The $\Omega_c(3120)$ as a molecular state and its analogy with the $\Omega(2012)$

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N. Ikeno, W. H. Liang, and E. Oset, Phys. Rev. D 109, 054023 (2024).

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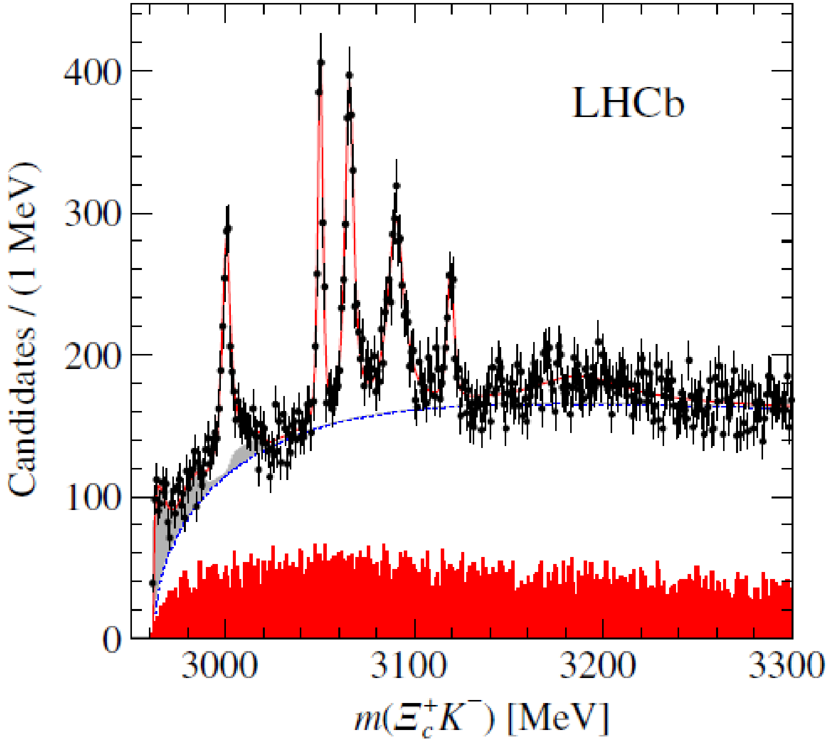
Discovery of excited Ω_c (=css) states by LHCb

- Five states of Ω_c in 2017

Phys. Rev. Lett. 118, 182001 (2017)

$\Omega_c(3000)^0$
$\Omega_c(3050)^0$
$\Omega_c(3066)^0$
$\Omega_c(3090)^0$
$\Omega_c(3119)^0$



- Two additional states of Ω_c in 2023

Phys. Rev. Lett. 131, 131902 (2023)

Resonance	m (MeV)	Γ (MeV)
$\Omega_c(3000)^0$	3000.44 ± 0.07	3.83 ± 0.23
$\Omega_c(3050)^0$	3050.18 ± 0.04	0.67 ± 0.17
$\Omega_c(3065)^0$	3065.63 ± 0.06	3.79 ± 0.20
$\Omega_c(3090)^0$	3090.16 ± 0.11	8.48 ± 0.44
$\Omega_c(3119)^0$	3118.98 ± 0.12	0.60 ± 0.63
$\Omega_c(3185)^0$	3185.1 ± 1.7	50 ± 7
$\Omega_c(3327)^0$	3327.1 ± 1.2	20 ± 5

- Many theoretical studies to understand the Ω_c nature

-Quark model, -Molecular picture, Lattice QCD,....

Works based on molecular perspective of Ω_c

G. Montana, A. Feijoo and A. Ramos, Eur. Phys. J. A 54, 64 (2018)
V. R. Debastiani, J. M. Dias, W. H. Liang and E. Oset, Phys. Rev. D97, 094035 (2018)

- Coupled channels of Meson-Baryon interaction
- Vector meson exchange interaction based on local hidden gauge approach

- $\Omega_c(3050)$, $\Omega_c(3090)$:
  $J^P = 1/2^-$ states by both works

- **$\Omega_c(3119)$:**
  Not obtained as a $J^p = 1/2^-$ state by both works

  $J^P = 3/2^-$ state by V. R. Debastiani et al.
  - Couples mostly to $\Xi_c^* \bar{K}$, $\Omega_c^* \eta$,
  - The mass was around 3125 MeV, and the width was zero

TABLE I. $J = 1/2$ states chosen and threshold mass in MeV.

States	$\Xi_c \bar{K}$	$\Xi_c' \bar{K}$	ΞD	$\Omega_c \eta$	ΞD^*	$\Xi_c \bar{K}^*$	$\Xi_c' \bar{K}^*$
Threshold	2965	3074	3185	3243	3327	3363	3472

TABLE II. $J = 3/2$ states chosen and threshold mass in MeV.

States	$\Xi_c^* \bar{K}$	$\Omega_c^* \eta$	ΞD^*	$\Xi_c \bar{K}^*$	$\Xi^* D$	$\Xi_c' \bar{K}^*$
Threshold	3142	3314	3327	3363	3401	3472

V. R. Debastiani et al., PRD 97, 094035 (2018)

$\Omega_c(3119)$ [$\Omega_c(3120)$ in PDG] and $\Omega(2012)$

J^P = 3/2$^-$ state by V. R. Debastiani, J. M. Dias, W. H. Liang, E. Oset, PRD97 (2018)

- couples mostly to $\Xi_c^* \bar{K}, \Omega_c^* \eta,$ - **the width was zero**

Transition potential from coupled channels

- $\Omega_c(3120)$ (=css) with $J^p=3/2^-$

$$\begin{array}{ccc} & \Xi_c^* \bar{K} \quad\quad \Omega_c^* \eta & \\ V = & \left(\boxed{\begin{array}{cc} F & \frac{4}{\sqrt{3}} F \\ \frac{4}{\sqrt{3}} F & 0 \end{array}} \right) & \begin{array}{c} \Xi_c^* \bar{K} \\ \Omega_c^* \eta \end{array} \end{array}$$

$$F = -\frac{1}{4f^2}(k^0 + k'^0)$$

k^0, k'^0 the energies of initial and final states

$\Longleftrightarrow$ There are analogies of $\Omega(2012)$

- $\Omega(2012)$ (=sss) with $J^p=3/2^-$

$$\begin{array}{cc} & \bar{K}\Xi^* \quad\quad \eta\Omega \quad\quad \bar{K}\Xi \\ V = & \left(\begin{array}{ccc} \boxed{\begin{array}{cc} 0 & 3F \\ 3F & 0 \end{array}} & \begin{array}{c} \alpha q_{\rm on}^2 \\ \beta q_{\rm on}^2 \end{array} \\ \begin{array}{cc} \alpha q_{\rm on}^2 & \beta q_{\rm on}^2 \end{array} & 0 \end{array} \right) \begin{array}{c} \bar{K}\Xi^* \\ \eta\Omega \\ \bar{K}\Xi \end{array} \end{array}$$

- $K\Xi$ channel is considered as decay width

R. Pavao and E. Oset, EPJC78(2018)
N. Ikeno, G. Toledo, E. Oset, PRD(2020)

=> In the $\Omega_c(3120)$ case, we can introduce the $\Xi_c \bar{K}$ channel in the D-wave by analogy with what was done in the $\Omega(2012)$ case

Our study of $\Omega_c(3120)$

N. Ikeno, W. H. Liang, and E. Oset, PRD109, 054023 (2024)

- We like to understand the nature of $\Omega_c(3120)$ based on the molecular picture

- Apply the analogy of the $\Omega(2012)$ studies

R. Pavao and E. Oset, Eur. Phys. J. C78, 857 (2018).
N. Ikeno, G. Toledo, and E. Oset, Phys. Rev. D 101, 094016 (2020).
N. Ikeno, W. H. Liang, G. Toledo, and E. Oset, Phys. Rev.D 106, 034022 (2022).

- We retake the work by V. R. Debastiani et al., PRD 97 (2018), and we introduce the $\Xi_c\bar{K}$ channel in the D-wave

- We evaluate the mass, width of $\Omega_c(3120)$, and the partial decay widths into $\Xi_c\bar{K}$ and $\pi\Xi_c\bar{K}$

Study of the $\Omega(2012)$ as a molecular state

R. Pavao and E. Oset, Eur. Phys. J. C78, 857 (2018).

N. Ikeno, G. Toledo, and E. Oset, Phys. Rev. D 101, 094016 (2020).

N. Ikeno, W. H. Liang, G. Toledo, and E. Oset, Phys. Rev.D 106, 034022 (2022).

Discovery of Ω(2012) by Belle: Excited state of Ω (=sss)

In 2018, Belle reported a new state Ω(2012) state: PRL121(2018)052003

$$M_{\Omega(2012)} = 2012.4 \pm 0.7 \pm 0.6\ \mathrm{MeV}$$

$$\Gamma_{\Omega(2012)} = 6.4^{+2.5}_{-2.0} \pm 1.6\ \mathrm{MeV}$$

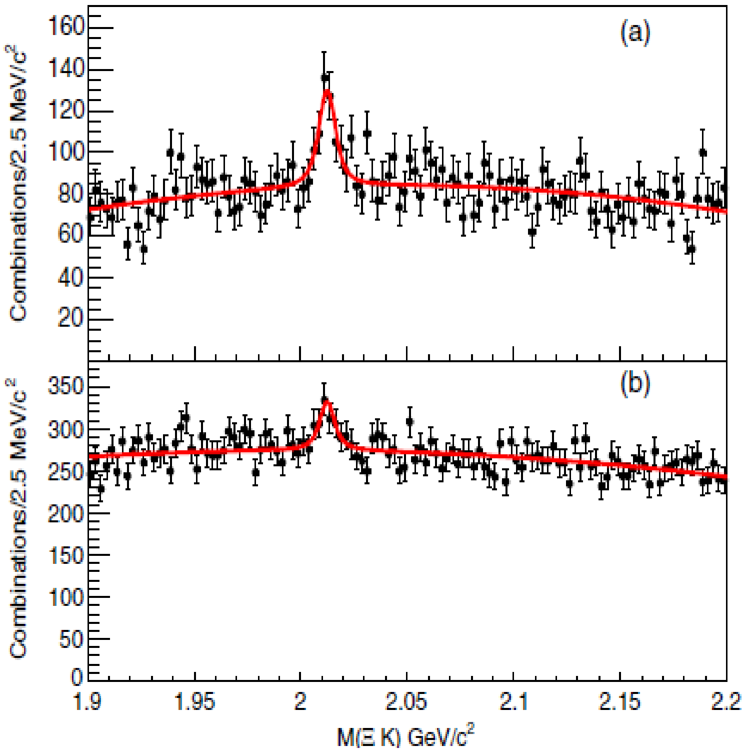


FIG. 2. The (a) $\Xi^0 K^-$ and (b) $\Xi^- K^0_S$ invariant mass distributions in data taken at the $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ resonance energies.

This prompted many theoretical studies of the Ω(2012) nature

- Quark model pictures
- Molecular pictures based on the meson-baryon interaction

Ω(2012) is dynamically generated as a molecular state from the $\bar{K}\Xi^*$ and ηΩ coupled channels interaction

$J^P = 3/2^-$ state

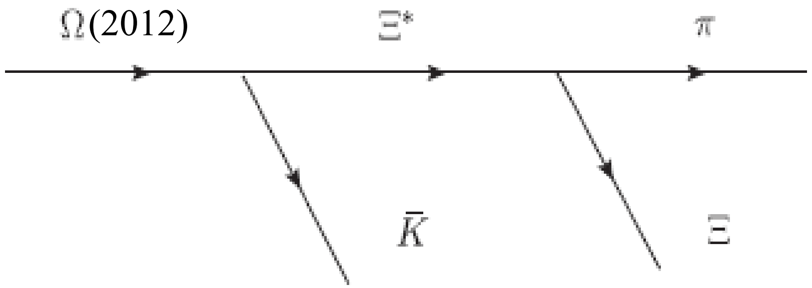
To test the molecular nature of Ω(2012), the Belle performed some tests, particularly looking at **the decay** into $\bar{K}\pi\Xi$, a signal of the $\bar{K}\Xi^*$ component of the state.

Belle experiment of Ω(2012)

- In **2019**, Belle reported a ratio of the Ω(2012) decay: PRD100(2019)032006

$$\mathcal{R}^{\Xi\pi K}_{\Xi K} = \frac{\mathcal{B}(\Omega(2012) \to \Xi(1530)(\to \Xi\pi)K)}{\mathcal{B}(\Omega(2012) \to \Xi K)} < 11.9\%$$

$\Omega(2012) \to \pi\bar{K}\Xi$

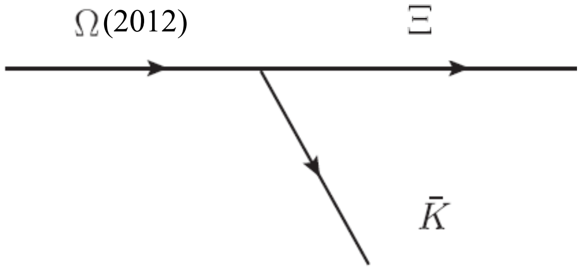


Three-body ΞπK decay width is significantly smaller than that of two-body ΞK decay width

⇒ Seems challenging result for the molecular picture nature

- In **2022**, a reanalysis of data (different cut): arXiv:2207.03090

$$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}} = 0.97 \pm 0.24 \pm 0.07$$



⇒ Consistent with the molecular interpretation

⇒ Strong support for the molecular picture

- In **2021**, Ω(2012) has been observed by the Ω_c decay: PRD104(2021)052005

$$\frac{\mathcal{B}(\Omega_c^0 \to \pi^+ \Omega(2012)^-) \times \mathcal{B}(\Omega(2012)^- \to K^-\Xi^0)}{\mathcal{B}(\Omega_c^0 \to \pi^+ K^- \Xi^0)} = (9.6 \pm 3.2 \pm 1.8)\%,$$

Ω(2012): Coupled channels approach

R. Pavao and E. Oset, EPJC78(2018)
N. Ikeno, G. Toledo, E. Oset, PRD(2020)

3 channels: $\boxed{\bar{K}\Xi^*, \eta\Omega \text{ (s-wave)}, \ \bar{K}\Xi \text{ (d-wave)}}$

- Bethe-Salpeter equation:

$$T = [1 - VG]^{-1} V$$

- Transition potential: Ω^* $J^p=3/2^-$

$$\begin{array}{cc} & \begin{array}{ccc} \bar{K}\Xi^* & \eta\Omega & \bar{K}\Xi \end{array} \\ V = & \begin{pmatrix} 0 & 3F & \alpha q_{\rm on}^2 \\ 3F & 0 & \beta q_{\rm on}^2 \\ \alpha q_{\rm on}^2 & \beta q_{\rm on}^2 & 0 \end{pmatrix} \end{array} \begin{array}{c} \bar{K}\Xi^* \\ \eta\Omega \\ \bar{K}\Xi \end{array}$$

$$F = -\frac{1}{4f^2}(k^0 + k'^0) \qquad q_{\rm on} = \frac{\lambda^{1/2}\left(s, m_{\bar{K}}^2, m_{\Xi}^2\right)}{2\sqrt{s}}$$

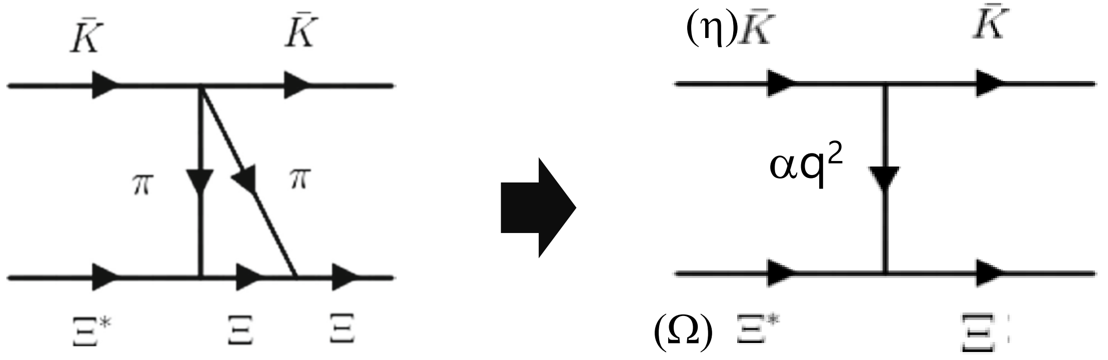
k^0, k'^0 the energies of initial and final states

- Diagonal potential is null
- Non-diagonal potential is nonzero.

- s-wave potentials between $\bar{K}\Xi^*$ and $\eta\Omega$: taken from chiral Lagrangian of

S. Sarkar, E. Oset, M.J. Vicente Vacas, NPA750(2005) 294
E.E. Kolomeitsev, M.F.M. Lutz, PLB 585 (2004) 243–252

- d-wave potential between $\bar{K}\Xi$ and $\bar{K}\Xi^*$ or $\eta\Omega$: described in terms of α, β: free parameters



A possible d-wave diagram for the $\bar{K}\Xi^*$ -> $\bar{K}\Xi$ transition

We do not make a model
Estimates done by M. P. Valderrama, PRD98,054009 (2018).

$G_{K^-\Xi^*}$ function accounting for $\Xi^* \to \pi\Xi$ decay

- Meson-Baryon loop function G:

$$G(\sqrt{s}) = \begin{pmatrix} G_{\bar{K}\Xi^*}(\sqrt{s}) & 0 & 0 \\ 0 & G_{\eta\Omega}(\sqrt{s}) & 0 \\ 0 & 0 & G_{\bar{K}\Xi}(\sqrt{s}) \end{pmatrix}$$

q_{max}: cut off parameter

For s-wave channel

$$G_i(\sqrt{s}) = \int_{|\boldsymbol{q}|<q_{\max}} \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_i(\boldsymbol{q})} \frac{M_i}{E_i(\boldsymbol{q})} \frac{1}{\sqrt{s} - \omega_i(\boldsymbol{q}) - E_i(\boldsymbol{q}) + i\epsilon}$$

for $i = \bar{K}\Xi^*, \eta\Omega$

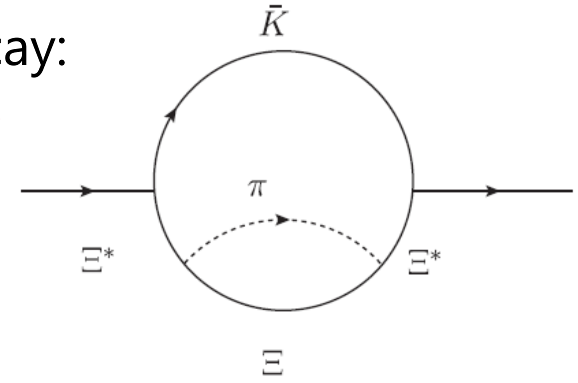
For d-wave channel

$$G_{\bar{K}\Xi}(\sqrt{s}) = \int_{|\boldsymbol{q}|<q'_{\max}} \frac{d^3q}{(2\pi)^3} \frac{(q/q_{\rm on})^4}{2\omega_{\bar{K}}(\boldsymbol{q})} \frac{M_\Xi}{E_\Xi(\boldsymbol{q})} \frac{1}{\sqrt{s} - \omega_{\bar{K}}(\boldsymbol{q}) - E_\Xi(\boldsymbol{q}) + i\epsilon}$$

- We take into account the Ξ^* mass distribution due to its width for $\Xi^* \to \pi\Xi$ decay:

  $G_{K^-\Xi^*}$ is convolved with the Ξ^* mass distribution: $\Omega(2012) \to \pi K\Xi$ decay

$$\tilde{G}_{\bar{K}\Xi^*}(\sqrt{s}) = \frac{1}{N} \int_{M_{\Xi^*}-\Delta M_{\Xi^*}}^{M_{\Xi^*}+\Delta M_{\Xi^*}} d\tilde{M} \left(-\frac{1}{\pi}\right) \mathrm{Im}\left(\frac{1}{\tilde{M} - M_{\Xi^*} + i\frac{\Gamma_{\Xi^*}}{2}}\right) G_{\bar{K}\Xi^*}(\sqrt{s}, m_{\bar{K}}, \tilde{M})$$



=> Comparison of the $\Omega(2012)$ with/without convolution gives us the estimate of $\Omega(2012)$ decay width into $\bar{K}\Xi$ and $\pi\bar{K}\Xi$ decay channels :

$$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}} = \frac{\Gamma_{\Omega^*\to\pi\bar{K}\Xi}}{\Gamma_{\Omega^*\to\bar{K}\Xi}} = \frac{\Gamma_{\Omega^*,\mathrm{con}} - \Gamma_{\Omega^*,\mathrm{non}}}{\Gamma_{\Omega^*,\mathrm{non}}}$$

$\Gamma_{\rm con}$: $G_{K^-\Xi^*}$ with convolution (accounts for $K\Xi$ and $\pi K\Xi$ decays)

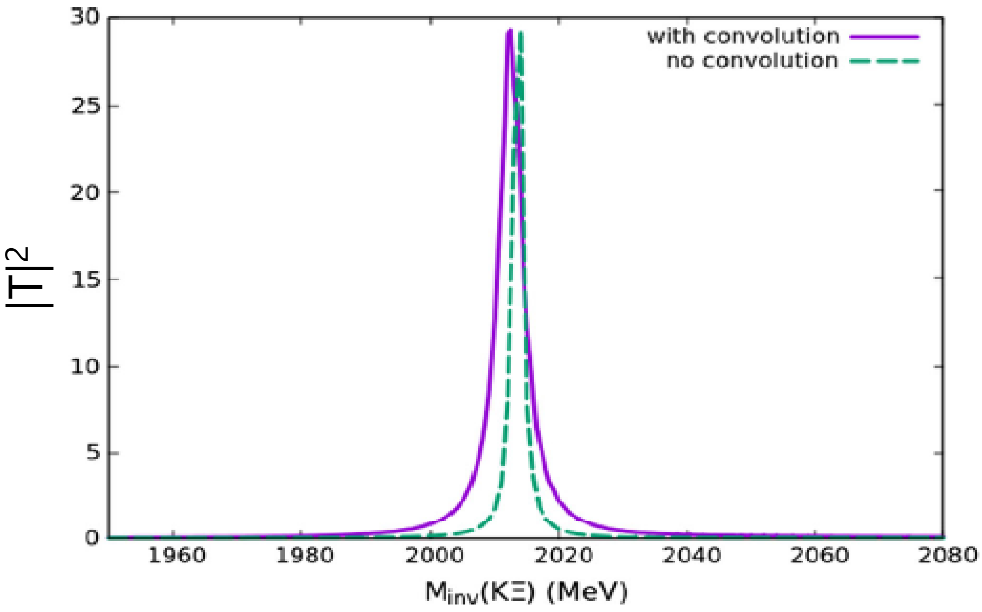
$\Gamma_{\rm non}$: $G_{K^-\Xi^*}$ without convolution (only for $K\Xi$ decay)

Calculated Ω(2012) mass, width, and decay ratio

We make a fit to the experimental data by changing the α, β, q_{max} parameters.

$$M_{\Omega(2012)} = 2012.4 \pm 0.7 \pm 0.6\ \mathrm{MeV} \qquad \Gamma_{\Omega(2012)} = 6.4^{+2.5}_{-2.0} \pm 1.6\ \mathrm{MeV}$$

- Result by R. Pavao and E. Oset, EPJC78(2018)



α (MeV^{-3})	β (MeV^{-3})	$q_{\max} = q'_{\max}$ (MeV)
4.0×10^{-8}	1.5×10^{-8}	735

- Result with convolution

$$m_{\Omega^*} = 2012.37\ \mathrm{MeV}, \quad \Gamma_{\Omega^*} = 6.24\ \mathrm{MeV}.$$

- Result without convolution

$$m^{(\text{no conv.})}_{\Omega^*} = 2013.5\ \mathrm{MeV}, \quad \Gamma^{(\text{no conv.})}_{\Omega^*} = 3.2\ \mathrm{MeV}.$$

$$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}} = \frac{\Gamma_{\Omega^*\to\pi\bar{K}\Xi}}{\Gamma_{\Omega^*\to\bar{K}\Xi}} = \frac{\Gamma_{\Omega^*,\mathrm{con}} - \Gamma_{\Omega^*,\mathrm{non}}}{\Gamma_{\Omega^*,\mathrm{non}}} = 0.95$$

=> Good agreement with the latest Belle result $\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}} = 0.97 \pm 0.24 \pm 0.07$

Limitation of calculated ratio R

We also find an acceptable solution in terms of natural values for the q_{max}, α, β parameters which reproduce fairly well the experimental data in 2019 $\frac{\Gamma_{\Omega}(\pi\bar{K}\Xi)}{\Gamma_{\Omega,\bar{K}\Xi}} < 11.9\,\%$

- Results of Set1-3 by N. Ikeno, G. Toledo, and E. Oset, PRD101, 094016 (2020)

  Similar results in J. Lu, C. Zeng, E. Wang, J. Xie, and L. Geng, Eur. Phys. J. C, 361 (2020)

	Pavao&Oset	Set 1	Set 2	Set 3
$q_{\rm max}(\bar{K}\Xi^*)$ [MeV]	735	735	775	735
$q_{\rm max}(\eta\Omega)$ [MeV]	735	735	710	750
α [10^{-8} MeV^{-3}]	4.0	-8.7	-8.7	-11.0
β [10^{-8} MeV^{-3}]	1.5	18.3	18.3	20.0
M_R [MeV]	2012.4	2012.7	2012.7	2012.6
Γ_R [MeV]	6.24	7.3	7.7	8.2
$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}}$	0.95	0.109	0.104	0.109

The molecular picture was pushed to the limit and showed that the ratio R could be made as small as 10%, but not smaller than 10%

Couplings g_i of different channels

- The couplings g_i of the $\Omega(2012)$ to the different channels are obtained from the residue of the T-matrix of the pole in the second Riemann sheet. Close to the pole

$$T_{ij} = \frac{g_i g_j}{z - z_R} \, (z, \text{complex energy}; \, z_R, \text{complex pole position}) \qquad g_i^2 = \lim_{z \to z_R} (z - z_R) T_{ii}; \;\; g_j = g_i \frac{T_{ij}}{T_{ii}} |_{z=z_R}.$$

- We also show the wave function at the origin for the s-wave states, wf($g_i G_i$), and the probability of each channel $-g_i^2 \frac{\partial G_i}{\partial \sqrt{s}}$

$$\sum_i (-) g_i^2 \frac{\partial G}{\partial \sqrt{s}} = 1,$$

	$\bar{K}\Xi^*$ (2027)	$\eta\Omega$ (2220)	$\bar{K}\Xi$ (1812)
g_i	$1.86 - i0.02$	$3.52 - i0.46$	$-0.42 + i0.12$
g_i (Pavao, Oset)	$2.01 + i0.02$	$2.84 - i0.01$	$-0.29 + i0.04$
$\mathrm{wf}_i(g_i G_i)$	$-34.05 - i1.10$	$-30.66 + i3.67$	$\dots$
$-g_i^2 \frac{\partial G_i}{\partial \sqrt{s}}$	$0.57 + i0.10$	$0.25 - i0.06$	$\dots$

- D. Gamermann, J. Nieves, E. Oset, and E. R. Arriola, PRD 81, 014029 (2010).
- F. Aceti, L.R. Dai, L.S. Geng, E. Oset and Y. Zhang, EPJA50, 57 (2014)
- S. Weinberg, Phys. Rev. 130, 776 (1963).
- T. Hyodo, D. Jido and A. Hosaka, PRC85,015201 (2012).
- T. Hyodo, Int. J. Mod. Phys. A28, 1330045 (2013).
- T. Sekihara, PRC104, 035202 (2021), etc.

The strength of the *wf* and the probability dominates for the $\bar{K}\Xi^*$ state.

Note, however, that the $\eta\Omega$ channel is required to bind Ω^* state since the diagonal potential of the $\bar{K}\Xi^*$ channel is zero and hence cannot produce any bound state by itself.

Study of the $\Omega_c(3120)$ as a molecular state

N. Ikeno, W. H. Liang, and E. Oset, Phys. Rev. D 109, 054023 (2024).

$\Omega_c(3120)$: Coupled channels approach

3 channels: $\Xi_c^*\bar{K}, \Omega_c^*\eta$ (*s*-wave), $\Xi_c\bar{K}$ (*d*-wave)

- Transition potential: Ω_c^* $J^p=3/2^-$

$$
\begin{array}{cc}
 & \begin{array}{ccc} \Xi_c^*\bar{K} & \Omega_c^*\eta & \Xi_c\bar{K} \end{array} \\
V = & \begin{pmatrix} \boxed{\begin{array}{cc} F & \frac{4}{\sqrt{3}}F \\ \frac{4}{\sqrt{3}}F & 0 \end{array}} & \begin{array}{c} \alpha q_{\rm on}^2 \\ \beta q_{\rm on}^2 \end{array} \\ \begin{array}{cc} \alpha q_{\rm on}^2 & \beta q_{\rm on}^2 \end{array} & 0 \end{pmatrix} \begin{array}{c} \Xi_c^*\bar{K} \\ \Omega_c^*\eta \\ \Xi_c\bar{K} \end{array}
\end{array}
$$

$$F = -\frac{1}{4f^2}(k^0 + k'^0) \qquad q_{\rm on} = \frac{\lambda^{1/2}(s, m_{\bar{K}}^2, m_{\Xi_c}^2)}{2\sqrt{s}}$$

k^0, k'^0 the energies of initial and final states

Attractive in the $\Xi_c^*\bar{K}$ channel

(↔ Diagonal potential is null in the $\Omega(2012)$ case)

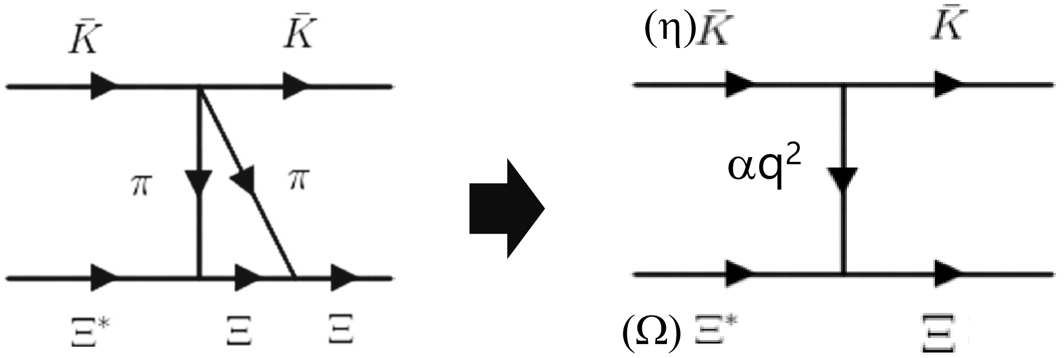
- Bethe-Salpeter equation:

$$T = [1 - VG]^{-1}V$$

- s-wave potentials between $\Xi_c^*\bar{K}, \Omega_c^*\eta$ taken from V. R. Debastiani, J. M. Dias, W. H. Liang, E. Oset, PRD97 (2018)

- d-wave potential between $\Xi_c\bar{K}$ and $\Xi_c^*\bar{K}$ or $\Omega_c^*\eta$ described in terms of α, β: free parameters

Similar way to R. Pavao and E. Oset, EPJC78(2018)



A possible d-wave diagram for the $\bar{K}\Xi^*$ -> $\bar{K}\Xi$ transition

We do not make a model

Effect of Ξ_c^* decay width

- Meson-Baryon loop function G: $\mathrm{diag}(G_{\Xi_c^*\bar{K}}, G_{\Omega_c^*\eta}, G_{\Xi_c\bar{K}})$

q_{max}: cut-off parameter

For s-wave channel of $\Xi_c^*\bar{K}, \Omega_c^*\eta$:

$$G_i(\sqrt{s}) = \int_{|\vec{q}|<q_{\max}} \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{1}{2\omega_i(q)} \frac{M_i}{E_i(q)} \frac{1}{\sqrt{s} - \omega_i(q) - E_i(q) + i\varepsilon}$$

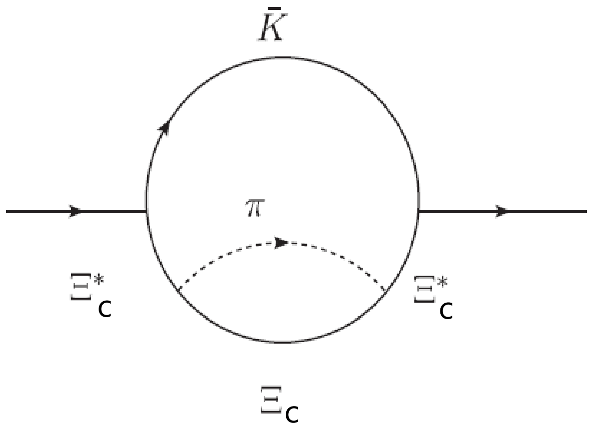
For d-wave channel of $\Xi_c\bar{K}$:

$$G_{\Xi_c\bar{K}}(\sqrt{s}) = \int_{|\vec{q}|<q_{\max}} \frac{\mathrm{d}^3 q}{(2\pi)^3} \left(\frac{q}{q_{\mathrm{on}}}\right)^4 \frac{1}{2\omega_{\bar{K}}(q)} \frac{M_{\Xi_c}}{E_{\Xi_c}(q)} \frac{1}{\sqrt{s} - \omega_{\bar{K}}(q) - E_{\Xi_c}(q) + i\varepsilon}$$

- We take into account Ξ_c^* decay width for $\Xi_c^* \to \pi\Xi_c$ decay

$$\tilde{G}_{\Xi_c^*\bar{K}}(\sqrt{s}) = \int_{|\vec{q}|<q_{\max}} \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{1}{2\omega_{\bar{K}}(q)} \frac{M_{\Xi_c^*}}{E_{\Xi_c^*}(q)} \frac{1}{\sqrt{s} - \omega_{\bar{K}}(q) - E_{\Xi_c^*}(q) + i\frac{\sqrt{s'}}{2E_{\Xi_c^*}(q)}\Gamma_{\Xi_c^*}(\sqrt{s'})}$$

$$\Gamma_{\Xi_c^*}(M_{\mathrm{inv}}) = \frac{M_{\Xi_c^*}}{M_{\mathrm{inv}}} \left(\frac{q'}{q'_{\mathrm{on}}}\right)^3 \Gamma_{\mathrm{on}} \theta(M_{\mathrm{inv}} - m_\pi - M_{\Xi_c})$$



- Result with Ξ_c^* decay width: Accounts for $K\Xi_c$ and $\pi K\Xi_c$ decays
- Result without Ξ_c^* decay width: Only for $K\Xi_c$ decay

$\Rightarrow$ Estimate $\Omega_c(3120)$ decay width into $K\Xi_c$ and $\pi K\Xi_c$ decay channels

Calculated mass and width of $\Omega_c(3120)$

We make a fit to the experimental data by changing the α, β, q_{max} parameters.

Exp. data: $M_{\Omega_c(3120)} = 3119.1 \pm 0.3 \pm 0.9^{+0.3}_{-0.5}$ MeV,

$\Gamma_{\Omega_c(3120)} = 1.1 \pm 0.8 \pm 0.4$ MeV.

We get a good fit to the data with the parameters

$$q_{\max} = 674.6 \text{ MeV}, \quad \alpha = 2.6 \times 10^{-8} \text{ MeV}^{-3},$$
$$\beta = 2.0 \times 10^{-9} \text{ MeV}^{-3}.$$

Pole position appears at $(3119.13 + i0.54)$ MeV

$\Rightarrow$ The width is 1.08 MeV in agreement with the data

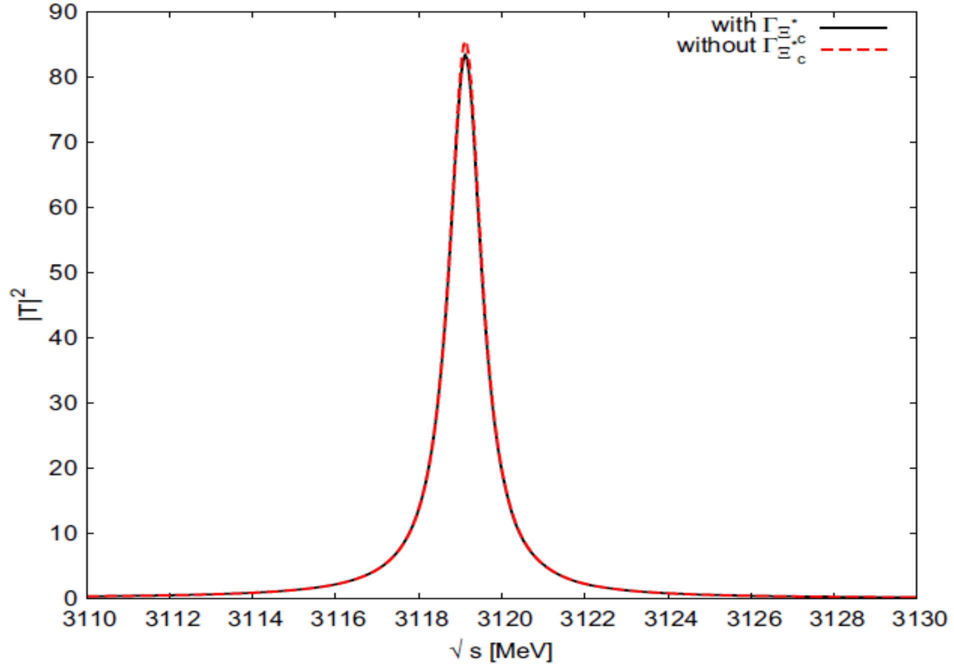


FIG. 2. $|T_{\Xi_c^* \bar{K}}|^2$ as a function of $\sqrt{s}$ in the cases with $\Gamma_{\Xi_c^*}$ and without $\Gamma_{\Xi_c^*}$, respectively.

Effect of Ξ_c^* decay width is very small in Figure

$\Rightarrow$ Unlike in the case of the $\Omega(2012)$, we cannot determine the $\Omega_c(3120) \to \Xi_c^* \bar{K} \to \Xi_c \pi \bar{K}$ in this way $\Rightarrow$ We use another way

Couplings g_i, wf at the origin(g_iG_i), probability $-g_i^2\frac{\partial G_i}{\partial\sqrt{s}}$

- $\Omega_c(3120)$

	$\Xi_c^*\bar{K}$	$\Omega_c^*\eta$	$\Xi_c\bar{K}$
g_i	$2.06-i0.02$	$2.09-i0.01$	-0.138
g_iG_i	$-36.77+i0.17$	$-17.64+i0.06$	
$-g_i^2\frac{\partial G_i}{\partial\sqrt{s}}$	0.63	0.10	

- $\Omega(2012)$ R. Pavao and E. Oset, EPJC78(2018)

	$\Xi^*\bar{K}$	$\Omega\eta$	$\Xi\bar{K}$
g_i	$2.01+i0.02$	$2.84-i0.01$	-0.29
g_iG_i	$-37.11+i0.55$	$-24.95+i0.38$	
$-g_i^2\frac{\partial G_i}{\partial\sqrt{s}}$	0.64	0.16	

The couplings are defined at the pole as

$$g_ig_j=\lim_{\sqrt{s}\to\sqrt{s_p}}(\sqrt{s}-\sqrt{s_p})T_{ij},$$

with $\sqrt{s_p}$ the energy of the pole

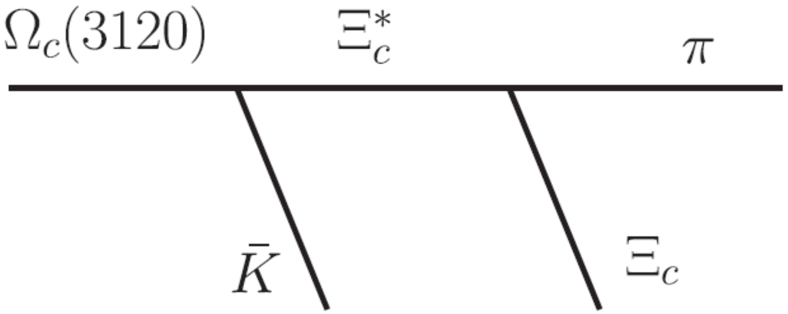
$\Xi_c^*\bar{K}$ has the largest probability of around 63% and $\Omega_c^*\eta$ around 10%

$\Rightarrow$ Largely molecular state

The results of $\Omega_c(3120)$ are similar to those of $\Omega(2012)$.

Partial decay widths into $\Xi_c \bar{K} \pi$ and $\Xi_c \bar{K}$

- Mechanism for $\Omega_c(3120)$ to decay into $\Xi_c\bar{K}\pi$ via primary decay into $\Xi_c^*\bar{K}$

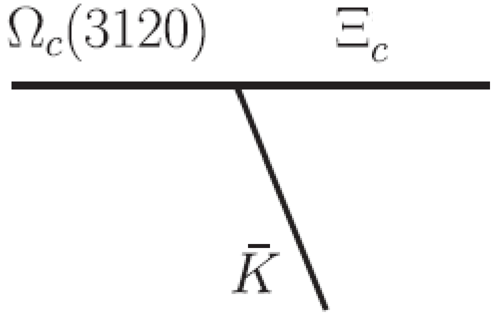


$$\frac{d\Gamma_{\Omega_c}}{dM_{\rm inv}(\pi\Xi_c)} = \frac{1}{(2\pi)^3}\frac{M_{\Xi_c}}{M_{\Omega_c}}\, p_{\bar{K}}\,\tilde{p}_\pi\, |t_{\Omega_c\to\pi\bar{K}\Xi_c}|^2$$

$$t_{\Omega_c\to\pi\bar{K}\Xi_c} = g_{\Omega_c,\bar{K}\Xi_c^*}\,\frac{1}{M_{\rm inv}(\pi\Xi_c) - M_{\Xi_c^*} - i\Gamma_{\Xi_c^*}/2}\, g_{\Xi_c^*,\pi\Xi_c}\,\tilde{p}_\pi$$

$$\Gamma_{\Omega_c\to\Xi_c\pi\bar{K}} = 0.03\ \mathrm{MeV}$$

- $\Omega_c(3120)$ to decay into $\Xi_c\bar{K}$



$$\Gamma_{\Omega_c\to\Xi_c\bar{K}} = \frac{1}{2\pi}\frac{M_{\Xi_c}}{M_{\Omega_c}}\, g^2_{\Omega_c,\Xi_c\bar{K}}\, p'_{\bar{K}} = 0.90\ \mathrm{MeV}$$

=> Sum of them is $\mathbf{\Gamma_{\Omega c}}$ **~ 1 MeV**

The width of $\Omega_c(3120)$ decay to $\Xi_c\bar{K}\pi$ is much smaller than the $\bar{K}\pi\Xi$ in the case of the $\Omega(2012)$. The small ratio of 3% is challenging in the present experimental errors

Scattering length a_j and effective range $r_{0,j}$

- $\Omega_c(3120)$

[fm]	a_j	$r_{0,j}$
$\Xi_c^* \bar{K}$	$1.45 - i0.07$	$-0.08 - i0.01$
$\Omega_c^* \eta$	$0.44 - i0.09$	$0.26 + i0.01$

- $\Omega(2012)$

[fm]	a_j	$r_{0,j}$
$\Xi^* \bar{K}$	$1.69 - i0.17$	$-0.37 - i0.01$
$\Omega \eta$	$0.51 - i0.09$	$0.25 - i0.03$

Scattering length

$$-\frac{1}{a_j} = -\frac{8\pi\sqrt{s}}{2M_j}(T_{jj})^{-1}\Big|_{\sqrt{s_{\rm th}},j}$$

Effective range

$$r_{0,j} = \frac{1}{\mu_j}\frac{\partial}{\partial\sqrt{s}}\left[-\frac{8\pi\sqrt{s}}{2M_j}(T_{jj})^{-1} + ik_j\right]_{\sqrt{s_{\rm th}},j}$$

These magnitudes can be determined experimentally, something feasible nowadays, for instance, measuring correlation functions

Discussion on the nature of Tcc and X(3872) from these information

J. Song, L. R. Dai and E. Oset, PRD 108,114017 (2023): L. R. Dai, J. Song and E. Oset, PLB846, 138200 (2023)

Summary

- We have studied the $\Omega_c(3120)$ based on the molecular picture
- The $\Omega_c(3120)$ mostly couples to $\Xi_c^* \bar{K}, \Omega_c^* \eta$ channels
- The state with $J^P = 3/2^-$ decays to $\Xi_c \bar{K}$ in the D-wave and we included this decay channel in our approach
- Evaluation of the fraction of the $\Omega_c(3120)$ width that goes into $\Xi_c \bar{K} \pi$ by the analogous analysis of $\Omega(2012)$ to see the nature of the molecular state

$\Rightarrow$ Small ratio of about 3% is obtained due to a relatively big binding, compared to its analogous $\Omega(2012)$ state

- As an alternative, the scattering length, and effective length, together with BE, and width of $\Omega_c(3120)$ will help to understand the nature of $\Omega_c(3120)$

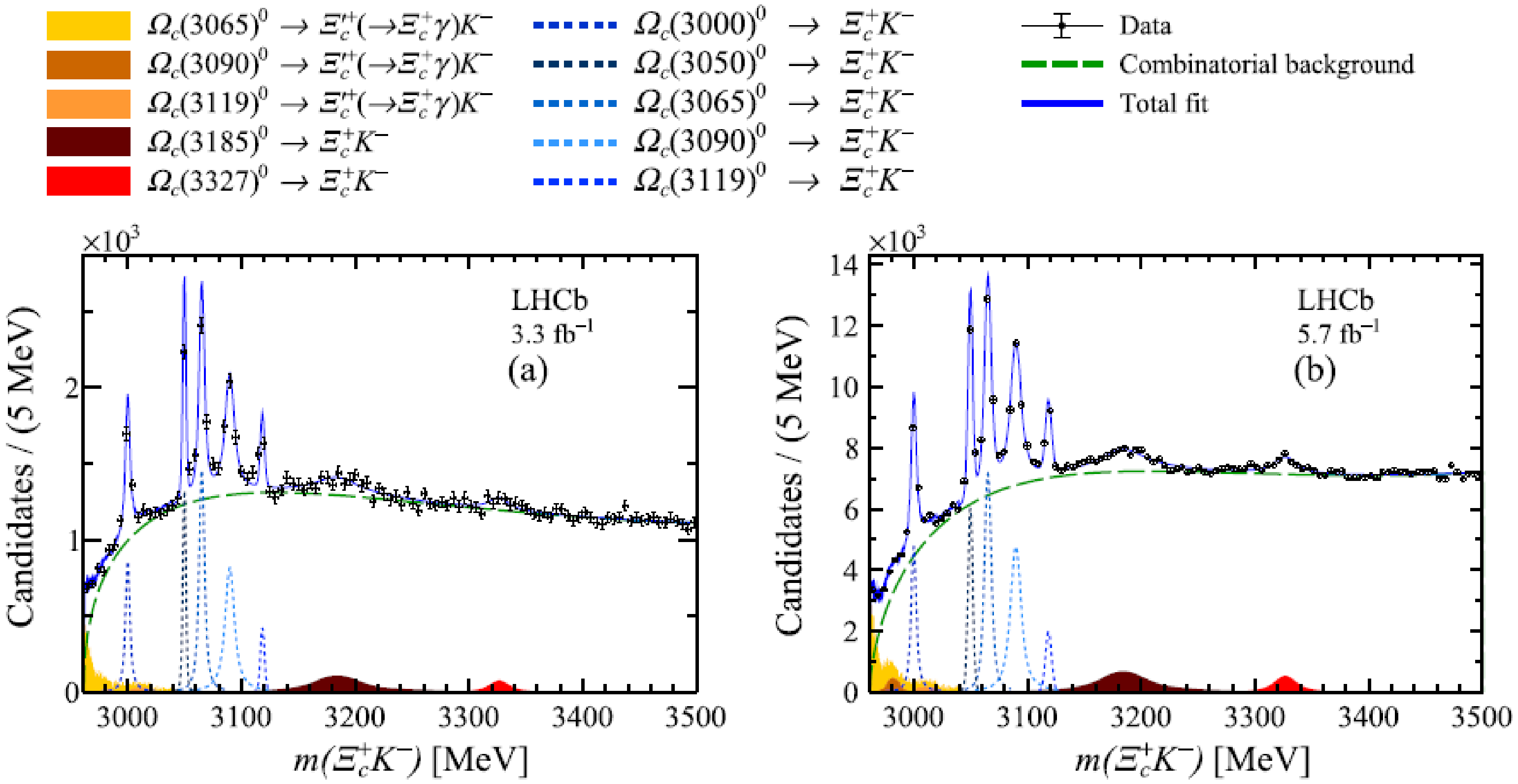


FIG. 1. Invariant-mass distribution of the $\Omega_c(X)^0$ candidates in (a) dataset 1 and (b) dataset 2, with the fit results overlaid. A bin width of 5 MeV is used for plotting. The previously observed excited Ω_c^0 states are shown in blue dashed lines. The $\Omega_c(3185)^0$ state is shown in the brown area, and the $\Omega_c(3327)^0$ state is shown in the red area. Three feed-down components are shown as the yellow areas, while the green long-dashed line corresponds to the combinatorial background.

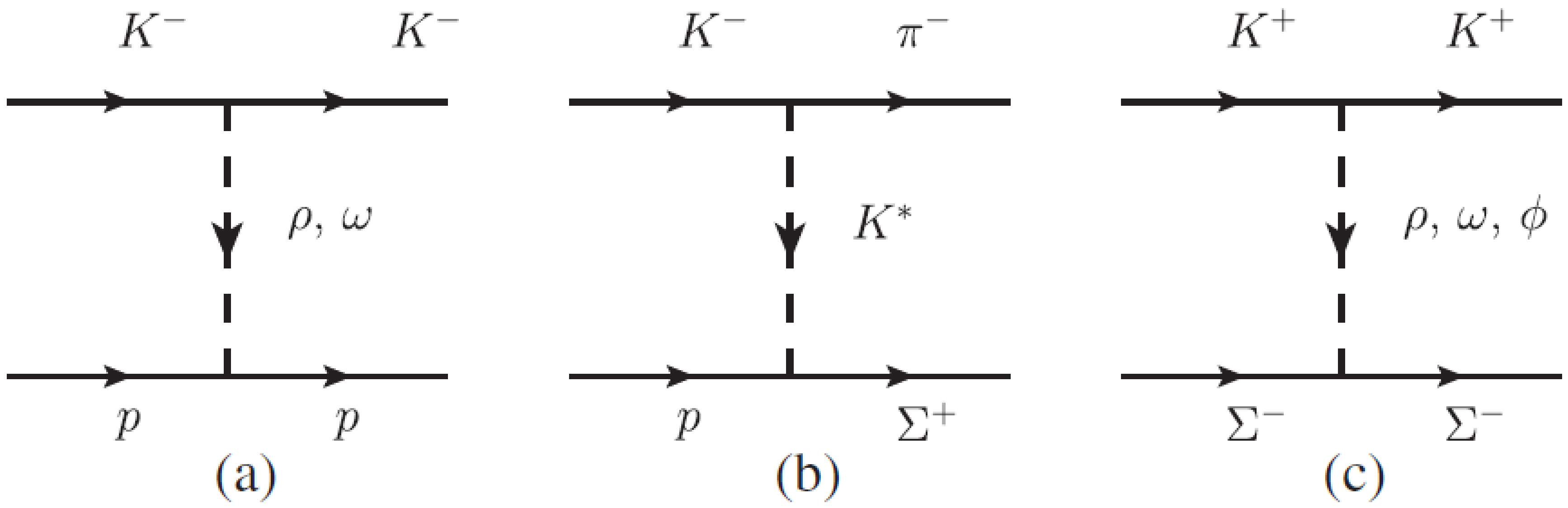


FIG. 1. Vector exchange in the meson-baryon interaction.

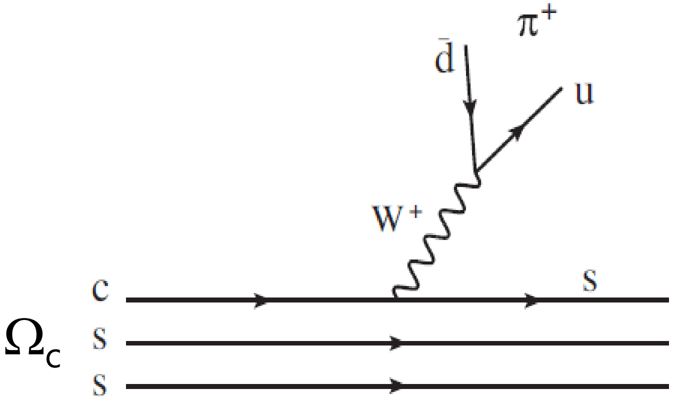
The $\Omega_c \to \pi^+\Omega(2012) \to \pi^+(K^-\Xi^0)$ reaction

- Belle result: $$\frac{\mathcal{B}(\Omega_c^0 \to \pi^+\,\Omega(2012)^-) \times \mathcal{B}(\Omega(2012)^- \to K^-\Xi^0)}{\mathcal{B}(\Omega_c^0 \to \pi^+K^-\Xi^0)} = (9.6 \pm 3.2 \pm 1.8)\%,$$

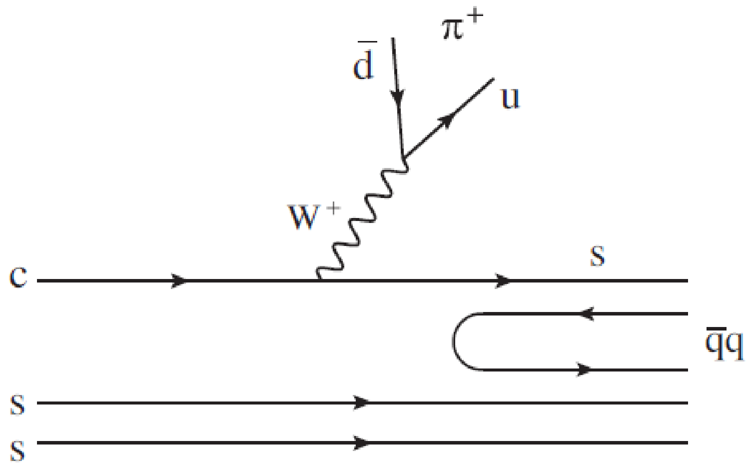
- We study a mechanism for $\Omega_c \to \pi^+\Omega(2012)$ production through an external emission weak decay, where the $\Omega(2012)$ is dynamically generated from the interaction of $\bar{K}\Xi^*$ and $\eta\Omega$, with $\bar{K}\Xi$ as the main decay channel.

N. Ikeno, W. H. Liang, G. Toledo, and E. Oset, PRD 106, 034022 (2022).

Microscopic picture for $\Omega_c \to \pi^+sss$



Hadronization of an ss pair



$$sss \to \sum_i s\bar{q}_i q_i ss = \sum_i P_{3i} q_i ss$$

where $$P \equiv \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \sqrt{\frac{2}{3}}\eta' \end{pmatrix}$$

$$sss \to K^-\underline{uss} + \bar{K}^0\underline{dss} - \frac{\eta}{\sqrt{3}}\underline{sss}$$

We obtain $\bar{K}\Xi^*$, $\bar{K}\Xi$, $\eta\Omega$

- Weak interaction vertices: $V_P = C(q^0 + \vec{\sigma}\cdot\vec{q})$

$\mathcal{L}_{W,\pi} \sim W^\mu \partial_\mu \phi$. $\mathcal{L}_{\bar{q}Wq} \sim (\bar{q}_{\rm fin} W_\mu \gamma^\mu (1-\gamma_5) q_{\rm in}$

C: unknown constant

K. Miyahara, et al., PRC95, 035212 (2017) ;
V. R. Debastiani, et al., PRD97, 094035 (2018)

W: Weight for the matrix elements of $\Omega_c \uparrow\uparrow\uparrow$ going to π^+ and the different final states

$$K^-\Xi^{*0}(S_z = 3/2):\ W = \frac{1}{\sqrt{3}}C(q^0 + q_z),$$

$$\bar{K}^0\Xi^{*-}(S_z = 3/2):\ W = \frac{1}{\sqrt{3}}C(q^0 + q_z),$$

$$\eta\Omega(S_z = 3/2):\ W = -\frac{1}{\sqrt{3}}C(q^0 + q_z),$$

$$K^-\Xi^{*0}(S_z = 1/2):\ W = \frac{1}{3}Cq_+,$$

$$\bar{K}^0\Xi^{*-}(S_z = 1/2):\ W = \frac{1}{3}Cq_+,$$

$$\eta\Omega(S_z = 1/2):\ W = -\frac{1}{3}Cq_+,$$

$$K^-\Xi^0(S_z = 1/2):\ W = \frac{\sqrt{2}}{3}Cq_+$$

$$\bar{K}^0\Xi^-(S_z = 1/2):\ W = -\frac{\sqrt{2}}{3}Cq_+,$$

$$V_P = C(q^0 + \vec{\sigma}\cdot\vec{q})$$

the matrix element of

$$\vec{\sigma}\cdot\vec{q} = \sigma_+ q_- + \sigma_- q_+ + \sigma_z q_z,$$

$$\sigma_+ = \frac{1}{2}(\sigma_x + i\sigma_y), \qquad \sigma_- = \frac{1}{2}(\sigma_x - i\sigma_y),$$

$$q_+ = q_x + iq_y, \qquad q_- = q_x - iq_y,$$

q^0 is the energy of the π^+ and q its three-momentum

We can take the z direction in the π^+ direction, and when integrating over the π^+ angles, we get the angle averaged values of q_z^2, $q^0 q_z$, and $|q_+|^2$,

$$q_z^2 \to \frac{1}{3}\vec{q}^2, \qquad q^0 q_z \to 0, \qquad |q_+|^2 = q_x^2 + q_y^2 \to \frac{2}{3}\vec{q}^2.$$

internal emission

We can also have $\bar{K}^0\pi^+\Xi^-$, $\bar{K}^0\pi^+\Xi^{*-}$ production via internal emission, although suppressed by a color factor around 1/3.

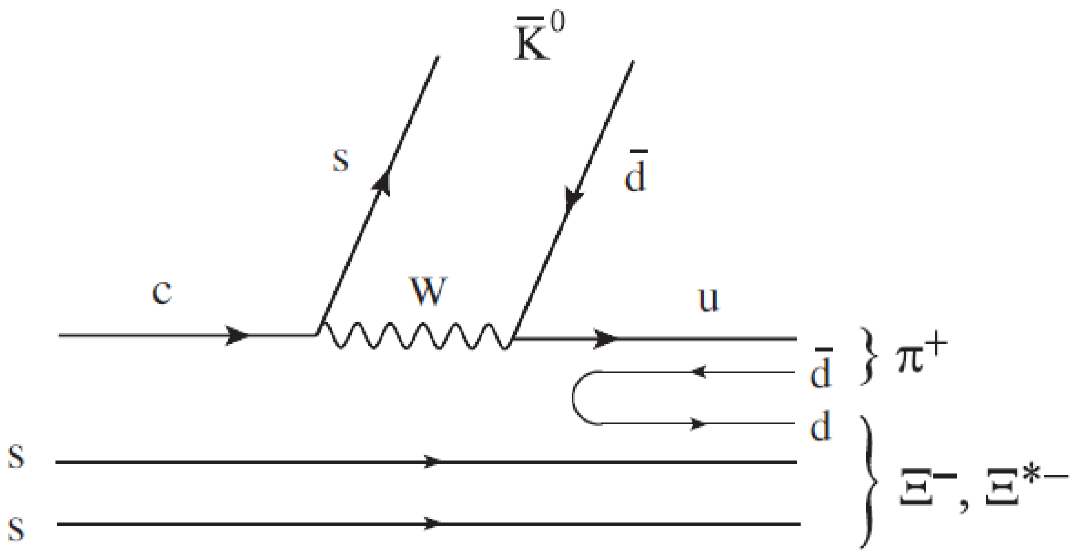


FIG. 4. Mechanism for internal emission for $\bar{K}^0\pi^+\Xi^-$, $\bar{K}^0\pi^+\Xi^{*-}$.

If we look at the $\pi^+K^-\Xi^0$ final state production, we just have an external emission