Mixture of Hadronic Molecules and Quarkonium Core

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- Composite particles of quarks and gluons.
- Strong interaction is important.
- QCD cannot be solved by perturbation theory except in the high energy region.
- Effective theory is important.

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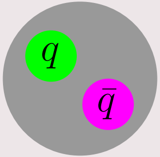
- Normally $q\bar{q}$
- π, K, D, and so on.



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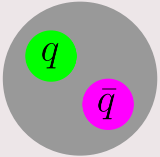
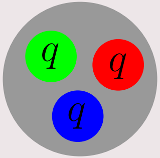


Baryon

- Fermion
- Normally qqq
- Proton, Neutron, and so on.



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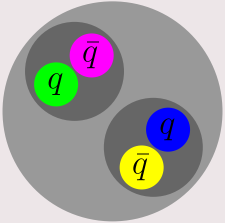
Hadronic molecular state



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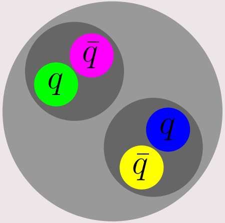
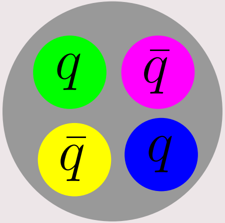


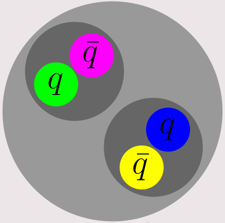
Compact state



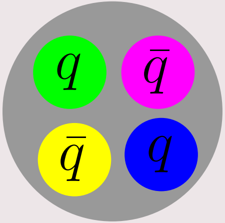
Exotic Hadron

- Difficult to explain of $q\bar{q}$ or qqq.
- More complex structure than normal hadrons.
- Various states are considered, such as hadronic molecular states and compact states.
- In recent years, new exotic hadrons have been reported almost every year.

Hadronic molecular state



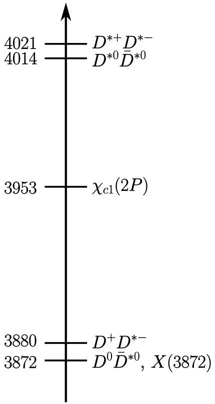
Compact state



What is $X(3872)$?

- One of the best known exotic hadrons.
- Reported by the Belle experiment in 2003, and later reported in various experiments.

  (CDF(2004), D0(2004), BaBar(2005), LHCb(2012), CMS(2013), BESIII(2014), ATLAS,(2017))
- $J^{PC} = 1^{++}$, $c\bar{c}q\bar{q}(q = u, d)$?



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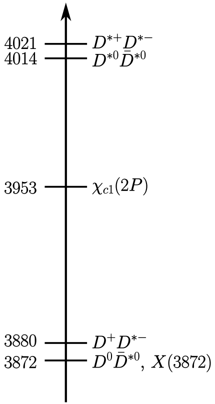

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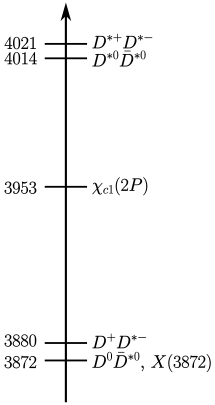
- $J^{PC} = 1^{++}$, $c\bar{c}q\bar{q}(q = u, d)$?
- Hadronic molecular state? Compact state?
- Very close to the threshold of $D^0\bar{D}^{*0}$.

  $m_{X(3872)} - m_{D^0\bar{D}^{*0}} = -0.04$ MeV

  (Particle Data Group, PTEP **2022**(2022)083C01)

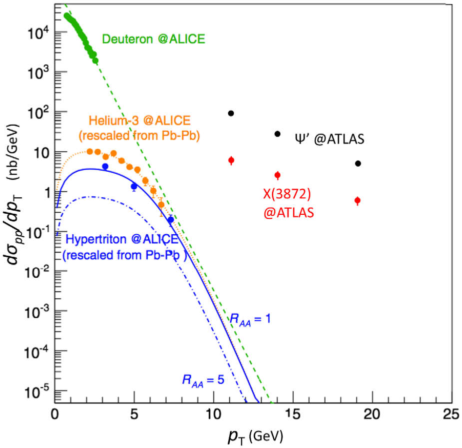


Is $X(3872)$ a hadronic molecule?

- Also close to 3953 MeV, the quark model mass prediction of $\chi_{c1}(2P)$.

  (S. Godfrey and N. Isgur, Phys. Rev. D **32**(1985)189)

- $\chi_{c1}(2P)$ is $c\bar{c}$ meson, and it is also $J^{PC} = 1^{++}$.
- Some experimental data suggest a structure different from hadronic molecules.
- **Can we explain $X(3872)$ by hadronic molecules + $\chi_{c1}(2P)$?**

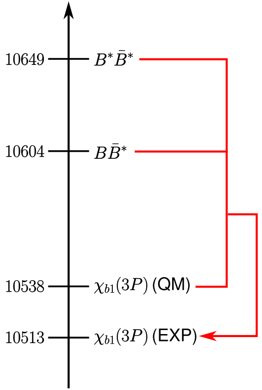


Esposito et al, Phys. Rev. D **92**(2015)034028,

Olsen et al, Rev. Mod. Phys. **90**(2018)015003

What is X_b?

- Hidden-bottom tetraquark, the botmonium counterpart of $X(3872)$.
- $b\bar{b}q\bar{q}$
- Undiscovered?



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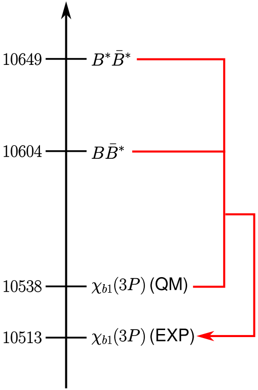
(S. Godfrey and K. Moats, Phys. Rev. D **92**(2015)054034)



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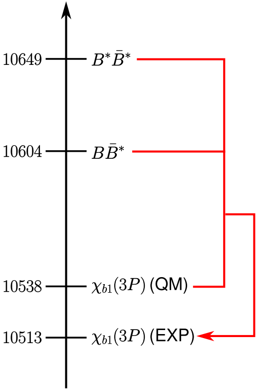
  (S. Godfrey and K. Moats, Phys. Rev. D **92**(2015)054034)
- The experimental value of the mass of $\chi_{b1}(3P)$ is $10\,513$ MeV.

  (Particle Data Group, PTEP **2022**(2022)083C01)
- Is it possible to regard $\chi_{b1}(3P)$ reported in the experiment as X_b?



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Core-molecular hybrid model

- Mixture state of quarkonium core ($\chi_{c1}(2P)$, $\chi_{b1}(3P)$) and hadronic molecules ($D^{(*)}\bar{D}^{(*)}$, $B^{(*)}\bar{B}^{(*)}$).

$$\mathcal{H}\Psi = E\Psi \tag{1}$$

$$\Psi = \begin{pmatrix} c_1\left|[D^0\bar{D}^{*0}](S)\right\rangle \\ c_2\left|[D^+D^{*-}](S)\right\rangle \\ c_3\left|[D^0\bar{D}^{*0}](D)\right\rangle \\ c_4\left|[D^+D^{*-}](D)\right\rangle \\ c_5\left|D^{*0}\bar{D}^{*0}(D)\right\rangle \\ c_6\left|D^{*+}D^{*-}(D)\right\rangle \\ c_7\left|\chi_{c1}(2P)\right\rangle \end{pmatrix} \tag{2}$$

$$\mathcal{H} = \begin{pmatrix} H_0 + V_{\mathrm{OPEP}} & \mathcal{U}^\dagger \\ \mathcal{U} & m_{\chi_{c1}(2P)} - (m_{D^0} + m_{D^{*0}}) \end{pmatrix} \tag{3}$$

Interaction Lagrangian

$P^{(*)}$

$$\mathcal{L}_{HHM}^{(Q)} = g\,\mathrm{Tr}\left[H_b^{(Q)}\gamma^\mu\gamma^5 A_{ba\mu}\bar{H}_a^{(Q)}\right] \tag{4}$$

$$H_a^{(Q)} = \frac{1+\not{v}}{2}(\gamma_\mu P_a^{*\mu} - \gamma^5 P_a) \tag{5}$$

$$\bar{H}_a^{(Q)} = (\gamma_\mu P_a^{*\mu\dagger} + \gamma^5 P_a^\dagger)\frac{1+\not{v}}{2} \tag{6}$$

$$\langle 0|P|P\rangle = \sqrt{m_P} \tag{7}$$

$$\langle 0|P^{*\mu}|P^*\rangle = \sqrt{m_{P^*}}\epsilon^\mu \tag{8}$$

$\bar{P}^{(*)}$

$$\mathcal{L}_{HHM}^{(\bar{Q})} = g\,\mathrm{Tr}\left[\bar{H}_a^{(\bar{Q})}\gamma^\mu\gamma^5 A_{ab\mu}H_b^{(\bar{Q})}\right] \tag{9}$$

$$H_a^{(\bar{Q})} = \left(\bar{P}_{a\mu}^*\gamma^\mu - \bar{P}_a\gamma^5\right)\frac{1-\not{v}}{2} \tag{10}$$

$$\bar{H}_a^{(\bar{Q})} = \frac{1-\not{v}}{2}\left(\bar{P}_{a\mu}^{*\dagger}\gamma^\mu + \bar{P}_a^\dagger\gamma^5\right) \tag{11}$$

$$\langle 0|\bar{P}|\bar{P}\rangle = \sqrt{m_{\bar{P}}} \tag{12}$$

$$\langle 0|\bar{P}^{*\mu}|\bar{P}^*\rangle = \sqrt{m_{\bar{P}^*}}\epsilon^\mu \tag{13}$$

$$A_\mu = \frac{i}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger) \simeq -\frac{\partial_\mu(\boldsymbol{\pi}\cdot\boldsymbol{\tau})}{2f_\pi} \tag{14}$$

OPEP

- Consider One-Pion-Exchange-Potential (OPEP) as an interaction of hadronic molecules ($D^{(*)}\bar{D}^{(*)}$, $B^{(*)}\bar{B}^{(*)}$).

$$V_{\rm OPEP} = \frac{1}{3}\left(\frac{g}{2f_\pi}\right)^2 \begin{pmatrix} C & 2C & -\sqrt{2}T & -2\sqrt{2}T & -\sqrt{6}T & -2\sqrt{6}T \\ 2C & C & -2\sqrt{2}T & -\sqrt{2}T & -2\sqrt{6}T & -\sqrt{6}T \\ -\sqrt{2}T & -2\sqrt{2}T & C+T & 2C+2T & -\sqrt{3}T & -2\sqrt{3}T \\ -2\sqrt{2}T & -\sqrt{2}T & 2C+2T & C+T & -2\sqrt{3}T & -\sqrt{3}T \\ -\sqrt{6}T & -2\sqrt{6}T & -\sqrt{3}T & -2\sqrt{3}T & C-T & 2C-2T \\ -2\sqrt{6}T & -\sqrt{6}T & -2\sqrt{3}T & -\sqrt{3}T & 2C-2T & C-T \end{pmatrix} \tag{15}$$

$$C(r) = \frac{m_\pi^2}{4\pi}\left(\frac{e^{-m_\pi r}}{r} - \frac{e^{-\Lambda r}}{r} - \frac{\Lambda^2 - m_\pi^2}{2\Lambda}e^{-\Lambda r}\right) \tag{16}$$

$$T(r) = \left(3 + 3m_\pi r + (m_\pi r)^2\right)\frac{e^{-m_\pi r}}{4\pi r^3} - \left(3 + 3\Lambda r + (\Lambda r)^2\right)\frac{e^{-\Lambda r}}{4\pi r^3} + \frac{m_\pi^2 - \Lambda^2}{2}(1+\Lambda r)\frac{e^{-\Lambda r}}{4\pi r} \tag{17}$$

Core-molecule mixing potential

- Quarkonium core ($\chi_{c1}(2P)$, $\chi_{b1}(3P)$) and S-waves of hadronic molecules ($D^{(*)}\bar{D}^{(*)}(S)$, $B^{(*)}\bar{B}^{(*)}(S)$) couple in core-molecule mixing potential.

  (M. Takizawa and S. Takeuchi, PTEP **2013**(2013)093D01)

$$\mathcal{H} = \begin{pmatrix} H_0 + V_{\mathrm{OPEP}} & \mathcal{U}^\dagger \\ \mathcal{U} & m_{\chi_{c1}(2P)} - (m_{D^0} + m_{D^{*0}}) \end{pmatrix} \tag{18}$$

$$\mathcal{U} = \begin{pmatrix} U & U & 0 & 0 & 0 & 0 \end{pmatrix} \tag{19}$$

$$\begin{aligned} \left\langle \chi_{c1}(2P) \middle| U \middle| [D^0\bar{D}^{*0}](S) \right\rangle &= \int d^3x \, \langle \chi_{c1}(2P)|U|\boldsymbol{x}\rangle \left\langle \boldsymbol{x} \middle| [D^0\bar{D}^{*0}](S) \right\rangle \\ &= \int d^3x \, g_{c\bar{c}} \sqrt{2}\pi \Lambda_q^{\frac{3}{2}} \frac{e^{-\Lambda_q r}}{r} Y_l^m(\theta,\phi) \left\langle \boldsymbol{x} \middle| [D^0\bar{D}^{*0}](S) \right\rangle \end{aligned} \tag{20}$$

Contents

$X(3872)$ in the core-molecular hybrid model

- Think of it as a mixture $\chi_{c1}(2P)$, $D^0\bar{D}^{*0}(S)$, $D^+D^{*-}(S)$, $D^0\bar{D}^{*0}(D)$, $D^+D^{*-}(D)$, $D^{*0}\bar{D}^{*0}(D)$, and $D^{*+}D^{*-}(D)$.

	$g_{c\bar{c}}$	Λ_q (MeV)	g	Λ (MeV)	BE (MeV) (input)
$D^{(*)}\bar{D}^{(*)}$			0.55	1834	0.04
$D^{(*)}\bar{D}^{(*)}\&\chi_{c1}(2P)$	0.04935	300	0.55	1130	0.04
$D^{(*)}\bar{D}^{(*)}\&\chi_{c1}(2P)$	0.04686	500	0.55	1130	0.04
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- When hadronic molecules only, the OPEP cutoff Λ is determined to reproduce the binding energy (BE).
- When the quarkonium core is included, we estimate $\Lambda = 1130$ MeV from hadron size. (S. Yasui and K. Sudoh, Phys. Rev. D **80**(2009)034008)
- Determine the $g_{c\bar{c}}$ that reproduces BE at $\Lambda_q = 300$ MeV, 500 MeV, and 1000 MeV.

	$g_{c\bar{c}}$	Λ_q (MeV)	g	Λ (MeV)	BE (MeV) (input)
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Mixing ratio of $X(3872)$

	Λ_q (MeV)	$\chi_{c1}(2P)$	$D^0\bar{D}^{*0}(S)$	$D^+D^{*-}(S)$	$D^0\bar{D}^{*0}(D)$	$D^+D^{*-}(D)$	$D^{*0}\bar{D}^{*0}(D)$	$D^{*+}D^{*-}(D)$
$D^{(*)}\bar{D}^{(*)}$			**95.1 %**	3.8 %	0.2 %	0.2 %	0.3 %	0.3 %
$D^{(*)}\bar{D}^{(*)}\&\chi_{c1}(2P)$	300	2.2 %	**95.0 %**	2.8 %	0.1 %	0.1 %	0.0 %	0.1 %
$D^{(*)}\bar{D}^{(*)}\&\chi_{c1}(2P)$	500	3.5 %	**92.8 %**	3.4 %	0.1 %	0.1 %	0.1 %	0.1 %
$D^{(*)}\bar{D}^{(*)}\&\chi_{c1}(2P)$	1000	6.4 %	**89.0 %**	4.1 %	0.1 %	0.1 %	0.1 %	0.1 %

- The mixing ratios of $D^0\bar{D}^{*0}(S)$ are about $90\,\%$ and is a principal component.

Mixing ratio of $X(3872)$

	Λ_q (MeV)	$\chi_{c1}(2P)$	$D^0\bar{D}^{*0}(S)$	$D^+D^{*-}(S)$	$D^0\bar{D}^{*0}(D)$	$D^+D^{*-}(D)$	$D^{*0}\bar{D}^{*0}(D)$	$D^{*+}D^{*-}(D)$
$D^{(*)}\bar{D}^{(*)}$			95.1 %	3.8 %	0.2 %	0.2 %	0.3 %	0.3 %
$D^{(*)}\bar{D}^{(*)}\&\chi_{c1}(2P)$	300	**2.2 %**	95.0 %	2.8 %	0.1 %	0.1 %	0.0 %	0.1 %
$D^{(*)}\bar{D}^{(*)}\&\chi_{c1}(2P)$	500	**3.5 %**	92.8 %	3.4 %	0.1 %	0.1 %	0.1 %	0.1 %
$D^{(*)}\bar{D}^{(*)}\&\chi_{c1}(2P)$	1000	**6.4 %**	89.0 %	4.1 %	0.1 %	0.1 %	0.1 %	0.1 %

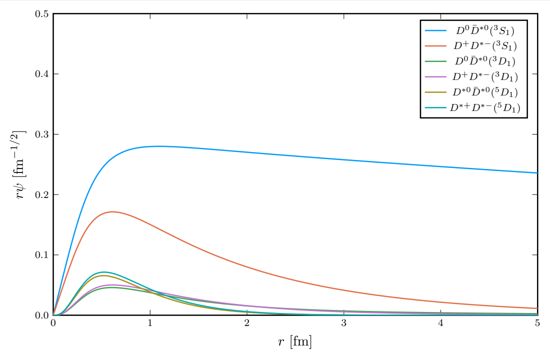
- The mixing ratios of $D^0\bar{D}^{*0}(S)$ are about $90\,\%$ and is a principal component.
- The mixing ratios of the quarkonium core are a few $\%$, small but not negligible.
- Because it has large attraction, and when it is included, $X(3872)$ is bound even with reasonable Λ.

Expectation value of potential energy of $X(3872)$ at $\Lambda_q = 500$ MeV.

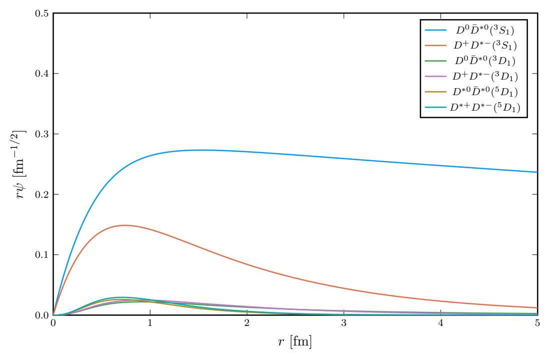
$D^0\bar{D}^{*0}(S)$	$D^+D^{*-}(S)$	$D^0\bar{D}^{*0}(D)$	$D^+D^{*-}(D)$	$D^{*0}\bar{D}^{*0}(D)$	$D^{*+}D^{*-}(D)$	$\chi_{c1}(2P)$	
0.053	0.056	-0.051	-0.115	-0.104	-0.236	-1.737	$D^0\bar{D}^{*0}(S)$
0.056	0.016	-0.061	-0.034	-0.129	-0.073	-1.120	$D^+D^{*-}(S)$
-0.051	-0.061	0.003	0.007	-0.006	-0.013	0	$D^0\bar{D}^{*0}(D)$
-0.115	-0.034	0.007	0.004	-0.013	-0.007	0	$D^+D^{*-}(D)$
-0.104	-0.129	-0.006	-0.013	-0.004	-0.008	0	$D^{*0}\bar{D}^{*0}(D)$
-0.236	-0.073	-0.013	-0.007	-0.008	-0.005	0	$D^{*+}D^{*-}(D)$
-1.737	-1.120	0	0	0	0	0	$\chi_{c1}(2P)$

Wave functions of $X(3872)$

$D^{(*)}\bar{D}^{(*)}$



$D^{(*)}\bar{D}^{(*)}\&\chi_{c1}(2P)$ $\Lambda_q = 500$ MeV



- When the quarkonium core is included, the $D^*\bar{D}^*$ component is reduced.
- Isospin symmetry is broken.

Contents

X_b in the core-molecular hybrid model

- Estimate OPEP cutoff $\Lambda = 1080$ MeV from hadron size.

  (S. Yasui and K. Sudoh, Phys. Rev. D **80**(2009)034008)
- When hadronic molecules only, we find out if X_b is bound.

	$g_{b\bar{b}}$	Λ_q (MeV)	g	Λ (MeV)
$B^{(*)}\bar{B}^{(*)}$			0.55	1080
$B^{(*)}\bar{B}^{(*)}\&\chi_{b1}(3P)$	0.04935	300	0.55	1080
$B^{(*)}\bar{B}^{(*)}\&\chi_{b1}(3P)$	0.04686	500	0.55	1080
$B^{(*)}\bar{B}^{(*)}\&\chi_{b1}(3P)$	0.04571	1000	0.55	1080

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- When the quarkonium core is included, for the calculation we use the mass of the quark model of $\chi_{b1}(3P)$.
- Compare the resulting mass of X_b with the experimental mass of $\chi_{b1}(3P)$.

	$g_{b\bar{b}}$	Λ_q (MeV)	g	Λ (MeV)
$B^{(*)}\bar{B}^{(*)}$			0.55	1080
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$B^{(*)}\bar{B}^{(*)}\&\chi_{b1}(3P)$	0.04686	500	0.55	1080
$B^{(*)}\bar{B}^{(*)}\&\chi_{b1}(3P)$	0.04571	1000	0.55	1080

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- When hadronic molecules only, we find out if X_b is bound.
- When the quarkonium core is included, for the calculation we use the mass of the quark model of $\chi_{b1}(3P)$.
- Compare the resulting mass of X_b with the experimental mass of $\chi_{b1}(3P)$.
- Use the same values as $X(3872)$ for $g_{b\bar{b}}$ and Λ_q.

	$g_{b\bar{b}}$	Λ_q (MeV)	g	Λ (MeV)
$B^{(*)}\bar{B}^{(*)}$			0.55	1080
$B^{(*)}\bar{B}^{(*)}\&\chi_{b1}(3P)$	0.04935	300	0.55	1080
$B^{(*)}\bar{B}^{(*)}\&\chi_{b1}(3P)$	0.04686	500	0.55	1080
$B^{(*)}\bar{B}^{(*)}\&\chi_{b1}(3P)$	0.04571	1000	0.55	1080

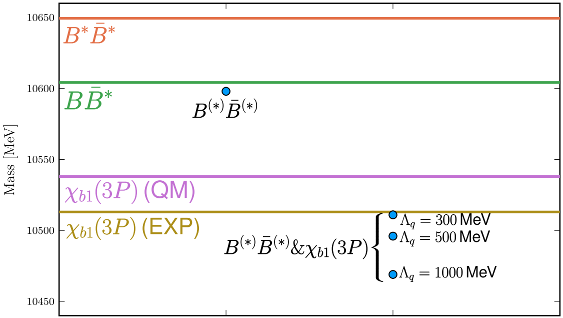
X_b in the core-molecular hybrid model

- Estimate OPEP cutoff $\Lambda = 1080$ MeV from hadron size.

  (S. Yasui and K. Sudoh, Phys. Rev. D **80**(2009)034008)
- When hadronic molecules only, we find out if X_b is bound.
- When the quarkonium core is included, for the calculation we use the mass of the quark model of $\chi_{b1}(3P)$.
- Compare the resulting mass of X_b with the experimental mass of $\chi_{b1}(3P)$.
- Use the same values as $X(3872)$ for $g_{b\bar{b}}$ and Λ_q.

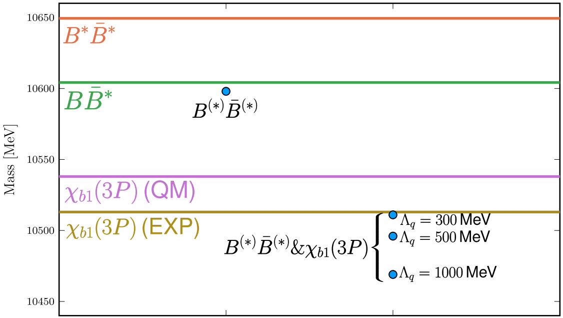
	$g_{b\bar{b}}$	Λ_q (MeV)	g	Λ (MeV)	m_{X_b} (MeV) (**output**)
$B^{(*)}\bar{B}^{(*)}$			0.55	1080	**10598**
$B^{(*)}\bar{B}^{(*)}\&\chi_{b1}(3P)$	0.04935	300	0.55	1080	**10511**
$B^{(*)}\bar{B}^{(*)}\&\chi_{b1}(3P)$	0.04686	500	0.55	1080	**10496**
$B^{(*)}\bar{B}^{(*)}\&\chi_{b1}(3P)$	0.04571	1000	0.55	1080	**10469**

Mass of X_b



- When hadronic molecules only, the mass of X_b is 6 MeV below the $B\bar{B}^*$ threshold.

Mass of X_b



- When hadronic molecules only, the mass of X_b is 6 MeV below the $B\bar{B}^*$ threshold.
- When the quarkonium core is included, then $10\,470$–$10\,510$ MeV.
- The experimentally reported $\chi_{b1}(3P)$ may be X_b, but the Λ_q dependence is too large to tell from this analysis only.

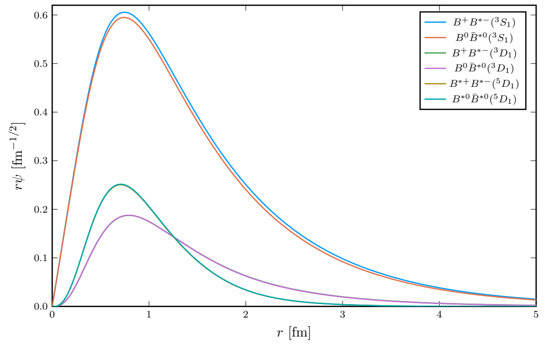
Mixing ratio of X_b

	Λ_q (MeV)	$\chi_{b1}(3P)$	$B^+B^{*-}(S)$	$B^0\bar{B}^{*0}(S)$	$B^+B^{*-}(D)$	$B^0\bar{B}^{*0}(D)$	$B^{*+}B^{*-}(D)$	$B^{*0}\bar{B}^{*0}(D)$
$B^{(*)}\bar{B}^{(*)}$			42.9 %	40.8 %	3.4 %	3.4 %	4.7 %	4.7 %
$B^{(*)}\bar{B}^{(*)}\&\chi_{b1}(3P)$	300	78.4 %	9.9 %	9.9 %	0.3 %	0.3 %	0.6 %	0.6 %
$B^{(*)}\bar{B}^{(*)}\&\chi_{b1}(3P)$	500	76.3 %	11.0 %	10.9 %	0.3 %	0.3 %	0.6 %	0.6 %
$B^{(*)}\bar{B}^{(*)}\&\chi_{b1}(3P)$	1000	77.6 %	10.6 %	10.6 %	0.2 %	0.2 %	0.4 %	0.4 %

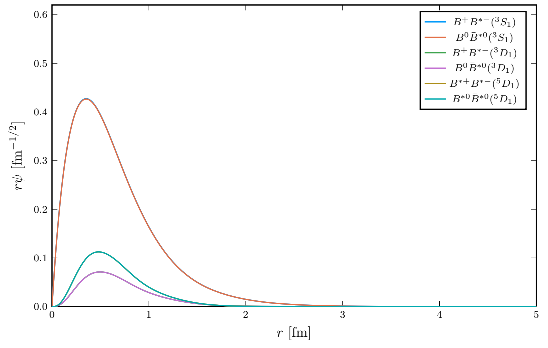
- When hadronic molecules only, the mass of X_b is 6 MeV below the $B\bar{B}^*$ threshold.
- When the quarkonium core is included, then $10\,470$–$10\,510$ MeV.
- The experimentally reported $\chi_{b1}(3P)$ may be X_b, but the Λ_q dependence is too large to tell from this analysis only.

Wave functions of X_b

$B^{(*)}\bar{B}^{(*)}$



$B^{(*)}\bar{B}^{(*)}\&\chi_{b1}(3P)$ $\Lambda_q = 500$ MeV



- The inclusion of the quarkonium core changes the shape of the wavefunction significantly because the quarkonium core is the principal component.
- Isospin symmetry is not broken.

Summary

- $X(3872)$
  - When the quarkonium core is included, it is bound with a reasonable OPEP cutoff Λ.
  - The mixing ratio of $D^0\bar{D}^{*0}(S)$ is about $90\,\%$ and is a principal component.
  - The mixing ratio of the $\chi_{c1}(2P)$ is a few $\%$, but it is not negligible, because it has large attraction.

Summary

- $X(3872)$
  - When the quarkonium core is included, it is bound with a reasonable OPEP cutoff Λ.
  - The mixing ratio of $D^0\bar{D}^{*0}(S)$ is about $90\,\%$ and is a principal component.
  - The mixing ratio of the $\chi_{c1}(2P)$ is a few $\%$, but it is not negligible, because it has large attraction.
- X_b
  - The structure is very different depending on whether it is coupled to the quarkonium core or not.
  - When hadronic molecules only, it is bound and its mass is $10\,598$ MeV.
  - When the quarkonium core is included, the mass of $10\,470$–$10\,510$ MeV, which is close to the experimental value $10\,513$ MeV for the mass of $\chi_{b1}(3P)$.
  - This analysis only does not tell us whether the experimentally reported $\chi_{b1}(3P)$ can be regarded as X_b, because the Λ_q dependence is too large.

Outlook

- Use more realistic core-molecule mixing potentials such as $^3P_0$ pair creation model.
- Consider meson exchanges such as ρ and ω other than π.
- Consider the resonance state.
- Applying the core-molecule mixed model to exotic hadrons other than $X(3872)$ and X_b.

Back Up

Interaction Lagrangian

$$\mathcal{L}_{\pi P P^*} = -\frac{g}{f_\pi}\left(P_b^{*\mu} P_a^\dagger + P_b P_a^{*\mu\dagger}\right)\partial_\mu(\boldsymbol{\pi}\cdot\boldsymbol{\tau})_{ba} \tag{21}$$

$$\mathcal{L}_{\pi P^* P^*} = i\frac{g}{f_\pi}\epsilon^{\mu\nu\rho\sigma} v_\mu P_{b\nu}^* P_{a\rho}^{*\dagger}\partial_\sigma(\boldsymbol{\pi}\cdot\boldsymbol{\tau})_{ba} \tag{22}$$

$$\mathcal{L}_{\pi \bar{P} \bar{P}^*} = \frac{g}{f_\pi}\left(\bar{P}_a^{*\dagger\mu} \bar{P}_b + \bar{P}_a^\dagger \bar{P}_b^{*\mu}\right)\partial_\mu(\boldsymbol{\pi}\cdot\boldsymbol{\tau})_{ab} \tag{23}$$

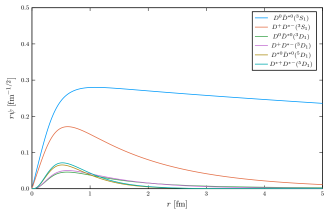
$$\mathcal{L}_{\pi \bar{P}^* \bar{P}^*} = -i\frac{g}{f_\pi}\epsilon^{\mu\nu\rho\sigma} v_\mu \bar{P}_{a\nu}^{*\dagger} \bar{P}_{b\rho}^*\partial_\sigma(\boldsymbol{\pi}\cdot\boldsymbol{\tau})_{ab} \tag{24}$$

Mass

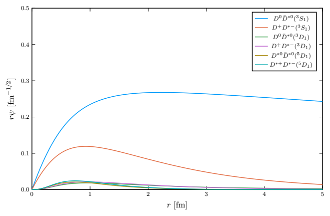
	$X(3872)$	D^0	D^+	D^{*0}	D^{*+}	$\chi_{c1}(2P)$
Mass [MeV]	3871.65	1864.84	1869.66	2006.85	2010.26	3953

	B^+	B^0	B^{*+}	B^{*0}	$\chi_{b1}(3P)$ (QM)
Mass [MeV]	5279.34	5279.66	5324.71	5324.71	10538

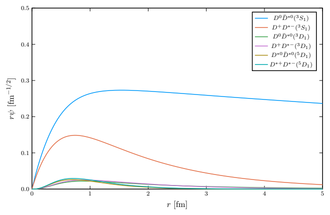
$D^{(*)}\bar{D}^{(*)}$



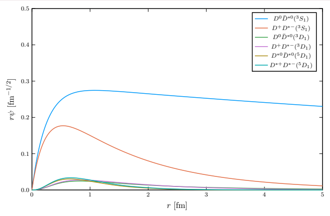
$D^{(*)}\bar{D}^{(*)}\&\chi_{c1}(2P)$ $\Lambda_q = 300$ MeV



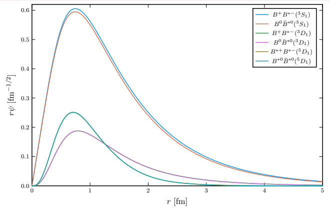
$D^{(*)}\bar{D}^{(*)}\&\chi_{c1}(2P)$ $\Lambda_q = 500$ MeV



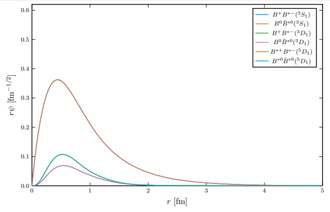
$D^{(*)}\bar{D}^{(*)}\&\chi_{c1}(2P)$ $\Lambda_q = 1000$ MeV



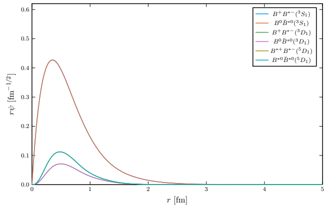
$D^{(*)}\bar{D}^{(*)}$



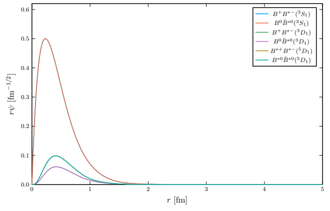
$D^{(*)}\bar{D}^{(*)}\&\chi_{c1}(2P)$ $\Lambda_q = 300$ MeV



$D^{(*)}\bar{D}^{(*)}\&\chi_{c1}(2P)$ $\Lambda_q = 500$ MeV



$D^{(*)}\bar{D}^{(*)}\&\chi_{c1}(2P)$ $\Lambda_q = 1000$ MeV



Expectation value of kinetic energy when considering only hadronic molecules of $X(3872)$

$D^0\bar{D}^{*0}(S)$	$D^+D^{*-}(S)$	$D^0\bar{D}^{*0}(D)$	$D^+D^{*-}(D)$	$D^{*0}\bar{D}^{*0}(D)$	$D^{*+}D^{*-}(D)$	
2.709						$D^0\bar{D}^{*0}(S)$
	1.797					$D^+D^{*-}(S)$
		0.753				$D^0\bar{D}^{*0}(D)$
			0.911			$D^+D^{*-}(D)$
				1.953		$D^{*0}\bar{D}^{*0}(D)$
					2.343	$D^{*+}D^{*-}(D)$

Expectation value of potential energy when considering only hadronic molecules of $X(3872)$

$D^0\bar{D}^{*0}(S)$	$D^+D^{*-}(S)$	$D^0\bar{D}^{*0}(D)$	$D^+D^{*-}(D)$	$D^{*0}\bar{D}^{*0}(D)$	$D^{*+}D^{*-}(D)$	
0.071	0.080	-0.264	-0.575	-0.647	-1.411	$D^0\bar{D}^{*0}(S)$
0.080	0.025	-0.352	-0.192	-0.880	-0.479	$D^+D^{*-}(S)$
-0.264	-0.352	0.031	0.069	-0.074	-0.162	$D^0\bar{D}^{*0}(D)$
-0.575	-0.192	0.069	0.037	-0.162	-0.088	$D^+D^{*-}(D)$
-0.647	-0.880	-0.074	-0.162	-0.060	-0.131	$D^{*0}\bar{D}^{*0}(D)$
-1.411	-0.479	-0.162	-0.088	-0.131	-0.071	$D^{*+}D^{*-}(D)$

Expectation value of kinetic energy of $X(3872)$ at $\Lambda_q = 300$ MeV

	$D^0\bar{D}^{*0}(S)$	$D^+D^{*-}(S)$	$D^0\bar{D}^{*0}(D)$	$D^+D^{*-}(D)$	$D^{*0}\bar{D}^{*0}(D)$	$D^{*+}D^{*-}(D)$	$\chi_{c1}(2P)$
$D^0\bar{D}^{*0}(S)$	1.410						
$D^+D^{*-}(S)$		0.750					
$D^0\bar{D}^{*0}(D)$			0.079				
$D^+D^{*-}(D)$				0.109			
$D^{*0}\bar{D}^{*0}(D)$					0.166		
$D^{*+}D^{*-}(D)$						0.226	
$\chi_{c1}(2P)$							1.800

Expectation value of potential energy of $X(3872)$ at $\Lambda_q = 300$ MeV

$D^0\bar{D}^{*0}(S)$	$D^+D^{*-}(S)$	$D^0\bar{D}^{*0}(D)$	$D^+D^{*-}(D)$	$D^{*0}\bar{D}^{*0}(D)$	$D^{*+}D^{*-}(D)$	$\chi_{c1}(2P)$	
0.041	0.038	-0.035	-0.081	-0.069	-0.160	-1.183	$D^0\bar{D}^{*0}(S)$
0.038	0.010	-0.038	-0.022	-0.077	-0.045	-0.617	$D^+D^{*-}(S)$
-0.035	-0.038	0.002	0.005	-0.004	-0.008	0	$D^0\bar{D}^{*0}(D)$
-0.081	-0.022	0.005	0.003	-0.008	-0.005	0	$D^+D^{*-}(D)$
-0.069	-0.077	-0.004	-0.008	-0.002	-0.005	0	$D^{*0}\bar{D}^{*0}(D)$
-0.160	-0.045	-0.008	-0.005	-0.005	-0.003	0	$D^{*+}D^{*-}(D)$
-1.183	-0.617	0	0	0	0	0	$\chi_{c1}(2P)$

Expectation value of kinetic energy of $X(3872)$ at $\Lambda_q = 500\,$MeV

$D^0\bar{D}^{*0}(S)$	$D^+D^{*-}(S)$	$D^0\bar{D}^{*0}(D)$	$D^+D^{*-}(D)$	$D^{*0}\bar{D}^{*0}(D)$	$D^{*+}D^{*-}(D)$	$\chi_{c1}(2P)$	
2.095							$D^0\bar{D}^{*0}(S)$
	1.342						$D^+D^{*-}(S)$
		0.121					$D^0\bar{D}^{*0}(D)$
			0.158				$D^+D^{*-}(D)$
				0.263			$D^{*0}\bar{D}^{*0}(D)$
					0.341		$D^{*+}D^{*-}(D)$
						2.855	$\chi_{c1}(2P)$

Expectation value of kinetic energy of $X(3872)$ at $\Lambda_q = 1000$ MeV

	$D^0\bar{D}^{*0}(S)$	$D^+D^{*-}(S)$	$D^0\bar{D}^{*0}(D)$	$D^+D^{*-}(D)$	$D^{*0}\bar{D}^{*0}(D)$	$D^{*+}D^{*-}(D)$	$\chi_{c1}(2P)$
$D^0\bar{D}^{*0}(S)$	3.439						
$D^+D^{*-}(S)$		2.616					
$D^0\bar{D}^{*0}(D)$			0.166				
$D^+D^{*-}(D)$				0.209			
$D^{*0}\bar{D}^{*0}(D)$					0.375		
$D^{*+}D^{*-}(D)$						0.468	
$\chi_{c1}(2P)$							5.231

Expectation value of potential energy of $X(3872)$ at $\Lambda_q = 1000$ MeV

$D^0\bar{D}^{*0}(S)$	$D^+D^{*-}(S)$	$D^0\bar{D}^{*0}(D)$	$D^+D^{*-}(D)$	$D^{*0}\bar{D}^{*0}(D)$	$D^{*+}D^{*-}(D)$	$\chi_{c1}(2P)$	
0.067	0.078	-0.068	-0.149	-0.143	-0.318	-2.942	$D^0\bar{D}^{*0}(S)$
0.078	0.025	-0.086	-0.048	-0.191	-0.106	-2.291	$D^+D^{*-}(S)$
-0.068	-0.086	0.004	0.009	-0.008	-0.017	0	$D^0\bar{D}^{*0}(D)$
-0.149	-0.048	0.009	0.005	-0.017	-0.010	0	$D^+D^{*-}(D)$
-0.143	-0.191	-0.008	-0.017	-0.005	-0.012	0	$D^{*0}\bar{D}^{*0}(D)$
-0.318	-0.106	-0.017	-0.010	-0.012	-0.006	0	$D^{*+}D^{*-}(D)$
-2.942	-2.291	0	0	0	0	0	$\chi_{c1}(2P)$

Expectation value of kinetic energy when considering only hadronic molecules of X_b

	$B^+B^{*-}(S)$	$B^0\bar{B}^{*0}(S)$	$B^+B^{*-}(D)$	$B^0\bar{B}^{*0}(D)$	$B^{*+}B^{*-}(D)$	$B^{*0}\bar{B}^{*0}(D)$
$B^+B^{*-}(S)$	5.569					
$B^0\bar{B}^{*0}(S)$		5.540				
$B^+B^{*-}(D)$			3.098			
$B^0\bar{B}^{*0}(D)$				3.118		
$B^{*+}B^{*-}(D)$					7.994	
$B^{*0}\bar{B}^{*0}(D)$						8.062

Expectation value of potential energy when considering only hadronic molecules of X_b

$B^+B^{*-}(S)$	$B^0\bar{B}^{*0}(S)$	$B^+B^{*-}(D)$	$B^0\bar{B}^{*0}(D)$	$B^{*+}B^{*-}(D)$	$B^{*0}\bar{B}^{*0}(D)$	
0.223	0.437	-0.904	-1.810	-2.093	-4.204	$B^+B^{*-}(S)$
0.437	0.215	-1.775	-0.889	-4.112	-2.065	$B^0\bar{B}^{*0}(S)$
-0.904	-1.775	0.195	0.390	-0.408	-0.820	$B^+B^{*-}(D)$
-1.810	-0.889	0.390	0.195	-0.817	-0.410	$B^0\bar{B}^{*0}(D)$
-2.093	-4.112	-0.408	-0.817	-0.289	-0.580	$B^{*+}B^{*-}(D)$
-4.204	-2.065	-0.820	-0.410	-0.580	-0.291	$B^{*0}\bar{B}^{*0}(D)$

Expectation value of kinetic energy of X_b at $\Lambda_q = 300$ MeV

$B^+B^{*-}(S)$	$B^0\bar{B}^{*0}(S)$	$B^+B^{*-}(D)$	$B^0\bar{B}^{*0}(D)$	$B^{*+}B^{*-}(D)$	$B^{*0}\bar{B}^{*0}(D)$	$\chi_{b1}(3P)$	
3.882							$B^+B^{*-}(S)$
	3.898						$B^0\bar{B}^{*0}(S)$
		0.597					$B^+B^{*-}(D)$
			0.597				$B^0\bar{B}^{*0}(D)$
				1.762			$B^{*+}B^{*-}(D)$
					1.764		$B^{*0}\bar{B}^{*0}(D)$
						-51.781	$\chi_{b1}(3P)$

Expectation value of potential energy of X_b at $\Lambda_q = 300$ MeV

$B^+B^{*-}(S)$	$B^0\bar{B}^{*0}(S)$	$B^+B^{*-}(D)$	$B^0\bar{B}^{*0}(D)$	$B^{*+}B^{*-}(D)$	$B^{*0}\bar{B}^{*0}(D)$	$\chi_{b1}(3P)$	
0.089	0.178	-0.239	-0.478	-0.644	-1.289	-10.782	$B^+B^{*-}(S)$
0.178	0.089	-0.477	-0.238	-1.286	-0.643	-10.756	$B^0\bar{B}^{*0}(S)$
-0.239	-0.477	0.031	0.061	-0.076	-0.153	0	$B^+B^{*-}(D)$
-0.478	-0.238	0.061	0.031	-0.153	-0.076	0	$B^0\bar{B}^{*0}(D)$
-0.644	-1.286	-0.076	-0.153	-0.063	-0.126	0	$B^{*+}B^{*-}(D)$
-1.289	-0.643	-0.153	-0.076	-0.126	-0.063	0	$B^{*0}\bar{B}^{*0}(D)$
-10.782	-10.756	0	0	0	0	0	$\chi_{b1}(3P)$

Expectation value of kinetic energy of X_b at $\Lambda_q = 500\,\text{MeV}$

	$B^+B^{*-}(S)$	$B^0\bar{B}^{*0}(S)$	$B^+B^{*-}(D)$	$B^0\bar{B}^{*0}(D)$	$B^{*+}B^{*-}(D)$	$B^{*0}\bar{B}^{*0}(D)$	$\chi_{b1}(3P)$
$B^+B^{*-}(S)$	7.058						
$B^0\bar{B}^{*0}(S)$		7.072					
$B^+B^{*-}(D)$			0.702				
$B^0\bar{B}^{*0}(D)$				0.703			
$B^{*+}B^{*-}(D)$					2.073		
$B^{*0}\bar{B}^{*0}(D)$						2.075	
$\chi_{b1}(3P)$							-50.421

Expectation value of potential energy of X_b at $\Lambda_q = 500$ MeV.

$B^+B^{*-}(S)$	$B^0\bar{B}^{*0}(S)$	$B^+B^{*-}(D)$	$B^0\bar{B}^{*0}(D)$	$B^{*+}B^{*-}(D)$	$B^{*0}\bar{B}^{*0}(D)$	$\chi_{b1}(3P)$	
0.118	0.235	-0.280	-0.560	-0.767	-1.535	-16.128	$B^+B^{*-}(S)$
0.235	0.117	-0.559	-0.279	-1.531	-0.766	-16.099	$B^0\bar{B}^{*0}(S)$
-0.280	-0.559	0.033	0.066	-0.083	-0.167	0	$B^+B^{*-}(D)$
-0.560	-0.279	0.066	0.033	-0.167	-0.083	0	$B^0\bar{B}^{*0}(D)$
-0.767	-1.531	-0.083	-0.167	-0.070	-0.140	0	$B^{*+}B^{*-}(D)$
-1.535	-0.766	-0.167	-0.083	-0.140	-0.070	0	$B^{*0}\bar{B}^{*0}(D)$
-16.128	-16.099	0	0	0	0	0	$\chi_{b1}(3P)$

Expectation value of kinetic energy of X_b at $\Lambda_q = 1000$ MeV

$B^+B^{*-}(S)$	$B^0\bar{B}^{*0}(S)$	$B^+B^{*-}(D)$	$B^0\bar{B}^{*0}(D)$	$B^{*+}B^{*-}(D)$	$B^{*0}\bar{B}^{*0}(D)$	$\chi_{b1}(3P)$	
14.862							$B^+B^{*-}(S)$
	14.869						$B^0\bar{B}^{*0}(S)$
		0.617					$B^+B^{*-}(D)$
			0.617				$B^0\bar{B}^{*0}(D)$
				1.820			$B^{*+}B^{*-}(D)$
					1.822		$B^{*0}\bar{B}^{*0}(D)$
						−51.260	$\chi_{b1}(3P)$

Expectation value of potential energy of X_b at $\Lambda_q = 1000$ MeV

	$B^+B^{*-}(S)$	$B^0\bar{B}^{*0}(S)$	$B^+B^{*-}(D)$	$B^0\bar{B}^{*0}(D)$	$B^{*+}B^{*-}(D)$	$B^{*0}\bar{B}^{*0}(D)$	$\chi_{b1}(3P)$
$B^+B^{*-}(S)$	0.140	0.280	-0.244	-0.487	-0.682	-1.365	-26.869
$B^0\bar{B}^{*0}(S)$	0.280	0.140	-0.486	-0.243	-1.363	-0.682	-26.839
$B^+B^{*-}(D)$	-0.244	-0.486	0.024	0.048	-0.062	-0.124	0
$B^0\bar{B}^{*0}(D)$	-0.487	-0.243	0.048	0.024	-0.124	-0.062	0
$B^{*+}B^{*-}(D)$	-0.682	-1.363	-0.062	-0.124	-0.053	-0.106	0
$B^{*0}\bar{B}^{*0}(D)$	-1.365	-0.682	-0.124	-0.062	-0.106	-0.053	0
$\chi_{b1}(3P)$	-26.869	-26.839	0	0	0	0	0