Seeing Invisible

Study of Invisible World of Sub-atomic Particles

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Plan

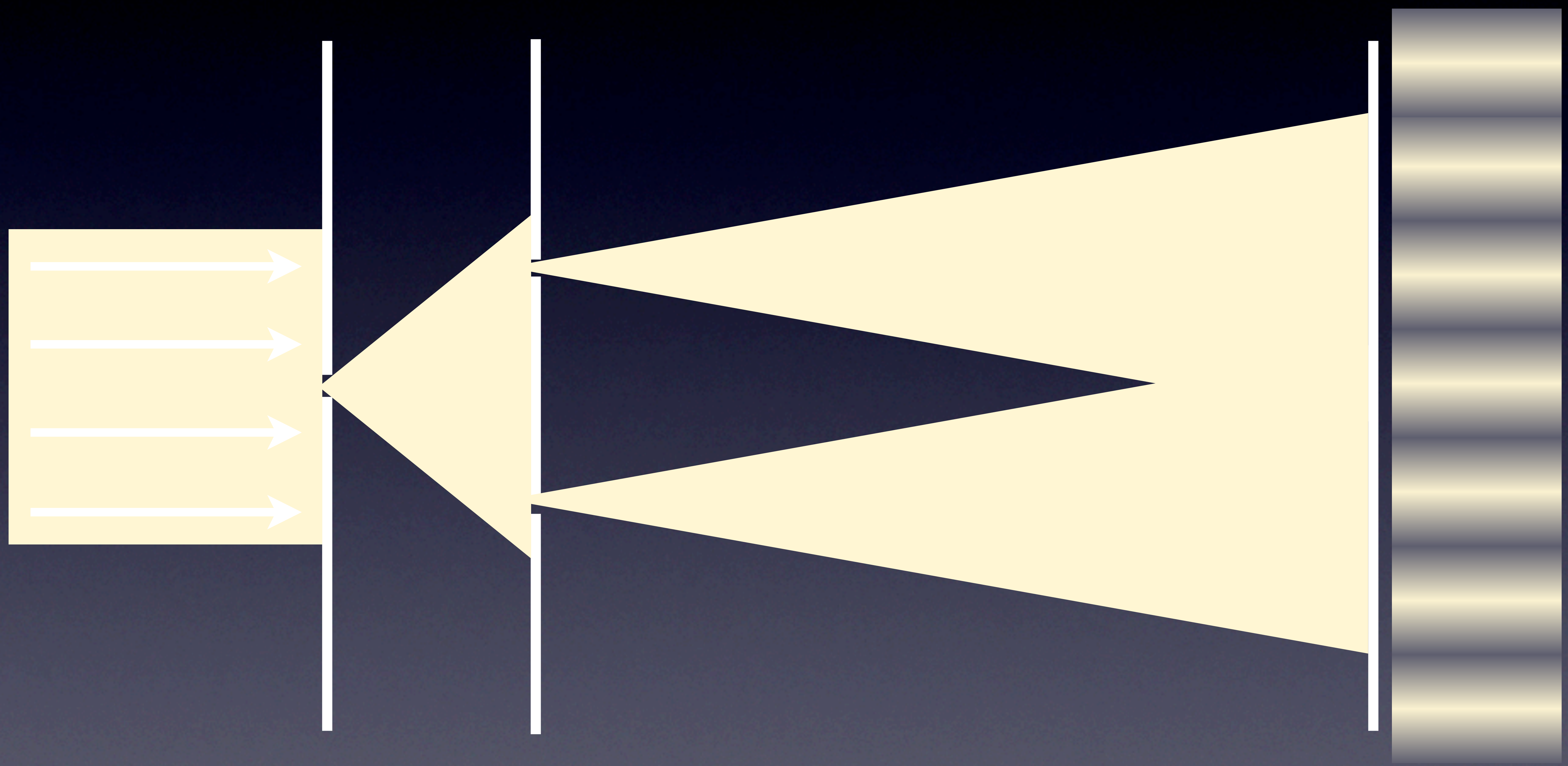
- Diffraction of Light
- General Features of Nuclear Physics Experiments
- Elastic Electron Scattering
- A Few Pictures
- Summary

What is vision?

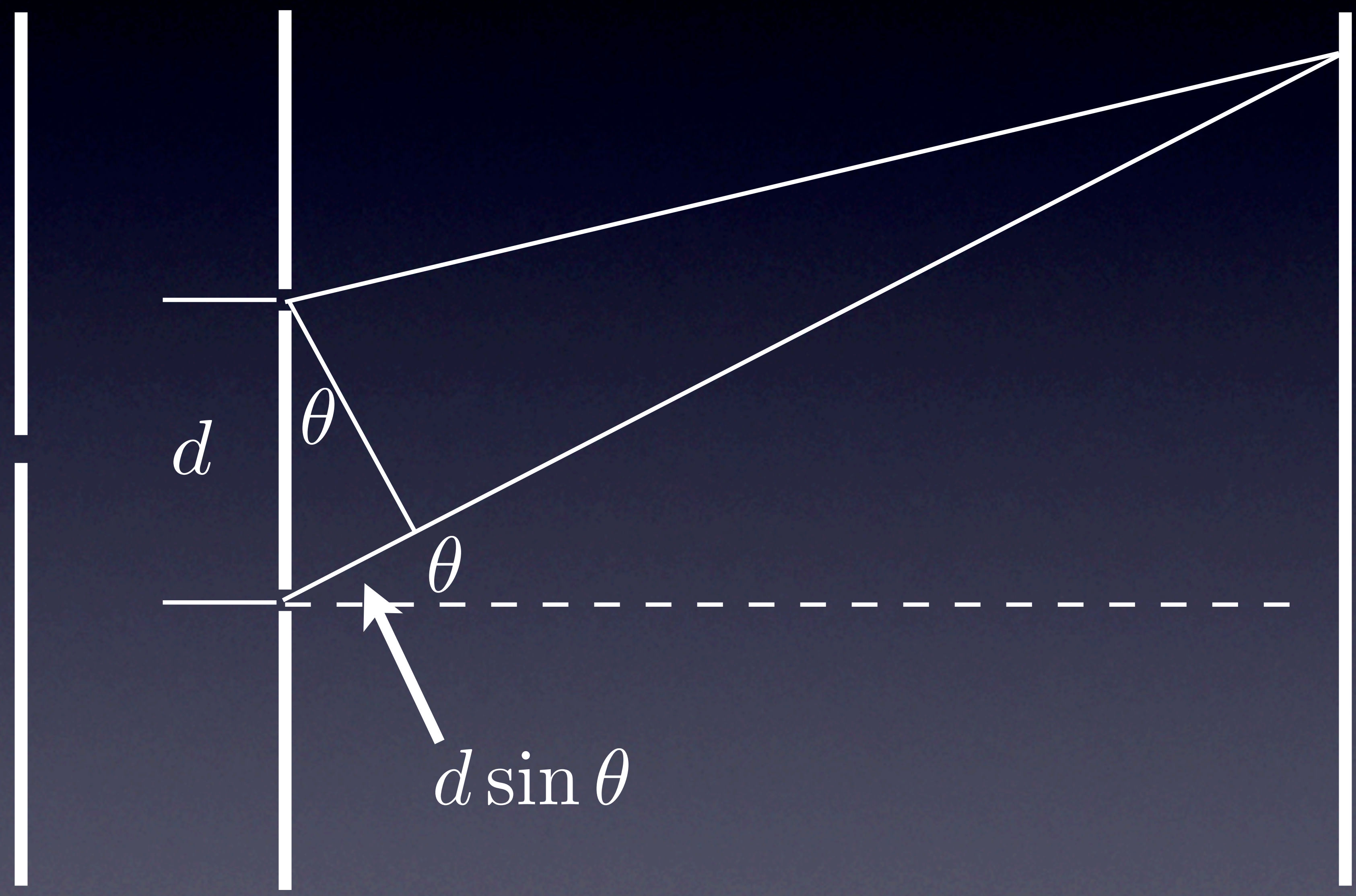
- Send a bunch of photons into an object
- Detect the scattered photons with our eyes
- What about the nucleons?
  - Too small to see with our eyes (10^{-15}m = 1 fm)

How to see tiny things?

Young’s Double Slit



Young's Double Slit



Wavy Patterns

$$A(\theta) = A_0 \sin\left(\frac{2\pi r}{\lambda}\right) + A_0 \sin\left(\frac{2\pi r}{\lambda} + \frac{2\pi d \sin\theta}{\lambda}\right)$$

$$= 2A_0 \cos\left(\frac{\pi d \sin\theta}{\lambda}\right) \sin\left(\frac{2\pi(r + d\sin\theta)}{\lambda}\right)$$

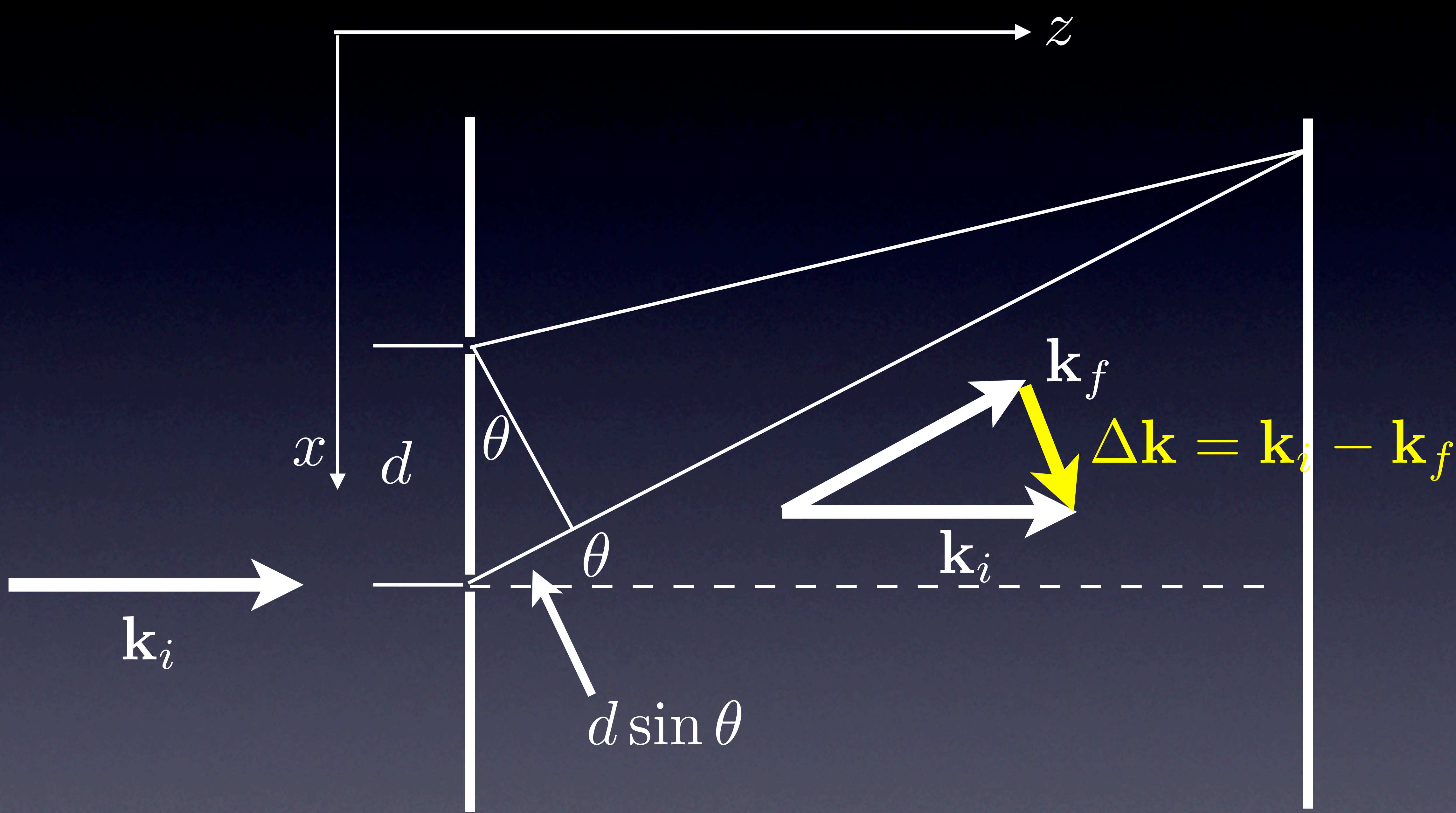
$$I = |A(\theta)|^2 = 4I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

Dark spots when $d \sin\theta = \left(n + \frac{1}{2}\right)\lambda$

Move to Quantum World

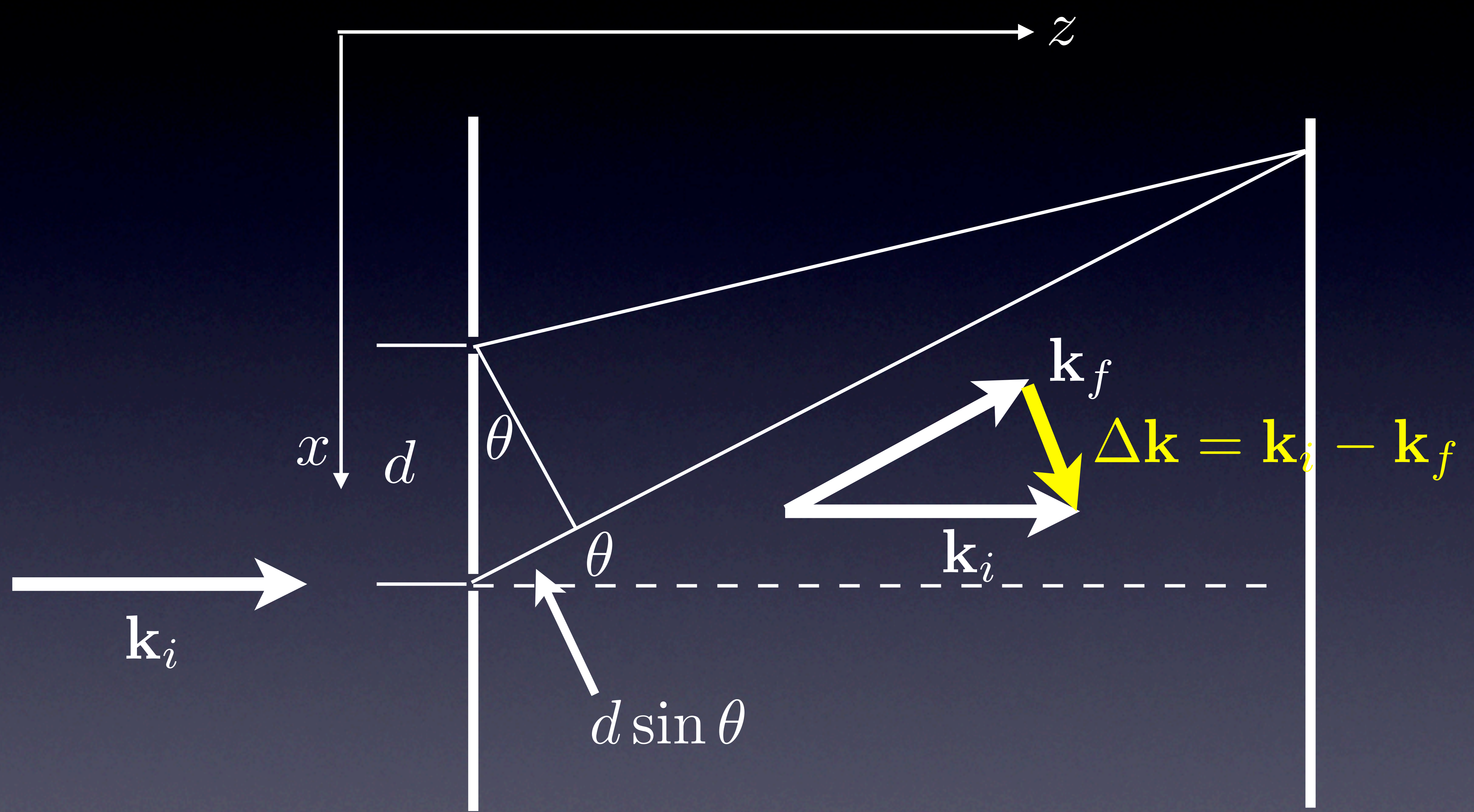
- In quantum world,
  wave = particle, particle = wave
  wavelength (λ) ~ momentum (p)
- According to De Broglie $\lambda = \dfrac{h}{p}$
- Then we can use $\dfrac{2\pi}{\lambda} = \dfrac{p}{\hbar} = k$

Double Slit in Momentum Space



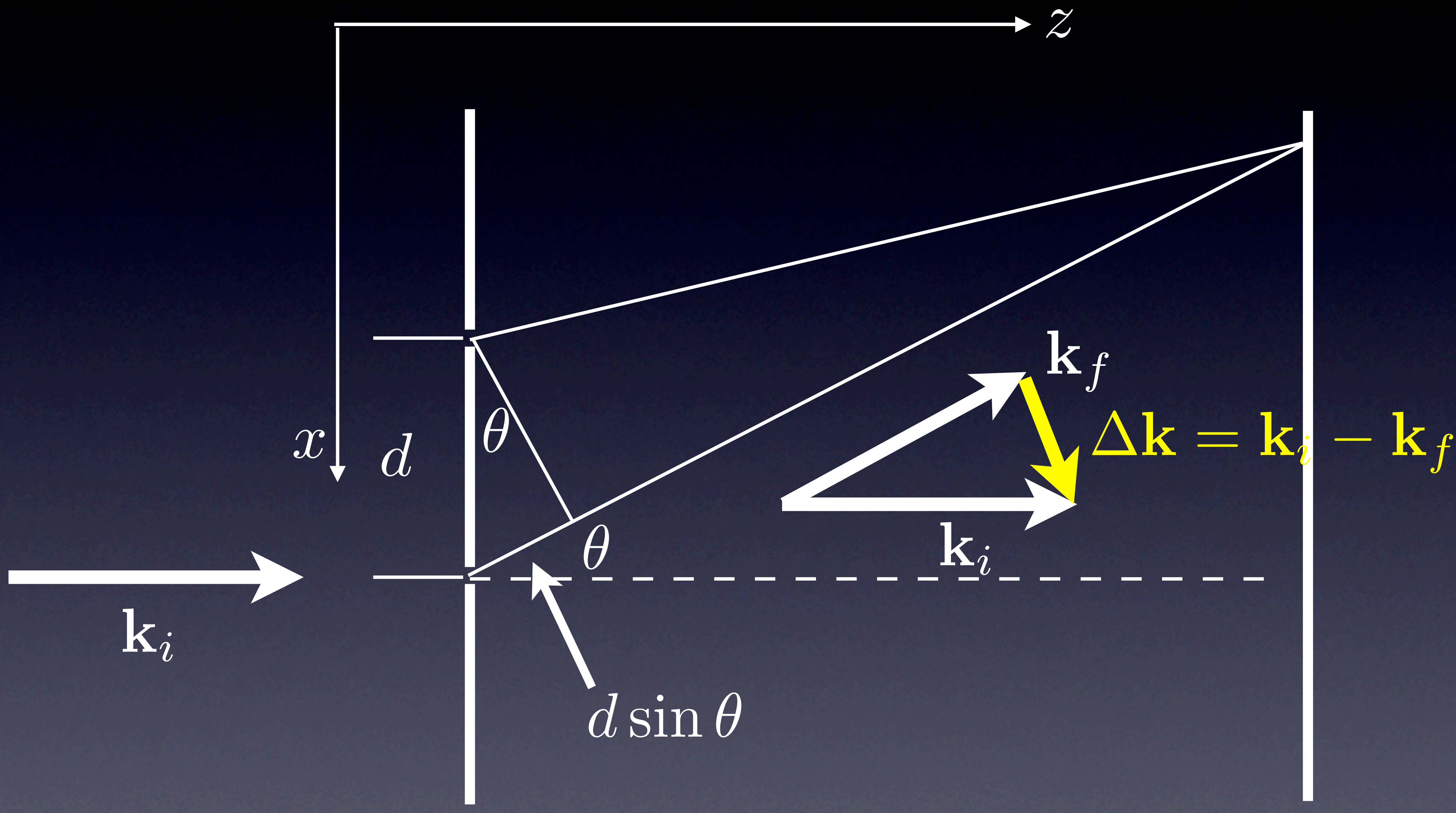
$$\mathbf{k}_i = |\mathbf{k}|\hat{\mathbf{z}} \qquad \mathbf{k}_f = -|\mathbf{k}|\sin\theta\hat{\mathbf{x}} + |\mathbf{k}|\cos\theta\hat{\mathbf{z}}$$

Double Slit in Momentum Space



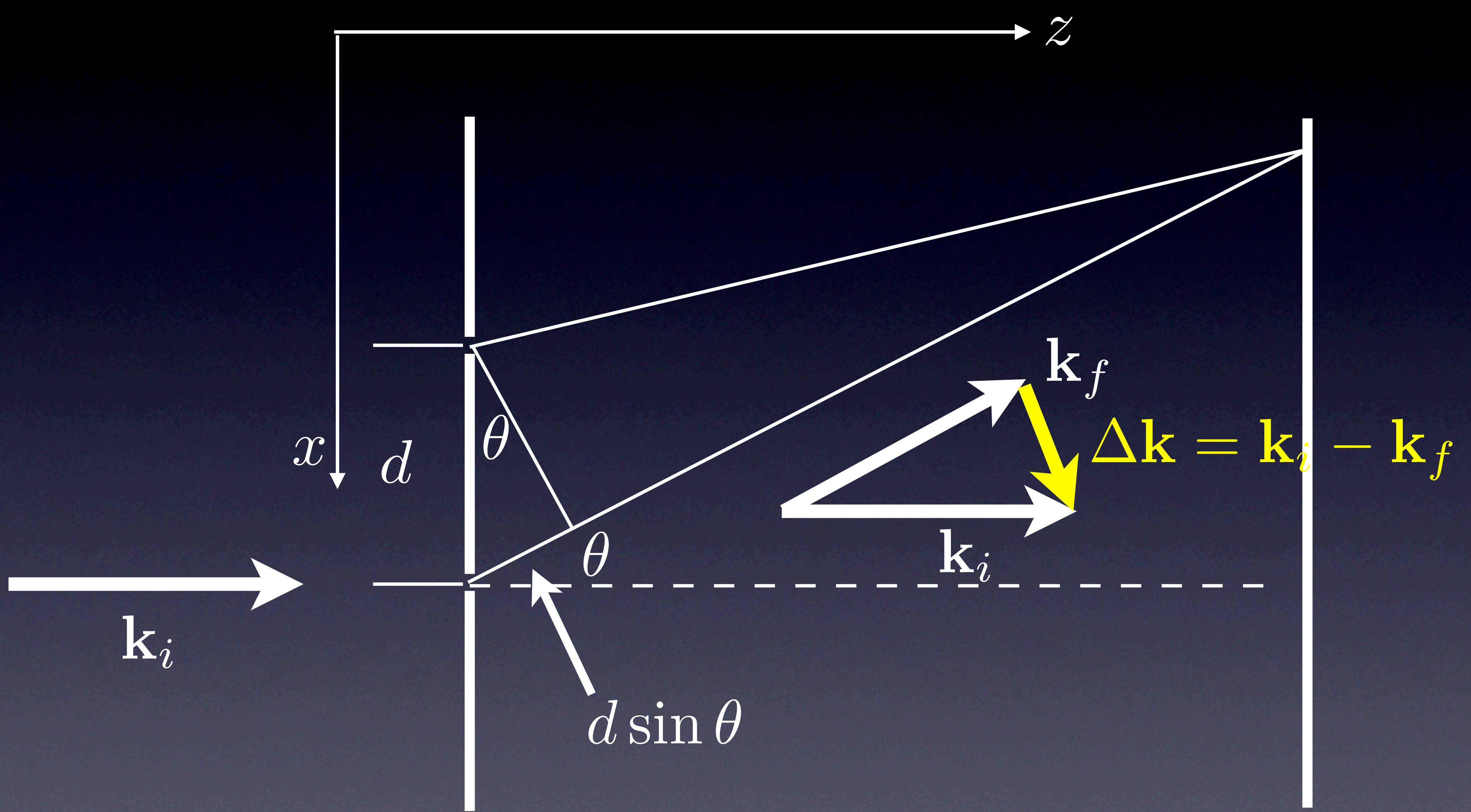
$$\mathbf{\Delta k} = |\mathbf{k}| \sin\theta \hat{\mathbf{x}} + |\mathbf{k}|(1 - \cos\theta)\hat{\mathbf{z}}$$

Double Slit in Momentum Space



$$\Delta\mathbf{k} = |\mathbf{k}| \sin\theta \hat{\mathbf{x}} + |\mathbf{k}|(1 - \cos\theta)\hat{\mathbf{z}} \qquad |\mathbf{k}| = \frac{2\pi}{\lambda}$$

Double Slit in Momentum Space



$$\mathbf{\Delta k} = \frac{2\pi}{\lambda}\sin\theta\hat{\mathbf{x}} + \cdots$$

Wavy Patterns, again

$$A(\theta) = 2A_0 \cos\left(\frac{\pi d \sin\theta}{\lambda}\right) \sin\left(\frac{2\pi(r + d\sin\theta)}{\lambda}\right)$$

Using $\mathbf{\Delta k} = \frac{2\pi}{\lambda}\sin\theta\hat{\mathbf{x}} + \cdots$ $\qquad \frac{\pi d \sin\theta}{\lambda} = \mathbf{\Delta k}\cdot\left(\frac{d}{2}\hat{\mathbf{x}}\right)$

$$\begin{aligned} A(\theta) &= A_0\left[\cos\left(\mathbf{\Delta k}\cdot\frac{d}{2}\hat{\mathbf{x}}\right) + \cos\left(\mathbf{\Delta k}\cdot\frac{-d}{2}\hat{\mathbf{x}}\right)\right] \\ &\quad \times \sin\left(\frac{2\pi(r + d\sin\theta)}{\lambda}\right) \end{aligned}$$

Generalization

- For a distribution of diffraction holes

$$A(\theta) = \sum_{n=0}^{N} A_0 \exp\left(i\Delta\mathbf{k}\cdot\mathbf{x}\right) e^{ikr}$$

- For continuous distribution of scattering centers

$$A(\theta) = A_0 \left[\int_V \rho(\mathbf{x}) \exp\left(i\Delta\mathbf{k}\cdot\mathbf{x}\right)\, d^3\mathbf{x}\right] e^{ikr}$$

Form Factor

Intensity vs Form Factor

$$F(\Delta \mathbf{k}) = \int_V \rho(\mathbf{x}) \exp(i \Delta \mathbf{k} \cdot \mathbf{x}) \, d^3\mathbf{x}$$

Form Factor

$$A(\theta) = A_0 F(\Delta \mathbf{k}) e^{ikr}$$

Amplitude

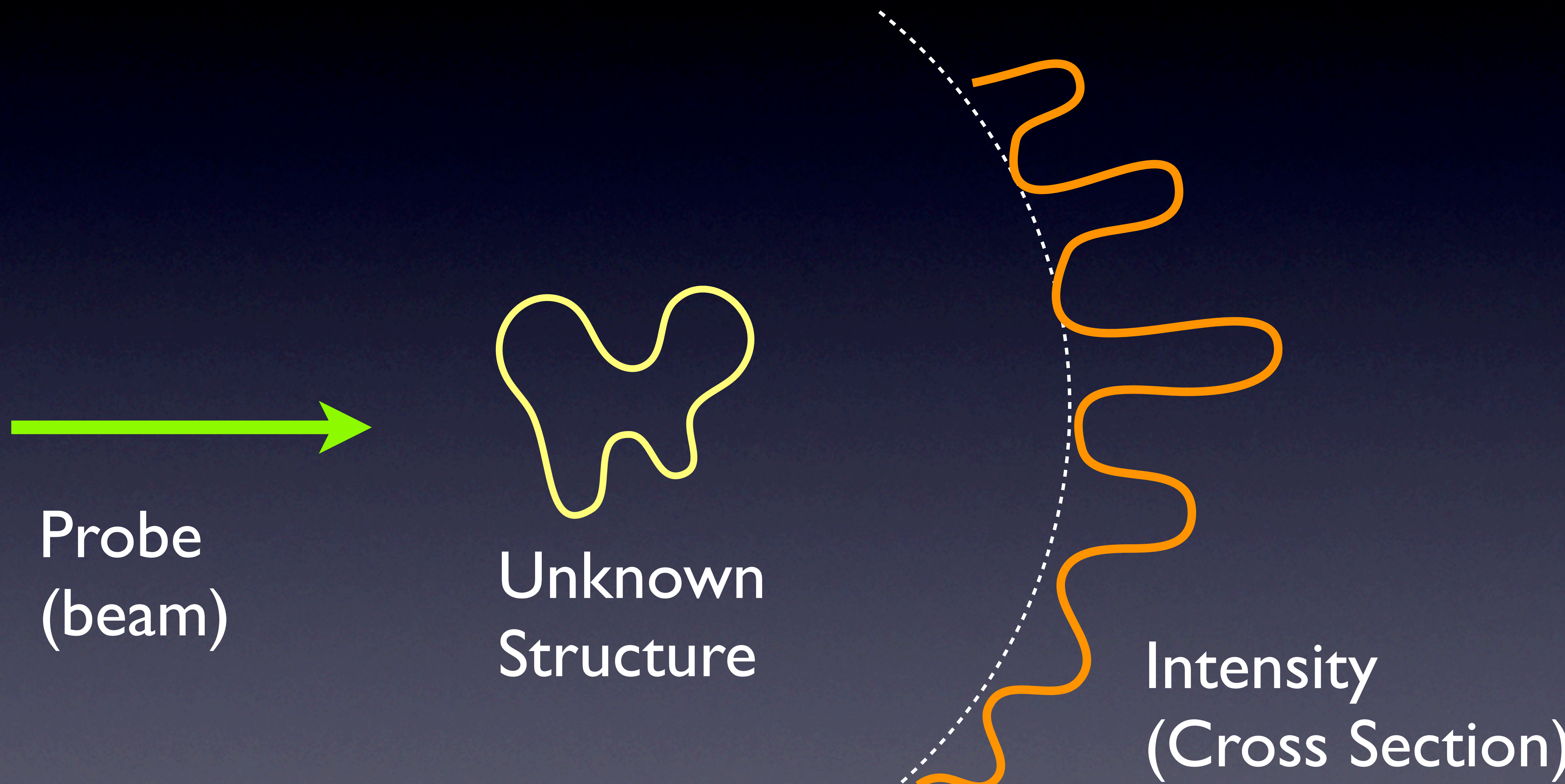
$$\begin{aligned} I(\theta) &= |A(\theta)|^2 \\ &= A_0^2 |F(\Delta \mathbf{k})|^2 \end{aligned}$$

Intensity

$$\rho(\mathbf{x}) = \int_V F(\mathbf{k}) \exp(-i \mathbf{k} \cdot \mathbf{x}) \, d^3\mathbf{k}$$

Internal Structure

Nuclear Experiments



Nuclear Experiments

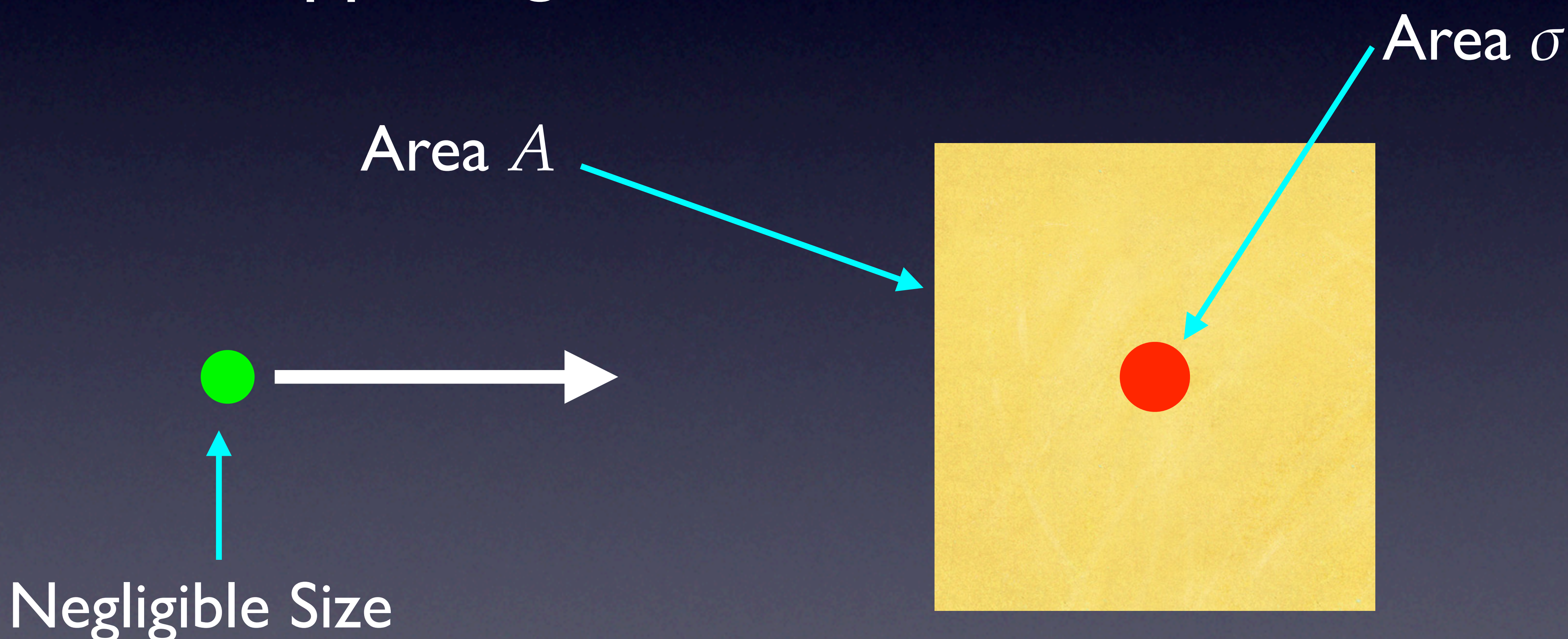
- Beam
  - photon, electron, proton, nucleus
  - pion, muon, kaon, positron, anti-proton
  - unstable nuclei (RI beam)
- Usually requires **accelerators**

Nuclear Experiments

- Target
  - proton (hydrogen)
  - stable nuclei
  - neutron???
- solid, liquid, gas

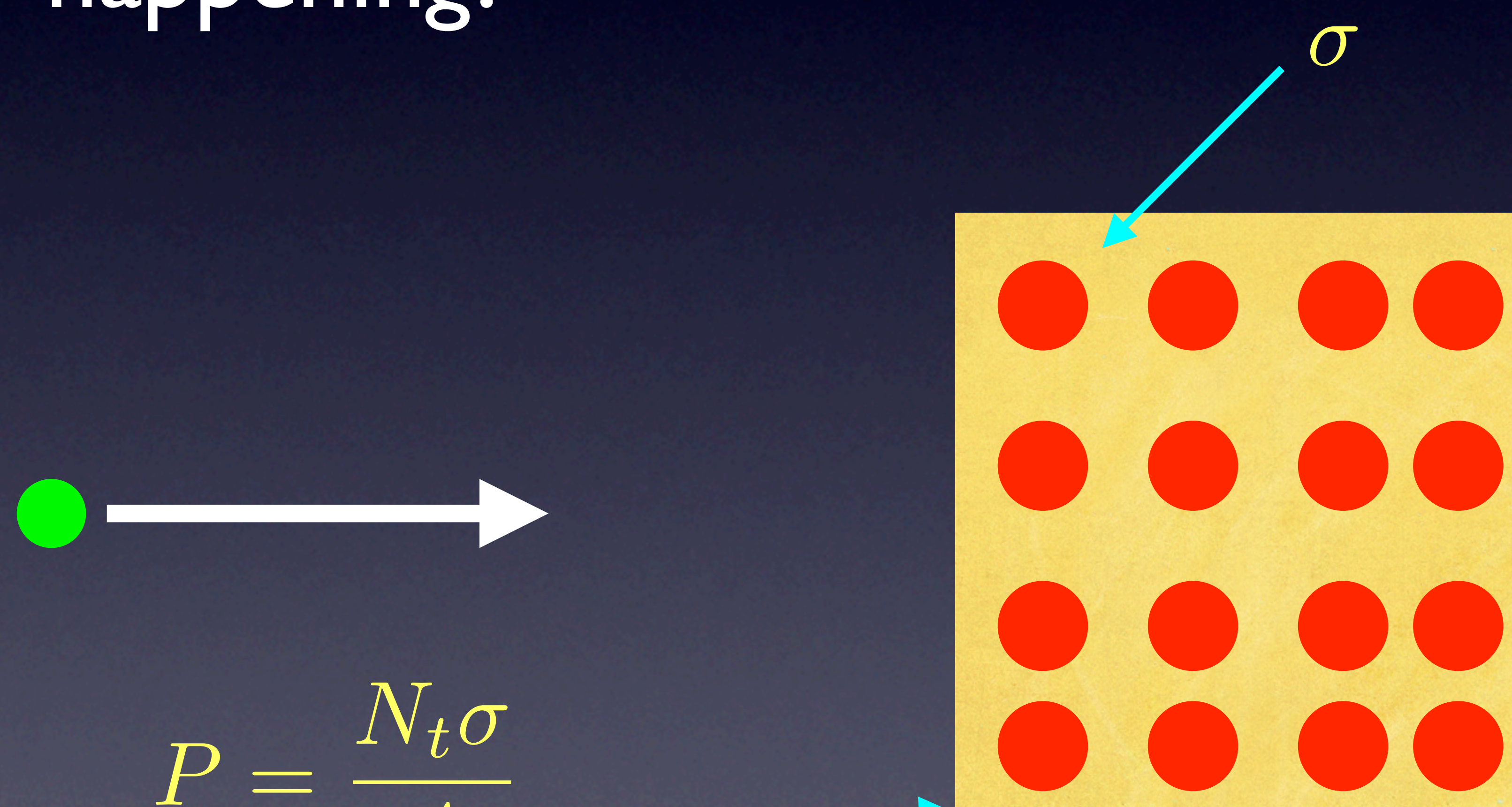
Cross Section

- What is the "probability" of some happening?



Cross Section

- What is the "probability" of some happening?



$$P = \frac{N_t \sigma}{A}$$

Cross Sections

- Number of Happenings

$$N_e = N_i \cdot P \qquad P = \frac{N_t \sigma}{A}$$

$$= N_i \frac{N_t \sigma}{A}$$

$$\sigma = \frac{N_e}{N_i \frac{N_t}{A}}$$

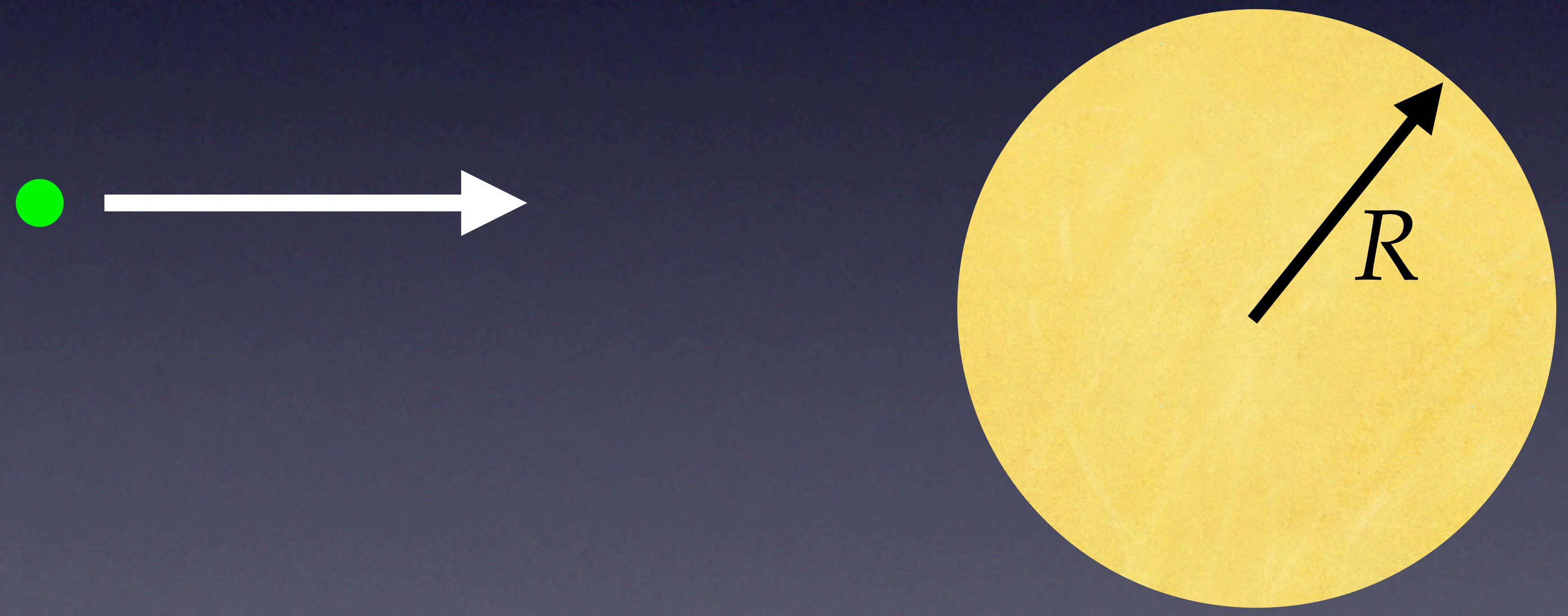
Cross Section

$$\sigma = \frac{\text{Number of Scattered Particles}}{(\text{Number of Incident Particles*Number of Scattering Center})/\text{Area}}$$

- Unit : area
  - cm^2, etc
  - barn = 10^{-24} cm^2
  - mb, μb, nb, pb

Quiz 1

- There is a ball of radius R, what is the cross section of collision with tiny bullets (negligible size)?



Quiz 2

- What is the cross section for meteorites collide with the earth?
  1. πR^2
  2. smaller than πR^2
  3. greater than πR^2

Quiz 3

- Supernova Explosion SN1987A (Feb. 23)
  - Distance to earth: 140 kly = 1.3×10^{21}m
- Release of 10^{57} neutrinos
- Kamiokande detector: 2000 tons of water
- neutrino-proton cross section: $2\times10^{-45}\text{m}^2$
- How many neutrinos to be detected?

Answer

$$N_e = N_i \cdot P$$

$$= \frac{N_i N_t \sigma}{A}$$

$$N_t = \frac{2000_{\text{ton}} \times 10^{3}_{\text{kg}} \times 10^{3}_{\text{g}} \times 6.02 \times 10^{23}_{\text{mol}^{-1}}}{18_{\text{g mol}^{-1}}} \times 2$$

$$= 1.33 \times 10^{32}$$

$$N_e = \frac{10^{57} \times 1.33 \times 10^{32} \times 2 \times 10^{-45}_{\text{m}^2}}{4\pi \times (1.3 \times 10^{21})^{2}_{\text{m}^2}}$$

$$\simeq 12$$

Compton Scattering

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$ Kinematics

Klein-Nishina Formula

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_e^2 \left(\frac{\lambda}{\lambda'}\right)^2 \left[\frac{\lambda}{\lambda'} + \frac{\lambda'}{\lambda} - \sin^2\theta\right]$$ Dynamics

$$x(t) = x(0) + v(0)t + \frac{1}{2}at^2$$ *vs.* $$F = ma$$

Nuclear Experiments

- Intensities (or cross sections)
  - **Interaction** between the beam and the target
  - Measuring various particles and properties

Nuclear Experiments

- Measuring various particles and properties
  - beam, target
  - “new” things
    - newly produced particles
    - fragments of the beam or target
  - properties
    - energy and/or angular distributions
    - polarization (or spin direction)
    - mass, lifetime, etc

Notation

- Target (beam, detected particles) undetected
- $^{12}C(p, \gamma)^{13}N$
  - Target = ^{12}C
  - Beam = p (proton)
  - Detected = γ (photon)
  - Undetected = ^{13}N

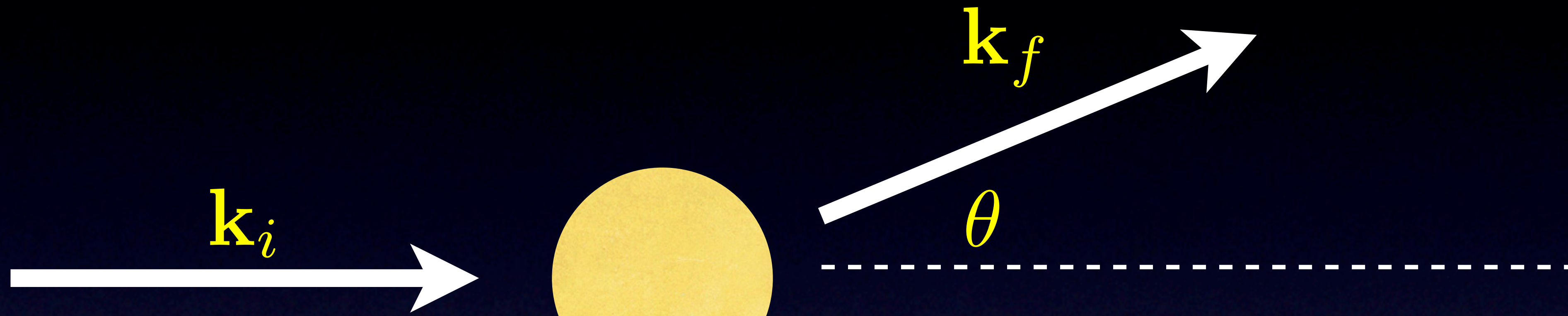
Another Example

- $^{10}B(p, \alpha\gamma)^{7}Be$
- Target: ^{10}B
- Beam: p (proton)
- Detected: γ (photon)
- Undetected: ^{7}Be, α (assumed)

Quiz 4

- What is the following experiment?
- $p(^{12}C, {}^{13}N\gamma)$

Scattering of Electrons



$$\Delta \mathbf{k} = \mathbf{q} = \mathbf{k}_i - \mathbf{k}_f$$

- Electron's charge interacts with charge distribution inside the target

$$F(\mathbf{q}) = \int_V \rho(\mathbf{x}) e^{i\mathbf{q}\cdot\mathbf{x}}\, d^3\mathbf{x}$$

- Form factor = Fourier transform of the charge distribution

Homework

- Calculate the form factor for uniform density sphere of radius R
  - In other words, do the following integral.

$$F(\mathbf{q}) = \int_V \rho(\mathbf{x}) e^{i\mathbf{q}\cdot\mathbf{x}}\, d^3\mathbf{x}$$

with

$$\rho(r) = \begin{cases} 1 & r \leq R, \\ 0 & r > R. \end{cases}$$

Some Hints

- We can take **q** in z direction (why?)
- $\mathbf{q} \cdot \mathbf{x} = q\, r \cos\theta$

$$F(q) = \int \rho(r) \exp(iqr\cos\theta) r^2 dr \sin\theta d\theta d\phi$$

$$= 2\pi \int_0^R dr \int_{-1}^{+1} \exp(iqr\cos\theta) d(\cos\theta) r^2 dr$$

On Carbon Nucleus

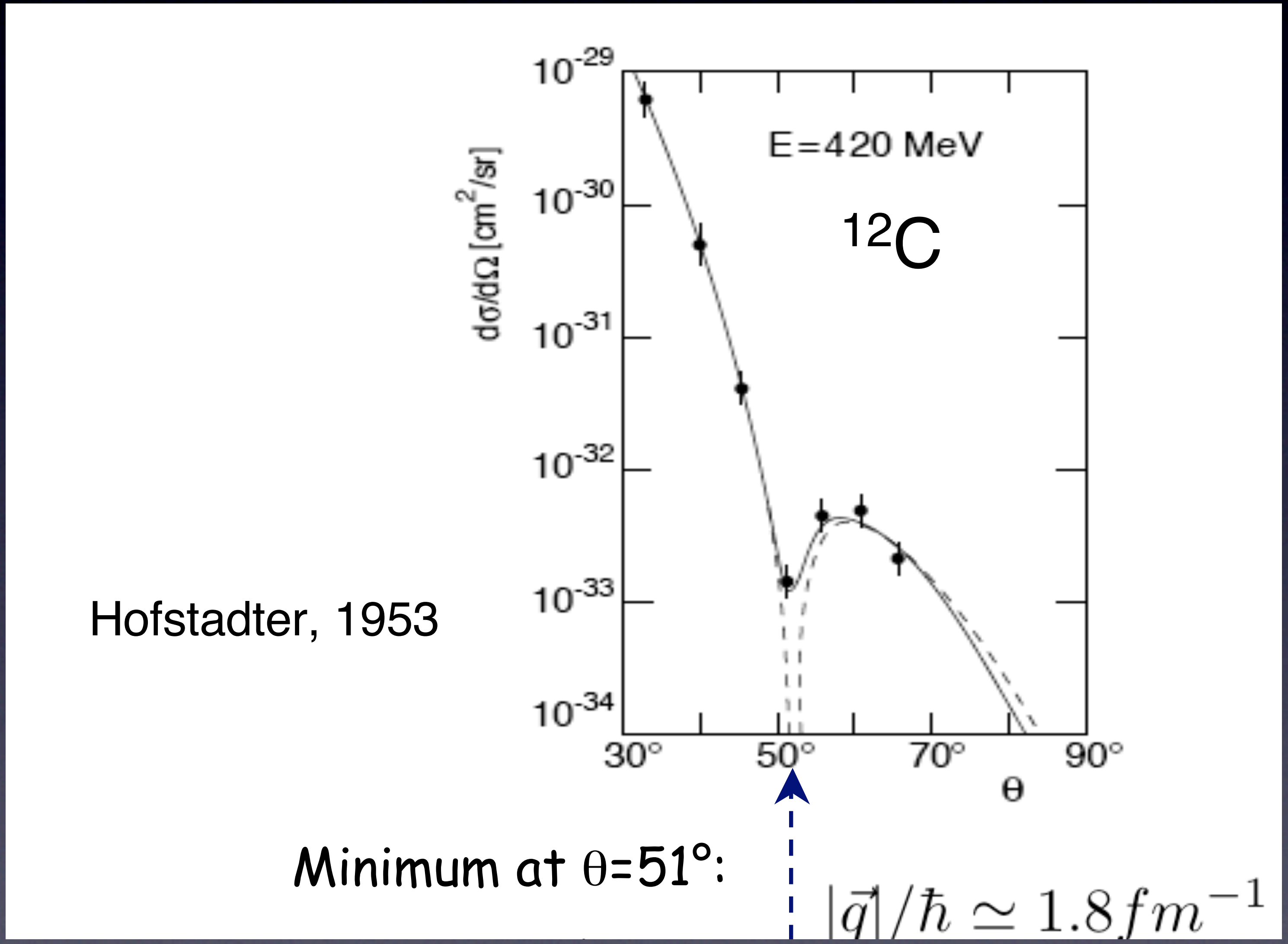
- For a uniformly charged sphere

$$F(q) = \frac{3}{\alpha^3}(\sin\alpha - \alpha\cos\alpha)$$

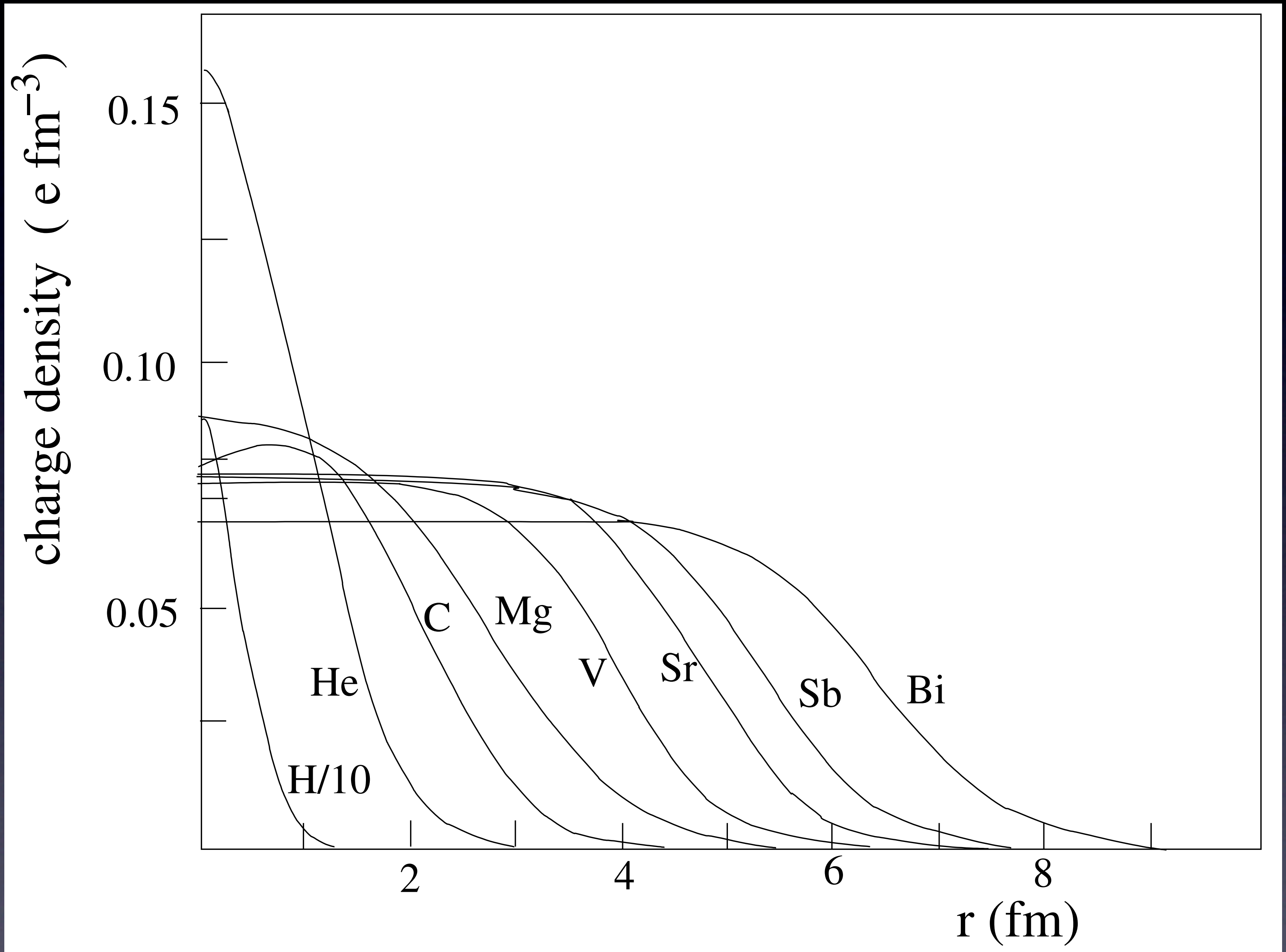
$$\alpha = |\mathbf{q}|R/\hbar$$

- From the position of the first minimum

  R ~ 2.5 fm
  or 2.5×10^{-15}m



Nuclear Radius



$$\sqrt{<r^2>} = r_0 A^{1/3} \quad r_0 = 0.94\text{fm}$$

On the Proton

- Electrons also have spin, so does the proton
- Two form factors for
  - charge distribution $G_E(q^2)$ $\quad q^2 = \Delta E^2 - \Delta \mathbf{k}^2$
  - spin(magnetization) distribution $G_M(q^2)$
- Electric and Magnetic form factors

$$\pi^- + \pi^+$$

- Spin 0 + Spin (Rutherford Scattering)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left(\frac{1 + E'/E}{2} \right)^2$$

$e^- + \pi^+$

- Spin 1/2 + Spin 0 (Mott Scattering)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left(\frac{E'}{E} \right) \cos^2 \frac{\theta}{2}$$

$$e^- + \mu^+$$

- Spin 1/2 + Spin 1/2

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left(\frac{E'}{E} \right) \left\{ \cos^2 \frac{\theta}{2} + \frac{-q^2}{2M^2} \sin^2 \frac{\theta}{2} \right\}$$

Summary

Spin 0 on Spin 0	$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left(\frac{1 + E'/E}{2} \right)^2$
Spin 1/2 on Spin 0	$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left(\frac{E'}{E} \right) \cos^2 \frac{\theta}{2}$
Spin 1/2 on Spin 1/2	$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left(\frac{E'}{E} \right) \left\{ \cos^2 \frac{\theta}{2} + \frac{-q^2}{2M^2} \sin^2 \frac{\theta}{2} \right\}$

Cross Section

$$e + p \to e' + p$$

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \frac{E'}{E} \cos^2\left(\frac{\theta}{2}\right) \left(\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\left(\frac{\theta}{2}\right) \right) \\ \tau &= -q^2/4M^2 \end{aligned}$$

- Separation of the two form factors
  - Measure the cross section at two different angles
  - keeping $-q^2$ constant

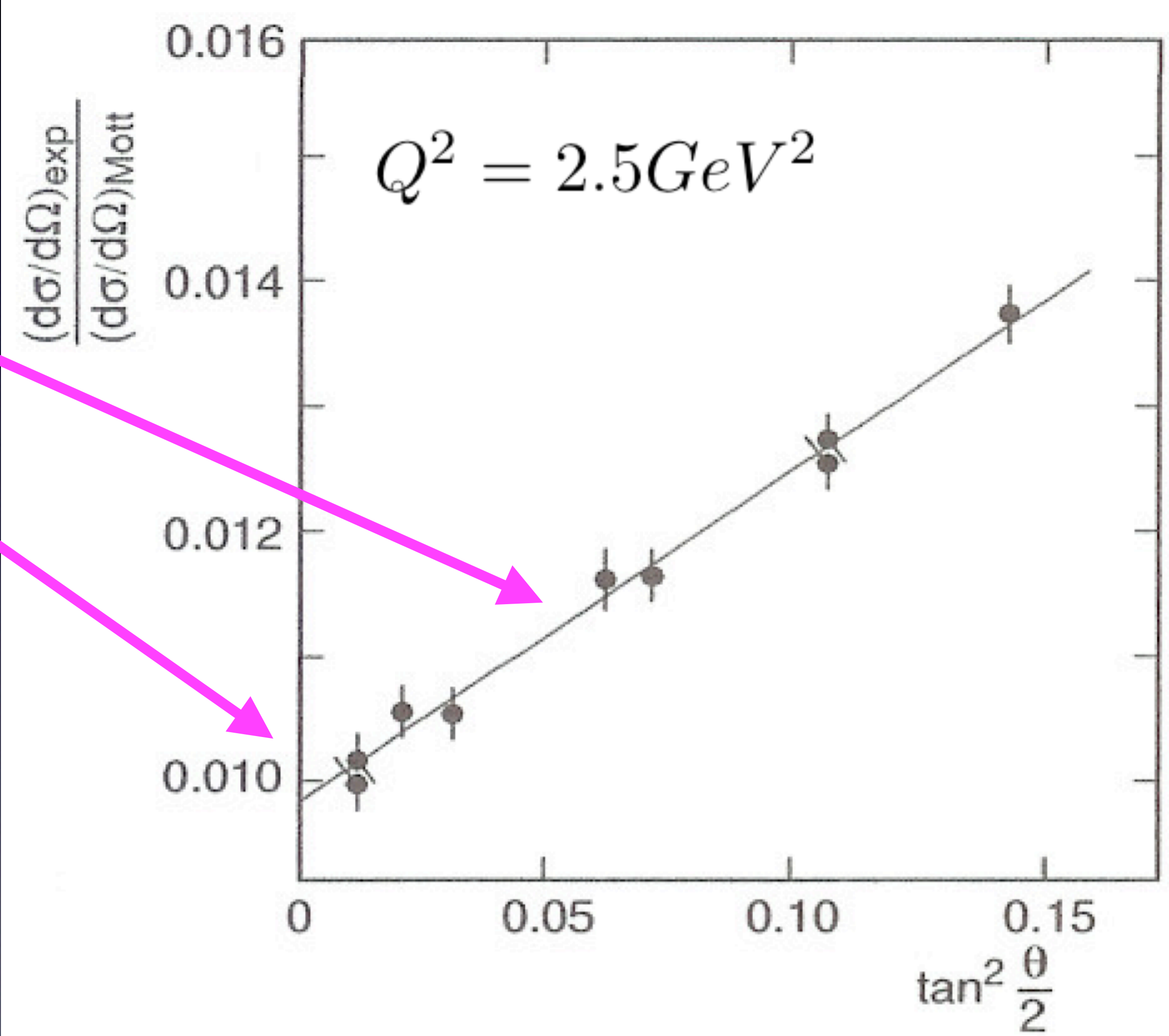
Rosenbluth Separation

Slope $2\tau G_M^2$

Intercept

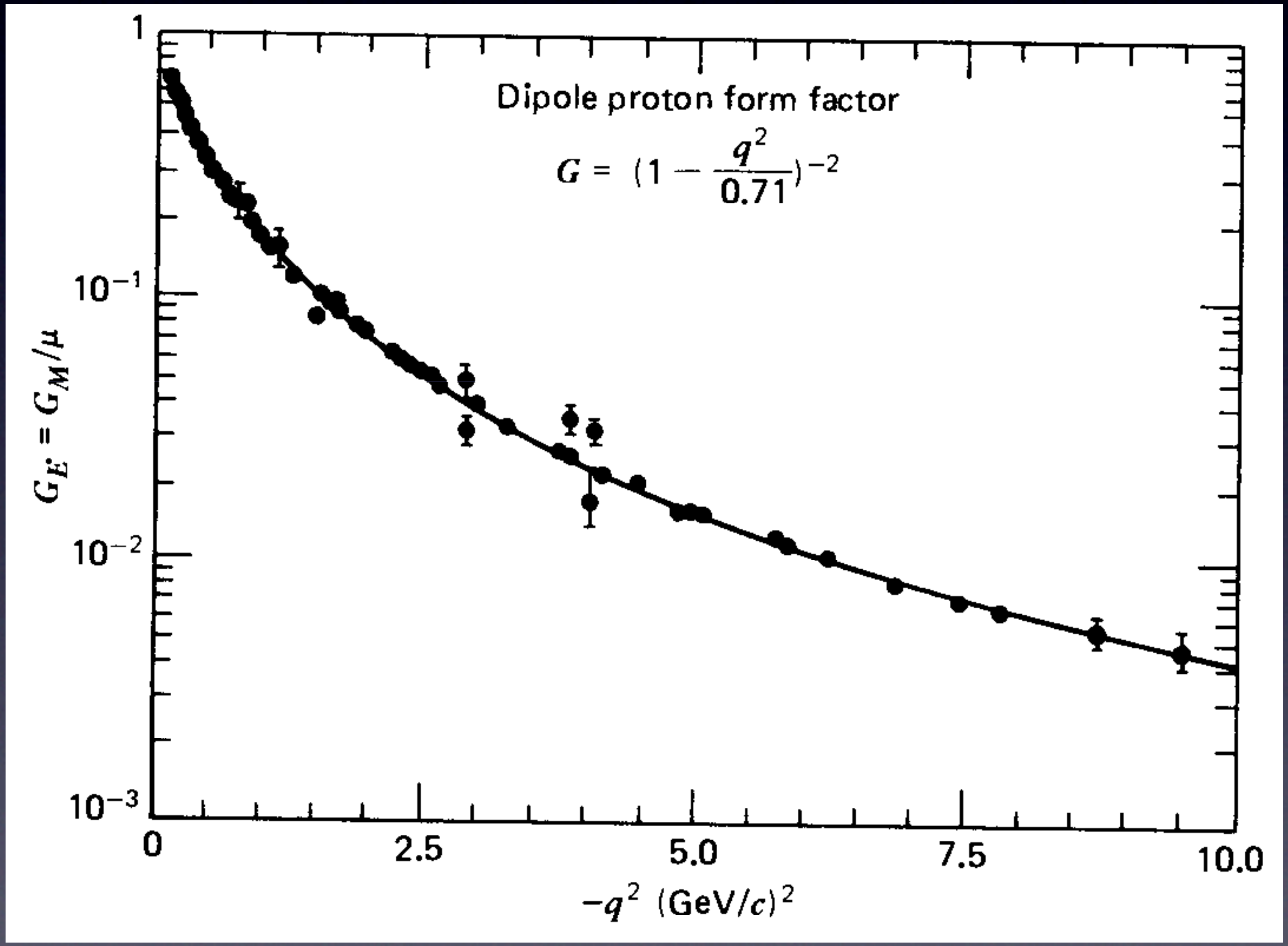
$(G_E^2 + \tau G_M^2)/(1+\tau)$

$$\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\left(\frac{\theta}{2}\right)$$



R.E. Taylor, Proc. Int. Symp. on electron and photon interactions at High Energies (Stanford 1967)

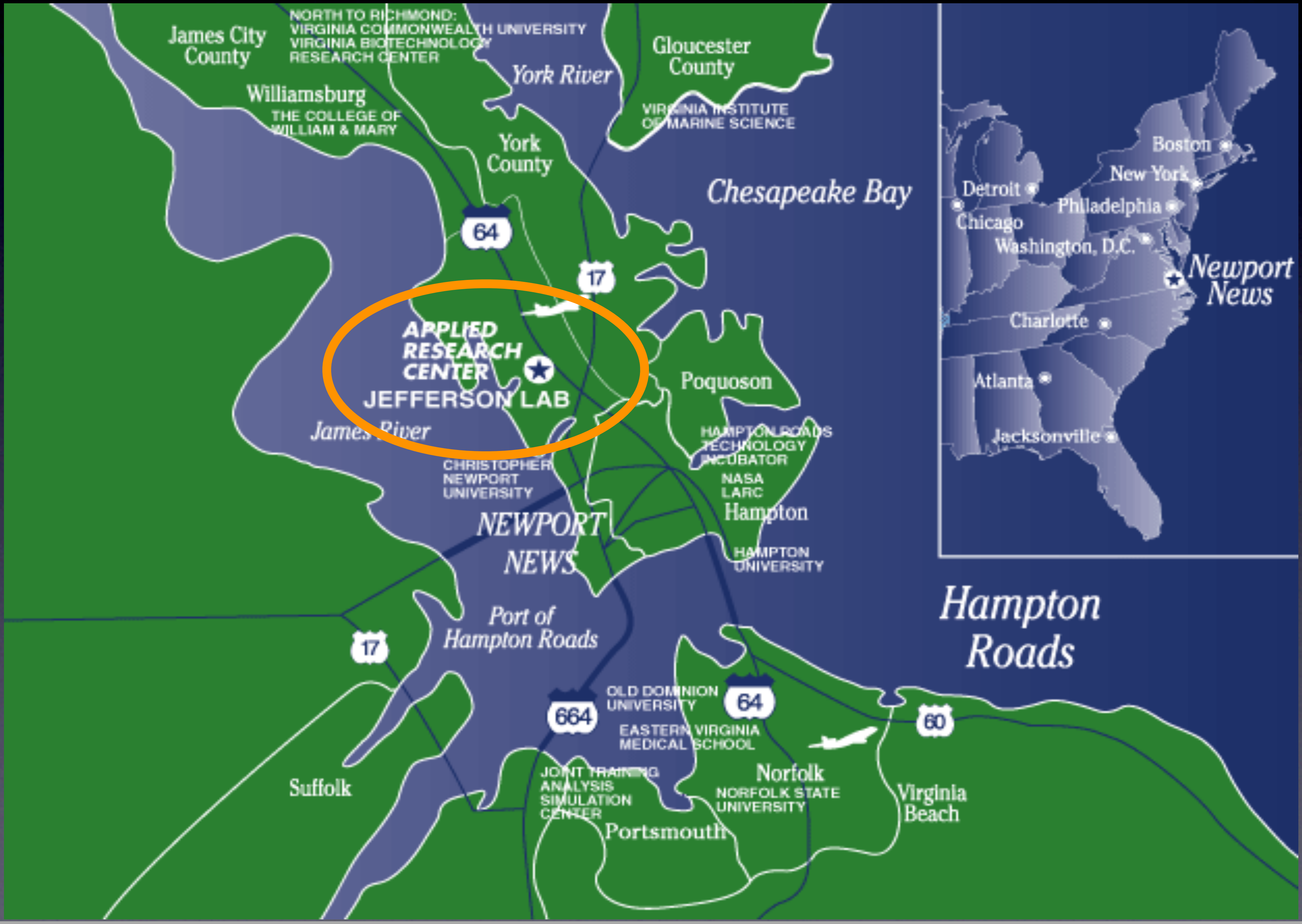
Form Factors of the Proton



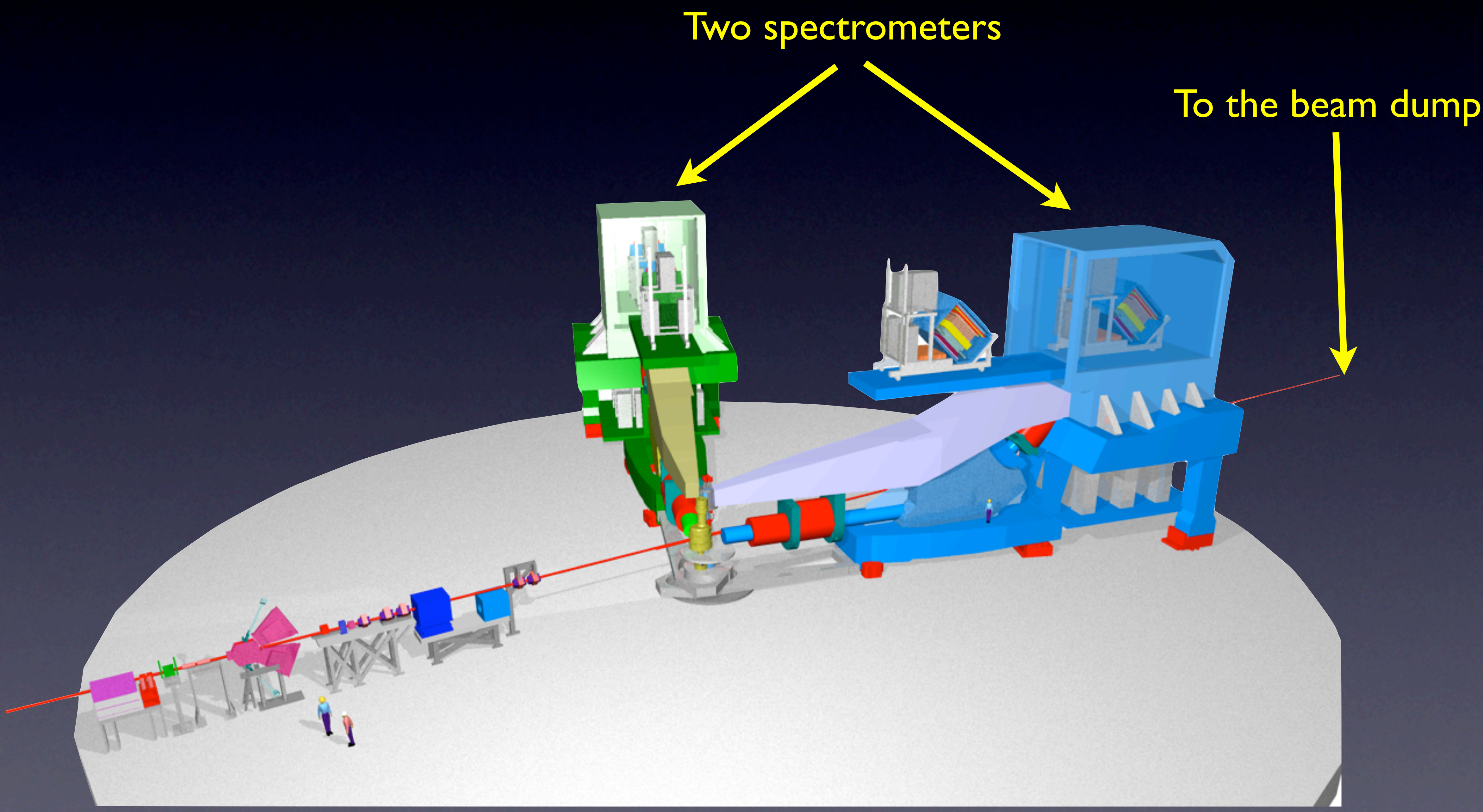
- $G_E = G_M/\mu$
- Radius of the proton

$$\sqrt{< r^2 >} = 0.81\text{fm}$$

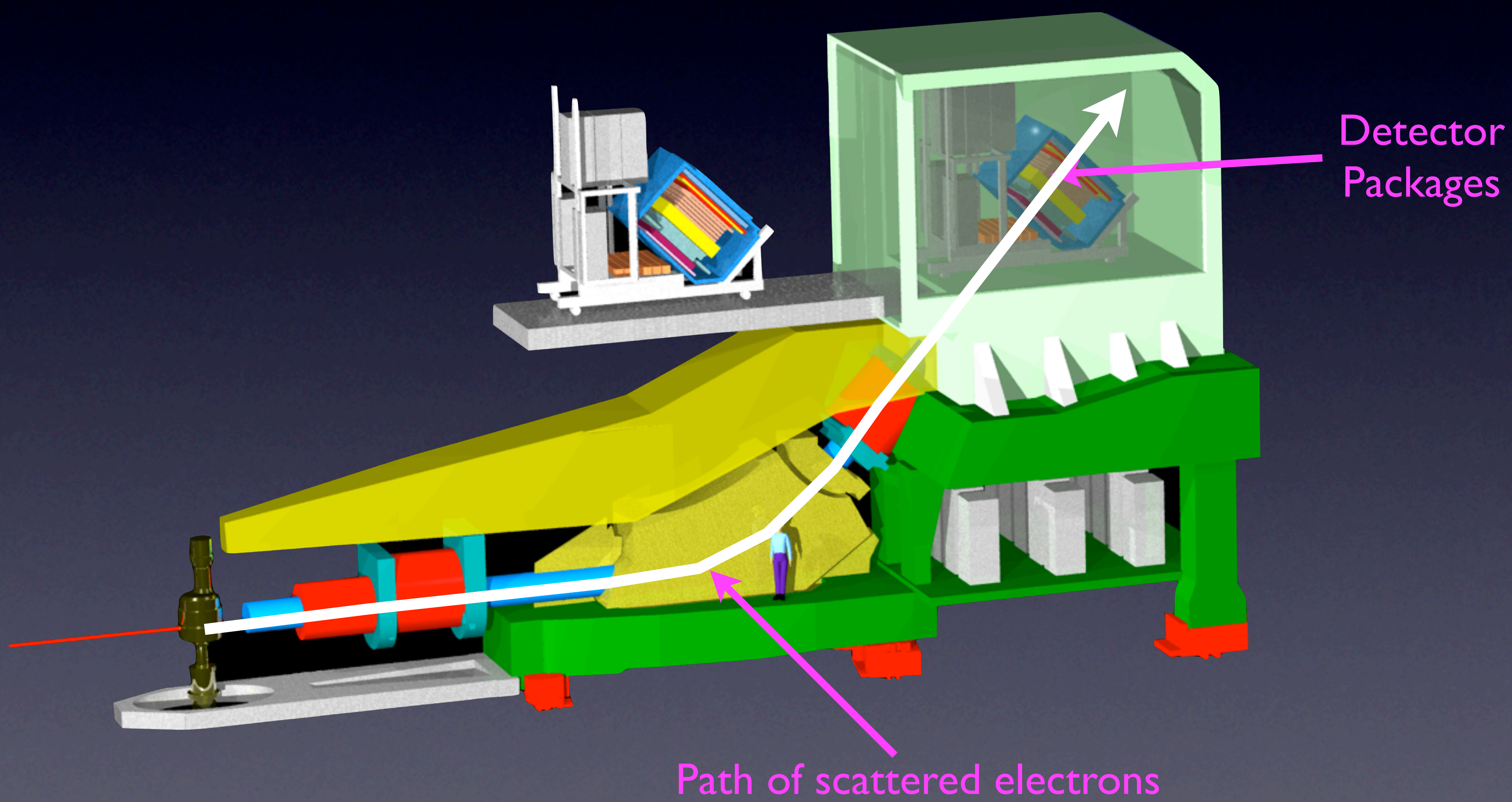
Jefferson Lab



Jefferson Lab Hall-A

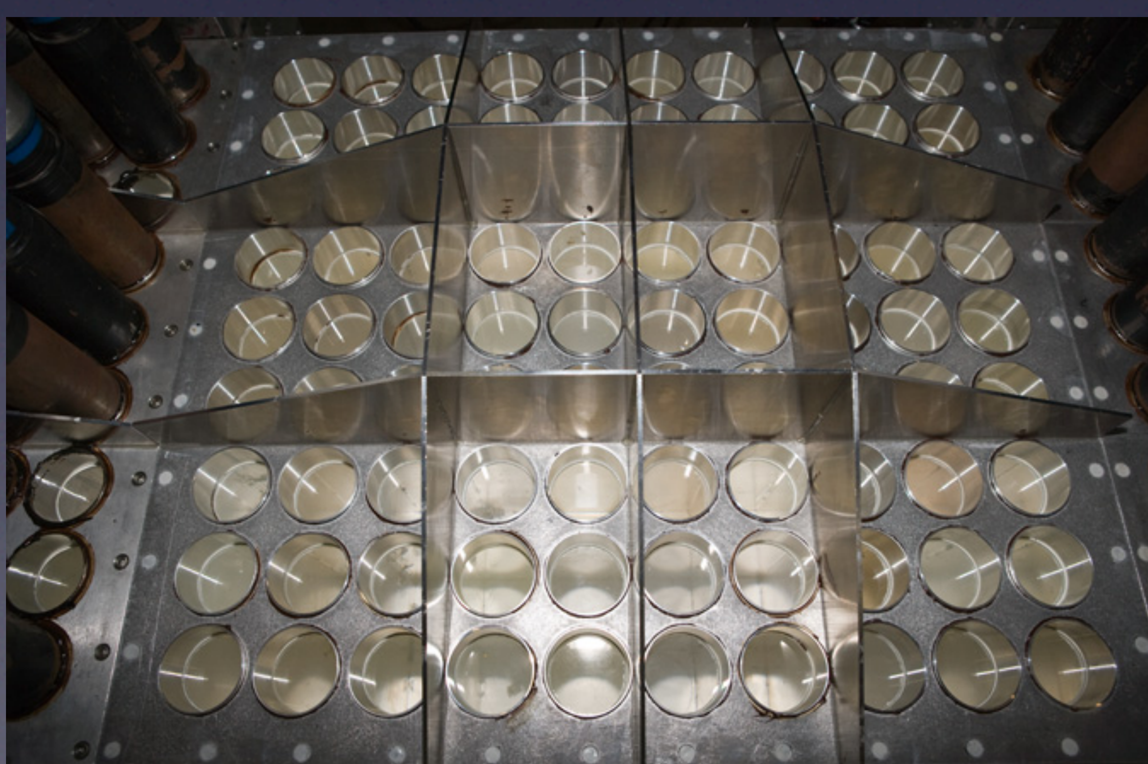


Spectrometer



Experimental Hall A

Installation of the new detector



Summary

- Nuclear or Subatomic Physics
  - Study of interactions of the nuclei
  - Study of the structure of the nuclei
- beam on target
  - Measurement of various “things”
  - Infer interactions/structure