Large-scale shell model study of β^- -decay properties of N = 126, 125 nuclei along the *r*-process path

Anil Kumar

Center for Computational Sciences University of Tsukuba



Collaborators:

N. Shimizu (CCS, Tsukuba Univ.), Y. Utsuno (JAEA), P. C. Srivastava (IITR), and C. Yuan (SYSU)

[RIBF ULIC mini-WS] Structure of neutron-rich matter revealed by beta decay at RIKEN Tokyo, 29-30 July, 2024

Anil Kumar (CCS, TU)

Shell Model Studies in Beta Decays of N = 126 isotones



Introduction

2 Shell model calculations



4 Results

5 Summary and Conclusions

Nucleosynthesis of heavy elements: competition b/w neutron capture and β decay

The origin of most atomic nuclei with masses heavier than the iron group elements is attributed to **neutron capture** nucleosynthesis

slow neutron capture process (s-process)

neutron capture << beta decay.

- Isotopes near stability are synthesized.

rapid neutron capture process (r-process)

neutron capture >> beta decay.

- Explosive environment!
- High temperatures ($T \approx 10^9 \text{ K}$), and neutron densities (> 10^{20} neutrons/cm³).
- Isotopes far from stability are synthesized.

Nucleosynthesis of heavy elements: competition b/w neutron capture and β decay

The origin of most atomic nuclei with masses heavier than the iron group elements is attributed to **neutron capture** nucleosynthesis





Fig. taken: A. Arcones et al., Astron Astrophys Rev 31, 1 (2023).

T. Kajino et al., Progress in Particle and Nuclear Physics 107, 109 (2019).

Shell Model Studies in Beta Decays of N = 126 isotones

Nucleosynthesis of heavy elements: competition b/w neutron capture and <u>*B*</u> decav

The origin of most atomic nuclei with masses heavier than the iron group elements is attributed to **neutron capture** nucleosynthesis





- About half of the elements heavier than Fe are produced by **r-process**
- Lots of neutron-rich nuclei involved around A ~ 195, for ex. Au. Pt....
- Poor experimental information about the beta decay around $A \sim 195$ ٩
- Fig. taken: A. Arcones et al., Astron Astrophys Rev 31, 1 (2023).
- T. Kajino et al., Progress in Particle and Nuclear Physics 107, 109 (2019).

Aim of this study



Fig. taken: A. Evdokimov et al., "XII International Symposium on Nuclei in the Cosmos", PoS, (2012).



Fig. taken: A. Evdokimov et al., "XII International Symposium on Nuclei in the Cosmos", PoS, (2012).



To investigate the β -decay properties like half-lives $(T_{1/2})$ and β -delayed neutron emission probabilities (P_n)

Competition b/w first-forbidden and Gamow-Teller transitions

To discuss the systematic study of Gamow-Teller strength distributions

Recent developments in large-scale shell model calculations like methodology, effective interactions, and the advent of computational resources offer the opportunity to conduct demanding large-scale calculations

Fig. taken: A. Evdokimov et al., "XII International Symposium on Nuclei in the Cosmos", PoS, (2012).



To investigate the β -decay properties like half-lives $(T_{1/2})$ and β -delayed neutron emission probabilities (P_n)

Competition b/w first-forbidden and Gamow-Teller transitions

To discuss the systematic study of Gamow-Teller strength distributions

Recent developments in large-scale shell model calculations like methodology, effective interactions, and the advent of computational resources offer the opportunity to conduct demanding large-scale calculations

- Many nuclei are particularly important to the *r*-process nucleosynthesis
- The predicted β -decay half-lives play crucial role in determining the *r*-process time scale around the 3rd-peak

Fig. taken: A. Evdokimov et al., "XII International Symposium on Nuclei in the Cosmos", PoS, (2012).

Anil Kumar (CCS, TU)

Model space in the present calculation



Model space in the present calculation



Inclusion of **first-forbidden** β decay

 For nuclei within this region, protons and neutrons occupy different shells with different parity

Model space in the present calculation



Inclusion of **first-forbidden** β decay

- For nuclei within this region, protons and neutrons occupy different shells with different parity
- When undergoing with β decay, only one Gamow-Teller transition $\nu 0h_{9/2} \rightarrow \pi 0h_{11/2}$ possible with same parity ($\Delta J = 0, \pm 1, \Delta \pi = N_0$)

Model space in the present calculation



Inclusion of **first-forbidden** β decay

- For nuclei within this region, protons and neutrons occupy different shells with different parity
- When undergoing with β decay, only one Gamow-Teller transition $\nu 0h_{9/2} \rightarrow \pi 0h_{11/2}$ possible with same parity ($\Delta J = 0, \pm 1, \Delta \pi = N_0$)
- Due to parity change between two different shells of proton and neutron, becomes important to the consideration of first-forbidden transitions ($\Delta J = 0, 1, 2, \Delta \pi = \text{Yes}$)

Model space in the present calculation



Inclusion of **first-forbidden** β decay

- For nuclei within this region, protons and neutrons occupy different shells with different parity
- When undergoing with β decay, only one Gamow-Teller transition $\nu 0h_{9/2} \rightarrow \pi 0h_{11/2}$ possible with same parity ($\Delta J = 0, \pm 1, \Delta \pi = N_0$)
- Due to parity change between two different shells of proton and neutron, becomes important to the consideration of first-forbidden transitions ($\Delta J = 0, 1, 2, \Delta \pi = Yes$)

Hamiltonian

Kuo-Herling hole: KHHE Hamiltonian [1] and modified in [2]

Shell Model code:

KSHELL: MPI + OpenMP hybrid code [3]



[2] C. Yuan et al., Phys. Rev. C 106, 044314 (2022).

[3] N. Shimizu et al., Computer Physics Communications 244, 372 (2019).

Anil Kumar (CCS, TU)

Shell Model Studies in Beta Decays of N = 126 isotones

Previous shell-model studies of β **-decay near** N = 126

T. Suzuki et al.

• N = 126 isotones with Z = 64 - 78

Previous

- Utilized original KHHE Hamiltonian
- Truncated model space used for N = 126 isotones

Present

- Utilized slightly modified KHHE Hamiltonian by C. Yuan et al.
- Performed full model space calculations for N = 126 isotones
- Extend N = 126 isotones chain to proton deficient side
- Also included N = 125 isotones chain in addition to N = 126 isotones
- Discuss the distribution of Gamow-Teller strength

Q. Zhi et al.

• N = 126 isotones with Z = 66 - 73



Shell Model Studies in Beta Decays of N = 126 isotones

$Q(\beta^{-})$ values of N = 126, 125 isotones



Shell model predicted $Q(\beta^-)$ values of N = 126, 125 isotones are compared with available experimental data and with different theoretical model calculations

$$Q(\beta^{-}) = E_{\text{g.s.}}^{\text{par.}} - E_{\text{g.s.}}^{\text{dau.}} + \delta m,$$

where the $\delta m = 0.782$ MeV.

- Shell model excellently reproduced the available experimental values
- Odd-even staggering is also observed similar to that observed in the FRDM19 and KTUY05 models
- Demonstrating a consistent trend with other theoretical models, such as FRDM19 and KTUY05, the predicted values are slightly smaller on the proton-deficient side.
- FRDM19: P. Möller et al., Atomic Data and Nuclear Data Tables 125, 1 (2019).
- KTUY05: H. Koura et al., Prog. of Theo. Phys. 113, 305 (2005).

Anil Kumar (CCS, TU)

Partial half-life:

$$t_{1/2} = \frac{\kappa}{f}$$
, where $k = 6144$ sec,

and f is the dimensionless integrated shape function, which can be expressed as

$$f = \int_{1}^{w_0} C(w) (w^2 - 1)^{1/2} w (w_0 - w)^2 F_0(Z, w) dw,$$

where *w* is the total energy of the electron and w_0 is the maximum energy of *w*. The theoretical shape factor C(w) is defined as

$$C(w) = \sum_{k_e, k_v, K} \lambda_{k_e} \Big[M_K(k_e, k_v)^2 + m_K(k_e, k_v)^2 - \frac{2\gamma_{k_e}}{k_e w_e} M_K(k_e, k_v) m_K(k_e, k_v) \Big].$$

The shape factor C(w) for the Gamow-Teller transition is defined as

$$C(w) = \frac{|\mathcal{M}_{\rm GT}|^2}{2J_i + 1}.$$
H. Behrens, W. Bühring, *Electron Radial Wave Functions and Nuclear Beta-Decay* (Clarendon Press, Oxford, 1982).

Anil Kumar (CCS, TU)

Shell Model Studies in Beta Decays of N = 126 isotones

For first-forbidden β decay, the form of the shape factor C(w) can be written in simple way

$$C(w) = K_0 + K_1 w + K_{-1} w^{-1} + K_2 w^2,$$

where, the coefficients $K_n(n = -1, 0, 1, 2)$ depend on the first-forbidden nuclear matrix elements.

Summary of GT and FF nuclear matrix elements, where $\lambda = -g_A/g_V = 1.2701(25)$, $C_1 = \sqrt{4\pi/3}Y_1$, and $C = 1/\sqrt{2J_i + 1}$, $E_\gamma = Q(\beta^-) + \Delta E_C - \delta m$.

Fransition	Rank	Notations	Nuclear matrix element (NME)	NME in non-relativistic approximation
GT	0	$\mathcal{M}_{\mathrm{GT}}$	$\lambda \langle f \boldsymbol{\sigma} t_{-} i \rangle$	
FF	0	\mathcal{M}_0^S	$\lambda\sqrt{3}\langle f ir[C_1\otimes \boldsymbol{\sigma}]^0t i\rangle C$	
		\mathcal{M}_0^T	$\lambda\sqrt{3}\langle f \gamma_5t i\rangle C$	$-\lambda\sqrt{3}\langle f (i/M_N)[\boldsymbol{\sigma}\otimes\nabla]^0t i\rangle C$
	1	x	$-\langle f irC_1t i\rangle C$	
		<i>ξ</i> ′ y	$-\langle f \boldsymbol{\alpha}t_{-} i\rangle C$	$E_{\gamma}x$
		и	$\lambda \sqrt{2} \langle f ir[C_1 \otimes \boldsymbol{\sigma}]^1 t i \rangle C$	
	2	z	$-2\lambda \langle f ir[C_1\otimes \boldsymbol{\sigma}]^2 t i\rangle C$	

To make a comparison conveniently between experiment and theory, we define the average shape factor by

$$\overline{(C(W))} = \frac{f}{f_0} = \frac{6144 \ s}{f_0 t}$$

with f_0

$$f_0 = \int_1^{W_0} (W^2 - 1)^{1/2} W(W_0 - W)^2 F_0(Z, W) dW$$

To make a comparison conveniently between experiment and theory, we define the average shape factor by

$$\overline{(C(W))} = \frac{f}{f_0} = \frac{6144 \ s}{f_0 t}$$

with f_0

$$f_0 = \int_1^{W_0} (W^2 - 1)^{1/2} W(W_0 - W)^2 F_0(Z, W) dW$$

Total half-life

$$\frac{1}{T_{1/2}} = \sum_f \frac{1}{t_{i \to f}}$$



To make a comparison conveniently between experiment and theory, we define the average shape factor by

$$\overline{(C(W))} = \frac{f}{f_0} = \frac{6144 \ s}{f_0 t}$$

with f_0

$$f_0 = \int_1^{W_0} (W^2 - 1)^{1/2} W(W_0 - W)^2 F_0(Z, W) dW$$

Total half-life

$$\frac{1}{T_{1/2}} = \sum_f \frac{1}{t_{i \to f}}$$

 β -delayed neutron emission probabilities:

$$P_n = \left(\sum_{E_f \ge S_n} \frac{1}{t_{i \to f}}\right) \left/ \left(\sum_{\text{all } f} \frac{1}{t_{i \to f}}\right)\right.$$

Anil Kumar (CCS, TU)

Shell Model Studies in Beta Decays of N = 126 isotones



July 30, 2024

- The β -decay half-lives are evaluated by including both the Gamow-Teller and the first-forbidden transitions.
- GT strengths are calculated using the Lanczos strength function method [1].
- The moment

$$S_k = \sum_{\nu} (E_{\nu} - E_i)^k |\langle \nu | \hat{O} | i \rangle|^2$$

up to a sufficiently large k should be calculated. The Lanczos algorithm guarantees the correct moment up to k = 2n - 1 with n Lanczos iteration starting with $\vec{u_1} = \hat{O}|i\rangle$.

• In this work, the GT strengths calculated with 250 Lanczos iterations to confirm sufficiently converged results for N = 126, 125 isotones.

- The β -decay half-lives are evaluated by including both the Gamow-Teller and the first-forbidden transitions.
- GT strengths are calculated using the Lanczos strength function method [1].
- The moment

$$S_k = \sum_{\nu} (E_{\nu} - E_i)^k |\langle \nu | \hat{O} | i \rangle|^2$$

up to a sufficiently large k should be calculated. The Lanczos algorithm guarantees the correct moment up to k = 2n - 1 with n Lanczos iteration starting with $\vec{u_1} = \hat{O}|i\rangle$.

- In this work, the GT strengths calculated with 250 Lanczos iterations to confirm sufficiently converged results for N = 126, 125 isotones.
- In case of first-forbidden β decay: due to the involvement of six operators, the first-forbidden strength has been obtained by diagonalization of the Hamiltonian and calculated strength for 50 eigenvalues for each state in the daughter nuclei

- The β -decay half-lives are evaluated by including both the Gamow-Teller and the first-forbidden transitions.
- GT strengths are calculated using the Lanczos strength function method [1].
- The moment

$$S_k = \sum_{\nu} (E_{\nu} - E_i)^k |\langle \nu | \hat{O} | i \rangle|^2$$

up to a sufficiently large k should be calculated. The Lanczos algorithm guarantees the correct moment up to k = 2n - 1 with n Lanczos iteration starting with $\vec{u_1} = \hat{O}|i\rangle$.

- In this work, the GT strengths calculated with 250 Lanczos iterations to confirm sufficiently converged results for N = 126, 125 isotones.
- In case of first-forbidden β decay: due to the involvement of six operators, the first-forbidden strength has been obtained by diagonalization of the Hamiltonian and calculated strength for 50 eigenvalues for each state in the daughter nuclei
- Full model space diagonalized:
 - N = 126 isotones with Z = 52 79 and
 - N = 125 isotones with Z = 52 59 and Z = 72 79

- The β -decay half-lives are evaluated by including both the Gamow-Teller and the first-forbidden transitions.
- GT strengths are calculated using the Lanczos strength function method [1].
- The moment

$$S_k = \sum_{\nu} (E_{\nu} - E_i)^k |\langle \nu | \hat{O} | i \rangle|^2$$

up to a sufficiently large k should be calculated. The Lanczos algorithm guarantees the correct moment up to k = 2n - 1 with n Lanczos iteration starting with $\vec{u_1} = \hat{O}|i\rangle$.

- In this work, the GT strengths calculated with 250 Lanczos iterations to confirm sufficiently converged results for N = 126, 125 isotones.
- In case of first-forbidden β decay: due to the involvement of six operators, the first-forbidden strength has been obtained by diagonalization of the Hamiltonian and calculated strength for 50 eigenvalues for each state in the daughter nuclei
- Full model space diagonalized:
 - N = 126 isotones with Z = 52 79 and
 - N = 125 isotones with Z = 52 59 and Z = 72 79
- Due to the limitation of computational resources, monopole-based truncation is applied on N = 125isotones with Z = 60 - 71

Lanczos method for strength distribution: Example ⁴⁸Ca



Results from: E. Caurier et al., Rev. Mod. Phys. 77, 427 (2005).

Lanczos method for strength distribution: Example ⁴⁸Ca



• After 50 iterations, a good distribution is achieved, despite notable variations in individual strengths between 50 and 1000 iterations.

Results from: E. Caurier et al., Rev. Mod. Phys. 77, 427 (2005).

Anil Kumar (CCS, TU)

Effective operators: Gamow-Teller

To make a comparison of the shell model calculated β decay rate with the experimental one, we used effective operators in the calculations

 $\hat{O}^{\text{eff}} = q \times \hat{O}^{\text{free}}$

where, the bare operator \hat{O}^{free} is multiplied by a scaling factor (a name coined as a **quenching factor** in several earlier studies.)

To address the quenching factor in GT transitions in this study:

ransitions for quenching factor ($q_{\rm GT} = 0.54$)									
Transition	$ \mathcal{M}_{ m GT} $		$\log f_0 t$						
	Exp.	SM ($q = 1$)	SM ($q = 0.54$)	Exp.	SM ($q = 1$)	SM ($q = 0.54$)			
199 Pt(5/2 ⁻) \rightarrow^{199} Au(7/2 ⁻)	0.114	0.297	0.160	6.45(1)	5.62	6.16			
$^{200}\mathrm{Au}^{m}(12^{-}) \rightarrow ^{200}\mathrm{Hg}(11^{-})$	0.349	0.608	0.327	6.1(3)	5.6	6.2			

Anil Kumar (CCS, TU)

Effective operators: First-forbidden

- The operators of rank 0, 1, and 2 in the first-forbidden β decay also required a quenching factor.
- To obtain quenching factors for all FF operators, we have minimized the chi-square function between theoretical and experimental average shape factors by including unique and non-unique first-forbidden transitions (two unique and sixteen non-unique transitions).



Shell Model Studies in Beta Decays of N = 126 isotones

Result: Half-lives of experimentally known β decay

Shell-model predicted half-lives (sec) in comparison to the existing experimental data

		<i>N</i> = 12	<i>N</i> = 126		
	²⁰² Ir	²⁰³ Pt	²⁰⁴ Au	²⁰⁴ Pt	²⁰⁵ Au
Present	4.7	20.6	25.4	13.9	23.7
Exp	15(3)	22(4)	37.2(8)	16^{+6}_{-5}	32.5(14)
SM 2018				38.3	

We introduce r as a measure of deviation

$$r = \log_{10}(T_{1/2}^{\text{calc}}/T_{1/2}^{\text{exp}}),$$

and its mean value and standard deviation can be written as

$$\bar{r} = \frac{1}{n} \sum_{i=1}^{n} r_i,$$

and



Exp: A. I. Morales et al., Phys. Rev. Lett. **113**, 022702 (2014).

SM 2018: T. Suzuki et al., The Astrophysical Journal 859, 133 (2018).

Discrepancies of shell-model half-lives from the experimental ones $\overline{\bar{r}} = \sigma = 10^{\overline{r}} = 10^{\sigma} \text{ n}$

Anil Kumar (CCS, TU)

July 30, 2024

Result: Half-lives of N = 126 **isotones**



Present shell-model predicted β -decay half-lives for N = 126 isotones are compared with different theoretical model calculations and existing experimental data.



Result: Half-lives of N = 126 **isotones**



Present shell-model predicted β -decay half-lives for N = 126 isotones are compared with the previous shell model calculated and existing experimental data.

SM: T. Suzuki et al., The Astrophysical Journal 859, 133 (2018).

SM: Q. Zhi et al., Phys. Rev. C 87, 025803 (2013).Exp.: A. I. Morales et al., Phys. Rev. Lett. 113, 022702 (2014).

Result: Half-lives of N = 125 **isotones**



Present shell-model predicted β -decay half-lives for N = 126 isotones are compared with different theoretical model calculations and existing experimental data.

 FRDM19: P. Möller et al., Atomic Data and Nuclear Data Tables 125, 1 (2019).
 Exp.: A. I. Morales et al., Phys. Rev. Lett. 113, 022702 (2014).

 KTUY05: H. Koura et al., Prog. of Theo. Phys. 113, 305 (2005).
 Shell Model Studies in Beta Decays of N = 126 isotones
 July 30, 2024
 18/24

Result: Contribution of GT and FF transitions in Half-lives



The effect of first-forbidden transitions in the total half-lives of N = 126 (top) and N = 125 (bottom) isotones

Result: Contribution of GT and FF transitions in Half-lives



- The effect of first-forbidden transitions in the total half-lives of N = 126 (top) and N = 125 (bottom) isotones
- GT transition dominated by the $v0h_{9/2} \rightarrow \pi 0h_{11/2}$



Anil Kumar (CCS, TU)

Shell Model Studies in Beta Decays of N = 126 isotones

Result: Gamow-Teller strength distributions of N = 126 isotones



- The GT strength peaks are observed at low excitation energies between 3-6 MeV and 5-7 MeV for even-Z and odd-Z of parent nuclei, respectively.
- This peak, dominated by the $\nu 0h_{9/2} \rightarrow \pi 0h_{11/2}$ transition, is enhanced on the proton deficient side because the Pauli-blocking effect caused by the occupying the valence proton $0h_{11/2}$ orbit is weakened.

Result: Total Gamow-Teller strength distributions as a function of $\pi 0h_{11/2}$ orbit



As the proton number increases, the proton $0h_{11/2}$ orbit becomes occupied, and simultaneously, the sum of GT strength decreases due to the **Pauli blocking effect**, reaching almost zero near Z = 82.

Anil Kumar (CCS, TU)

Result: β -delayed neutron emission probabilities $P_n(\%)$



 No experimental information available about the β-delayed neutron emission probabilities P_n(%) in this region

FRDM19: P. Möller et al., Atomic Data and Nuclear Data Tables 125, 1 (2019).

KTUY05: H. Koura et al., Prog. of Theo. Phys. 113, 305 (2005).

Shell Model Studies in Beta Decays of N = 126 isotones

Result: β -delayed neutron emission probabilities $P_n(\%)$



- No experimental information available about the β-delayed neutron emission probabilities P_n(%) in this region
- Even-odd staggering due to phase space

FRDM19: P. Möller et al., Atomic Data and Nuclear Data Tables 125, 1 (2019).

KTUY05: H. Koura et al., Prog. of Theo. Phys. 113, 305 (2005).

Result: β -delayed neutron emission probabilities $P_n(\%)$



- No experimental information available about the β-delayed neutron emission probabilities P_n(%) in this region
- Even-odd staggering due to phase space

FRDM19: P. Möller et al., Atomic Data and Nuclear Data Tables 125, 1 (2019).

KTUY05: H. Koura et al., Prog. of Theo. Phys. 113, 305 (2005).

Summary:

• Good agreement between shell model predicted and available experimental data.

Summary:

- Good agreement between shell model predicted and available experimental data.
- The contribution from first-forbidden transitions are important, especially for nuclei around N = 126 region.

Summary:

- Good agreement between shell model predicted and available experimental data.
- The contribution from first-forbidden transitions are important, especially for nuclei around N = 126 region.
- The Gamow-Teller strength exhibits systematic distributions as a function of proton number.

Summary:

- Good agreement between shell model predicted and available experimental data.
- The contribution from first-forbidden transitions are important, especially for nuclei around N = 126 region.
- The Gamow-Teller strength exhibits systematic distributions as a function of proton number.
- The present study of β -decay properties of waiting point nuclei around $A \approx 195$ will add more information in the third *r*-process abundance peak distributions.

Summary:

- Good agreement between shell model predicted and available experimental data.
- The contribution from first-forbidden transitions are important, especially for nuclei around N = 126 region.
- The Gamow-Teller strength exhibits systematic distributions as a function of proton number.
- The present study of β -decay properties of waiting point nuclei around $A \approx 195$ will add more information in the third *r*-process abundance peak distributions.

Future:

• Further, we will analyze the impact of the present calculated β -decay half-lives and β -delayed neutron emission probability on the r-process abundance distribution (with Nobuya Nishimura).

Anil Kumar (CCS, TU)

Shell Model Studies in Beta Decays of N = 126 isotones