The pelletron experiment

$^{12}C(p,\gamma)^{13}N$, $^{10}B(p, \alpha\gamma)^{7}Be$
$^{27}Al(p, p'\gamma_{12})^{27}Al$&$^{27}Al(p, \alpha\gamma)^{24}Mg$
$^{9}Be(p, \gamma)^{10}B$

H. Ishiyama

RNC/RIKEN

Nishina School 2024

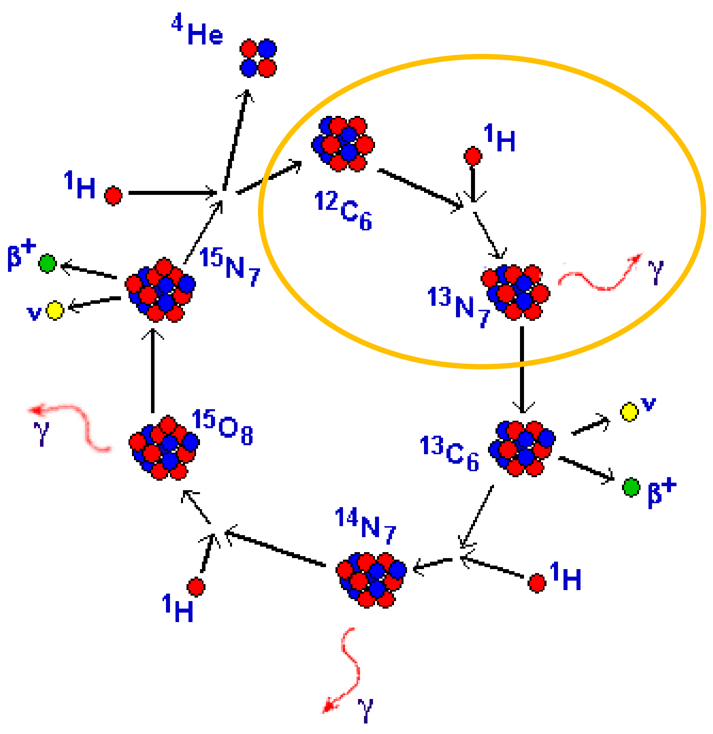
July 25th- Aug. 2nd

$^{12}C(p,\gamma)^{13}N$ $p + {}^{12}C \rightarrow \gamma + {}^{13}N$

- A reaction involved in the CNO cycle nuclear burning in **massive*** stars, …

- At low (astrophysical) energies, two dominant resonances are important.

CNO cycle



$$p + {}^{12}C \rightarrow {}^{13}N^* \rightarrow \gamma + {}^{13}N$$

* more massive than the sun

Appropriate energies of proton beams
from the accelerator (pelletron)?

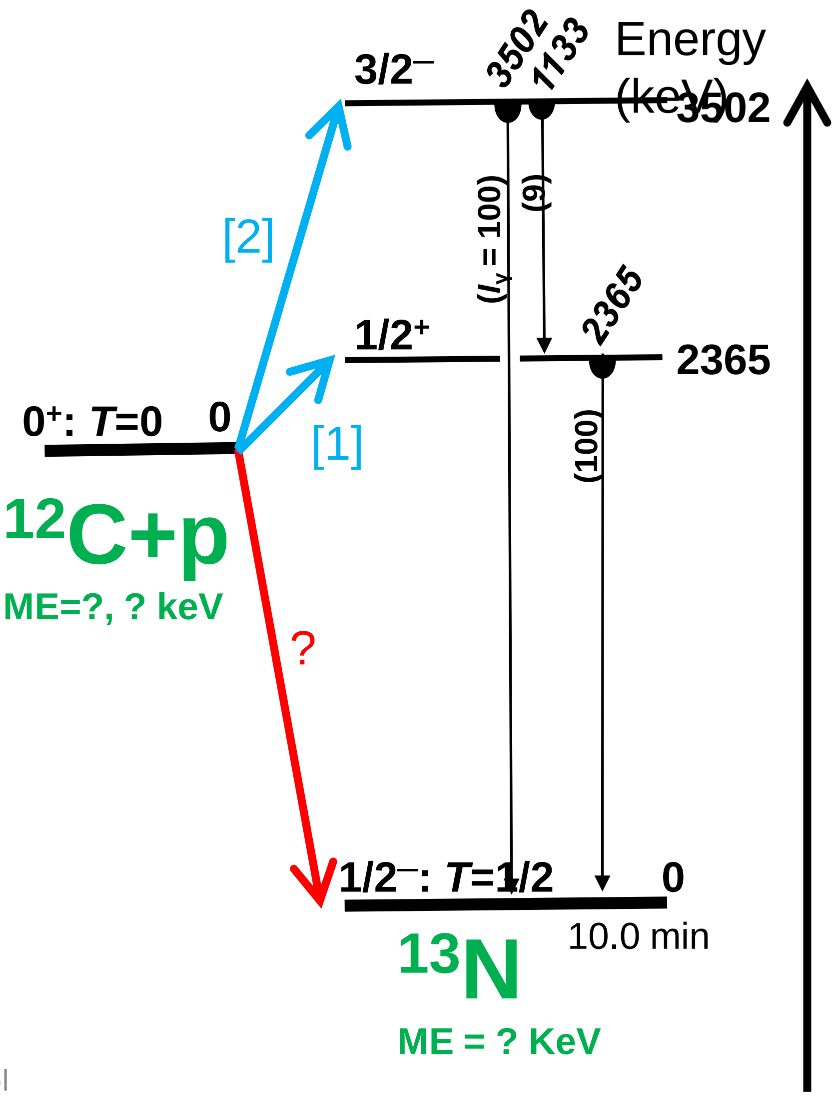
Question 1
energy (mass*) difference between ^{12}C+p & ^{13}N

- Energy levels of at the center-of-mass (CM) system

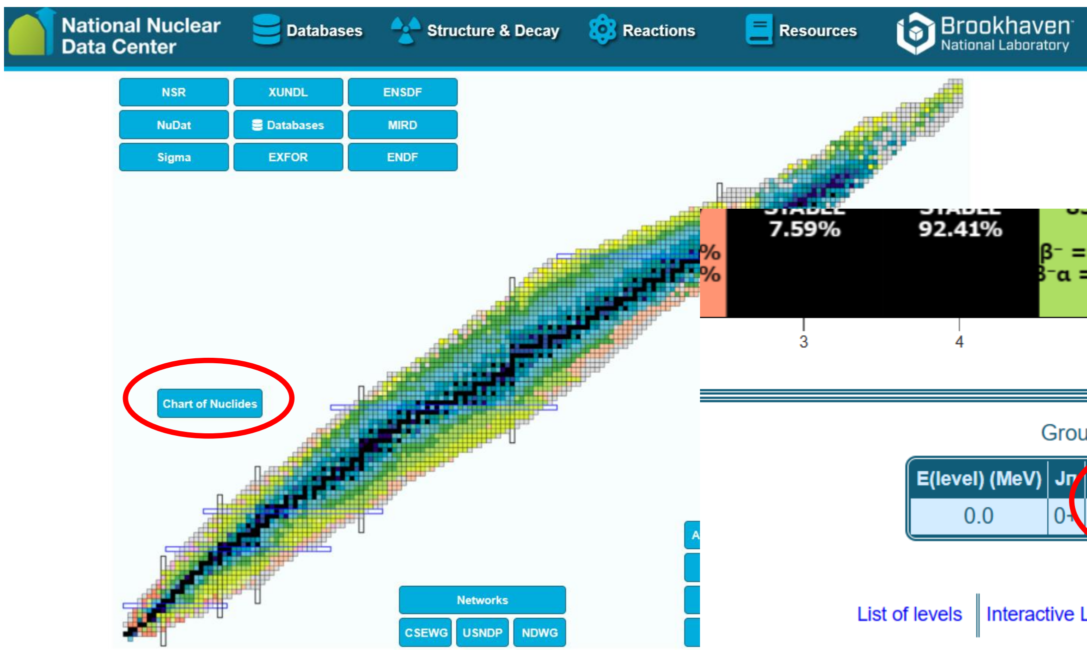
- mass difference between ^{12}C+p and ^{13}N is ? keV

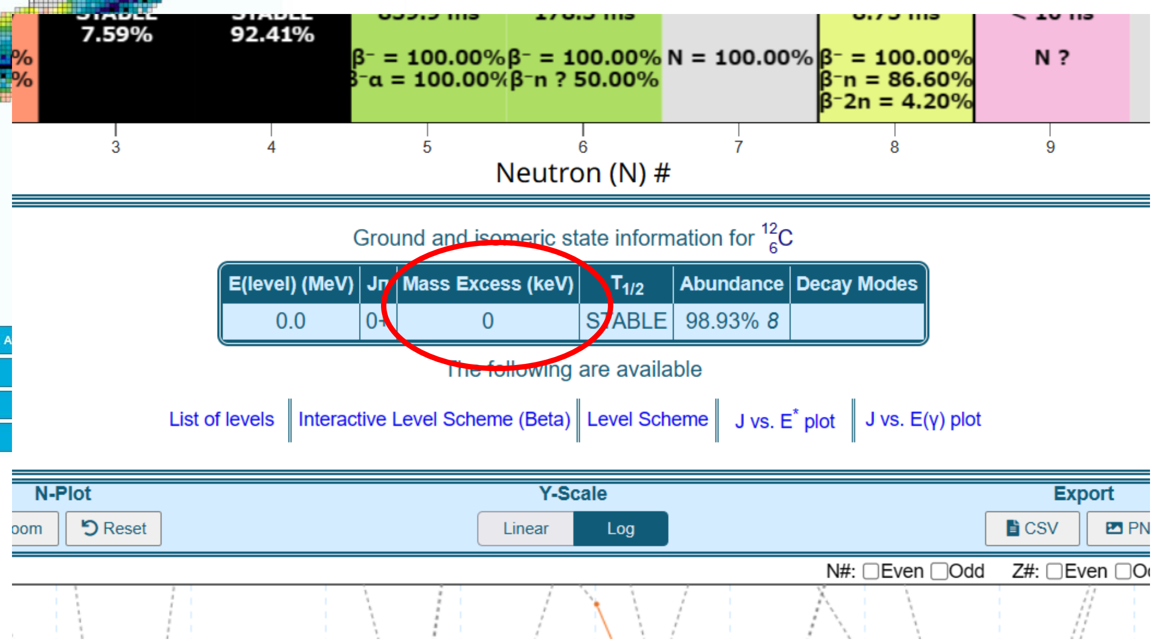
Get Mass Excess (mass) from NNDC.

ME: relative mass value to ^{12}C
(ME(^{12}C) = 0)



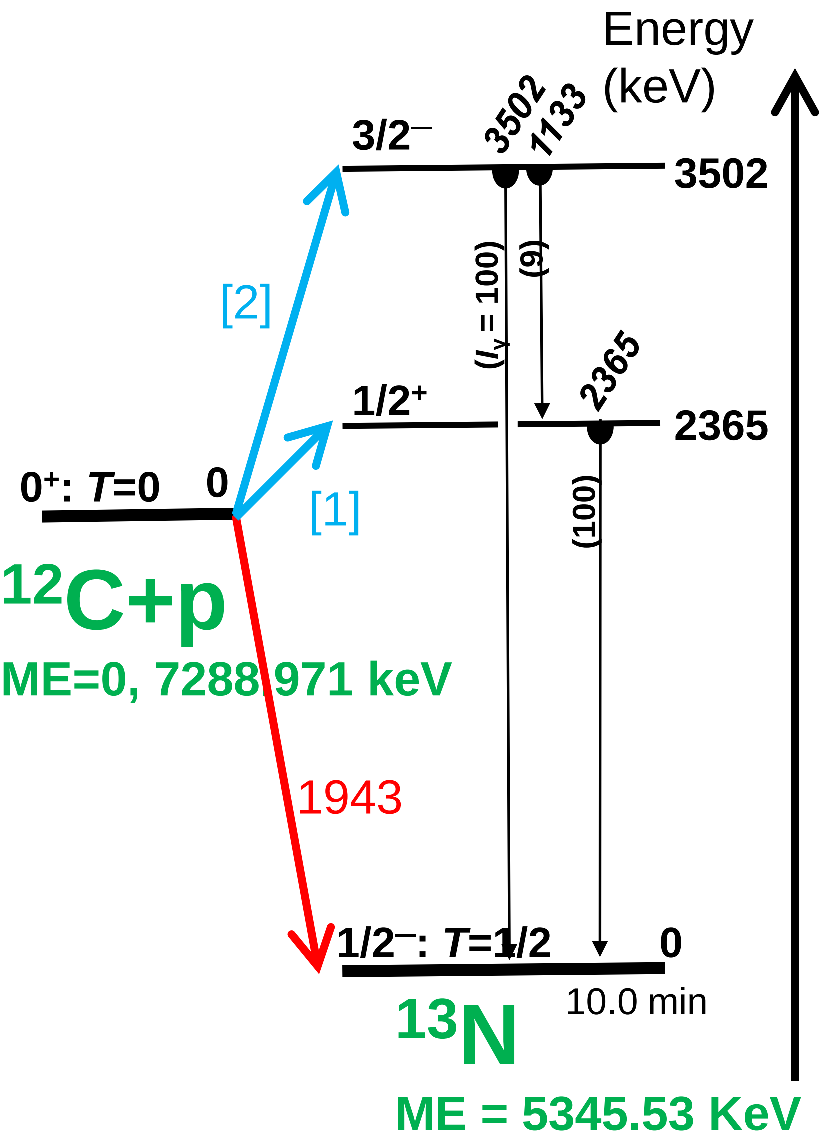

Google
NNDC
Google 検索
I'm Feeling Lucky


National Nuclear Data Center
Databases
Structure & Decay
Reactions
Resources
Brookhaven National Laboratory
NSR
XUNDL
ENSDF
NuDat
Databases
MIRD
Sigma
EXFOR
ENDF
Chart of Nuclides
Networks
CSEWG
USNDP
NDWG


7.59%
92.41%
β⁻ = 100.00%
β⁻α = 100.00%
β⁻ = 100.00%
β⁻n ? 50.00%
N = 100.00%
β⁻ = 100.00%
β⁻n = 86.60%
β⁻2n = 4.20%
N ?
Neutron (N) #
Ground and isomeric state information for $^{12}_{6}C$
E(level) (MeV)
Jπ
Mass Excess (keV)
T1/2
Abundance
Decay Modes
0.0
0+
0
STABLE
98.93% 8
The following are available
List of levels
Interactive Level Scheme (Beta)
Level Scheme
J vs. E* plot
J vs. E(γ) plot
N-Plot
Y-Scale
Export
Reset
Linear
Log
CSV
N#: Even Odd
Z#: Even

Question 1
energy (mass*) difference between ^{12}C+p & ^{13}N

- Energy levels of at the center-of-mass (CM) system
- mass difference between ^{12}C+p and ^{13}N is 1943 keV

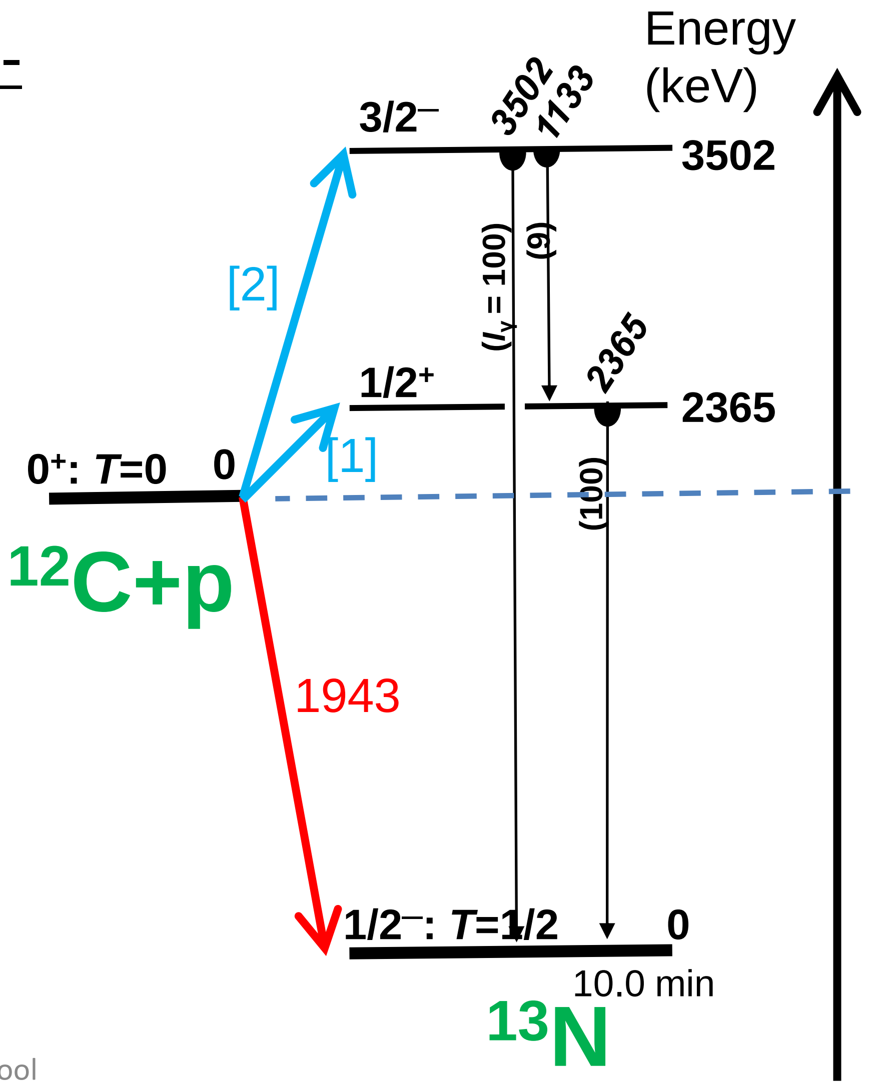


Question 1
energy (mass) difference between ^{12}C+p & ^{13}N

- Energy levels of at the center-of-mass (CM) system

- mass difference between ^{12}C+p and ^{13}N is 1943 keV

- Calculate the energy differences for [1] and [2]



Answers:
422 keV for [1]
1559 keV for [2]

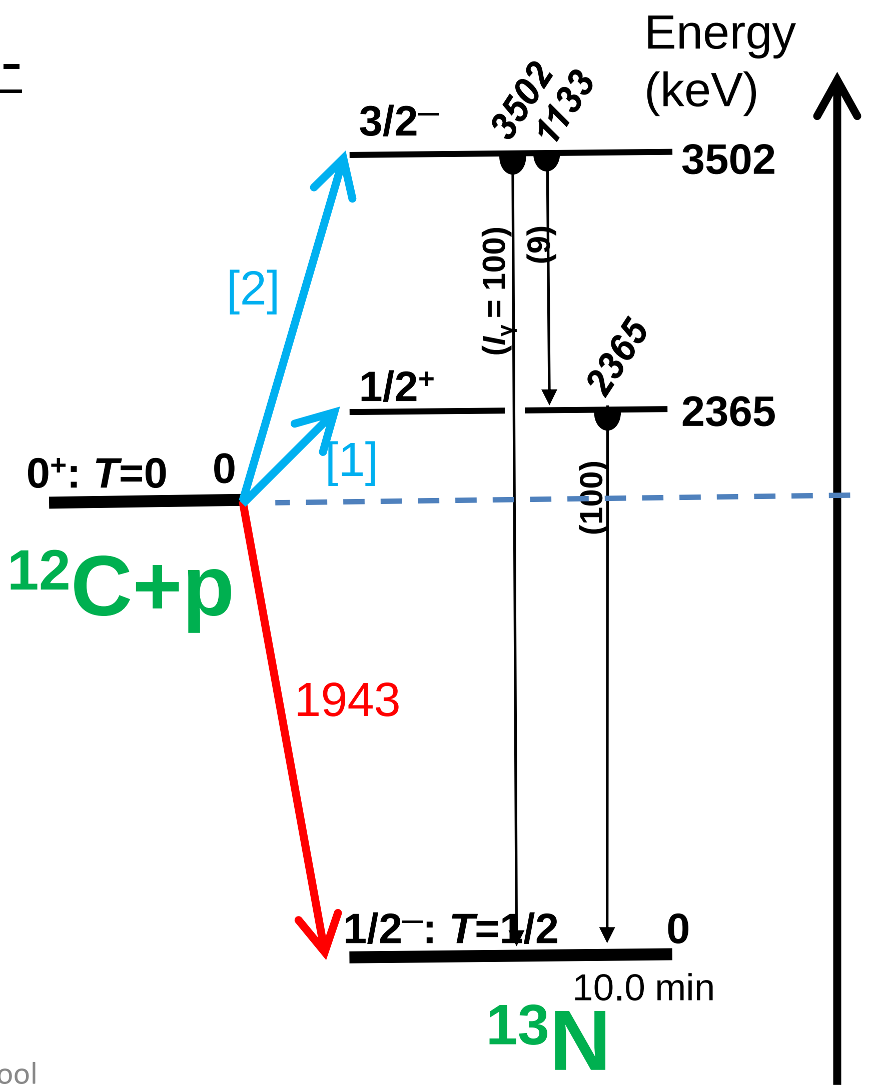
Question 1
energy (mass) difference between ^{12}C+p & ^{13}N

- Energy levels of at the <u>center-of-mass (CM) system</u>

- mass difference between ^{12}C+p and ^{13}N is 1943 keV

- Calculate the energy differences for [1] and [2]

Note:
The level at 2365 keV (or 3502 keV) is a **resonance**.



In the “laboratory” frame

nuclear reaction study with energetic beams

Question 2

proton kinetic energy necessary to populate a resonance?

Is it

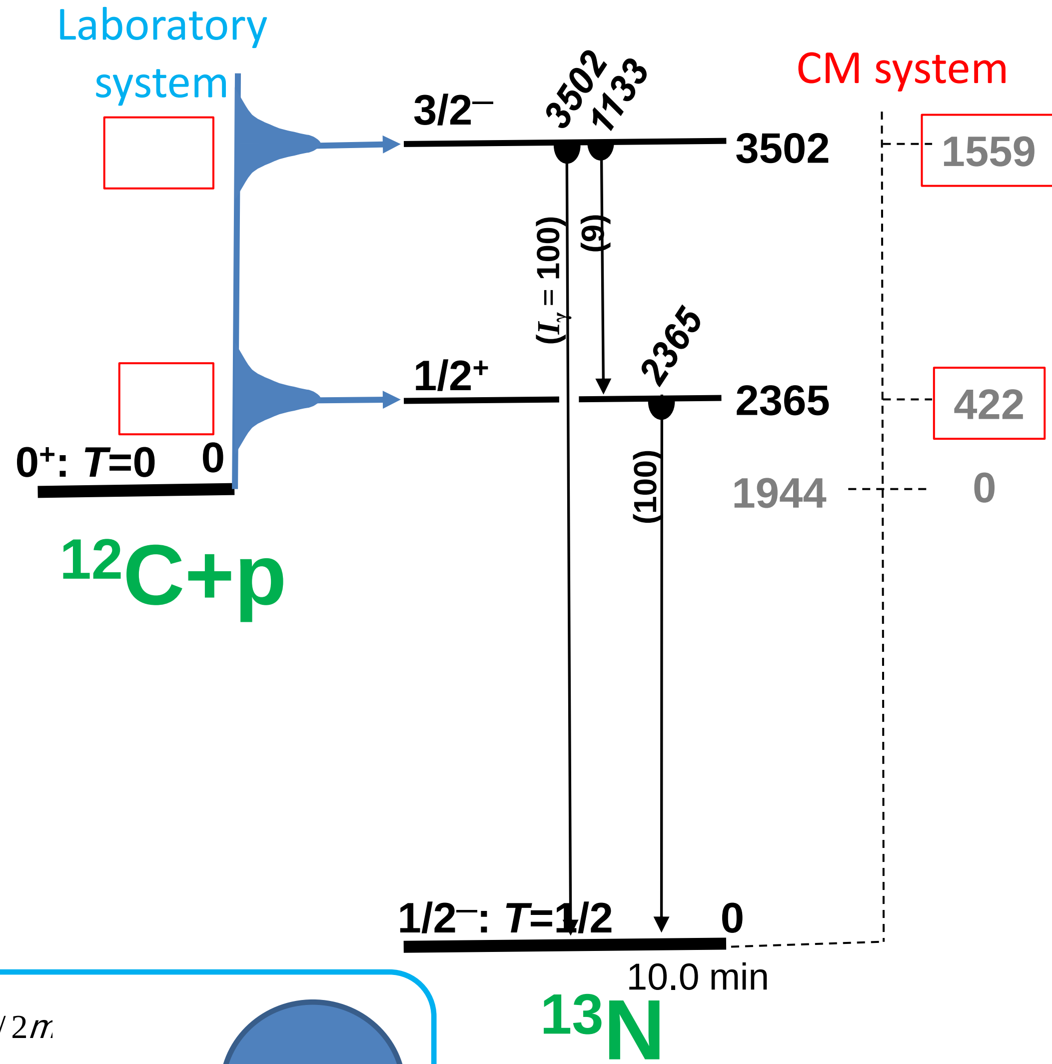
Laboratory <-> CM system :

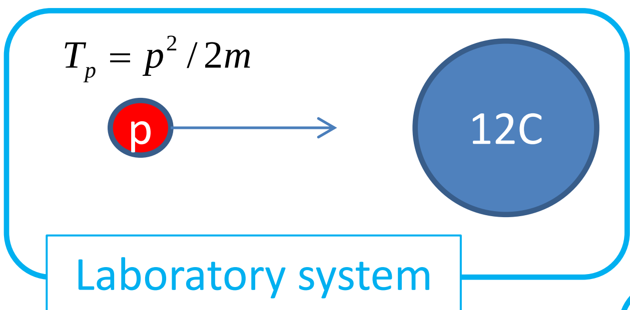
$$T_{\mathrm{CM}} = \frac{p_{\mathrm{rel}}^2}{2\mu}, \mu = \frac{m_1 m_2}{m_1 + m_2}, p_{rel} = \frac{m_2 p_1 - m_1 p_2}{m_1 + m_2}$$

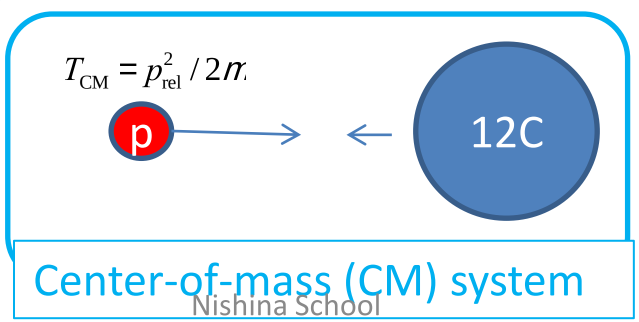
$$m_1 = m, m_2 \cong 12m, p_1 = p, p_2 = 0$$

$$\therefore \ T_{\mathrm{CM}} = \frac{p^2}{2m} \times \frac{12}{13} = T_p \times \frac{12}{13}$$

μ : reduced mass







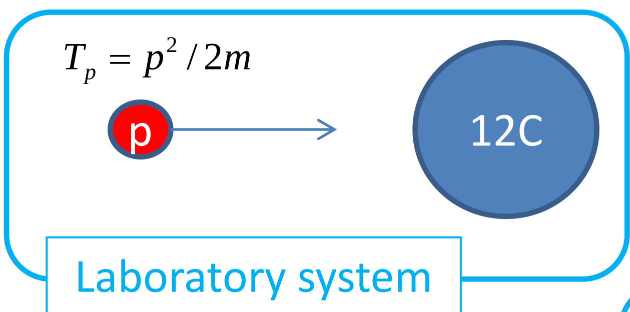
Laboratory <-> CM system :

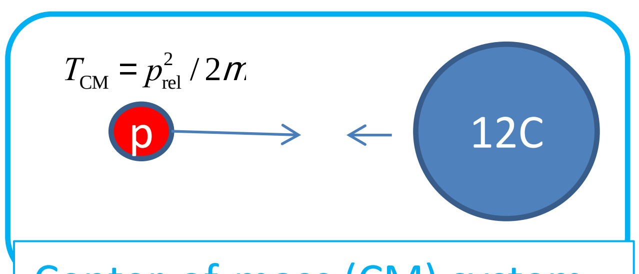
$$T_{\mathrm{CM}} = \frac{p_{\mathrm{rel}}^2}{2m},\ m = \frac{m_1 m_2}{m_1 + m_2},\ p_{rel} = \frac{m_2 p_1 - m_1 p_2}{m_1 + m_2}$$

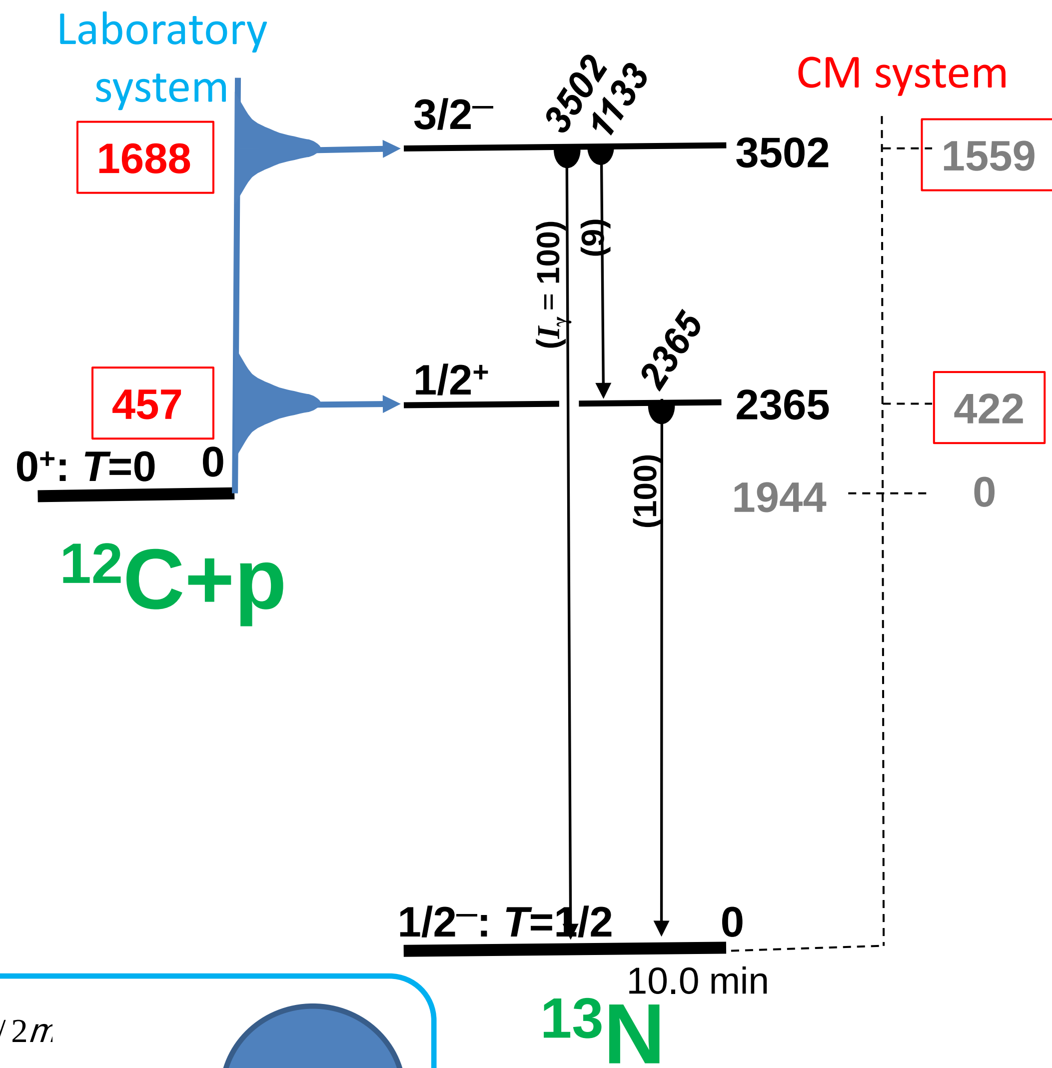
$$m_1 = m,\ m_2 \approx 12m,\ p_1 = p,\ p_2 = 0$$

$$\therefore\ T_{\mathrm{CM}} = \frac{p^2}{2m} \times \frac{12}{13} = T_p \times \frac{12}{13}$$

μ : reduced mass

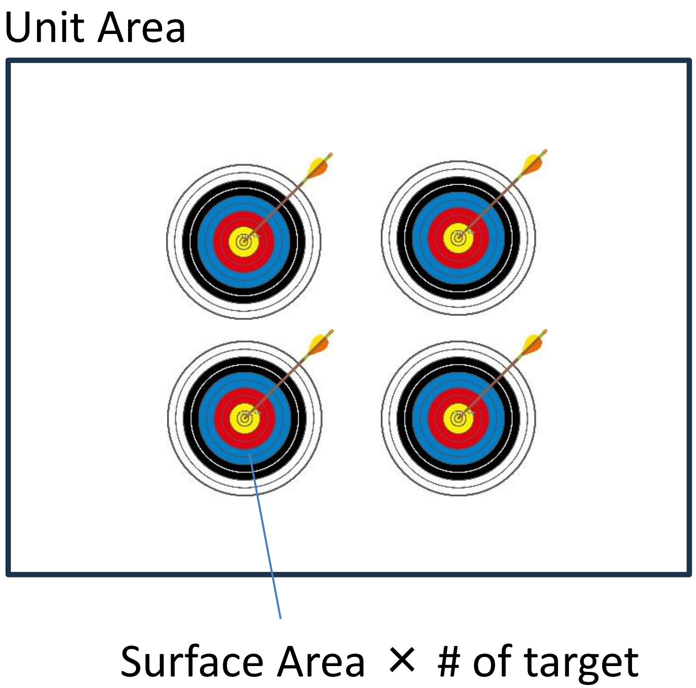






How do we measure the **cross sections**?

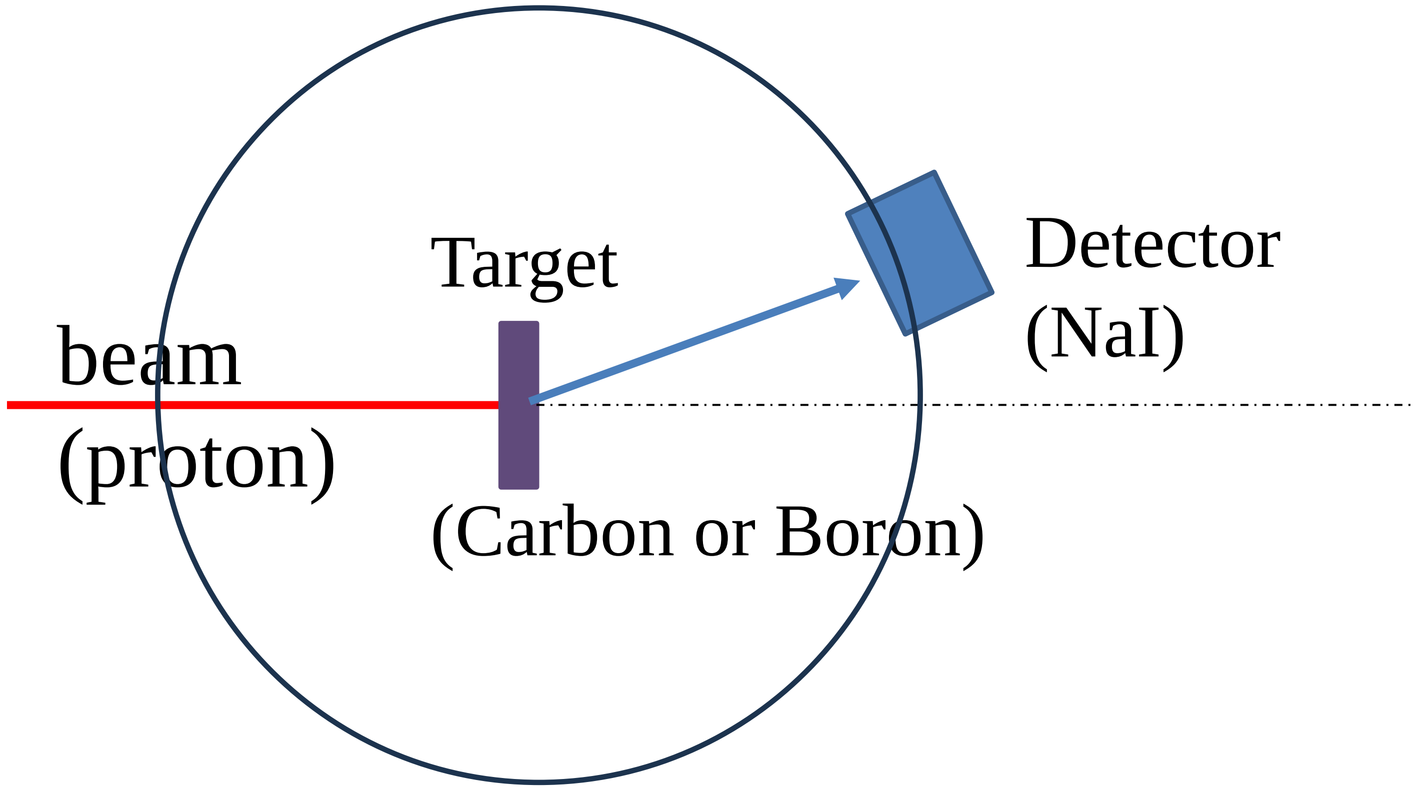
Cross Section?



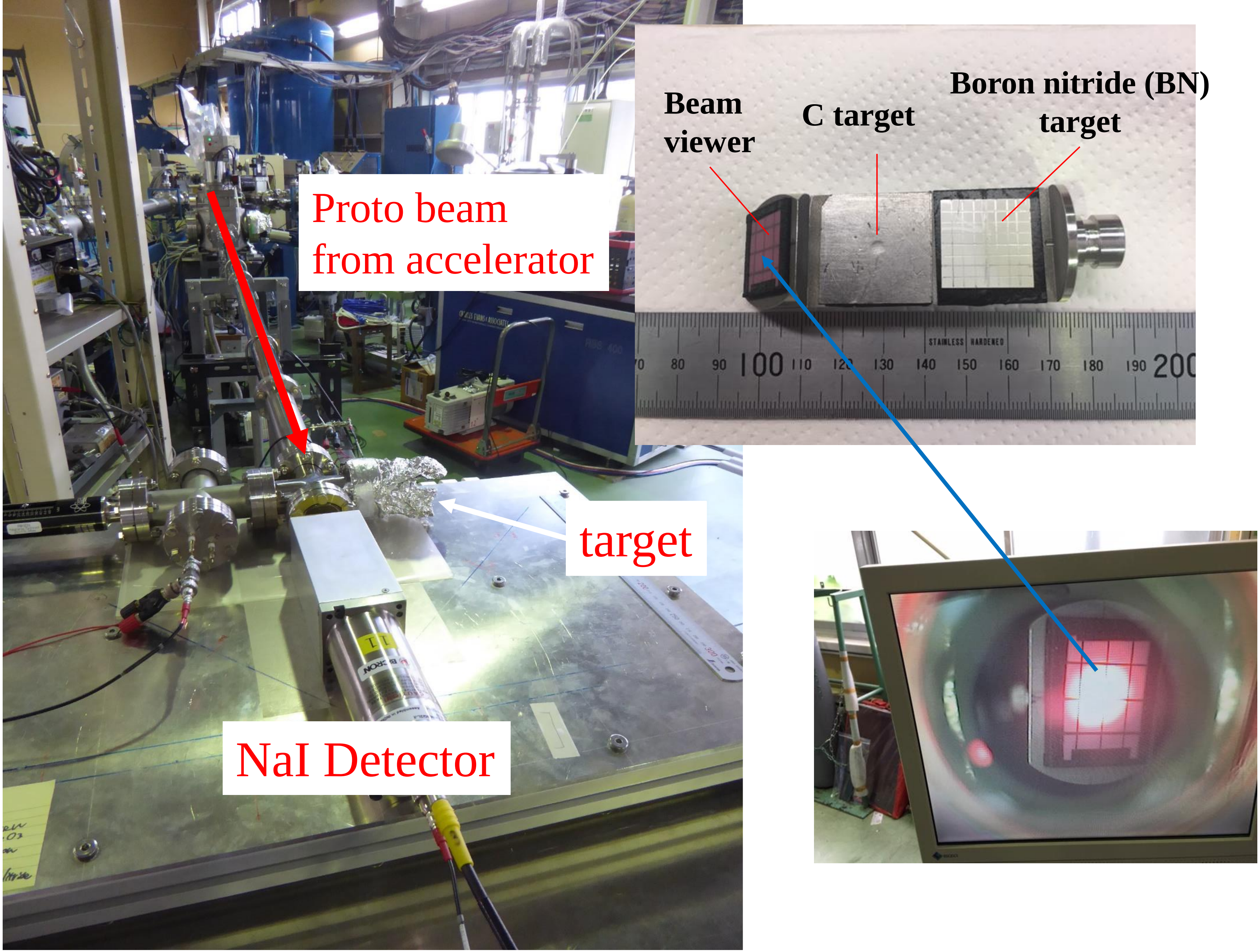
How many arrows hit targets?
= (# of arrows) × (# of target/unit area) × (Area of a target)

$$N_r = N_b \times N_t \times \sigma$$

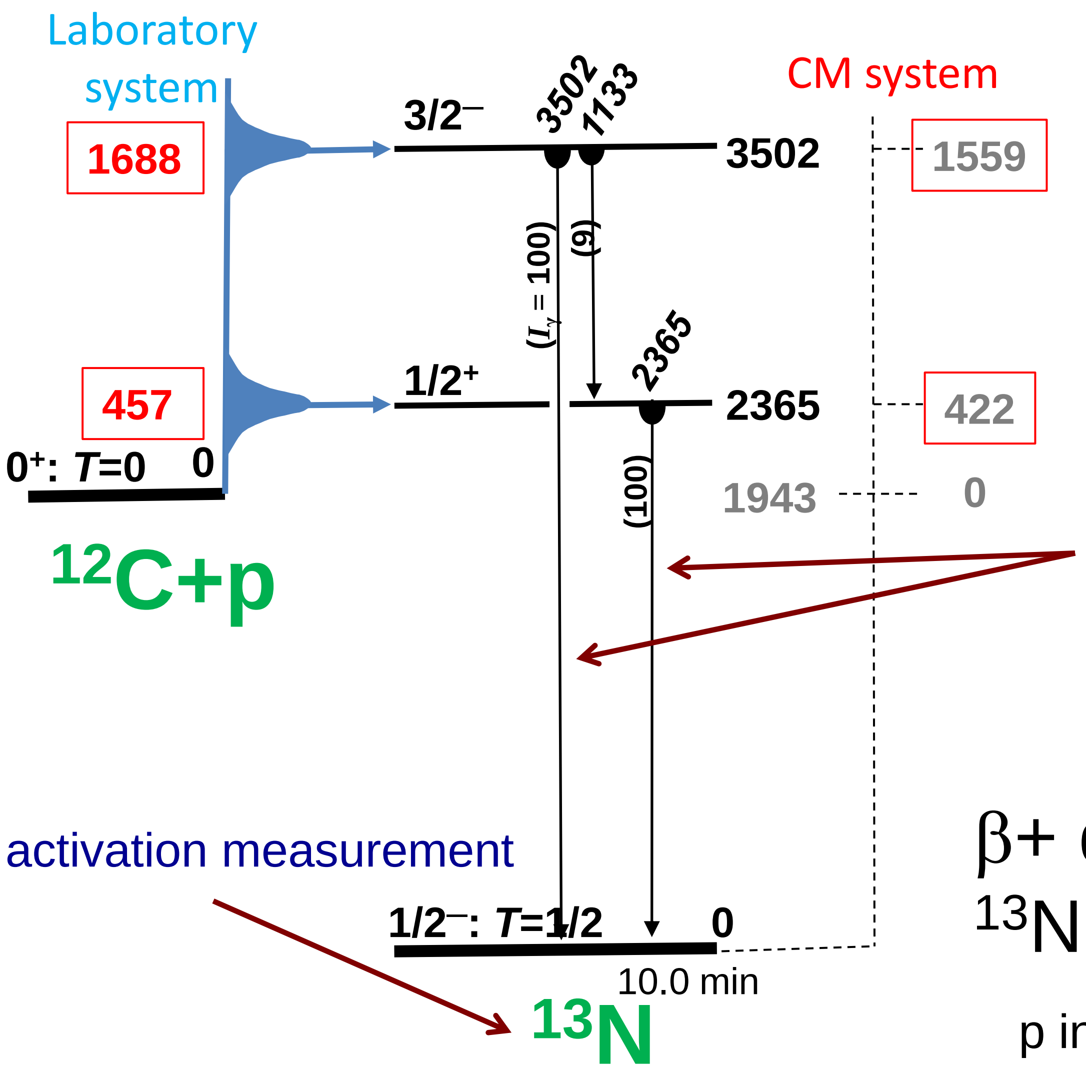
How many reactions occur?
= (# of beams) × (# of target/unit area) × (Cross section)



$$N_\gamma = N_b \times N_t \times d\sigma/d\Omega \times d\Omega \times \varepsilon \times P\gamma$$


Proto beam
from accelerator
target
NaI Detector
Beam
viewer
C target
Boron nitride (BN)
target

“in-beam” and “activation”



In-beam measurement

β+ decay

$e^+ + e^- \rightarrow 2\gamma$ (511 keV*)

* $m_e c^2$

$^{13}N \rightarrow {}^{13}C + e^+ + \nu$

p in ^{13}N decays! (to n)

Remember a question of Sakurai san

Extraction of the (resonant capture) **cross section** of $^{12}C(p,\gamma)^{13}N$

Important parameters

1. The number of protons : N_b
2. The number of ^{12}C : N_t
3. The number of the 1st or 2nd resonance populated : N_r
4. The number of emitted γ: N_γ

Extraction of the (resonant capture) cross section of $^{12}C(p,\gamma)^{13}N$ -- more in details --

I : electric current of the beam
e : electron charge
t : measurement time
σ : cross section of p+12C -> 13N*(2nd :3502 keV) reaction
ρ : density of target carbon foil
T : thickness of target carbon foil
N_A : Avogadro number
P_γ : decay branching ratio of the 3502 keV resonance
Ω : solid angle of NaI(Tl)
ε : photo peak efficiency of 3502 keV

$$
\left\{
\begin{aligned}
N_b &= I \cdot t / e \\
N_t &= N_A \cdot T \cdot \rho / 12 \\
N_r &= N_b \cdot N_t \cdot \sigma \\
N_\gamma &= N_r \cdot P_\gamma \cdot (\Omega / 4\pi) \cdot \varepsilon
\end{aligned}
\right.
$$

[Parameters]

I : electric current of the beam

e : electron charge

t : measurement time

σ : cross section of p+12C -> 13N*(2^{nd} :3502 keV) reaction

ρ : density of target carbon foil

T : thickness of target carbon foil

N_A : Avogadro number

P_γ : decay branching ration of 3502 keV

Ω : solid angle of NaI(Tl)

ε : photo peak efficiency of 3502 keV

$$\sigma = N_\gamma \cdot 12 \cdot 4\pi / (I / e \cdot t \cdot N_A \cdot T \cdot \rho \cdot P_\gamma \cdot \Omega \cdot \varepsilon)$$

$$d\sigma / d\Omega = N_\gamma \cdot 12 / (I / e \cdot t \cdot N_A \cdot T \cdot \rho \cdot P_\gamma \cdot \Omega \cdot \varepsilon)$$

O : should be measured during the experiment

Design of the experiments
Yield estimation

- the yield of the measurement to check the feasibility

$$d\sigma / d\Omega = N_\gamma \cdot 12 / \left(I / e \cdot t \cdot N_A \cdot T \cdot \rho \cdot P_\gamma \cdot \Omega \cdot \varepsilon \right)$$ **[μbarn/sr]**

- Let’s estimate the yield of N_γ this afternoon!

Useful information for yield estimation

- Assumption
  - Typical beam current : **I = 200 [nA]**
  - Typical beam irradiation time : **20 - 30 [min.]**

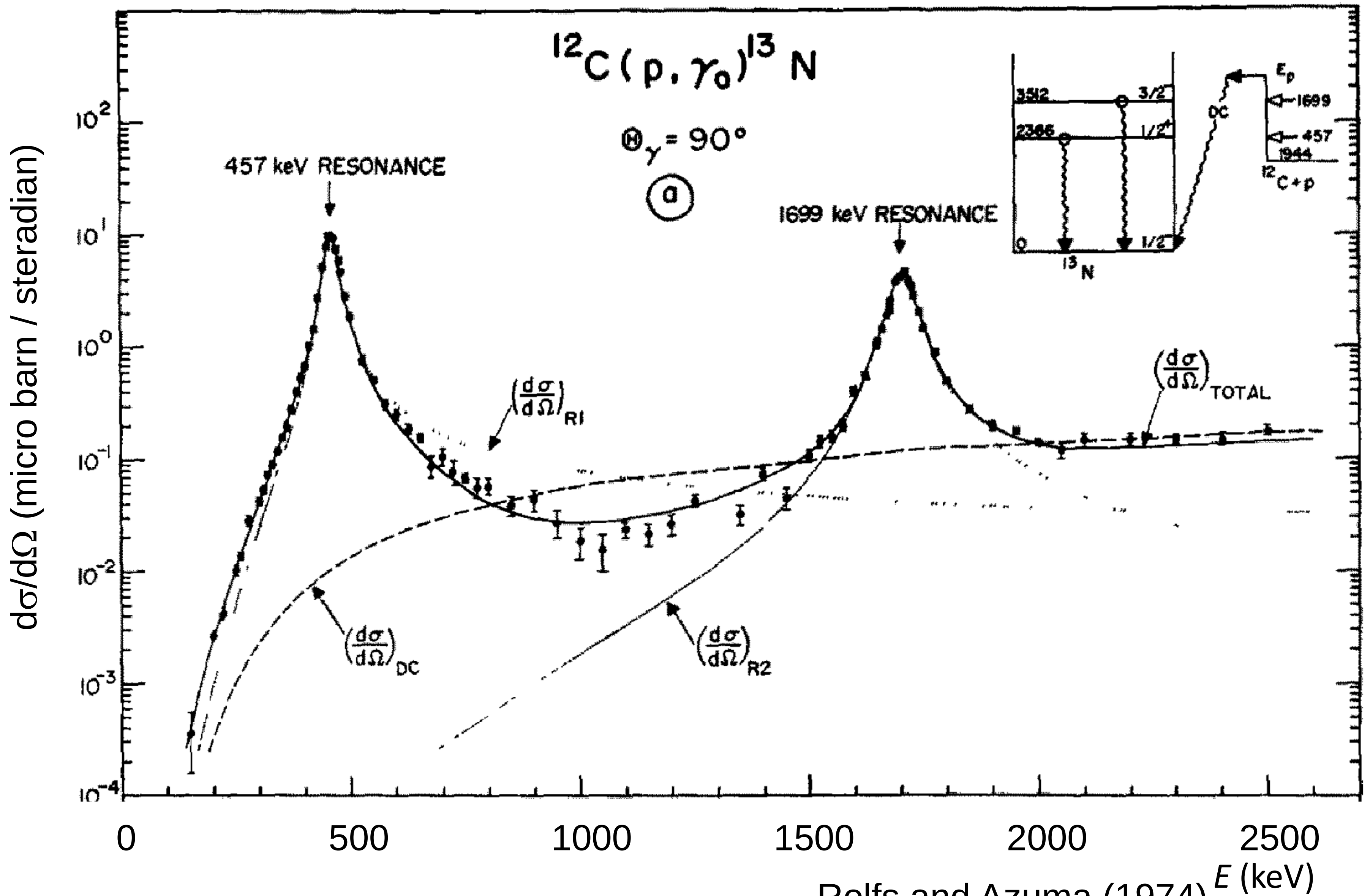
- Please consider and check whether **the measurement time (t)** and **the distance (L)** of NaI(Tl) to be set against the target are realistic or not.
  You can select at L =10, 40 cm
  Be not beyond about 500 counts/sec, for γ ray counting rate.

- 1 mb = 10^{-27} [cm^2]

- **Proton beam energy = 2 MeV**

- Target thickness >> proton range : "Thick target method"

$^{12}C(p,\gamma)^{13}N$ cross section in literatures

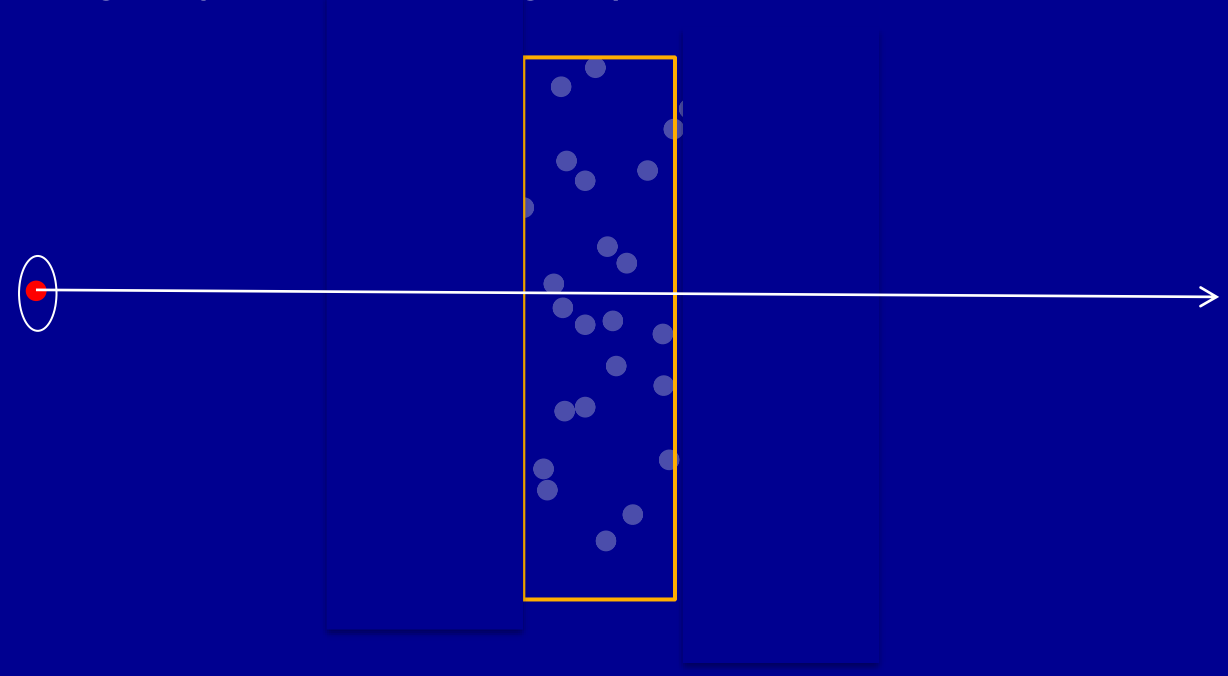


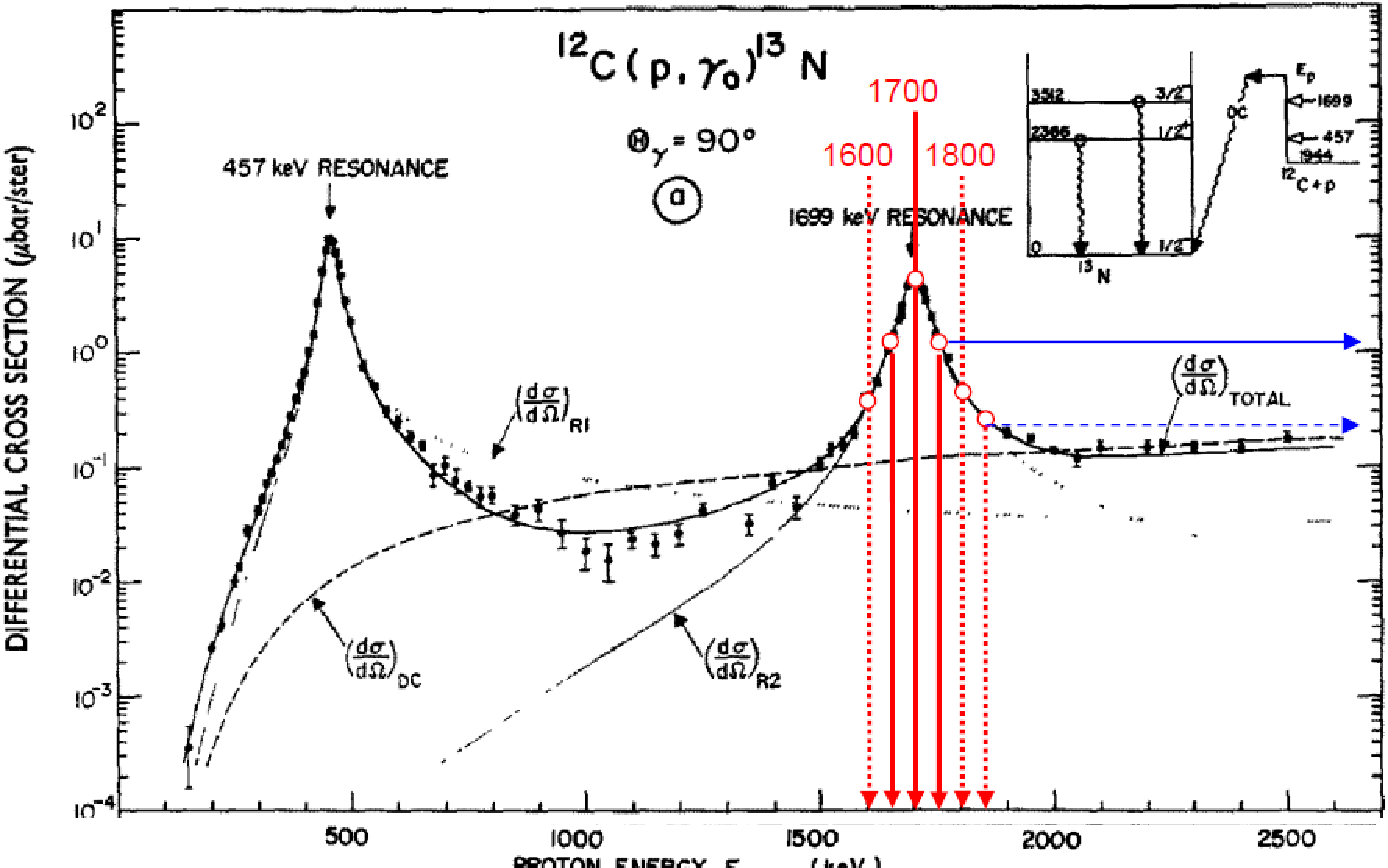
Rolfs and Azuma (1974)

The usual method for a certain energy interval

Target: thin so that the p energy-change (loss) is small.

σ: cross section (area of the imaginary circle in the figure)

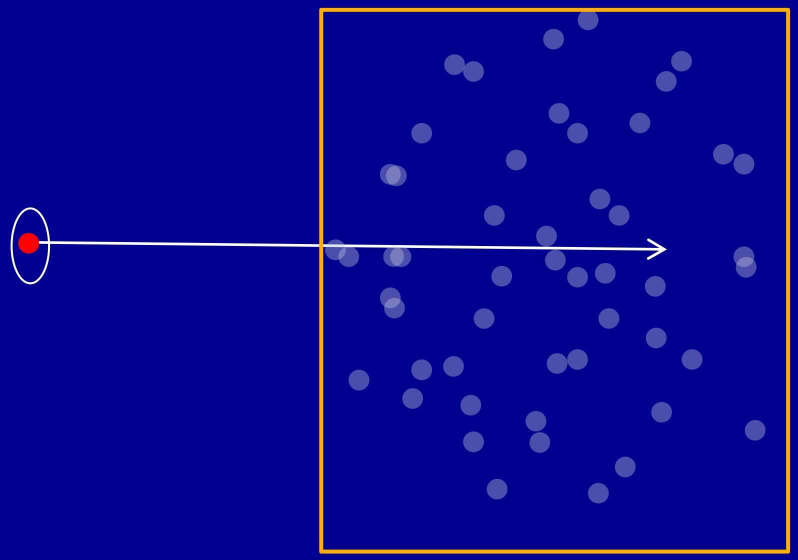



$^{12}C(p,\gamma_0)^{13}N$
$\Theta_\gamma = 90°$
(a)
457 keV RESONANCE
1699 keV RESONANCE
1600
1700
1800
$\left(\frac{d\sigma}{d\Omega}\right)_{R1}$
$\left(\frac{d\sigma}{d\Omega}\right)_{DC}$
$\left(\frac{d\sigma}{d\Omega}\right)_{R2}$
$\left(\frac{d\sigma}{d\Omega}\right)_{TOTAL}$
DIFFERENTIAL CROSS SECTION (μbar/ster)
PROTON ENERGY E_{LAB} (keV)
3512
2366
3/2
1/2
0
^{13}N
DC
E_p
1699
457
1944
$^{12}C+p$

The thick target method

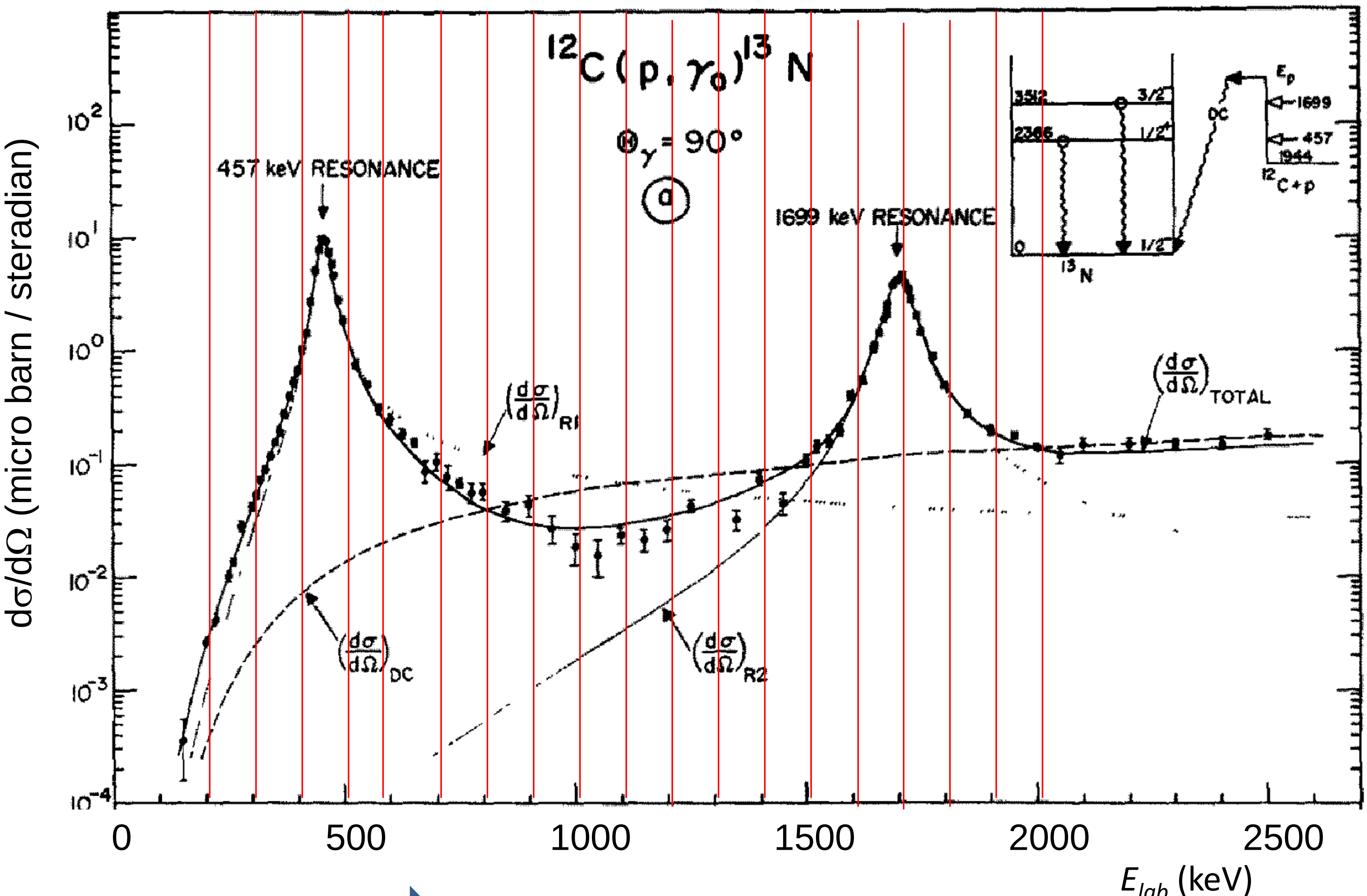
Target: thick enough to stop the beam

σ: cross section (area of the imaginary circle in the figure)



γ rays can be emitted in various different proton energies.

→ cross section integrated over a certain energy range.



Covert to target thickness from proton range

Rolfs and Azuma (1974)

Calculation for Range → SRIM code (free soft)

http://www.srim.org/

======= Target Composition ========

Atom Name	Atom Numb	Atomic Percent	Mass Percent
C	6	100.00	100.00

=================================

Proton beam in Carbon

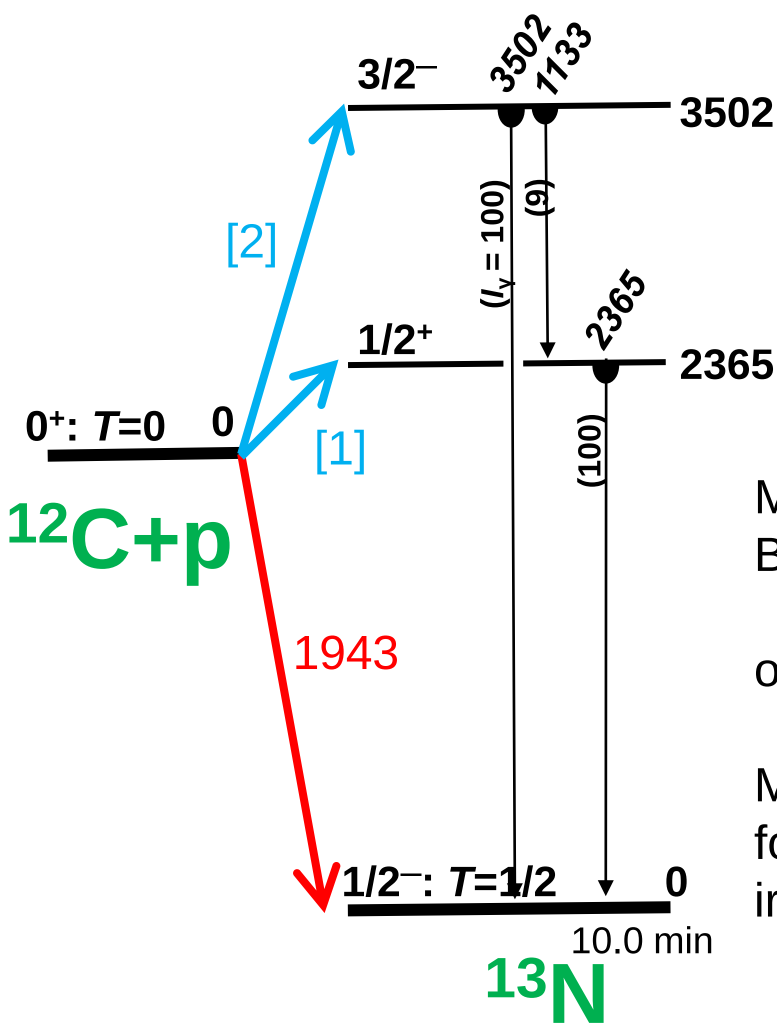
Bragg Correction = 0.00%

Stopping Units = MeV / (mg/cm2)

See bottom of Table for other Stopping units

Ion Energy	dE/dx Elec.	dE/dx Nuclear	Projected Range	Longitudinal Straggling	Lateral Straggling
500.00 keV	3.534E-01	2.738E-04	4.40 um	1837 A	1974 A
550.00 keV	3.334E-01	2.524E-04	5.04 um	2092 A	2182 A
600.00 keV	3.160E-01	2.342E-04	5.72 um	2347 A	2402 A
650.00 keV	3.007E-01	2.186E-04	6.44 um	2601 A	2632 A
700.00 keV	2.872E-01	2.051E-04	7.19 um	2856 A	2873 A
800.00 keV	2.640E-01	1.828E-04	8.80 um	3732 A	3385 A
900.00 keV	2.450E-01	1.651E-04	10.54 um	4553 A	3934 A
1.00 MeV	2.291E-01	1.507E-04	12.41 um	5350 A	4520 A
1.10 MeV	2.163E-01	1.387E-04	14.40 um	6134 A	5138 A
1.20 MeV	2.035E-01	1.286E-04	16.51 um	6915 A	5788 A
1.30 MeV	1.923E-01	1.199E-04	18.74 um	7703 A	6473 A
1.40 MeV	1.825E-01	1.124E-04	21.11 um	8501 A	7191 A
1.50 MeV	1.737E-01	1.058E-04	23.59 um	9309 A	7943 A
1.60 MeV	1.659E-01	9.998E-05	26.20 um	1.01 um	8727 A
1.70 MeV	1.588E-01	9.481E-05	28.93 um	1.10 um	9543 A
1.80 MeV	1.524E-01	9.017E-05	31.77 um	1.18 um	1.04 um
2.00 MeV	1.412E-01	8.219E-05	37.81 um	1.48 um	1.22 um

In-beam or activation



Measuring γ rays during
BOT (the proton beams on the target)

or

Measuring the ^{13}N activity (511 keV photons following the β^+ decay) after the beam irradiation

Radioactive decay

$N(t) = N_0 \exp[-t/\tau]$ ($N_0 e^{-t/\tau}$)

N: number of the initial nucleus (survived)

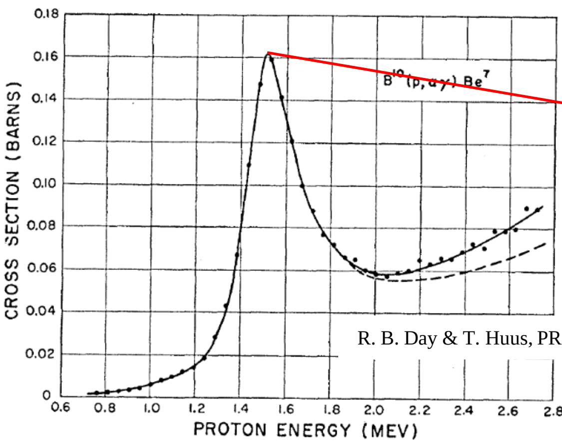
or number of decay per unit time

τ: mean life

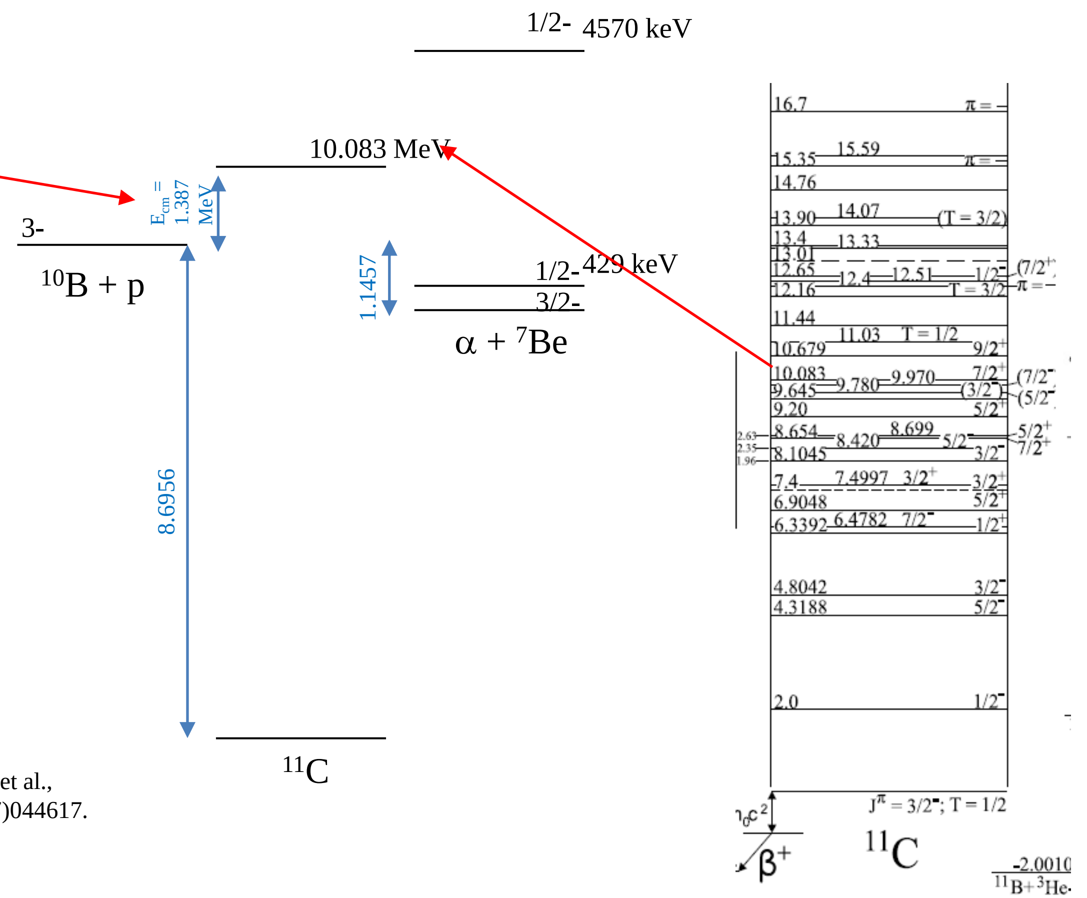
$t_{1/2}$ = half life --- $N(t_{1/2}) = N_0/2$

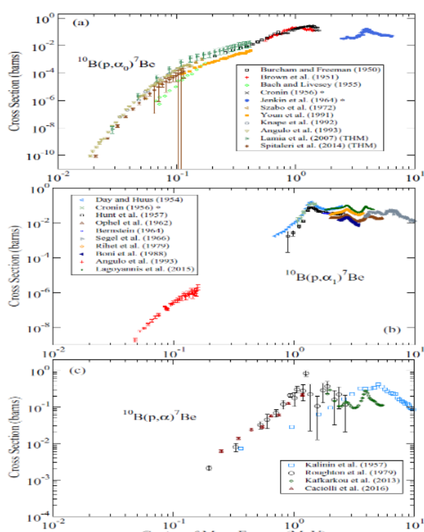
Q: Estimate the counts after 20 min. proton irradiation.

$^{10}B(p, \alpha\gamma)^{7}Be$



R. B. Day & T. Huus, PR 95(1954)1003





M. Wiescher, et al.,
PRC 95 (2017)044617.

Note)
Density of current Boron Nitride target
d = 2.06 g/cm^{3}

$^{27}Al(p, p_1\gamma)^{27}Al$, $^{27}Al(p, p_2\gamma)^{27}Al$ and $^{27}Al(p, \alpha\gamma)^{24}Mg$ reactions

Nuclear Instruments and Methods in Physics Research B 394 (2017) 28–32

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journal homepage: www.elsevier.com/locate/nimb

Thick target yields of proton induced gamma-ray emission from Al, Si and P

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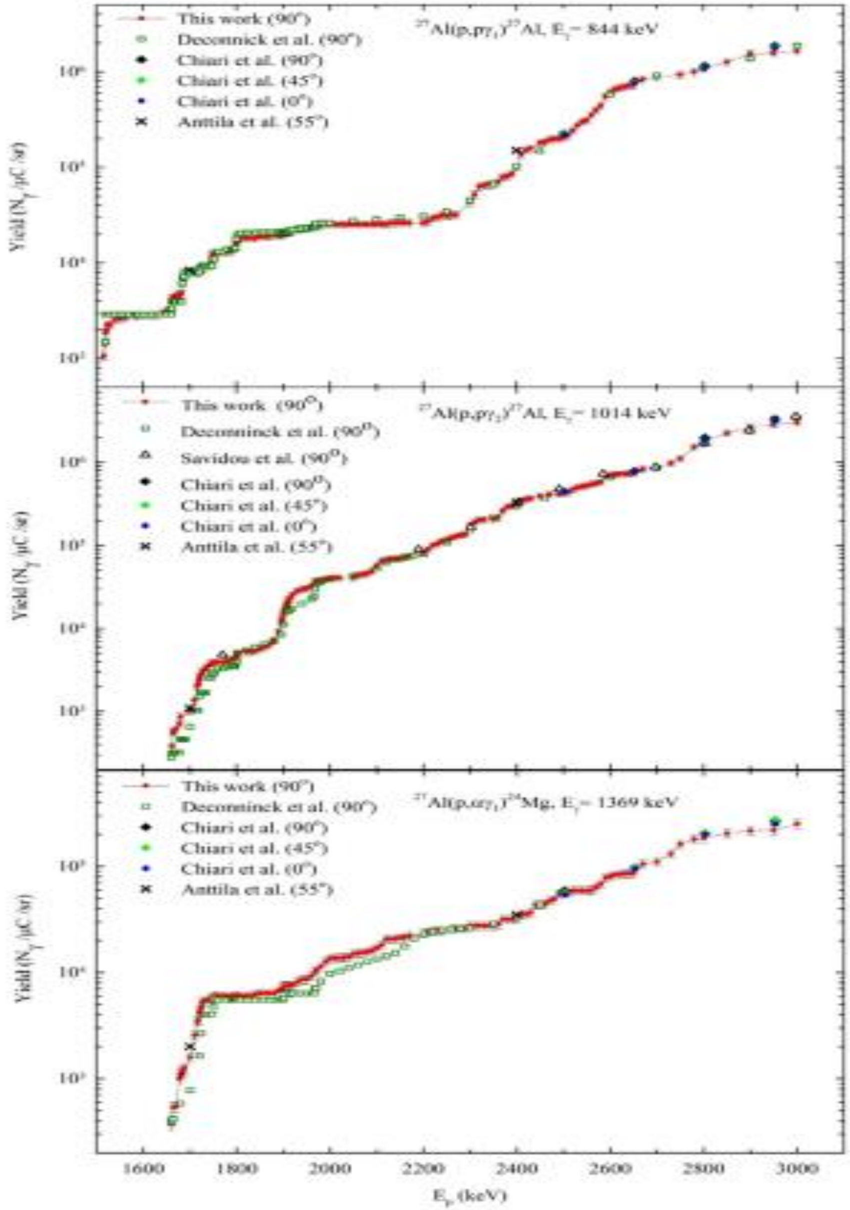
ABSTRACT

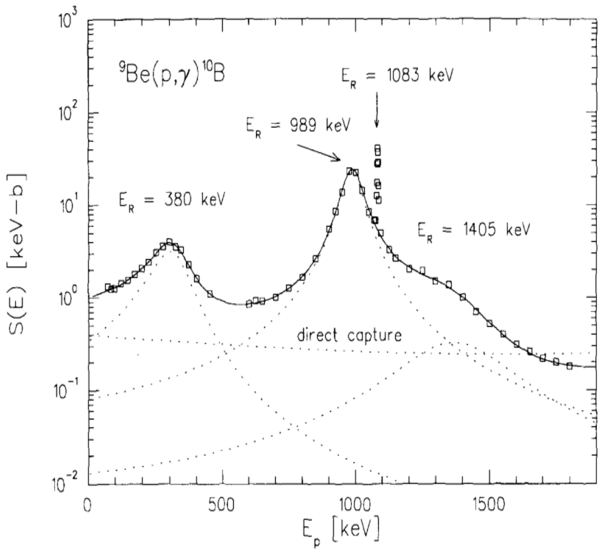
Thick target excitation yield curves of gamma-rays from the reactions $^{27}Al(p,p'\gamma)^{27}Al$ ($E_\gamma = 844$ and 1014 keV), $^{27}Al(p,\alpha\gamma)^{27}Al$ ($E_\gamma = 1369$ keV), $^{28}Si(p,p'\gamma)^{28}Si$ ($E_\gamma = 1779$ keV), $^{29}Si(p,p'\gamma)^{29}Si$ ($E_\gamma = 1273$ keV) and $^{31}P(p,p'\gamma)^{31}P$ ($E_\gamma = 1266$ keV) were measured by bombarding pure-element targets with protons at energies below 3 MeV. Gamma-rays were detected with a High Purity Ge detector placed at an angle of 90° with respect to the beam direction. The obtained thick target gamma-ray yields were compared with the previously published data. The overall systematic uncertainty of the thick target yield values



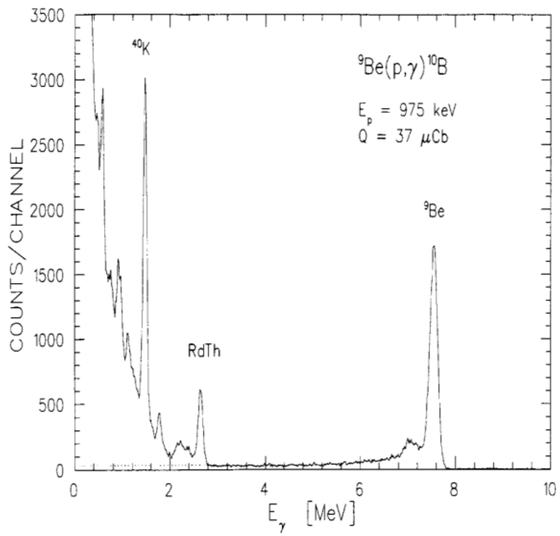
γ ray yields on thick Al target (Nγ/μC/sr on literature)

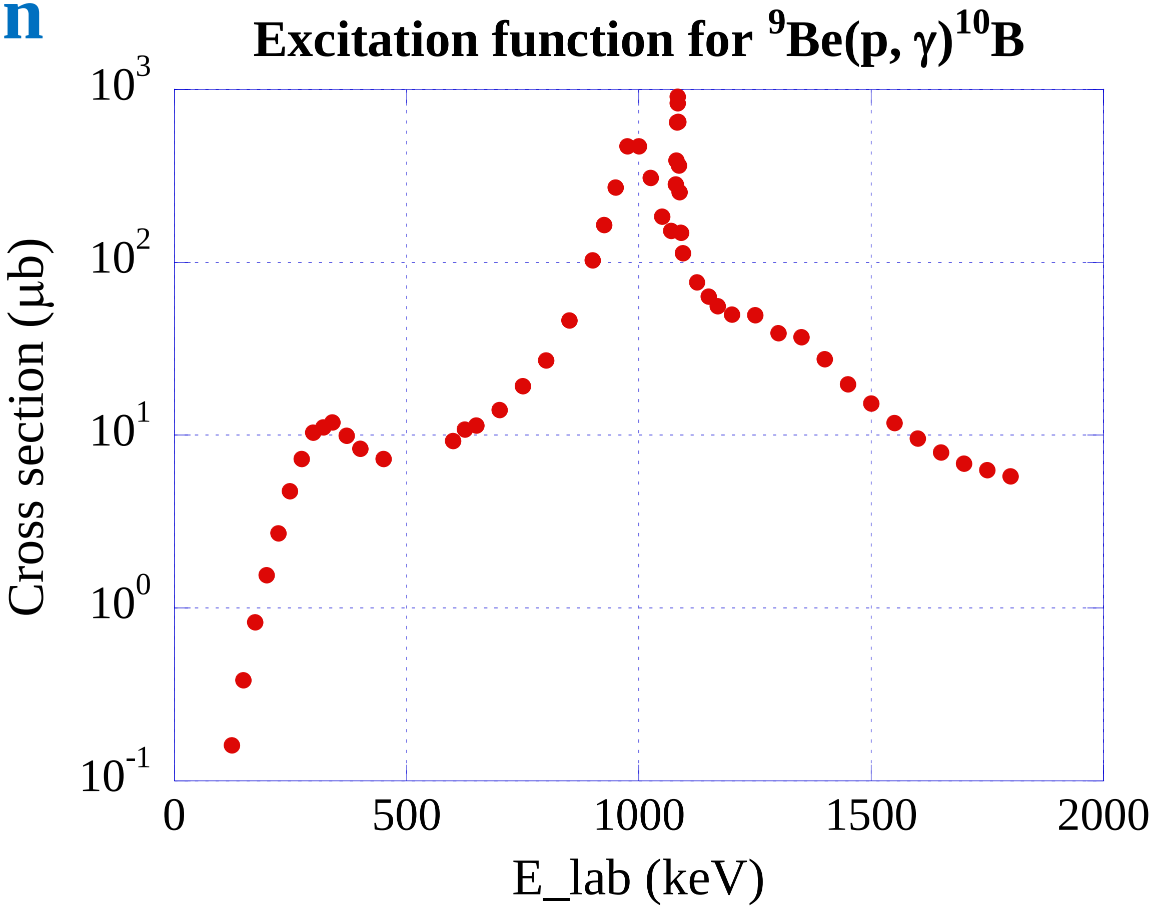
⇒ Can estimate Nγ without # of target

$^{9}Be(p, \gamma)^{10}B$ reaction



D. Zahnow et al. / Nuclear Physics A 589 (1995) 95–105





Solid angle (unit: steradian)

Beam intensity: We assume **200 nA**.
(elementary charge: 1.6×10^{-19} C:1 A = 1 C/s)

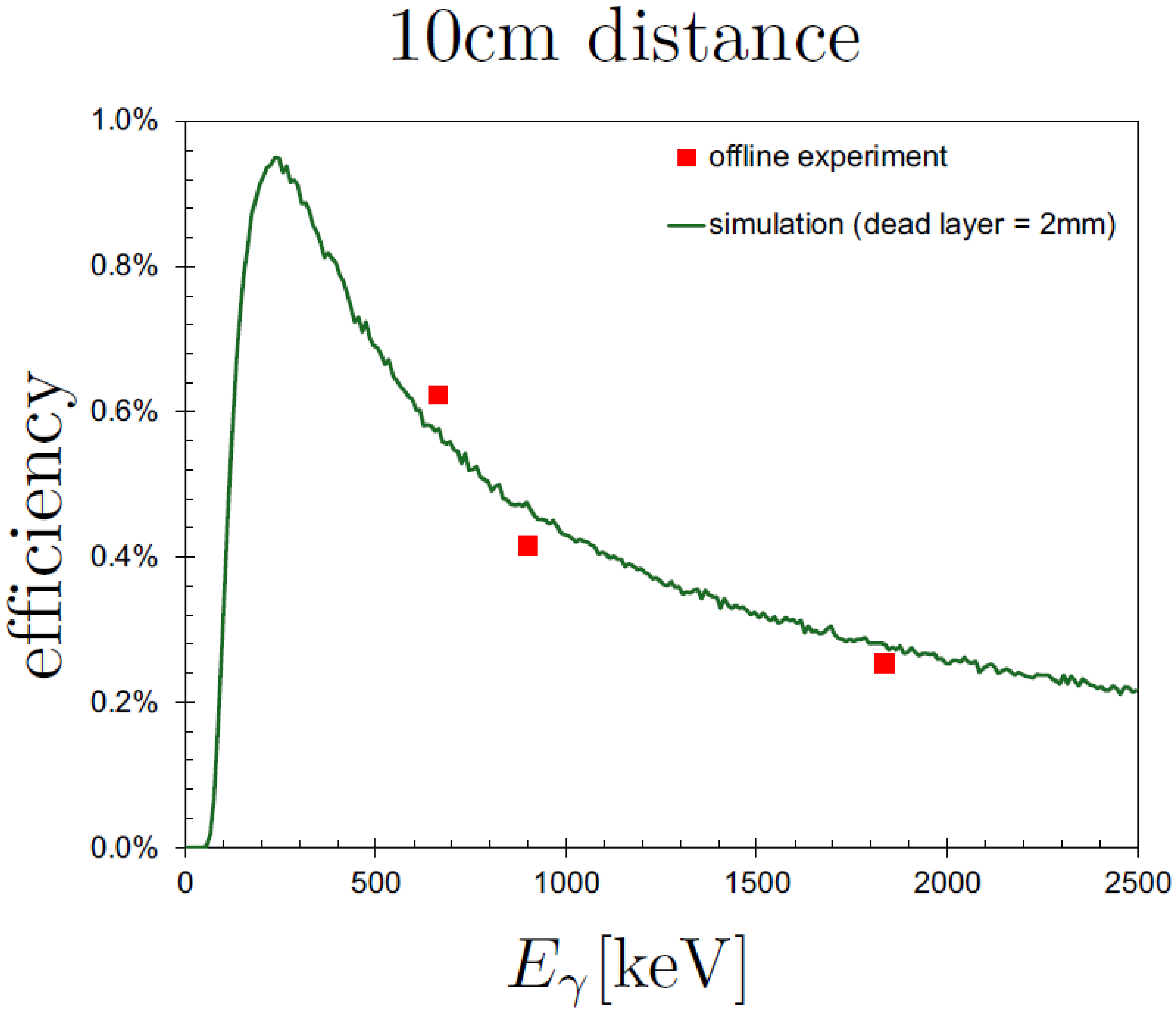
Size of the NaI(Tl) crystal: 6x6x12 cm^2

"Target thickness"
divide into 10 or 20 slices.

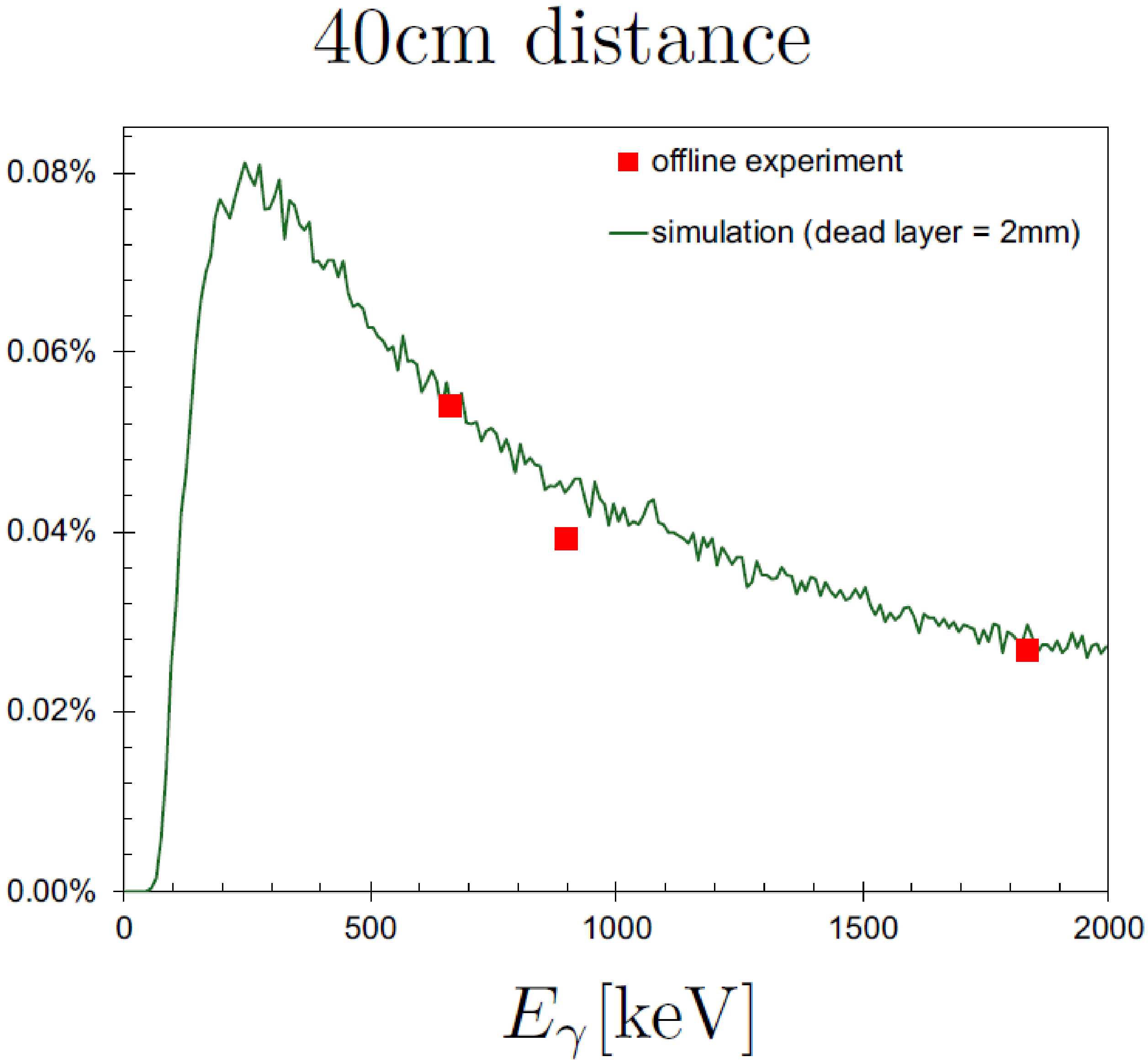
Photo (full-energy-peak) efficiency of the NaI(Tl) including solid angle => Use following figures

Which distance from the target to the center of NaI(Tl) scintillator should be?
L = 10 cm or 40 cm

NaI Detector efficiency at L = 10 cm



NaI Detector efficiency at L = 40 cm



Wide energy range (L = 10 cm)

In case of 9Be(p, γ) reaction, the efficiency is more complex.
Please ask Yoshida san and Iimura san.