

# Open issues in numerical modeling of core-collapse supernova

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解説

### 重力崩壊型超新星の物理——研究の現状と今後の課題

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重力崩壊型超新星爆発は、宇宙で起こる大規模な爆発現象である。爆発を駆動している星の中心付近では、高密度（核密度）かつ高温（10 MeV以上）環境が実現され、強い力（弱い力・電磁力・重力）という自然界で働く4つの基本的な力全てが爆発機構に関わっており、理論物理学の観点からも興味深い。爆発によって重元素の生成と宇宙空間への放出が起こるため、宇宙の化学組成を決める重要な天体現象である。また、爆発後には中性子星やブラックホールなどの高密度天体を残すことから、宇宙で起こる様々な宇宙の高エネルギー天体現象とも密接に関連する。このように、重力崩壊型超新星爆発の研究は非常に学際的な分野であり、素・核・宇宙・天文学などの幅広い分野の研究者らによって、実験・観測・理論・シミュレーションなどの様々なアプローチにより研究が行われている。超新星爆発を駆動している中心エンジンには、密着の物理過程が詳細に絡み合った系である。その爆発機構は複雑で、理論宇宙物理学の難問の一つとして位置づけられてきた。しかし、ここ10年ほどの間に、超新星爆発の理論は著しく進展した。特に、理論計算（数値シミュレーション）においては、それまで爆発の再現に失敗していたのに対し、近年では十分に成功するモデルが多く報告されている。こうした進展の一つの理由は、計算能力的向上と数値計算手法の発展のおかげで、より正確に詳細な物理過程を取り込んだ多次元ニュートリノ輻射流体計算が実行可能になったことである。例えば、第一原理計算に最も近いとされる、ボルツマン方程式を直接解く多次元輻射流体計算が「高価」などのスパコンで

少数のモデルに対して実行されている一方、近似的なニュートリノ輸送法を用いた多次元計算がより多くのモデルに対して系統的に行われている。また、ニュートリノと物質との強い相互作用の扱いについても精緻化が進み、例えば核子の weak current における形状因子やストレンジネスの影響、さらには多体効果なども、シミュレーションでは取り込まれている。シミュレーションが、長時間かつ様々なタイプの天体質量に対して系統的に行えるようになり、観測値の定量的な推定が行えるようになってきたことも、近年の重要な進歩である。実際、過去の超新星理論モデルとは違い、最終的な爆発エネルギーの値や形成される中性子星の質量や半径などが定量的に議論できるようになってきた。電磁波・重力波・ニュートリノに関する理論モデルの精度も相対的に上がり、マルチメッセンジャー天文学の発展にも貢献している。このように超新星爆発の研究は、近年著しく発展したが、それでも超新星爆発機構が完全に解明されたわけではない。実際、現在考慮されているニュートリノ反応の取り扱いは不確定性が大きく、それが爆発可否に影響する可能性がある。また、ニュートリノ集団運動に代表される量子運動論的な効果は、現在の誰も個人超新星爆発計算にも取り込まれておらず、現在の超新星爆発の理解を定量化してしまおうとしない、ニュートリノ反発計算の精度を上げ、量子運動論的ニュートリノ輻射輸送計算に基づいた超新星モデルの再構築が、今後10年の超新星爆発の理論的研究の主要なターゲットになるだろう。

用語解説

大規模計算 超新星爆発現象の再現に必要となる計算資源を必要とする。大規模計算は本論文および本誌上の掲載記事の特長を特徴とする。

中性子星 主に中性子から構成されている半径約10km程度の、質量は太陽よりも重く、密度は極めて高い天体である。超新星爆発後、残った星の中心部分に形成される。

ニュートリノ輻射流体シミュレーション ニュートリノと物質の相互作用を考慮し、物質の運動方程式を連立して、ニュートリノ輸送を同時に解く数値計算。以下に、従来の天体物理学の数値シミュレーションと異なる点を挙げる。ほとんどの場合、超新星爆発は、超新星爆発の中心部から始まる。

ニュートリノ集団運動 超新星爆発の中心部では、ニュートリノの自己相互作用が非常に重要となる。ニュートリノの自己相互作用は、超新星爆発の中心部で、ニュートリノの集団運動を特徴づける。ニュートリノの自己相互作用は、超新星爆発の中心部で、ニュートリノの集団運動を特徴づける。



## Review

### Physical mechanism of core-collapse supernovae that neutrinos drive

By Shoichi YAMADA,<sup>\*1,\*2,†</sup> Hiroki NAGAKURA,<sup>\*3</sup> Ryuichiro AKAHO,<sup>\*1</sup> Akira HARADA,<sup>\*4</sup> Shun FURUSAWA,<sup>\*5</sup> Wakana IWAKAMI,<sup>\*1</sup> Hirotada OKAWA,<sup>\*6</sup> Hideo MATSUFURU<sup>\*7</sup> and Kohsuke SUMIYOSHI<sup>\*8</sup>

(Edited by Katsuhiko SATO, M.J.A.)

**Abstract:** The current understanding of the mechanism of core-collapse supernovae (CCSNe), one of the most energetic events in the universe associated with the death of massive stars and the main formation channel of compact objects such as neutron stars and black holes, is reviewed for broad readers from different disciplines of science who may not be familiar with the object. Therefore, we emphasize the physical aspects than the results of individual model simulations, although large-scale high-fidelity simulations have played the most important roles in the progress we have witnessed in the past few decades. It is now believed that neutrinos are the most important agent in producing the commonest type of CCSNe. The so-called neutrino-heating mechanism will be the focus of this review and its crucial ingredients in micro- and macrophysics and in numerics will be explained one by one. We will also try to elucidate the remaining issues.

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## CCSN explosions can be reproduced in numerical simulations

Nagakura et al. 2019

See also:

Takiwaki's talk for a general review

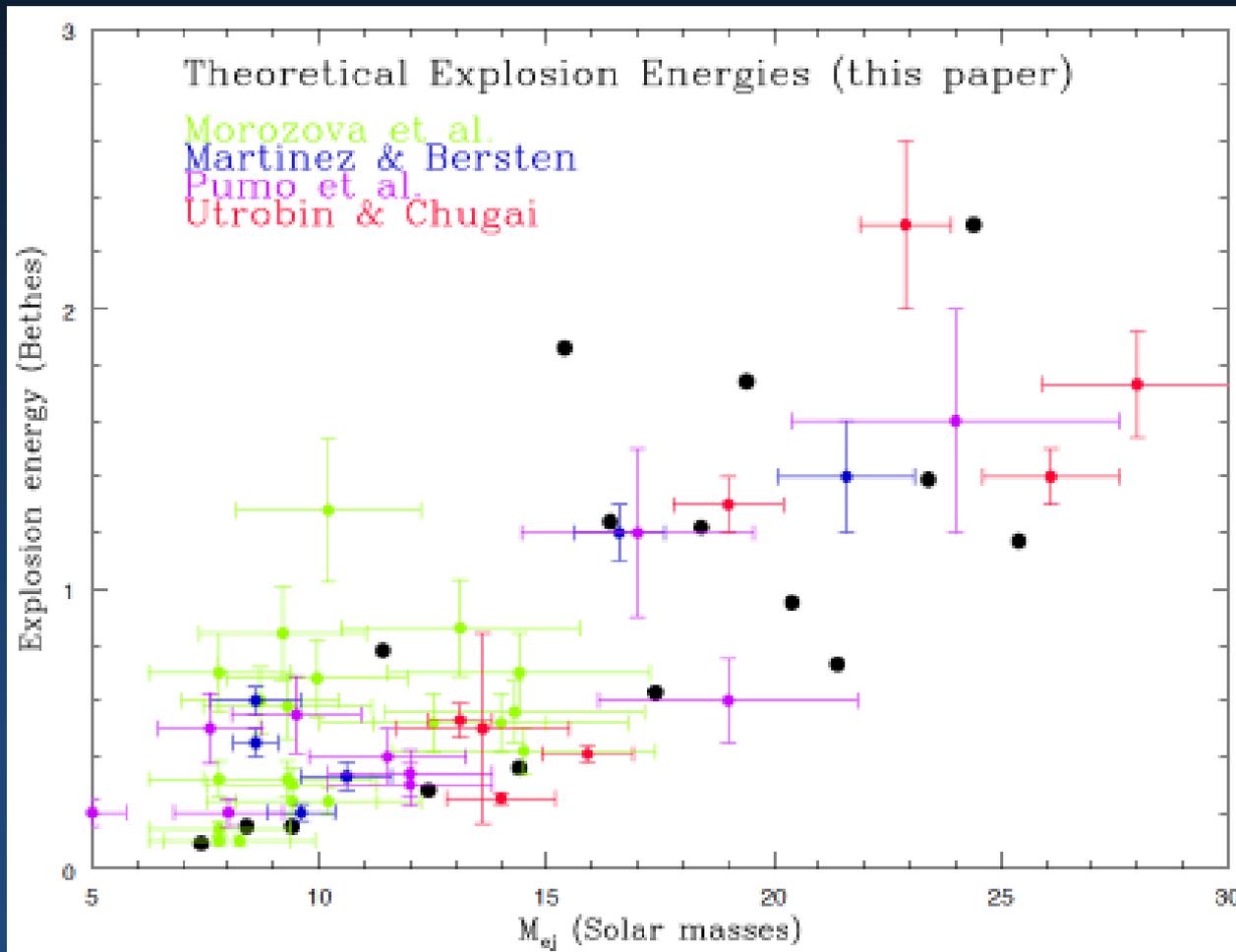
Wang's talk for more recent results

Boccioli's poster for systematic studies

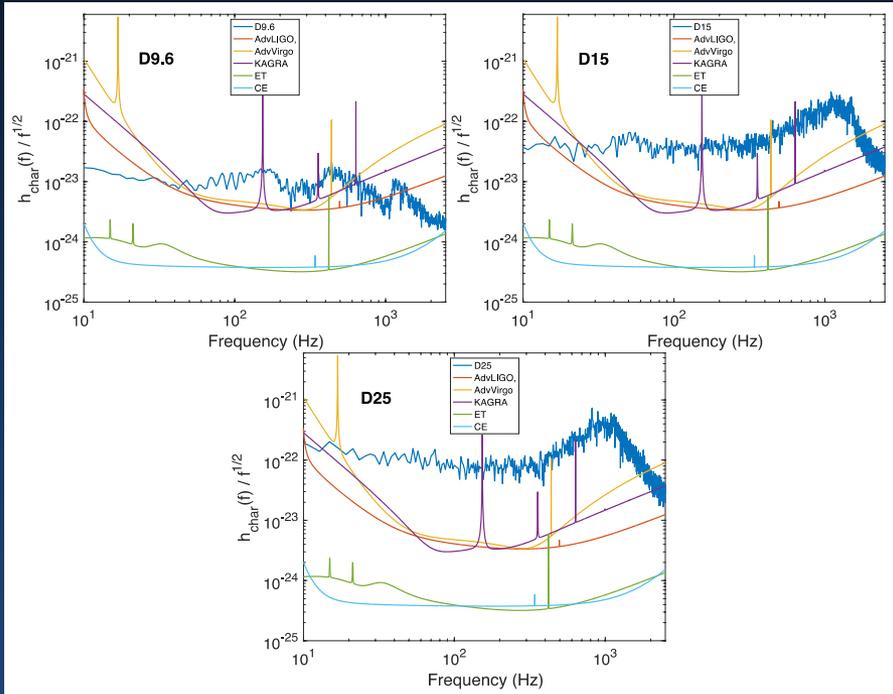


# Comparison between theory (CCSN simulations) and observations

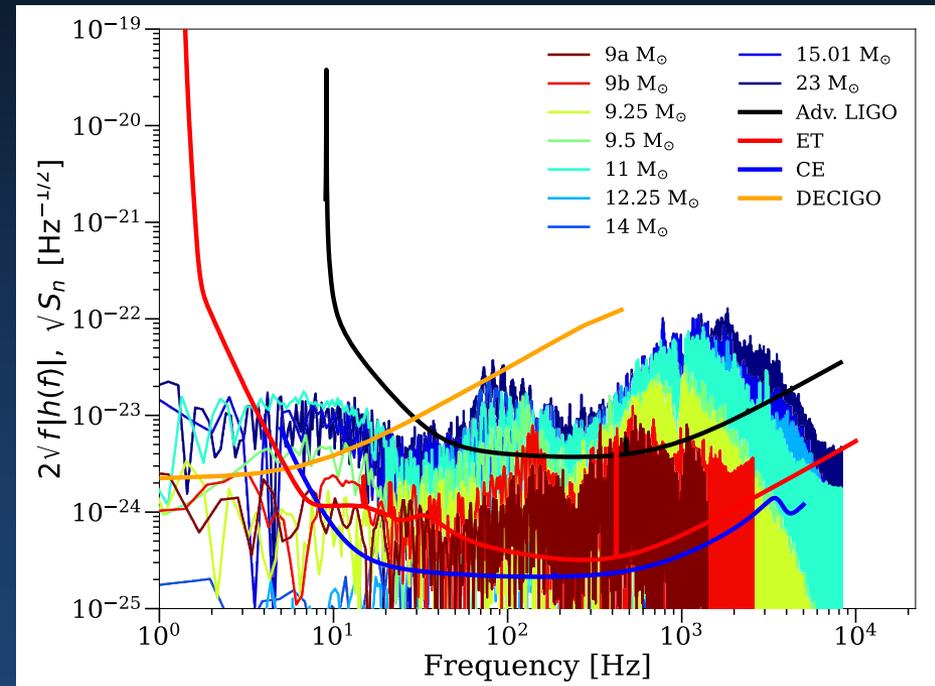
Explosion energy:



# Gravitational waves:



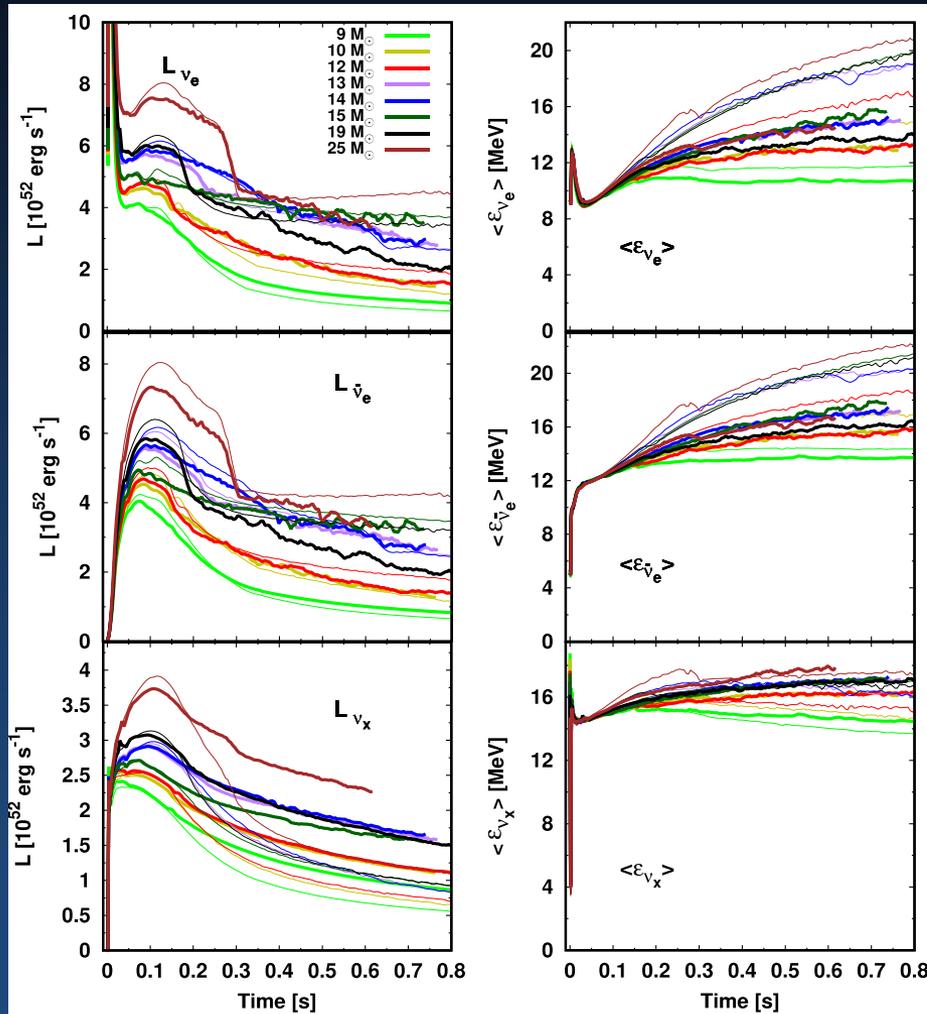
Mezzacappa et al. 2022



Vartanyan et al. 2023

→ See Sotani's talk: PNS asteroseismology for CCSN GWs.

# Neutrino signal:



Nagakura et al. 2021

1. Explosion models have low neutrino luminosity than those with non-explosions  
(due to less accretion components)

2. The average energy of electron-type neutrinos and their anti-partners are lower in 3D than 1D.

3. Neutrino luminosity of heavy-leptonic neutrinos are higher in 3D than 1D.  
(due to PNS convection)

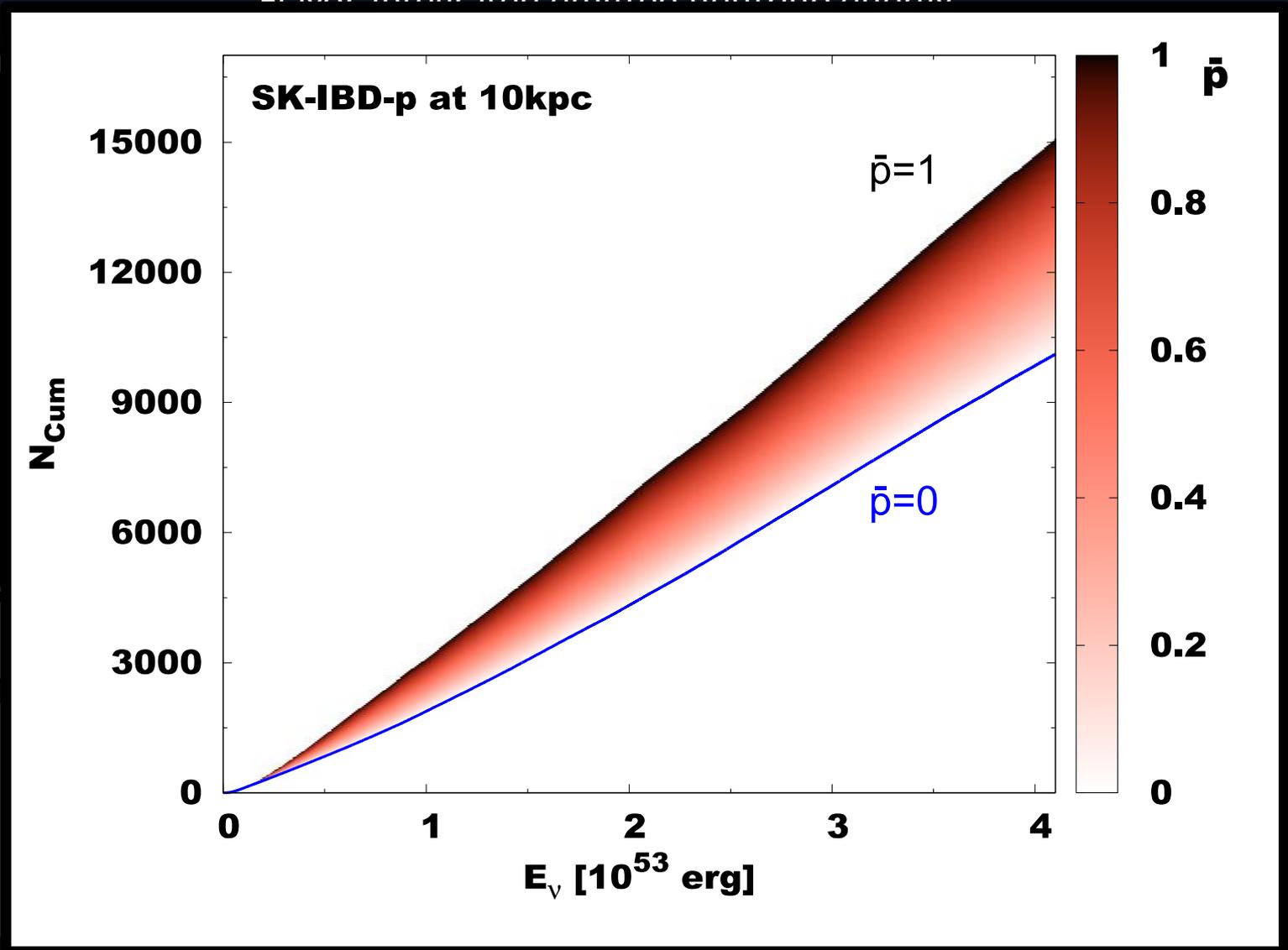
See also

Suwa's talk for neutrino signal in the late phase  
Nakanishi's poster for neutrino detection

# Correlation between TONE ( $E_\nu$ ) and $N_{cum}$ in neutrino detector

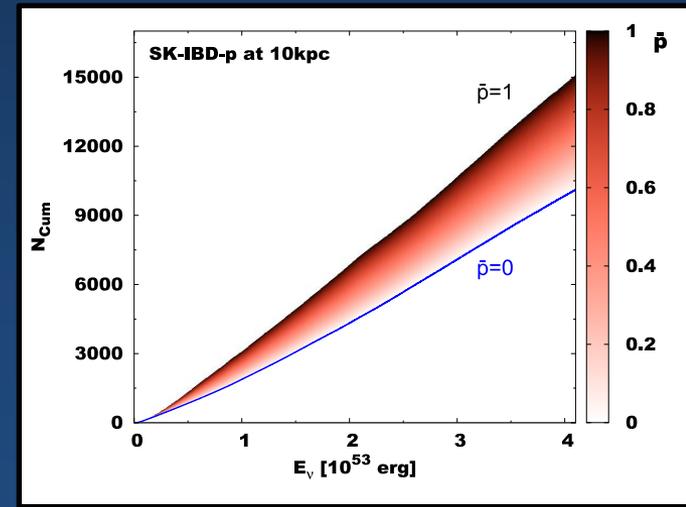
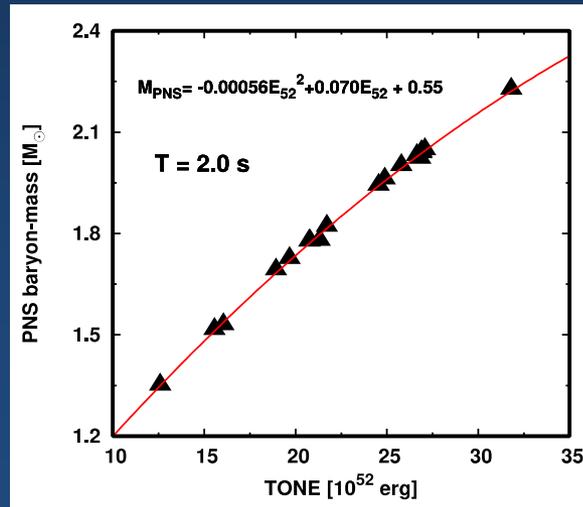
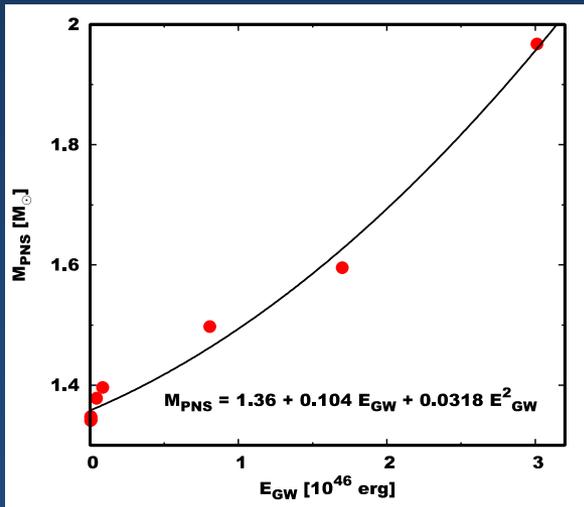
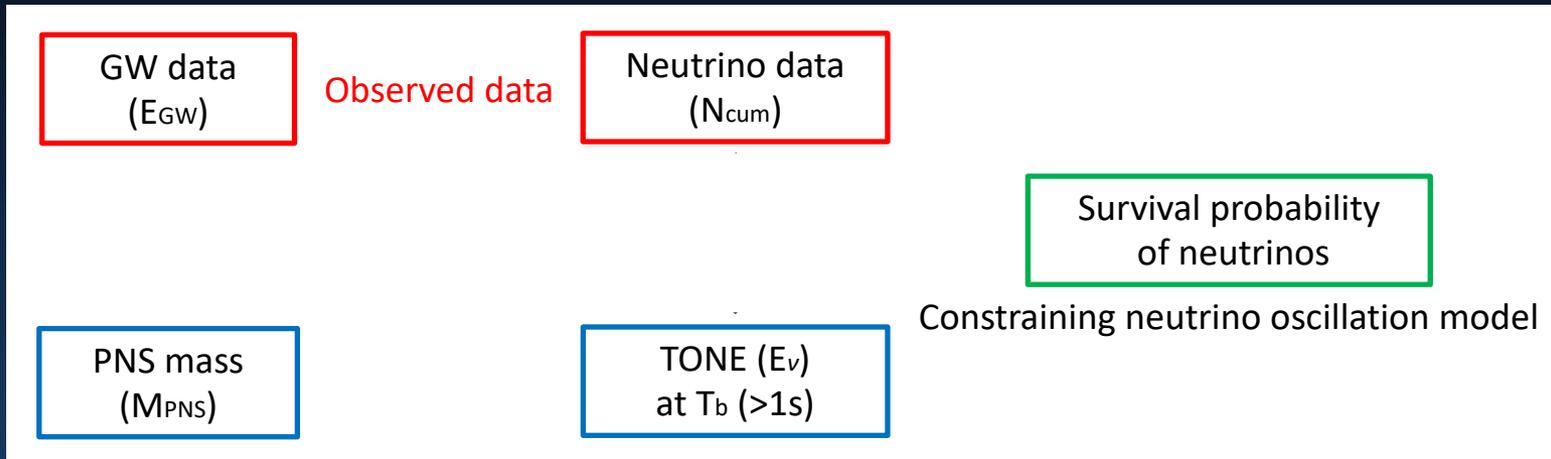
Flavor integrated emitted neutrino energy

Nagakura et al. 2021



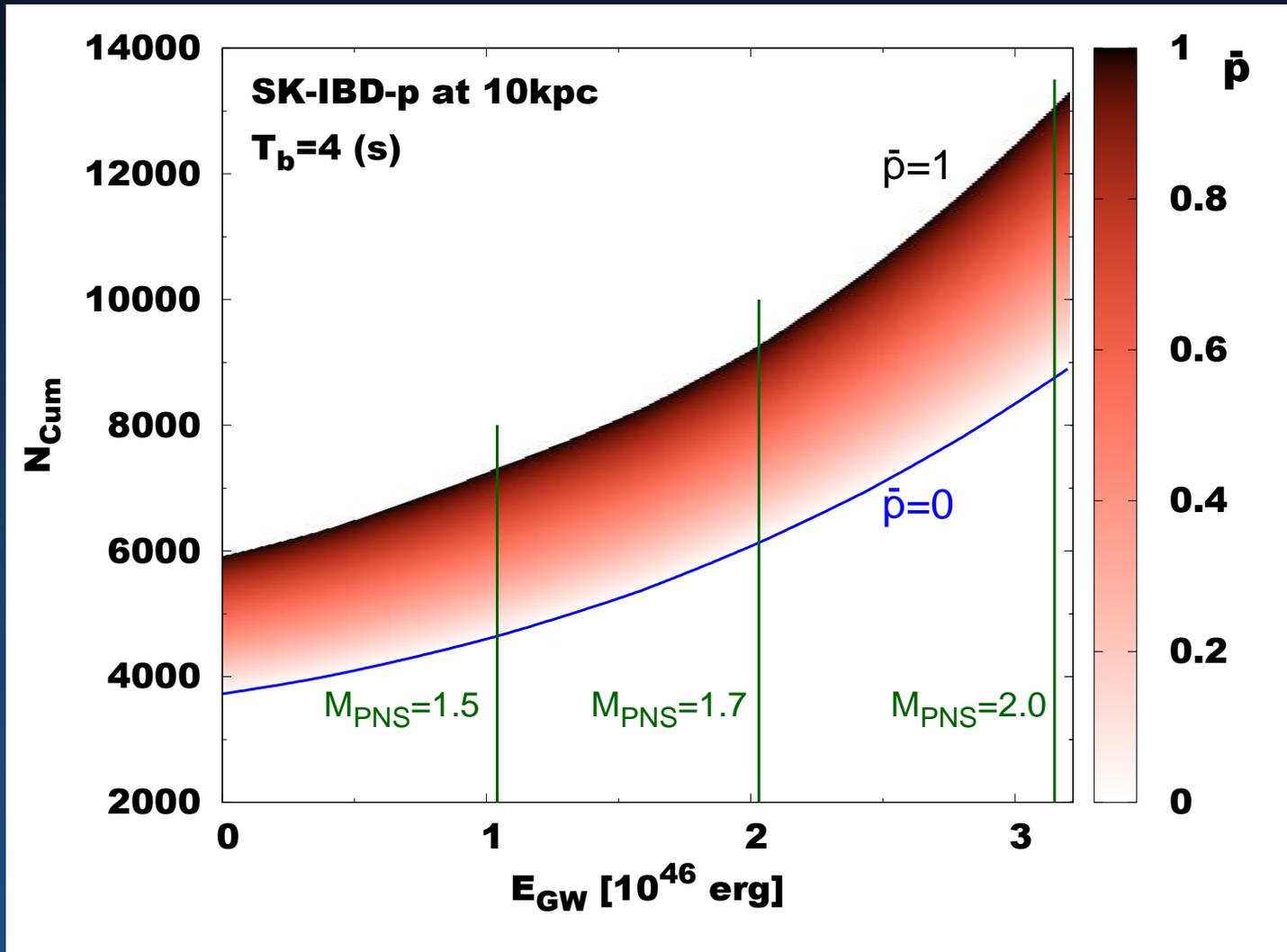
# Constraining neutrino oscillation in CCSNe by joint analysis of GW and neutrino signal

Nagakura and Vartanyan 2024



# Constraining neutrino oscillation in CCSNe by joint analysis of GW and neutrino signal

Nagakura and Vartanyan 2024

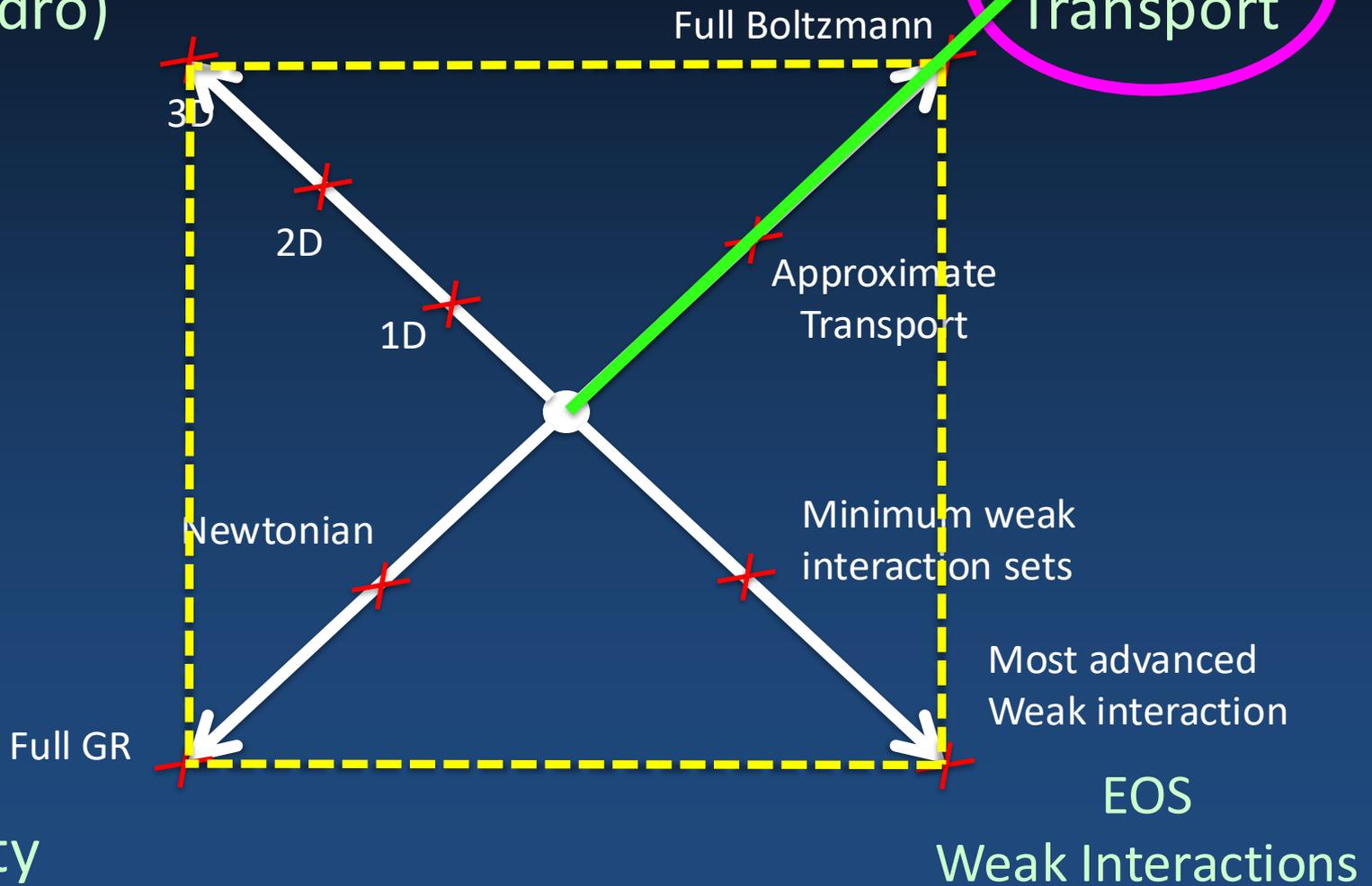


- There remain many issues in CCSN simulations

Dimensionality  
(for Hydro)

Beyond Boltzmann (QKE)

Neutrino  
Transport



Gravity

# General relativistic full Boltzmann neutrino transport

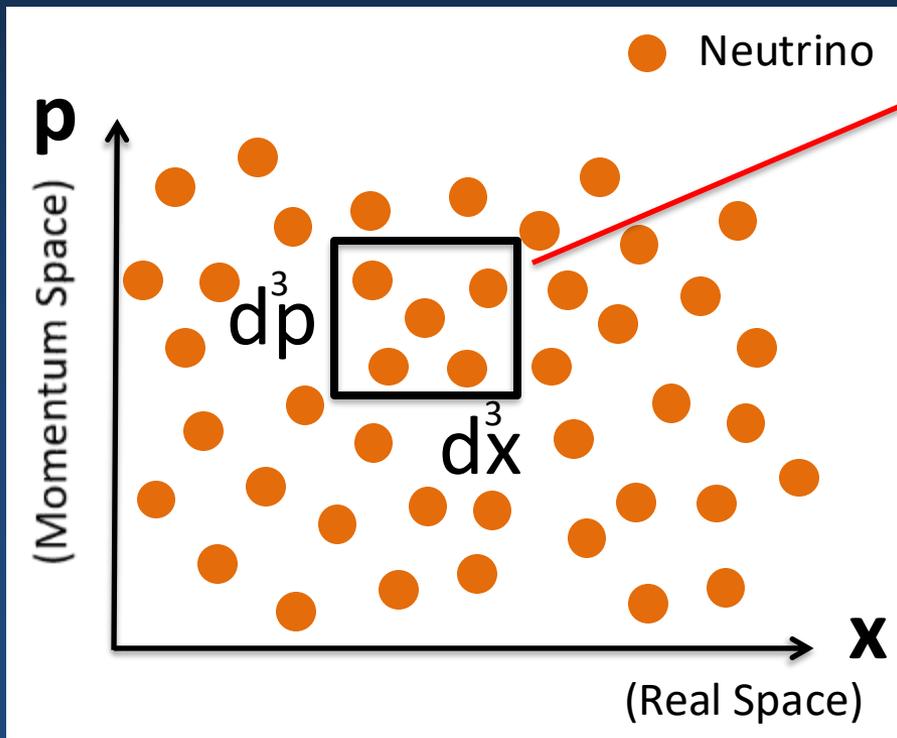
$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau} \frac{\partial f}{\partial p^i} = \left( \frac{\delta f}{\delta \tau} \right)_{\text{col}},$$

(Time evolution + Advection Term)

(Collision Term)

Nagakura et al. 2014, 2017, 2019  
Akaho et al. 2021

6 dimensional Phase Space



$$dN = f(t, \mathbf{p}, \mathbf{x}) d^3 p d^3 x$$

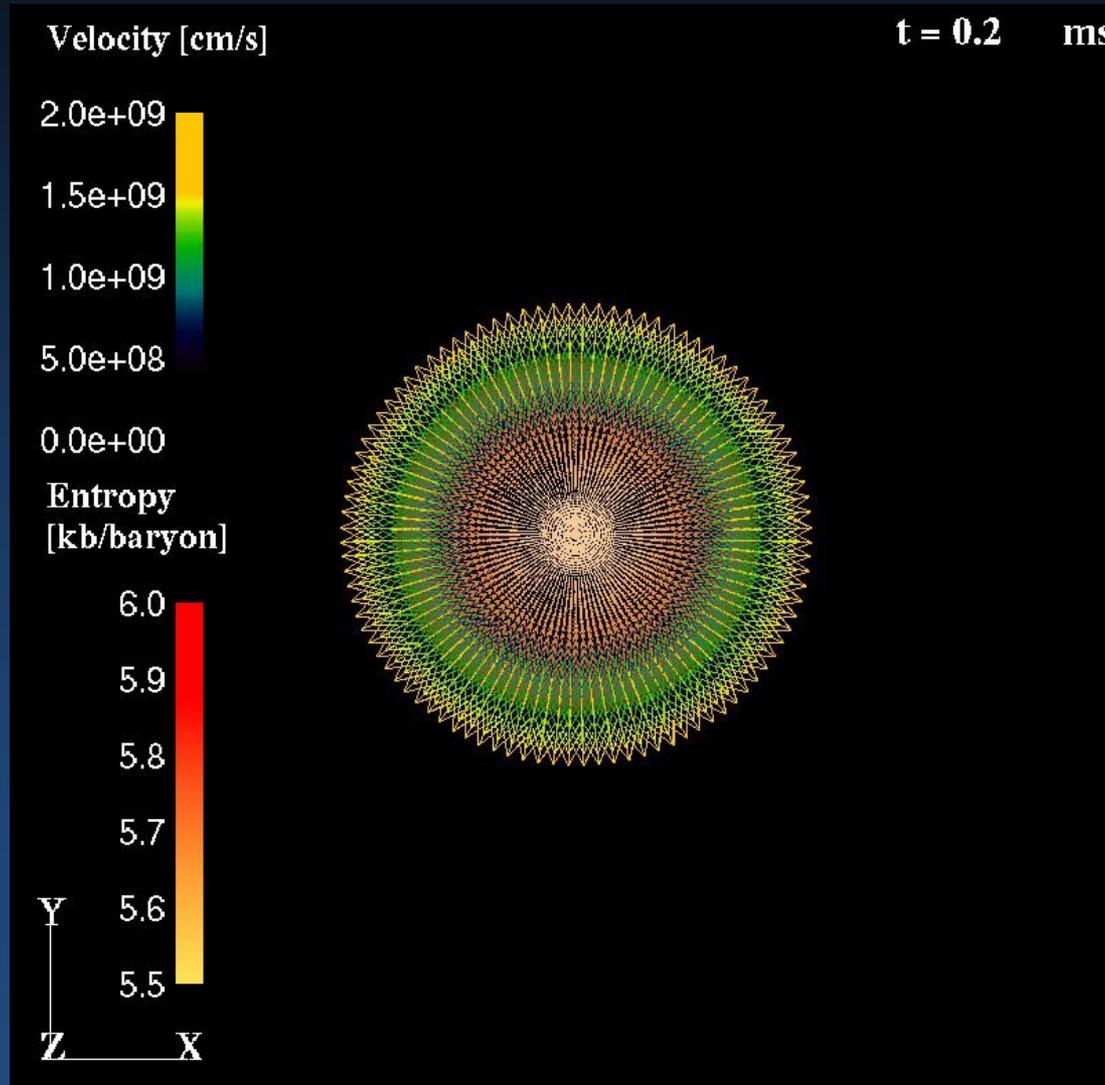
Conservative form of GR Boltzmann eq.

$$\begin{aligned} & \frac{1}{\sqrt{-g}} \frac{\partial(\sqrt{-g} \nu^{-1} p^\alpha f)}{\partial x^\alpha} \Big|_{q(t)} + \frac{1}{\nu^2} \frac{\partial}{\partial \nu} (-\nu f p^\alpha p_\beta \nabla_\alpha e^\beta_{(0)}) \\ & + \frac{1}{\sin \bar{\theta}} \frac{\partial}{\partial \bar{\theta}} \left( \nu^{-2} \sin \bar{\theta} f \sum_{j=1}^3 p^\alpha p_\beta \nabla_\alpha e^\beta_{(j)} \frac{\partial \ell_{(j)}}{\partial \bar{\theta}} \right) \\ & + \frac{1}{\sin^2 \bar{\theta}} \frac{\partial}{\partial \bar{\varphi}} \left( \nu^{-2} f \sum_{j=2}^3 p^\alpha p_\beta \nabla_\alpha e^\beta_{(j)} \frac{\partial \ell_{(j)}}{\partial \bar{\varphi}} \right) = S_{\text{rad}}, \end{aligned}$$

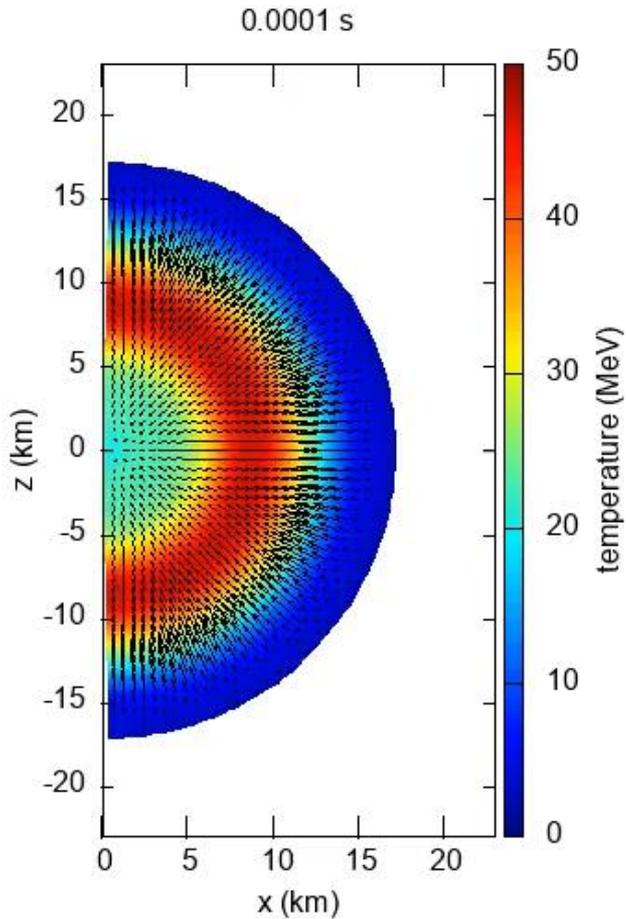
Shibata and HN et al. 2014  
(See also Cardall et al. 2013)

# - 3D CCSN simulations with full Boltzmann neutrino transport

Iwakami et al. 2020, 2021

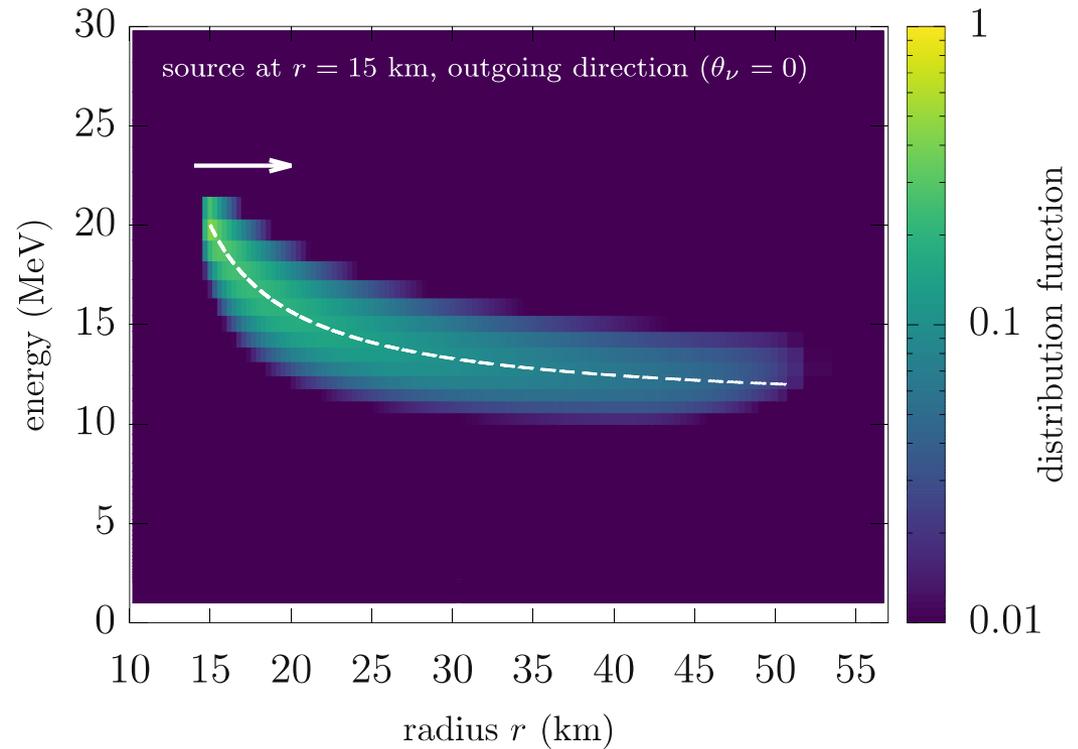


# GR simulations with full Boltzmann neutrino transport



Akaho et al. 2023

## Gravitational redshift in Black hole spacetime



Akaho et al. 2020

# Quantum Kinetics neutrino transport:

Vlasenko et al. 2014, Volpe 2015,  
Blaschke et al. 2016, Richers et al. 2019

$$p^\mu \frac{\partial f^{(-)}}{\partial x^\mu} + \frac{dp^i}{d\tau} \frac{\partial f^{(-)}}{\partial p^i} = -p^\mu u_\mu \overline{S}_{\text{col}}^{(-)} + ip^\mu n_\mu [\overline{H}^{(-)}, f^{(-)}],$$

Transport terms  
(same as Boltz eq.)

Collision term

Oscillation term

f is not a  
"distribution function"

## Density matrix

$$f^{(-)} = \begin{bmatrix} f_{ee}^{(-)} & f_{e\mu}^{(-)} & f_{e\tau}^{(-)} \\ f_{\mu e}^{(-)} & f_{\mu\mu}^{(-)} & f_{\mu\tau}^{(-)} \\ f_{\tau e}^{(-)} & f_{\tau\mu}^{(-)} & f_{\tau\tau}^{(-)} \end{bmatrix}$$

## Hamiltonian

$$\overline{H}^{(-)} = \overline{H}_{\text{vac}}^{(-)} + \overline{H}_{\text{mat}}^{(-)} + \overline{H}_{\nu\nu}^{(-)},$$

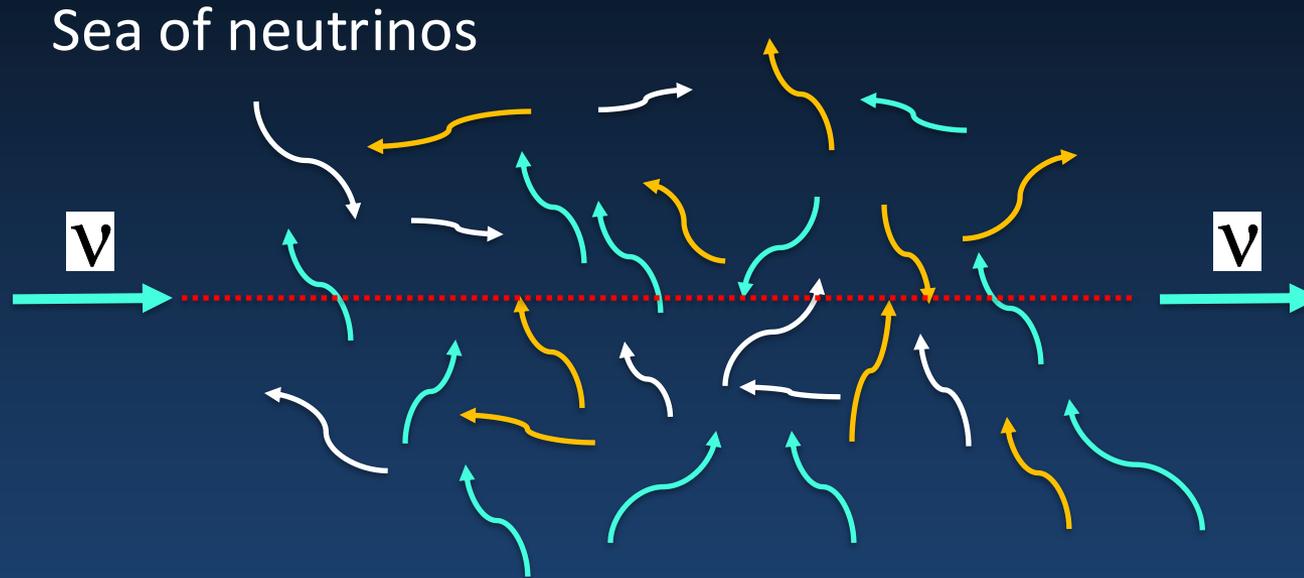
$$H_{\text{vac}} = \frac{1}{2\nu} U \begin{bmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{bmatrix} U^\dagger,$$

$$H_{\text{mat}} = D \begin{bmatrix} V_e & 0 & 0 \\ 0 & V_\mu & 0 \\ 0 & 0 & V_\tau + V_{\mu\tau} \end{bmatrix},$$

$$H_{\nu\nu} = \sqrt{2}G_F \int \frac{d^3q'}{(2\pi)^3} \left(1 - \sum_{i=1}^3 \ell'_{(i)} \ell_{(i)}\right) (f(q') - \bar{f}^*(q')),$$

# Neutrino oscillation induced by self-interactions

Pantalone 1992



1. Refractions by self-interactions induce neutrino flavor conversions, which is analogy to matter effects (e.g., MSW resonance).
2. The oscillation timescale is much shorter than the global scale of CCSN/BNSM.
3. Collective neutrino oscillation induced by neutrino-self interactions commonly occurs in CCSNe and BNSM environments.

# Rich flavor-conversion phenomena driven by neutrino-neutrino self-interactions

## - Slow-mode (Duan et al. 2010)

- Energy-dependent flavor conversion occurs.
- The frequency of the flavor conversion is proportional to

$$\sqrt{\omega\mu}$$

Vacuum:	$\omega = \frac{\Delta m^2}{2E_\nu}$ ,
Matter:	$\lambda = \sqrt{2}G_F n_e$ ,
Self-int:	$\mu = \sqrt{2}G_F n_\nu$ ,

## - Fast-mode (FFC) (Sawyer 2005)

- Collective neutrino oscillation in the limit of  $\omega \rightarrow 0$ .
- The frequency of the flavor conversion is proportional to
- Anisotropy of neutrino angular distributions drives the fast flavor-conversion.

$$\mu$$

## - Collisional instability (Johns 2021)

- Asymmetries of matter interactions between neutrinos and anti-neutrinos drive flavor conversion.

$$\text{Im} \left[ \frac{\Gamma - \bar{\Gamma}}{2} \pm \frac{\mu S}{(\mu D)^2 + 4} \right] - \frac{\Gamma + \bar{\Gamma}}{2}$$

$\Gamma$ : Matter-interaction rate

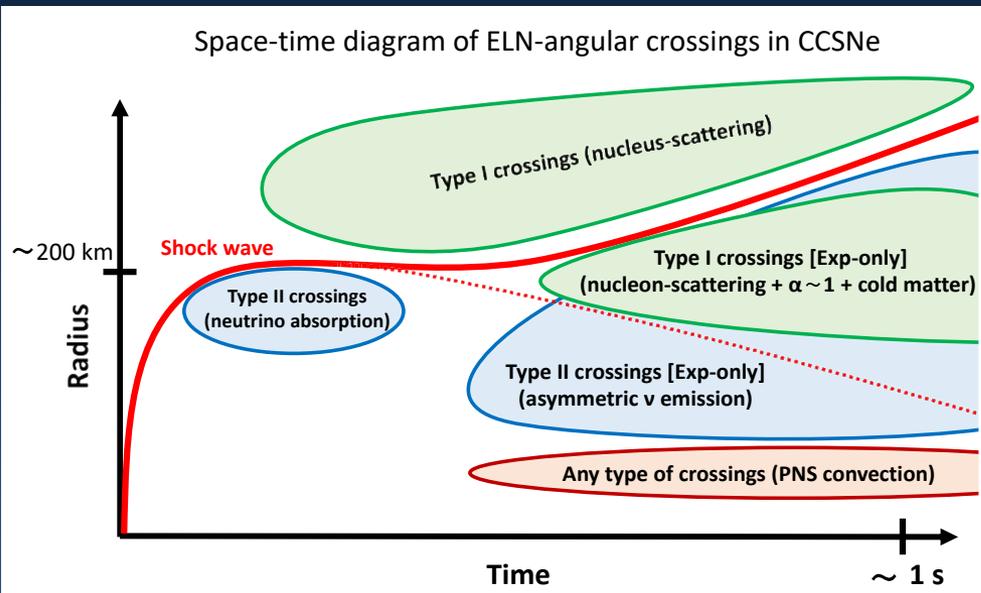
## - Matter-neutrino resonance (Malkus et al. 2012)

- The resonance potentially occur in BNSM/Collapsar environment (but not in CCSN).
- Essentially the same mechanism as MSW resonance.

$$|\lambda + \mu| \sim |\omega|$$

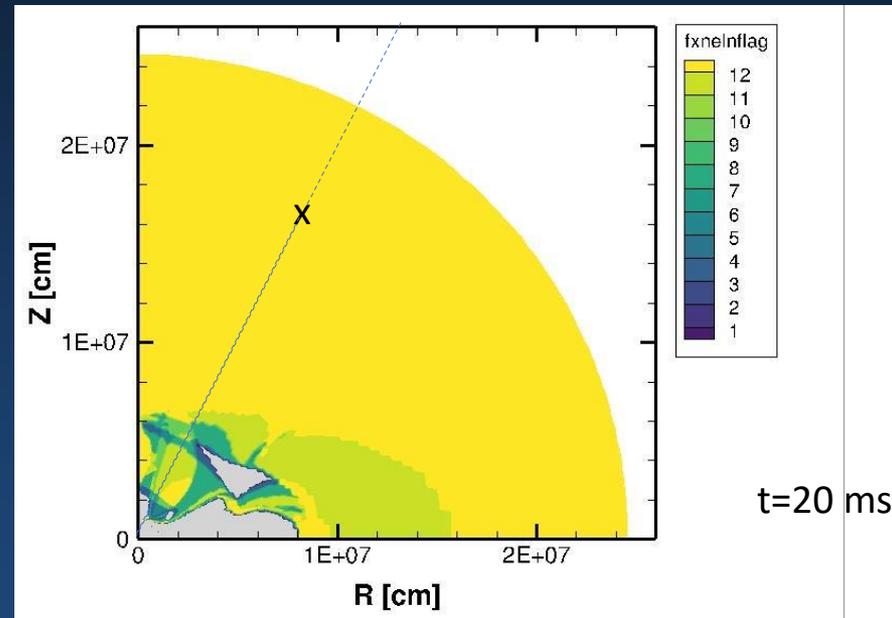
# FFC occurs in both CCSN and BNSM

## Core-collapse supernova



Nagakura et al. PRD 2021

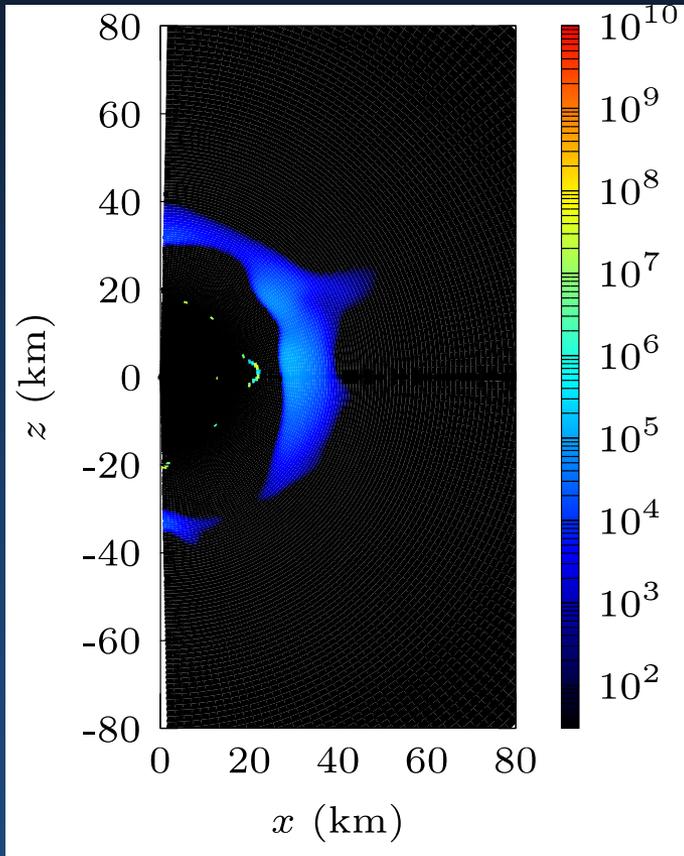
## Binary neutron star merger



Sumiyoshi et al. in prep

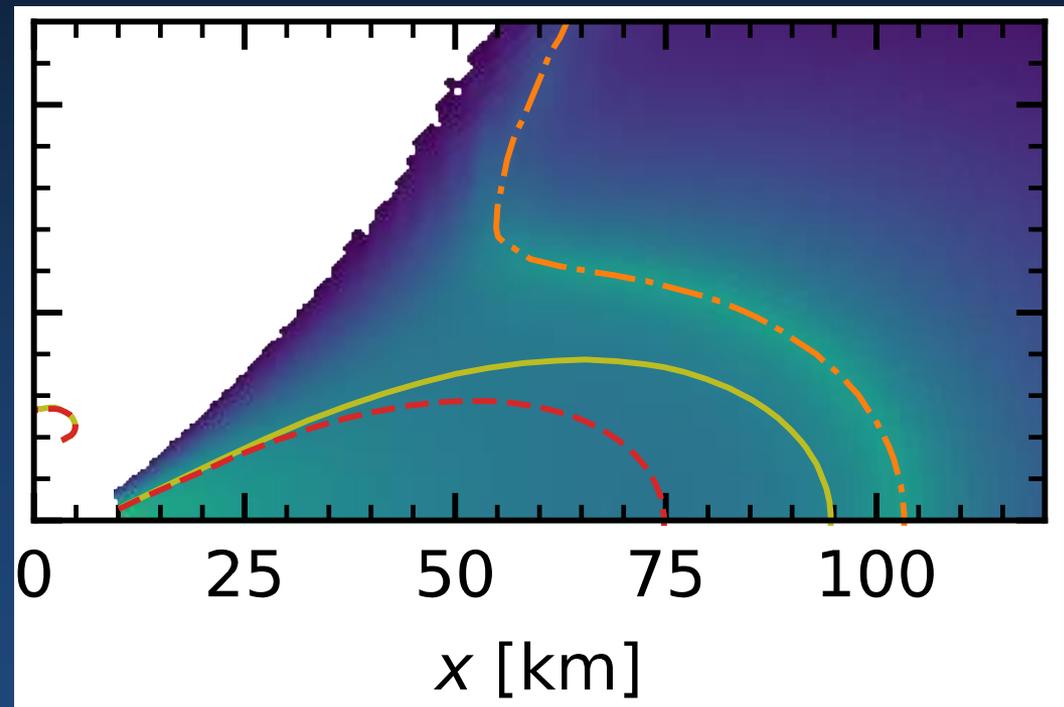
# Collisional instability also occurs in both CCSN and BNSM

## Core-collapse supernova



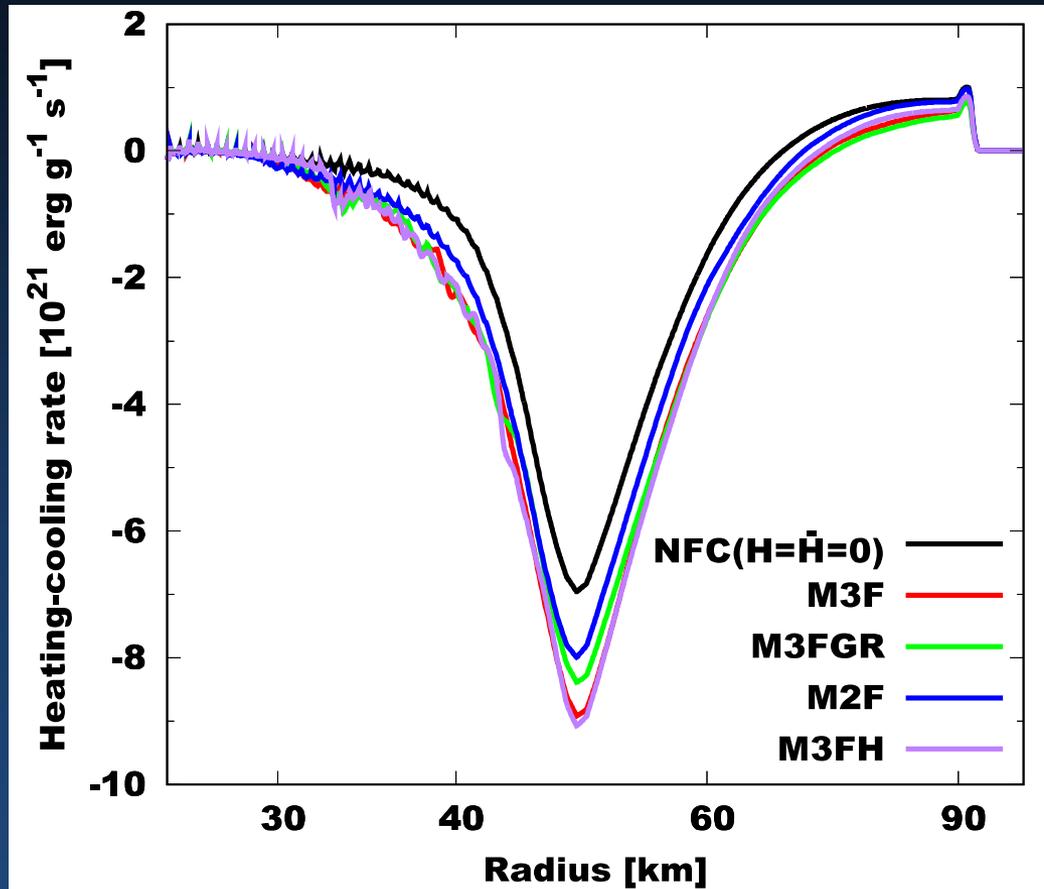
Akaho et al. 2023

## Binary neutron star merger



Xiong et al. 2022

# - Global Simulations of FFC (in CCSN) Nagakura PRL 2023



## Numerical setup:

Collision terms are switched on.

Fluid-profiles are taken from a CCSN simulation.

General relativistic effects are taken into account.

A wide spatial region is covered.

Three-flavor framework

Neutrino-cooling is enhanced by  $\sim 30\%$   
Neutrino-heating is suppressed by  $\sim 40\%$

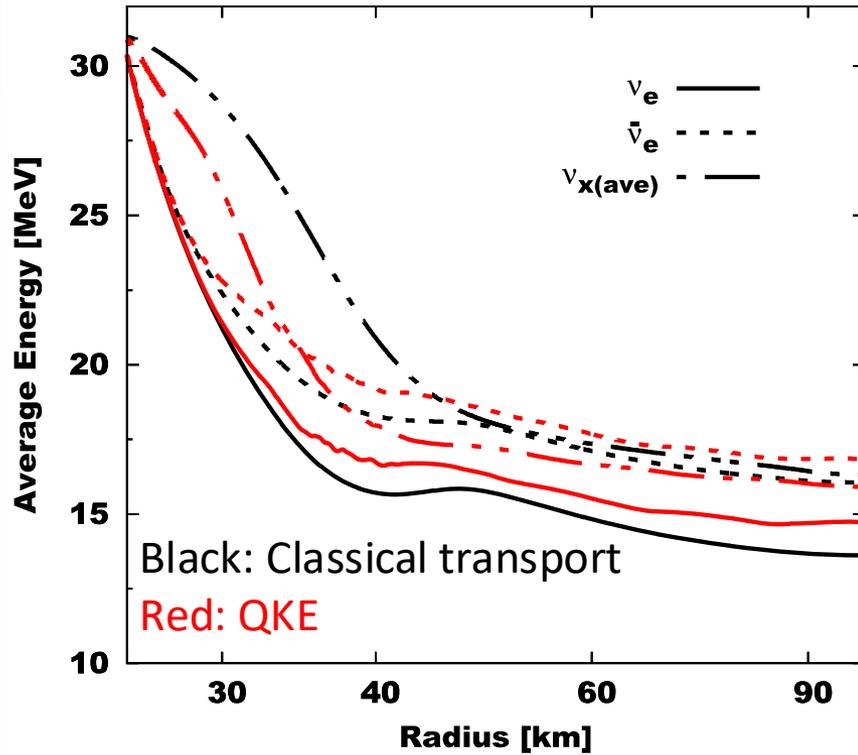


Huge impact on shock revival

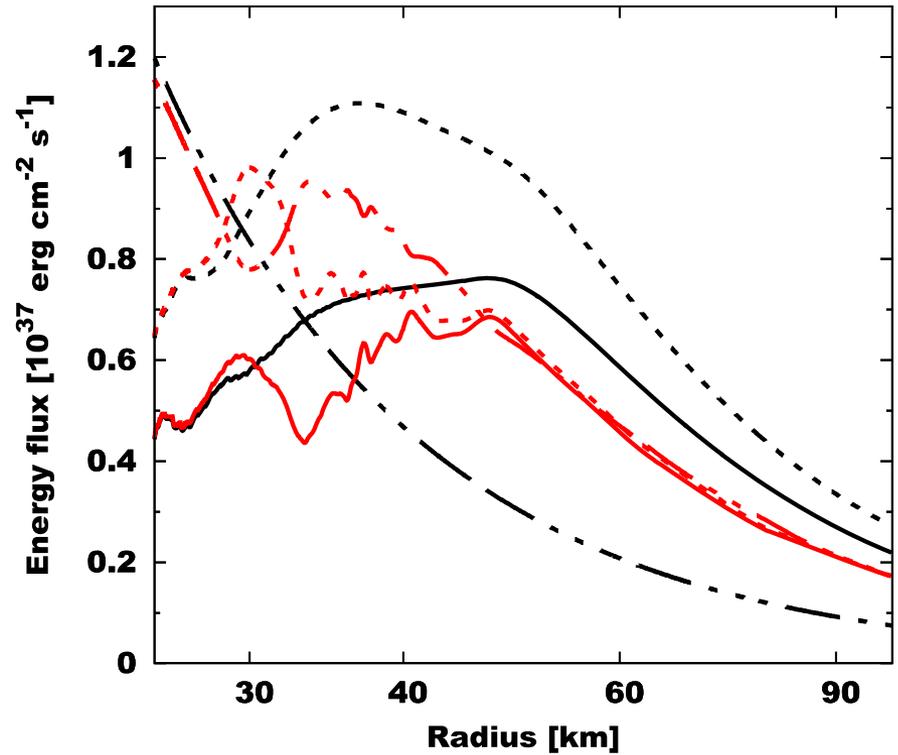
See also Mori's poster

# - Global Simulations of FFC (in CCSN) Nagakura PRL 2023

Average energy

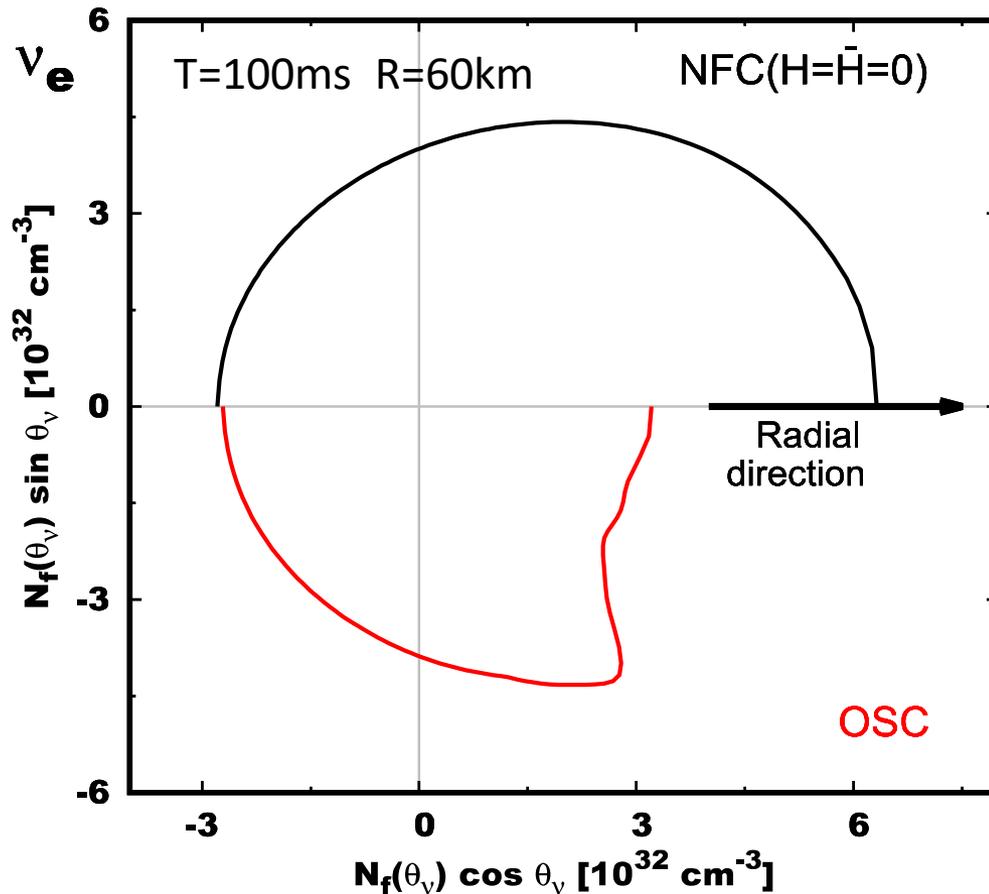


Energy flux



# - Global Simulations of FFC (in CCSN) Nagakura PRL 2023

Neutrino angular distributions



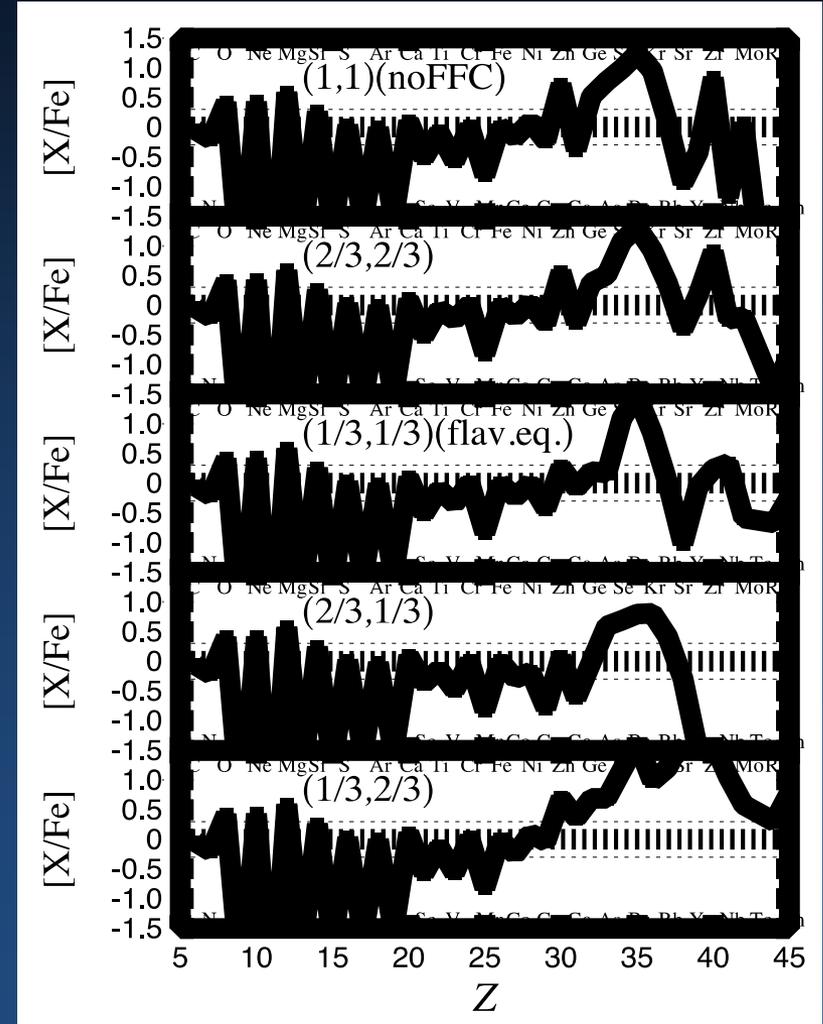
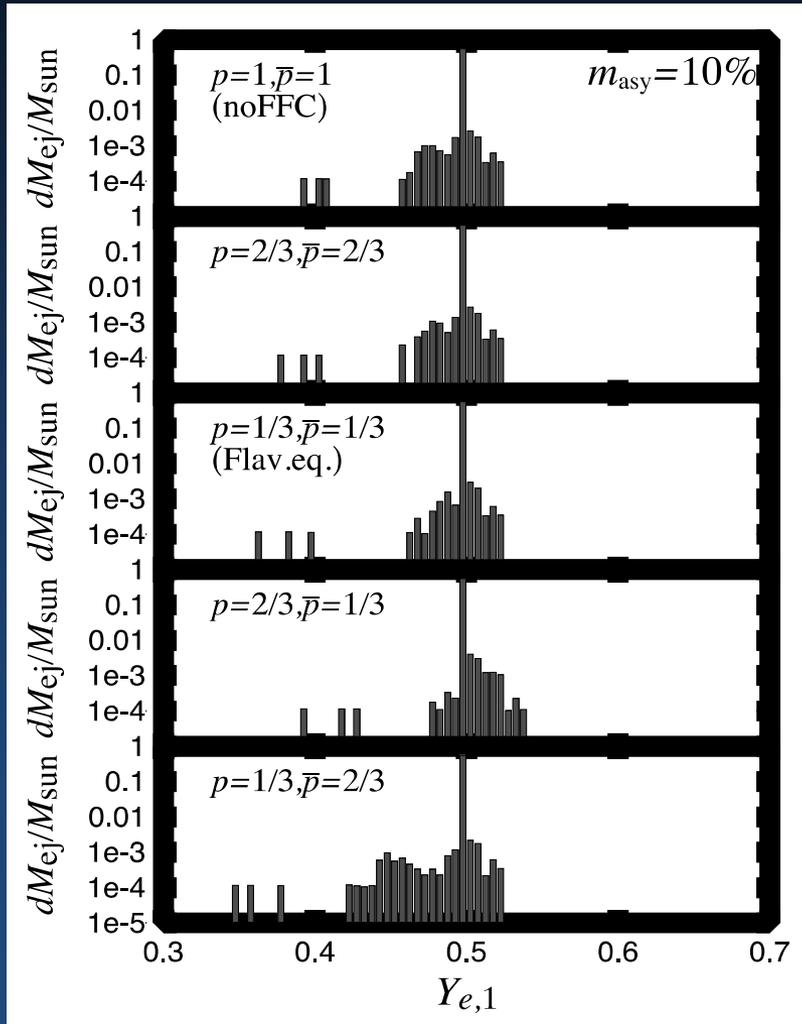
Angular distributions become very complicated.



Requiring multi-angle treatments in neutrino transport

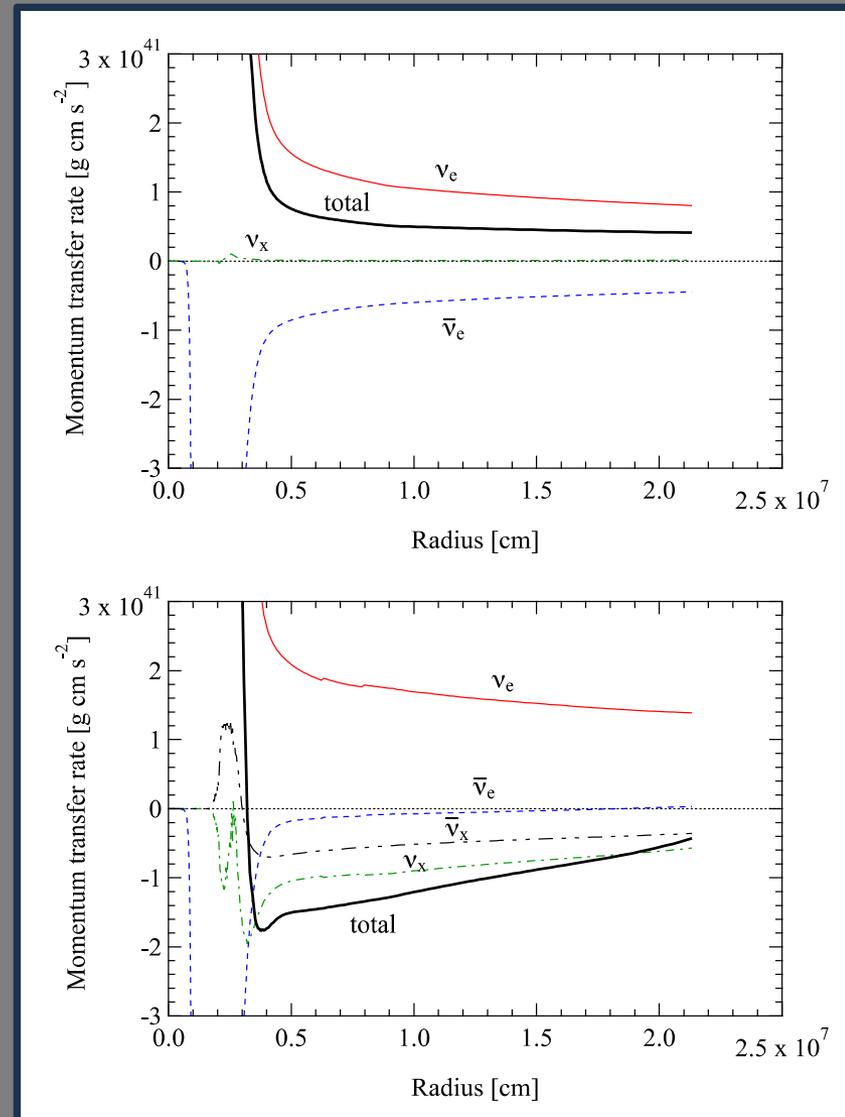
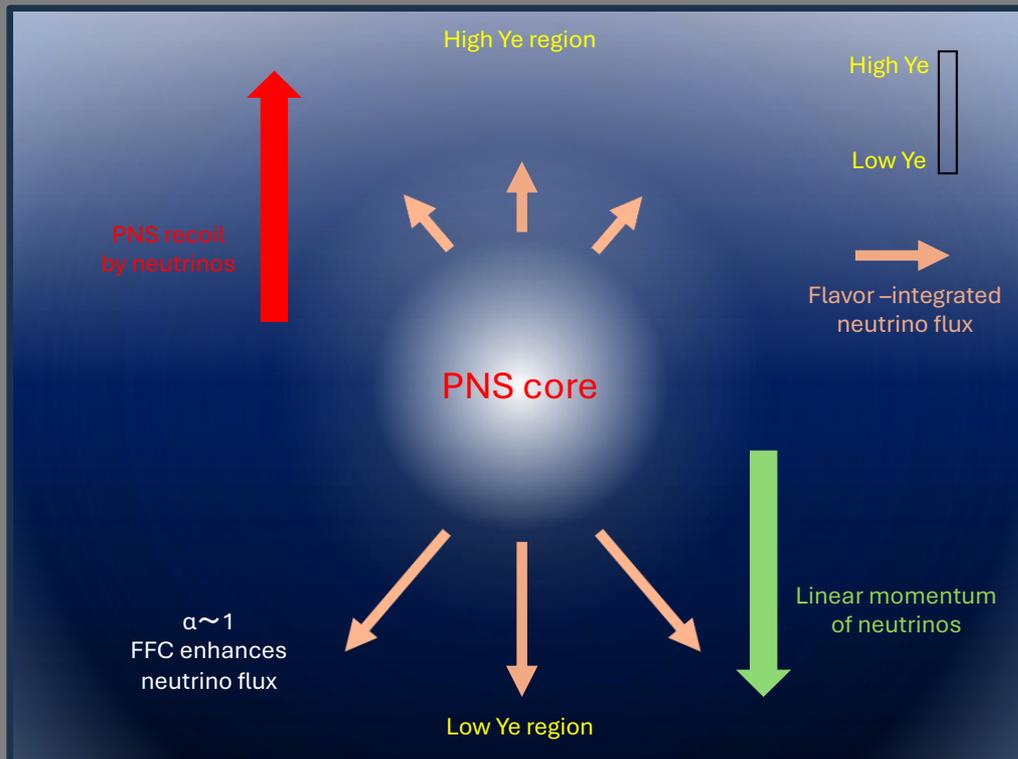
# - FFC can change explosive nucleosynthesis in CCSN ejecta

Fujimoto and Nagakura 2023



# - Neutron star kick powered by neutrino flavor conversions

Nagakura and Sumiyoshi 2024

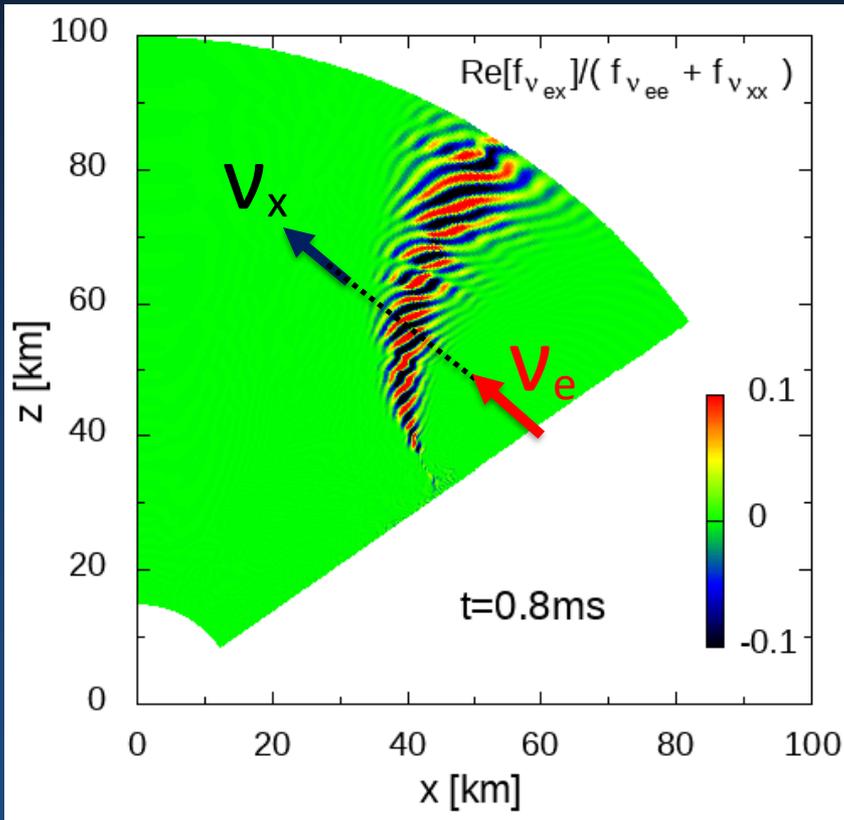


# - Global simulations of FFC (in binary neutron star merger)

Appearance of flavor swap and EXZS:

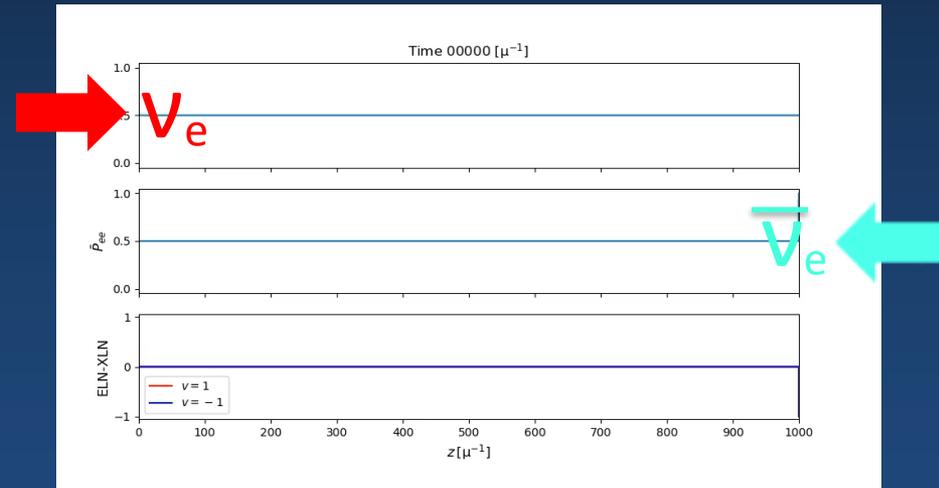
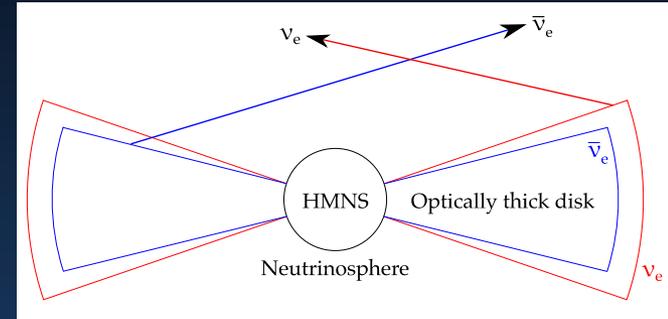
Flavor coherency

Nagakura 2023



Colliding-beam model

Zaizen and Nagakura 2024

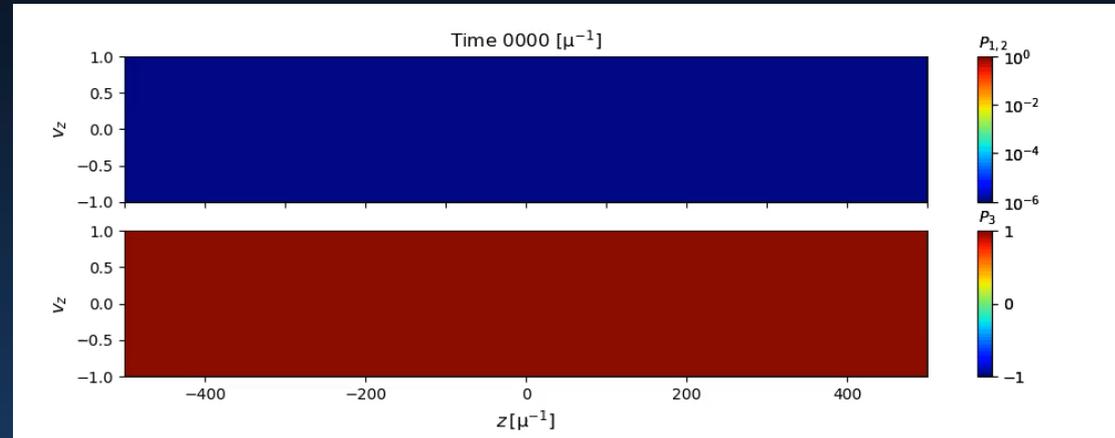
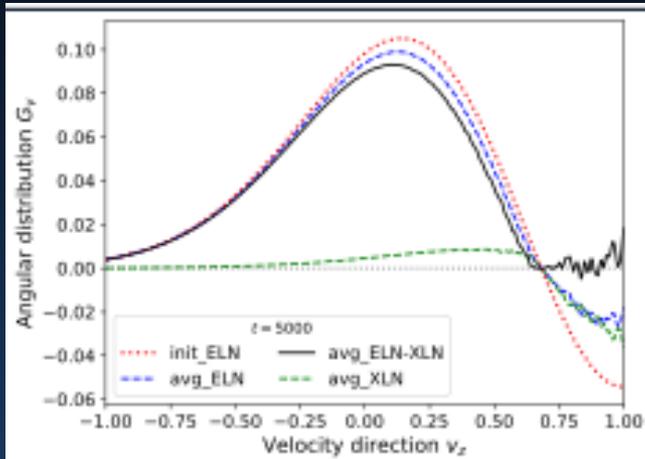


$$\partial_{t'}^2 P_3 \sim -4\mu^2 \left( 1 - (P_3)^2 \right)$$

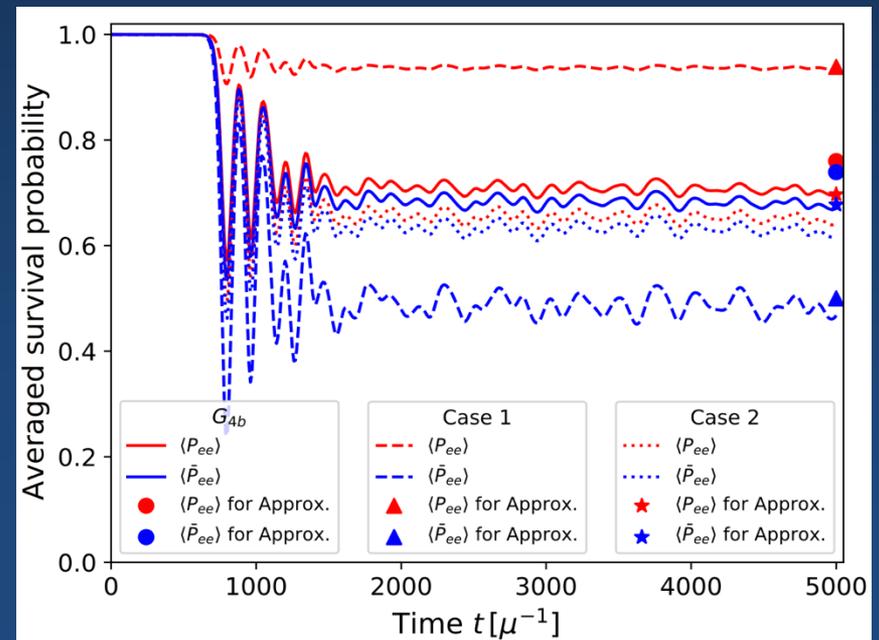
Neutrino flavor swap are inevitable.

# - Analytic scheme to determine asymptotic states of FFC

Zaizen and Nagakura 2022



Conservation law of neutrinos  
+  
Stability condition  
(disappearance of ELN-XLN crossings)

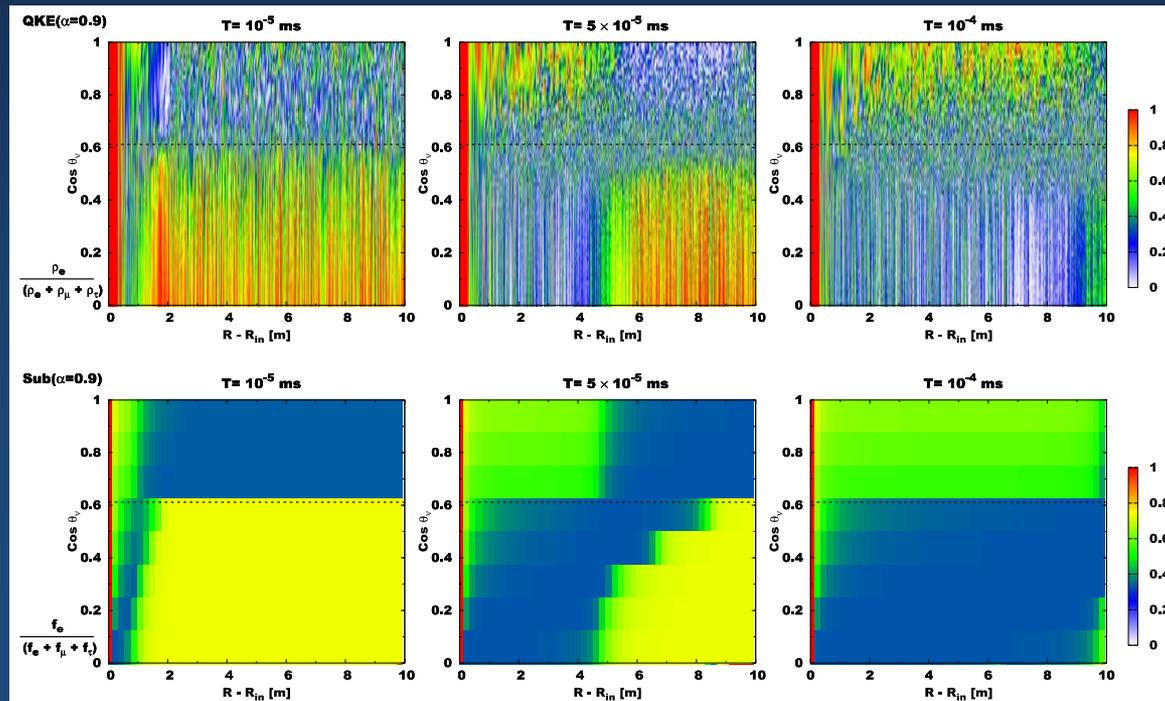


# - BGK Subgrid model Nagakura et al. 2024

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau} \frac{\partial f}{\partial p^i} = -p^\mu u_\mu S + ip^\mu n_\mu [H, f] \quad : \text{Full QKE}$$

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau} \frac{\partial f}{\partial p^i} = -p^\mu u_\mu S + p^\mu n_\mu \frac{1}{\tau_a} (f - f^a) \quad : \text{Relaxation-time approximation}$$

Radial-angular distributions for survival probability of electron-type neutrinos



## Summary:

- ✓ Remarkable progress on numerical modeling of CCSN have been made during the last decade.
- ✓ Observable signals can be discussed with realistic theoretical models.
- ✓ However, there are still many uncertainties in input physics. Neutrino quantum kinetics is the greatest one in the current CCSN theory.
- ✓ These uncertainties should not be overlooked, as they may be a game-changing ingredient.
- ✓ BGK subgrid model can offer a way to incorporate effects of flavor conversions into classical CCSN simulations.