Spin-1 quarkonia in a rotating frame and their spin contents

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Hyungjoo Kim

WPI-SKCM2, Hiroshima University

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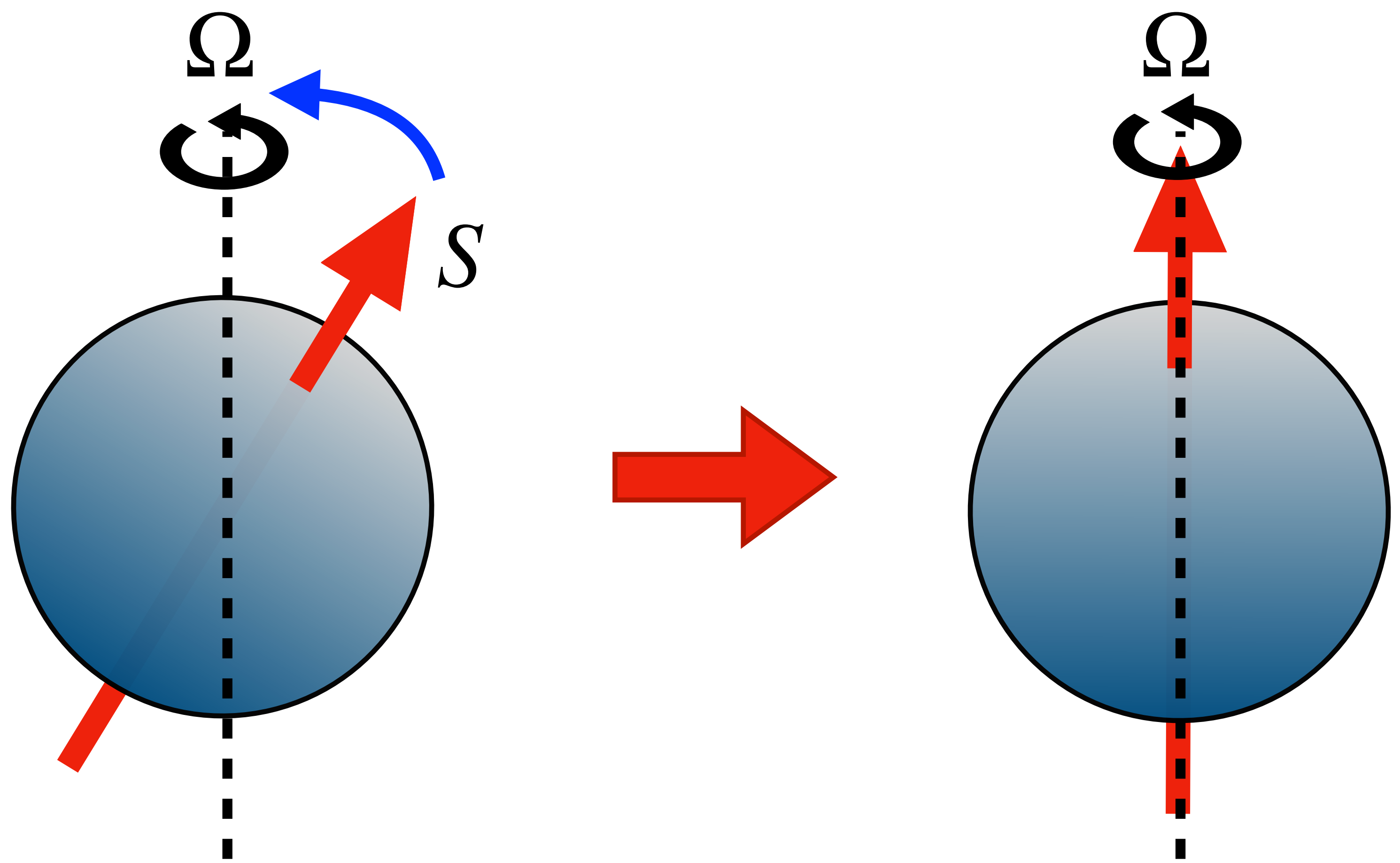
Spin-Rotation Coupling(SRC)



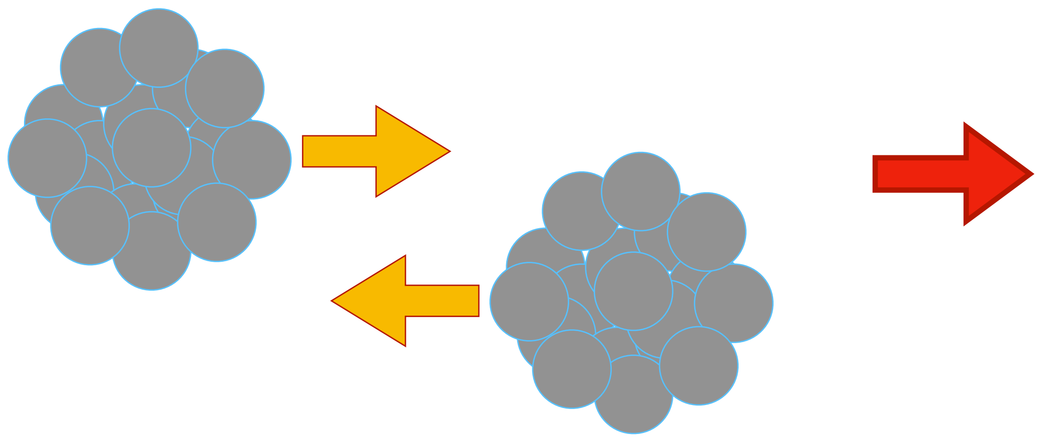
$$H_{SRC} = H_r - H_i = -S \cdot \Omega$$

H_i : Inertial frame energy
H_r : Rotating frame energy
S : Spin
Ω : Angular velocity

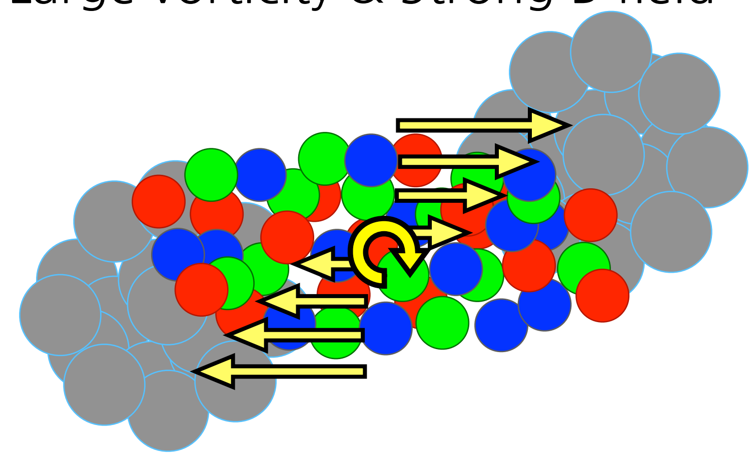
Vector meson spin alignment

In non-central HICs,

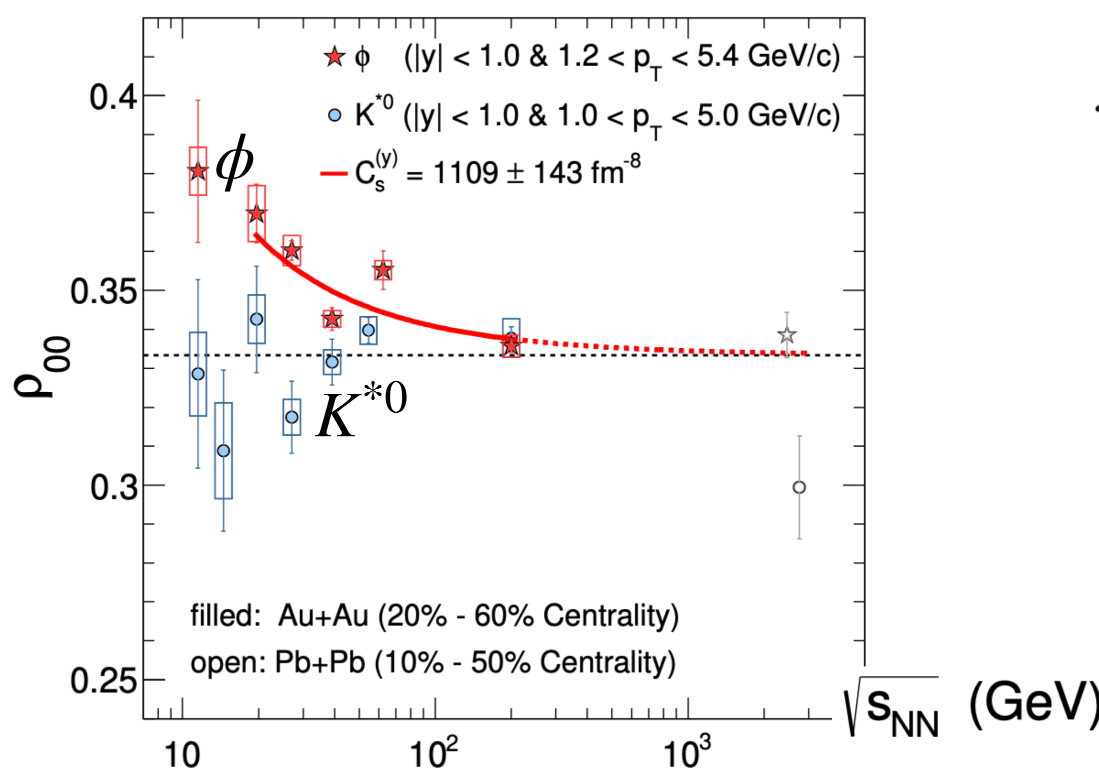
initial angular momentum



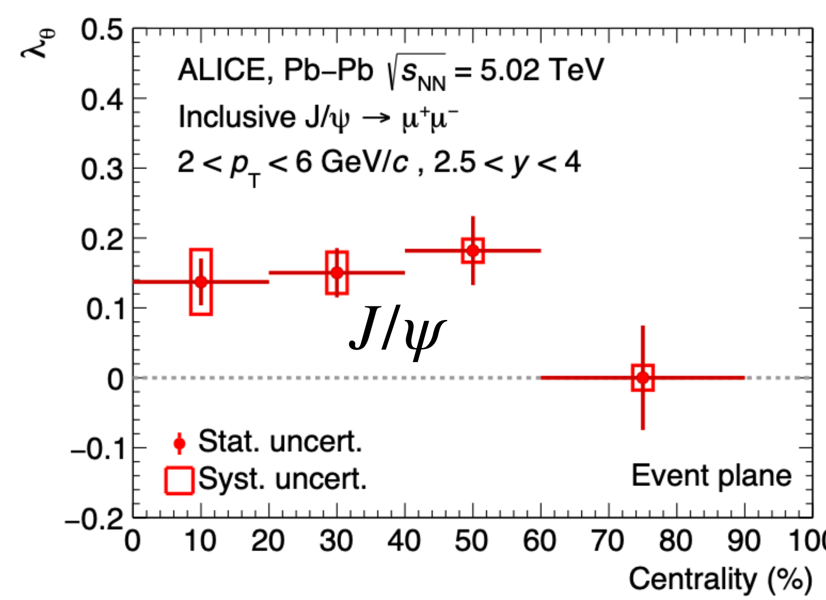
Large vorticity & Strong B field



STAR, arXiv:2204.02302 (2022)



ALICE, arXiv:2204.10171 (2022)



=> Detailed mechanism is complex and still not clearly understood.

Q. $\mathrm{H_{SRC}} = -\,\mathrm{S}\cdot\Omega$ is universal for any spin?

- Spin-1/2: Dirac eq. in a rotating frame using G.R.

$$\left[i\partial\!\!\!/_x + gA\!\!\!/(x) + \boldsymbol{\Sigma}\cdot\boldsymbol{\Omega} - m\right]\Psi(x) = 0 \text{ where } \boldsymbol{\Sigma} = \gamma^0(\mathbf{S}+\mathbf{L})$$

$$\Rightarrow\ \mathrm{H_{SRC}} = -\,\mathrm{S}\cdot\Omega \text{ for spin-1/2}$$

- Spin-1: No strict derivation based on G.R. until recently

- **PRD102(2020)12,125028 - J.Kapusta, E.Rrapaj, S.Rudaz**

  - Proca eq. for massive spin-1 particle using G.R.

  - $\mathrm{H_{SRC}} = -\left(\frac{1}{2}\right)\mathrm{S}\cdot\Omega$ for spin-1!

  - contradictory to naive expectation and quark model

- **Motivation**

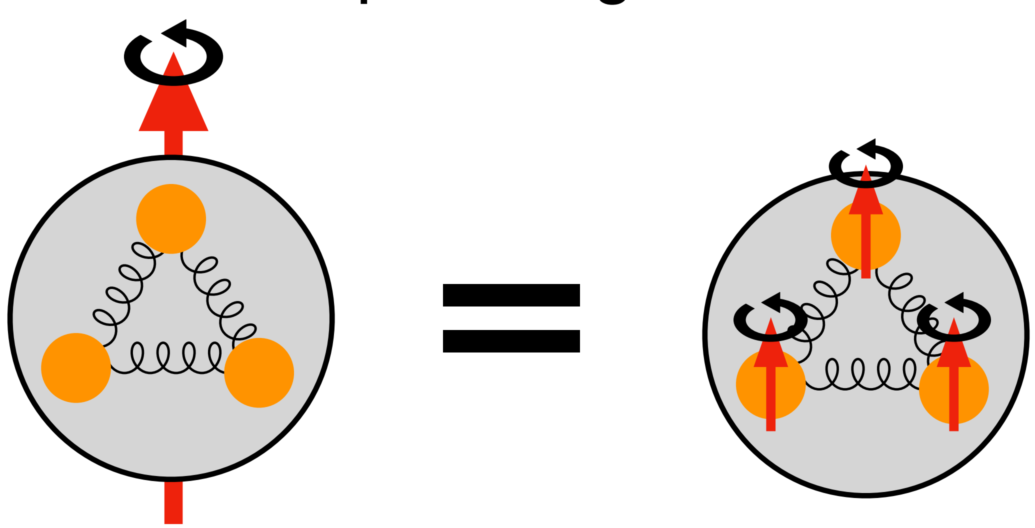
=> Clarify the strength of SRC for spin-1 particle in a different way!

Outline

- We study SRC of (the simplest) spin-1 heavy $Q\bar{Q}$ system
- Introduce a free parameter "g_Ω" which indicates the strength of SRC,

$$\boxed{H_{SRC} = -\, g_\Omega \, S \cdot \Omega}$$

- "Total SRC = All reaction of quark + gluon in a rot frame"



- We prove that $g_\Omega = g_\Omega^{\rm quark}(Q^2) + g_\Omega^{\rm gluon}(Q^2) = 1$ for spin-1 $Q\bar{Q}$ system
- Each component of g_Ω carried by quarks and gluons = Spin content
- We study spin contents of J/ψ, $\Upsilon(1S)$ for V and χ_{c1}, χ_{b1} for AV

How to extract g_Ω?

1. Describe the correlation function in a rotating frame

$$\Pi^{\mu\nu}(q) = i \int d^4x e^{iqx} \langle 0 \,|\, \mathrm{T}\{\mathrm{j}^{\mu}(\mathrm{x})\mathrm{j}^{\nu}(0)\} \,|\, 0 \rangle$$

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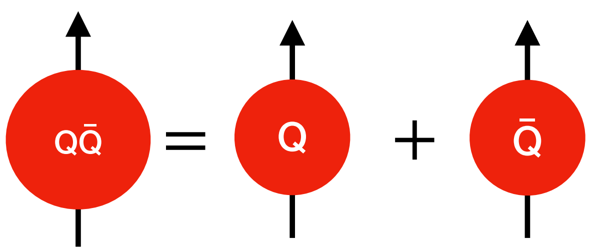
$$\Pi^{\mu\nu}(q) = i\int d^4x e^{iqx}\langle 0|\mathrm{T}\{\mathrm{j}^\mu(\mathrm{x})\mathrm{j}^\nu(0)\}|0\rangle$$

$$\epsilon_\mu^+ = (0,1,i,0)/\sqrt{2}$$

2. Pick out a right circularly polarized state => $\Pi^+(\omega) = \epsilon_\mu^+\epsilon_\nu^{+*}\Pi^{\mu\nu}(\omega,0)$



$$|11\rangle = |\frac{1}{2}\frac{1}{2}\rangle + |\frac{1}{2}\frac{1}{2}\rangle$$

$$|10\rangle = |\frac{1}{2}\frac{1}{2}\rangle + |\frac{1}{2}-\frac{1}{2}\rangle$$

$$|1-1\rangle = |\frac{1}{2}-\frac{1}{2}\rangle + |\frac{1}{2}-\frac{1}{2}\rangle$$

Energy of $|11\rangle$ will be shifted by '$-\Omega$' i.e. $\omega \rightarrow \omega - \Omega$

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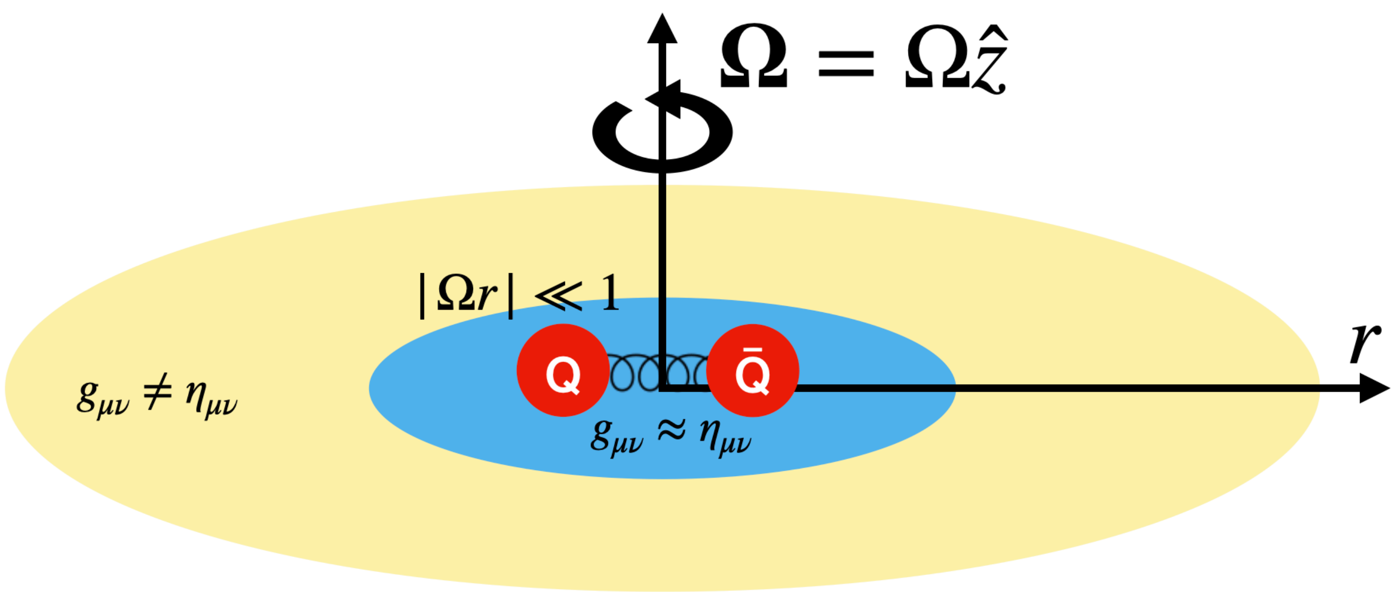

- No external OAM
- Expand $\boldsymbol{\Omega}$ linear term

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2. Pick out a right circularly polarized state => $\Pi^+(\omega) = \epsilon_\mu^+ \epsilon_\nu^{+*} \Pi^{\mu\nu}(\omega, 0)$

3. Put the system at the center of the rotation => $q_\mu = (\omega, \vec{0})$

4. Up to linear terms in Ω

$$\Pi^+(\omega) = \omega^2\, \Pi^{vac}(\omega^2) + \omega\, \Pi^{rot}(\omega^2)\, \Omega + \mathcal{O}(\Omega^2)$$

Π^{vac} : ordinary vacuum invariant ftn. vacuum properties ex) mass

Π^{rot} : new function appearing in a rotating frame. spin information

How to extract g_Ω?

1. Describe the correlation function in a rotating frame

$$\Pi^{\mu\nu}(q) = i\int d^4x e^{iqx}\langle 0|\,\mathrm{T}\{\mathrm{j}^\mu(\mathrm{x})\mathrm{j}^\nu(0)\}\,|0\rangle$$

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Π^{vac} : ordinary vacuum invariant ftn. vacuum properties ex) mass

Π^{rot} : new function appearing in a rotating frame. spin information

5. Extract g_Ω by comparing two different descriptions of Π^{rot}

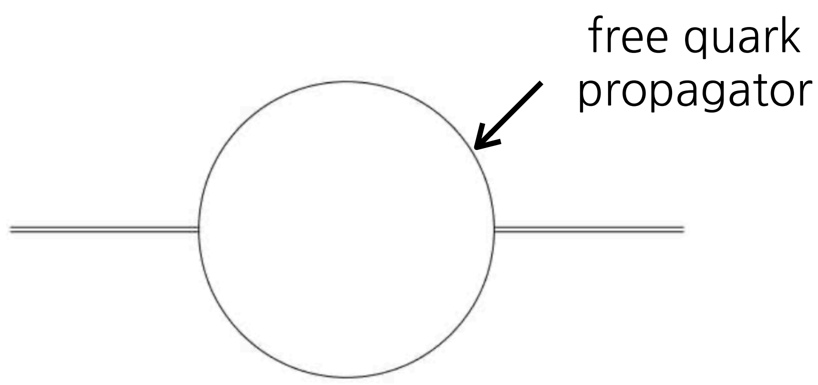
   (a) Directly compute Feynman diagrams in a rotating frame

   (b) Phenomenological derivation from Π^{vac}

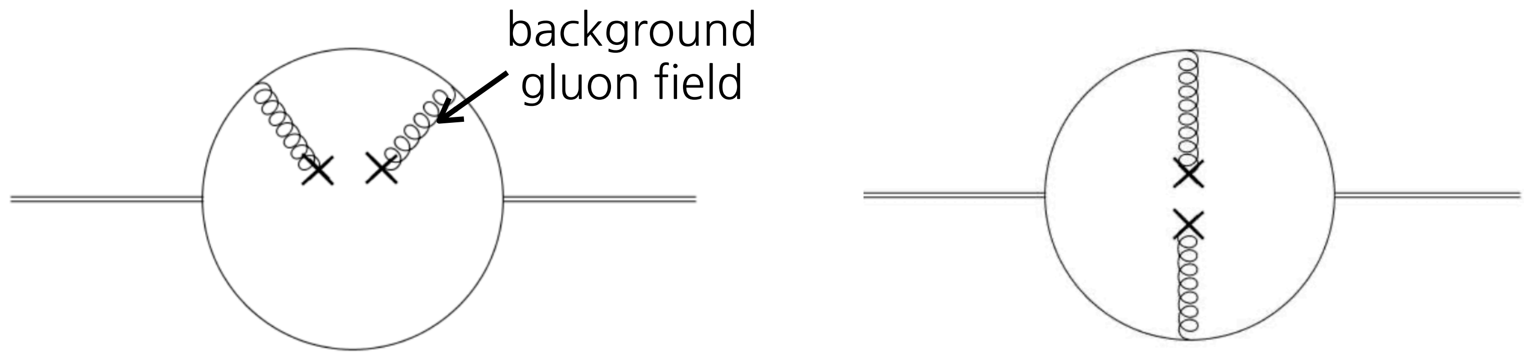
Method (a) - direct computation in a rotating frame

Feynman diagrams in Operator Product Expansion(OPE)

- Leading perturbative diagram



- Leading non-perturbative diagrams : Gluon condensates $\langle (\alpha_s/\pi)G^2 \rangle$



- Compute in an inertial frame $\rightarrow$ Ω independent terms $\rightarrow$ Π^{vac}
- Compute in a rotating frame $\rightarrow$ collect Ω linear terms $\rightarrow$ Π^{rot}

Quarks in a rotating frame

- Recall Dirac eq. in a rotating frame

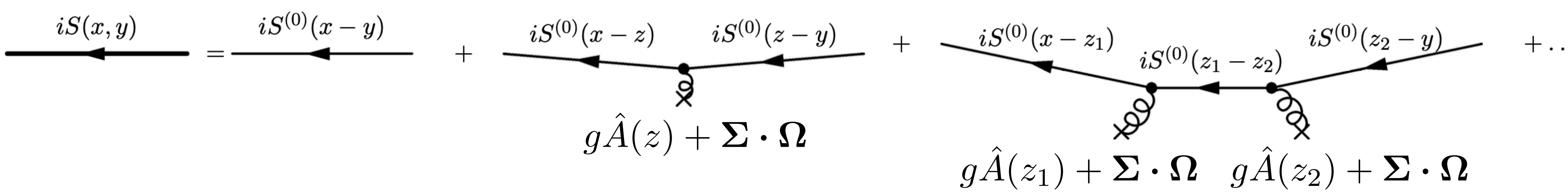
$$\left[i\partial\!\!\!/_x + gA\!\!\!/(x) + \boldsymbol{\Sigma}\cdot\boldsymbol{\Omega} - m\right]\Psi(x) = 0 \quad \text{where} \quad \boldsymbol{\Sigma} = \gamma^0(\mathbf{S}+\mathbf{L})$$

- Quark propagator in a rotating frame

$$\left[i\partial\!\!\!/_x + gA\!\!\!/(x) + \boldsymbol{\Sigma}\cdot\boldsymbol{\Omega} - m\right]S(x) = \delta(x)$$

  - It is difficult to find full propagator
  - We can expand in terms of 'g' and 'Ω'

$$S^{\text{full}} \approx S^{(0)} + S^{(0)}\left[gA\!\!\!/ + \Sigma\cdot\Omega\right]S^{(0)} + S^{(0)}\left[gA\!\!\!/ + \Sigma\cdot\Omega\right]S^{(0)}\left[gA\!\!\!/ + \Sigma\cdot\Omega\right]S^{(0)} + \cdots$$



Gluons in a rotating frame

- Covariant derivatives in curved space-time(Γ^a_{bc} : Christoffel symbols)

$$D_c G_{ab} = \partial_c G_{ab} - \Gamma^d_{ca} G_{db} - \Gamma^d_{cb} G_{ad}$$

- Fock-Schwinger gauge($x^\mu A_\mu = 0$) in curved space-time

$$\begin{aligned} A_\mu(x) &= -\frac{1}{2} x^\nu G_{\mu\nu} - \frac{1}{3} x^\nu x^\alpha \partial_\alpha G_{\mu\nu} + \cdots \\ &= -\frac{1}{2} x^\nu G_{\mu\nu} - \frac{1}{3} x^\nu x^\alpha D_\alpha G_{\mu\nu} \underbrace{- \frac{1}{3} x^\nu x^\alpha (\Gamma^d_{\alpha\mu} G_{d\nu} + \Gamma^d_{\alpha\nu} G_{\mu d})}_{\text{additional contribution in curved space-time}} + \cdots \end{aligned}$$

- $\Gamma^2_{01} = \Omega, \Gamma^1_{02} = -\Omega$ in a rotating frame.

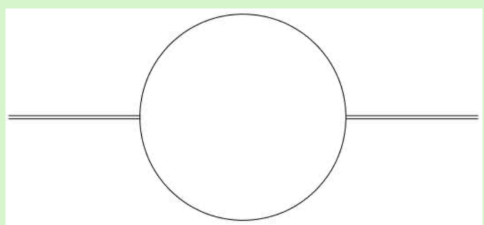
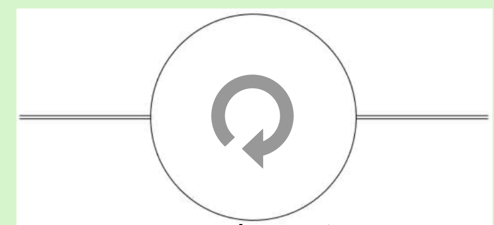
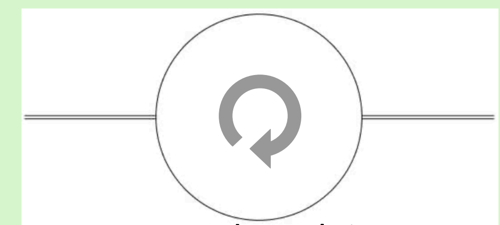
$$\not{A}^{\text{new}}(x) = -\frac{1}{3} x^\nu x^\alpha \gamma^\mu (\Gamma^d_{\alpha\mu} G_{d\nu} + \Gamma^d_{\alpha\nu} G_{\mu d}) \propto \vec{x} \times (\vec{E} \times \vec{B}) \cdot \Omega = J_g \cdot \Omega$$

- Kapusta et al. thought that $D_c G_{ab} = \partial_c G_{ab}$ in a rotating frame.
  (Their result might be wrong)

Summary of method (a)

$$S_{\text{quark}} \approx S^{(0)} + S^{(0)}\big[\underbrace{g\not{A} + \Sigma\cdot\Omega}_{(\mathrm{S_q} + \mathrm{L_q} + \mathrm{J_g})\,\Omega}\big]S^{(0)} + \cdots$$

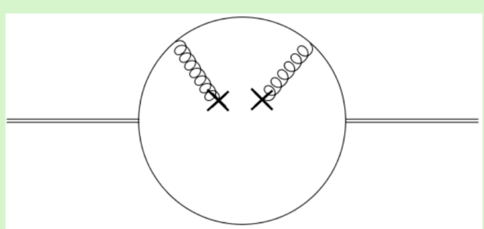
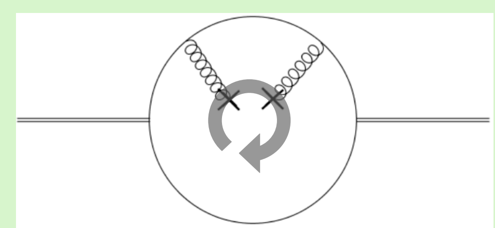
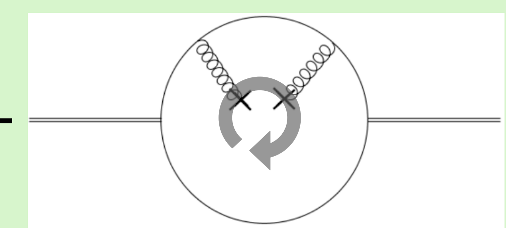
perturbative

$\Pi^{vac} =$  $\Rightarrow$ $\Pi^{rot}_{(a)} =$  $+$ 

quark spin

quark orbit

non-perturbative

$\Pi^{vac} =$  $\Rightarrow$ $\Pi^{rot}_{(a)} =$  $+$ 

quark(spin+orbit)

gluon

In a rot frame, we can compute $\Pi^{rot}_{(a)}$ while decomposing the given diagrams into quark and gluon AM according to their origin

Method (b) - Phenomenological derivation

- In an inertial frame

$$\Pi^{+}(\omega) = \epsilon_{+}^{\mu *}\epsilon_{+}^{\nu}\Pi_{\mu\nu}(\omega,0) = \omega^2\,\Pi^{vac}(\omega^2)$$

- Energy shift of all right circularly polarized state in a rotating frame

$$=> \omega \to \omega - g_{\Omega}\,\Omega \qquad (\because H_{\rm SRC} = -\,g_{\Omega}\,S\cdot\Omega)$$

$$\begin{aligned}\Pi^{+}(\omega + g_{\Omega}\Omega) &= (\omega + g_{\Omega}\Omega)^2\,\Pi^{vac}((\omega + g_{\Omega}\Omega)^2) \\ &= \omega^2\,\Pi^{vac}(\omega^2) + \omega\,\Pi^{rot}(\omega^2)\,\Omega + \mathcal{O}(\Omega^2)\end{aligned}$$

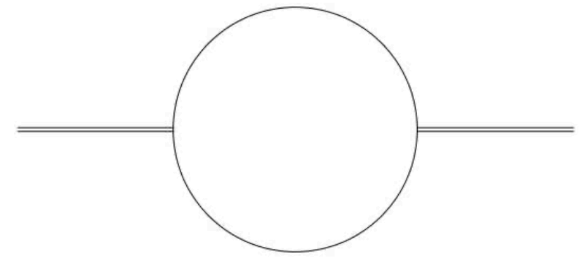
- Simple expression of rotating part in terms of vacuum invariant ftn.

$$\Pi^{rot}_{(b)}(\omega^2) = 2\underset{\text{unknown}}{\underline{g_{\Omega}}}\left\{\Pi^{vac}(\omega^2) + \omega^2\frac{\partial\Pi^{vac}(\omega^2)}{\partial\omega^2}\right\}$$

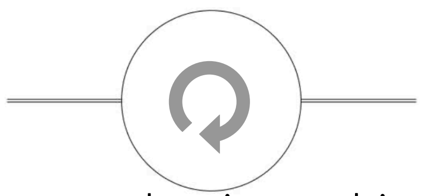
=> We can directly derive Π^{rot} from Π^{vac} but it includes unknown g_{Ω}

g_Ω in perturbative region

Inertial frame

$$\Pi^{vac}_{pert} =$$ 

Rotating frame

$$\Pi^{rot}_{(a)} =$$ 

quark spin + orbit

description (a)
all responses of quarks in a rot frame
quark's spin + orbital AM

=

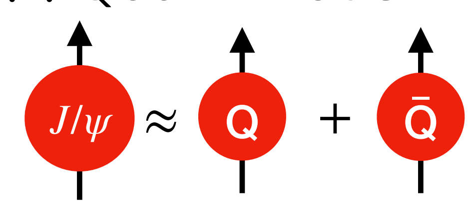
$$\Pi^{rot}_{(b)} = 2g_\Omega \left\{ \Pi^{vac}(\omega^2) + \omega^2 \frac{\partial \Pi^{vac}(\omega^2)}{\partial \omega^2} \right\}$$

description (b)
phen. derivation based on the energy shift of total system by "-$g_\Omega \Omega$".

$g_\Omega = 1$ in the perturbative region

When two free quarks form a spin-1 state in a rel. way,
they follow $\mathrm{H_{SRC}} = -\mathrm{S} \cdot \Omega$

c.f. Quark Model



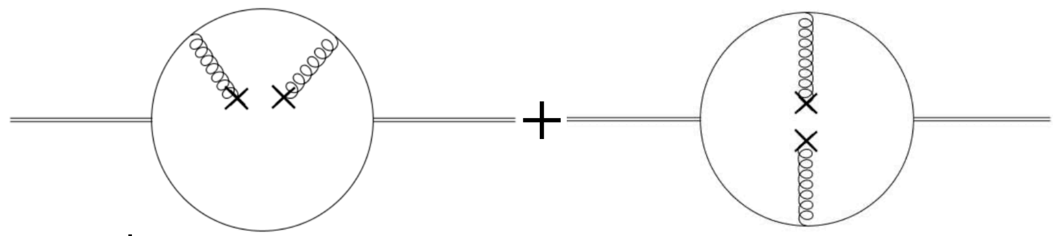
$$|11\rangle \sim |\frac{1}{2}\frac{1}{2}\rangle + |\frac{1}{2}\frac{1}{2}\rangle$$

$$H_r = m_{J/\psi} - \Omega$$

g_Ω in non-perturbative region

Inertial frame

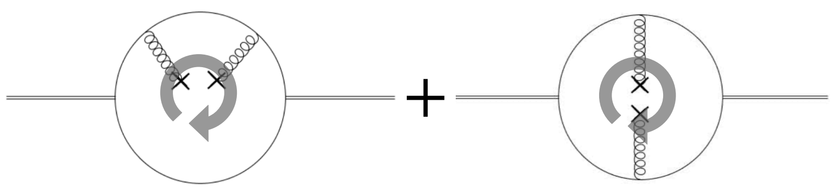
$$\Pi^{vac}_{G0} =$$



G0 : gluon condensate

Rotating frame

$$\Pi^{rot}_{(a)} =$$



quark spin+ quark orbit + gluon

description (a)
all responses of quarks, gluons in a rot frame

=

$$\Pi^{rot}_{(b)} = 2g_\Omega \left\{ \Pi^{vac}(\omega^2) + \omega^2 \frac{\partial \Pi^{vac}(\omega^2)}{\partial \omega^2} \right\}$$

description (b)
phen. derivation based on the energy shift of total system by "-$g_\Omega \Omega$".

$$\boxed{g_\Omega = 1 \text{ in the non-perturbative region}}$$

Even in non-pert region, spin-1 system follows $\mathrm{H_{SRC}} = -\mathbf{S} \cdot \mathbf{\Omega}$

Physical meaning of $g_\Omega = 1$?

Method (b)

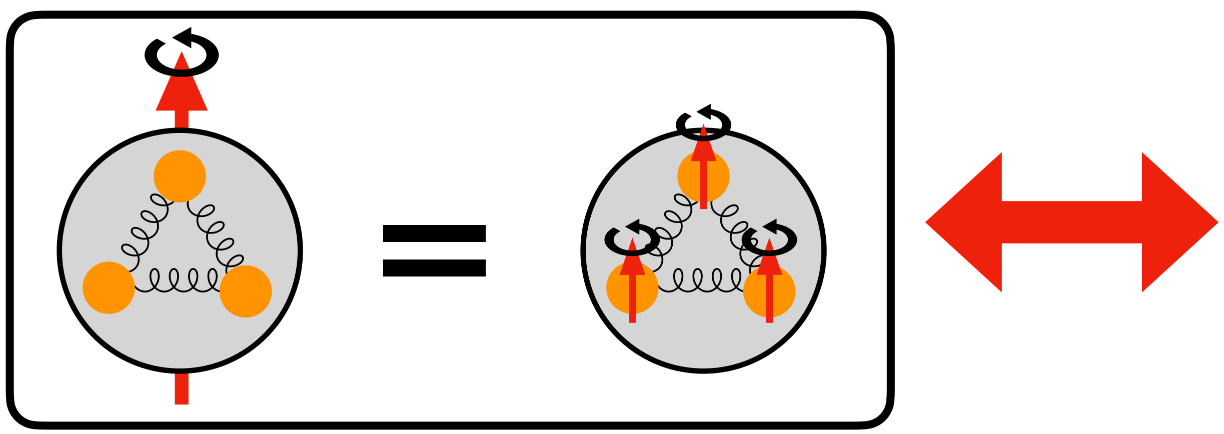
= SRC of the total system

= $g_\Omega \langle \vec{S} \rangle$ where $\vec{S}$ is spin-1 operator where $\langle \cdots \rangle = \int d^4x e^{iq\cdot x} \langle 0 | T[j(x)\cdots j(0)] | 0 \rangle$

Method (a)

= Ω linear terms in all responses of quarks and gluon in a rotating frame

= $\langle \vec{J}_{\mathrm{QCD}} \rangle$ where $\vec{J}_{\mathrm{QCD}} = \int d^3x (\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma_5\psi + \psi^\dagger(\vec{x}\times(-i\vec{D}))\psi + \vec{x}\times(\vec{E}\times\vec{B}))$



$$\boxed{g_\Omega \langle \vec{S} \rangle = \langle \vec{J}_{\mathrm{QCD}} \rangle}$$

Therefore, we can conclude that $g_\Omega = \langle \vec{J}_{\mathrm{QCD}} \rangle / \langle \vec{S} \rangle$

- $g_\Omega = 1$ means $\langle \vec{S} \rangle = \langle \vec{J}_{\mathrm{QCD}} \rangle$
- This should be valid for any Feynman diagram ($\because$ AM conservation)

Application - g_Ω of ground states

From Kallen-Lehmann(or spectral) rep,

"$g_\Omega = 1$" is universal for all physical states that can couple to $j^\mu(x)$.

If we can extract the ground state,

=> Fraction of g_Ω carried by each a.m. inside the ground state

$$g_\Omega^{\text{ground}} = \frac{\langle J_{\text{QCD}} \rangle}{\langle S_{\text{tot}} \rangle} = \frac{\langle S_q \rangle + \langle L_k \rangle + \langle L_p \rangle + \langle J_g \rangle}{\langle S_{\text{tot}} \rangle} = 1$$

$S_q = \frac{1}{2}\gamma^1\gamma^2$: quark spin,

$L_k = r \times p$: kinetic part of quark orbital a.m,

$L_p = r \times gA$: potential part of quark orbital a.m,

$J_g = r \times (E \times B)$: gluon total a.m.

=> Spin content of the ground state

Result - spin contents of spin-1 quarkonia

With the help of 'QCD sum rule' + simple 'pole+continuum' ansatz.

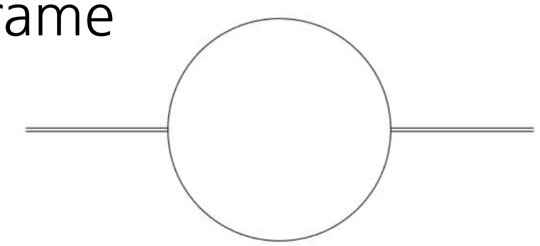
			Vector (%)			Axial (%)		
			S-wave	$\Upsilon(1S)$	J/ψ	P-wave	χ_{b1}	χ_{c1}
Quark	spin	S_q	100	92	88	50	43	40
Quark	$r \times p$	L_k	0	7.6	11	50	57	61
Quark	$r \times gA$	L_p	0	0.003	0.2	0	-0.001	0.08
Gluon	$r \times (E \times B)$	J_g	0	0.015	0.8	0	-0.005	-1.5

- Total sum of 4 pieces becomes 1
- Classical picture from the naive Q.M.
  S-wave: quark spin(100%) , P-wave: quark spin(50%) quark oam(50%)
- Spin contents are slightly different from the classical picture.
  As the quark mass becomes lighter, spin contents deviate more from the classical picture
  ex) J/ψ is considered as S-wave but quarks do not carry all of the total spin
  $\Upsilon(1S)$ is still comparable with the classical picture

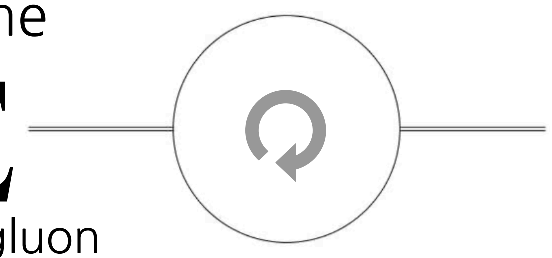
Summary

- For a given Feynman diagram in an inertial frame, there is a counterpart in a rotating frame

inertial frame

$\Pi^{vac} =$ 

rotating frame

$\Pi^{rot} = \sum_{\text{quark, gluon}}$ 

$$\Pi^{rot}(\omega^2) = 2\left\{\Pi^{vac}(\omega^2) + \omega^2 \frac{\partial \Pi^{vac}(\omega^2)}{\partial \omega^2}\right\} \text{ with } \mathrm{g}_\Omega = 1$$

$$\langle \vec{S} \rangle = \langle \vec{J}_{\mathrm{QCD}} \rangle$$

1. We prove that spin-1 composite systems follow $\mathrm{H}_{\mathrm{SRC}} = -\,\mathrm{S} \cdot \Omega$
2. We examine spin contents of ground states in a relativistic way using QCD Sum Rules

Future plans, possible extensions

1. Light quark system

vector mesons

	Q.M.	$\Upsilon(1S)$	J/ψ	ρ, ω, ϕ
S_q	100	92	88	?
L_k	0	7.6	11	?
L_p	0	0.003	0.2	?
J_g	0	0.015	0.8	?

nucleons

	Q.M.	p, n
S_q	100	?
L_k	0	?
L_p	0	?
J_g	0	?

2. vacuum -> medium

$\langle 0 | \cdots | 0 \rangle \rightarrow \langle \Omega | \cdots | \Omega \rangle$ Lorentz symmetry broken

3. uniform rotation -> local vorticity

4. away from the center, 3-momentum, finite size effect, non-inertial effects, etc

Back up