PHSD basic concept

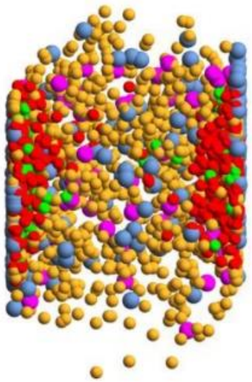
Elena Bratkovskaya

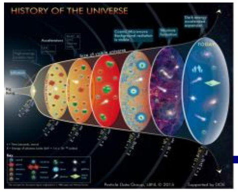
(GSI, Darmstadt & Uni. Frankfurt)

E16 Workshop @ Taiwan

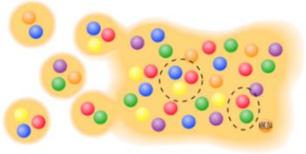
Institute of Physics, Academia Sinica, Taiwan

9–10 Sept 2024

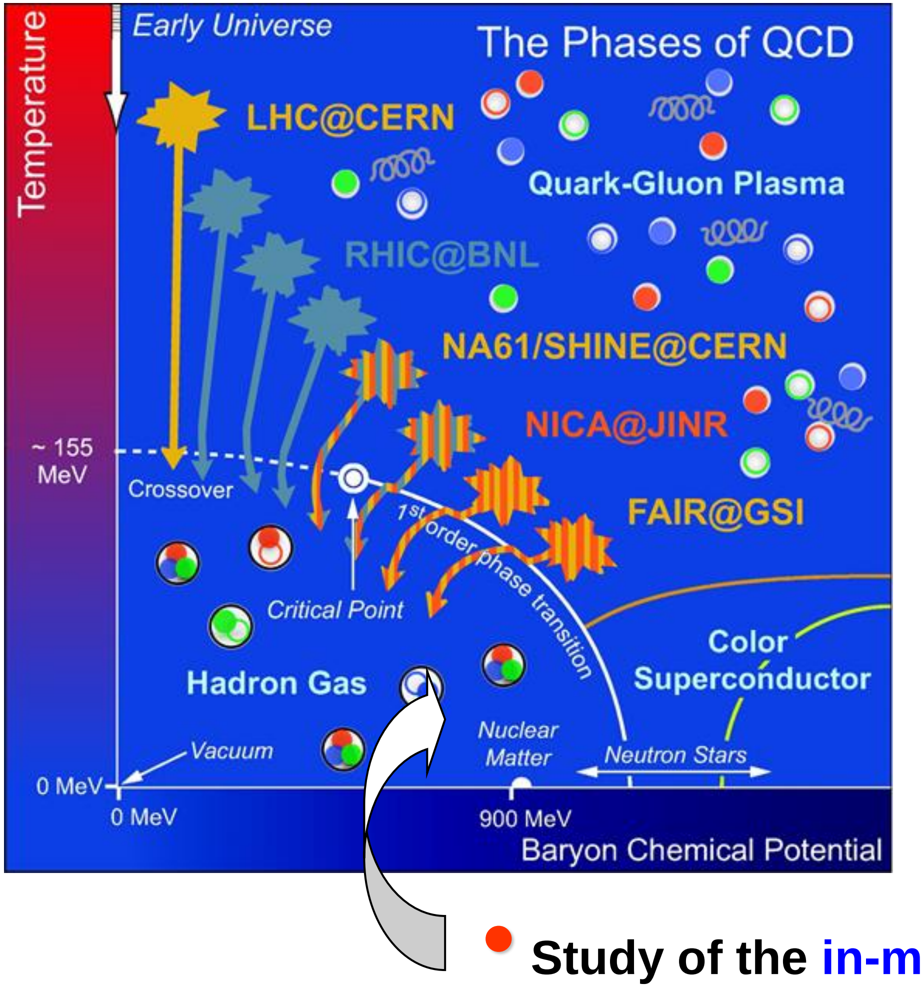


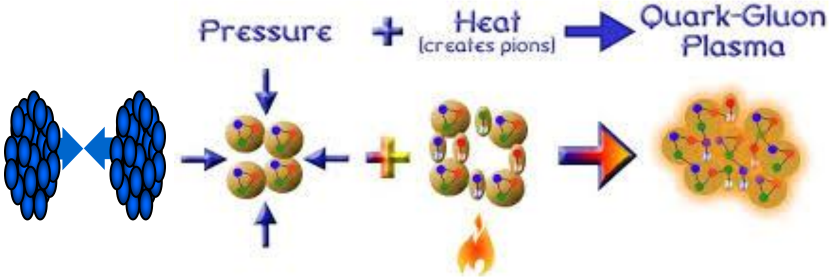


The ‚holy grail‘ of heavy-ion physics



The phase diagram of QCD → thermal properties of QCD in the (T, μ_B) plain



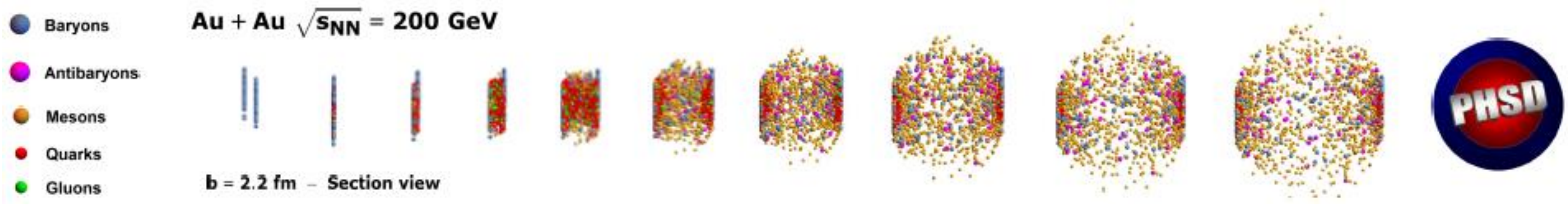


- **Equation-of-State of hot and dense matter?**
- **Study of the phase transition from hadronic to partonic matter – Quark-Gluon-Plasma**
- **Search for a critical point**
- **Search for signatures of chiral symmetry restoration**
- **Study of the in-medium properties of hadrons at high baryon density and temperature**

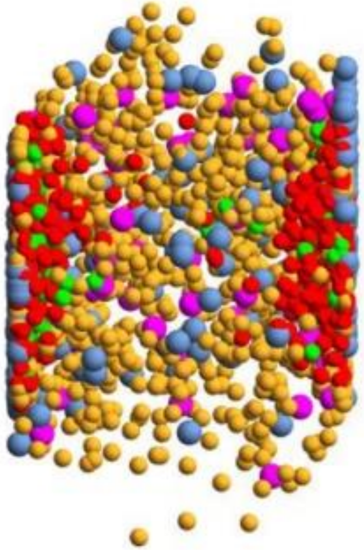
Dynamical description of heavy-ion collisions

The goal:
to study the properties of strongly interacting matter under extreme conditions from a microscopic point of view

Realization:
to develop a dynamical microscopic transport approach
1) applicable for strongly interacting systems,
which includes:
2) phase transition from hadronic matter to QGP
3) chiral symmetry restoration



Development fo the microscopic transport theory: from BUU to Kadanoff-Baym dynamics



History: semi-classical BUU equation


Ludwig Boltzmann

Boltzmann-Uehling-Uhlenbeck equation (non-relativistic formulation)
- propagation of particles in the **self-generated Hartree-Fock mean-field potential *U(r,t)*** with an **on-shell collision term:**

$$\frac{\partial}{\partial t} f(\vec r,\vec p,t) + \frac{\vec p}{m}\vec\nabla_{\vec r}\, f(\vec r,\vec p,t) - \vec\nabla_{\vec r}U(\vec r,t)\,\vec\nabla_{\vec p}\, f(\vec r,\vec p,t) = \left(\frac{\partial f}{\partial t}\right)_{coll}$$

collision term:
elastic and
inelastic reactions

$f(\vec r,\vec p,t)$ is the **single particle phase-space distribution function**
- probability to find the particle at position *r* with momentum *p* at time *t*

- self-generated **Hartree-Fock mean-field potential:**

$$U(\vec r,t) = \frac{1}{(2\pi\hbar)^3}\sum_{\beta_{occ}}\int d^3r'\, d^3p\;\; V(\vec r-\vec r',t)\;f(\vec r',\vec p,t)\; + \;(Fock\;\;term)$$

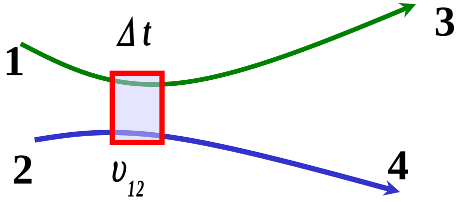
- **Collision term** for 1+2→3+4 (let‘s consider fermions) :

$$I_{coll} = \frac{4}{(2\pi)^3}\int d^3p_2\, d^3p_3\int d\Omega\;\;|\upsilon_{12}|\,\delta^3(\vec p_1+\vec p_2-\vec p_3-\vec p_4)\cdot\frac{d\sigma}{d\Omega}(1+2\to 3+4)\cdot P$$

Probability including **Pauli blocking of fermions:**

$$P = \underline{f_3 f_4(1-f_1)(1-f_2)} - \underline{f_1 f_2(1-f_3)(1-f_4)}$$

Gain term: 3+4→1+2   **Loss term: 1+2→3+4**



History: developments of relativistic transport models

Low energy HIC → High energy HIC

Non-relativistic semi-classical BUU

→ Relativistic transport models

‘Numerical simulation of medium energy heavy ion reactions’,
J. Aichelin and G. Bertsch, Phys.Rev.C 31 (1985) 1730-1738

‘Relativistic Vlasov-Uehling-Uhlenbeck model for heavy-ion collisions’
Che-Ming Ko, Qi Li, Phys.Rev. C37 (1988) 2270

‘Covariant Boltzmann-Uehling-Uhlenbeck approach for heavy-ion collisions’
Bernhard Blaettel, Volker Koch, Wolfgang Cassing, Ulrich Mosel, Phys.Rev. C38 (1988) 1767;

‘Relativistic BUU approach with momentum dependent mean fields’
T. Maruyama, B. Blaettel, W. Cassing, A. Lang, U. Mosel, K. Weber, Phys.Lett. B297 (1992) 228

‘The Relativistic Landau-Vlasov method in heavy ion collisions’
C. Fuchs, H.H. Wolter, Nucl.Phys. A589 (1995) 732

. . .

* Alternative to BUU: QMD – non-covariant EoM (contrary to BUU), but not a mean-field!

Covariant transport equation

- **Covariant relativistic on-shell BUU equation :**

  from many-body theory by connected Green functions in phase-space + mean-field limit for the propagation part (VUU)

$$\left\{ \left(\Pi_\mu - \Pi_\nu (\partial^p_\mu U^\nu_V) - m^*(\partial^p_\mu U^\nu_S)\right) \partial^\mu_x + \left(\Pi_\nu (\partial^x_\mu U^\nu_V) + m^*(\partial^x_\mu U^\nu_S)\right) \partial^\mu_p \right\} f(x,p) = I_{coll}$$

$$I_{coll} \equiv \sum_{2,3,4} \int d2\ d3\ d4\ \ [G^+G]_{1+2\to 3+4}\ \delta^4(\Pi + \Pi_2 - \Pi_3 - \Pi_4)$$

$$d2 \equiv \frac{d^3 p_2}{E_2}$$

$$\times \{\ f(x,p_3)\ f(x,p_4)\ (1 - f(x,p))\ (1 - f(x,p_2))$$

Gain term
3+4→1+2

$$-\ f(x,p)\ f(x,p_2)\ (1 - f(x,p_3))\ (1 - f(x,p_4))\ \}$$

Loss term
1+2→3+4

where $\partial^x_\mu \equiv (\partial_t, \vec{\nabla}_r)$

$m^*(x,p) = m + U_s(x,p)$ - effective mass

$\Pi_\mu(x,p) = p_\mu - U_\mu(x,p)$ - effective momentum

$U_s(x,p)$, $U_\mu(x,p)$ are scalar and vector part of particle self-energies

$\delta(\Pi_\mu \Pi^\mu - m^{*2})$ – mass-shell constraint

W. Ehehalt, W. Cassing, Nucl. Phys. A 602 (1996) 449

Dynamical transport model: collision terms

- ☐ **BUU eq. for different particles of type *i=1,…n***

Hadronic transport models:
BUU, IQMD, UrQMD, GiBUU,
HSD, JAM, SMASH, ...

$$\boxed{Df_i \equiv \frac{d}{dt} f_i = I_{coll}\left[f_1, f_2, \ldots, f_n\right]}$$

Drift term=Vlasov eq. collision term

$$i: \quad Baryons: \ p, n, \Delta(1232), N(1440), N(1535), \ldots, \Lambda, \Sigma, \Sigma^*, \Xi, \Omega; \ \Lambda_C$$

$$Mesons: \ \pi, \eta, K, \bar{K}, \rho, \omega, K^*, \eta', \phi, a_1, \ldots, D, \bar{D}, J/\Psi, \Psi', \ldots$$

➔ coupled set of BUU equations for different particles of type *i=1,…n*

$$\begin{cases} Df_N = I_{coll}\left[f_N, f_\Delta, f_{N(1440)}, \ldots, f_\pi, f_\rho, \ldots\right] \\ Df_\Delta = I_{coll}\left[f_N, f_\Delta, f_{N(1440)}, \ldots, f_\pi, f_\rho, \ldots\right] \\ \ldots \\ Df_\pi = I_{coll}\left[f_N, f_\Delta, f_{N(1440)}, \ldots, f_\pi, f_\rho, \ldots\right] \\ \ldots \end{cases}$$

Elementary hadronic interactions

Consider **all possible interactions – eleastic and inelastic collisions -** for the sytem of ***(N,R,m)***, where ***N***-nucleons, ***R***- resonances, ***m***-mesons, and **resonance decays**

Low energy collisions:

- **binary 2←→2 and 2←→3(4) reactions**
- **1←→2 : formation and decay of baryonic and mesonic resonances**

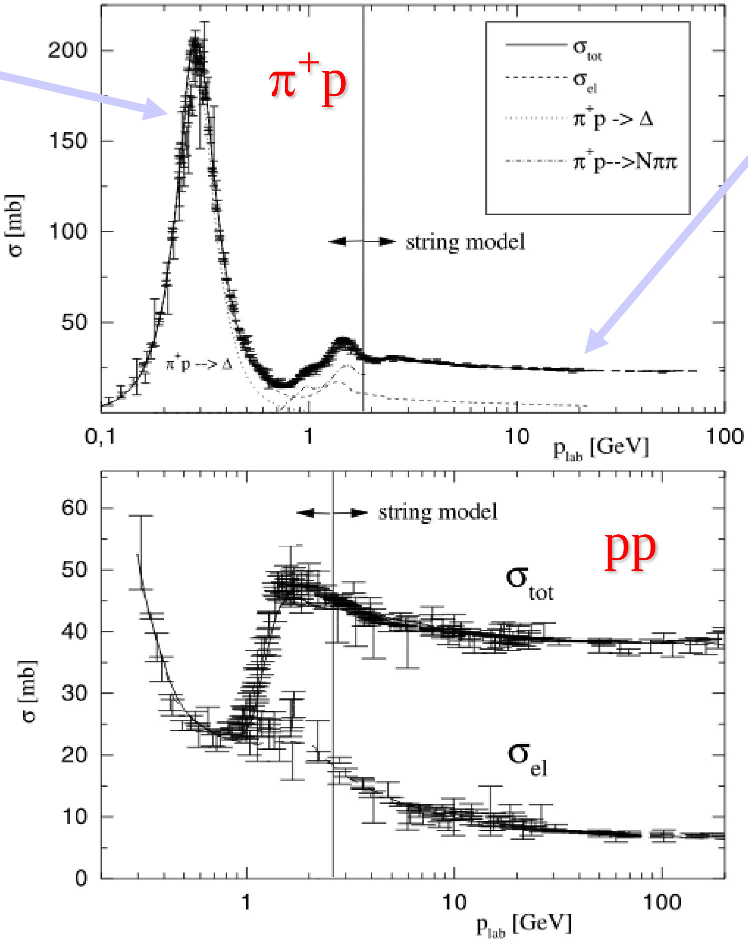
BB ←→ B´B´
BB ←→ B´B´m
mB ←→ m´B´
mB ←→ B´
mm ←→ m´m´
mm ←→ m´ . . .

Baryons:

B = p, n, Δ(1232), N(1440), N(1535), ...

Mesons:

M = π, η, ρ, ω, φ, ...



High energy collisions:

(above $s^{1/2}$~2.5 GeV)

Inclusive particle production:

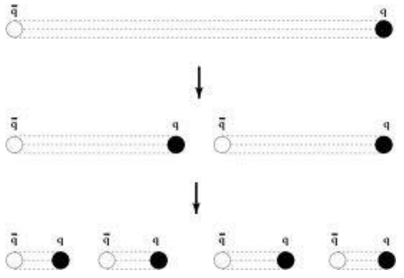
BB→X , mB→X, mm→X

X =many particles

described by

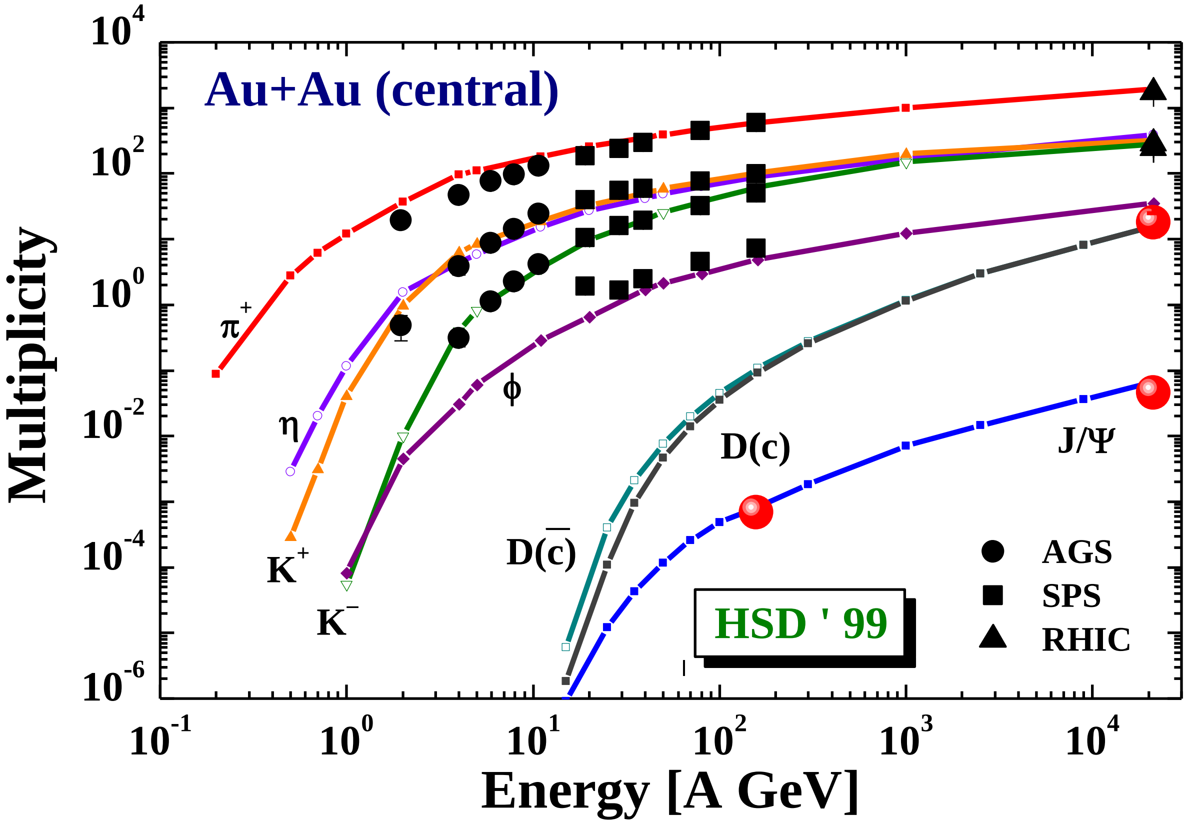
string formation and decay
(string = excited color singlet states *q-qq*, *q-qbar*)

using LUND string model



Microscopic transport model for heavy-ion collisions

- very good description of particle production in pp, pA, pA, AA reactions
- unique description of nuclear dynamics from low (~100 MeV) to ultrarelativistic (>20 TeV) energies



HSD predictions from 1999; data - from the new millenium

W. Cassing, E.B., Phys.Rept. 308 (1999) 65

From weakly to strongly interacting systems

Properties of matter (on hadronic and partonic levels) in heavy-ion collisions:
QGP – strongly interacting system! Degrees of freedom – dressed partons
Hadronic matter – in-medium effects – modification of hadron properties
at finite T,μ_B (vector mesons, strange mesons)

Many-body theory:
Strong interaction ➔ large width = short life-time
➔ broad spectral function ➔ quantum object

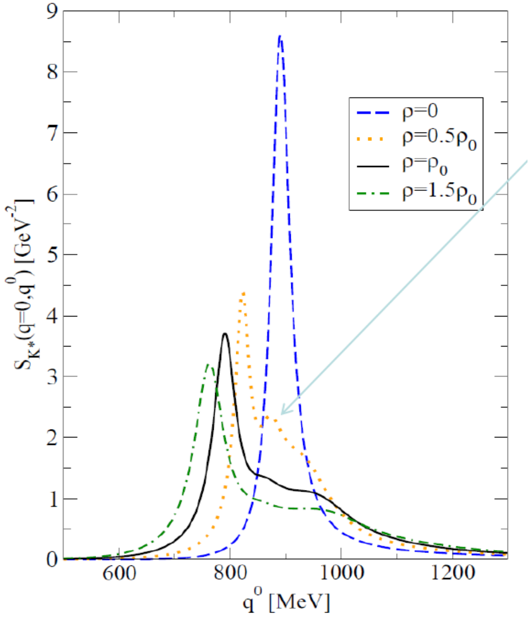
- **How to describe the dynamics of broad strongly interacting quantum states in transport theory?**

- ☐ **semi-classical BUU**

first order gradient expansion of quantum Kadanoff-Baym equations

- ☐ **generalized transport equations based on Kadanoff-Baym dynamics**

Kbar* spectral function



Λ(1783)N^{-1} and Σ(1830)N^{-1} exitations

Barcelona / Valencia group

Dynamical description of strongly interacting systems

☐ **Quantum field theory ➔**

Kadanoff-Baym dynamics for resummed single-particle Green functions $S^<$

(1962)

$$\hat{S}^{-1}_{0x} \; S^{<}_{xy} \;=\; \Sigma^{ret}_{xz} \odot S^{<}_{zy} \;+\; \Sigma^{<}_{xz} \odot S^{adv}_{zy}$$

Integration over the intermediate spacetime

Green functions $S^<$ / self-energies Σ :

$$iS^{<}_{xy} = \eta \langle \{ \Phi^{+}(y) \Phi(x) \} \rangle$$

$$iS^{>}_{xy} = \langle \{ \Phi(y) \Phi^{+}(x) \} \rangle$$

$$iS^{c}_{xy} = \langle T^{c} \{ \Phi(x) \Phi^{+}(y) \} \rangle \quad - causal$$

$$iS^{a}_{xy} = \langle T^{a} \{ \Phi(x) \Phi^{+}(y) \} \rangle \quad - anticausal$$

$$S^{ret}_{xy} = S^{c}_{xy} - S^{<}_{xy} = S^{>}_{xy} - S^{a}_{xy} \quad - retarded$$

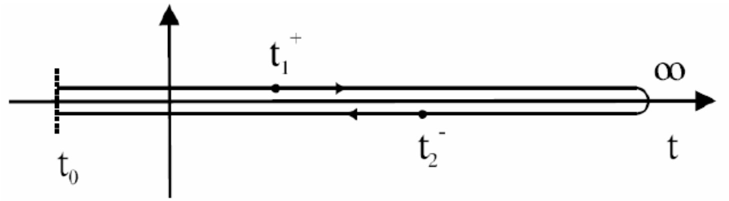
$$S^{adv}_{xy} = S^{c}_{xy} - S^{>}_{xy} = S^{<}_{xy} - S^{a}_{xy} \quad - advanced$$

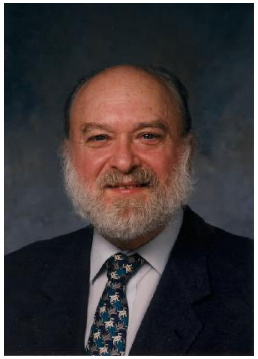
$$\hat{S}^{-1}_{0x} \equiv -(\partial^{\mu}_{x} \partial^{x}_{\mu} + M^{2}_{0})$$

$$\eta = \pm 1 (bosons / \; fermions)$$

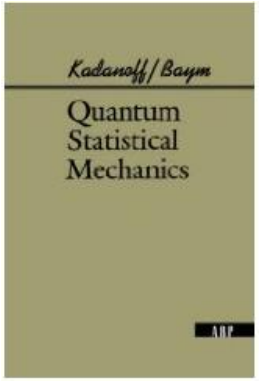
$$T^{a} (T^{c}) - (anti-) time - ordering \; operator$$

Real-time (Keldysh-) Contour





Leo Kadanoff



Gordon Baym

1st application for spacially homodeneous system with deformed Fermi sphere:
P. Danielewicz, Ann. Phys. 152, 305 (1984); … H.S. Köhler, Phys. Rev. 51, 3232 (1995); …

From Kadanoff-Baym equations to generalized transport equations

After the first-order gradient expansion of the Wigner transformed Kadanoff-Baym equations and separation into the real and imaginary parts one gets:

Generalized transport equations (GTE):

drift term **Vlasov term** **backflow term** **collision term = ‚gain‘ - ‚loss‘ term**

$$\diamond\{P^2 - M_0^2 - Re\Sigma^{ret}_{XP}\}\{S^<_{XP}\} - \boxed{\diamond\{\Sigma^<_{XP}\}\{Re S^{ret}_{XP}\}} = \frac{i}{2}\left[\Sigma^>_{XP}S^<_{XP} - \Sigma^<_{XP}S^>_{XP}\right]$$

Backflow term incorporates the off-shell behavior in the particle propagation

! vanishes in the quasiparticle limit $A_{XP} \to \delta(p^2-M^2)$

- ☐ **Propagation of the Green‘s function $iS^<_{XP}=A_{XP}N_{XP}$, which carries information not only on the number of particles (N_{XP}), but also on their properties, interactions and correlations (via A_{XP})**

W. Botermans, R. Malfliet, Phys. Rep. 198 (1990) 115

- ☐ **Spectral function:**

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - Re\Sigma^{ret}_{XP})^2 + \Gamma^2_{XP}/4}$$

Reaction rate of particle (at space-time position X):

$$\Gamma_{XP} = -Im\,\Sigma^{ret}_{XP} = 2\,p_0\Gamma$$

where Γ is a ‚width‘ of spectral function

4-dimentional generalizaton of the Poisson-bracket:

$$\diamond\{F_1\}\{F_2\} := \frac{1}{2}\left(\frac{\partial F_1}{\partial X_\mu}\frac{\partial F_2}{\partial P^\mu} - \frac{\partial F_1}{\partial P_\mu}\frac{\partial F_2}{\partial X^\mu}\right)$$

- ☐ **Life time** $\tau = \frac{\hbar c}{\Gamma}$

W. Cassing , S. Juchem, NPA 665 (2000) 377; 672 (2000) 417; 677 (2000) 445

Generalalized testparticle off-shell equations of motion

W. Cassing , S. Juchem, NPA 665 (2000) 377; 672 (2000) 417; 677 (2000) 445

- Employ **testparticle Ansatz** for the real valued quantity $i\, S^{<}_{XP}$

$$F_{XP} \; = A_{XP} N_{XP} = \; i\, S^{<}_{XP} \; \sim \; \sum_{i=1}^{N} \delta^{(3)}(\vec{X} - \vec{X}_i(t)) \;\; \delta^{(3)}(\vec{P} - \vec{P}_i(t)) \;\; \delta(P_0 - \epsilon_i(t))$$

insert in generalized transport equations and determine equations of motion !

➔ Generalized testparticle Cassing-Juchem off-shell equations of motion for the time-like particles:

$$\frac{d\vec{X}_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[2\,\vec{P}_i + \vec{\nabla}_{P_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right],$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} Re\Sigma^{ret}_{i} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right],$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial Re\Sigma^{ret}_{(i)}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right],$$

with $F_{(i)} \equiv F(t, \vec{X}_i(t), \vec{P}_i(t), \epsilon_i(t))$

$$C_{(i)} = \frac{1}{2\epsilon_i} \left[\frac{\partial}{\partial \epsilon_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial}{\partial \epsilon_i} \Gamma_{(i)} \right]$$

Realized in PHSD

Note: the common factor $1/(1-C_{(i)})$ can be absorbed in an ‚eigentime‘ of particle (i) !

On-shell limits: from KB to BUU

☐ **Γ(X,P) → 0**

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - Re\Sigma^{ret}_{XP})^2 + \Gamma^2_{XP}/4}$$

quasiparticle approximation :

A_{XP} = 2 p δ(P^2-M_0^2)

☐ **Γ(X,P) such that**

$\nabla_X \Gamma = 0$ and $\nabla_P \Gamma = 0$ ⟺

E.g.: Γ = const

Γ=Γ_{vacuum} (M)

‚Vacuum' spectral function with constant or mass dependent width Γ:

i.e. spectral function A_{XP} does NOT change the shape (and pole position) during propagation through the medium (no density-, T-dependence)

In on-shell limits the ‚backflow term' - which incorporates the off-shell behavior in the particle propagation - vanishes:

$$\frac{d\vec{X}_i}{dt} = \frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[2\,\vec{P}_i + \vec{\nabla}_{P_i} Re\Sigma^{ret}_{(i)} + \xcancel{\frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)}} \right],$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} Re\Sigma^{ret}_i + \xcancel{\frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)}} \right],$$

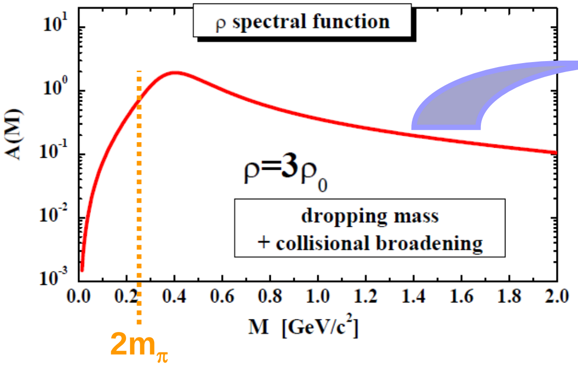
$$\frac{d\epsilon_i}{dt} = \frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial Re\Sigma^{ret}_{(i)}}{\partial t} + \xcancel{\frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t}} \right],$$

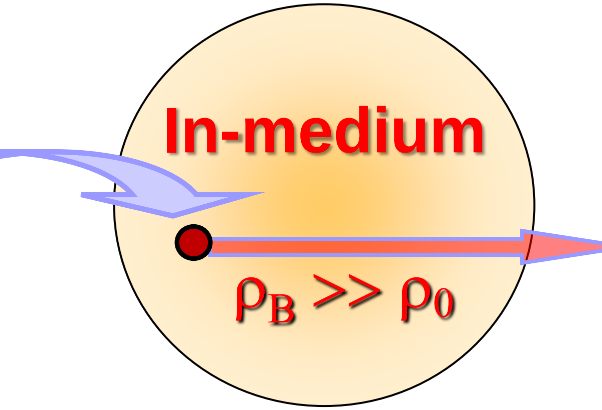
Hamilton equations of motion (independent on Γ) → BUU limit

W. Cassing, Eur. Phys. J. ST 168 (2009) 3

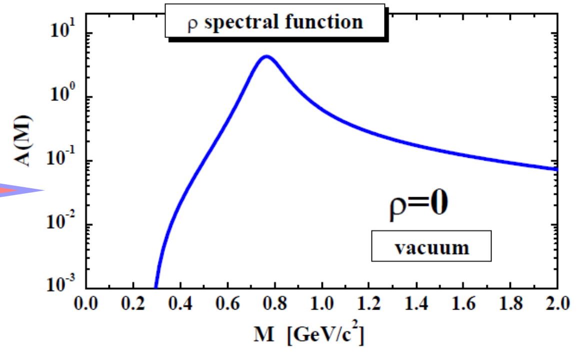
Off-shell vs. on-shell transport dynamics

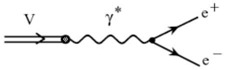
In-medium:
production of broad state



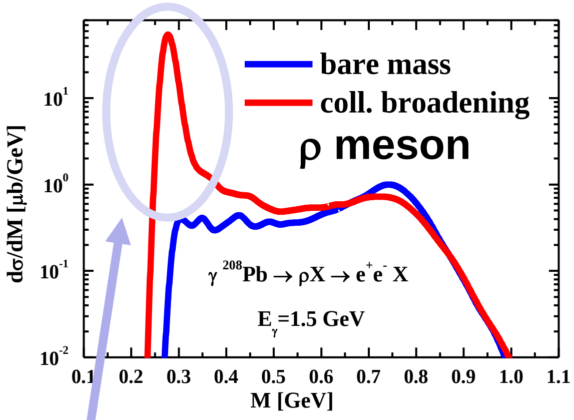


Vacuum ($\rho=0$) state





Dilepton invariant mass spectra

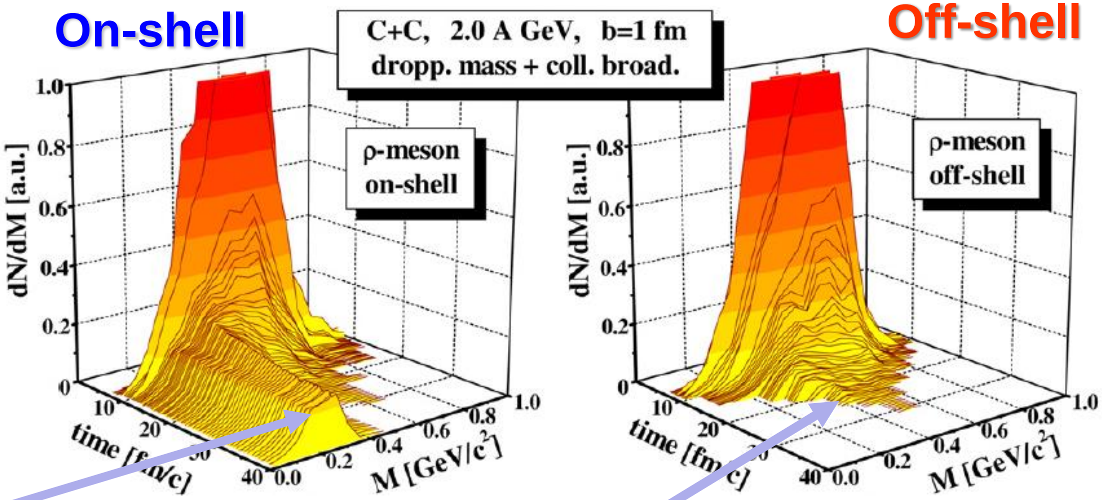


On-shell BUU:
low mass ρ and ω mesons live forever (and shine ‚fake‘ dileptons)!

(e)GiBUU: M. Effenberger et al, PRC60 (1999) 027601

Mass distribution of ρ mesons vs time

On-shell **Off-shell**



EB & Cassing, NPA 807 (2008) 214

The off-shell spectral function becomes on-shell in the vacuum dynamically by propagation through the medium!

Collision term in off-shell transport approach

Collision term for reaction 1+2->3+4:

spectral functions:

$$\underline{I_{coll}(X,\vec{P},M^2)} = Tr_2 Tr_3 Tr_4 \underline{A(X,\vec{P},M^2)A(X,\vec{P}_2,M_2^2)A(X,\vec{P}_3,M_3^2)A(X,\vec{P}_4,M_4^2)}$$

$$\underline{|G((\vec{P},M^2)+(\vec{P}_2,M_2^2)\to(\vec{P}_3,M_3^2)+(\vec{P}_4,M_4^2))|^2_{\mathcal{A},\mathcal{S}}}\ \delta^{(4)}(P+P_2-P_3-P_4)$$

$$[\underbrace{N_{X\vec{P}_3M_3^2}\,N_{X\vec{P}_4M_4^2}\,\bar{f}_{X\vec{P}M^2}\,\bar{f}_{X\vec{P}_2M_2^2}}_{\text{‚gain‘ term}} - \underbrace{N_{X\vec{P}M^2}\,N_{X\vec{P}_2M_2^2}\,\bar{f}_{X\vec{P}_3M_3^2}\,\bar{f}_{X\vec{P}_4M_4^2}}_{\text{‚loss‘ term}}]$$

with $\bar{f}_{X\vec{P}M^2} = 1+\eta\, N_{X\vec{P}M^2}$ and $\eta = \pm 1$ for bosons/fermions, respectively.

The trace over particles 2,3,4 reads explicitly

for fermions

$$Tr_2 = \sum_{\sigma_2,\tau_2} \frac{1}{(2\pi)^4}\int d^3P_2 \boxed{\frac{dM_2^2}{2\sqrt{\vec{P}_2^2+M_2^2}}}$$

for bosons

$$Tr_2 = \sum_{\sigma_2,\tau_2} \frac{1}{(2\pi)^4}\int d^3P_2 \boxed{\frac{dP_{0,2}^2}{2}}$$

additional integration

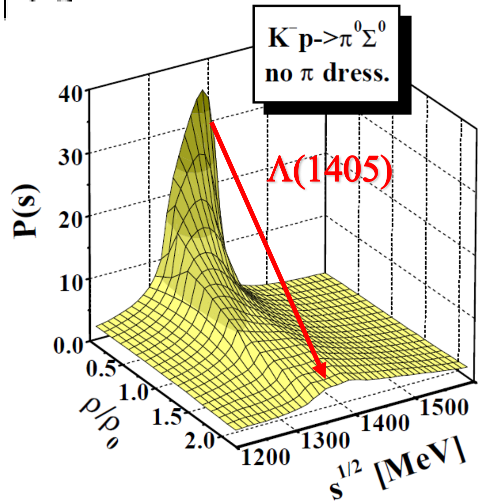
The off-shell transport approach and the particle spectral functions are fully determined once the in-medium transition amplitudes G are known in their off-shell dependence!

In-medium transition rates: G-matrix approach

Need to know in-medium transition amplitudes G and their off-shell dependence

$$|G((\vec{P}, M^2) + (\vec{P}_2, M_2^2) \rightarrow (\vec{P}_3, M_3^2) + (\vec{P}_4, M_4^2))|^2.$$

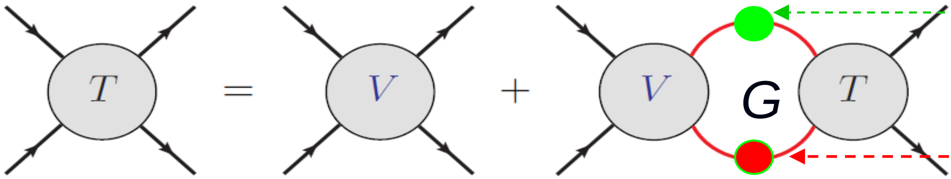
Coupled channel G-matrix approach



Transition probability :

$$\boxed{P_{1+2\rightarrow 3+4}(s) = \int d\cos(\theta) \frac{1}{(2s_1+1)(2s_2+1)} \sum_i \sum_\alpha G^\dagger G}$$

with G(p,ρ,T) - G-matrix from the solution of coupled-channel equations:



● **Meson selfenergy and spectral function**

● **Baryons: Pauli blocking and potential dressing**

$$\mathcal{T}_{ij}(\rho,T) = V_{ij} + V_{il}\, G_l(\rho,T)\, \mathcal{T}_{lj}(\rho,T)$$

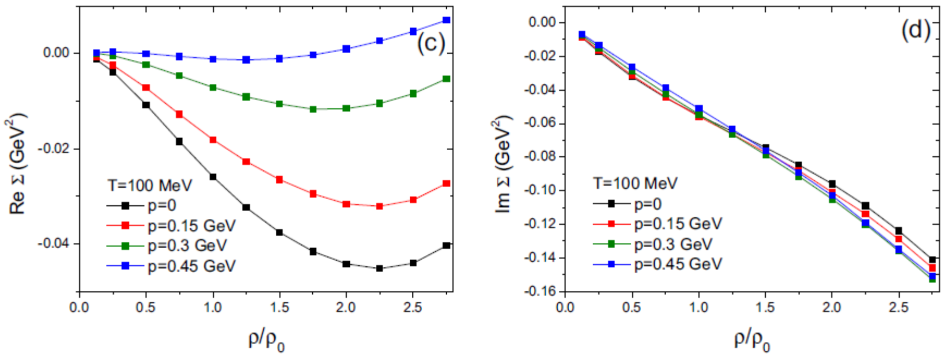
For strangeness:
W. Cassing, L. Tolos, E.B., A. Ramos, NPA727 (2003) 59; D. Cabrera, L. Tolos, J. Aichelin, E.B., PRC C90 (2014) 055207;
T. Song et al., PRC 103, 044901 (2021)

Off-shell dynamics for antikaons at SIS energies

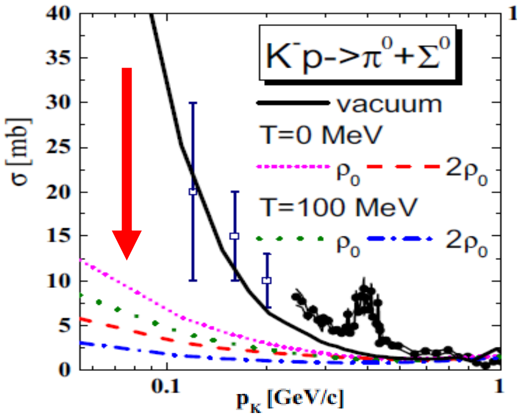
Spectral function of K$^-$ within the **G-matrix** approach:

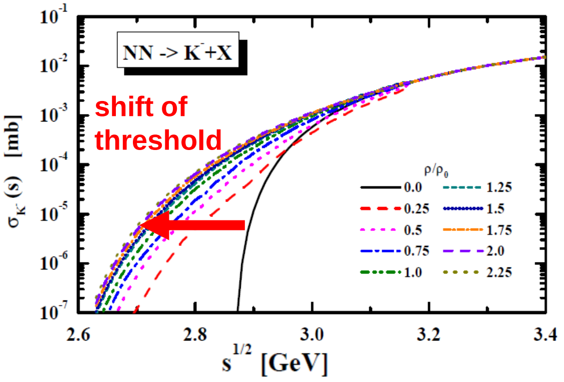
$$S_{\bar{K}}(k_0,\vec{k};T) = -\frac{1}{\pi}\frac{\mathrm{Im}\,\Sigma_{\bar{K}}(k_0,\vec{k};T)}{\left|k_0^2-\vec{k}^{\,2}-m_{\bar{K}}^2-\Sigma_{\bar{K}}(k_0,\vec{k};T)\right|^2}.$$

D. Cabrera et al., Phys.Rev.C 90 (2014) 055207

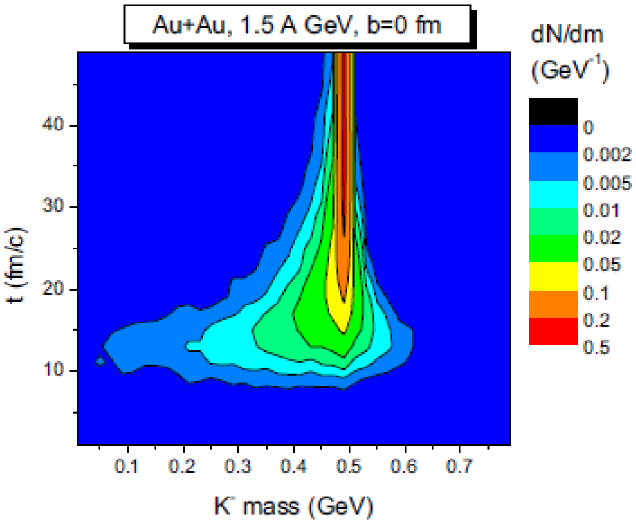


In-medium cross sections for K$^-$ production and absorption are strongly modified in the medium:

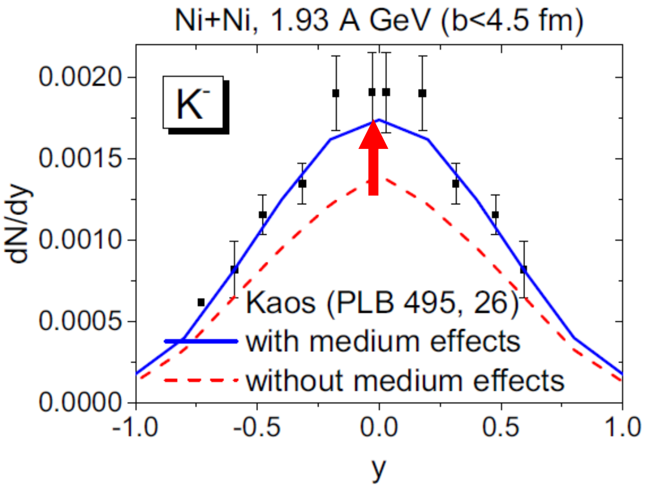




Time evolution of the K$^-$ masses



In-medium effects are mandatory for the description of experimental K$^-$ spectra



T. Song et al., PRC 103, 044901 (2021)

Advantages of Kadanoff-Baym dynamics vs Boltzmann

Kadanoff-Baym equations:

- ☐ propagate two-point Green functions
  $G^<(x,p) \rightarrow A(x,p)*N(x,p)$
  in 8 dimensions $x=(t,\vec{r})$ $p=(p_0,\vec{p})$
- ☐ $G^<$ carries information not only on the occupation number N_{XP}, but also on the particle properties, interactions and correlations via spectral function A_{XP}

Boltzmann equations

- ☐ propagate phase space distribution function $f(\vec{r},\vec{p},t)$ in 6+1 dimensions
- ☐ works well for small coupling
  = weakly interacting system,
  ➔ on-shell approach

- ☐ Applicable for strong coupling = strongly interacting system
- ☐ Dynamically generates a broad spectral function for strong coupling
- ☐ Includes memory effects (time integration) and off-shell transitions in collision term
- ☐ KB can be solved exactly for model cases such as Φ^4 – theory
- ☐ KB can be solved in 1st order gradient expansion in terms of generalized transport equations (in test particle ansatz) for realistic systems of HICs ➔ PHSD

W. Cassing, `*Transport Theories for Strongly-Interacting Systems'*,
Springer Nature: Lecture Notes in Physics 989, 2021
DOI: 10.1007/978-3-030-80295-0

Detailed balance on the level of 2←→n:
treatment of multi-particle collisions in transport approaches

W. Cassing, NPA 700 (2002) 618

Generalized off-shell collision integral for *n* ←→ *m* reactions:

$$I_{coll} = \sum_n \sum_m I_{coll}[n \leftrightarrow m]$$

$$I^i_{coll}[n \leftrightarrow m] =$$

$$\frac{1}{2} N_n^m \sum_\nu \sum_\lambda \left(\frac{1}{(2\pi)^4}\right)^{n+m-1} \int \left(\prod_{j=2}^{n} d^4p_j \;\; A_j(x,p_j)\right) \left(\prod_{k=1}^{m} d^4p_k \; A_k(x,p_k)\right)$$

$$\times A_i(x,p) \; W_{n,m}(p,p_j;i,\nu \mid p_k;\lambda) \; (2\pi)^4 \; \delta^4(p^\mu + \sum_{j=2}^{n} p_j^\mu - \sum_{k=1}^{m} p_k^\mu)$$

$$\times [\tilde{f}_i(x,p) \prod_{k=1}^{m} f_k(x,p_k) \prod_{j=2}^{n} \tilde{f}_j(x,p_j) - f_i(x,p) \prod_{j=2}^{n} f_j(x,p_j) \prod_{k=1}^{m} \tilde{f}_k(x,p_k)].$$

$\tilde{f} = 1 + \eta f$ **is Pauli-blocking or Bose-enhancement factors;**
η=1 for bosons and η=-1 for fermions

$W_{n,m}(p,p_j;i,\nu \mid p_k;\lambda)$ **is a transition matrix element squared**

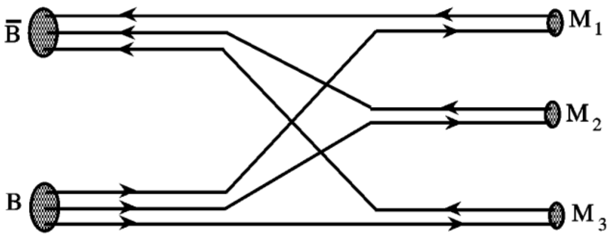
Multi-meson fusion in heavy-ion reactions

W. Cassing, NPA 700 (2002) 618
E. Seifert, W. Cassing, PRC 97 (2018) 024913, (2018) 044907

Multi-meson fusion reactions

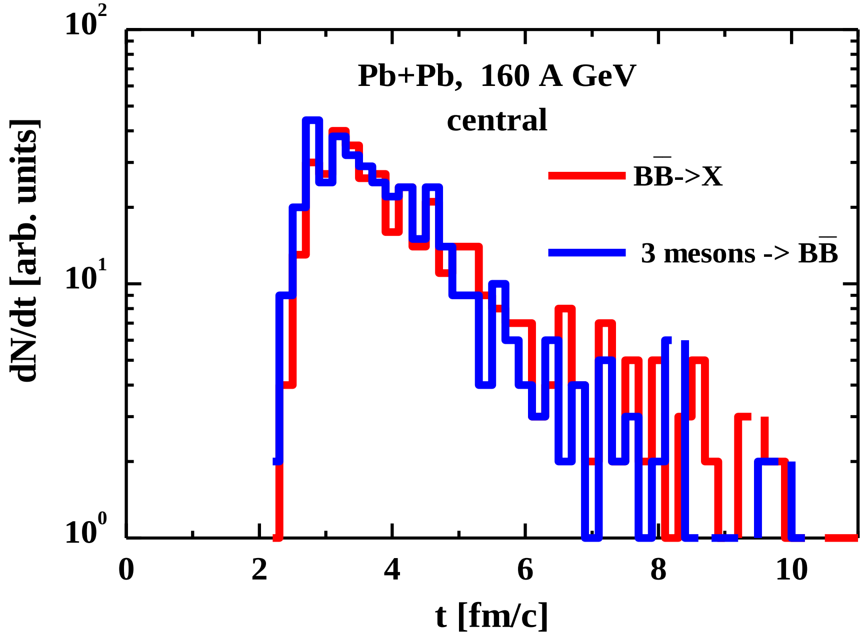
$m_1+m_2+\ldots+m_n \leftarrow\rightarrow$ B+Bbar

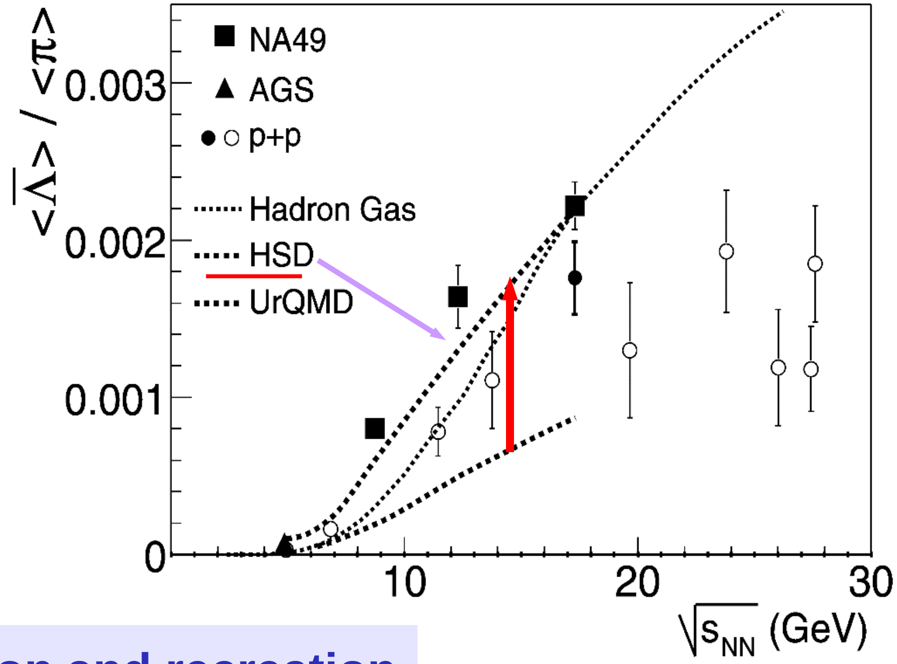
m=π,ρ,ω,.. B=p,Λ,Σ,Ξ,Ω, (>2000 channels)



2←→3

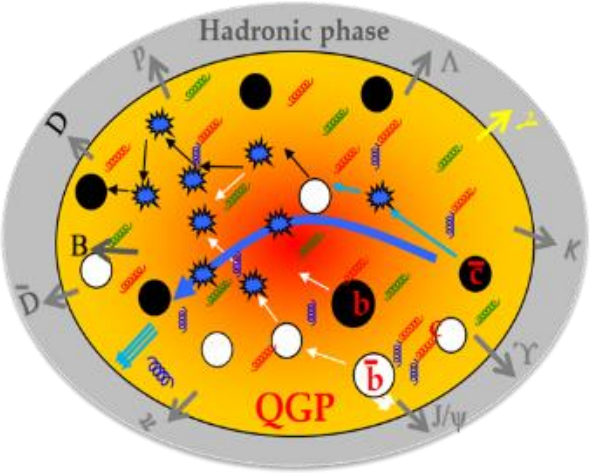
- ☐ important for anti-proton, anti-Λ, anti-Ξ, anti- Ω dynamics !



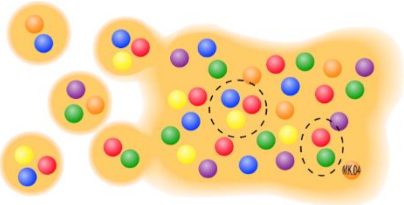


→ approximate equilibrium of annihilation and recreation

Modeling of sQGP in microscopic transport theory

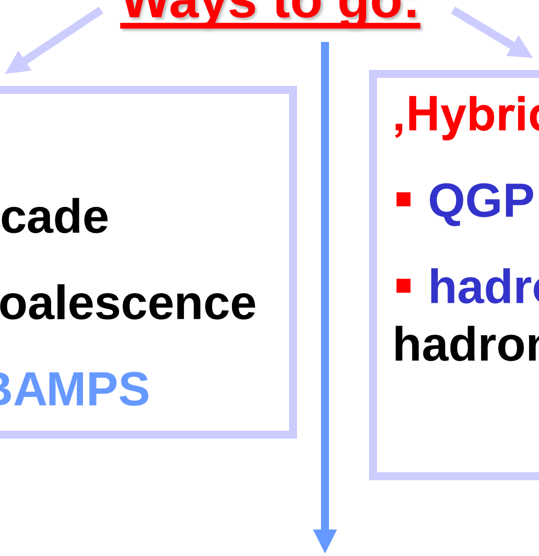


Goal: microscopic transport description of the partonic and hadronic phase



Problems:

- ☐ How to model a QGP phase in line with lQCD data?
- ☐ How to solve the hadronization problem?



Ways to go:

pQCD based models:

- QGP phase: pQCD cascade
- hadronization: quark coalescence

➔ AMPT, HIJING, BAMPS

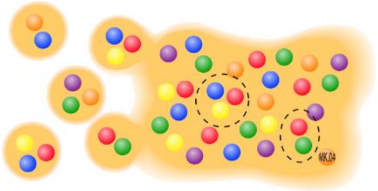
‚Hybrid‘ models:

- QGP phase: hydro with QGP EoS
- hadronic freeze-out: after burner - hadron-string transport model

➔ Hybrid-UrQMD

- microscopic transport description of the partonic and hadronic phase in terms of strongly interacting dynamical quasi-particles and off-shell hadrons

➔ PHSD



Degrees-of-freedom of QGP

For the microscopic transport description of the system one needs to know all degrees of freedom as well as their properties and interactions!

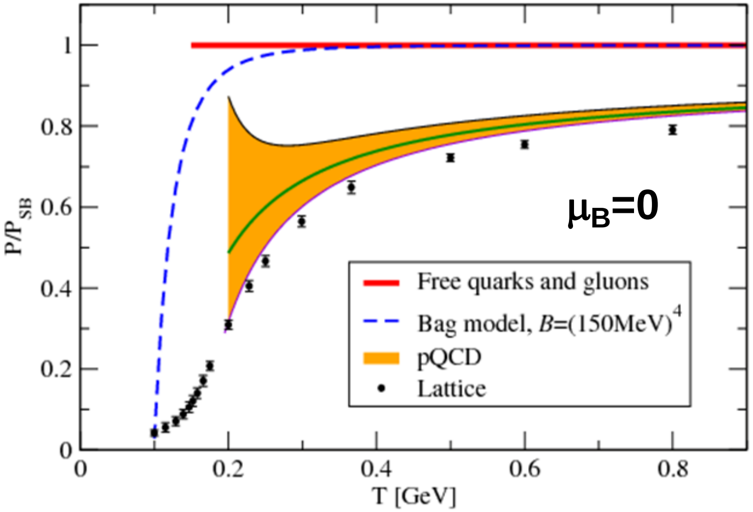
- **lQCD gives QGP EoS at finite μ_B**

! need to be interpreted in terms of degrees-of-freedom

pQCD:

- ☐ **weakly interacting system**
- ☐ **massless quarks and gluons**

How to learn about the degrees-of-freedom of QGP from HIC?
➔ microscopic transport approaches
➔ comparison to HIC experiments



Non-perturbative QCD ← pQCD

Thermal QCD
= QCD at high parton densities:

- ☐ **strongly interacting system**
- ☐ **massive quarks and gluons**

➔ quasiparticles
= effective degrees-of-freedom

QGP: Dynamical QuasiParticle Model (DQPM)

DQPM – effective model for the description of **non-perturbative** (strongly interacting) QCD based on **lQCD EoS**

Degrees-of-freedom: strongly interacting **dynamical quasiparticles** - quarks and gluons

Theoretical basis :

☐ ‚resummed' single-particle Green's functions → quark (gluon) propagator (2PI) : $G_q^{-1} = P^2 - \Sigma_q$

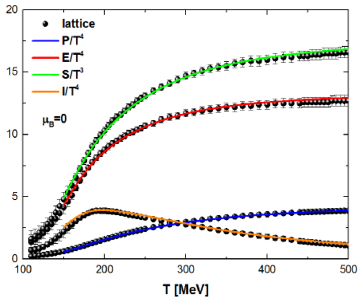
Properties of the quasiparticles are specified by scalar **complex self-energies:** $\Sigma_q = M_q^2 - i2\gamma_q\omega$

$Re\Sigma_q$: **thermal masses** (M_g, M_q); $Im\Sigma_q$: **interaction widths** (γ_g, γ_q) → spectral functions $\rho_q = -2ImG_q$

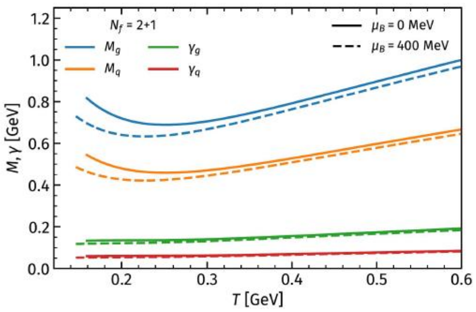
☐ introduce an **ansatz** (HTL; with few parameters) for the (T, μ_B) dependence of masses/widths

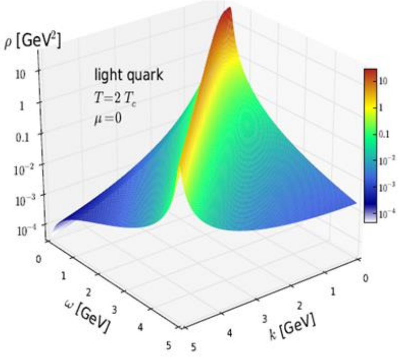
☐ evaluate the **QGP thermodynamics** in equilibrium using the Kadanoff-Baym theory

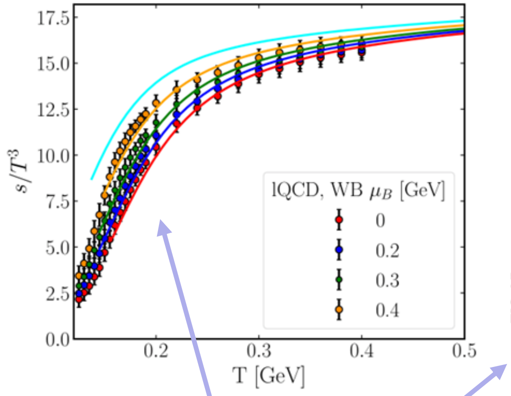
☐ fix DQPM parameters by comparison of the DQPM entropy density to **lQCD** at μ_B =0

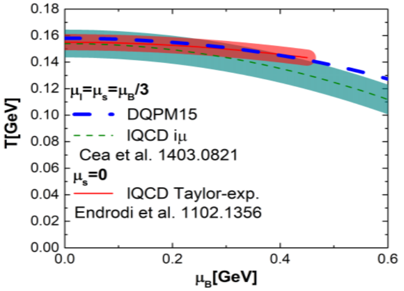


→ **Quasi-particle properties at (T, μ_B) :**









→ very good agreement with lQCD data for QGP thermodynamics at finite (T, μ_B)

- **DQPM** provides **mean-fields** (1PI) for q,g and **effective 2-body partonic interactions** (2PI); gives **transition rates** for the formation of **hadrons** → **sQGP in PHSD**

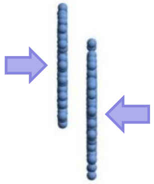
A. Peshier, W. Cassing, PRL 94 (2005) 172301; W. Cassing, NPA 791 (2007) 365: NPA 793 (2007), H. Berrehrah et al, Int.J.Mod.Phys. E25 (2016) 1642003; P. Moreau et al., PRC100 (2019) 014911; O. Soloveva et al., PRC101 (2020) 045203

Parton-Hadron-String-Dynamics (PHSD)

PHSD is a **non-equilibrium microscopic transport approach** for the description of **strongly-interacting hadronic and partonic matter** created in heavy-ion collisions

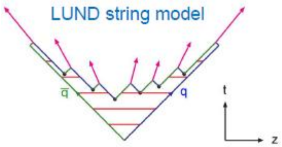
Dynamics: based on the solution of **generalized off-shell transport equations** derived from Kadanoff-Baym many-body theory

Initial A+A collision

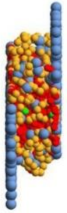


☐ **Initial A+A collisions** :

N+N → string formation → decay to pre-hadrons + leading hadrons



Partonic phase

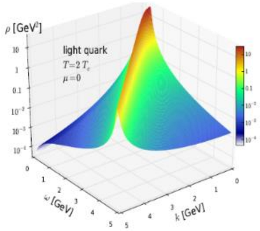


☐ **Formation of QGP stage** if local $\varepsilon > \varepsilon_{critical}$:

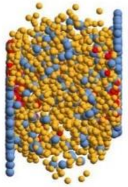
dissolution of pre-hadrons → partons

☐ **Partonic phase - QGP:**

QGP is described by the Dynamical QuasiParticle Model (DQPM) matched to reproduce lattice QCD EoS for finite T and μ_B (crossover)

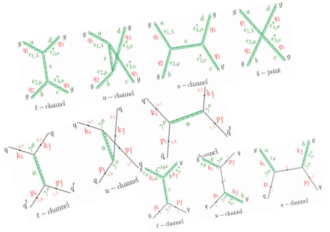


Hadronization

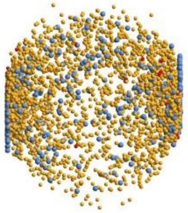


- **Degrees-of-freedom:** strongly interacting quasiparticles: massive quarks and gluons (g,q,q_{bar}) with sizeable collisional widths in a self-generated mean-field potential

- **Interactions:** (quasi-)elastic and inelastic collisions of partons

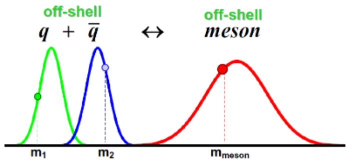


Hadronic phase



☐ **Hadronization** to colorless off-shell mesons and baryons:

Strict 4-momentum and quantum number conservation

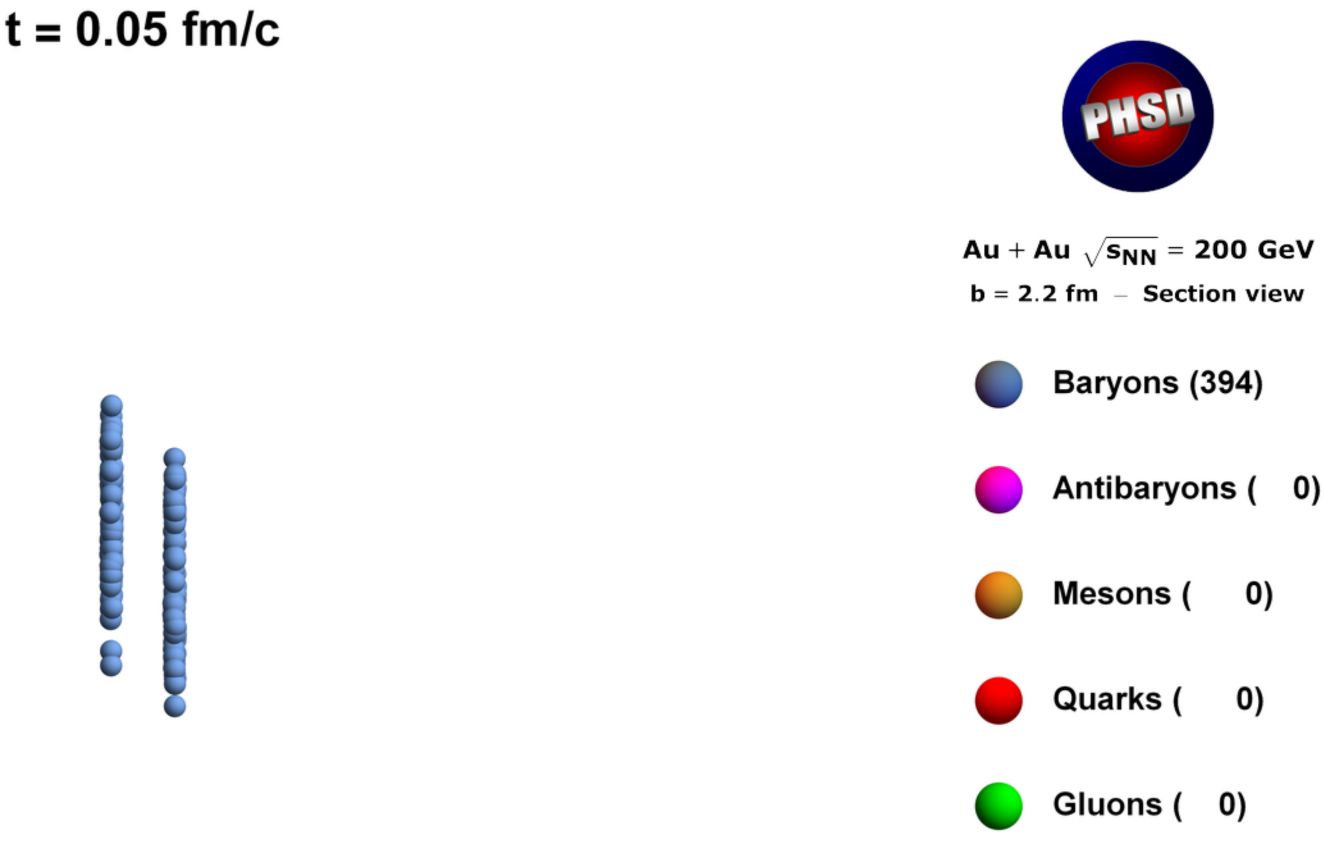


☐ **Hadronic phase: hadron-hadron interactions – off-shell HSD**

including *n* ←→ *m* selected reactions (for strangeness, anti-baryons, deuteron production)

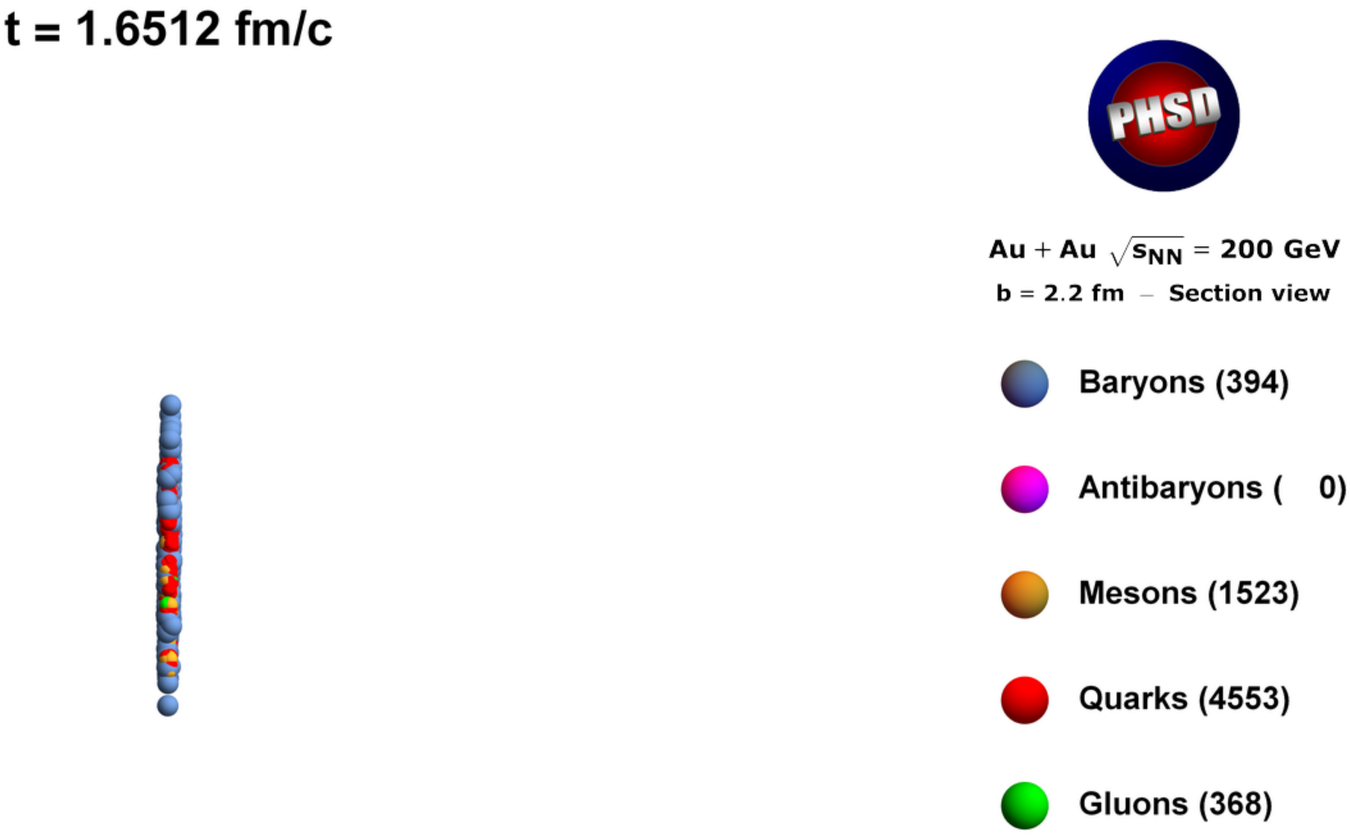
W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; W. Cassing, EPJ ST 168 (2009) 3

Stages of a collision in PHSD



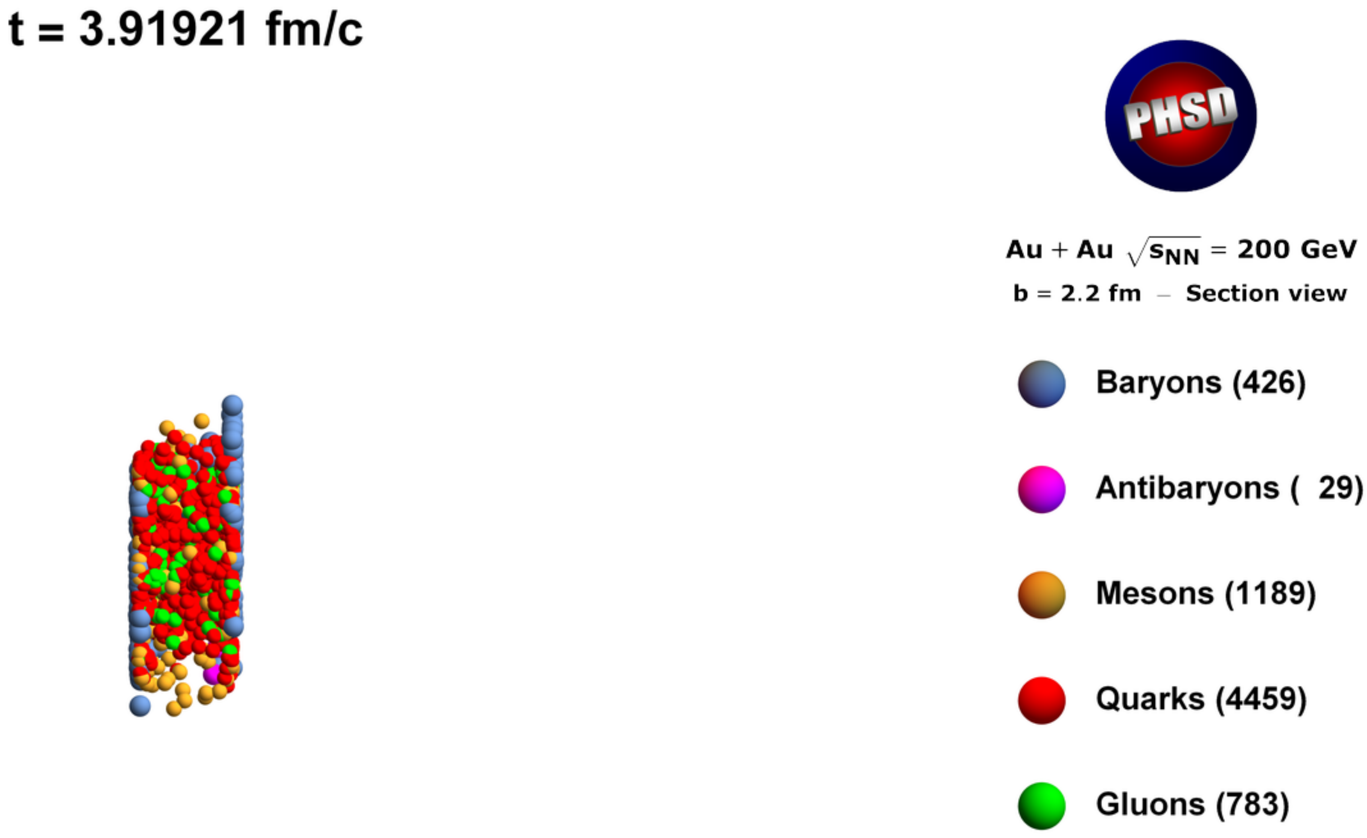
P.Moreau

Stages of a collision in PHSD



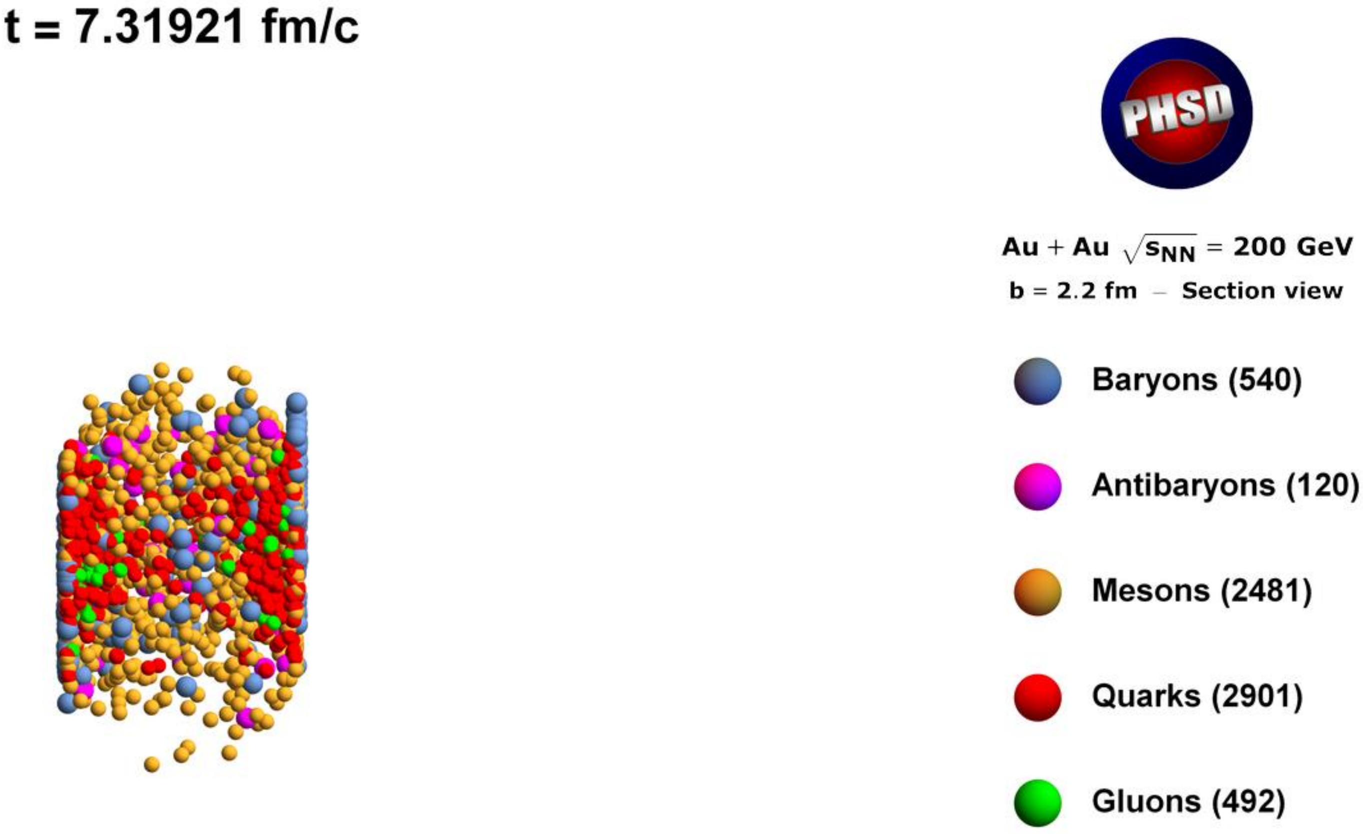
P.Moreau

Stages of a collision in PHSD



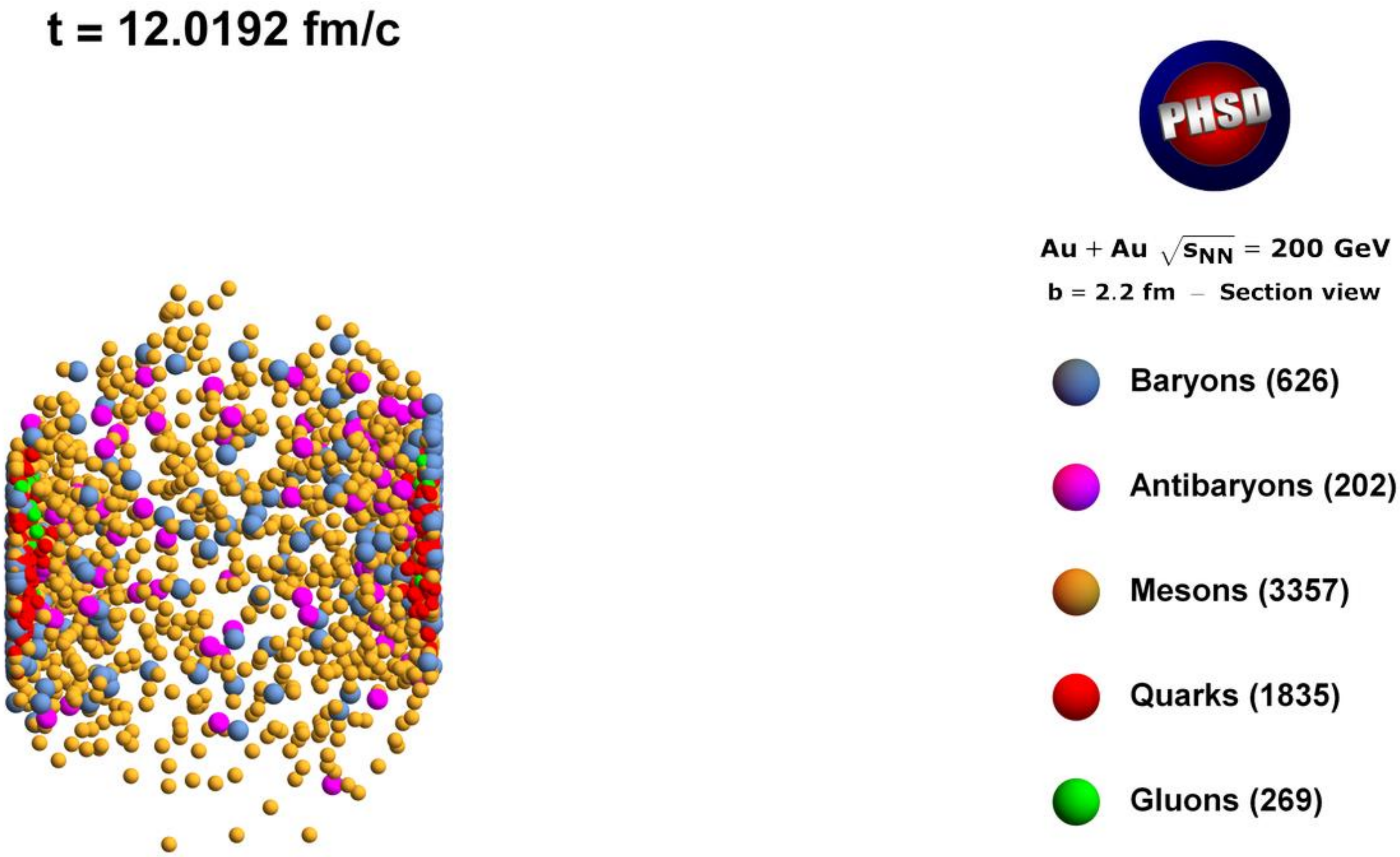
P.Moreau

Stages of a collision in PHSD



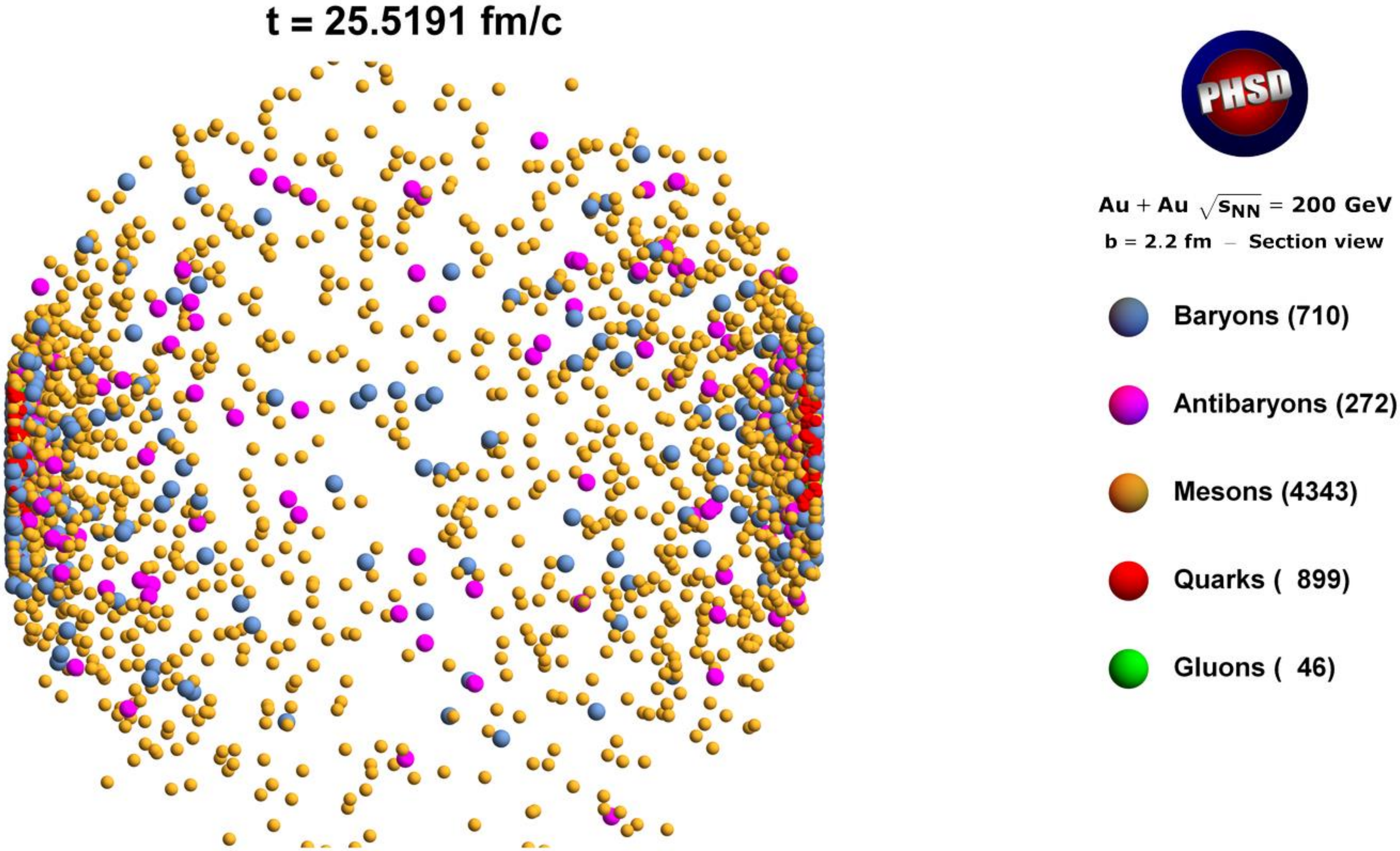
P.Moreau

Stages of a collision in PHSD



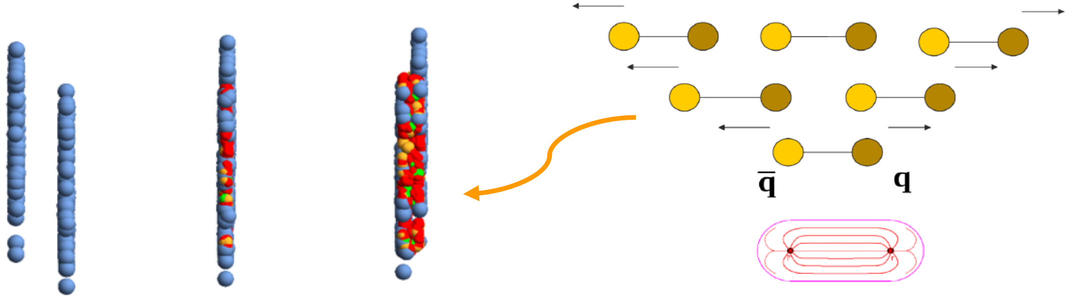
P.Moreau

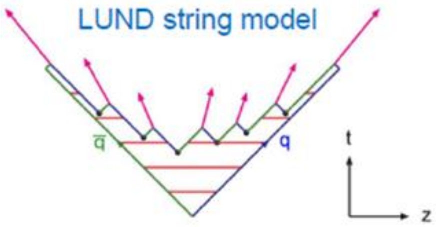
Stages of a collision in PHSD



P.Moreau

Modeling of the chiral symmetry restoration via Schwinger mechanism for string fragmentation in the initial phase of HIC

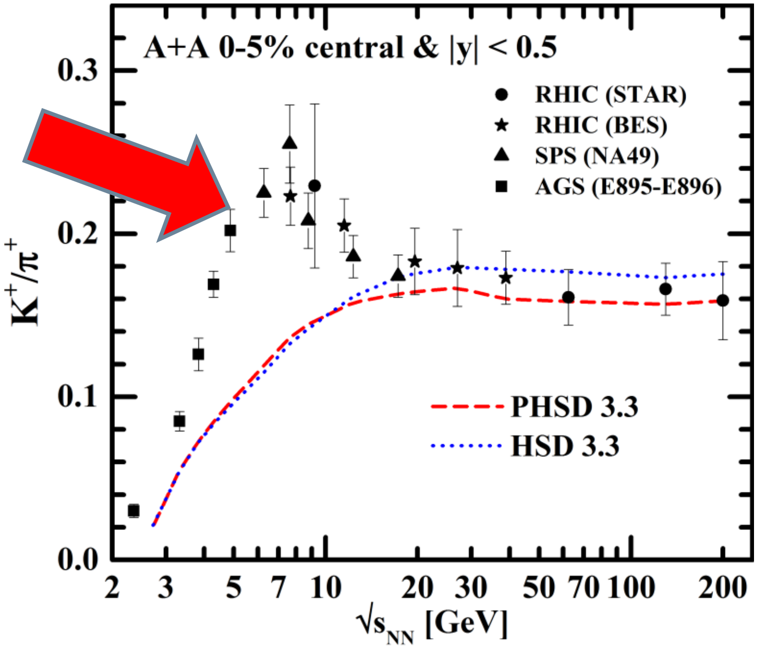




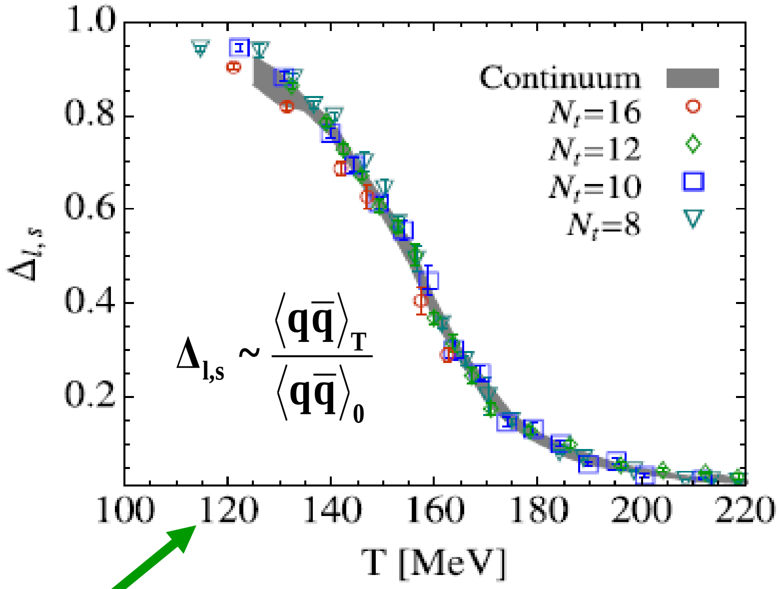
‚Flavour chemistry‘ of HIC: K^+/π^+ ‚horn‘ – 2015

PHSD: even when considering the creation of a QGP phase, the K^+/π^+ ‚horn‘ seen experimentally by NA49 and STAR at a bombarding energy ~30 A GeV (FAIR/NICA energies) remained unexplained (2015)!

➔ The origin of the ‘horn’ is not traced back to deconfinement ?!



lQCD BMW collaboration:



?

Can it be related to chiral symmetry restoration in the initial hadronic phase?!

W. Cassing, A. Palmese, P. Moreau, E.L. Bratkovskaya, PRC 93, 014902 (2016)

Scalar quark condensate in HIC

Non-linear $\sigma - \omega$ model:

$$\frac{\langle q\bar{q}\rangle}{\langle q\bar{q}\rangle_V} = 1 - \frac{\Sigma_\pi}{f_\pi^2 m_\pi^2}\rho_S - \sum_h \frac{\sigma_h \rho_S^h}{f_\pi^2 m_\pi^2}$$

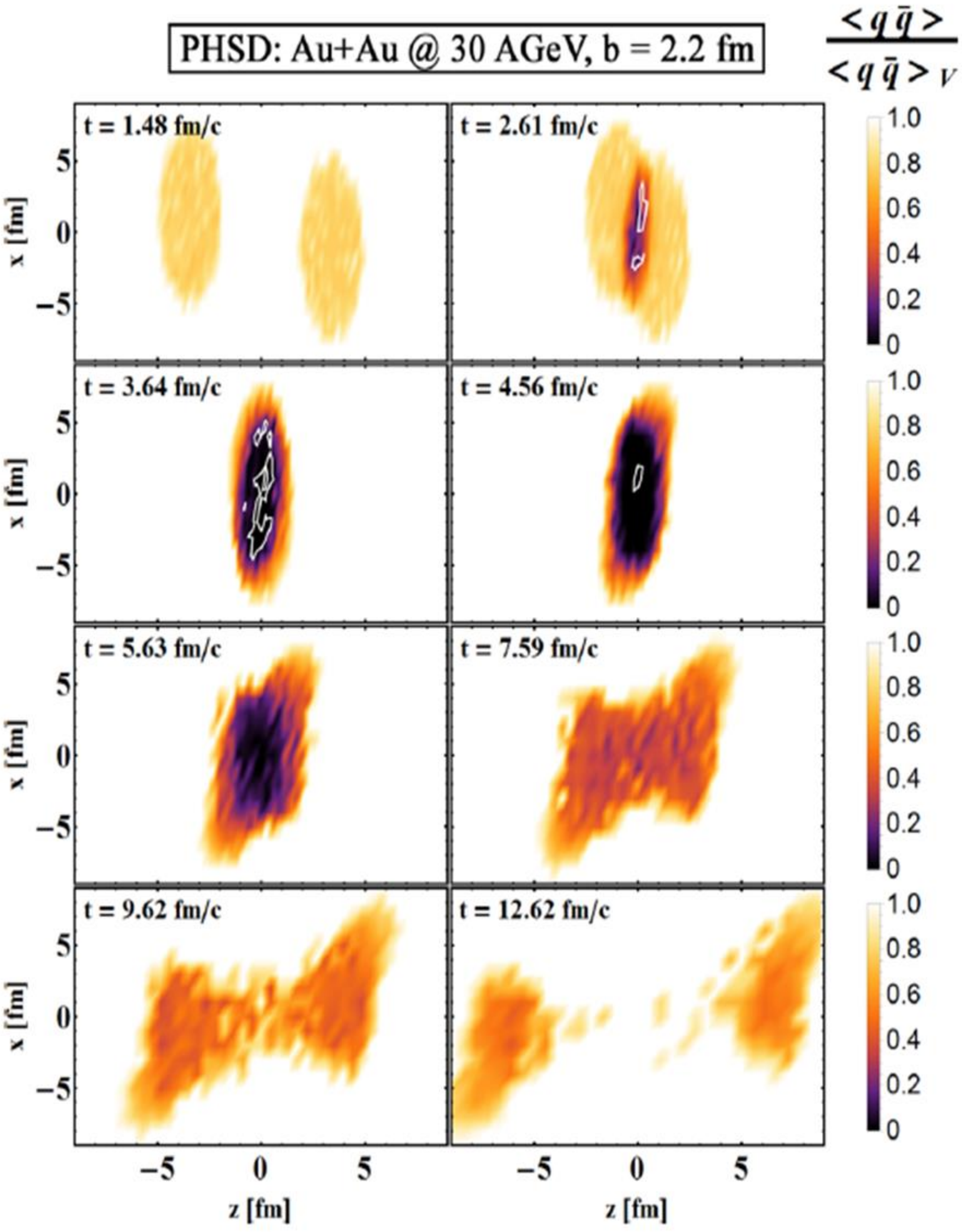
baryonic medium

mesonic medium

PHSD:
Ratio of the scalar quark condensate:

$$\frac{<q\,\bar{q}>}{<q\,\bar{q}>_V}$$

compared to the vacuum as a function of *x,z (y=0)* at different time *t* for central Au+Au collisions at 30 AGeV



- ☐ **restoration of chiral symmetry:**
  $\langle q\bar{q}\rangle/\langle q\bar{q}\rangle_V$ → 0

W. Cassing, A. Palmese, P. Moreau, E.L. Bratkovskaya, PRC 93, 014902 (2016), arXiv:1510.04120

Chiral symmetry restoration vs. deconfinement

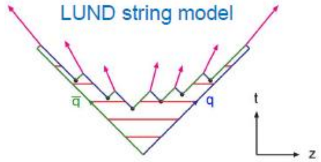
I. Initial stage of HICs:

Hadronic matter ➔ string formation

II. QGP

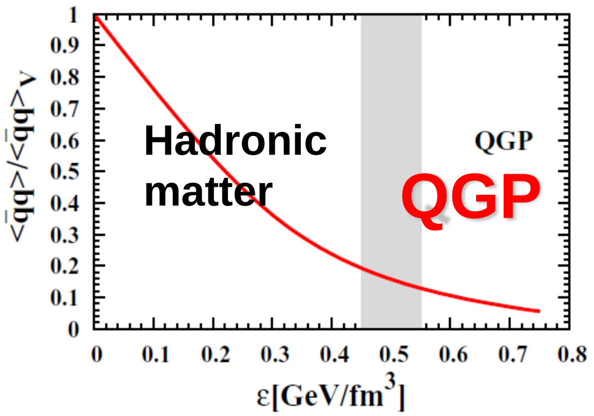
(time-like partons,
explicit partonic interactions)

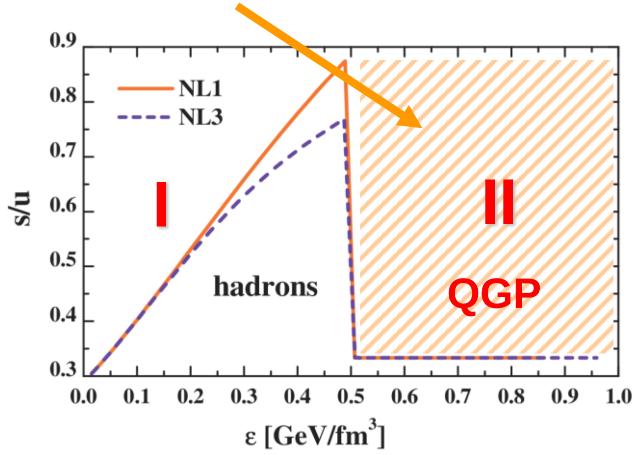
III. Hadronic phase



$$\frac{P(s\bar{s})}{P(u\bar{u})}=\frac{P(s\bar{s})}{P(d\bar{d})}=\gamma_S=\exp\left(-\pi\frac{m_S^{*2}-m_q^{*2}}{2\kappa}\right)$$

$$\mathbf{m}_q^*=\mathbf{m}_q^0+(m_q^V-m_q^0)\frac{<\mathbf{q\bar{q}}>}{<\mathbf{q\bar{q}}>_V}$$



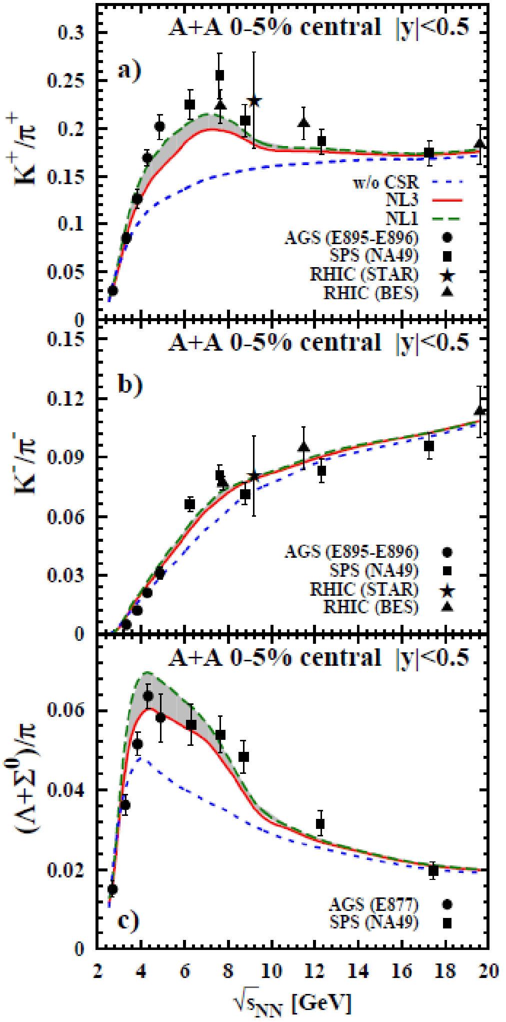


- ☐ Chiral symmetry restoration via Schwinger mechanism (and non-linear $\sigma-\omega$ model) changes the „flavour chemistry“ in string fragmentation (1PI):

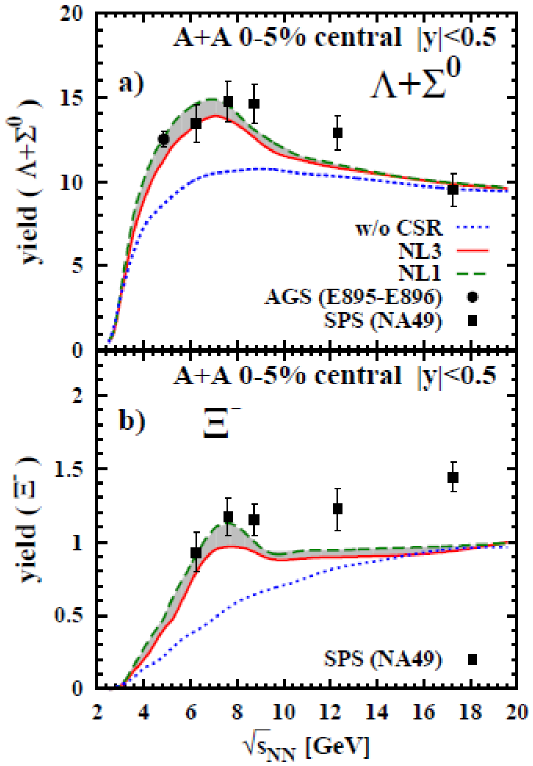
  $\langle q\bar{q}\rangle/\langle q\bar{q}\rangle_V$ → 0 ➔ m_s^* → m_s^0 ➔ s/u grows

➔ the strangeness production probability increases with the local energy density ε (up to ε_C) due to the partial chiral symmetry restoration!

Excitation function of hadron ratios and yields



A. Palmese et al., PRC94 (2016) 044912 , arXiv:1607.04073

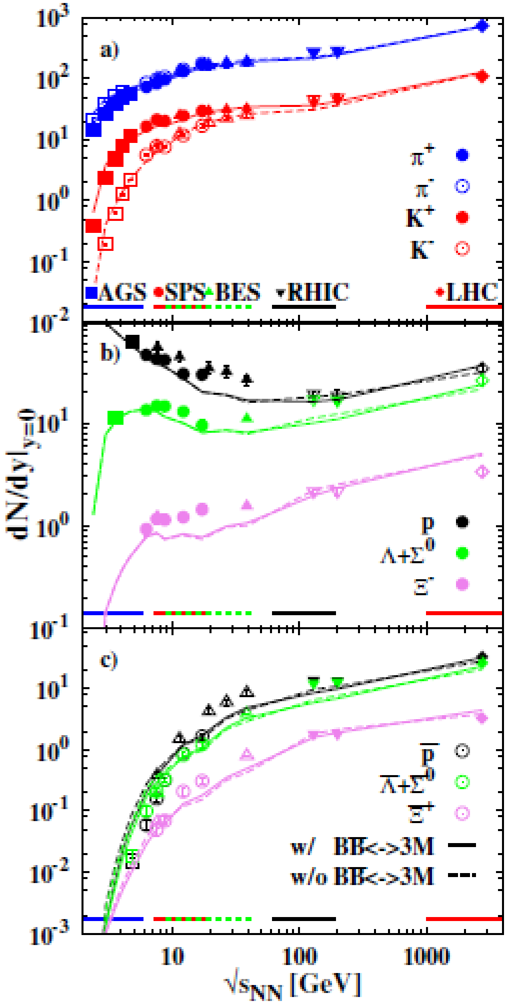


- **Influence of EoS: NL1 vs NL3 ➔ low sensitivity to the nuclear EoS**
- **Excitation function of the hyperons $\Lambda+\Sigma^0$ and Ξ^- show analogous peaks as K^+/π^+, $(\Lambda+\Sigma^0)/\pi$ ratios due to CSR**

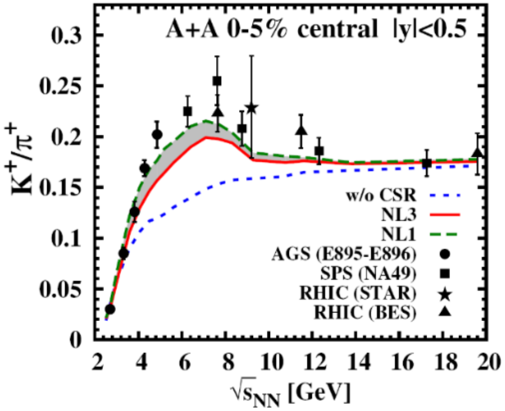
Chiral symmetry restoration leads to the enhancement of strangeness production in string fragmentation in the beginning of HICs in the hadronic phase.
➔The „horn“ structure is due to the interplay between CSR and deconfinement (QGP)

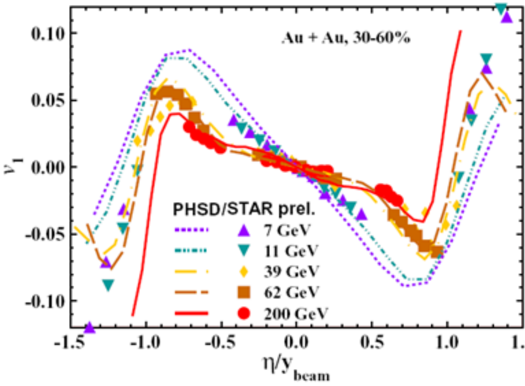
Non-equilibrium dynamics: description of A+A with PHSD

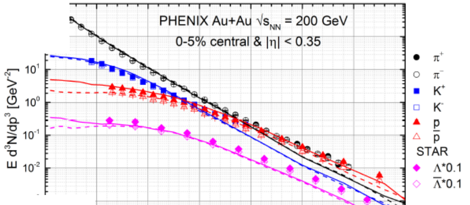
❑ PHSD: highlights

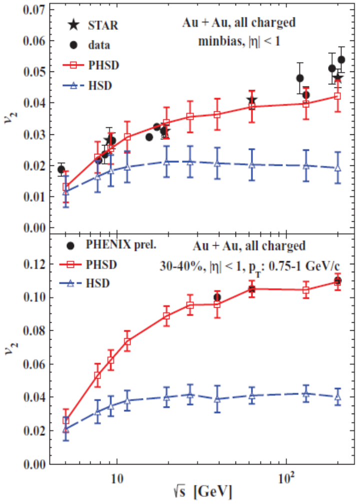


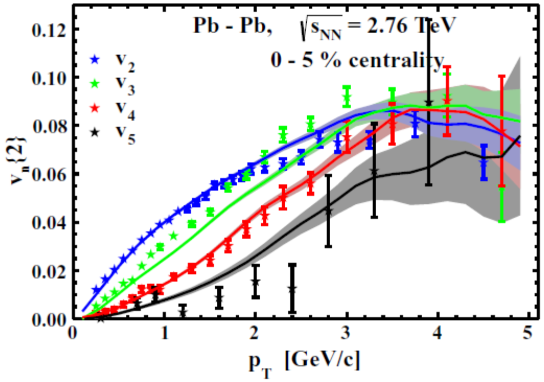
arXiv:1801.07557











PRC 85 (2012) 011902; JPG42 (2015) 055106

❑ **PHSD** provides a **good description of ‚bulk' observables** (y-, p_T-distributions, flow coefficients v_n, …) from SIS to LHC

Summary

The developments in the microscopic transport theory in the last decades - based on the solution of generalized transport equations derived from Kadanoff-Baym dynamics - made it applicable for the description of strongly-interaction hadronic and partonic matter created in p+A and heavy-ion collisions from SIS to LHC energies

Note: for the consistent description of HIC the input from lQCD and many-body theory is mandatory:
properties of partonic and hadronic degrees-of-freedom and their in-medium interactions