Large-scale shell model study of β^- -decay properties **of N** = **126***,* **125 nuclei along the** *r***-process path**

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Collaborators:

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[RIBF ULIC mini-WS] Structure of neutron-rich matter revealed by beta decay at RIKEN Tokyo, 29–30 July, 2024

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⁵ [Summary and Conclusions](#page-40-0)

Nucleosynthesis of heavy elements: competition b/w neutron capture and β decay

The origin of most atomic nuclei with masses heavier than the iron group elements is attributed to **neutron capture** nucleosynthesis

slow neutron capture process (s-process)

neutron capture *<<* **beta decay.**

− Isotopes near stability are synthesized.

rapid neutron capture process (r-process)

neutron capture *>>* **beta decay.**

- − Explosive environment!
- $-$ High temperatures ($T \approx 10^9$ *K*), and neutron densities $(> 10^{20} \text{ neutrons/cm}^3)$.
- − Isotopes far from stability are synthesized.

Nucleosynthesis of heavy elements: competition b/w neutron capture and β decay

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▶ Fig. taken: A. Arcones et al., Astron Astrophys Rev **³¹**, 1 (2023).

▶ T. Kajino et al., Progress in Particle and Nuclear Physics **¹⁰⁷**, 109 (2019).

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Nucleosynthesis of heavy elements: competition b/w neutron capture and β decay

The origin of most atomic nuclei with masses heavier than the iron group elements is attributed to **neutron capture** nucleosynthesis

- *About half of the elements heavier than Fe are produced by* **r-process**
- *Lots of neutron-rich nuclei involved around A* ∼ 195*, for ex. Au, Pt,...*
- *Poor experimental information about the beta decay around A* ∼ 195
- ▶ Fig. taken: A. Arcones et al., Astron Astrophys Rev **³¹**, 1 (2023).
- ▶ T. Kajino et al., Progress in Particle and Nuclear Physics **¹⁰⁷**, 109 (2019).

Aim of this study

To investigate the β -decay properties like half-lives $(T_{1/2})$ and β -delayed neutron emission probabilities (*Pn*)

Competition b/w first-forbidden and Gamow-Teller transitions

To discuss the systematic study of Gamow-Teller strength distributions

Recent developments in large-scale shell model calculations like methodology, effective interactions, and the advent of computational resources offer the opportunity to conduct demanding large-scale calculations

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- Many nuclei are particularly important to the *r*-process nucleosynthesis
- The predicted β -decay half-lives play crucial role in determining the *r*-process time scale around the 3rd-peak \bullet

Model space in the present calculation

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Hamiltonian

Kuo-Herling hole: KHHE Hamiltonian [1] and modified in [2]

Shell Model code:

KSHELL: MPI + OpenMP hybrid code [3]

▶ [2] C. Yuan et al., Phys. Rev. C **¹⁰⁶**, 044314 (2022).

▶ [3] N. Shimizu et al., Computer Physics Communications **²⁴⁴**, 372 (2019).

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Previous shell-model studies of β **-decay near** $N = 126$

T. Suzuki et al.

N = 126 isotones with *Z* = 64 − 78

Previous

- Utilized original KHHE Hamiltonian
- Truncated model space used for $N = 126$ isotones

Present

- Utilized slightly modified KHHE Hamiltonian by C. Yuan et al.
- \bullet Performed full model space calculations for $N = 126$ isotones
- \bullet Extend $N = 126$ isotones chain to proton deficient side
- Also included $N = 125$ isotones chain in addition to $N = 126$ isotones
-

Q. Zhi et al.

$$
N = 126
$$
 isotones with $Z = 66 - 73$

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$Q(\beta^-)$ values of $N = 126, 125$ isotones

Shell model predicted $Q(\beta^-)$ values of $N = 126$, 125 isotones are compared with available experimental data and with different theoretical model calculations

$$
Q(\beta^-) = E_{\text{g.s.}}^{\text{par.}} - E_{\text{g.s.}}^{\text{dau.}} + \delta m,
$$

where the $\delta m = 0.782$ MeV.

- Shell model excellently reproduced the available experimental values
- Odd-even staggering is also observed similar to that observed in the FRDM19 and KTUY05 models
- Demonstrating a consistent trend with other theoretical models, such as FRDM19 and KTUY05, the predicted values are slightly smaller on the proton-deficient side.
- FRDM19: P. Möller et al., Atomic Data and Nuclear Data Tables **125**, 1 (2019).
- ▶ KTUY05: H. Koura et al., Prog. of Theo. Phys. **¹¹³**, 305 (2005).

*B***-decay Theory**

Partial half-life:

$$
t_{1/2} = \frac{\kappa}{f}
$$
, where $k = 6144$ sec,

and *f* is the dimensionless integrated shape function, which can be expressed as

$$
f = \int_1^{w_0} C(w) (w^2 - 1)^{1/2} w (w_0 - w)^2 F_0(Z, w) dw,
$$

where *w* is the total energy of the electron and w_0 is the maximum energy of *w*. The theoretical shape factor $C(w)$ is defined as

$$
C(w) = \sum_{k_e, k_v, K} \lambda_{k_e} \Big[M_K(k_e, k_v)^2 + m_K(k_e, k_v)^2 - \frac{2\gamma_{k_e}}{k_e w_e} M_K(k_e, k_v) m_K(k_e, k_v) \Big].
$$

The shape factor $C(w)$ for the Gamow-Teller transition is defined as

$$
C(w) = \frac{|\mathcal{M}_{GT}|^2}{2J_i + 1}.
$$

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-decay Theory

For first-forbidden β decay, the form of the shape factor $C(w)$ can be written in simple way

$$
C(w) = K_0 + K_1 w + K_{-1} w^{-1} + K_2 w^2,
$$

where, the coefficients K_n ($n = -1, 0, 1, 2$) depend on the first-forbidden nuclear matrix elements.

Summary of GT and FF nuclear matrix elements, where $\lambda = -g_A/g_V = 1.2701(25)$, $C_1 = \sqrt{4\pi/3}Y_1$, and $C = 1/\sqrt{2J_i + 1}$, $E_{\gamma} = Q(\beta^-) + \Delta E_C - \delta m$.

-decay Theory

To make a comparison conveniently between experiment and theory, we define the average shape factor by

$$
\overline{(C(W))} = \frac{f}{f_0} = \frac{6144 \, s}{f_0 t}
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with f_0

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 β -delayed neutron emission probabilities:

$$
P_n = \left(\sum_{E_f \ge S_n} \frac{1}{t_{i \to f}}\right) / \left(\sum_{\text{all } f} \frac{1}{t_{i \to f}}\right)
$$

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Parent (*i*: initial) Daughter (*f*: final) A_ZX_N \mathbf{P}_n **PRIS Links** --------------------------- S_n $\frac{A}{Z+1}Y$ _{N−1}

- \bullet The β -decay half-lives are evaluated by including both the Gamow-Teller and the first-forbidden transitions.
- GT strengths are calculated using the Lanczos strength function method [1].
- The moment

$$
S_k = \sum_{\nu} (E_{\nu} - E_i)^k |\langle \nu | \hat{O} | i \rangle|^2
$$

up to a sufficiently large *k* should be calculated. The Lanczos algorithm guarantees the correct moment up to $k = 2n - 1$ with *n* Lanczos iteration starting with $\vec{u_1} = \hat{O}(i)$.

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- \bullet In case of first-forbidden β decay: due to the involvement of six operators, the first-forbidden strength has been obtained by diagonalization of the Hamiltonian and calculated strength for 50 eigenvalues for each state in the daughter nuclei
- Full model space diagonalized:
	- *N* = 126 isotones with *Z* = 52 − 79 and
	- *N* = 125 isotones with *Z* = 52 − 59 and *Z* = 72 − 79

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	- *N* = 125 isotones with *Z* = 52 − 59 and *Z* = 72 − 79
- Due to the limitation of computational resources, monopole-based truncation is applied on $N = 125$ isotones with $Z = 60 - 71$

Lanczos method for strength distribution: Example ⁴⁸**Ca**

Results from: E. Caurier et al., Rev. Mod. Phys. 77, 427 (2005).

Lanczos method for strength distribution: Example ⁴⁸**Ca**

After 50 iterations, a good distribution is achieved, despite notable variations in individual strengths between 50 and 1000 iterations.

Results from: E. Caurier et al., Rev. Mod. Phys. 77, 427 (2005).

Effective operators: Gamow-Teller

To make a comparison of the shell model calculated β decay rate with the experimental one, we used effective operators in the calculations

 $\hat{O}^{\text{eff}} = q \times \hat{O}^{\text{free}}$

where, the bare operator \hat{O}^{free} is multiplied by a scaling factor (a name coined as a **quenching factor** in several earlier studies.)

To address the quenching factor in GT transitions in this study:

Effective operators: First-forbidden

- \bullet The operators of rank 0, 1, and 2 in the first-forbidden β decay also required a quenching factor.
- To obtain quenching factors for all FF operators, we have minimized the chi-square function between theoretical and experimental average shape factors by including unique and non-unique first-forbidden transitions (**two unique and sixteen non-unique transitions**).

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Result: Half-lives of experimentally known β decay

Shell-model predicted half-lives (sec) in comparison to the existing experimental data

We introduce *r* as a measure of deviation

$$
r = \log_{10} (T_{1/2}^{\text{calc}} / T_{1/2}^{\text{exp}}),
$$

and its mean value and standard deviation can be written as

$$
\bar{r} = \frac{1}{n} \sum_{i=1}^{n} r_i,
$$

and

▶ Exp: A. I. Morales et al., Phys. Rev. Lett. **¹¹³**, 022702 (2014).

▶ SM 2018: T. Suzuki et al., The Astrophysical Journal **⁸⁵⁹**, 133 (2018).

Discrepancies of shell-model half-lives from the experimental ones \bar{r} σ $10^{\bar{r}}$ 10^{σ} n -0.18 0.17 0.66 1.48 5

Result: Half-lives of *N* = 126 **isotones**

Present shell-model predicted β -decay half-lives for $N = 126$ isotones are compared with different theoretical model calculations and existing experimental data.

Result: Half-lives of *N* = 126 **isotones**

Present shell-model predicted β -decay half-lives for $N = 126$ isotones are compared with the previous shell model calculated and existing experimental data.

SM: T. Suzuki et al., The Astrophysical Journal 859, 133 (2018).
 BM: Q. Zhi et al., Phys. Rev. C 87, 025803 (2013).

▶ Exp.: A. I. Morales et al., Phys. Rev. Lett. **¹¹³**, 022702 (2014).

Result: Half-lives of *N* = 125 **isotones**

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Result: Contribution of GT and FF transitions in Half-lives

half-lives of $N = 126$ (top) and $N = 125$ (bottom) isotones

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- half-lives of $N = 126$ (top) and $N = 125$ (bottom) isotones
- GT transition dominated by the $v0h_{9/2} \rightarrow \pi 0h_{11/2}$

- ▶ The GT strength peaks are observed at low excitation energies between 3-6 MeV and 5-7 MeV for even-*Z* and odd-*Z* of parent nuclei, respectively.
- This peak, dominated by the $v_0 h_{9/2} \rightarrow \pi 0 h_{11/2}$ transition, is enhanced on the proton deficient side because the Pauli-blocking effect caused by the occupying the valence proton $0h_{11/2}$ orbit is weakened.

Result: Total Gamow-Teller strength distributions as a function of $\overline{\pi}$ ⁰ h _{1/2} **orbit**

▶ As the proton number increases, the proton $0h_{11/2}$ orbit becomes occupied, and simultaneously, the sum of GT strength decreases due to the **Pauli blocking effect**, reaching almost zero near $Z = 82$.

Result: β -delayed neutron emission probabilities $P_n(\%)$

• No experimental information available about the β -delayed neutron emission probabilities P_n ^(%) in this region

FRDM19: P. Möller et al., Atomic Data and Nuclear Data Tables 125, 1 (2019).

▶ KTUY05: H. Koura et al., Prog. of Theo. Phys. **¹¹³**, 305 (2005).

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Future:

 \bullet Further, we will analyze the impact of the present calculated β -decay half-lives and β -delayed **neutron emission probability on the r-process abundance distribution (with Nobuya Nishimura).**

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