Magnetic Field Solenoidal Condition

$\nabla \cdot \boldsymbol{B} = 0$ must be satisfied in MHD phenomena.

However, this condition is not necessarily satisfied in numerical simulations calculating discrete quantities in space.

➡ Even a small nonzero value of divB can produce a large error in the solution of the MHD equations.

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induction equation: $\dfrac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}$

$$\frac{\partial B_r}{\partial t} = -(\nabla \times \boldsymbol{E})_r = \frac{\partial \cancel{E_\phi}}{\partial z} - \frac{1}{r}\cancel{\frac{\partial E_z}{\partial \phi}} = 0$$

$$\frac{\partial B_\phi}{\partial t} = -(\nabla \times \boldsymbol{E})_\phi = \frac{\partial E_z}{\partial r} - \frac{\partial E_r}{\partial z}$$

$$\frac{\partial B_z}{\partial t} = -(\nabla \times \boldsymbol{E})_z = \frac{1}{r}\cancel{\frac{\partial E_r}{\partial \phi}} - \frac{1}{r}\frac{\partial (r\cancel{E_\phi})}{\partial r} = 0$$

$\partial_\phi = 0$

$\boldsymbol{E}_\phi = -(\boldsymbol{v} \times \boldsymbol{B})_\phi = 0$

$\nabla \cdot \boldsymbol{B} = 0$ is automatically satisfied.

Time scale of field reversals

innermost stable circular orbit (ISCO): $R_g = GM/c^2$

$$R_{\rm base} \sim R_{\rm ISCO} \sim \text{a few } \times R_g$$

Keplerian orbital period:

$$\frac{GM}{r^2} = r{\Omega_K}^2 \qquad \Omega_K = \sqrt{\frac{GM}{r^3}}$$

$$T_{\rm base} = \frac{2\pi}{\Omega_K(R_{\rm base})} = \frac{2\pi R_g}{c}\left(\frac{R_{\rm base}}{R_g}\right)^{3/2}$$

$$R_{\rm base} = 10R_g \qquad T_{\rm base} \sim 200R_g/c$$

➡ $\tau_{\rm rev} \sim 10T_{\rm base} \sim 10^3 R_g/c$

rapidly rotating BH: $R_{\rm base} \sim R_{\rm ISCO} \sim R_g$ ➡ $\tau_{\rm rev} \sim 10^2 R_g/c$

Distance from Black hole

- axial length of the field reversal

$l_{\rm rev} \sim c\tau_{\rm rev} \sim 10^3 R_g$ rapid rotation: $l_{\rm rev} \sim 10^2 R_g$

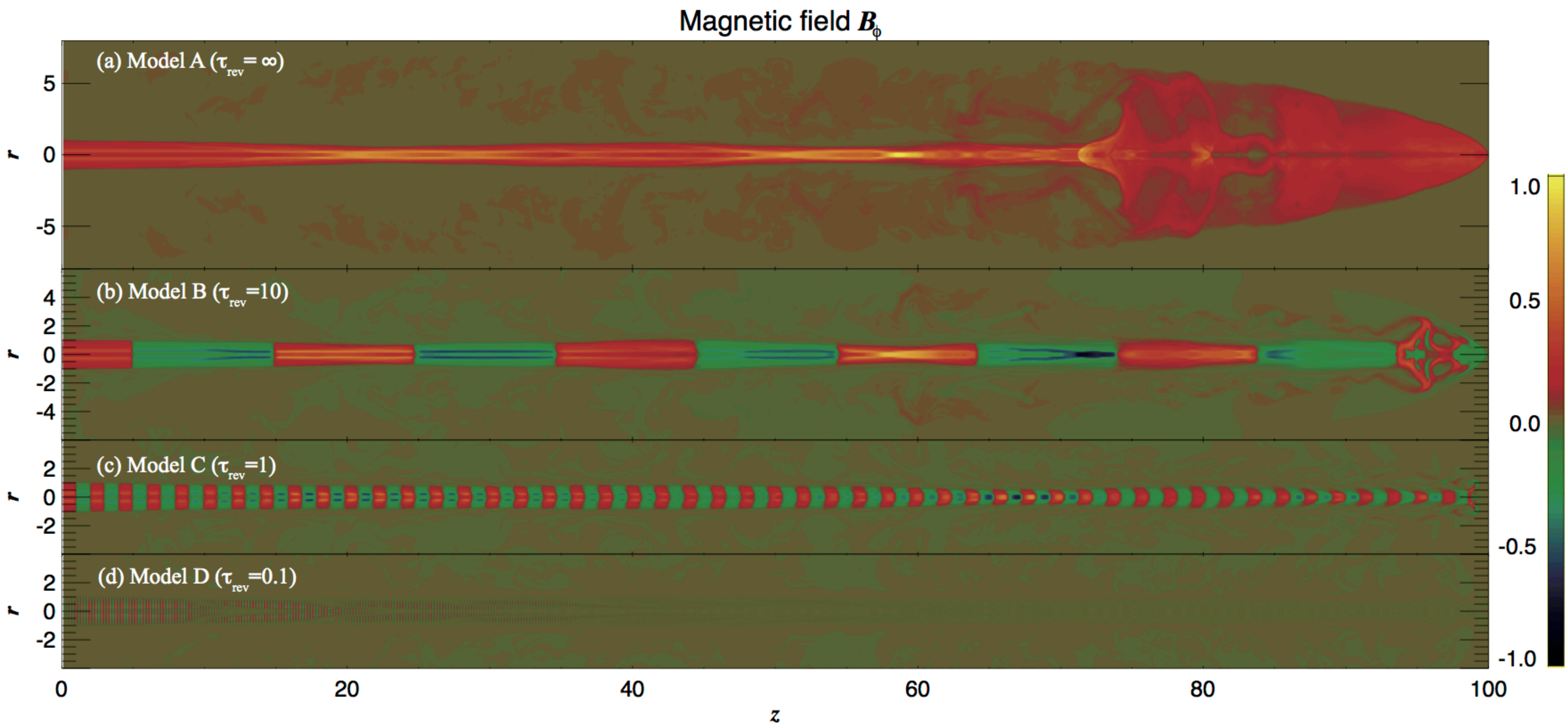
$r_j = 10R_g$	corresponds to	$\tau_{\rm rev} = 10$	model
$r_j = 100R_g$	corresponds to	$\tau_{\rm rev} = 1$	model
$r_j = 1000R_g$	corresponds to	$\tau_{\rm rev} = 0.1$	model

- simple conical jet ($\theta_j = 0.1$): $z = r_j/\tan\theta_j \sim r_j/\theta_j$

$\tau_{\rm rev} = 10$	- - - >	$z = 10^2 R_g$
$\tau_{\rm rev} = 1$	- - - >	$z = 10^3 R_g$
$\tau_{\rm rev} = 0.1$	- - - >	$z = 10^4 R_g$

AGN:	GRB:
$M = 10^8 M_{\rm sun}$	$M = 10 M_{\rm sun}$
$R_g \sim 10^{13}{\rm cm}$	$R_g \sim 10^6{\rm cm}$

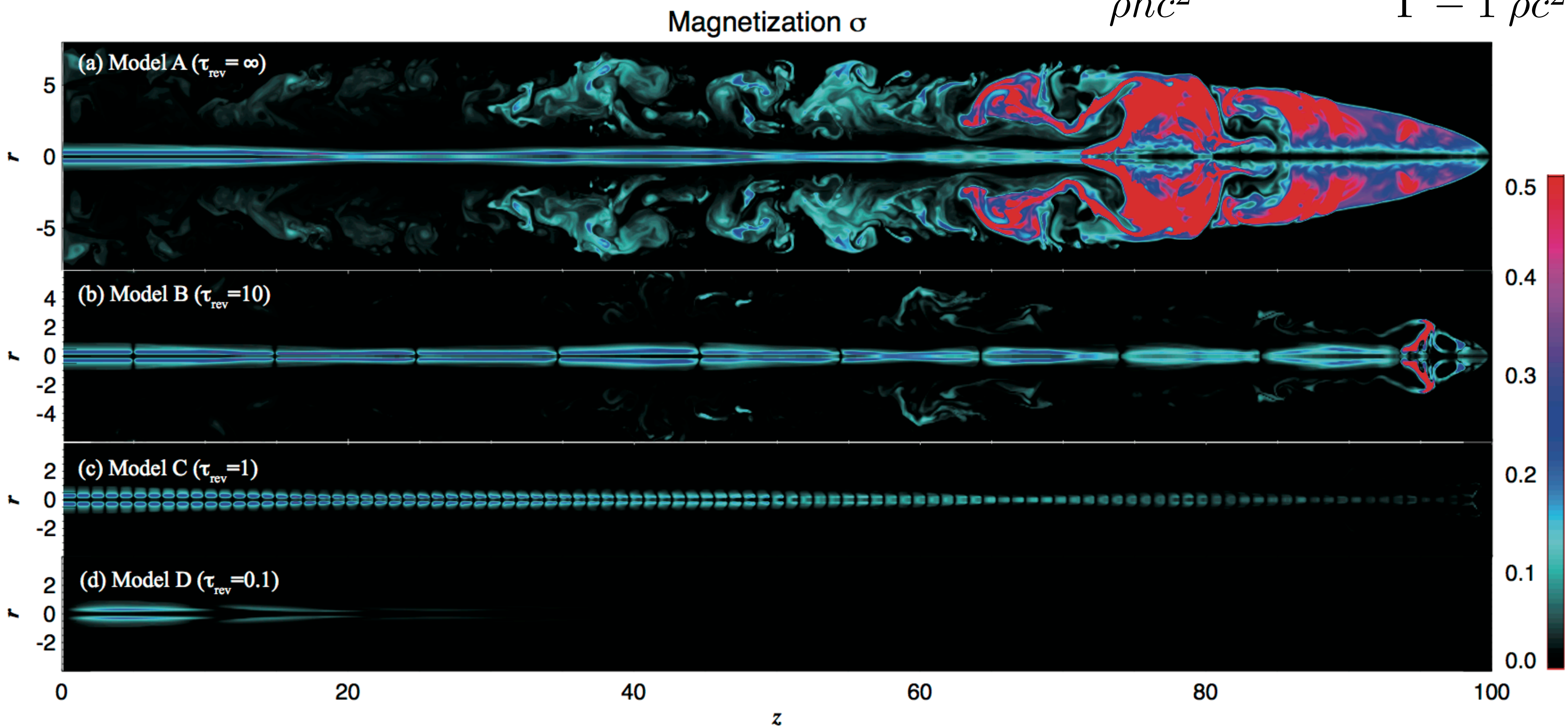
Comparison of B-field



- nose cone in model A
- We can not find strip pattern in model D.

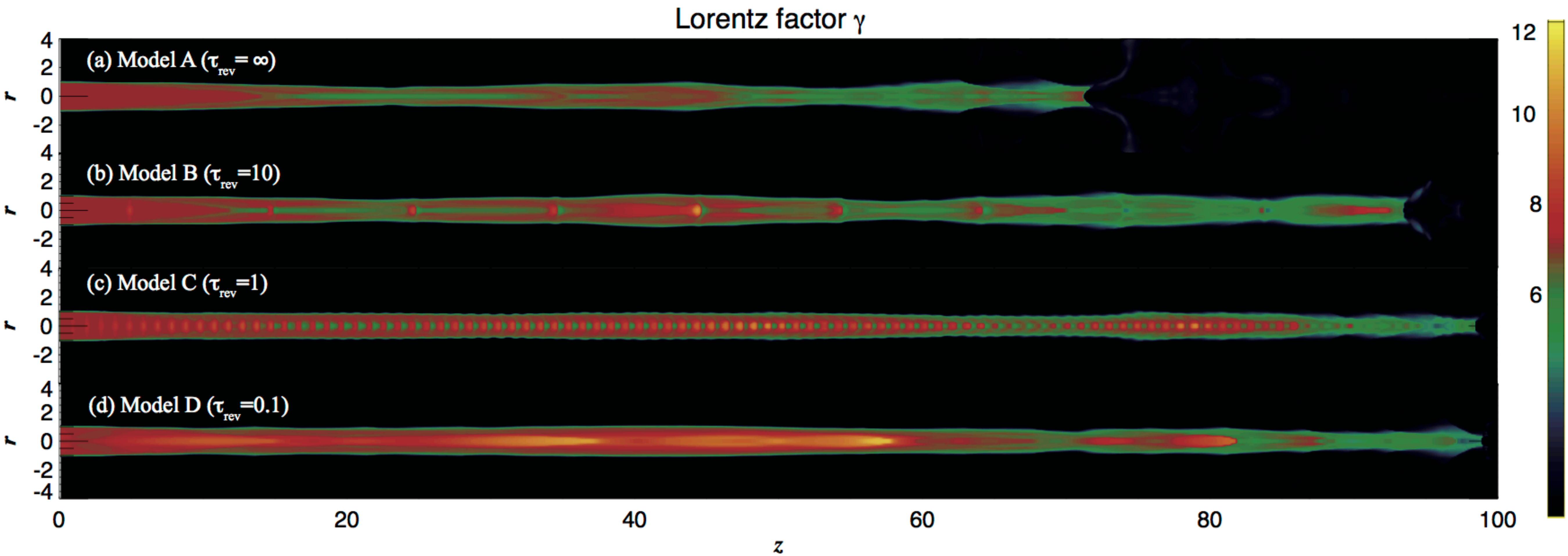
Comparison of magnetization

$$\sigma = \frac{b^2}{\rho h c^2} \quad h = 1 + \frac{\Gamma}{\Gamma - 1}\frac{P}{\rho c^2}$$



Magnetic energy is dissipated at the interface of field reversals.

Comparison of Lorentz factor

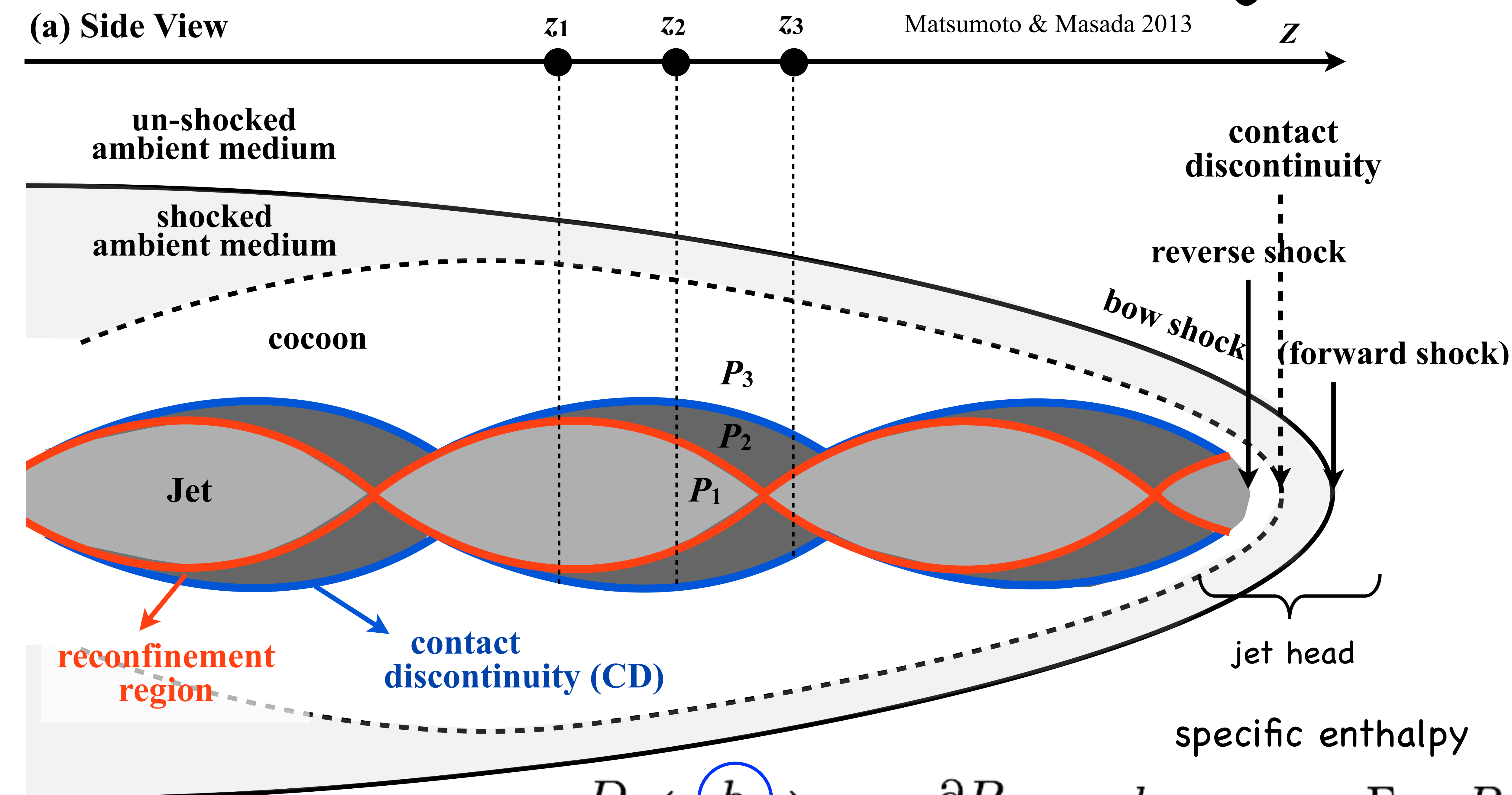


- energy conversion from magnetic energy into thermal energy

- acceleration at the reconfinement region

  thermal energy - - > kinetic energy

Rarefaction acceleration of jet



specific enthalpy

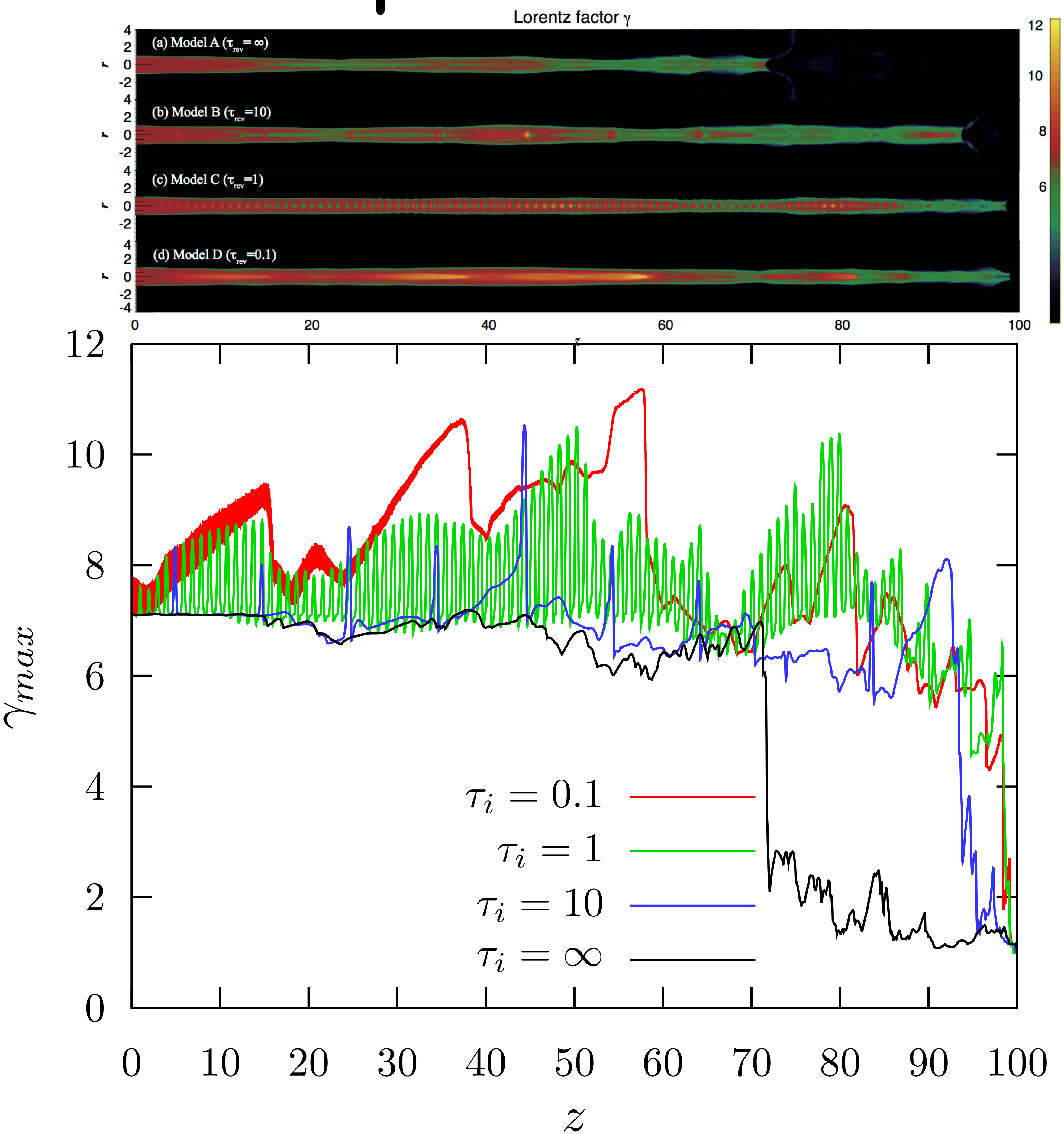
equation of motion: $\gamma\rho\frac{D}{Dt}\left(\gamma\frac{h}{c^2}v\right) = -\frac{\partial P}{\partial x}$ $\quad \frac{h}{c^2} = 1 + \frac{\Gamma}{\Gamma - 1}\frac{P}{\rho c^2}$

Bernoulli equation: $\gamma h \sim$ const.

When the jet radius expands, h decreases.

Then the Lorentz factor is accelerated.

Comparison of Lorentz factor



- Bernoulli equation:

$$(1+\sigma)\gamma = \gamma_{max}, \ \sigma = 0.4$$

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Summary

We addressed the dynamics of jets with toroidal magnetic field reversals through axisymmetric special relativistic magnetohydrodynamic simulations.

The magnetic energy at the interface of field reversals is dissipated when the jet propagates through a ambient medium.

The Lorentz factor of the jet is accelerated due to the in situ energy conversion from the magnetic energy to the kinetic energy, through the thermal energy.

When the aspect ratio between the jet radius and the axial length of the field reversals is 0.1, all magnetic energy of the jet is dissipated. On the other hand, when its ratio is greater than unity, the magnetic dissipation and the subsequent acceleration of the jet are observed locally at the interface of field reversals.

The inhomogeneity of the jet may be responsible for the variability of the light curve of the astrophysical jet such as the blazar and gamma-ray burst.