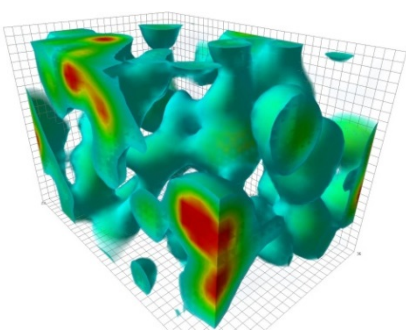
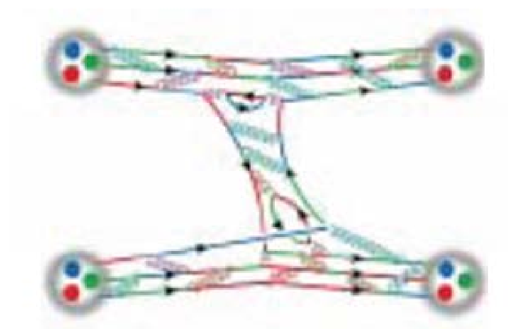
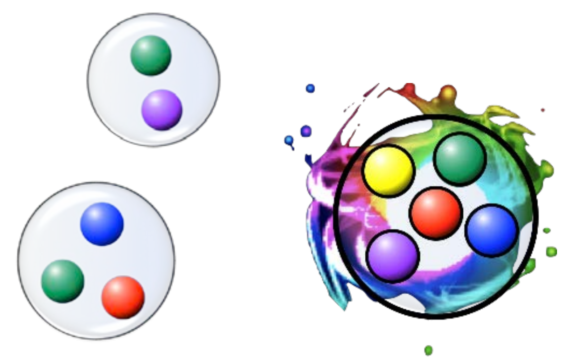
N-ϕ interaction from Lattice QCD

Takumi Doi

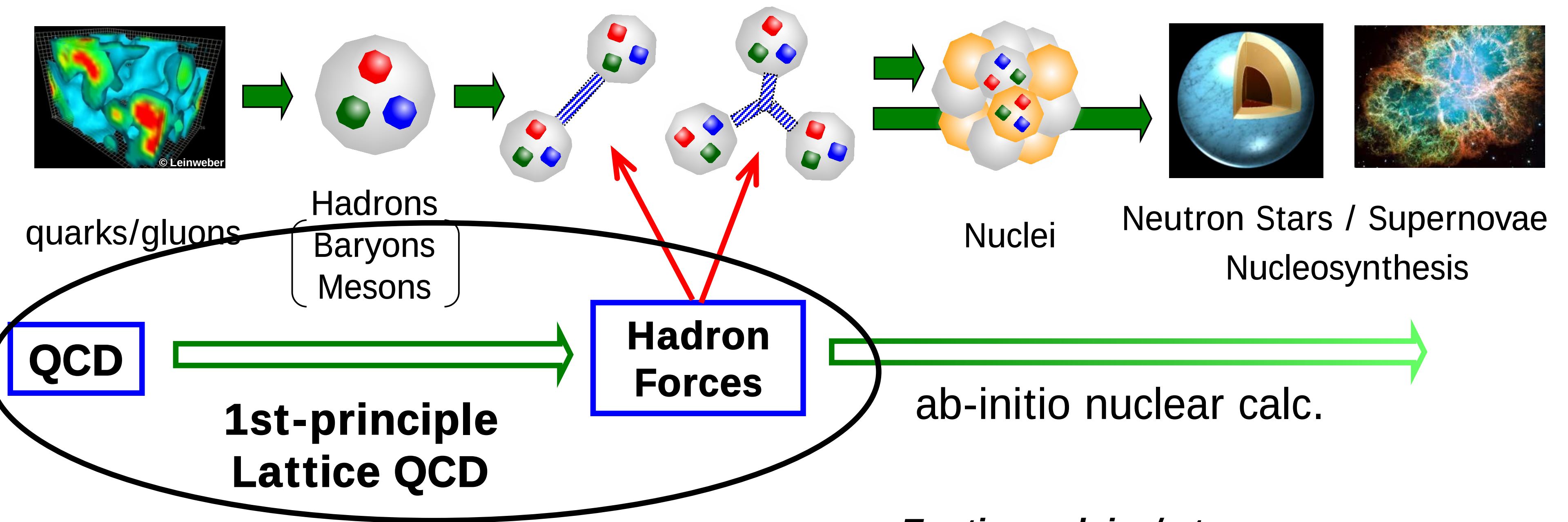
(RIKEN iTHEMS)







Exploring origin of matter from the theory of quarks

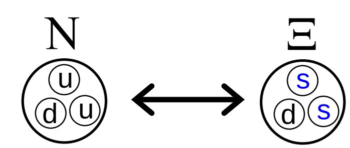


Exotic nuclei w/ strangeness

Nuclear Physics from QCD

Equation of State of dense matter

←→ Interactions w/ Hyperons



LQCD NΞ int

→ virtual state as a remnant of H-dibaryon

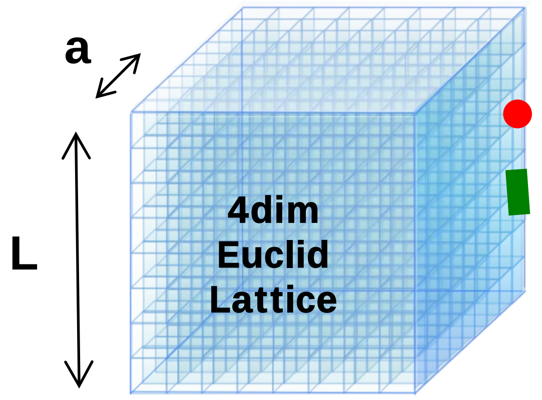
→ Ξ-hypernuclei (w/ Hiyama et al.)

→ femtoscopy (w/ ALICE Coll.)

General Hadron-Hadron Interactions
w/ strange (charm, bottom) quarks

Lattice QCD
First-principle calculation of QCD

$$Z = \int dU\, dq d\bar{q}\; e^{-S_E}$$



quarks on sites

$$q(x)$$

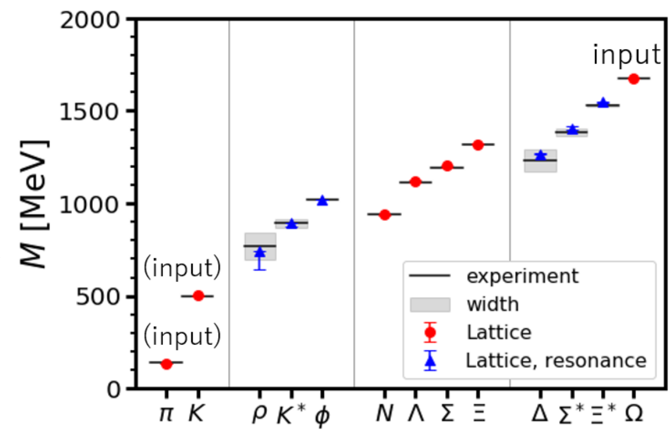
gluons on links

$$U_\mu(x, x+\mu) = \exp[-iaA_\mu]$$

K.G. Wilson
(1974)

- **Regularized system (finite a and L)**
- **Gauge-invariance manifest**
- **Fully-Nonperturbative**
- **DoF ~ $10^{9\text{-}10}$ ➔ Monte-Carlo w/ Euclid time**
  - Numerical calc by supercomputers

Thermodynamic limit (L → ∞)
& continuum limit (a → 0)

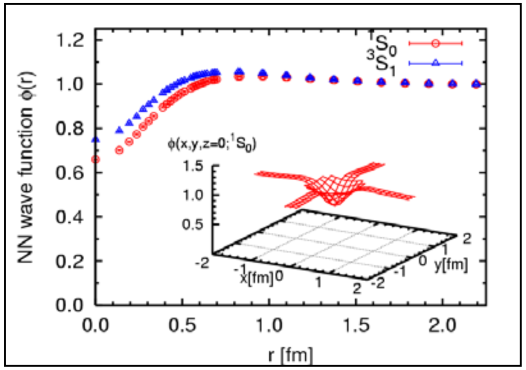


HAL Coll., arXiv:2406.16665

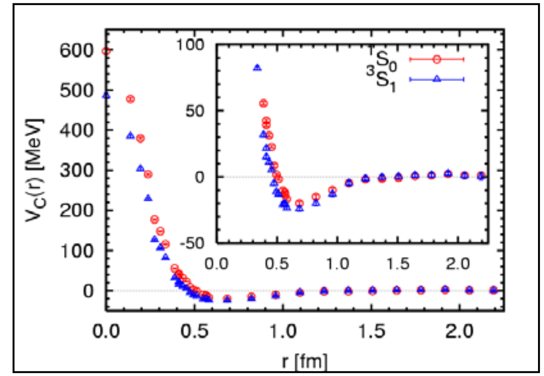
HAL QCD method

Lattice QCD

NBS wave func.



Lat Hadron Force



$$\psi_{\rm NBS}(\vec{r}) = \langle 0|H_1(\vec{r})H_2(\vec{0})|H_1(\vec{k})H_2(-\vec{k}), in\rangle$$
$$\simeq A_k \sin(kr - l\pi/2 + \delta_l(k))/(kr)$$

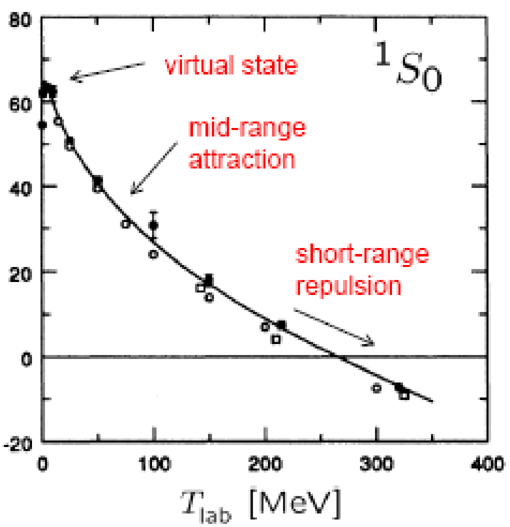
$$(k^2/m_N - H_0)\,\psi_{\rm NBS}(\vec{r}) = \int d\vec{r}\,' U(\vec{r},\vec{r}\,')\psi_{\rm NBS}(\vec{r}\,')$$

E-indep (& non-local) Potential

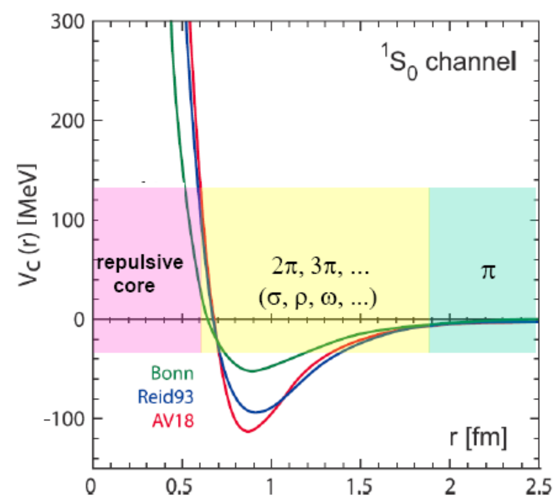
Analog to …

Scattering Exp.

Phase shifts



Phen. Potential



HAL QCD method

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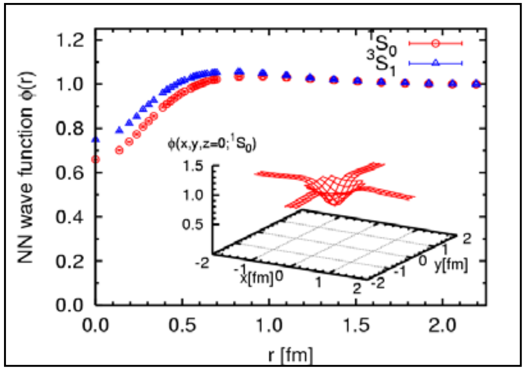
NBS wave func.



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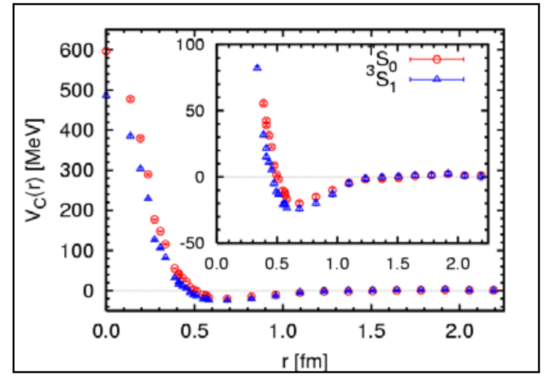

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$$\simeq A_k \sin(kr - l\pi/2 + \delta_l(k))/(kr)$$

$$\left(k^2/m_N - H_0\right)\psi_{\rm NBS}(\vec{r}) = \int d\vec{r}\,' U(\vec{r},\vec{r}\,')\psi_{\rm NBS}(\vec{r}\,')$$

- **Interacting kernel (“potential”) from spatial & temporal correlation**
  - Potential → Phase Shifts guaranteed by QFT

    Aoki-Hatsuda-Ishii PTP123(2010)89

- **Energy-indep potential**

  N.Ishii et al. (HAL Coll.) PLB712(2012)437
  - “Signal” from (elastic) excited states → Ground state saturation NOT required

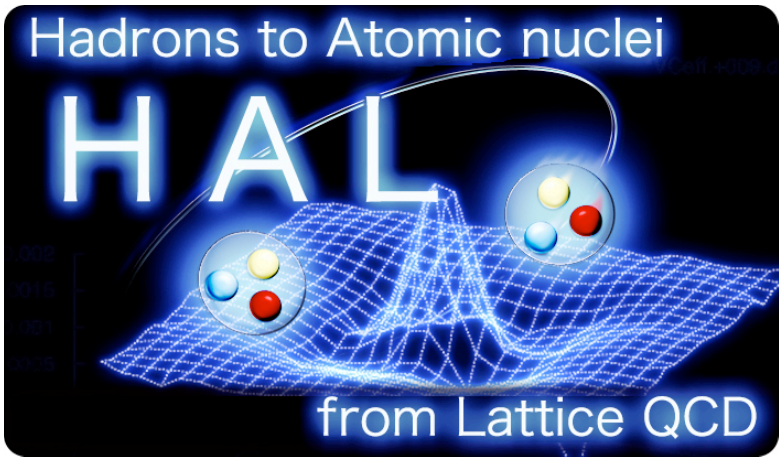
- **Coupled Channel formalism**

  S. Aoki et al. (HAL Coll.) Proc.Jpn.Acad.B87(2011)509
  - Physics above inelastic threshold → Essential for Hyperon-forces, Exotics

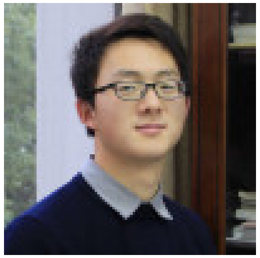
- **Extension to N-body systems (N>=3)**

  S. Aoki et al. (HAL Coll.) PRD88(2013)014036
  (non-rela approx.)

Hadrons to Atomic nuclei from Lattice QCD (HAL QCD Collaboration)



S. Aoki, T. M. Doi, E. Itou (Kyoto U.)
T. Aoyama (ISSP)
T. Doi, T. Hatsuda, Y. Lyu, W. A. Yamada, L. Wang, L. Zhang (RIKEN)
F. Etminan (U. of Birjand)
Y. Ikeda, N. Ishii, P. Junnarkar, H. Nemura, K. Sasaki (Osaka U.)
T. Inoue (Nihon U.)
K. Murakami (TITech)
K. Murase (Tokyo Metropolitan U.)
T. Sugiura (Rissho U.)
H. Tong (U. of Bonn)



「20XX年宇宙の旅」

from Quarks to Universe

+
I. Kanamori (RIKEN)
K.-I. Ishikawa (Hiroshima U.)
and many more

Challenge toward LQCD w/ physical mass

~2012

➔ lighter u,d-quark masses
(=lighter pion mass M_π)

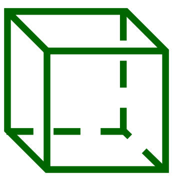
~2018

We were here

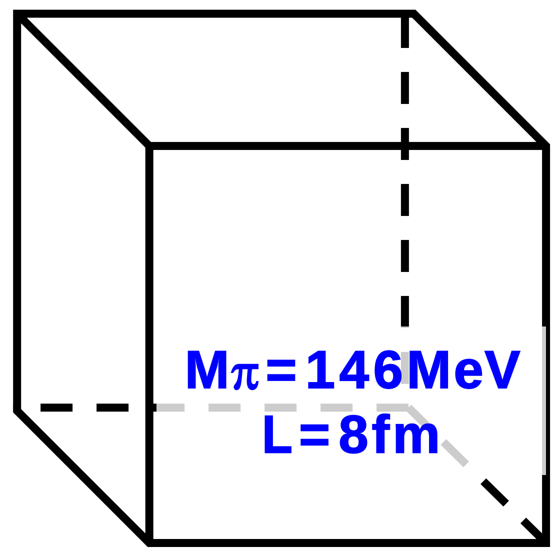
➔ more challenging

Phys. point

M_π=400MeV
L=3fm



Theoretical development
+
New supercomputer



Simulation w/
Unrealistic mass

K-computer (11PFlops)

Simulation w/
~Near physical mass

Challenge toward LQCD w/ physical mass

~2012

➔ lighter u,d-quark masses
(=lighter pion mass Mπ)

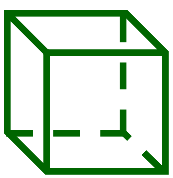
~2018

We were here

➔ more challenging

Phys. point

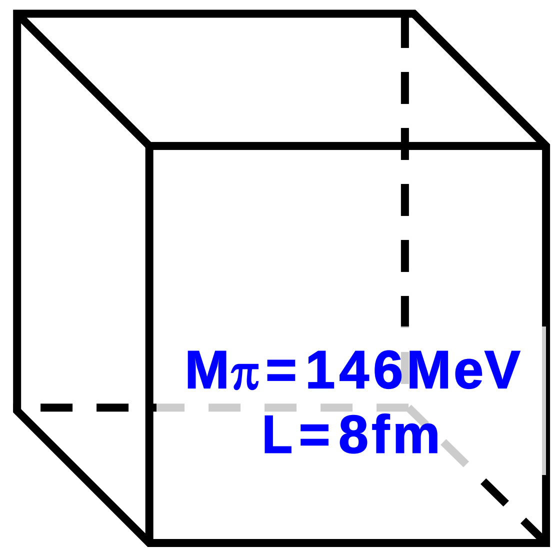
Mπ=400MeV
L=3fm



Theoretical development
+
New supercomputer

Simulation w/
Unrealistic mass

Fugaku (440PFlops)



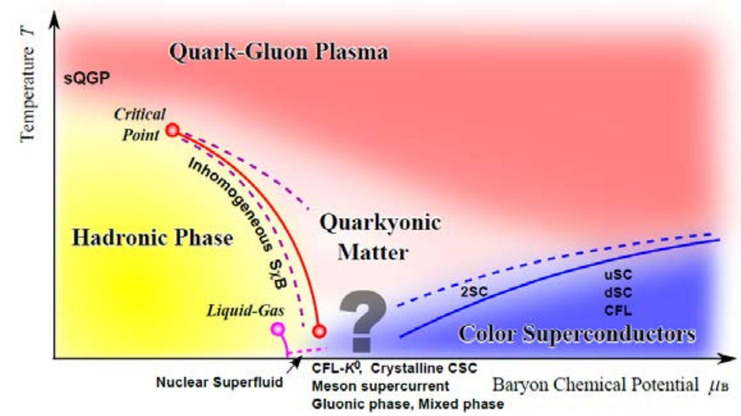
Simulation w/
~Near physical mass

Nϕ system

Medium effects on hadron properties

←→ partial restoration
of chiral symmetry

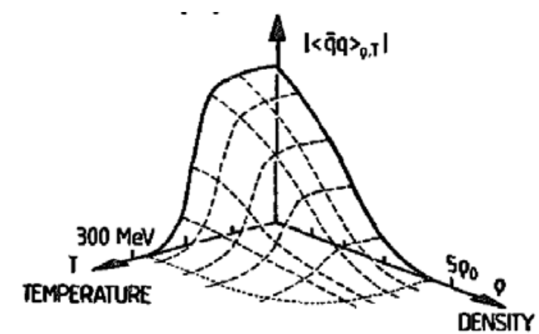
Hayano-Hatsuda, ReV. Mod. Phys. 82 (2010) 2949



Fukushima-Hatsuda, RPP74(2011)014001

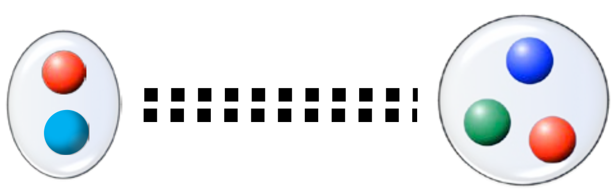
ϕ-meson is one of the ideal probes

Brown-Rho, RRL66(1991)2720
Hatsuda-Lee, PRC46(1992)R34
Gubler-Weise, NPA954(2016)125)



Weise, NPA553(1993)59

How color-dipole interacts w/ other hadrons?



2-pion-exchange?

Dipole-Dipole int: H. Fujii and D. Kharzeev, PRD60(1999)114039
Dipole-Nucleon int: J. Castella and G. Krein, PRD98(2018)014029

Nϕ system

Nϕ interaction (J=3/2,1/2)

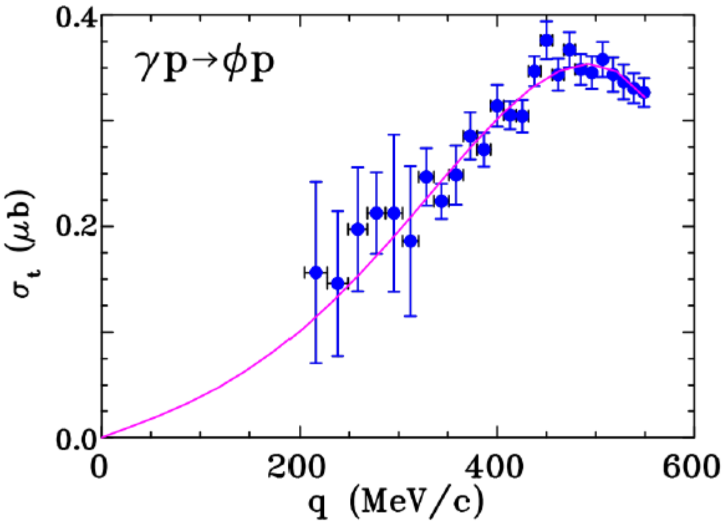
a < 0 : attractive
a > 0 : repulsive

$a_0^{\text{spin-ave}}$ [fm] (Re-part)

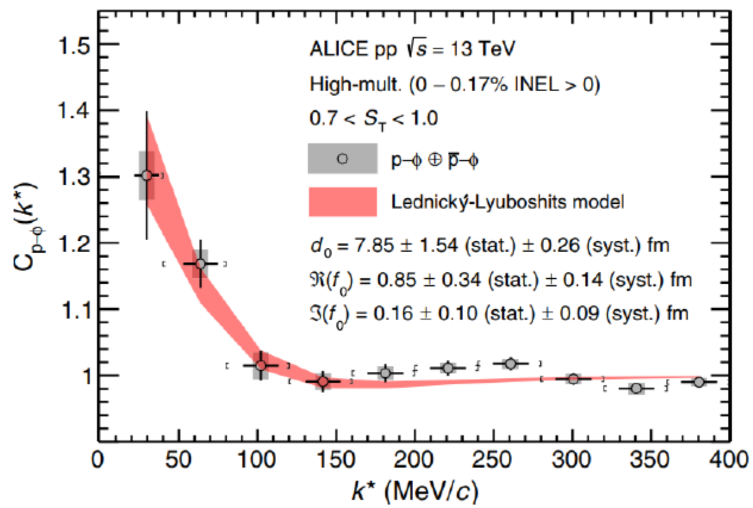
Theor.	Koike+	−0.15(02)	(QSR)
	Klingl+	+0.01	(VMD)
Expt.	CLAS	±0.063(10)	
	ALICE	−0.85(34)(14)	(r_{eff}=7.85(1.54)(0.26) fm)

Y. Koike+, PTP98(1997)631
F. Klingl+, NPA624(1997)527
I.I. Strakovsky+, PRC101(2020)045201
ALICE Coll., PRL127(2021)172301

CLAS



ALICE



First-principles calculation is highly required!

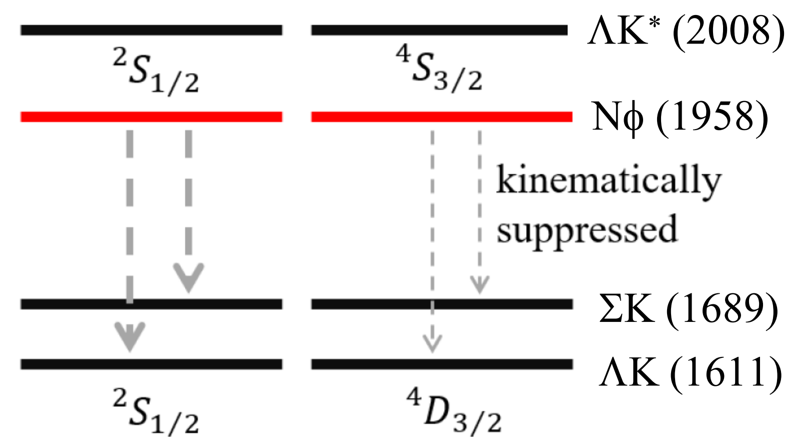
Nϕ system

[PDG]

Nϕ interaction from LQCD

ΛK, ΣK thresholds are open

Coupled channel calc expensive



J=3/2 : ΛK, ΣK suppressed by D-wave

J=1/2 : ΛK, ΣK S-wave mixing, no suppression

(3-body decay modes,
ϕ–decay modes neglected)

(ωN, ρN also neglected by OZI)

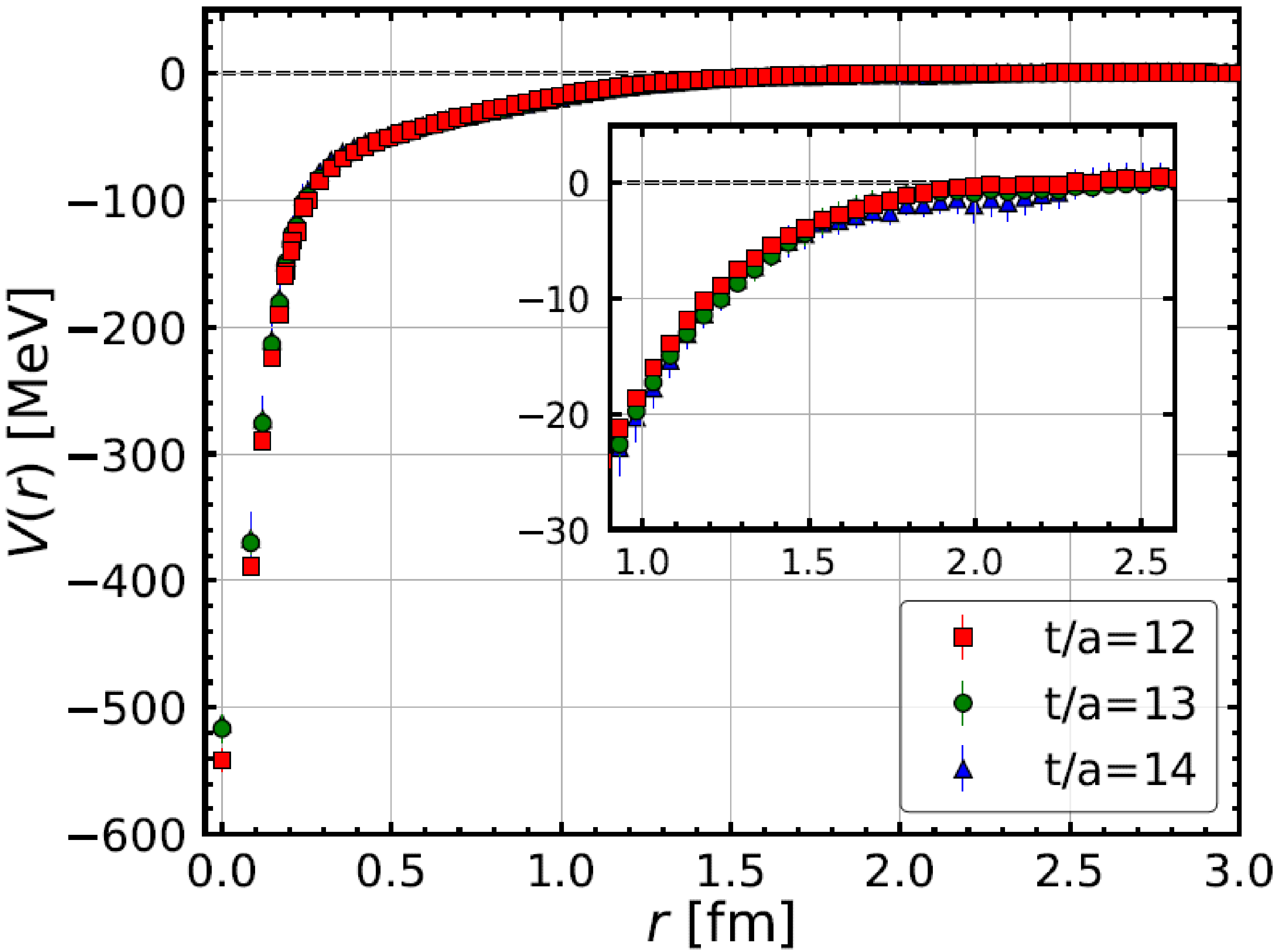
→ **We calculate Nϕ (J=3/2) w/ single-channel approximation**

Y. Lyu et al., PRD106(2022)074507

Hadron	Lattice [MeV]	Expt. [MeV]
π	146.4(4)	138.0
K	524.7(2)	495.6
ϕ	1048.0(4)	1019.5
N	954.0(2.9)	938.9

s-sbar annihilation neglected
ϕ → K-Kbar forbidden at this mass
ϕ→ 3π found to be negligible

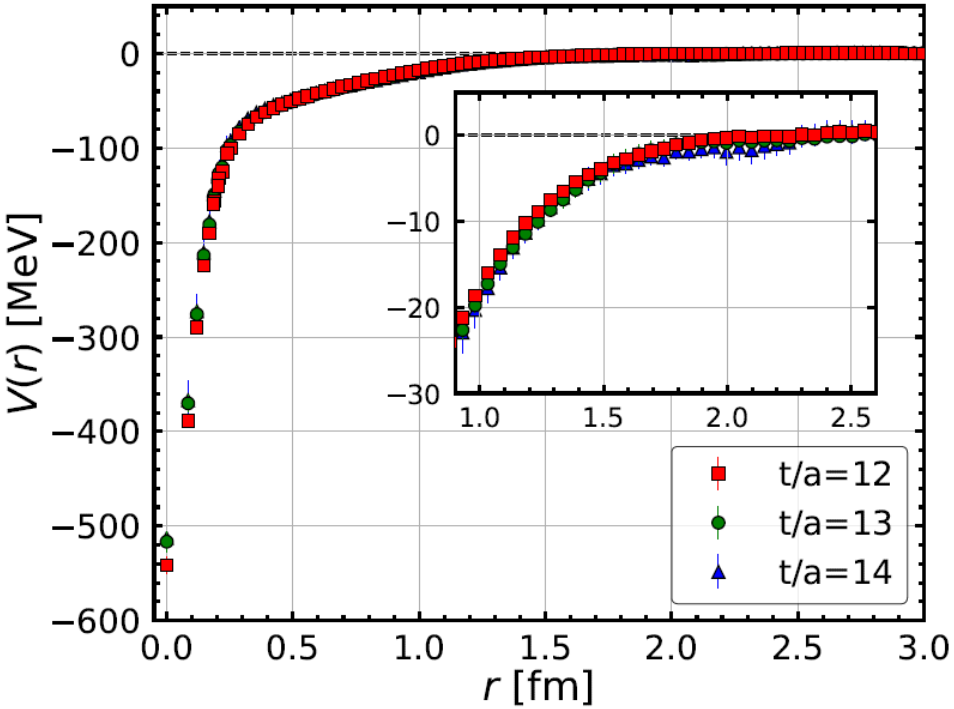
Nϕ system ($^4S_{3/2}$)



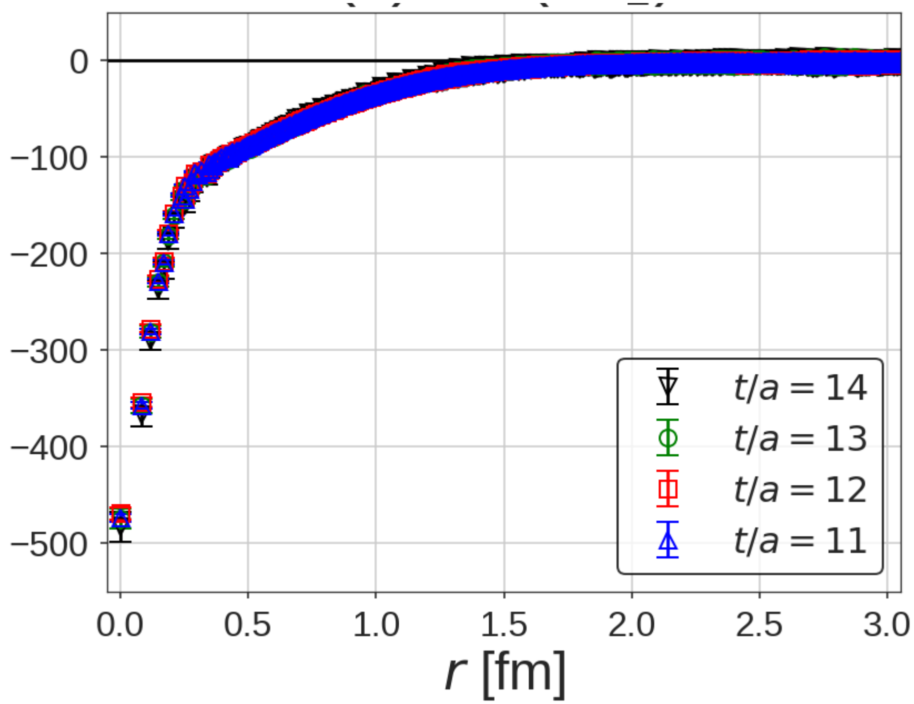
Attractive potential at all distances

Nϕ vs. NΩ potentials

Nϕ ($^4S_{3/2}$)



NΩ ($^5S_2$)



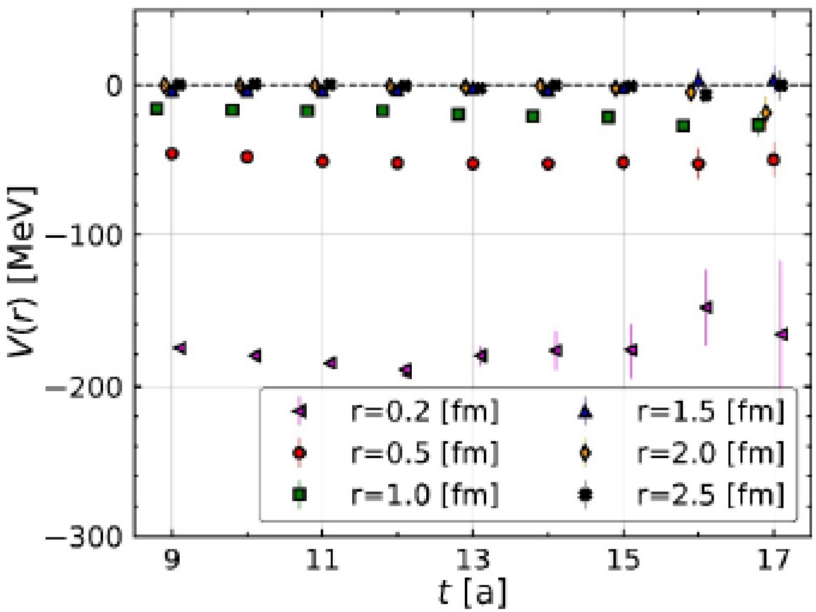
T. Iritani et al. (HAL Coll.), PLB792(2019)284

Similar long-range behavior observed

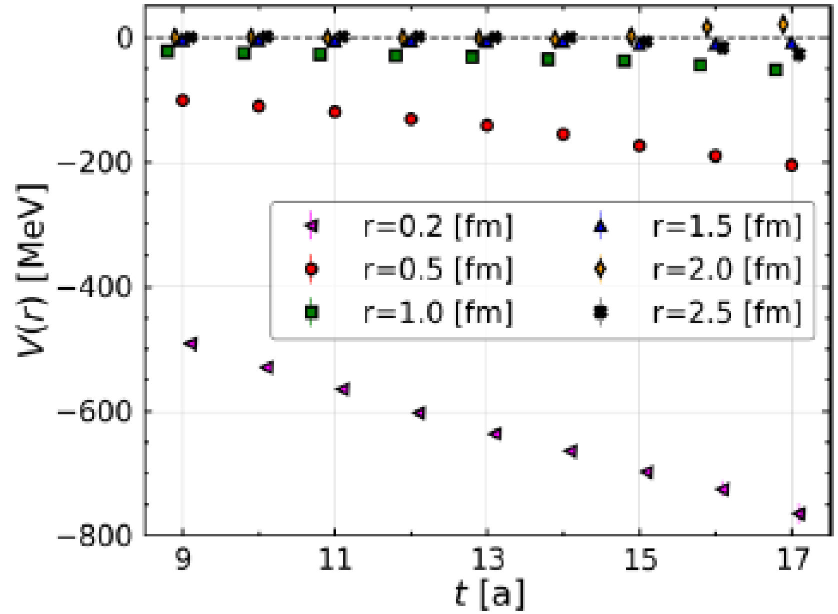
Effects of open channels?

Nϕ potentials in wider range of t

- $^4S_{3/2}$

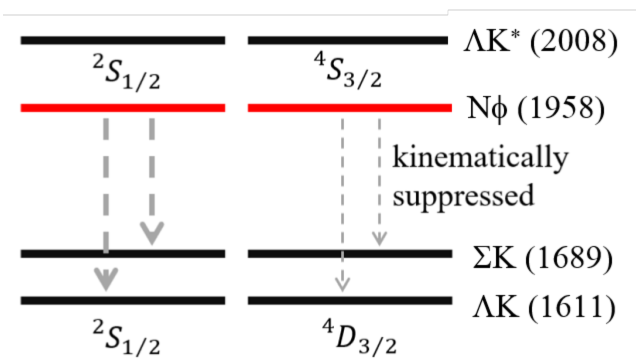


- $^2S_{1/2}$



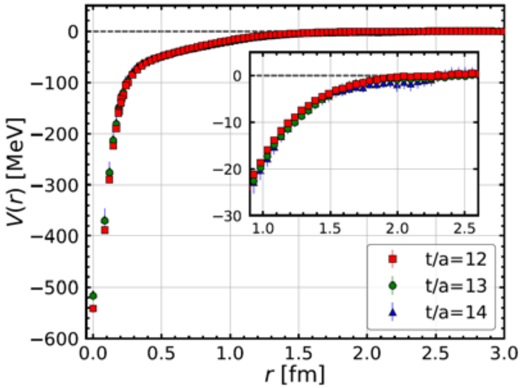
$^4S_{3/2}$: stable against t →
single channel study justified

$^2S_{1/2}$: clear t-dep observed

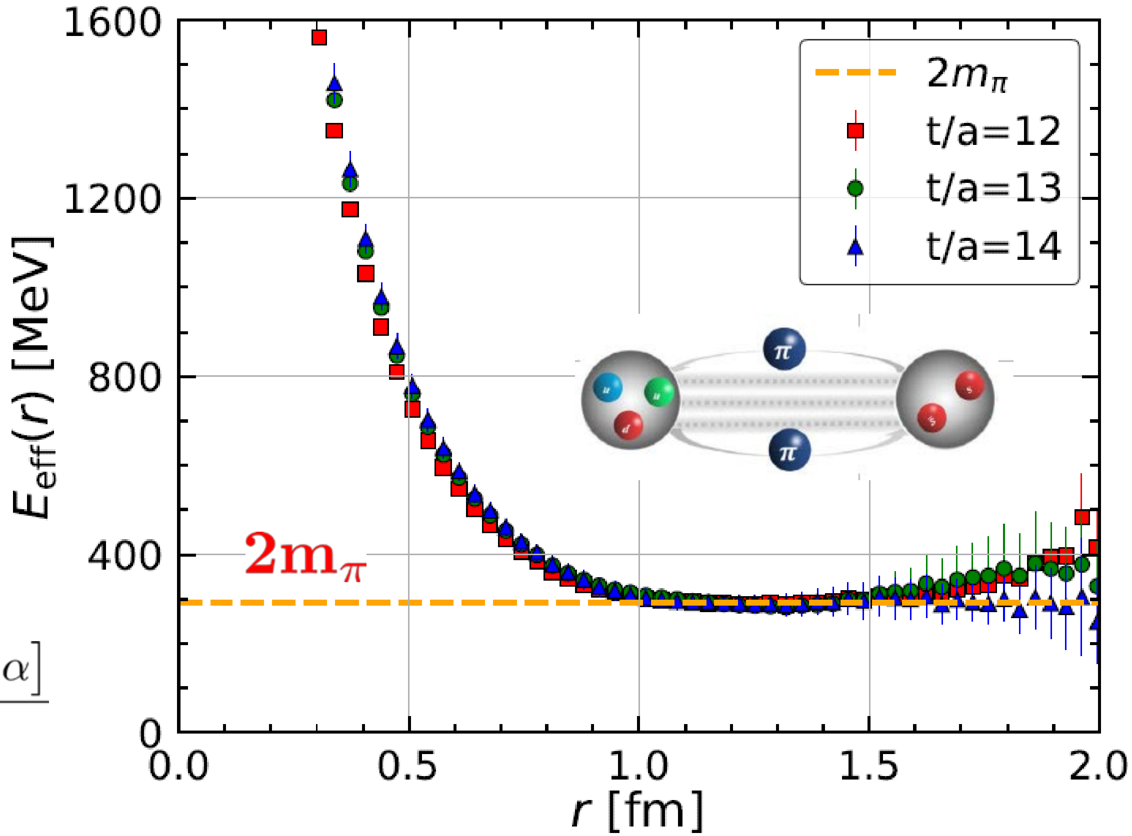


What is the origin of long-range attraction?

Potential



Tail structure of potential



$$V(r) \xrightarrow{r\to\infty} -\alpha \frac{e^{-2m_\pi r}}{r^2}$$

$$\Longrightarrow \quad E_{\rm eff}(r) = -\frac{\ln\left[-V(r)r^2/\alpha\right]}{r} \simeq 2m_\pi$$

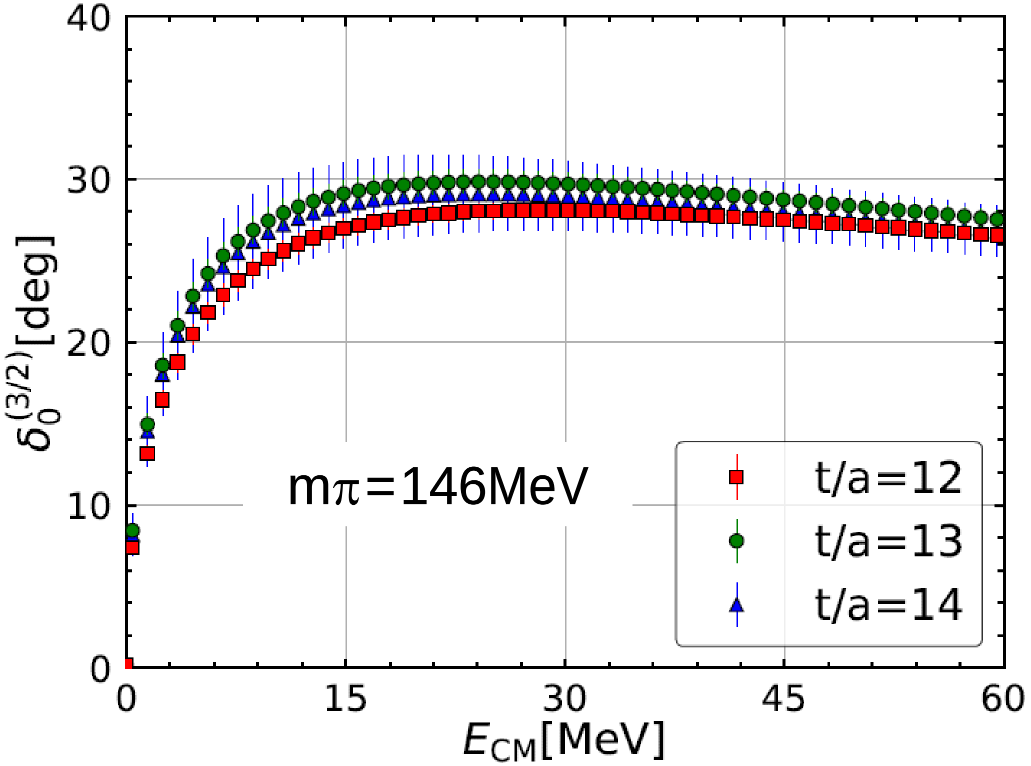
Tail is consistent w/ two-pion exchange (TPE) !

c.f. Fujii-Kharzeev, PRD60(1999)114039
Castella-Krein, PRD98(2018)014029

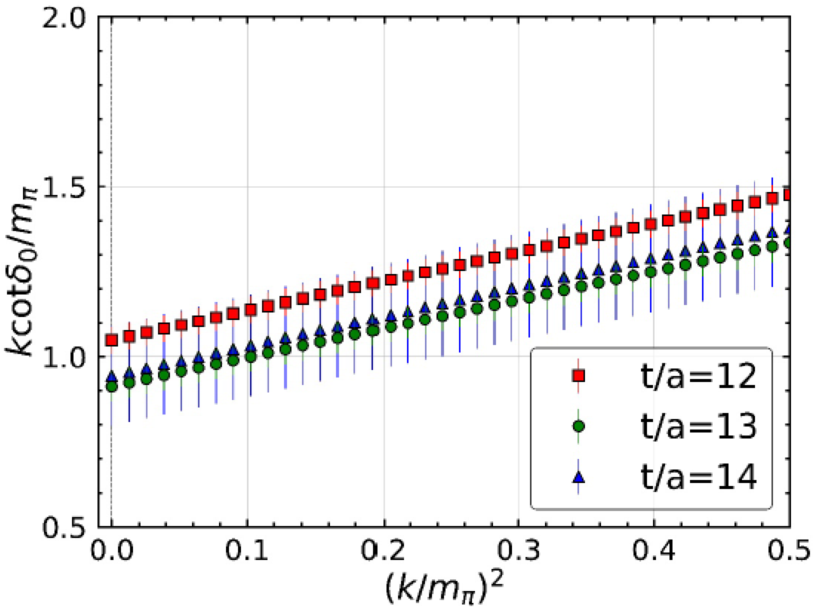
N.B. TPE is found to be important
for Tcc (D-D* int), too

Y. Lyu+, PRL131(2023)161901

Phase shifts of Nϕ ($^4S_{3/2}$)

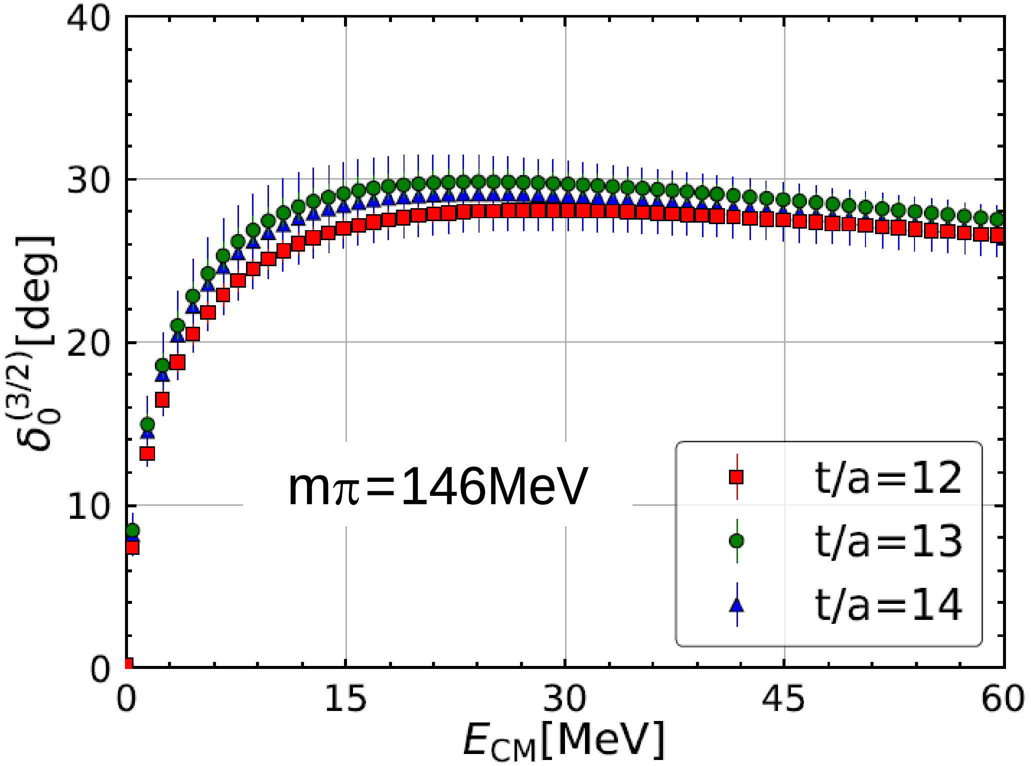


Effective Range Expansion



m_π [MeV]	$a_0^{(3/2)}$ [fm]	$r_{\rm eff}^{(3/2)}$ [fm]
146.4	$-1.43(23)_{\rm stat}(^{+36}_{-06})_{\rm syst}$	$2.36(10)_{\rm stat}(^{+02}_{-48})_{\rm syst}$

Phase shifts of Nϕ ($^4S_{3/2}$)



Fit the potential and solve Schrodinger eq. in infinite V

$$V_{\rm fit}(r) = \sum_{i=1,2} a_i e^{-(r/b_i)^2} + a_3 m_\pi^4 f(r;b_3)\frac{e^{-2m_\pi r}}{r^2}$$

EFT by Castella-Krein

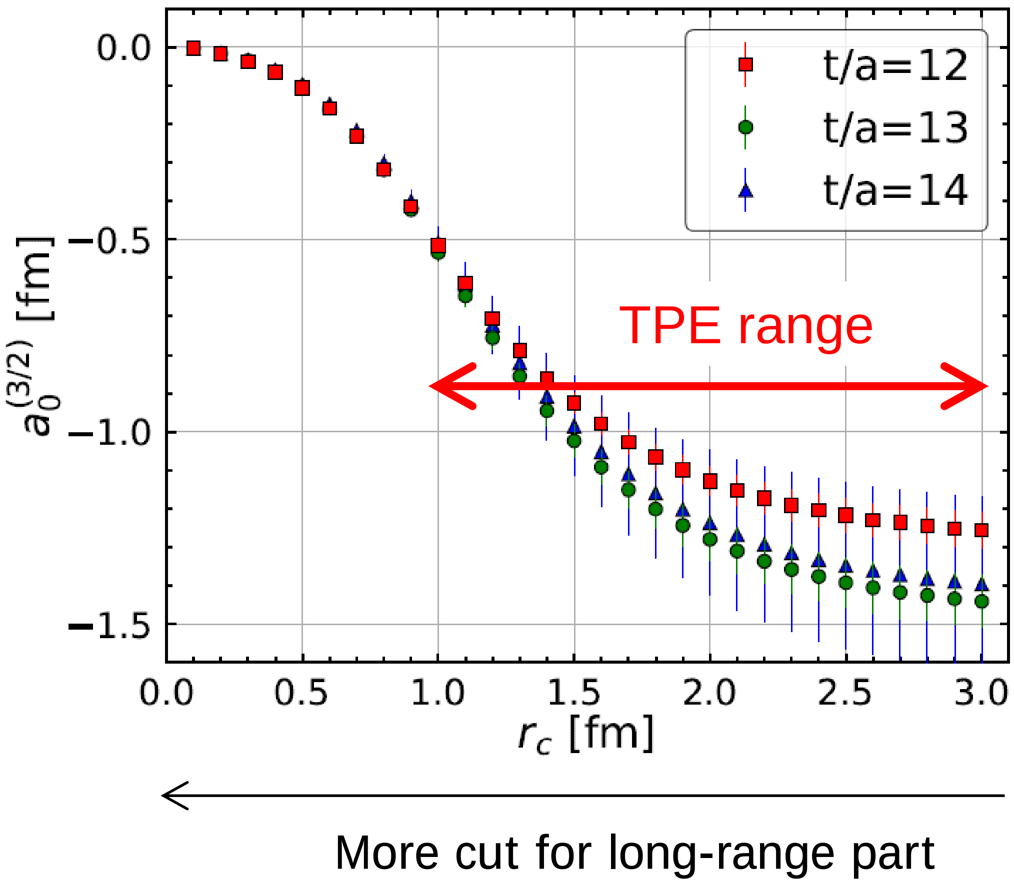
Fit w/ mπ = mπ(LQCD)

Semi-quantitative extrapolation by mπ → mπ(phys)

m_π [MeV]	$a_0^{(3/2)}$ [fm]	$r_{\rm eff}^{(3/2)}$ [fm]
146.4	$-1.43(23)_{\rm stat}\left(^{+36}_{-06}\right)_{\rm syst}$	$2.36(10)_{\rm stat}\left(^{+02}_{-48}\right)_{\rm syst}$
138.0	$\simeq -1.25$	$\simeq 2.49$

c.f. ALICE Coll. $a_0^{(\rm spin-ave)} = -0.85(34)(14)$ fm $r_{\rm eff}^{(\rm spin-ave)} = 7.85(1.54)(0.26)$ fm

Which is more important, long or short-range attraction?



Scattering length from potential w/ cut for long-range part

$$V(r; r_c) = \theta(r_c - r)V(r)$$

scattering length is dominated by TPE tail

we also test the cut of short-range part (r<0.1fm) → only ~2% effect

(→ This also indicates that discretization error is small)

New avenue for Hadron interactions by Combined analysis of LQCD & femtoscopy

Nϕ J=3/2 ← LQCD calc possible (ΛK, ΣK mixings suppressed by D-wave)

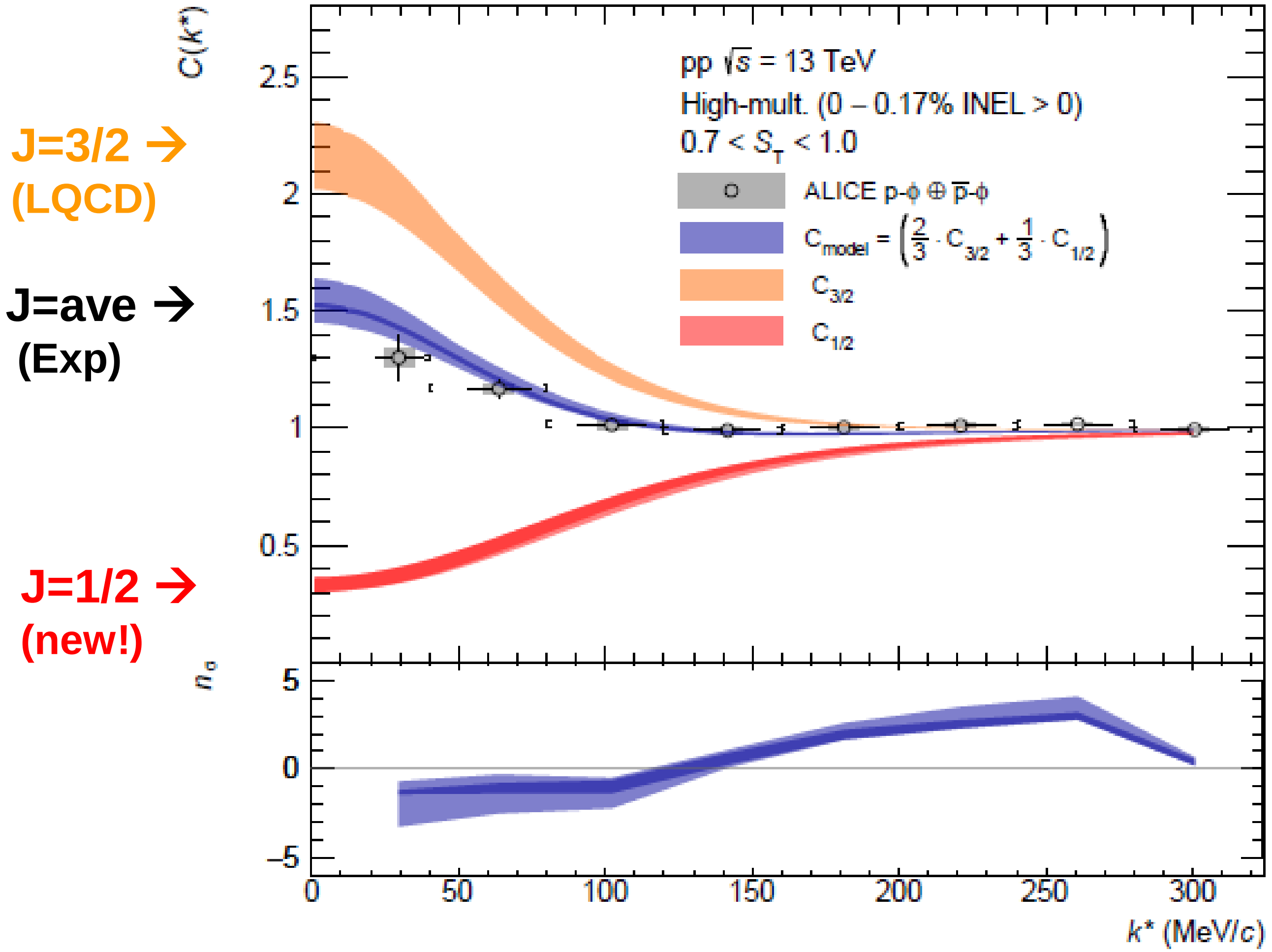
Nϕ J=1/2 ← LQCD challenging (ΛK, ΣK mixings are S-wave)

Nϕ spin average ← ALICE femtoscopy possible
(but spin projection difficult)

➔ By combining LQCD and ALICE femtoscopy,
we can extract Nϕ J=1/2

E. Chizzali+, PLB848(2024)138358

Nϕ system ($^2S_{1/2}$) from LQCD + Exp

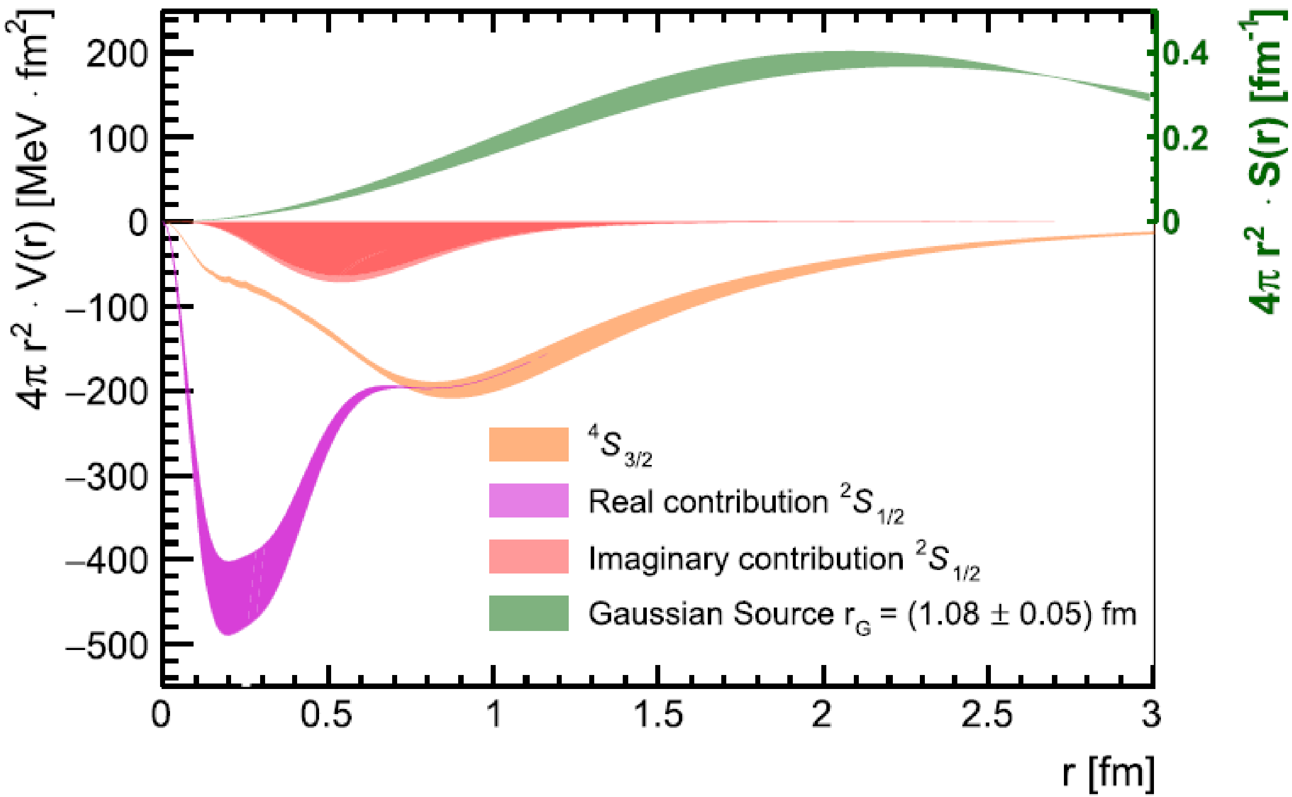


Correlation function measured by ALICE

Fit the femtoscopy data w/ parameters in potential in J=1/2

Nϕ system ($^2S_{1/2}$) from LQCD + Exp

Potential (w/ phase space factor of $4\pi r^2$)

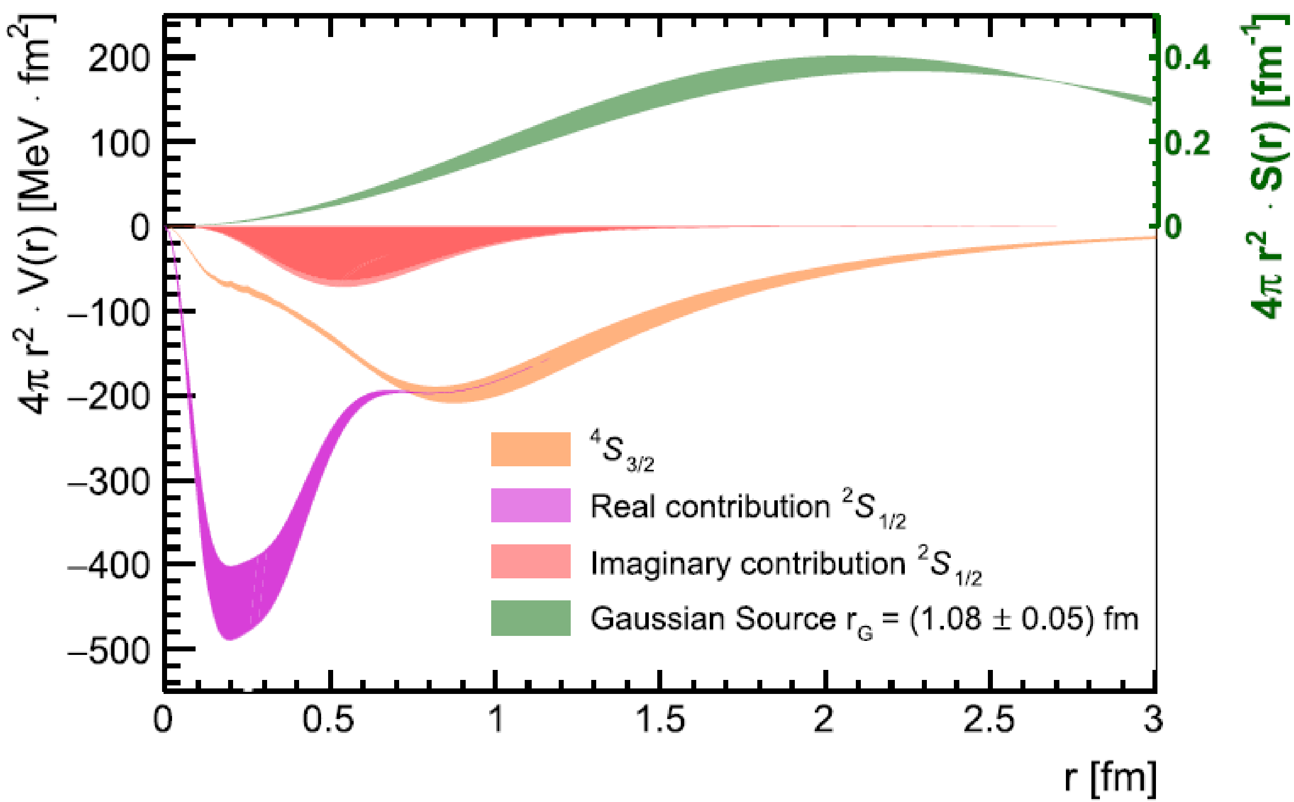


$$V_{N\phi}^{J=1/2}(r) = \beta\left(\sum_{i=1,2} a_i e^{-(r/b_i)^2}\right) + a_3 m_\pi^4 f(r;b_3)\frac{e^{-2m_\pi r}}{r^2} + i\gamma f(r;b_3)\frac{e^{-2m_K r}}{m_K r^2}$$

Short-range (phenomenological) | TPE (common for J=1/2, 3/2) | ΛK, ΣK effect by K-exchange

Nϕ system ($^2S_{1/2}$) from LQCD + Exp

Potential (w/ phase space factor of $4\pi r^2$)



Indication of Nϕ bound state in J=1/2

B.E. = 12.8-56.1 MeV

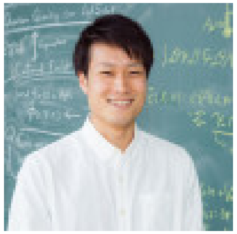
$$\mathrm{Re}\, f_0^{(1/2)} = -1.54^{+0.53}_{-0.53}(\mathrm{stat.})^{+0.16}_{-0.09}(\mathrm{syst.})\ \mathrm{fm},$$

$$\mathrm{Im}\, f_0^{(1/2)} = \ 0.00^{+0.35}_{-0.00}(\mathrm{stat.})^{+0.16}_{-0.00}(\mathrm{syst.})\ \mathrm{fm},$$

$$\mathrm{Re}\, d_0^{(1/2)} = +0.39^{+0.09}_{-0.09}(\mathrm{stat.})^{+0.02}_{-0.03}(\mathrm{syst.})\ \mathrm{fm},$$

$$\mathrm{Im}\, d_0^{(1/2)} = \ 0.00^{+0.00}_{-0.04}(\mathrm{stat.})^{+0.00}_{-0.02}(\mathrm{syst.})\ \mathrm{fm}.$$

Flavor-partner of N–ϕ: $N - c\bar{c}$ systems from LQCD

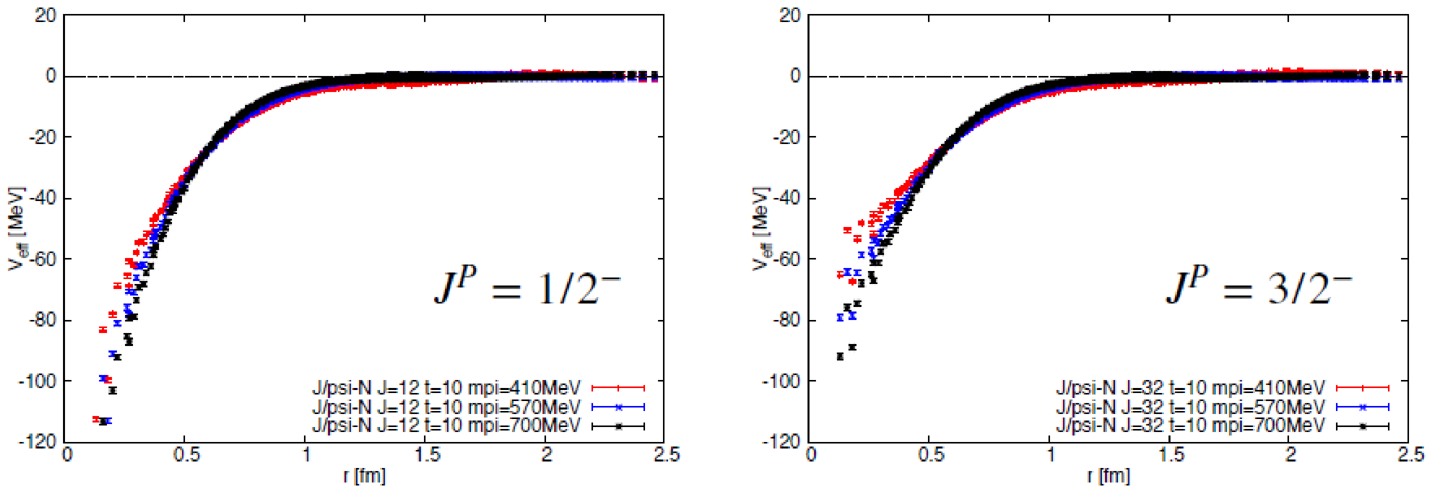


T. Sugiura

T. Sugiura et al., PoS LATTICE2018(2019)093

(See also Kawanai-Sasaki, PRD82(2010)091501)

- LQCD calc of N-J/Psi Interaction at mπ=410-700 MeV



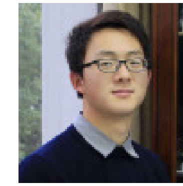
From talk slide @ 2019/12

Attractive at all distance! (but no bound state)

Effect of spin-spin int (due to breaking of heavy-quark sym)

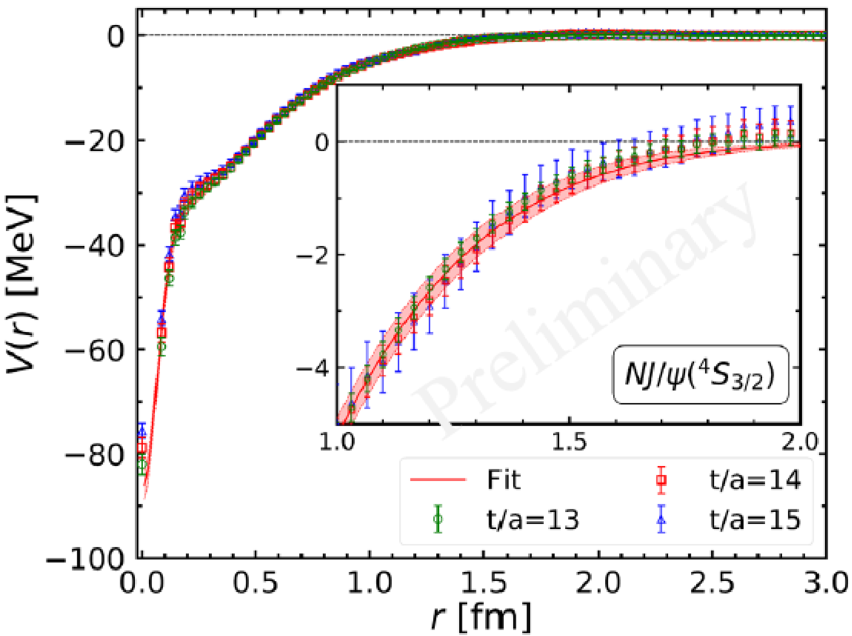
Next: Physical point simulation & study TPE
Coupled channel analysis toward Pc states

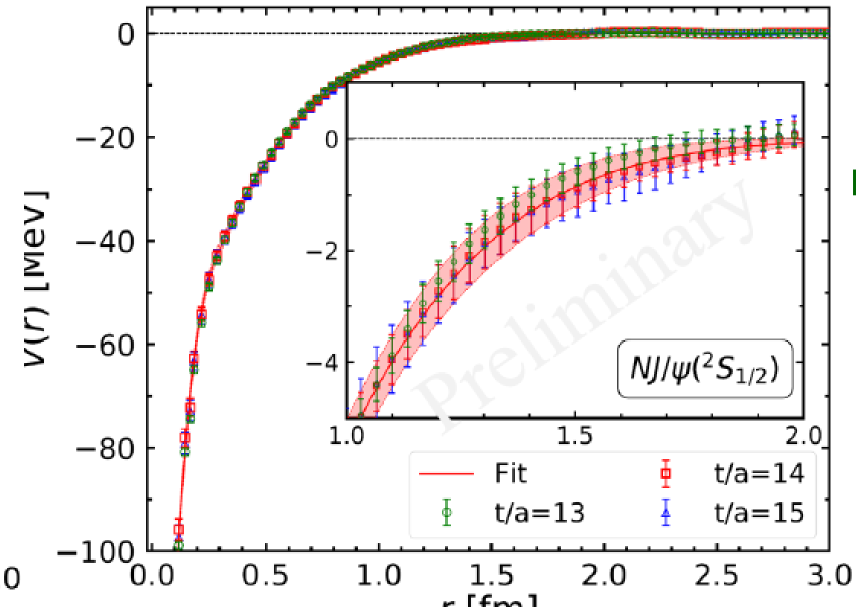
$N{-}c\bar{c}$ potentials near phys point

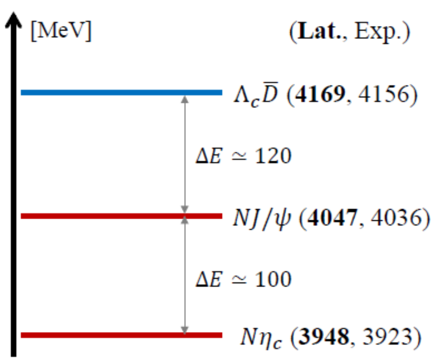


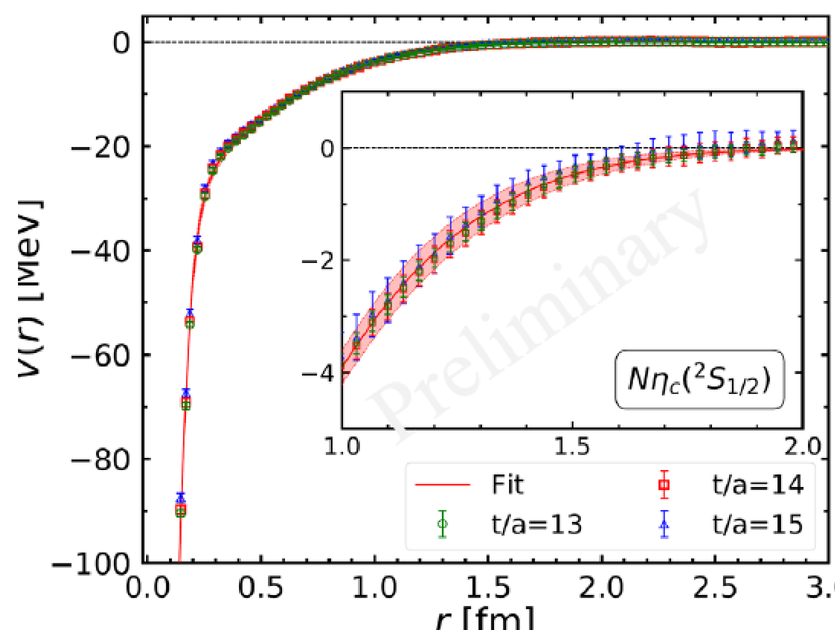
Y. Lyu

Slides from Lattice 2024



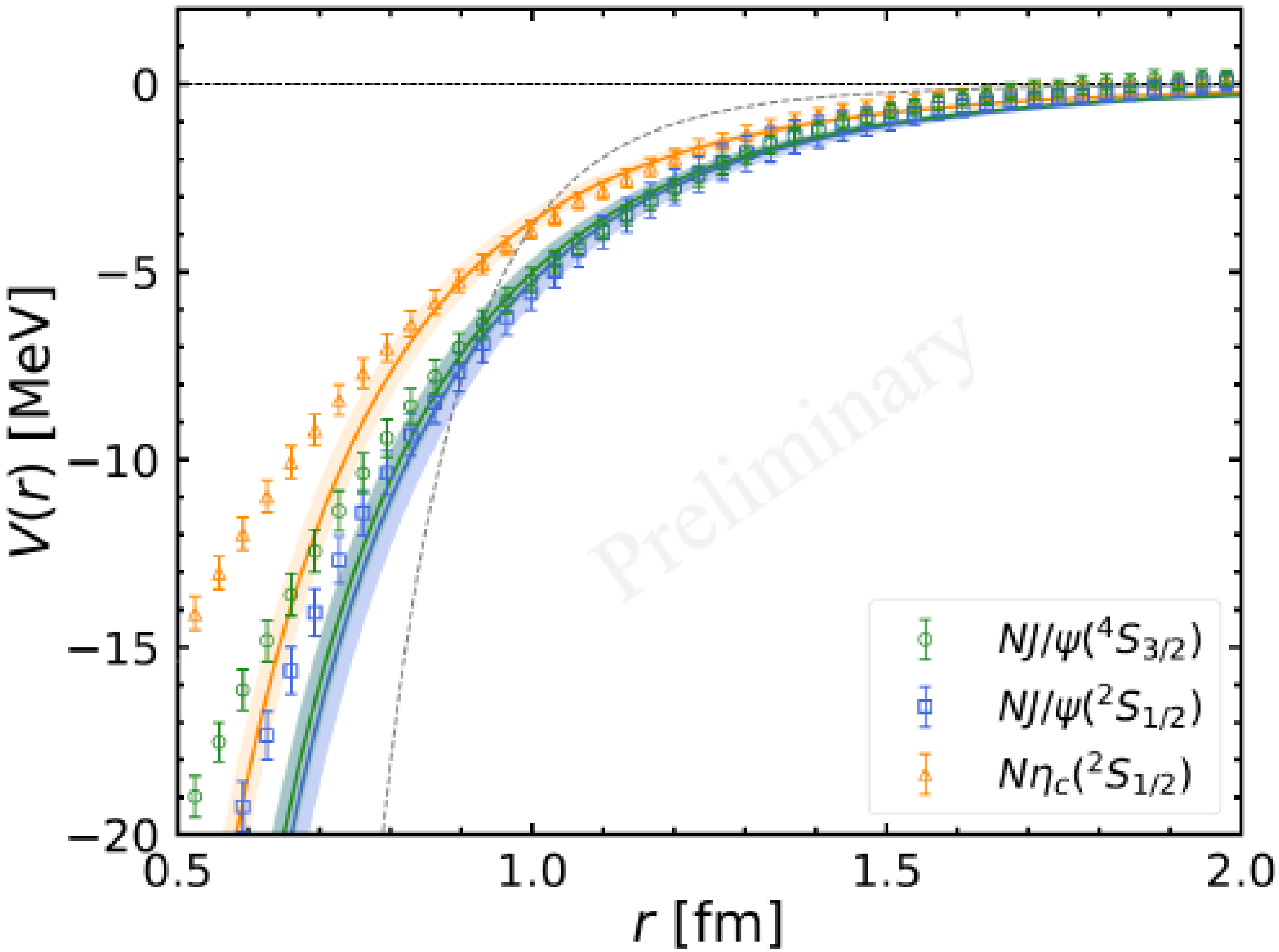






mπ=146MeV (Lat)

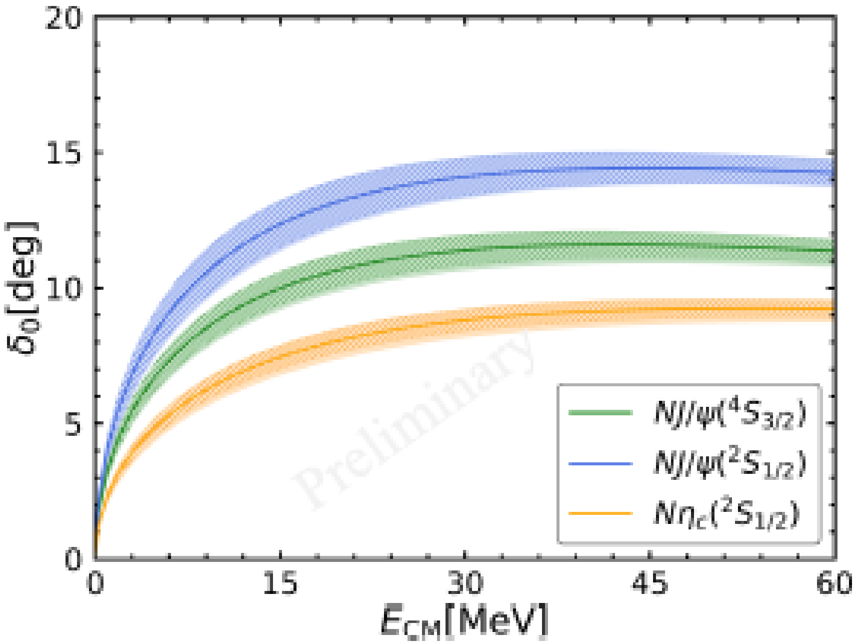
Long-range potentials of $N-c\bar{c}$



The long-range potentials are consistent with the two-pion-exchange (TPE)

Physical observables

- Scattering phase shifts



$$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_{\rm eff} k^2$$

channel	a_0 [fm]	$r_{\rm eff}$ [fm]
$NJ/\psi(^4S_{3/2})$	$0.30(2)\left(^{+0}_{-2}\right)$	$3.25(12)\left(^{+6}_{-9}\right)$
$NJ/\psi(^2S_{1/2})$	$0.38(4)\left(^{+0}_{-3}\right)$	$2.66(21)\left(^{+0}_{-10}\right)$
$N\eta_c(^2S_{1/2})$	$0.21(2)\left(^{+0}_{-1}\right)$	$3.65(20)\left(^{+0}_{-6}\right)$

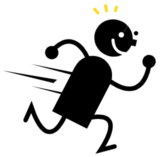
Preliminary

- A direct phenomenological application
  - The J/ψ mass modification in nuclear medium is related to the spin-averaged scattering length of N-J/ψ scattering A. Hayashigaki, Prog. Theor. Phys. 101 (1999)

$$\delta m_{J/\psi} \simeq -\frac{2\pi(m_N + m_{J/\psi})}{m_N m_{J/\psi}} a^{\text{spin-av}}_{J/\psi} \rho_{\rm nm} = -19(3)\ \text{MeV}$$

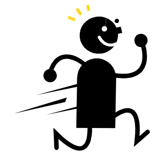
Challenge toward LQCD w/ physical mass

~2012

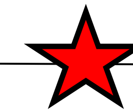


→ lighter u,d-quark masses
(=lighter pion mass M_π)

~2018



2023 !

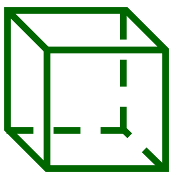


We were here

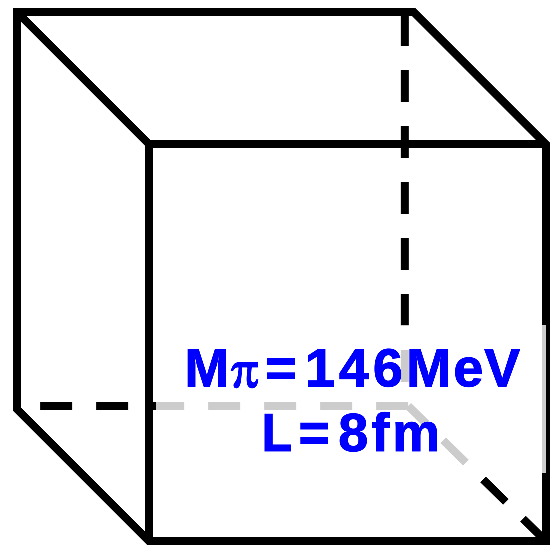
→ more challenging

Phys. point

M_π=400MeV
L=3fm



Theoretical development
+
New supercomputer



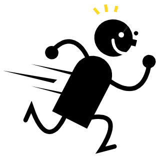
Fugaku (440PFlops)

Simulation w/
Unrealistic mass

Simulation w/
~Near physical mass

Challenge toward LQCD w/ physical mass

~2012



➔ lighter u,d-quark masses
(=lighter pion mass Mπ)

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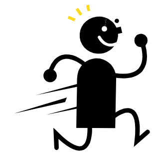
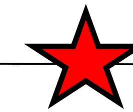


2023 !



We were here

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Phys. point

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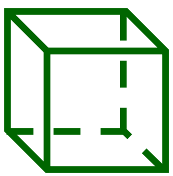
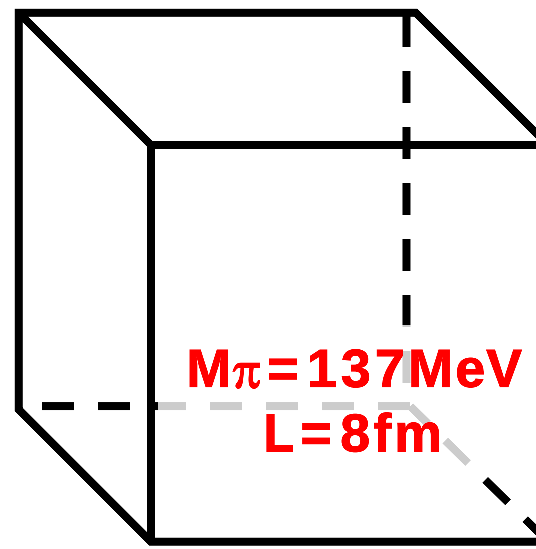


Theoretical development
+
New supercomputer

Fugaku (440PFlops)

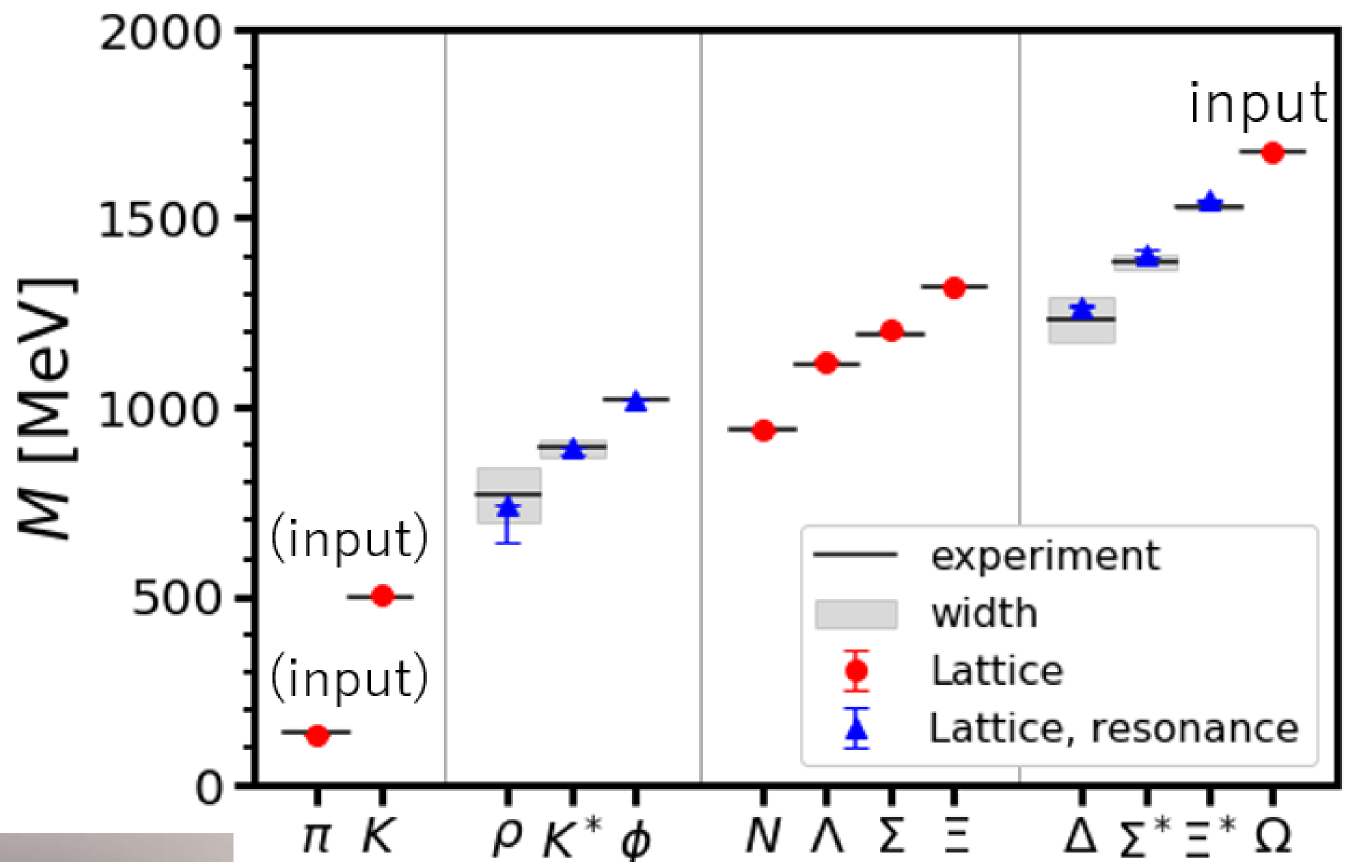


Simulation w/
Unrealistic mass

Simulation w/
the physical mass

LQCD simulations at the physical quark masses

(mπ=137MeV)

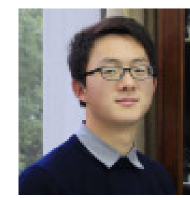
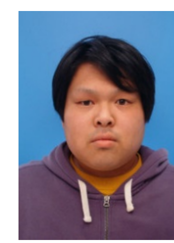
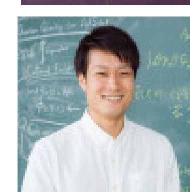


T. Aoyama et al. (HAL Coll.), arXiv:2406.16665

Fugaku (440PFlops)

Calc of Hadron Interactions in progress!



Summary

- Renaissance in hadron physics
  - Experiments (J-PARC & HEF-ex, LHC, Belle II, BES III, DAΦNE, …)
  - Theory by LQCD calc of Hadron Interactions
- First-principles LQCD calc near the physical point
  - HAL QCD method
  - New avenue by combined analysis of LQCD & femtoscopy
  - N-phi (& various meson-meson, meson-baryon, baryon-baryon systems)
    - Importance of Two-pion-exchange (TPE)
    - Possible N-phi bound state in J=1/2 ?
- Prospects
  - Physical point calc by Fugaku supercomputer
  - How ubiquitous is TPE in other systems?
  - From strange to charm/bottom sectors
    - N-J/Ψ , N-η_c interactions
    - Exotic hadrons