Transition GPDs and non-diagonal DVCS off spinless target in resonance region:

A unified framework for the meson case $\gamma^*\pi \to \gamma\pi\pi$

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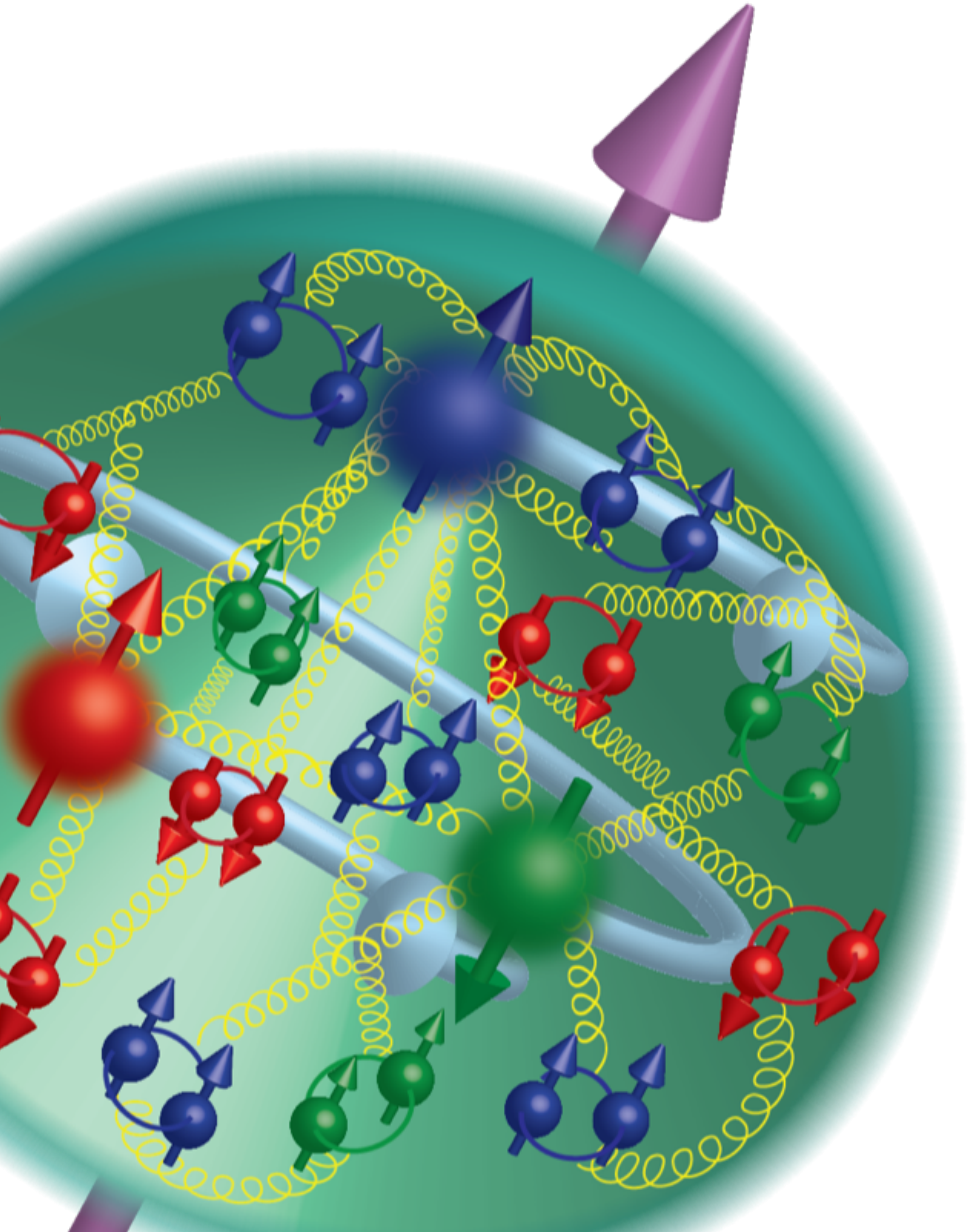
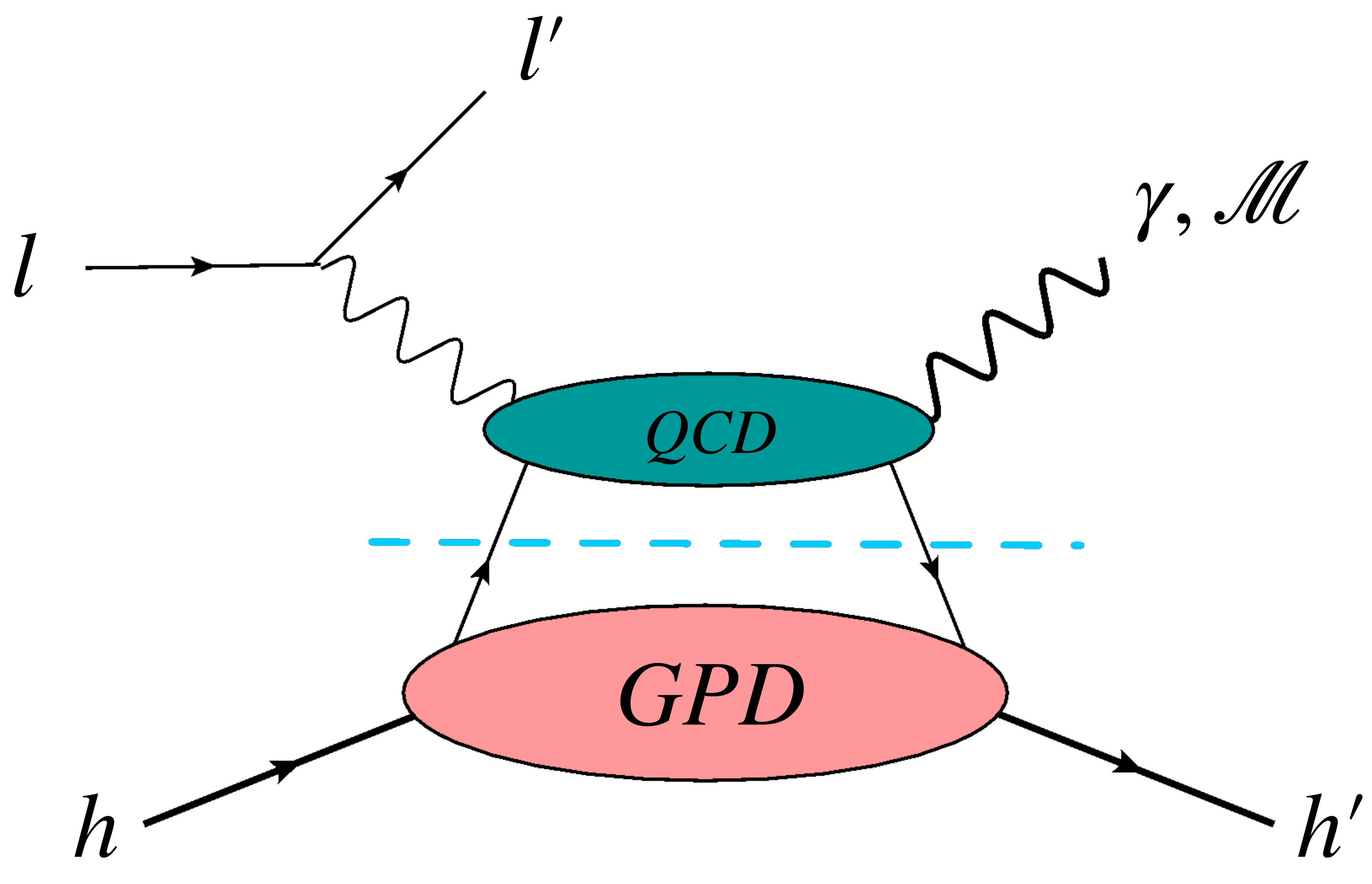
RIKEN Workshop on Quark Structure of Hadrons

3D Structure of Hadrons

Generalized parton distribution (GPD)

$$\frac{1}{2}\int \frac{dz^-}{2\pi} e^{iP^+z^-} \langle h' | \bar{\psi}(-z^-/2)[-z/2,z/2]\gamma^+\psi(z^-/2) | h\rangle \Big|_{z^+=z_\perp=0}$$





- (2+1)-D tomography in $(x, \vec{b}_T)$

  - Transverse spatial distribution of quarks in hadron resonances

- Form factors of local QCD operators with spin-J

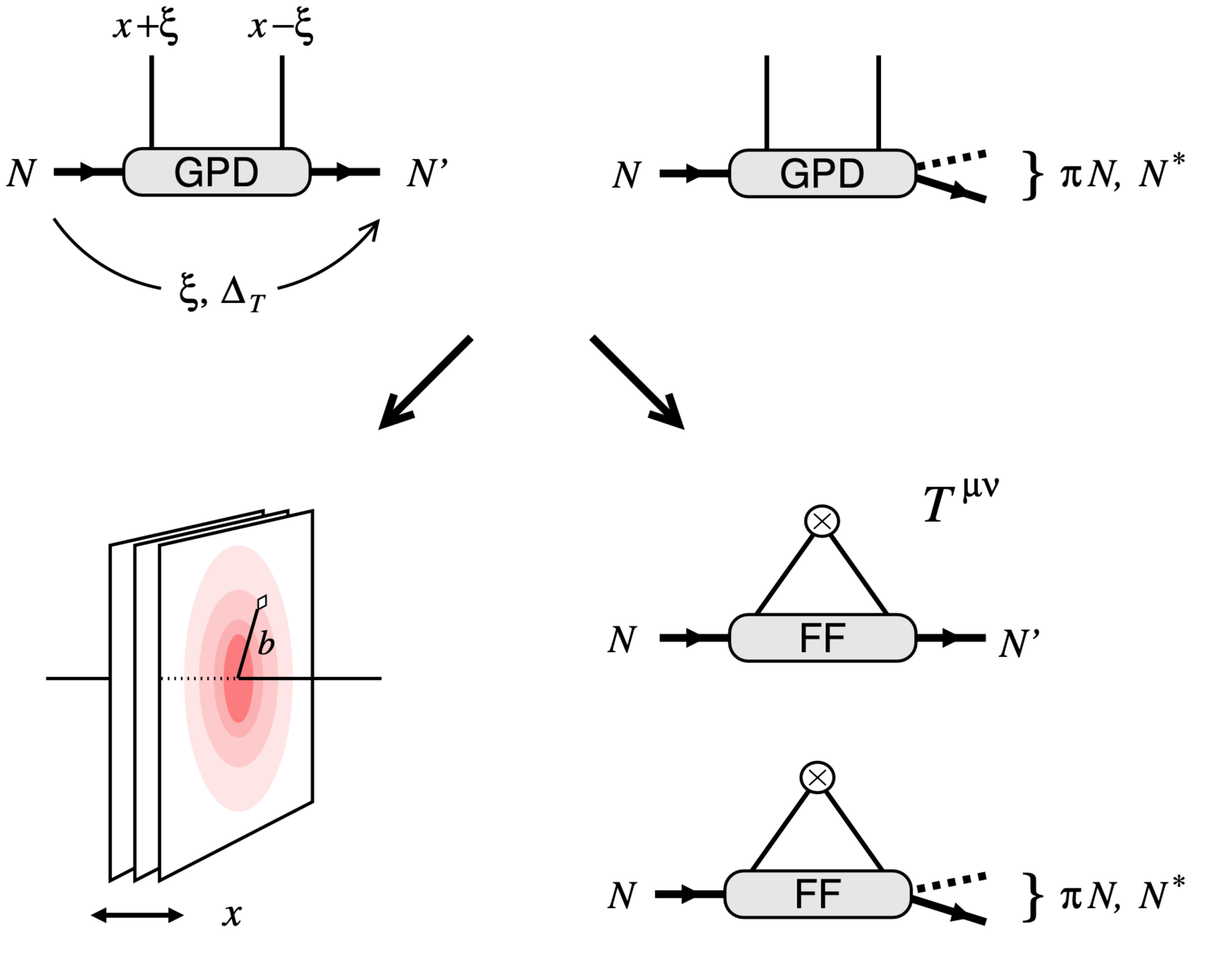
  - Mechanical properties such as the angular momentum, stress, etc.

D. Müller et. al., Fortsch. Phys. 42, 101 (1994)
A. V. Radyushkin, Phys. Rev. D 56, 5524 (1997)
X. Ji, Phys. Rev. Lett. 78, 610 (1997)
X. Ji, Phys. Rev. D 55, 7114 (1997)

$$\int dx\ x^{J-1} H(x, \xi, t)$$

→ Generalized FFs

Non-diagonal hard exclusive reactions



tomographic imaging
of nucleon/resonance

QCD energy–momentum tensor
angular momentum, mass

Recent developments in the non-diagonal hard exclusive reactions

S. Diehl et. al. (CLAS collaboration), Phys. Rev. Lett 131, 021901 (2023)

- Beam-spin asymmetry measurement of $\gamma^* p \to \pi^- \Delta^{++} \to \pi^- p \pi^+$ by the CLAS collaboration

K. Semenov-Tyan-Shanskiy and M. Vanderhaeghen, Phys. Rev. D 108, 034021 (2023)

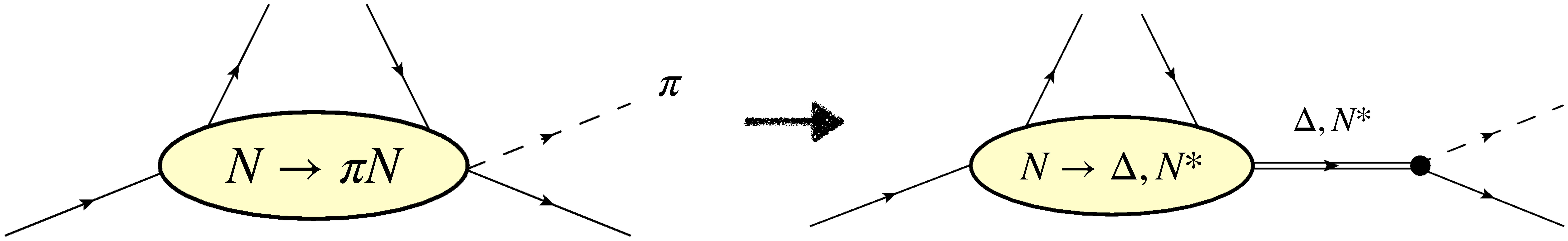
- DVCS of $\gamma^* N \to \gamma \pi N$ with $N \to \Delta, N^*$ transition GPDs

- DVMP of $\gamma^* p \to \pi^- \Delta^{++}$ with $p \to \Delta^{++}$ transition GPDs

P. Kroll and K. Passek-Kumericki, Phys. Rev. D 107, 054009 (2023)

S. Diehl et. al., White paper on *Exploring resonance structure with transition GPDs, arXiv:2405.15386*

Transition GPDs



I. Excitation of hadrons with non-local QCD probe (resonance region)

II. Natural test ground of the chiral dynamics (near-threshold region)

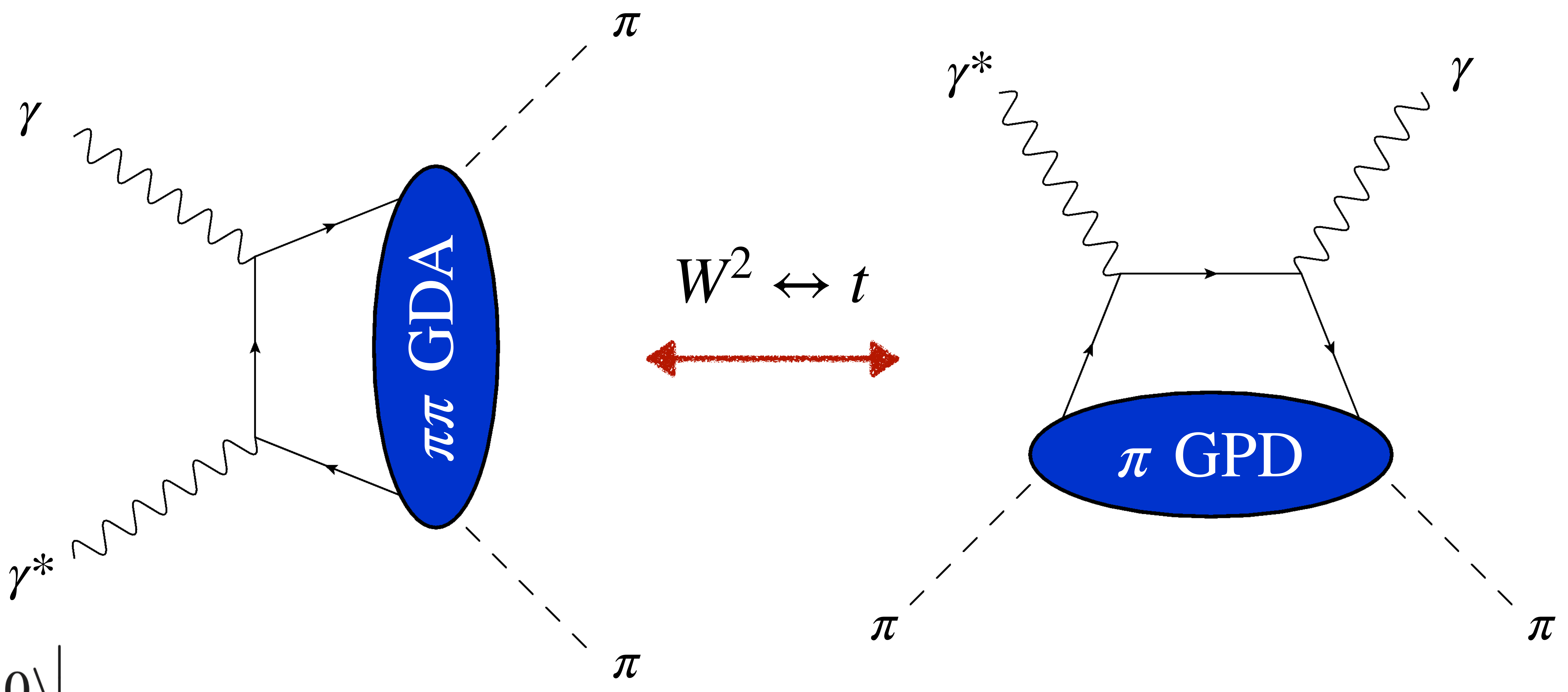
III. Non-diagonal matrix elements of the QCD energy-momentum tensor

✓ Transition GPDs such as $N \to \pi N$ GPDs depend on more arguments;
the invariant mass and *the decay angles* of produced hadronic states

Two-pion Generalized distribution amplitude

$$\gamma^*\gamma \to \pi\pi$$

Cross channel of $\gamma^*\pi \to \gamma\pi$



$$\Phi_q^{h\bar{h}}(z,\zeta,W^2) = \int \frac{dy^-}{2\pi} e^{i(2z-1)P^+y^-/2} \times \langle h(p)\bar{h}(p')|\bar{q}(-y/2)\gamma^+ q(y/2)|0\rangle\Big|_{y^+=\vec{y}_\perp=0}$$

M. Polyakov, Nucl. Phys. B 555, 231 (1999)
B. Lehmann-Droke et. al., Phys. Rev. D 63, 114001 (2001)

- Analytic continuation in $t \to W^2 > 0$: encodes $\pi\pi$ invariant mass spectra
- Dispersive analysis on $\pi\pi$ generalized distribution amplitude (GDA) in resonance region
- Application of $\pi\pi$ GDA for mechanical properties of pion   S. Kumano, Q.-T. Song, O. V. Teryaev, Phys. Rev. D 97, 014020 (2018)

Froissart-Gribov projection

M. Froissart, Phys. Rev. 123, 1053 (1961)
V. N. Gribov, Nucl. Phys. 22, 249 (1961)

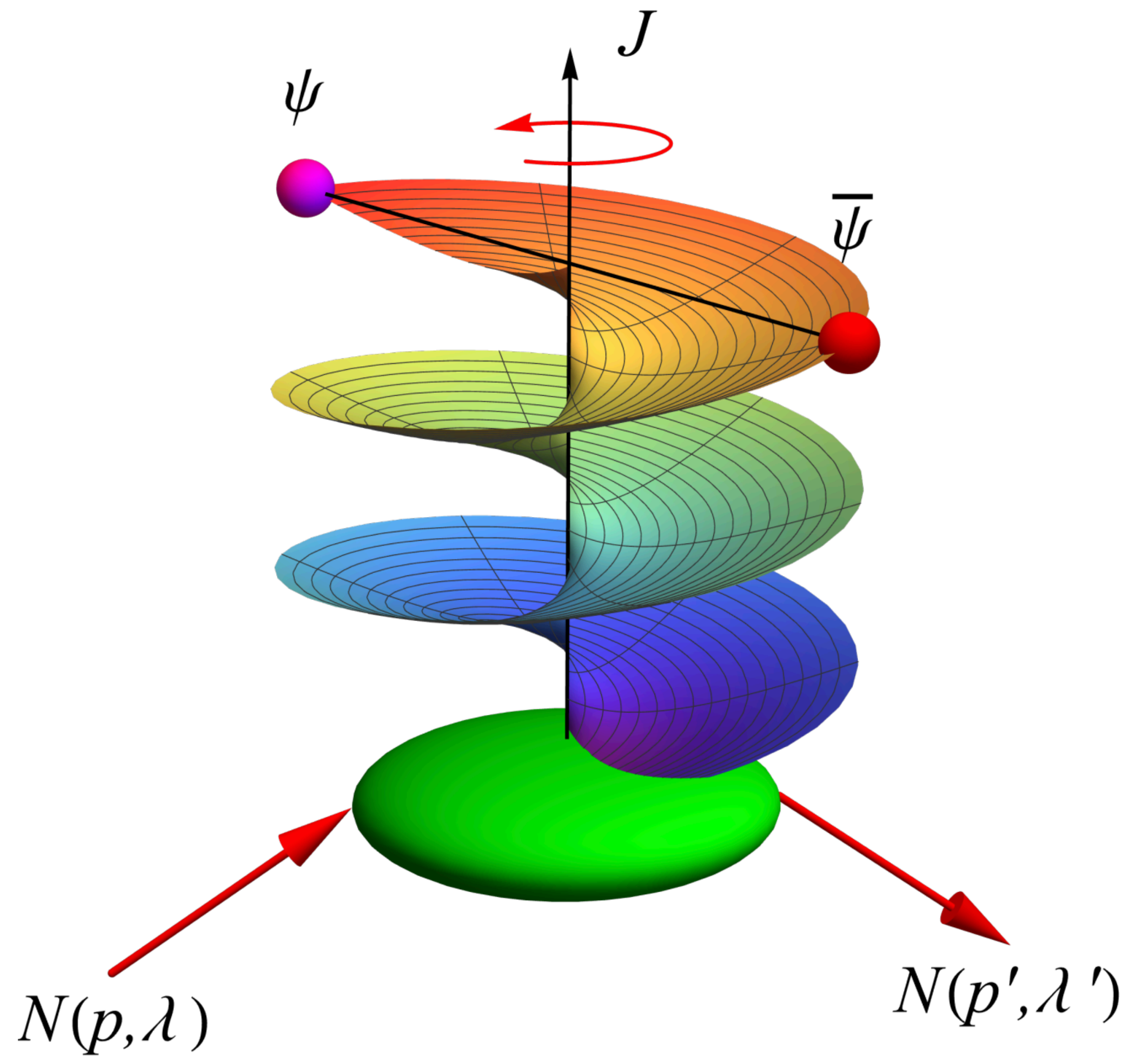
- Response of hadron to non-local QCD string-like probe by means of FG projection

K. M. Semenov-Tian-Shansky and P. Szajder, Phys. Rev. D 109, 054010 (2024)

- Cross channel PW expansion of Compton FF

$$\mathscr{H}(\cos\theta_t, t) = \sum_J F_J(t) P_J(\cos\theta_t)$$

$$\cos\theta_t \simeq -\frac{1}{\xi} + \mathcal{O}(1/Q^2)$$



- Dispersion relation for Compton FF

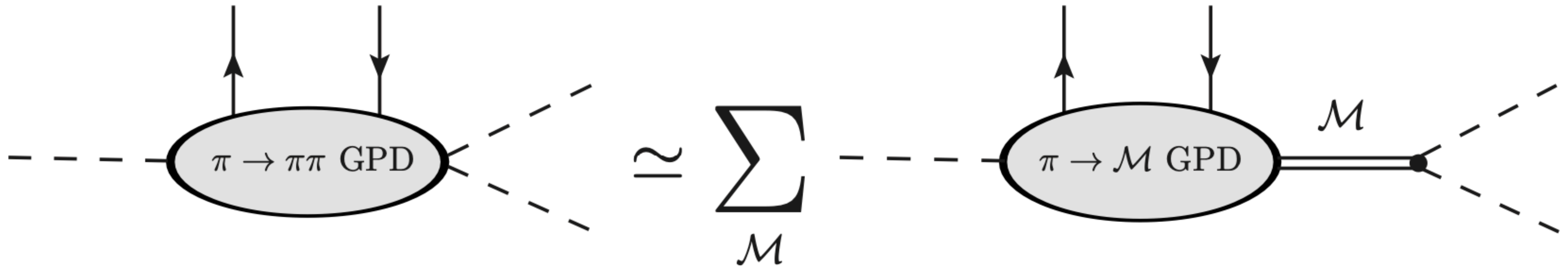
$$\mathrm{Re}\ \mathscr{H}(\xi, t) = \mathrm{P}\int_0^1 dx \frac{2xH(x,x,t)}{\xi^2 - x^2} + \text{subtraction term}$$

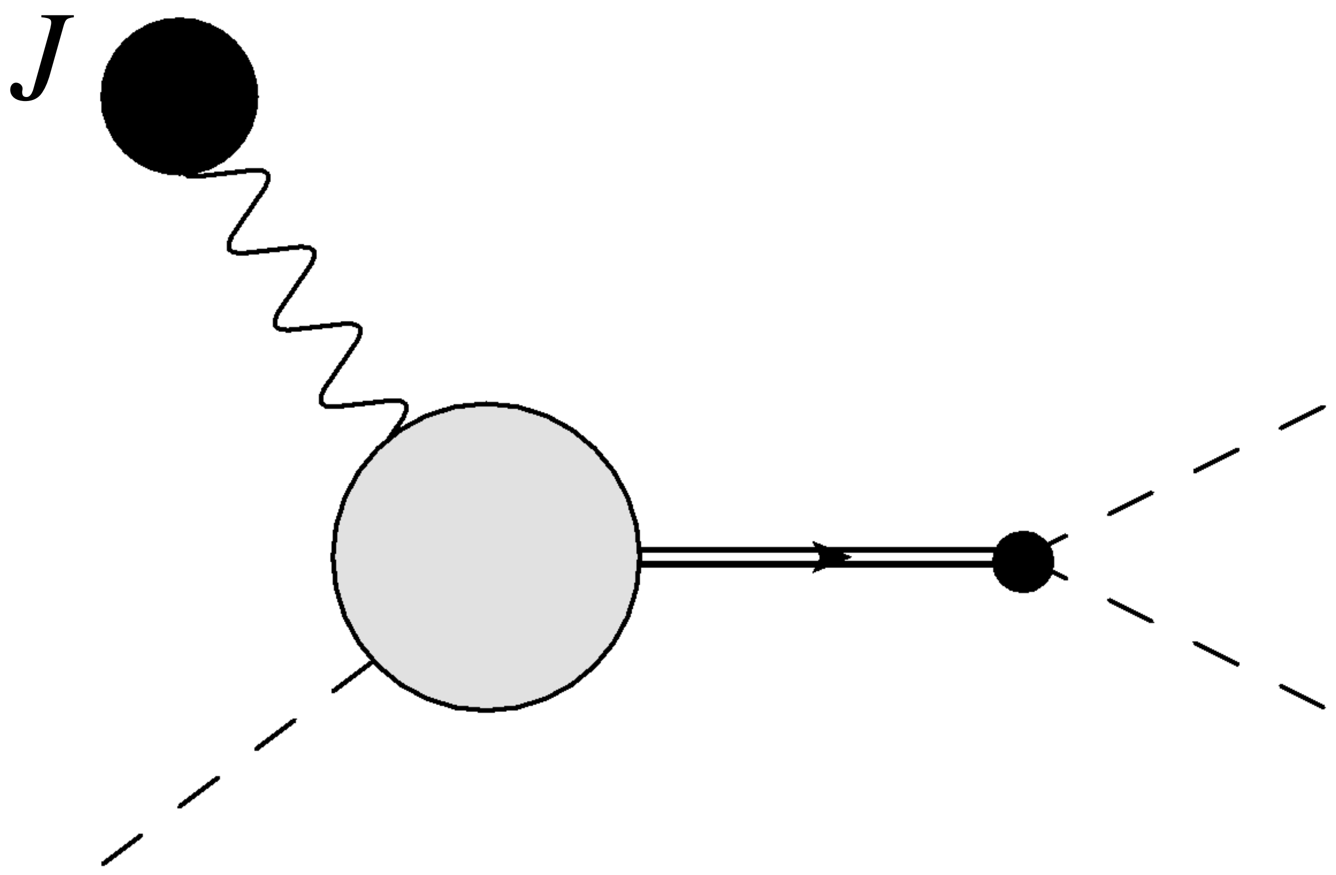
→ *FG projections can, in principle, be examined directly from measurements (spin asymmetries)*

$e^-\pi \to e^-\gamma\pi^+\pi^0$ DVCS

In this work, we study the non-diagonal DVCS of $\gamma^*\pi \to \gamma\pi\pi$ to avoid complications due to hadron spin.

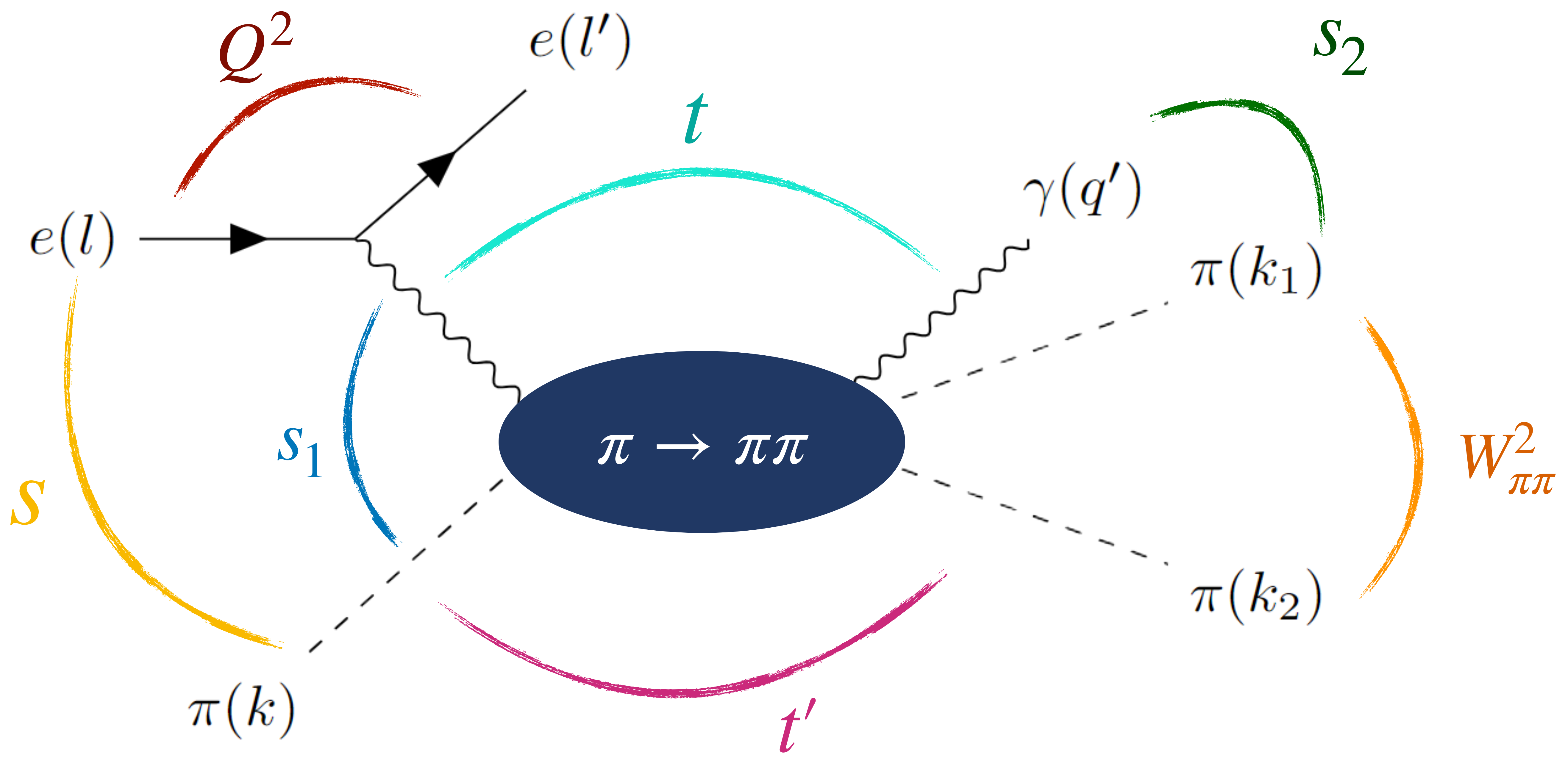
- Study the $\pi \to \rho$ contribution to the $\pi \to \pi\pi$ GPDs. Express the $e\pi \to e\gamma\pi\pi$ cross section and work out its angular distribution near $W_{\pi\pi} \simeq m_\rho$.
- Invariant mass distribution of $\pi \to \pi\pi$ GPDs from dispersive analysis
- Application of the Froissart-Gribov projection of the $\pi \to \pi\pi$ Compton FF

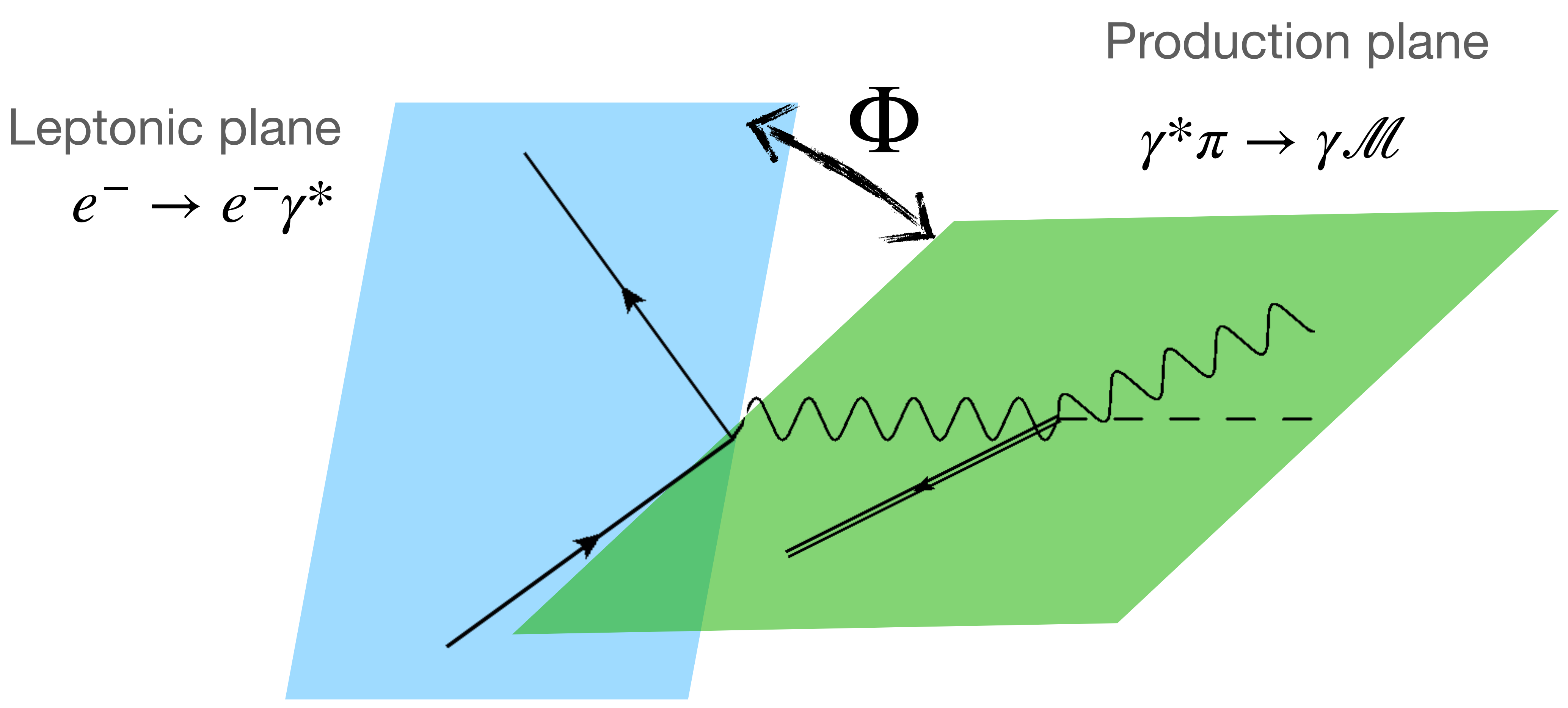




A preceding work to develop the framework of the partial wave analysis of transition GPDs for further generalization to $N \to \pi N$ GPDs

$e^- \pi \to e^- \gamma \pi^+ \pi^0$ DVCS





8 Kinematic variables

$$Q^2 = -q^2 = -(l-l')^2, \quad t = \left(p_{\pi\pi} - k\right)^2 = \Delta^2$$

$$x_B = \frac{Q^2}{2k\cdot q}, \quad y = \frac{k\cdot q}{k\cdot l}, \quad W^2_{\pi\pi} = p^2_{\pi\pi} = \left(k_1 + k_2\right)^2$$

$$\Omega^*_\pi = \left(\theta^*_\pi, \phi^*_\pi\right), \quad \Phi$$

Sudakov expansion of momenta in light cone vectors

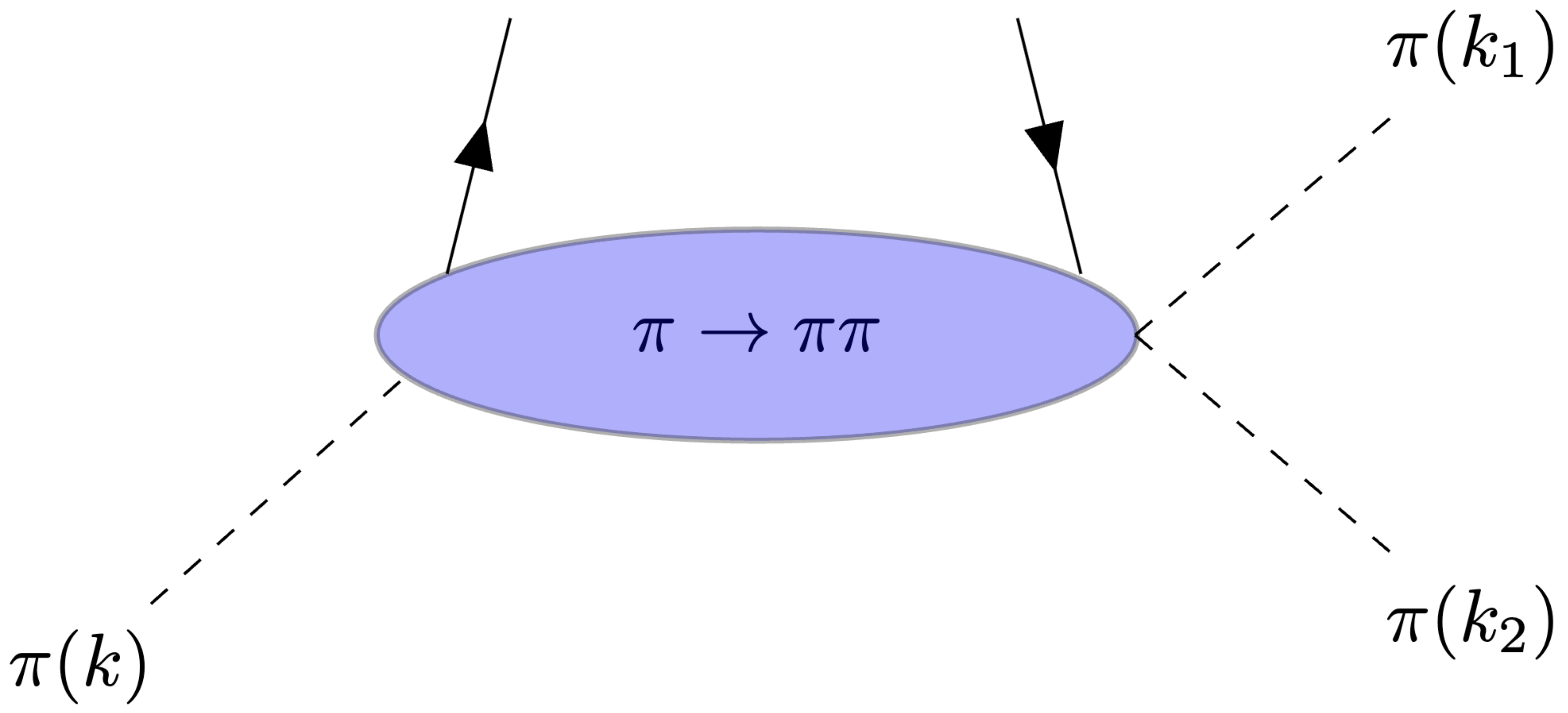
$$\begin{aligned}
\bar{P} &= \tilde{p} + \frac{\bar{P}^2}{2} n \\
q &= -2\xi'\tilde{p} + \frac{Q^2}{4\xi'} n \\
\Delta &= -2\xi\tilde{p} + \left(\bar{P}^2\xi + \frac{W^2_{\pi\pi} - m^2_\pi}{2}\right) n + \Delta_\perp \\
p_{\pi\pi} &= (1-\xi)\tilde{p} + \left[\frac{\bar{P}^2}{2}(1+\xi) + \frac{W^2_{\pi\pi} - m^2_\pi}{4}\right] n + \frac{\Delta_\perp}{2}
\end{aligned}$$

Our choice of kinematic variables: helicity angles in $\pi\pi$ CMS

$$t', s_2 \leftrightarrow \theta^*_\pi, \phi^*_\pi$$

$$\begin{aligned}
\cos\theta^*_\pi &= \left.\frac{\vec{q}\,'\cdot\vec{k}_2}{|\vec{q}\,'||\vec{k}_2|}\right|_{\vec{k}_1=-\vec{k}_2}, \\
\cos\varphi^*_\pi &= \left.\frac{(\vec{q}\,'\times\vec{k})\cdot(\vec{q}\,'\times\vec{k}_2)}{|\vec{q}\,'\times\vec{k}||\vec{q}\,'\times\vec{k}_2|}\right|_{\vec{k}_1=-\vec{k}_2}.
\end{aligned}$$

GPD parameterizations



Non-local twist-2 QCD operators

$$\begin{Bmatrix} \hat{O}_S \\ \hat{O}^d_V \end{Bmatrix}\left(-\frac{\lambda n}{2}, \frac{\lambda n}{2}\right) = \bar{\psi}\left(-\frac{\lambda n}{2}\right)\not{n}\begin{Bmatrix} 1 \\ \tau^d \end{Bmatrix}\psi\left(\frac{\lambda n}{2}\right)$$

$$\begin{Bmatrix} \hat{O}_{5S} \\ \hat{O}^d_{5V} \end{Bmatrix}\left(-\frac{\lambda n}{2}, \frac{\lambda n}{2}\right) = \bar{\psi}\left(-\frac{\lambda n}{2}\right)\not{n}\gamma_5\begin{Bmatrix} 1 \\ \tau^d \end{Bmatrix}\psi\left(\frac{\lambda n}{2}\right)$$

Twist-2 (Un)polarized isoscalar and isovector $\pi \to \pi\pi$ GPDs

$\sim |\sin\theta^*_\pi|\sin\phi^*_\pi$

$$\mathcal{F}(x)\langle\pi^b(k_1)\pi^c(k_2)|\begin{Bmatrix} \hat{O}_S \\ \hat{O}^d_V \end{Bmatrix}\left(-\frac{\lambda n}{2}, \frac{\lambda n}{2}\right)|\pi^a(k)\rangle = \frac{i\varepsilon(n,\bar{P},\Delta,k_1)}{f^3_\pi}\begin{Bmatrix} i\varepsilon^{abc}H_S^{\pi\to\pi\pi}(x,\xi,t;W^2_{\pi\pi},\theta^*_\pi,\phi^*_\pi) \\ \sum_{I=0}^{2} P^{I,bc}_{da}H_{V,I}^{\pi\to\pi\pi}(x,\xi,t;W^2_{\pi\pi},\theta^*_\pi,\phi^*_\pi) \end{Bmatrix}$$

$$\mathcal{F}(x)\langle\pi^b(k_1)\pi^c(k_2)|\begin{Bmatrix} \hat{O}_{5S} \\ \hat{O}^d_{5V} \end{Bmatrix}\left(-\frac{\lambda n}{2}, \frac{\lambda n}{2}\right)|\pi^a(k)\rangle = \frac{i}{f_\pi}\begin{Bmatrix} i\varepsilon^{abc}\tilde{H}_S^{\pi\to\pi\pi}(x,\xi,t;W^2_{\pi\pi},\theta^*_\pi,\phi^*_\pi) \\ \sum_{I=0}^{2} P^{I,bc}_{da}\tilde{H}_{V,I}^{\pi\to\pi\pi}(x,\xi,t;W^2_{\pi\pi},\theta^*_\pi,\phi^*_\pi) \end{Bmatrix}$$

$\mathscr{F}$(x) : Fourier transform

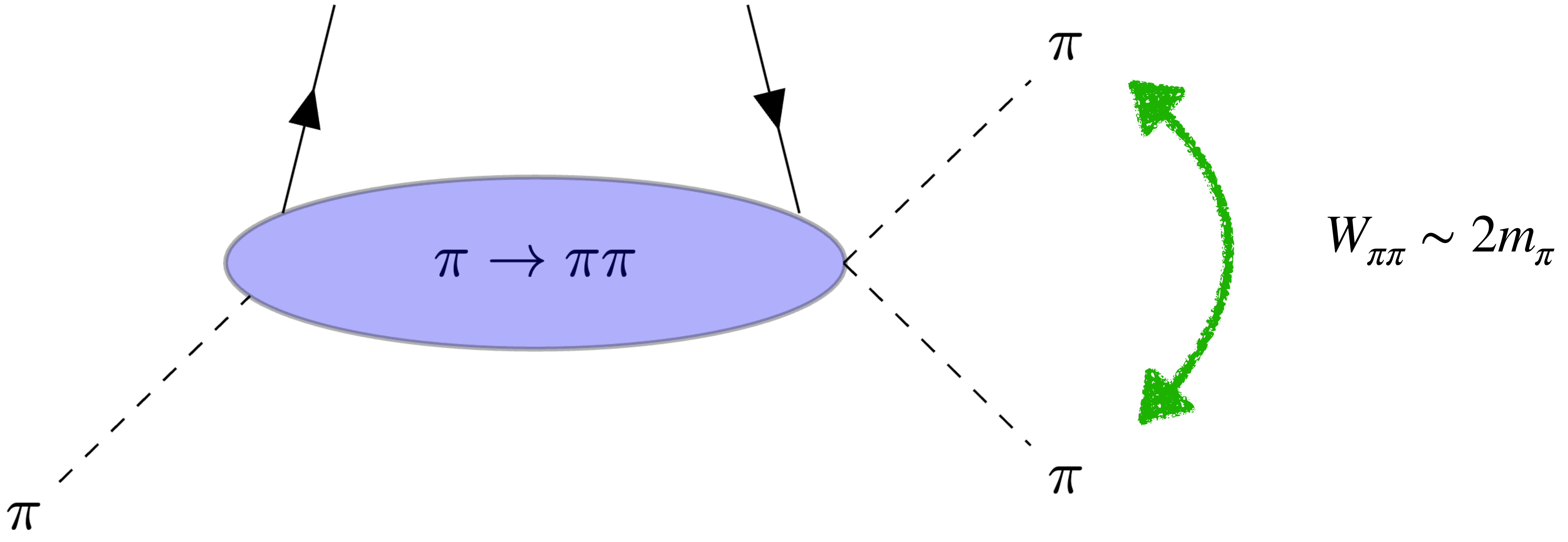
P^I : projector into isospin I

GPD near the soft-pion region

P. Pobylitsa, M. Polyakov, and M. Strikman, Phys. Rev. Lett. 87, 022001 (2001)

- Chiral dynamics provides parameter-free prediction of $\pi \to \pi\pi$ transition GPDs near the threshold in terms of the pion GPD.

- PCAC relation allows us to write the pion field in terms of the axial current and by the LSZ reduction *soft pion reduces to the chiral rotation of the operator.*



Soft-pion theorem

$$\left\langle \pi^b(k_1)\pi^c(k_2)\middle|\hat{O}\middle|\pi^a(k)\right\rangle = -\frac{i}{f_\pi}\left\langle \pi^c(k_2)\middle|\left[Q_5^b,\hat{O}\right]\middle|\pi^a(k)\right\rangle + \ldots,$$

↓

$$\tilde{H}_V^{\pi\to\pi\pi}(x,\xi,t;4m_\pi^2,\theta_\pi^*,\phi_\pi^*) \sim H_V^\pi\left(\frac{2x}{2-\alpha(1-\xi)},\frac{2\xi+\alpha(1-\xi)}{2-\alpha(1-\xi)},t'\right)$$

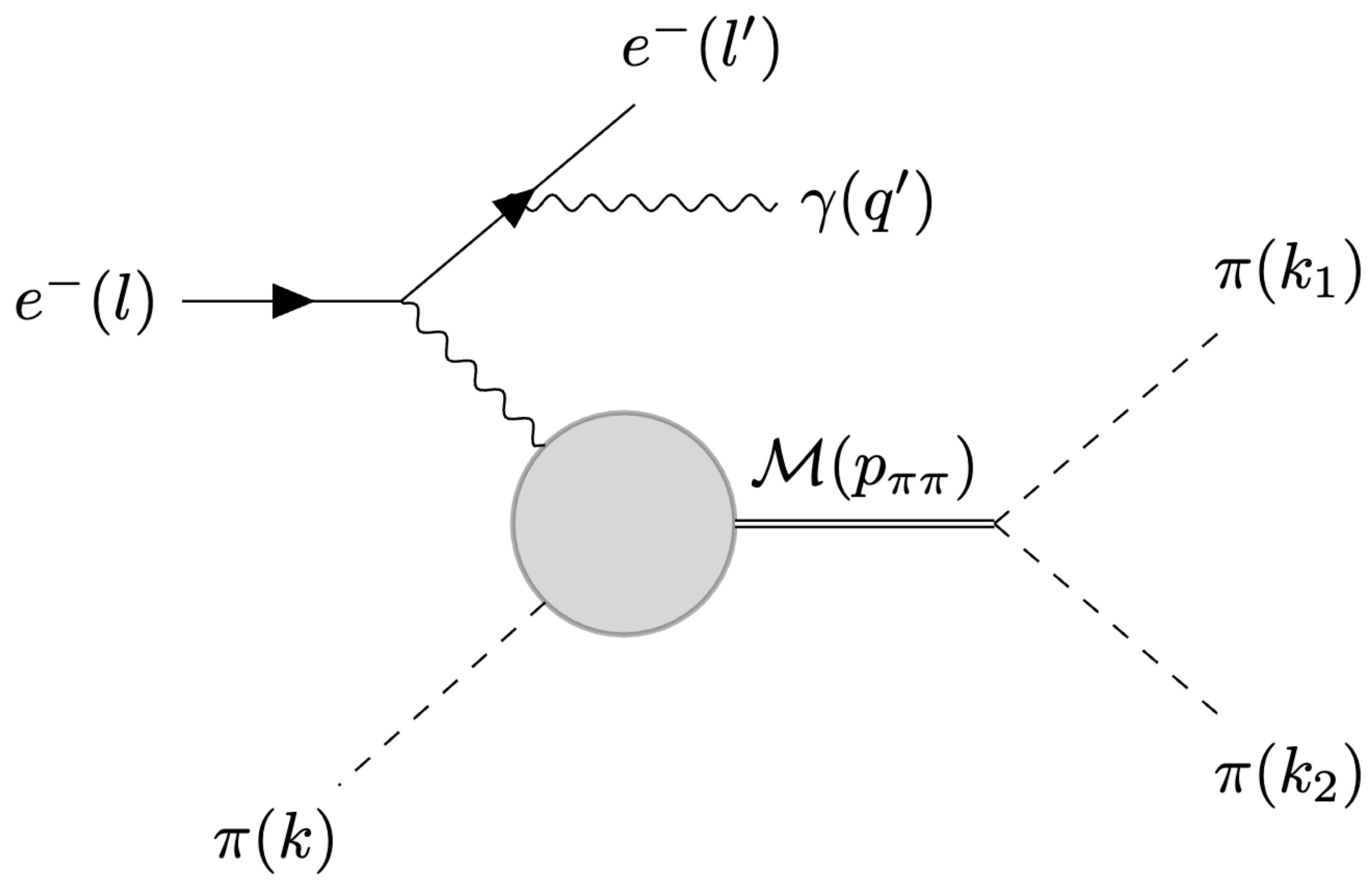
Longitudinal momentum distribution of final state pion

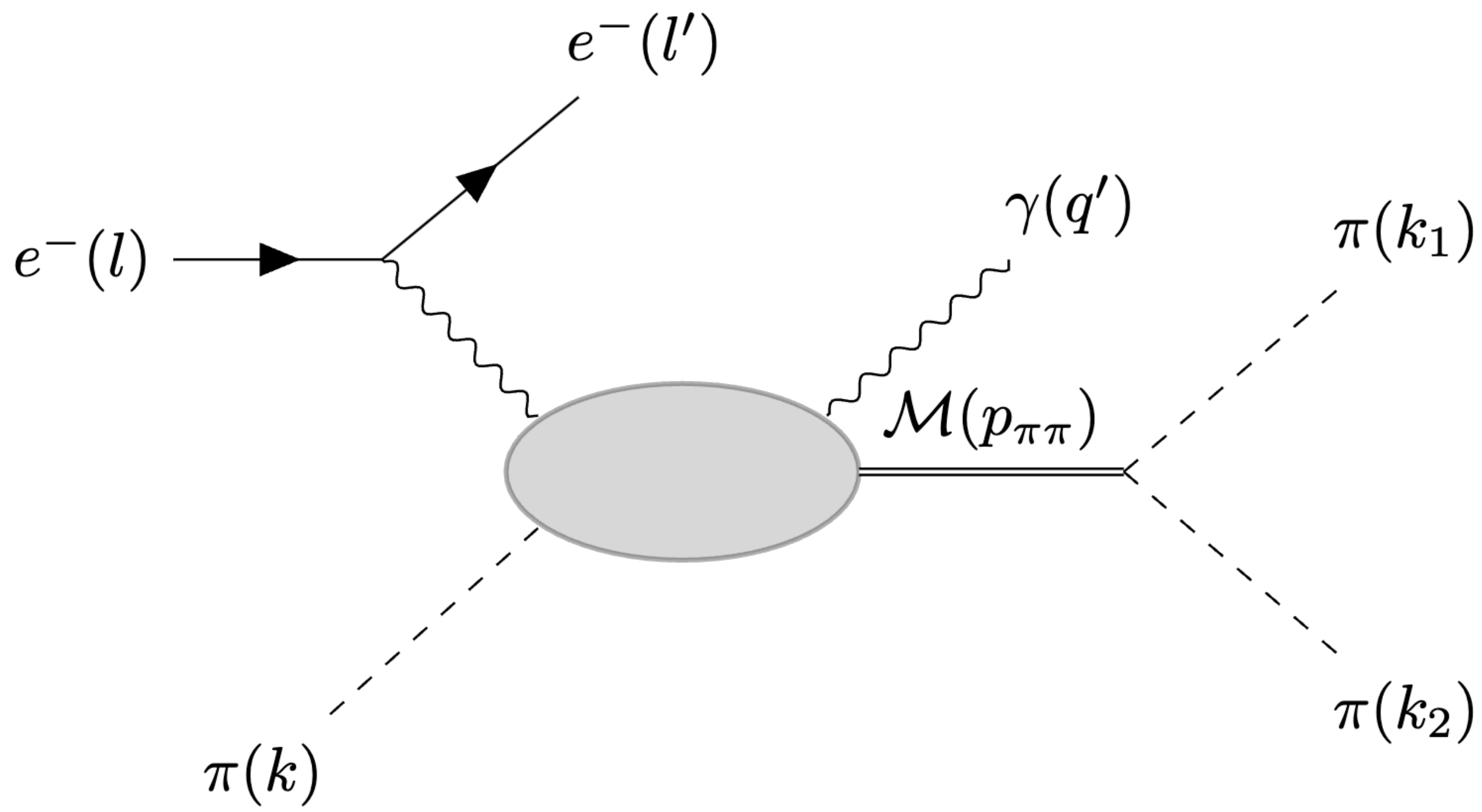
$$\alpha = \frac{1}{2}\left(1-\sqrt{1-\frac{4m_\pi^2}{W_{\pi\pi}^2}}\cos\theta_\pi^*\right)$$

H_V^π : Pion GPD

Vector meson resonance

- Investigate the effect due to intermediate $\rho(770)$ resonance
- Well-described by the Breit-Wigner distribution and $\rho\pi\pi$ effective vertex

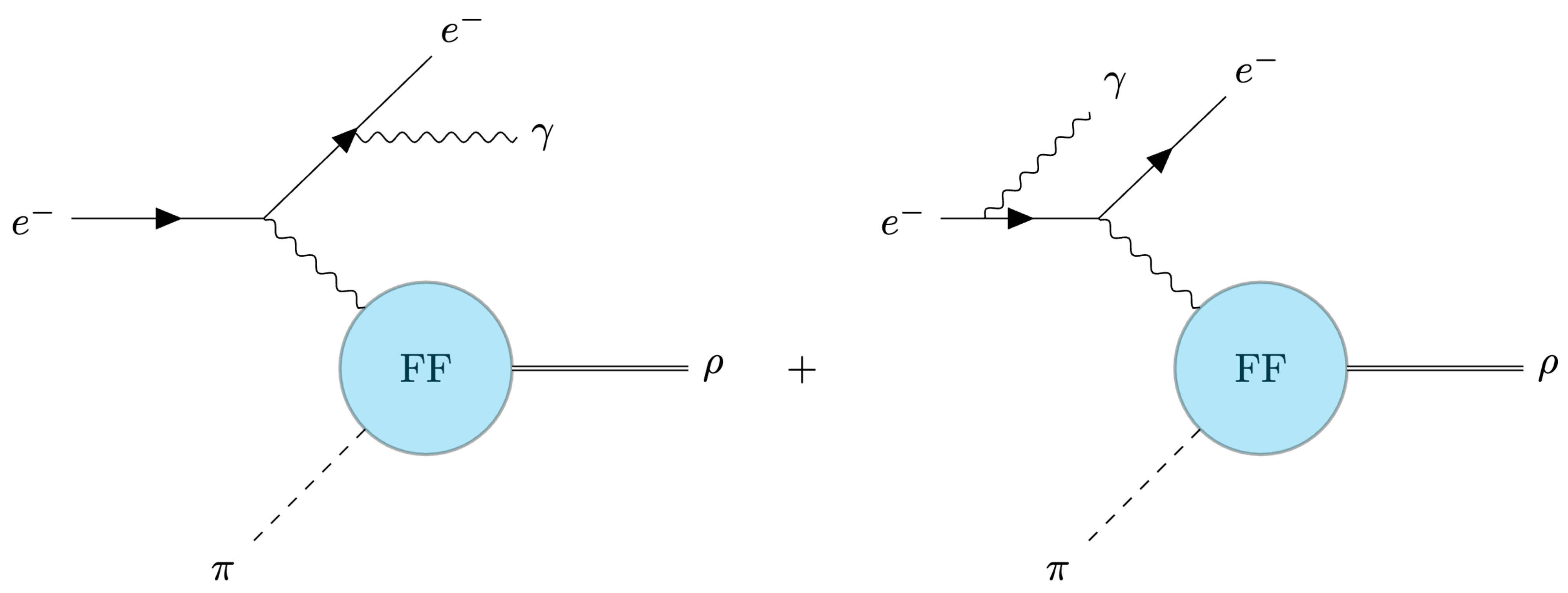




$$\mathcal{M}(e^-\pi \to e^-\gamma\rho \to e^-\gamma\pi\pi) = C_{\rm iso}g_{\rho\pi\pi}(k_1-k_2)_\mu \frac{i\sum_s \mathcal{E}^\mu(p_{\pi\pi},s)\mathcal{E}^{*\nu}(p_{\pi\pi},s)}{W_{\pi\pi}^2 - m_\rho^2 + im_\rho\Gamma_\rho}\mathcal{M}_\nu(e^-\pi \to e^-\gamma\rho)$$

Vector meson resonance

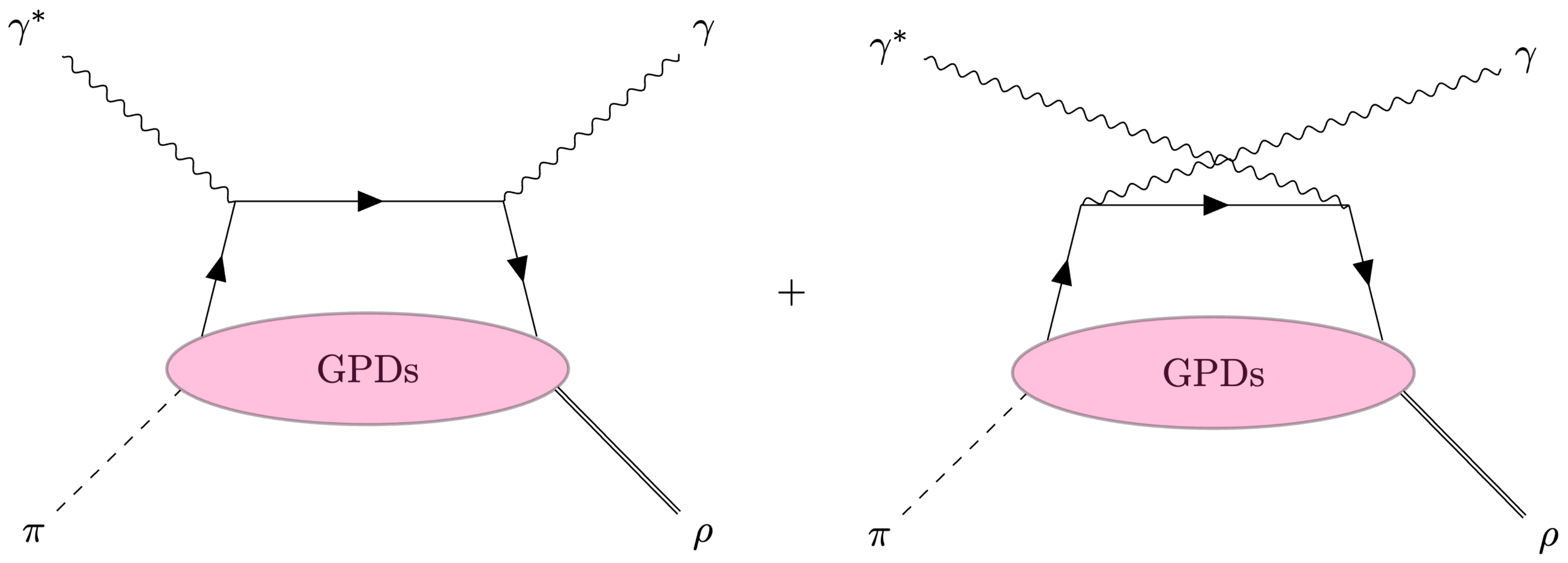
- Bethe-Heitler (BH) process



$$\mathcal{M}_{\rm BH}(e^-\pi \to e^-\gamma\rho) = \frac{e^3}{t}\varepsilon^{*\nu}(q',\lambda_\gamma)\bar{u}(l',\lambda'_e)\left[\gamma_\nu \frac{1}{\not{l}'+\not{q}'}\gamma_\mu + \gamma_\mu \frac{1}{\not{l}-\not{q}'}\gamma_\nu\right] u(l,\lambda_e)$$
$$\times \langle \rho(p_{\pi\pi},s)\,|\,J^\mu_{\rm em}(0)\,|\,\pi(k)\rangle\,,$$

~ $\pi \to \rho$ transition EM form factor

- DVCS process



$$H^{\mu\nu}_{\pi\to\rho} = -\frac{1}{2}g^{\mu\nu}_\perp \int_{-1}^{1} dx C^+(x,\xi)\frac{\varepsilon\left(n,\mathcal{E}^*(p_{\pi\pi},s),\bar{P},\Delta\right)}{m_\rho}H^{\pi\to\rho}(x,\xi,t)$$
$$+\frac{i}{2}\varepsilon^{\mu\nu}_\perp \int_{-1}^{1} dx C^-(x,\xi)\frac{1}{f_\pi}\left[(\mathcal{E}^*\cdot\Delta)\tilde{H}_1^{\pi\to\rho}(x,\xi,t) + m_\rho^2(\mathcal{E}^*\cdot n)\tilde{H}_2^{\pi\to\rho}(x,\xi,t)\right]$$

$\pi \to \rho$ transition GPDs

Comparing DVCS amplitude to the $\pi \to \pi\pi$ hadronic tensor yields $\pi \to \pi\pi$ GPDs near $W_{\pi\pi} = m_\rho$

Vector meson resonance

Unpolarized $\pi \to \pi\pi$ GPD near $W_{\pi\pi} = m_\rho$

$$H^{\pi\to\pi\pi}(x,\xi,t;W_{\pi\pi}^2,\theta_\pi^*,\phi_\pi^*)\Big|_{\rho(770)} = -2C_{\rm iso}\frac{f_\pi^3 g_{\rho\pi\pi}}{m_\rho}\frac{1}{W_{\pi\pi}^2 - m_\rho^2 + i m_\rho \Gamma_\rho} H^{\pi\to\rho}(x,\xi,t),$$

→

- $W_{\pi\pi}$-dist is solely governed by the Breit-Wigner
- Pseudotensor structure cancels $\mathrm{Y}_{1,-1}$

Polarized $\pi \to \pi\pi$ GPD near $W_{\pi\pi} = m_\rho$

$$\tilde{H}^{\pi\to\pi\pi}(x,\xi,\Delta^2;W_{\pi\pi}^2,\theta_\pi^*,\phi_\pi^*)\Big|_{\rho(770)} = C_{\rm iso} g_{\rho\pi\pi}\frac{1}{W_{\pi\pi}^2 - m_\rho^2 + i m_\rho \Gamma_\rho}\sqrt{1-\frac{4m_\pi^2}{W_{\pi\pi}^2}}\sqrt{\frac{4\pi}{3}}$$
$$\times\Big[\Big(R_{1,0}\tilde{H}_1^{\pi\to\rho}(x,\xi,\Delta^2) + m_\rho^2(1-\xi)\tilde{H}_2^{\pi\to\rho}(x,\xi,\Delta^2)\Big)\,\mathrm{Y}_{1,0}(\theta_\pi^*,\phi_\pi^*)$$
$$+ R_{1,1}\tilde{H}_1^{\pi\to\rho}(x,\xi,\Delta^2)\,\mathrm{Y}_{1,1}(\theta_\pi^*,\phi_\pi^*)\Big],$$

→

- Only the angular sturctures of $\mathrm{Y}_{l=1,m}$ remain
- Describe contribution due to the resonance of spin l

Partial-wave expansion of GPDs

Double PW expanded $\pi \to \pi\pi$ GPDs

$$
\begin{aligned}
H^{\pi\to\pi\pi}(x,\xi,t;W_{\pi\pi}^2,\theta_\pi^*,\phi_\pi^*) &= \frac{1}{|\sin\theta_\pi^*|\sin\phi_\pi^*}\sum_{l=1}^{\infty}\sum_{m=-l}^{-1} H^{l,m}(x,\xi,t;W_{\pi\pi}^2)\,\mathrm{Y}_{l,m}(\theta_\pi^*,\phi_\pi^*),\\
\tilde{H}^{\pi\to\pi\pi}(x,\xi,t;W_{\pi\pi}^2,\theta_\pi^*,\phi_\pi^*) &= \sum_{l=0}^{\infty}\sum_{m=0}^{l} \tilde{H}^{l,m}(x,\xi,t;W_{\pi\pi}^2)\,\mathrm{Y}_{l,m}(\theta_\pi^*,\phi_\pi^*).
\end{aligned}
$$

- PW expansion coefficients $H^{l,m}$ describe the intermediate resonance state of spin $J = l$.

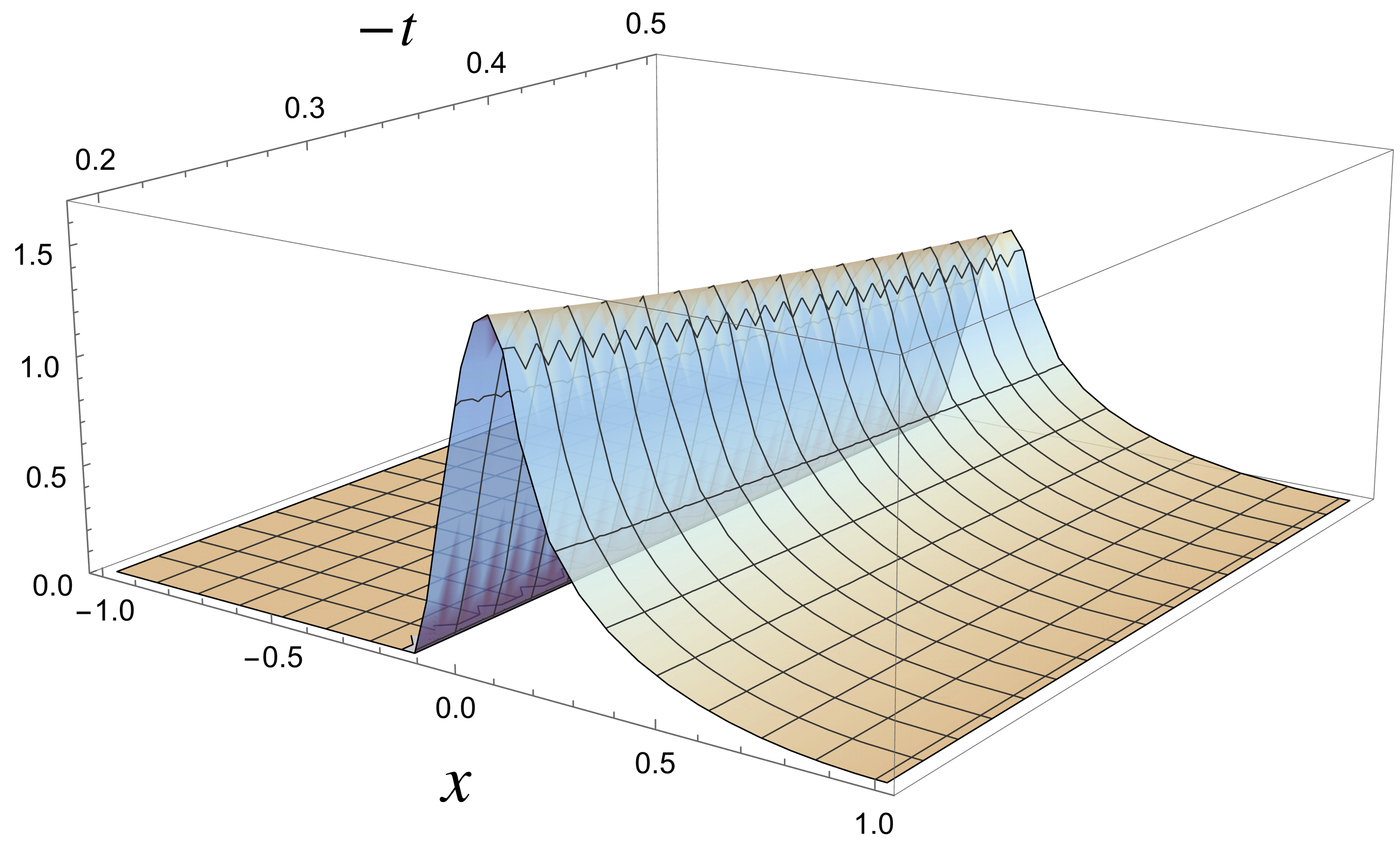
- Selection rules for m implies the unpolarized GPD is odd and polarized GPD is even in ϕ_π^*, which agree with the requirements of parity invariance.

Phenomenological models for GPDs

A. V. Radyushkin, Phys. Lett. B 449, 81 (1999)

(i) Unpolarized quark distribution

$$H^{\pi\to\rho}(x,\xi,t) = H_{DD}(x,\xi)F^{\rho\pi}(t)$$



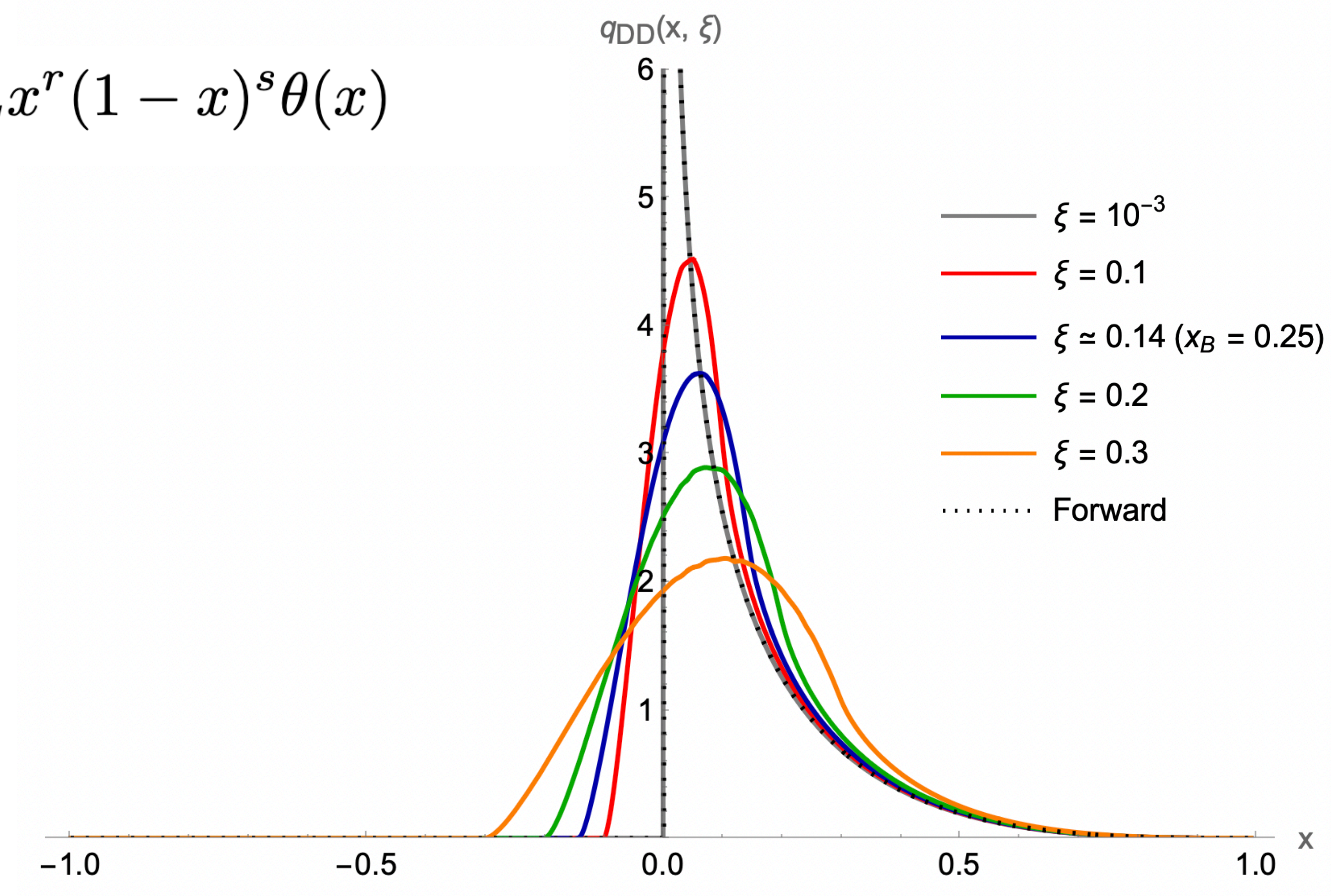
• Radyushkin double distribution

$$H_{DD}(x,\xi) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(x-\beta-\alpha\xi)h(\beta,\alpha)q(x)$$

With the profile function ($b=1$) and the quark distribution ($r=-0.5,\ s=2$)

$$h(\beta,\alpha) = \frac{1}{2^{2b+1}}\frac{\Gamma(2b+2)}{\Gamma^2(b+1)}\frac{[(1-|\beta|)^2-\alpha^2]^b}{(1-|\beta|)^{2b+1}}$$

$$q(x) = N_q x^r (1-x)^s \theta(x)$$



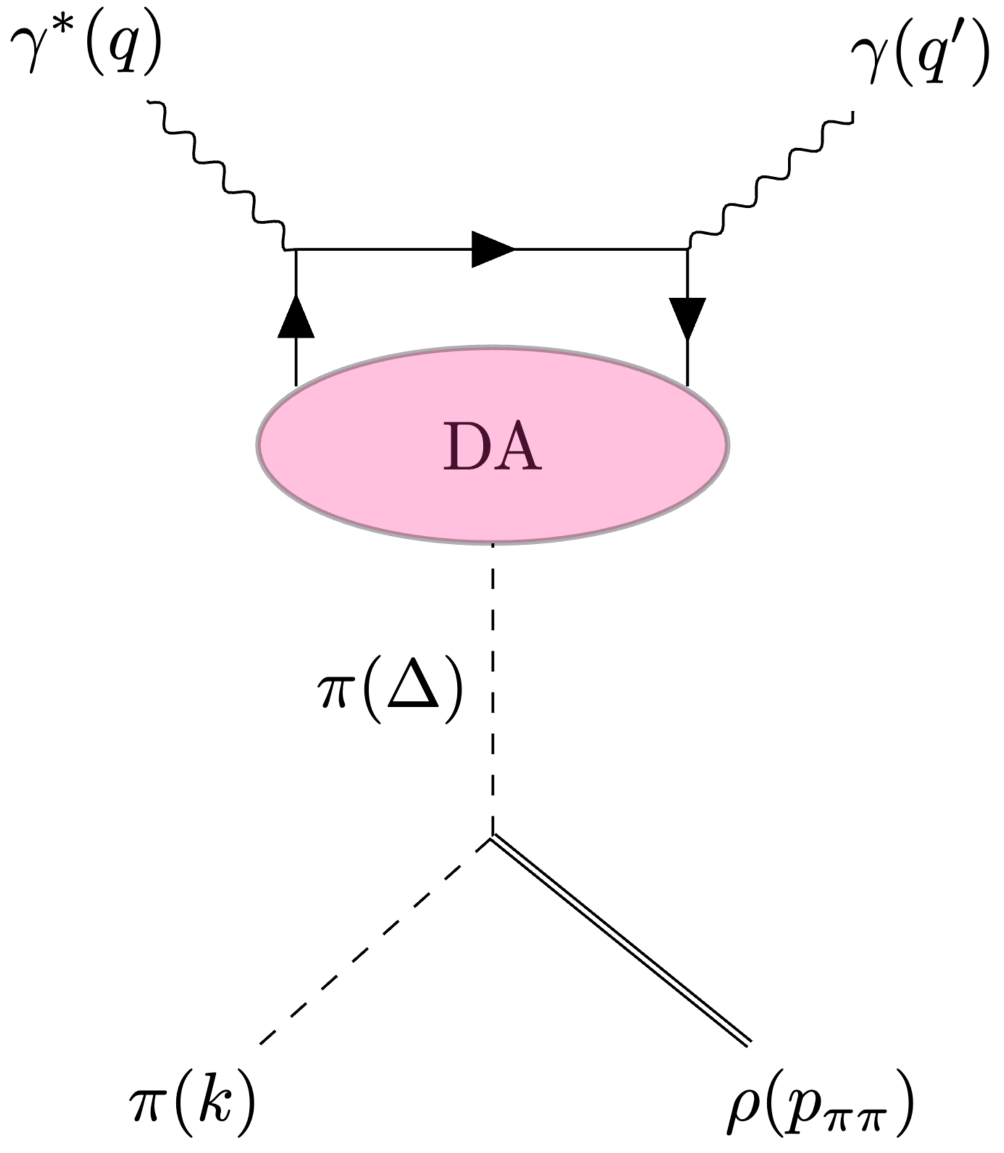
Phenomenological models for GPDs

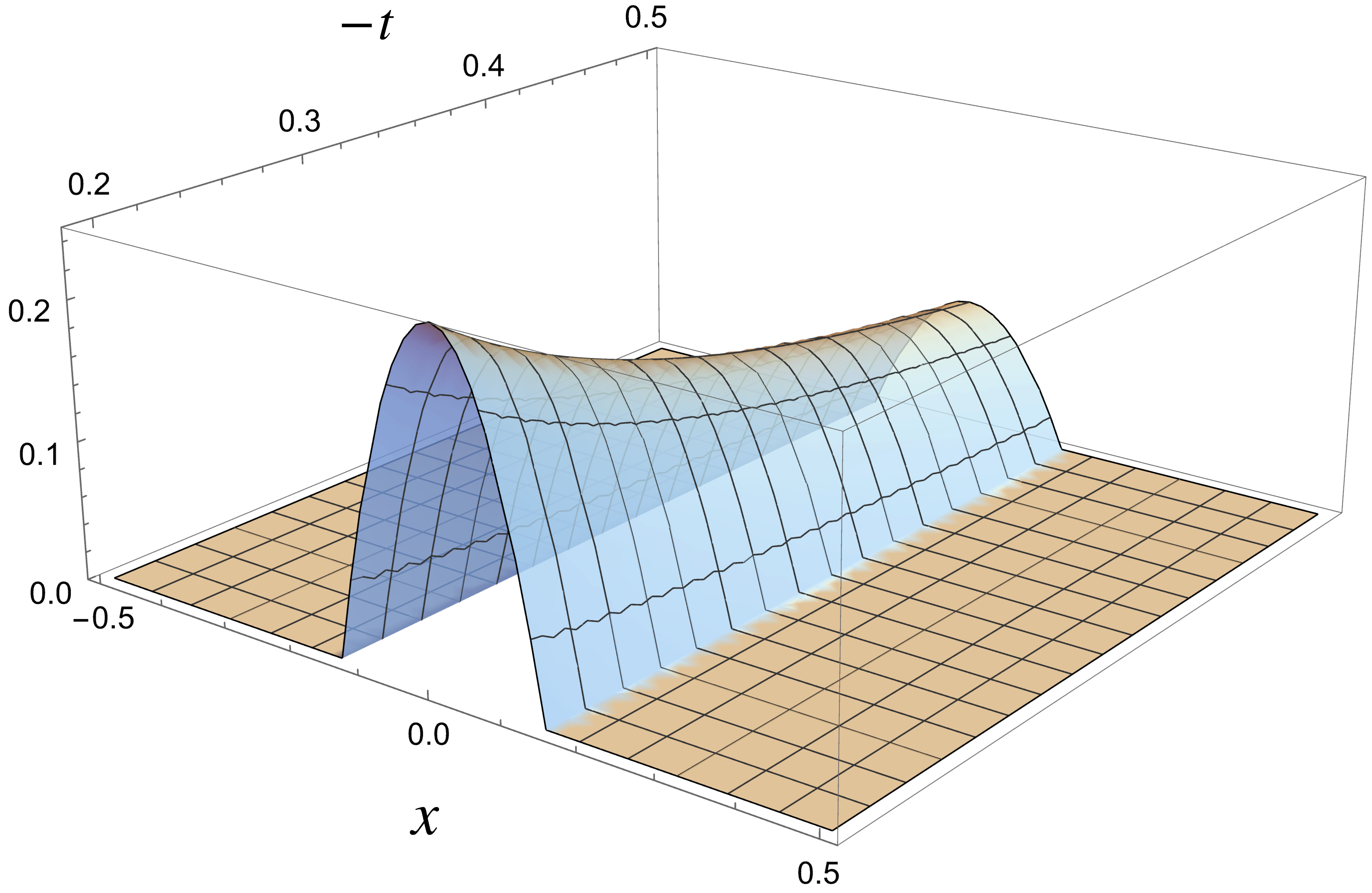
(ii) Pion pole dominance model

$$\tilde{H}_1^{\pi\to\rho}(x,\xi,t) = \phi_\pi\left(\frac{x}{\xi}\right)\theta(\xi - |x|)\frac{1}{3}\frac{4f_\pi^2 g_{\rho\pi\pi}}{m_\pi^2 - t}$$

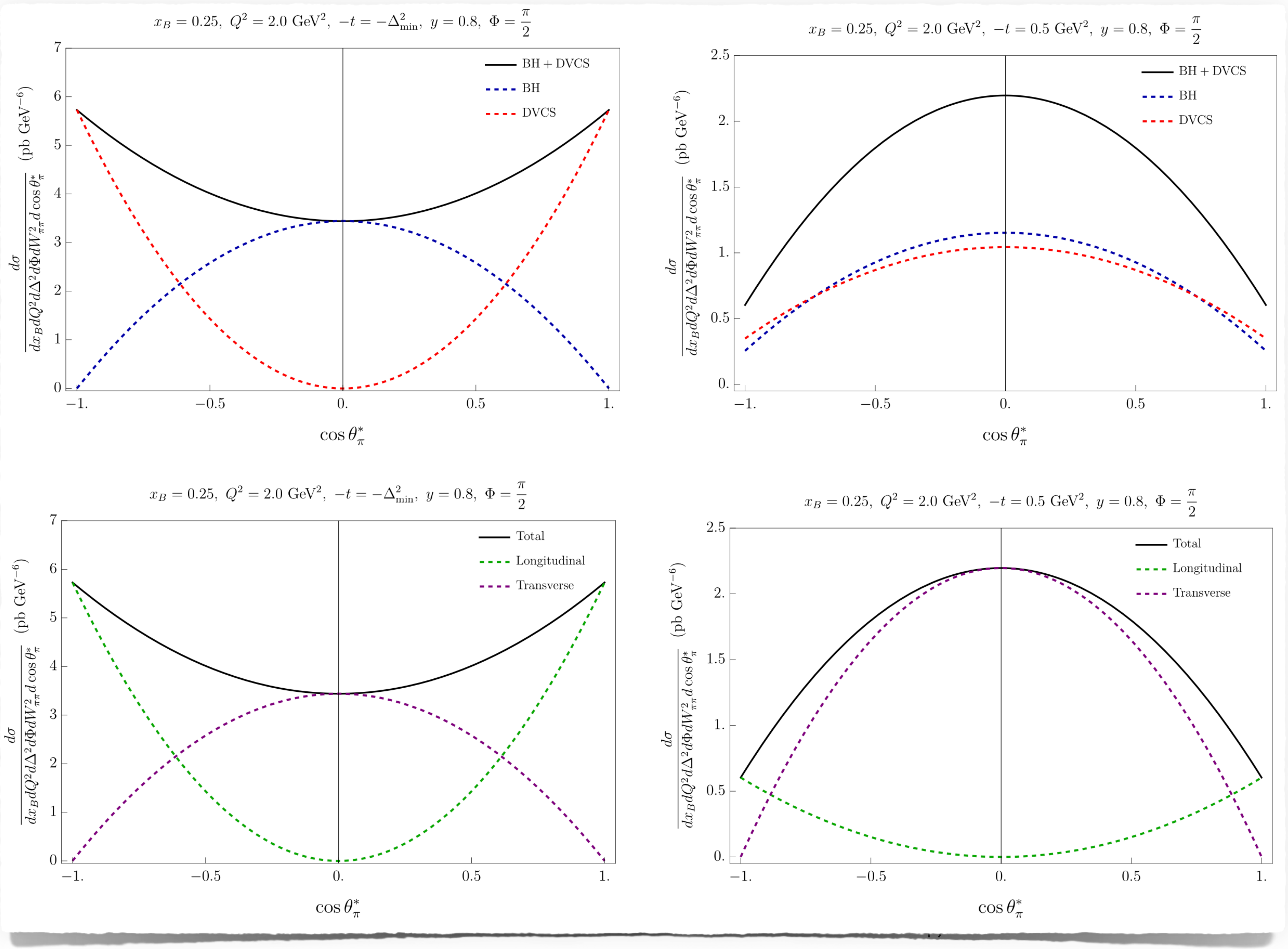
$$\tilde{H}_2^{\pi\to\rho}(x,\xi,t) = 0$$

$$\phi_\pi(u) = \frac{3}{4}(1-u^2)$$





$e\pi \to e\gamma\rho \to e\gamma\pi\pi$ cross section



$$\int_0^{2\pi} d\varphi^*_\pi \left|\mathcal{M}(e^-\pi \to e^-\gamma\rho \to e^-\gamma\pi\pi)\right|^2$$

$$= C^2_{\mathrm{iso}} g^2_{\rho\pi\pi} \frac{W^2_{\pi\pi} - 4m^2_\pi}{(W^2_{\pi\pi} - m^2_\rho)^2 + m^2_\rho \Gamma^2_\rho} \frac{4\pi}{3}$$

$$\times \sum_s \left|\mathcal{M}\left(e^-\pi \to e^-\gamma\rho(W_{\pi\pi}, s)\right)\right|^2$$

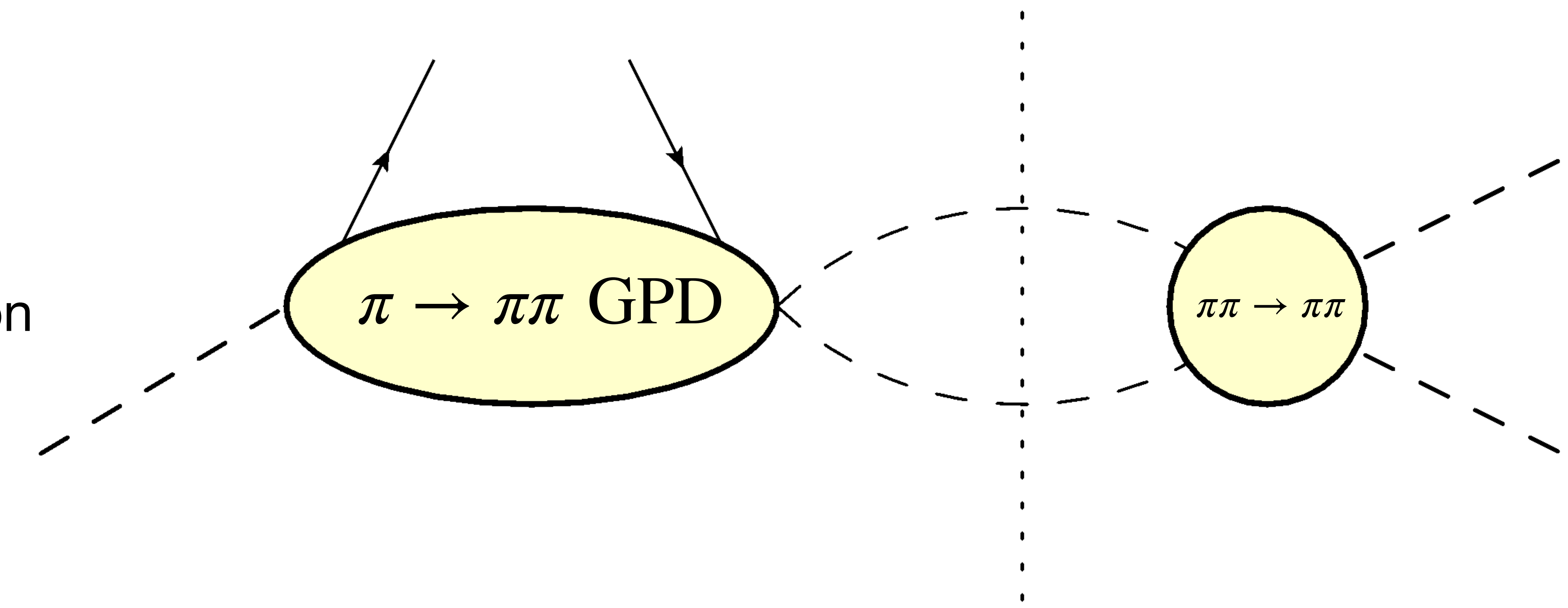
$$\times \left[\frac{3}{2}\cos^2\theta^*_\pi \delta_{s,0} + \frac{3}{4}\sin^2\theta^*_\pi\left(\delta_{s,1} + \delta_{s,-1}\right)\right]$$

Pion decay angular distribution estimated for $-t = -t_{min} \simeq 0.2\ \mathrm{GeV}^2$ and $-t = 0.5\ \mathrm{GeV}^2$

Dispersive analysis on GPD

Watson's final state interaction theorem

K. M. Watson, Phys. Rev. 95, 228 (1954)



✓ Imaginary part of GPD is given by discontinuity along the cut

✓ $\pi\pi$ invariant mass spectra of $\pi \to \pi\pi$ GPDs in resonance region from the dispersive analysis

$$\mathrm{Im}\,\mathcal{F}(x)\langle \pi_b(k_1)\pi_c(k_2)|\hat{O}_S\left(-\frac{\lambda n}{2},\frac{\lambda n}{2}\right)|\pi_a(p_\pi)\rangle$$
$$=\frac{1}{2!}\int d(\text{phase space})\mathcal{F}(x)\langle \pi_{b'}(k_1')\pi_{c'}(k_2')|\hat{O}_S\left(-\frac{\lambda n}{2},\frac{\lambda n}{2}\right)|\pi_a(p_\pi)\rangle^* P_{bc}^{I=1\,b'c'}$$
$$\times A_{\pi\pi}^{I=1}(k_1,k_2|k_1',k_2')$$

$$\mathrm{Im}\,\mathscr{M} \sim \mathrm{disc}\,\mathscr{M}$$

$$\longrightarrow \quad \mathrm{Im}H^{l,m}(x,\xi,t;W_{\pi\pi}^2) = \tan\delta_l^{I=1}(W_{\pi\pi}^2)\mathrm{Re}H^{l,m}(x,\xi,t;W_{\pi\pi}^2)$$

Dispersive analysis on GPD

Omnés representation

R. Omnés, Nuovo Cim. 8, 316 (1958)

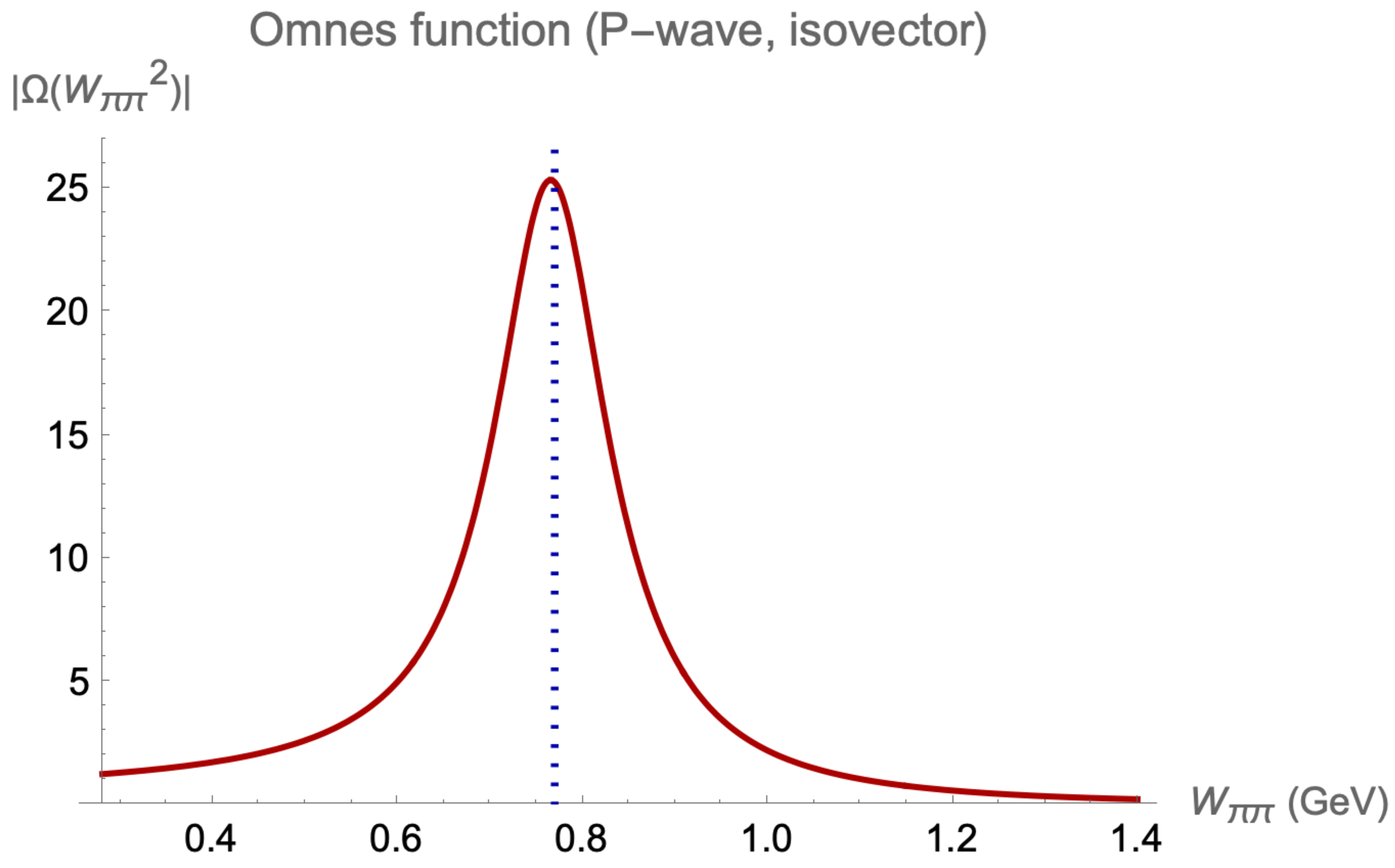
$$H^I_{l,m}(x,\xi,t;W^2_{\pi\pi}) = \sum_{n=0}^{N-1} \frac{w^{2n}}{n!}\frac{d^n}{dw^{2n}} H^I_{l,m}(x,\xi,t;W^2_{\pi\pi}=0) + \frac{w^{2N}}{\pi}\int_{4m_\pi^2}^{\infty} ds \frac{\mathrm{Im}H^I_{l,m}(x,\xi,t;s)}{s^N(s-W^2_{\pi\pi}-i\epsilon)}$$

➢ N-subtracted dispersion relation for the PW expanded $\pi \to \pi\pi$ GPDs

1-subtracted Omnés solution

$$H^I_{l,m}(x,\xi,t;W^2_{\pi\pi}) = H^I_{l,m}(x,\xi,t;W^2_{\pi\pi}=0)\Big|_{th} \Omega^I_l(W^2_{\pi\pi})$$

$$\Omega^I_l(W^2_{\pi\pi}) = \exp\left[\frac{W^2_{\pi\pi}}{\pi}\int_{4m_\pi^2}^{\infty} ds \frac{\delta^I_l(s)}{s(s-W^2_{\pi\pi}-i\epsilon)}\right]$$



Dispersive analysis on GPD

$H^{l,m}$ ansatz model at the threshold

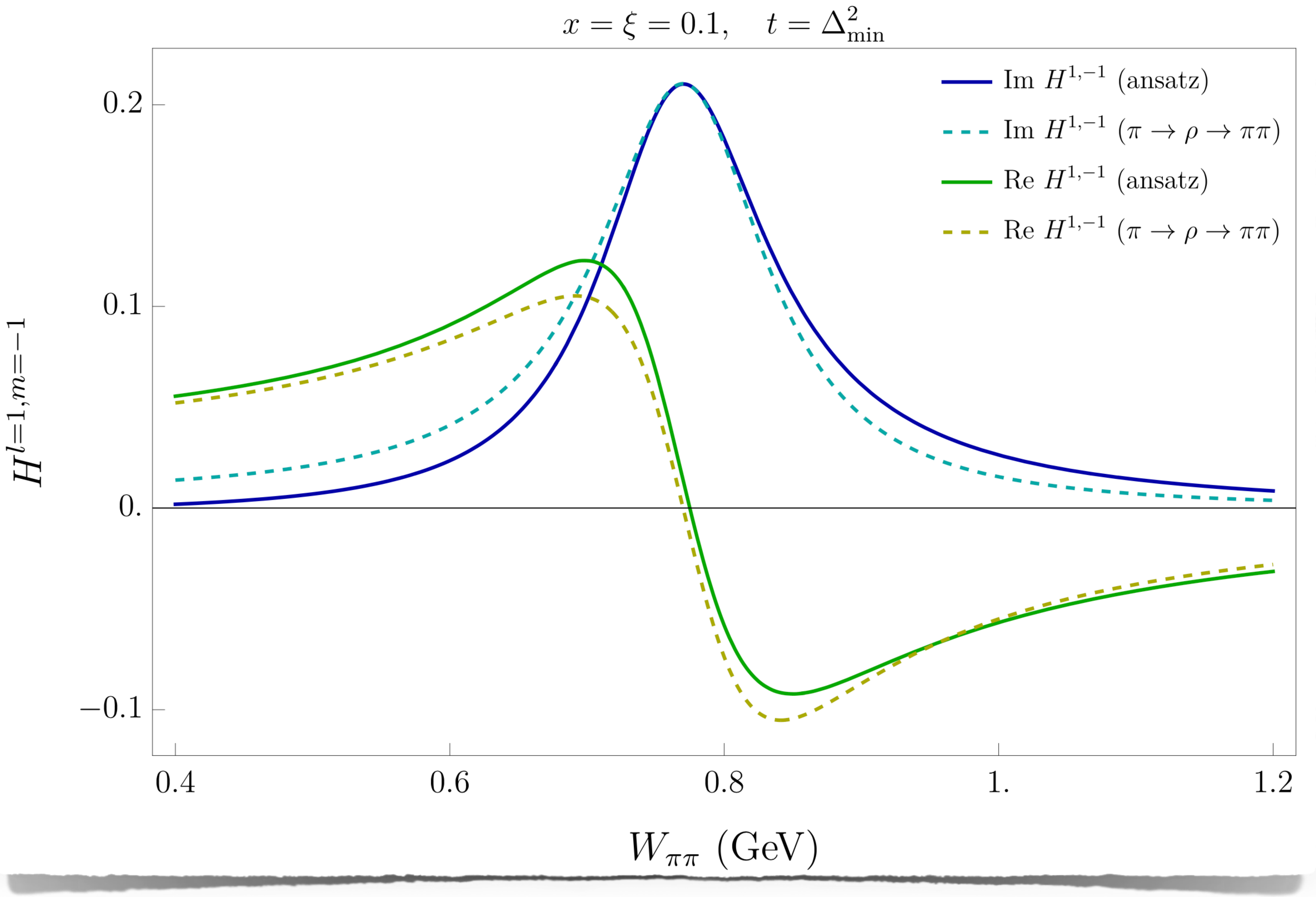
$$H^{l,m}(x, \xi, t; (W_{\pi\pi}^{th})^2) = N_{l,m} H_{DD}(x, \xi) F(t)$$

$$N_{1,-1} \simeq 0.02$$

$$F(\Delta^2) = \frac{1}{1 - \Delta^2/(0.63\ \mathrm{GeV}^2) + \Delta^4/(2.48\ \mathrm{GeV}^4)}$$

Parameters are fitted to $\pi \to \pi\pi$ GPD in the vicinity of $\rho(770)$

Imaginary part yields the $\pi\pi$ invariant mass spectra



$$H^{\pi\to\pi\pi}(x,\xi,t;W_{\pi\pi}^2,\theta_\pi^*,\phi_\pi^*)\Big|_{\rho(770)} = -2C_{\mathrm{iso}}\frac{f_\pi^3 g_{\rho\pi\pi}}{m_\rho}\frac{1}{W_{\pi\pi}^2 - m_\rho^2 + i m_\rho \Gamma_\rho} H^{\pi\to\rho}(x,\xi,t)$$

Froissart-Gribov projection

M. Froissart, Phys. Rev. 123, 1053 (1961)
V. N. Gribov, Nucl. Phys. 22, 249 (1961)

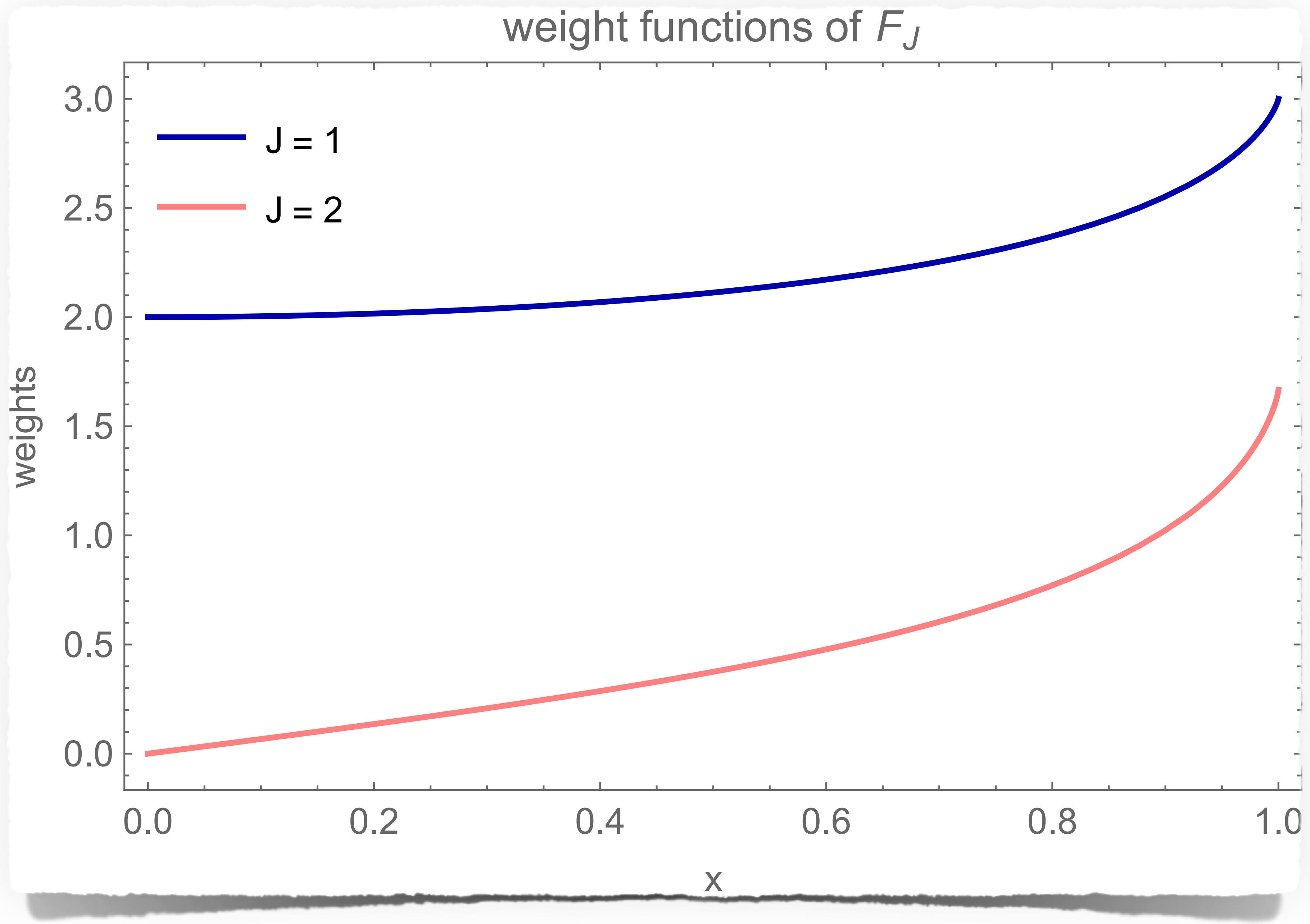
K. M. Semenov-Tian-Shansky and P. Szajder, Phys. Rev. D 109, 054010 (2024)

- Cross channel PW expansion of Compton FF

$$\mathscr{H}^{l,m}(\cos\theta_t, t, W_{\pi\pi}^2) = \sum_J F_J^{l,m}(t, W_{\pi\pi}^2) P_J(\cos\theta_t)$$

$$\cos\theta_t \simeq -\frac{1}{\xi} + \mathcal{O}(1/Q^2)$$

Ignoring mixing of the cross channel PWs



- FG projections for the (un)polarized $\pi \to \pi\pi$ Compton FFs

$$F_J^{\ell m}(t, W_{\pi\pi}^2) = 2\int_0^1 dx H^{\ell,m}(x, x, t, W_{\pi\pi}^2) \frac{2J+1}{J(J+1)} \frac{(-1)}{x} \sqrt{\frac{1}{x^2} - 1}\, Q_J^1(1/x)$$

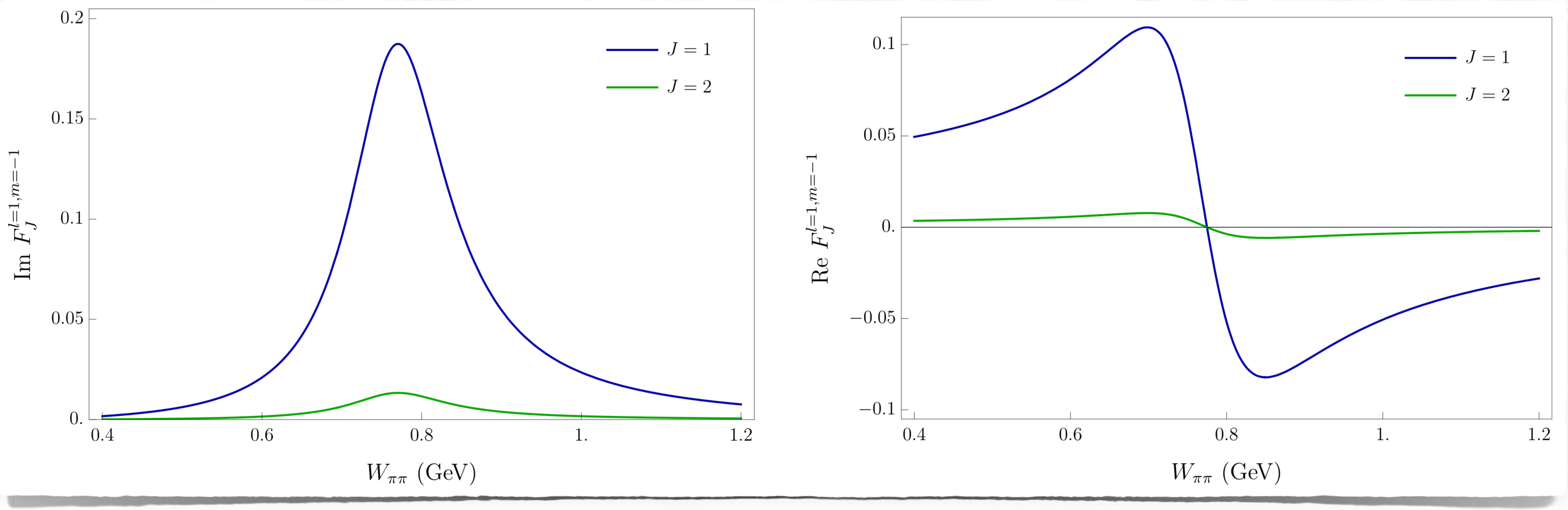
$$\tilde{F}_J^{\ell m}(t, W_{\pi\pi}^2) = 2\int_0^1 dx \tilde{H}^{\ell,m}(x, x, t, W_{\pi\pi}^2)(2J+1) \frac{Q_J(1/x)}{x^2}$$

Neumann integral formula for Legendre function of the 2nd kind

$$Q_J(z) = \frac{1}{2}\int_{-1}^{1} dz' \frac{P_J(z')}{z' - z}$$

- FG projection with $J \geq 1, -l \leq m < 0$ of the unpolarized $\pi \to \pi\pi$ Compton FF

$$F_J^{\ell m}(t, W_{\pi\pi}^2) = 2\int_0^1 dx H^{\ell,m}(x,x,t,W_{\pi\pi}^2)\frac{2J+1}{J(J+1)}\frac{(-1)}{x}\sqrt{\frac{1}{x^2}-1}\,Q_J^1(1/x)$$



Imaginary part of FG FF yields the $\pi\pi$ invariant mass spectra

Summary & Outlook

- We studied the angular structure of the $\pi \to \pi\pi$ GPDs and $e\pi \to e\gamma\pi\pi$ cross section in meson resonance region.

- We worked out the dispersive analysis on GPDs with the help of the double partial wave expansion and constructed simple ansatz for PW expanded GPDs, $H^{l,m}$

- Along with $H^{l,m}(x, x, t; W_{\pi\pi}^2)$ the Froissart-Gribov projection FFs can be evaluated, which encode hadronic transition from pion to spin-l state induced by the cross channel probe of a particular spin-J from the non-local QCD string operator.

- The FG projections come from the cross channel PW expansion of the Compton FFs which, in principle, can be extracted from the data

Thank you for your attention!