First-order phase transitions and gravitational wave production in the early Universe

Ryusuke Jinno (Kobe Univ.)

iTHEMS Cosmology Forum @Wako, 2024/9/27

KOBE UNIVERSITY

1 Overview

2 First-order phase transitions

3 Dynamics of bubbles

4 Gravitational waves

5 Recent topics

First-order phase transitions and gravitational wave production in the early Universe

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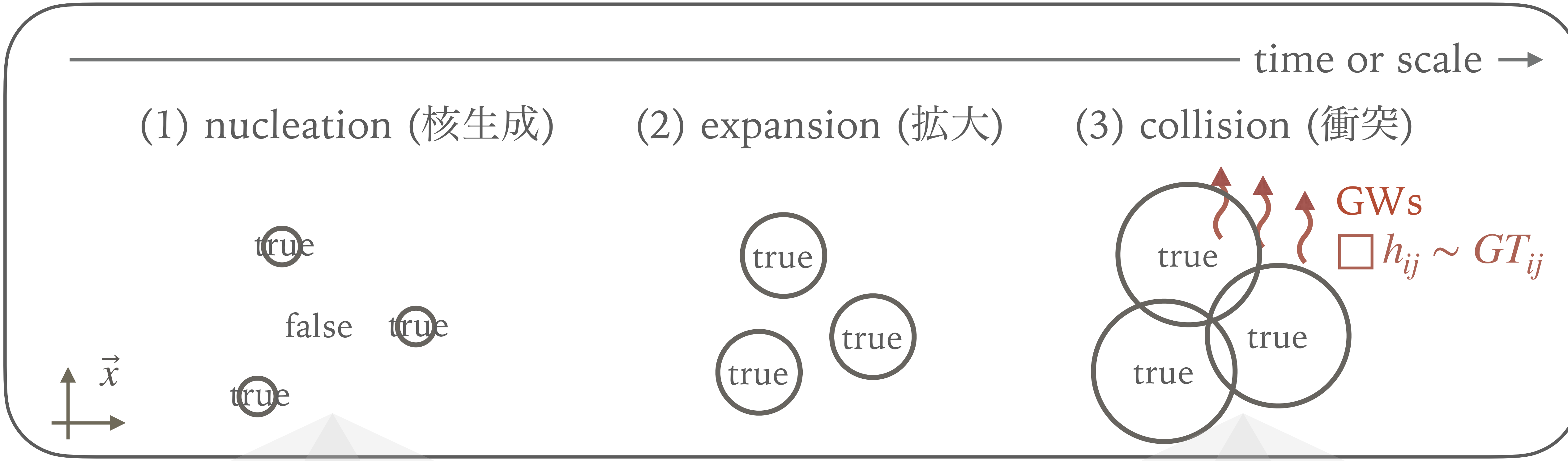
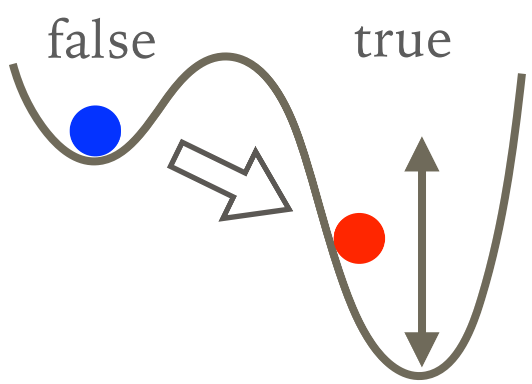
OVERVIEW

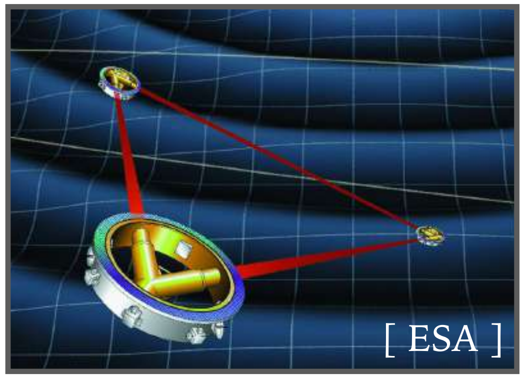
microphysics *macrophysics*



Physics of the Higgs sector



GW observations



OVERVIEW

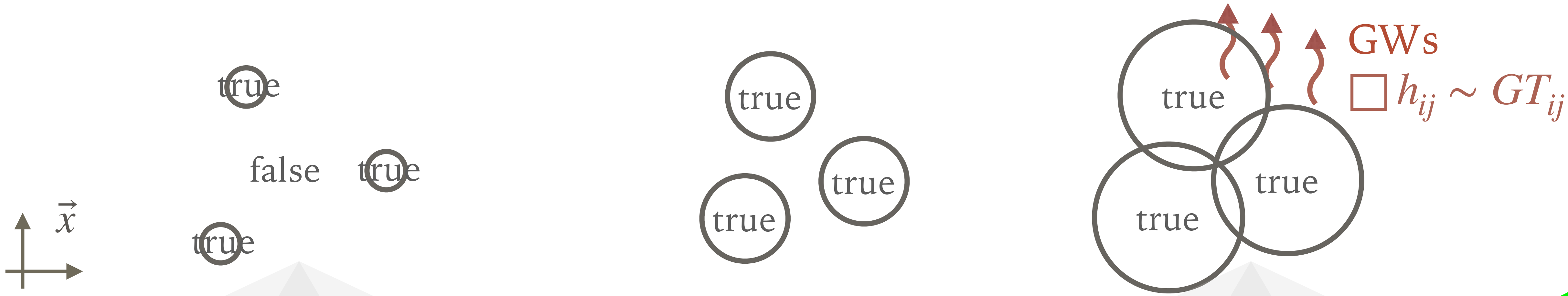
microphysics

Dynamics of bubbles

macrophysics

time or scale →

(1) nucleation (核生成) (2) expansion (拡大) (3) collision (衝突)



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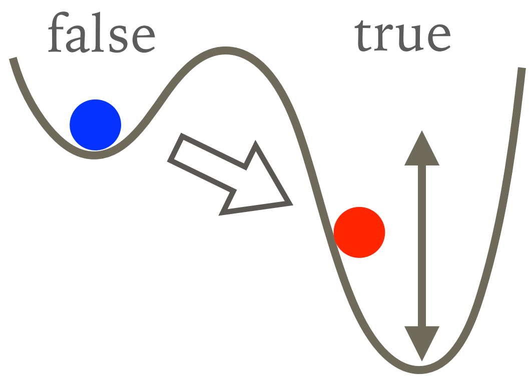
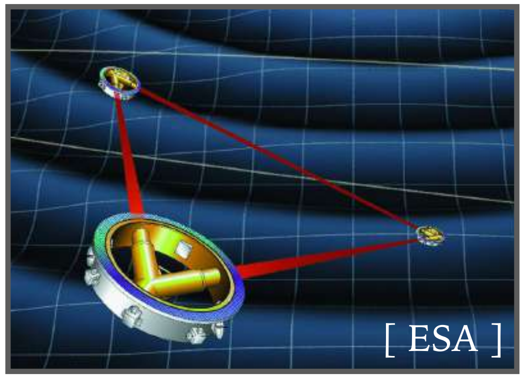


FOPTs in BSM

GW observations



GWs

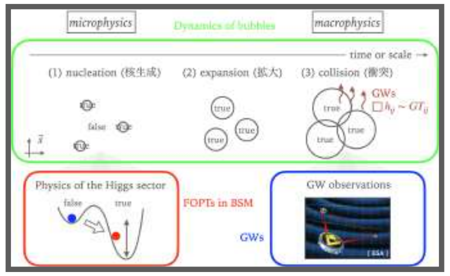
TALK PLAN

1. Intro

2. First-order phase transitions in beyond the Standard Model

~30min 3. Dynamics of bubbles

4. Gravitational wave production & observational prospects



~25min 5. Recent topics


1
Overview
2
First-order phase transitions
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Dynamics of bubbles
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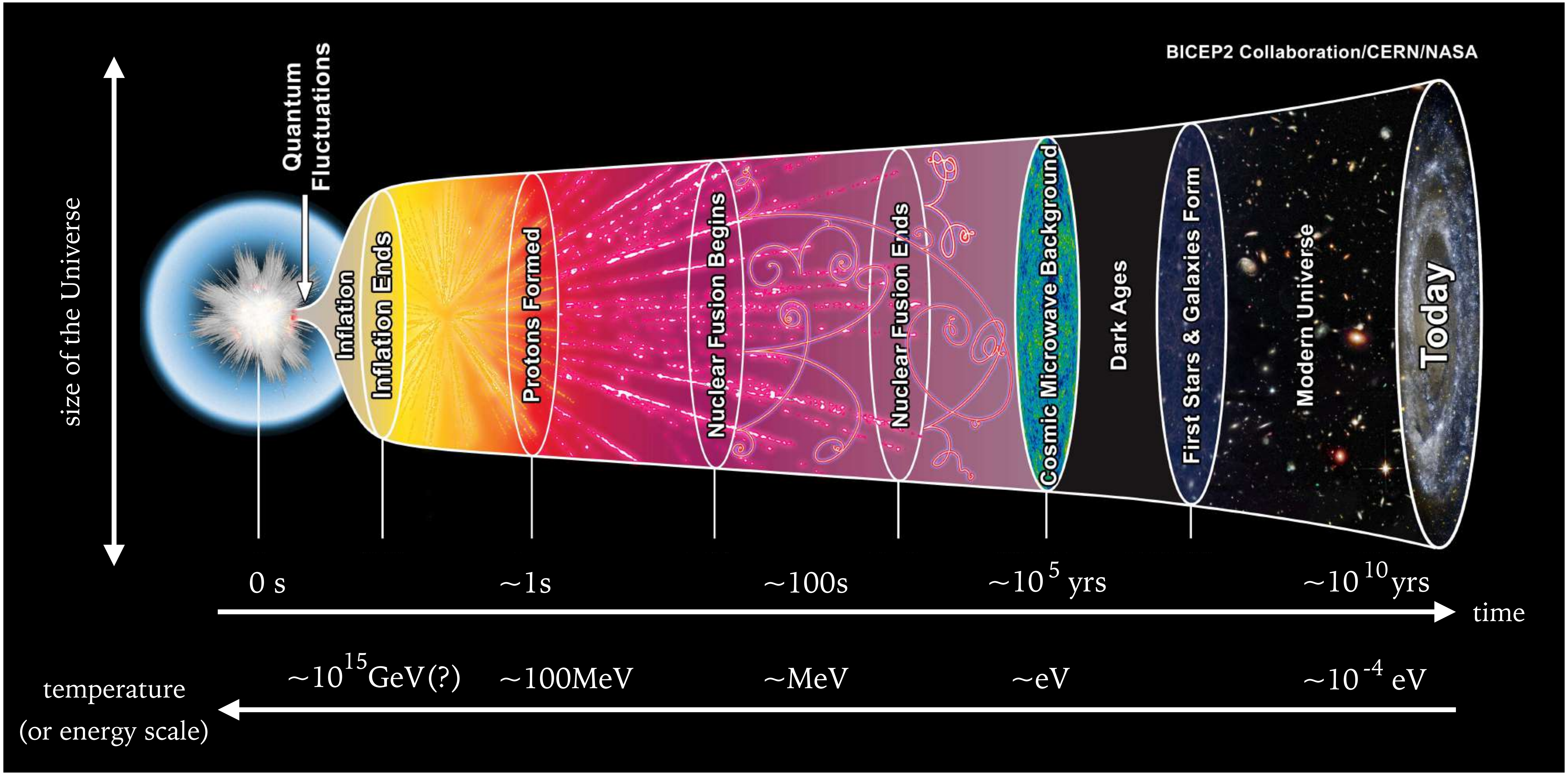
Gravitational waves
5
Recent topics

THERMAL HISTORY OF THE UNIVERSE



History of the Universe = History of cooling down

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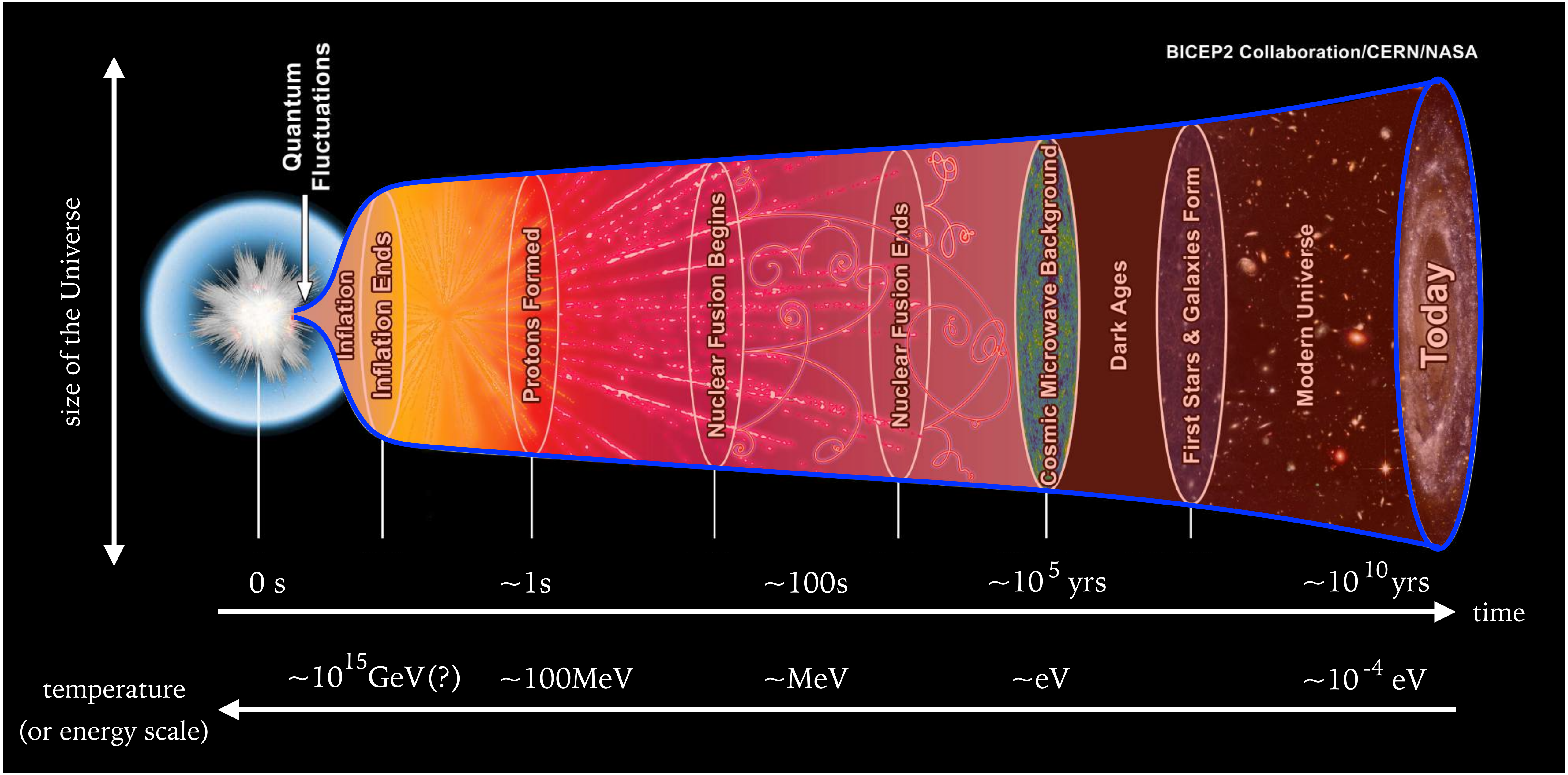


History of the Universe = History of cooling down

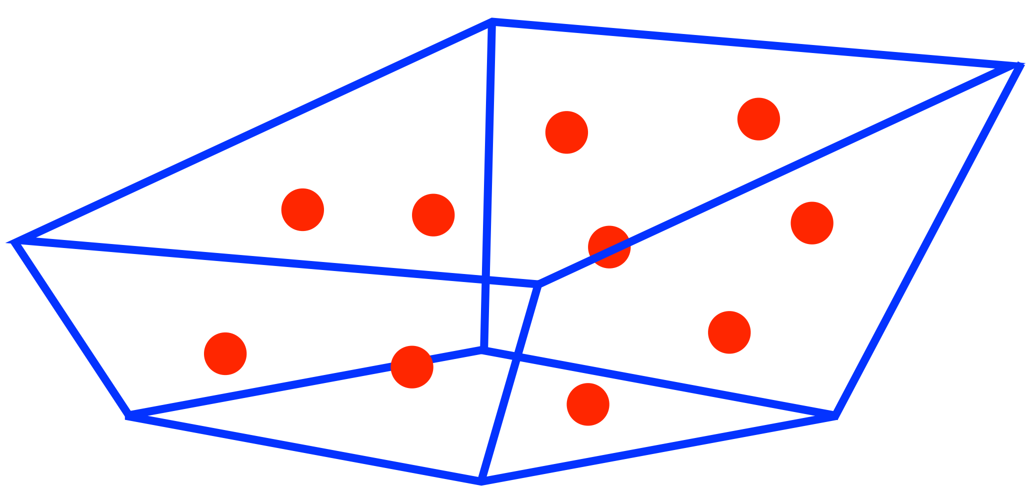
EINSTEIN EQUATION

➤ What describes the evolution of the Universe? → Einstein equation

カタチ $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ モノ

"*Space(-time) tells matter how to move. Matter tells space(-time) how to curve.*"

John Wheeler



SPACETIME

- Homogeneous and isotropic (over large scales) universe is described by the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric

$$ds^2 = -\,dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

$a(t)$: scale factor $\qquad H(t) \equiv \dfrac{\dot{a}(t)}{a(t)}$: Hubble parameter

- Temperature scales (roughly) inversely to the scale factor

$$\boxed{T(t) \sim a(t)^{-1}}$$

MATTER

➤ The Standard Model of particle physics

- Quarks and leptons form matter:

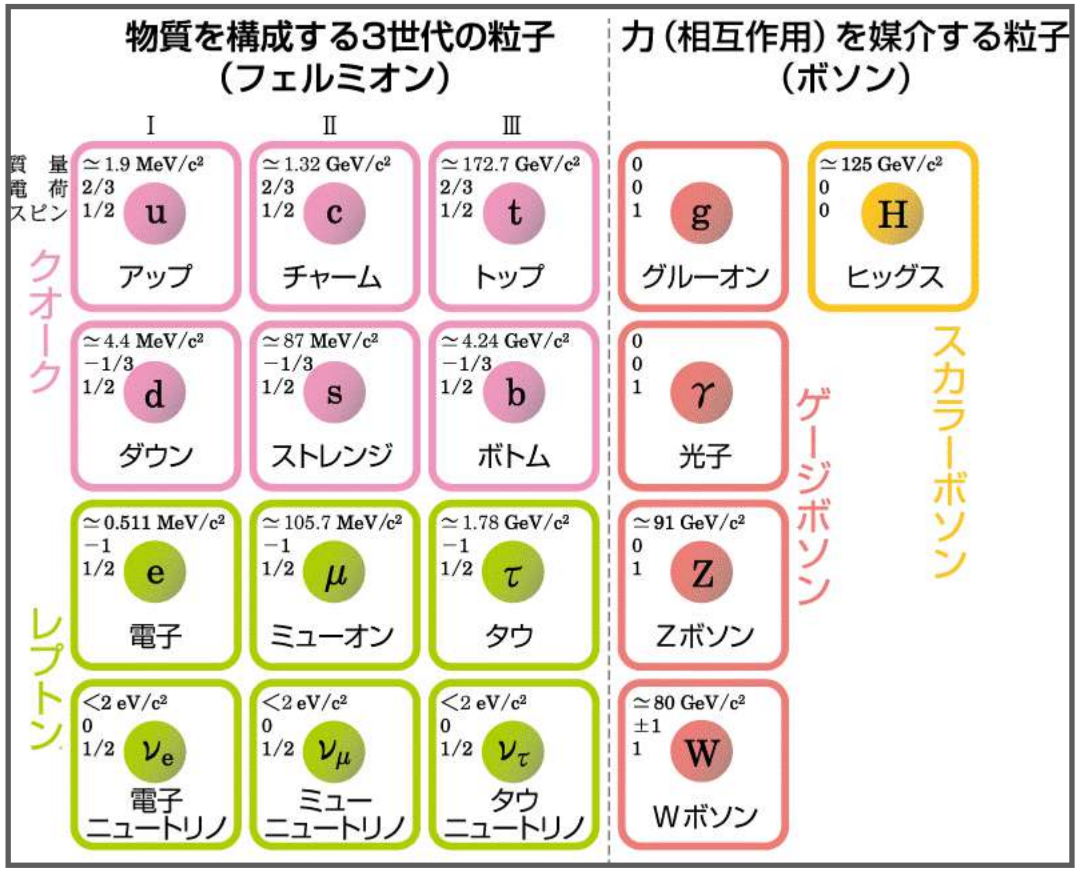
  They consist of 3 generations,

  which have different masses but similar properties

- Gauge bosons and a scalar boson mediate force

- The only scalar boson humans know is called Higgs boson:

  it gives masses to other particles by forming a condensate



[Wikimedia Commons, 天文学辞典]

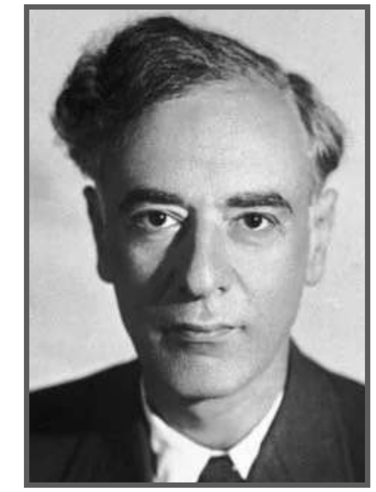
SOMETHING IS MISSING IN THE STANDARD MODEL

- Evidence/hint for beyond the Standard Model (BSM) physics

- Baryon asymmetry
- Dark matter
- Dark energy
- Inflation
- Unification of forces
- Mass hierarchy
- Neutrino masses
- Strong CP

PHASE TRANSITIONS

➤ Classification of phase transitions (a la Landau)

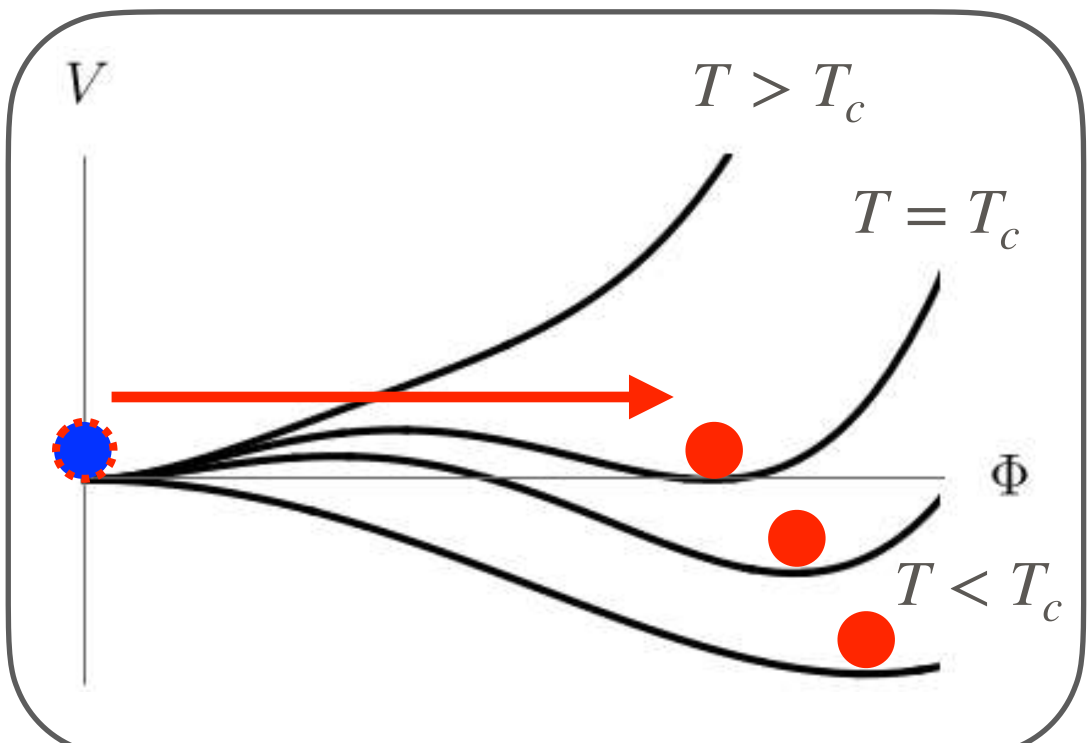


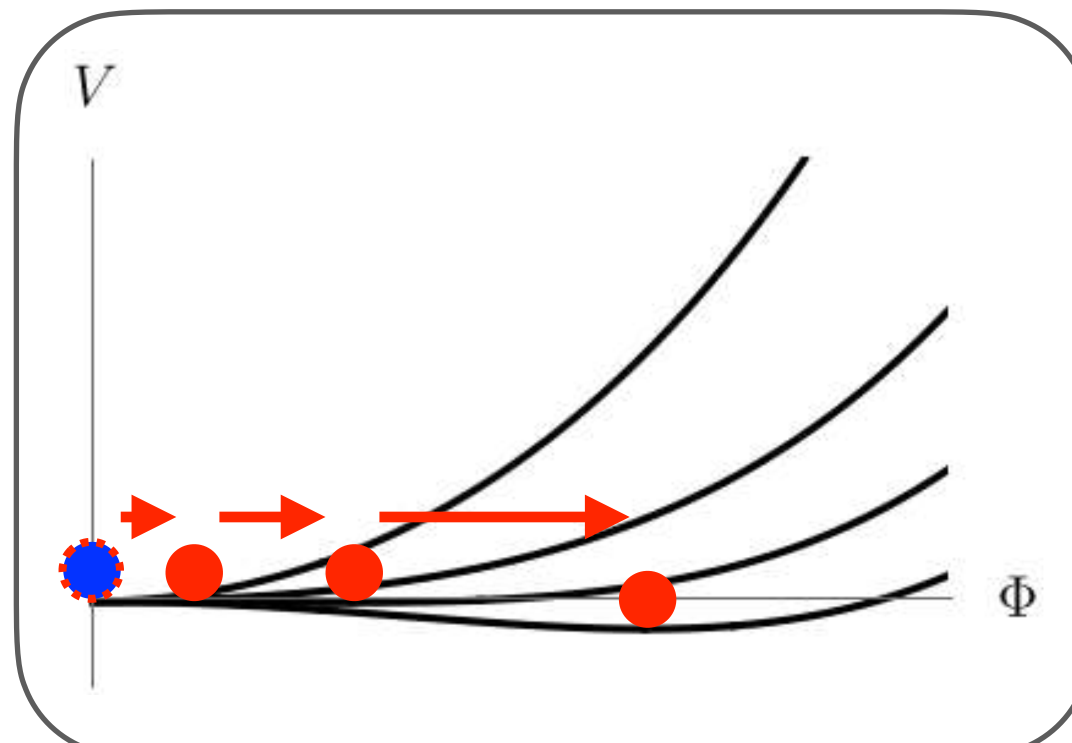
[Landau, from Wikipedia]

Order parameter (= scalar field)

discontinuous ↙ ↘ continuous

first-order | second-order

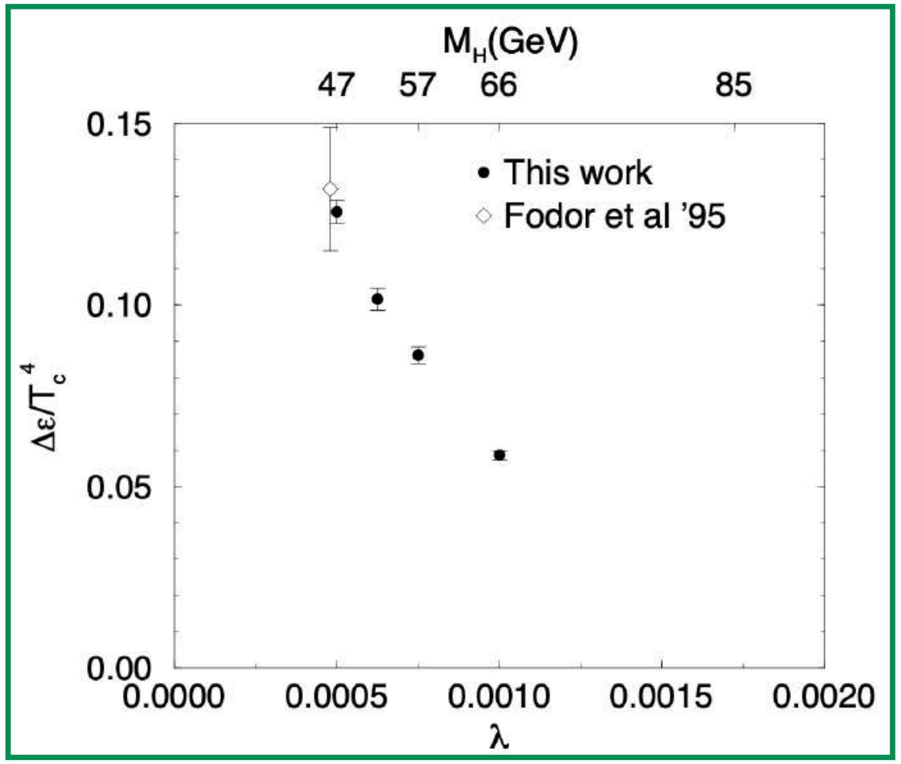




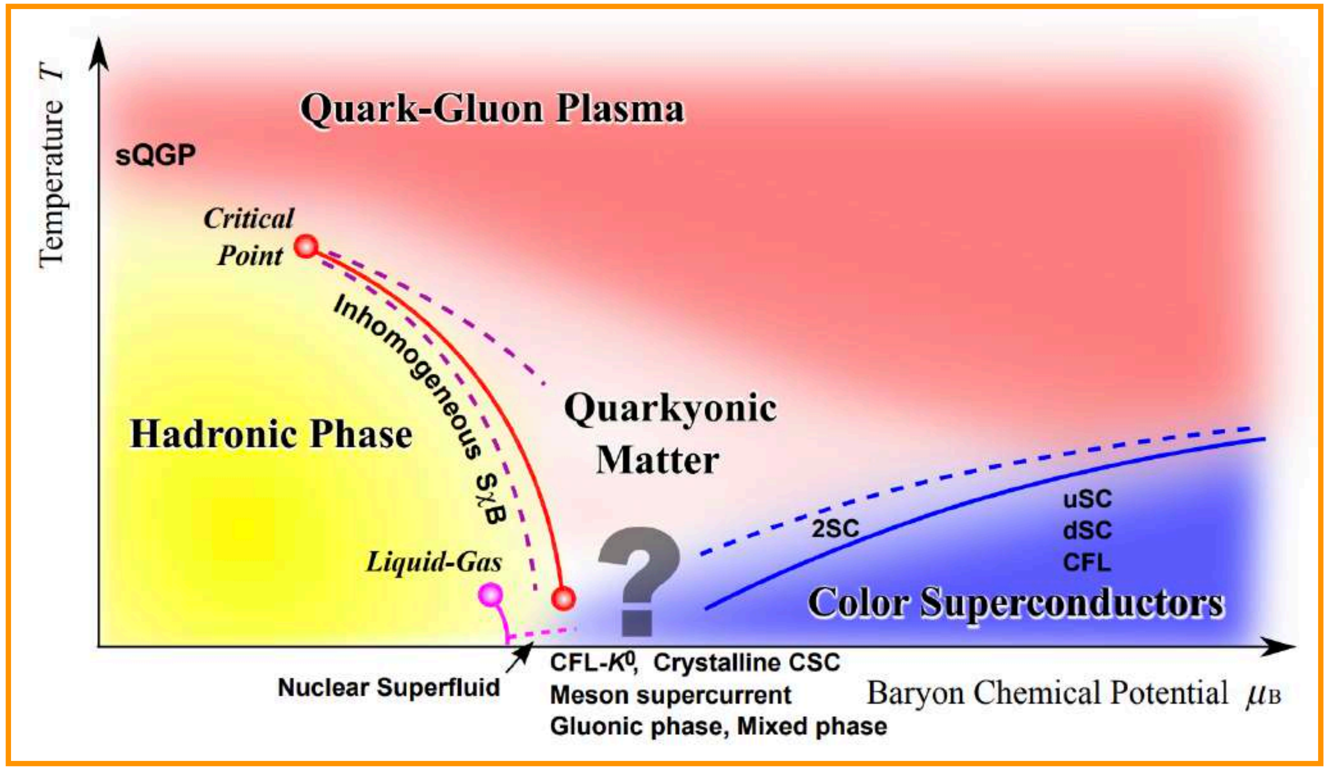
THERMAL HISTORY OF THE UNIVERSE

➤ Two candidates for FOPTs in the Standard Model (SM)

Electroweak "phase transition" & QCD "phase transition"



[Aoki '97]
see also
[Kajantie, Laine, Rummukainen, Shaposhnikov '96]
[Karsch, Neuhaus, Patkós, Rank '97]



[Fukushima, Hatsuda '11]

→ Unfortunately, both are crossover, meaning they are not even phase transitions

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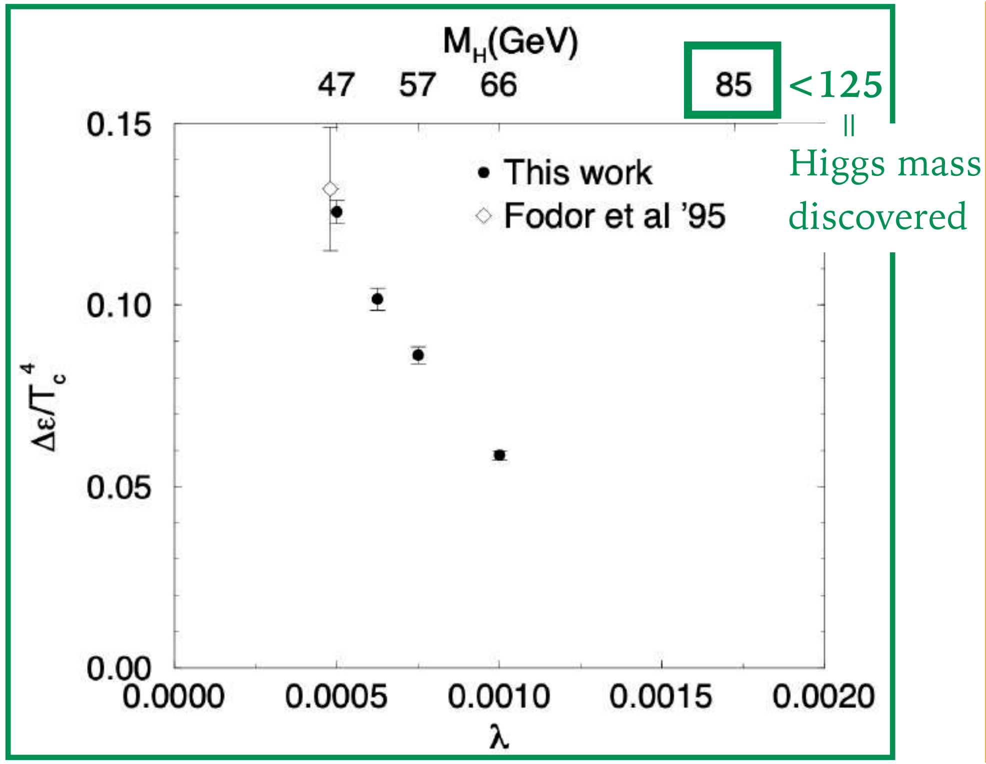

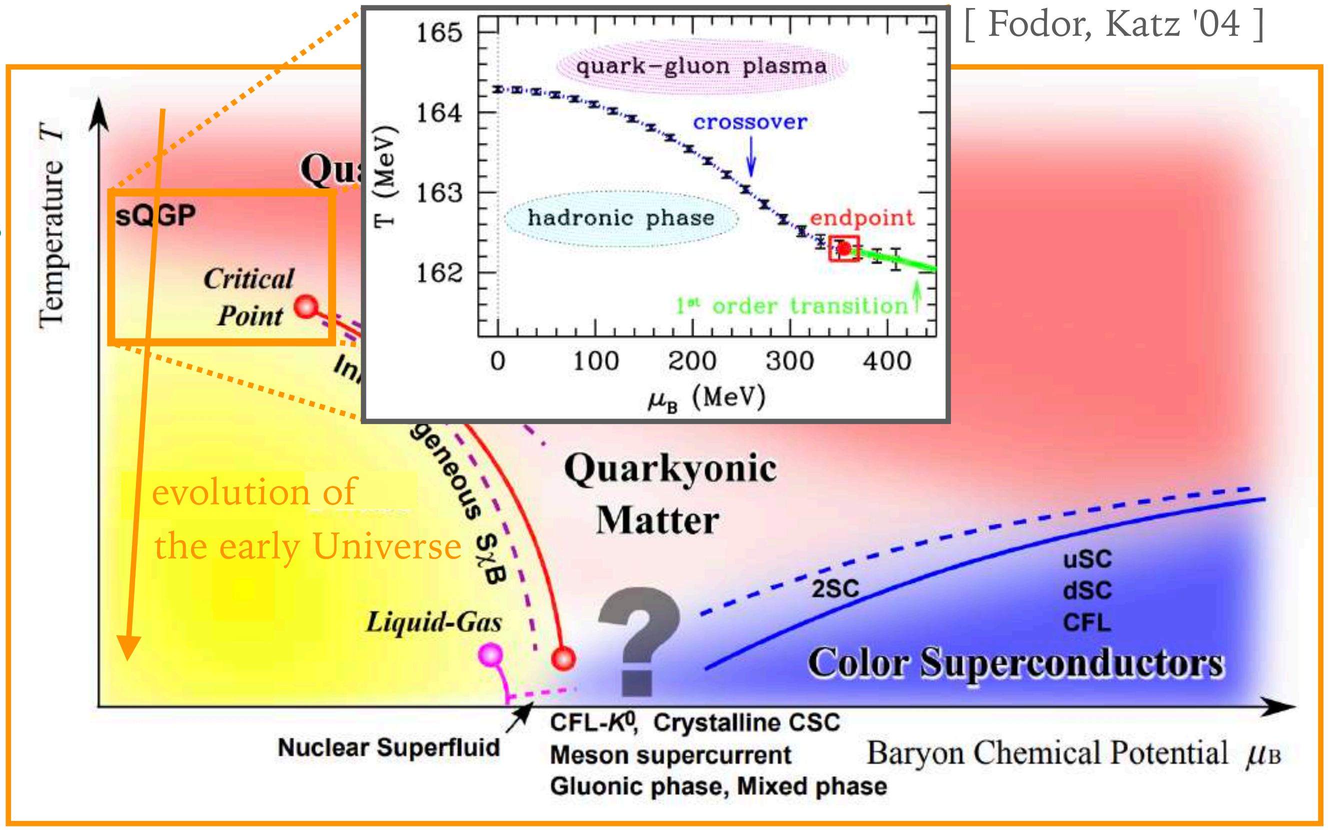

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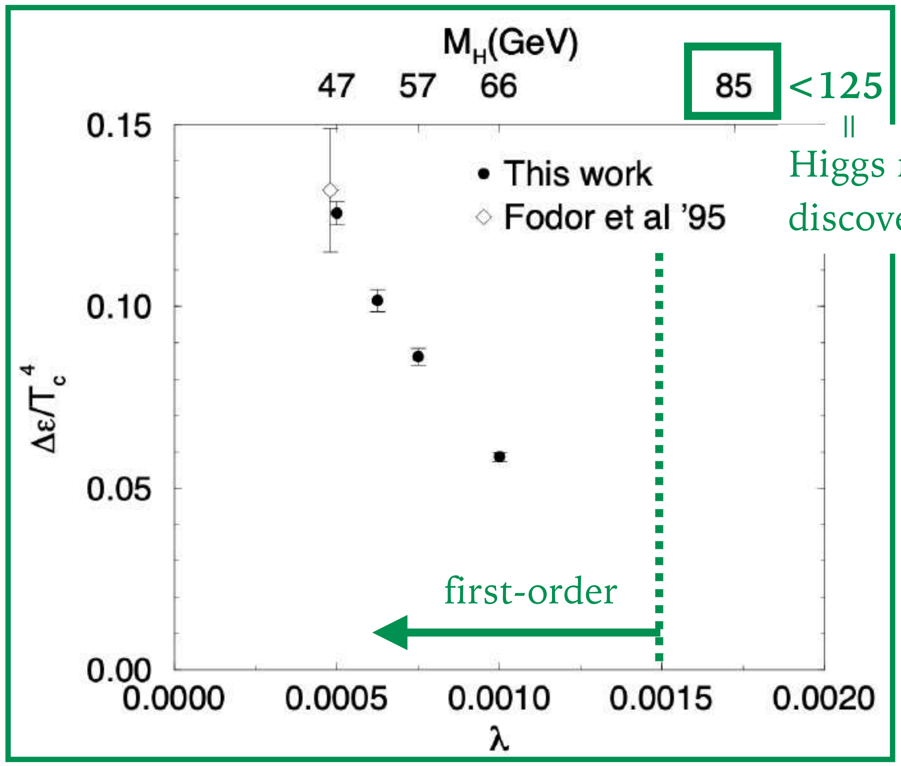

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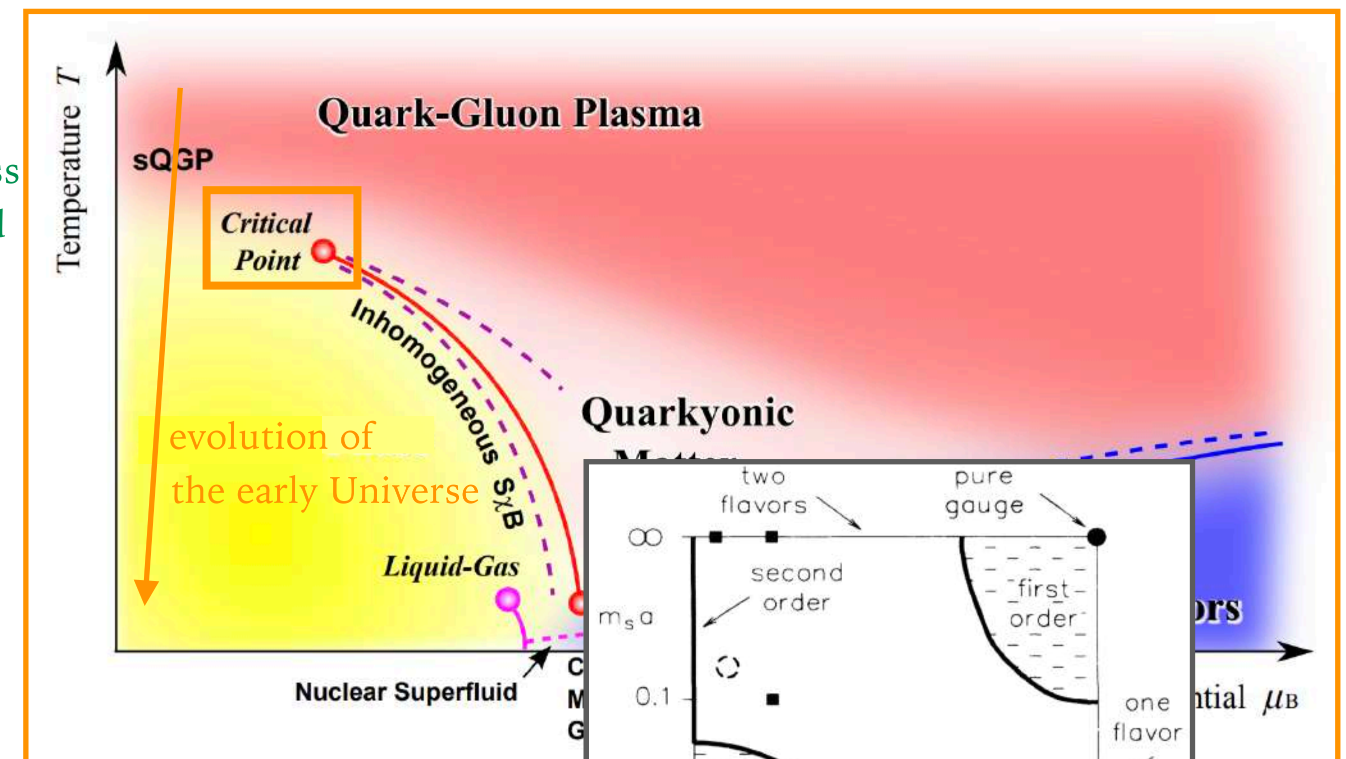

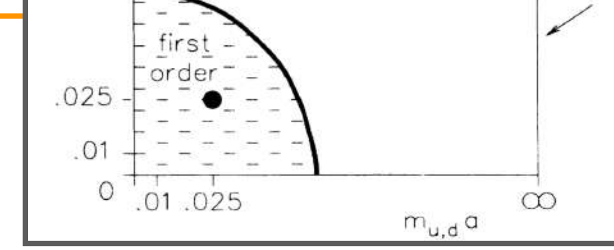

uda '11]

[Pisarski, Wilczek '84]
[Brown et al., '90]

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MOTIVATIONS TO CONSIDER FIRST-ORDER PHASE TRANSITIONS

- The vast energy scale the Universe might have experienced from inflation ($\lesssim 10^{15}\mathrm{GeV}$) down to the present ($\sim 10^{-4}\mathrm{eV}$)
- Spontaneous symmetry breaking that might have happened

  - Breaking of the GUT group (→ GUT)

  - Breaking of Peccei-Quinn symmetry $U(1)_{\mathrm{PQ}}$ (→ strong CP)

  - Breaking of B-L symmetry $U(1)_{\mathrm{B-L}}$ (→ neutrino masses)

  - Breaking of dark groups (→ dark matter?)

- Testability of the process in the coming 10-20 yrs with GWs

TRADITIONAL MOTIVATION TO CONSIDER FOPT

- Baryon asymmetry of the Universe (BAU)

  = Why more baryons than antibaryons?

Galaxy

Antigalaxy

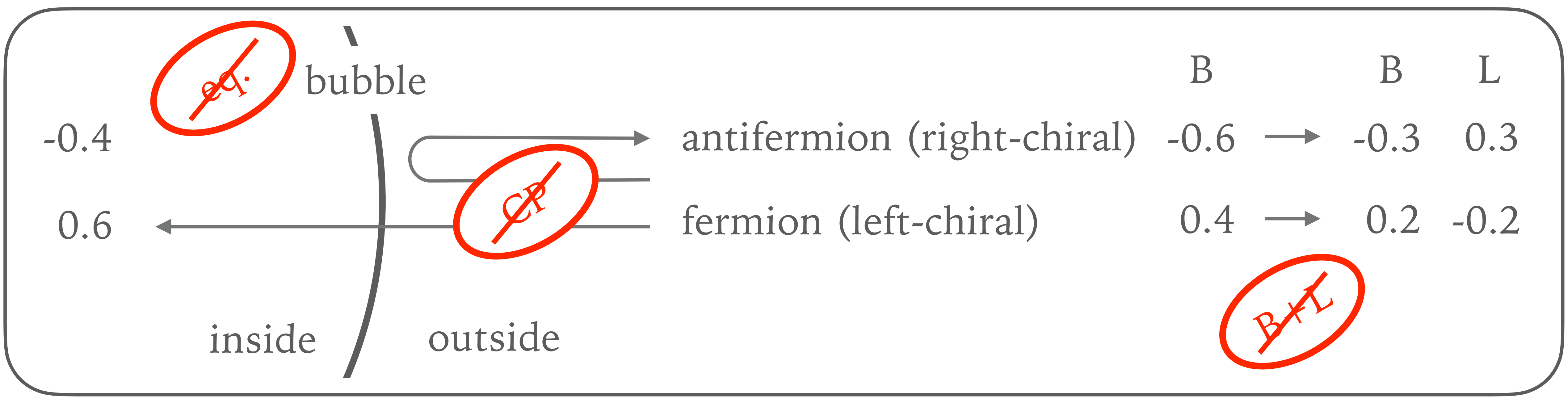
- 3 conditions to generate baryon asymmetry (Sakharov's conditions)

[Sakharov '67]

1) B violation 2) C&CP violation 3) Interactions out of thermal equilibrium

TRADITIONAL MOTIVATION TO CONSIDER FOPT

- Part of Sakharov's conditions are satisfied if an FOPT occurs (called electroweak baryogenesis) [Kuzmin, Rubakov, Shaposhnikov '85]



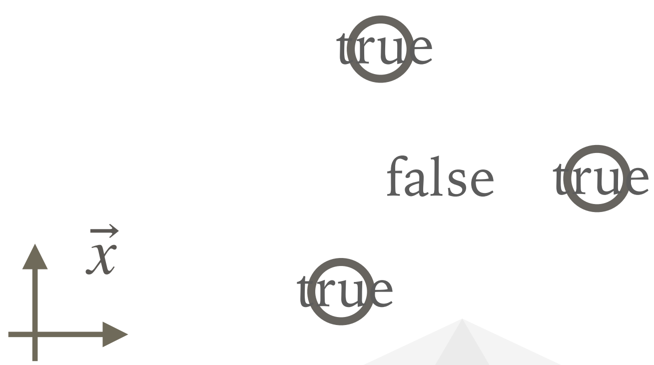
- However, electric dipole moments put stringent constraints

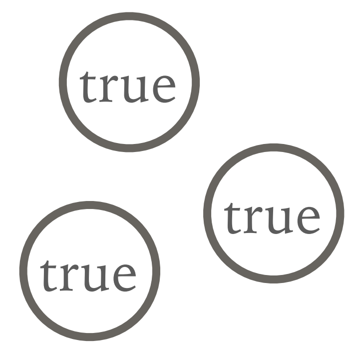
OVERVIEW

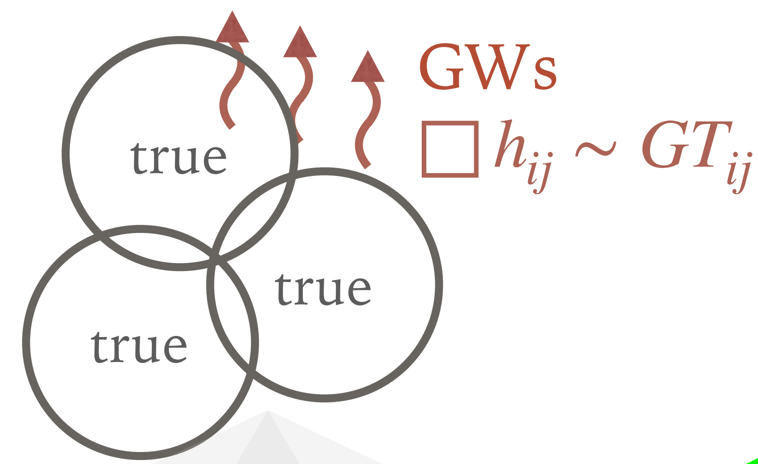
microphysics | Dynamics of bubbles | *macrophysics*

time or scale →

(1) nucleation (核生成) | (2) expansion (拡大) | (3) collision (衝突)







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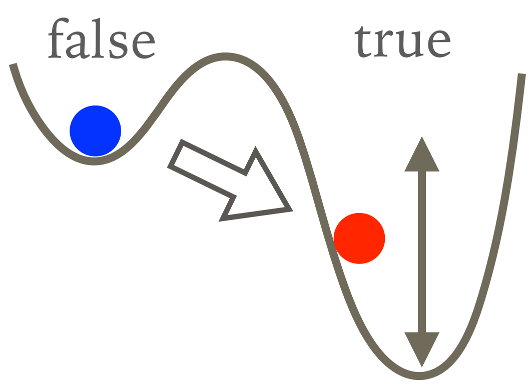

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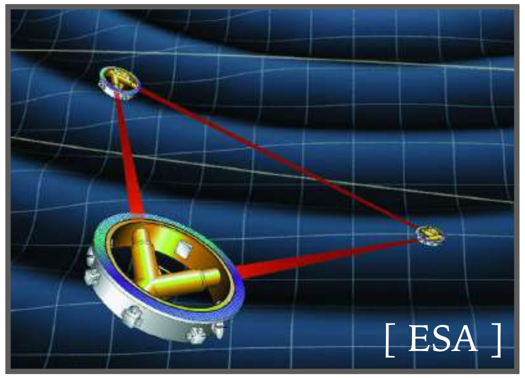
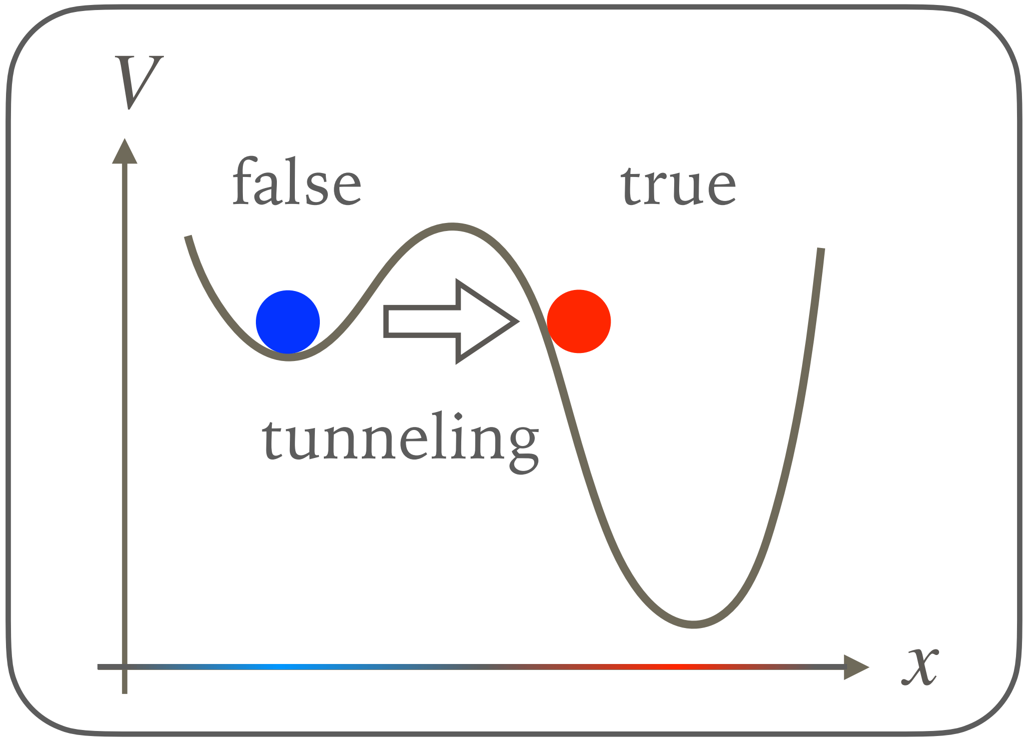
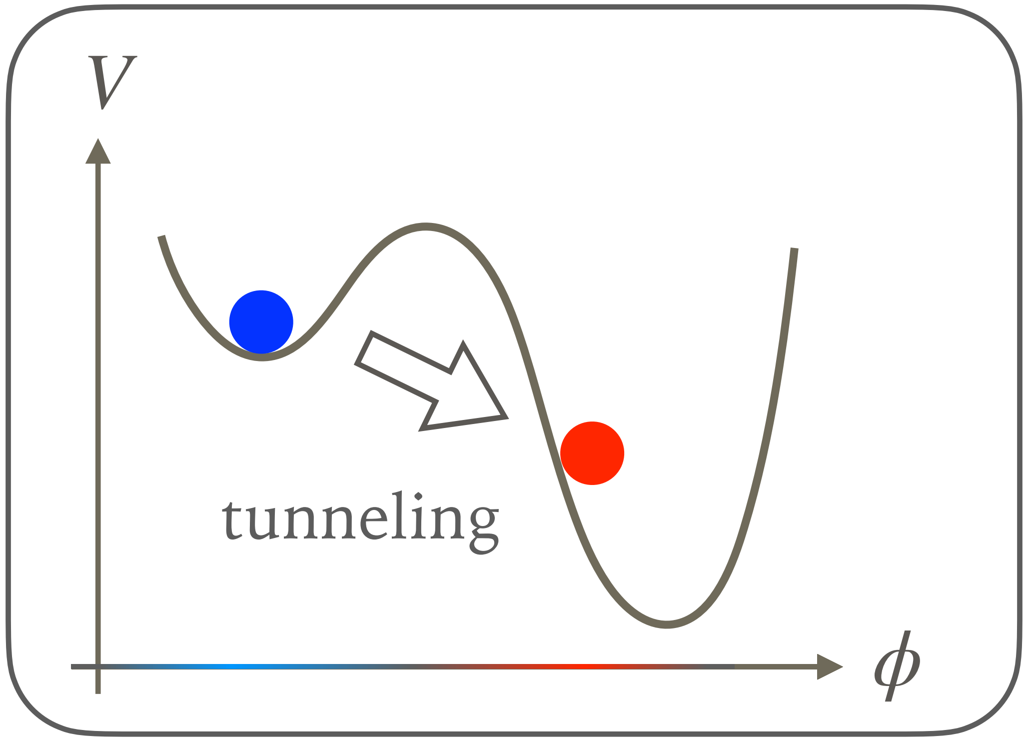


TUNNELING IN QUANTUM MECHANICS AND QFT

Quantum mechanics



Quantum field theory



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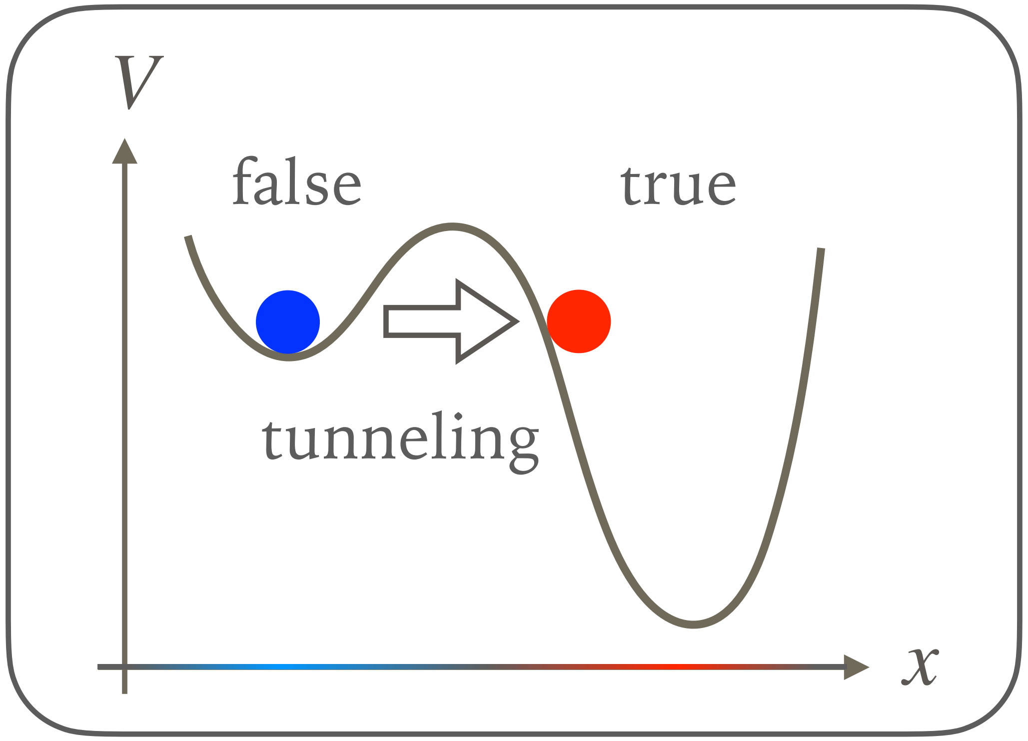

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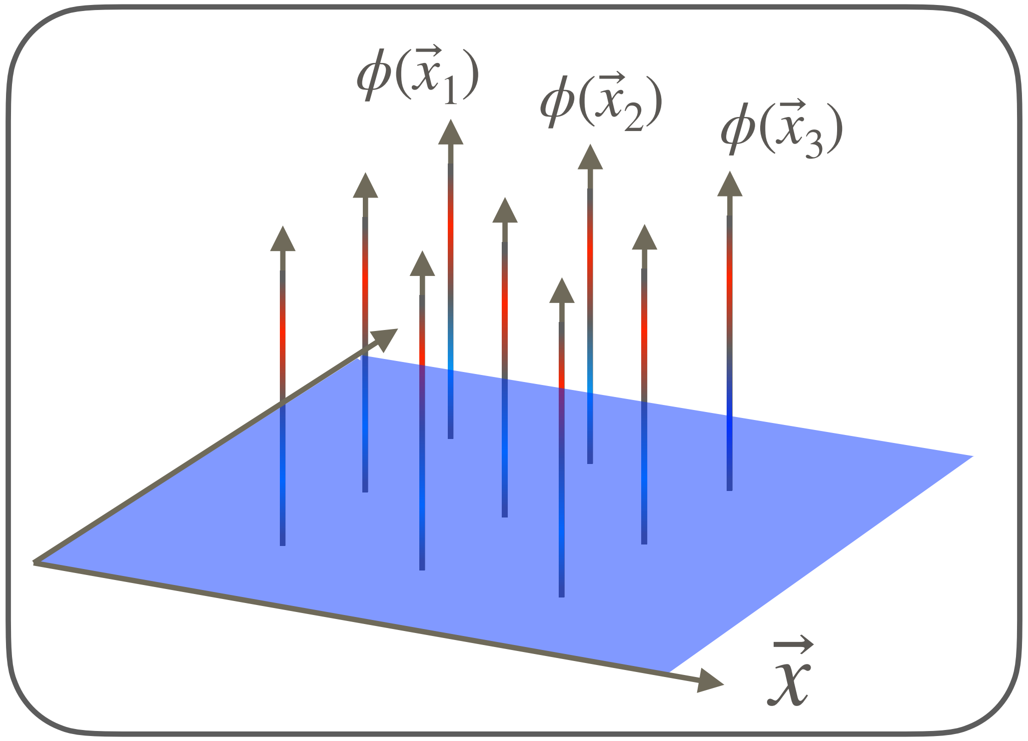

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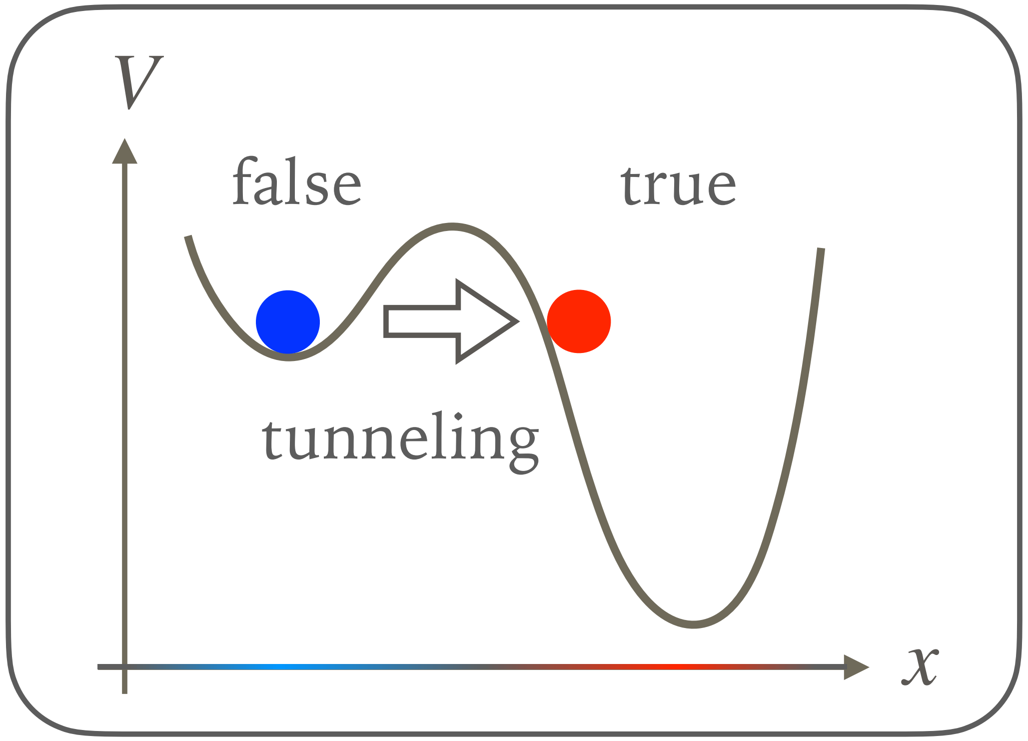

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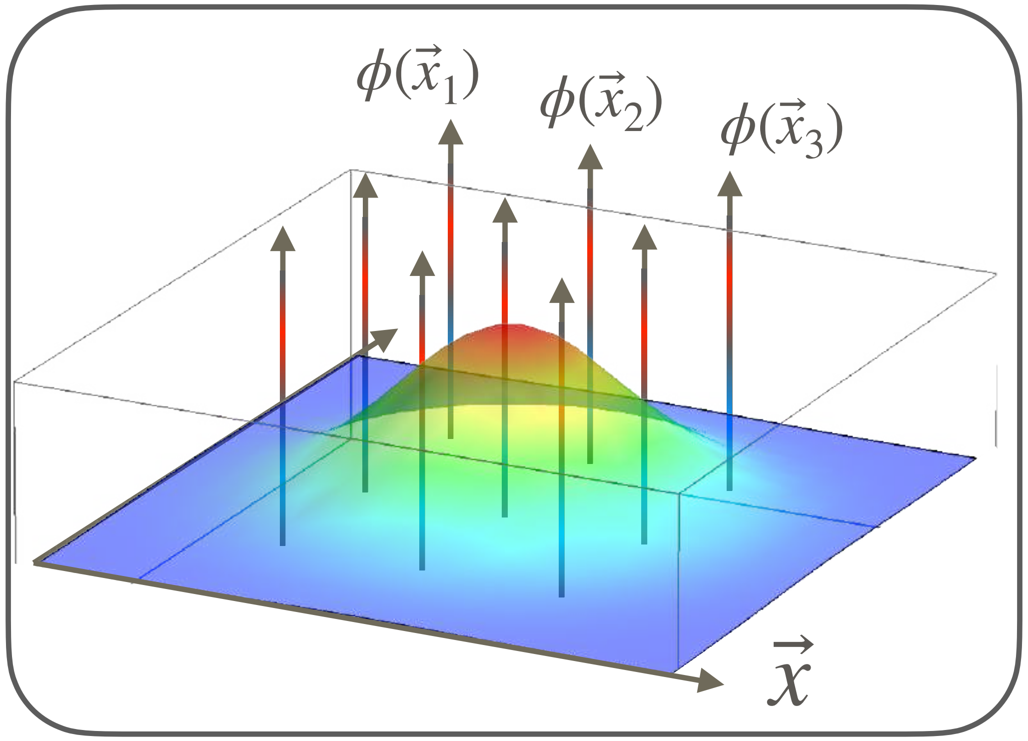
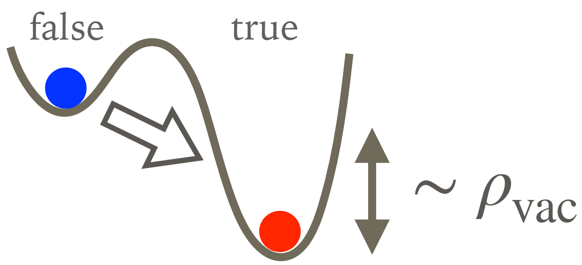


tunneling (nucleation, 核生成)

BUBBLE EXPANSION



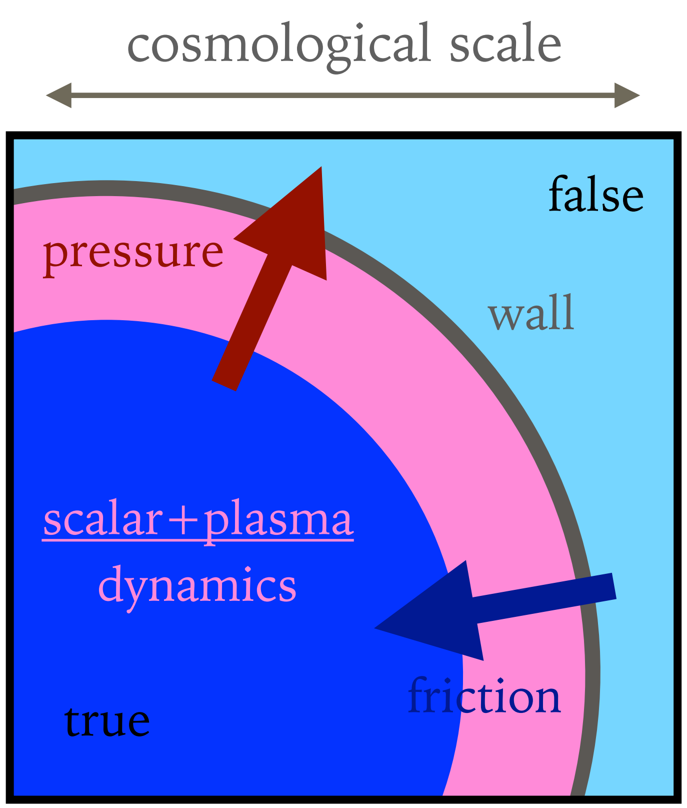
➤ "Pressure vs. Friction" determines the behavior:

(1) Pressure: wall is pushed by the released energy

Determined by $\alpha \equiv \rho_{\rm vac}/\rho_{\rm plasma}$

see e.g. [Espinosa et al. '10, Hindmarsh et al. '15, Giese et al. '20]

(2) Friction: wall is pushed back by plasma particles



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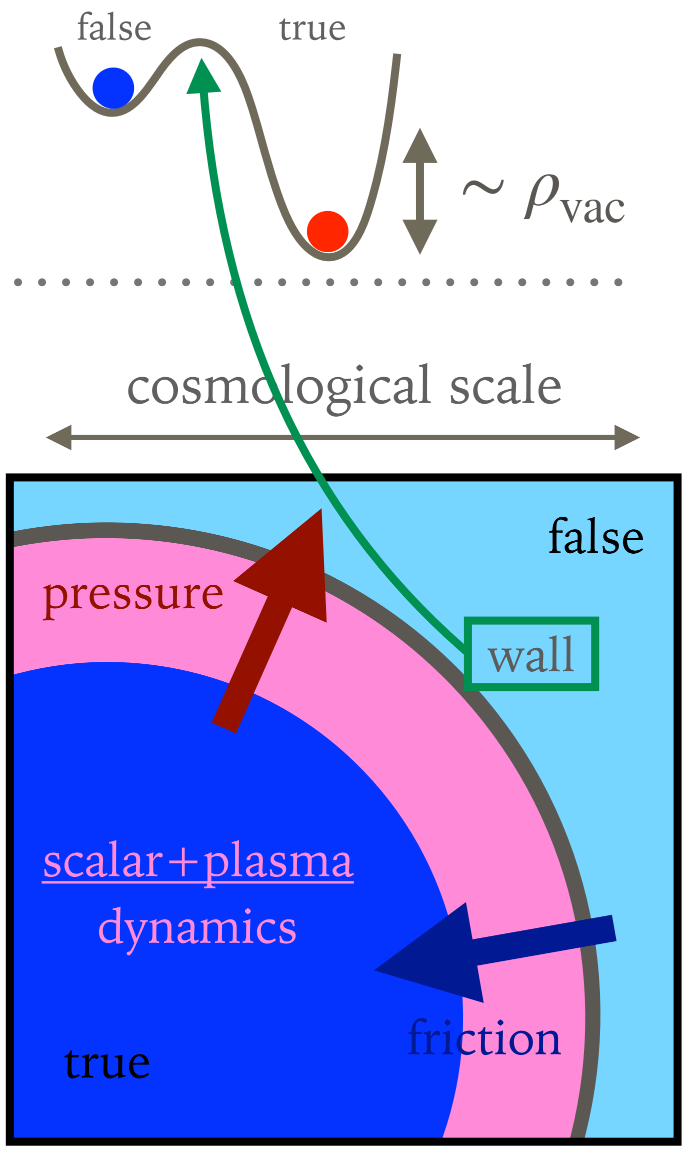

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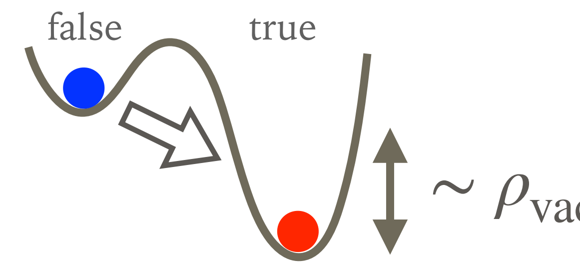

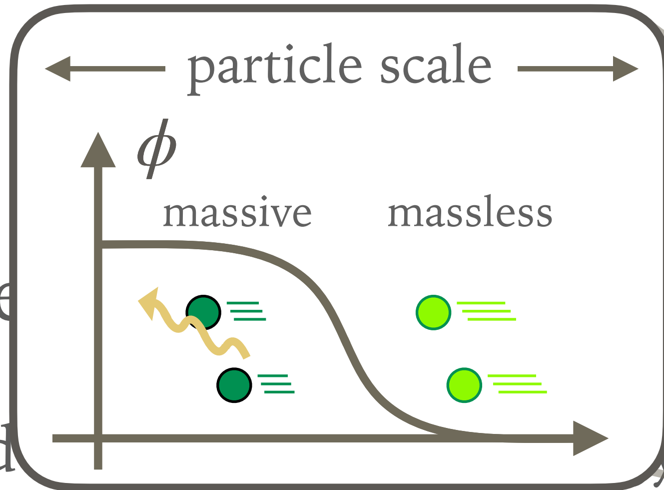

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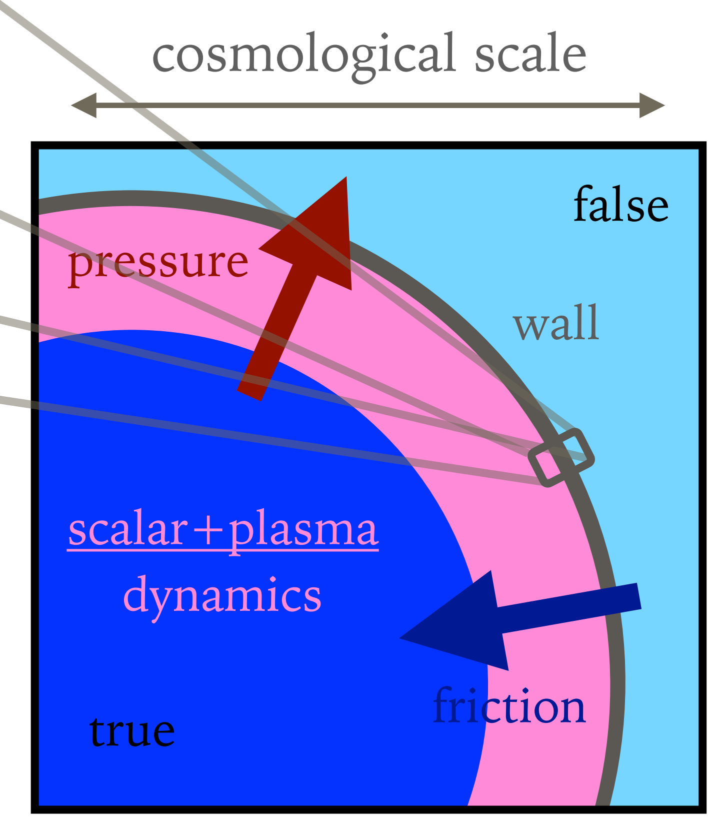

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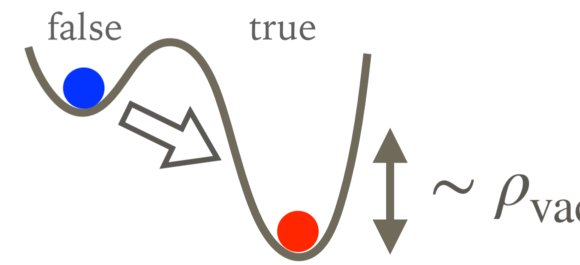

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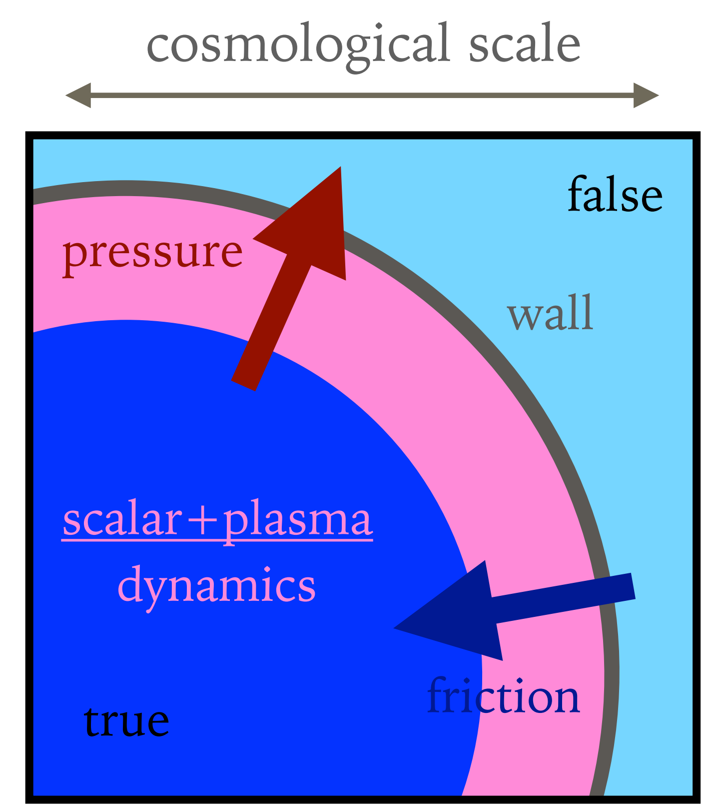
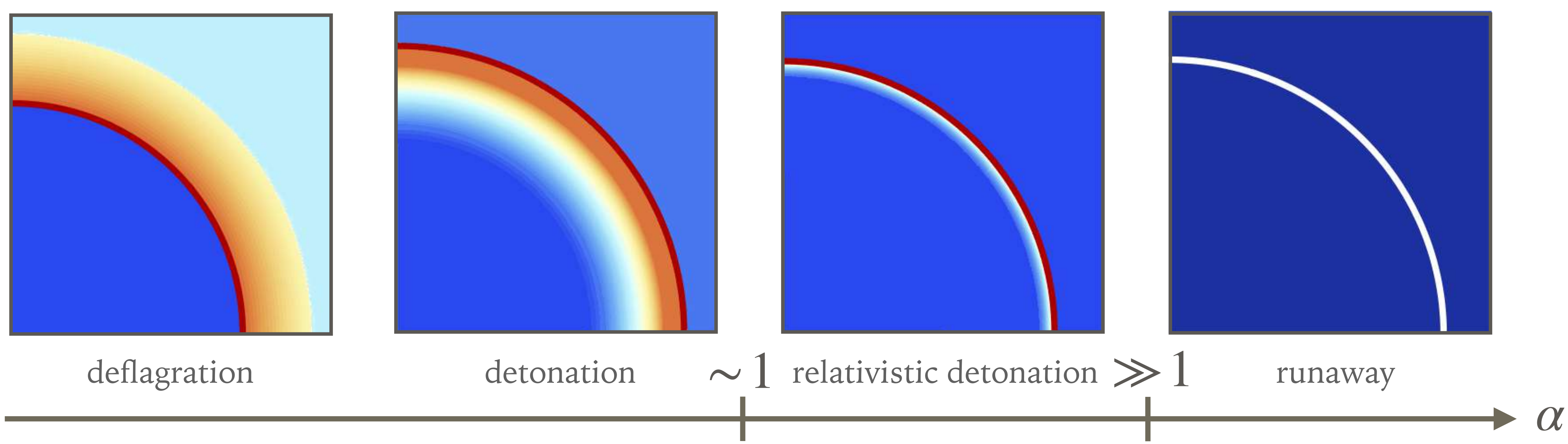


- Different types of bubble expansion



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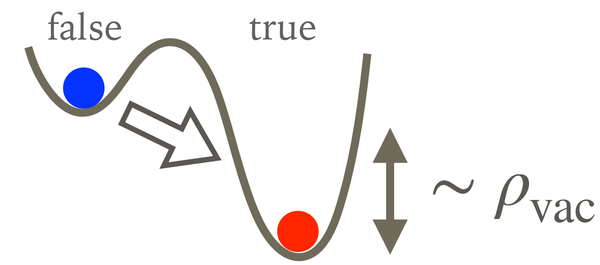
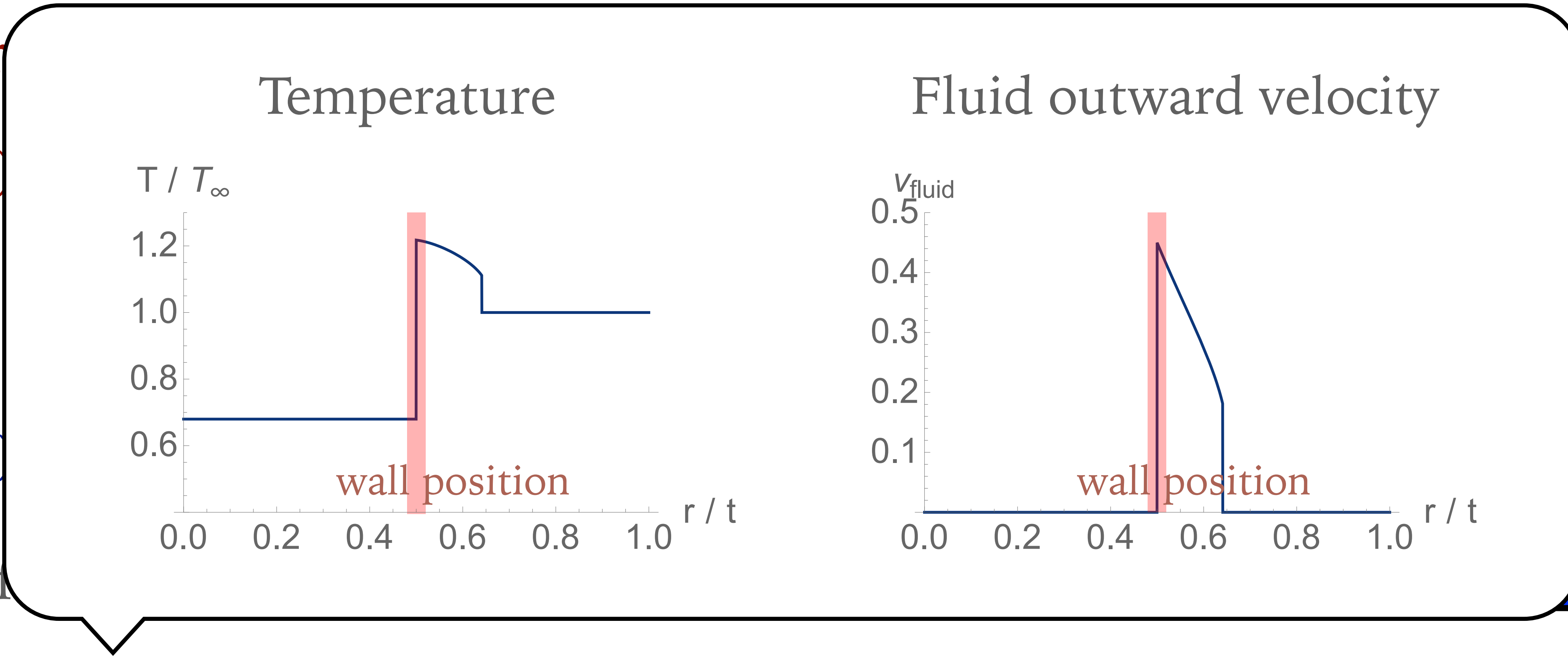


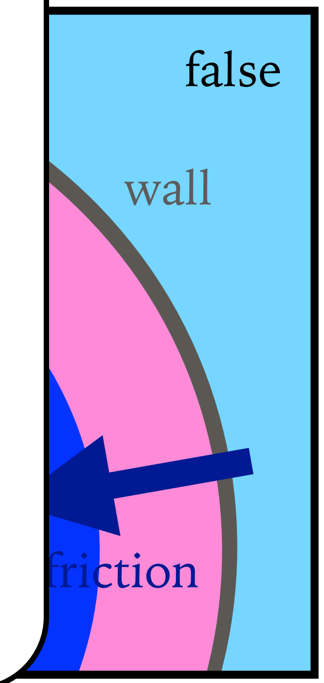
➤ "Pr … l scale

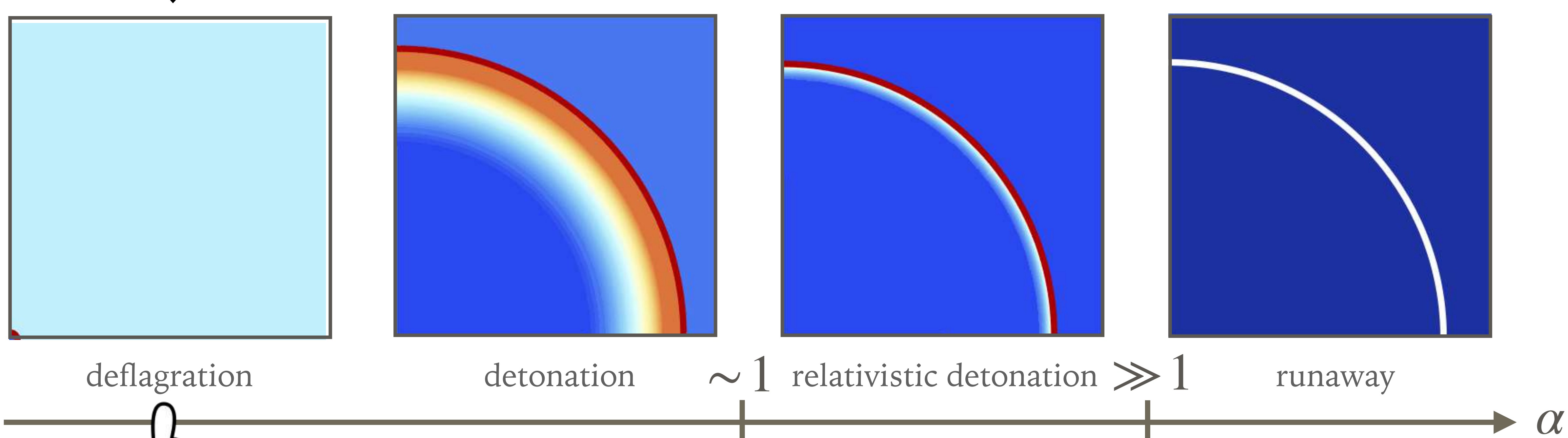
(1)

(2)

➤ Di







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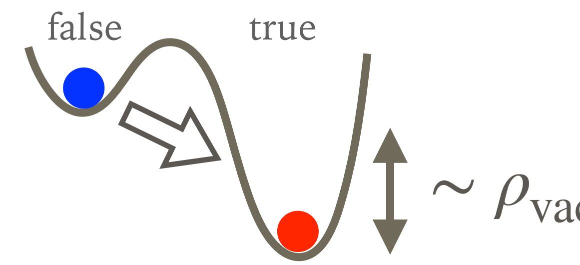

➤ "Pr

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(2)

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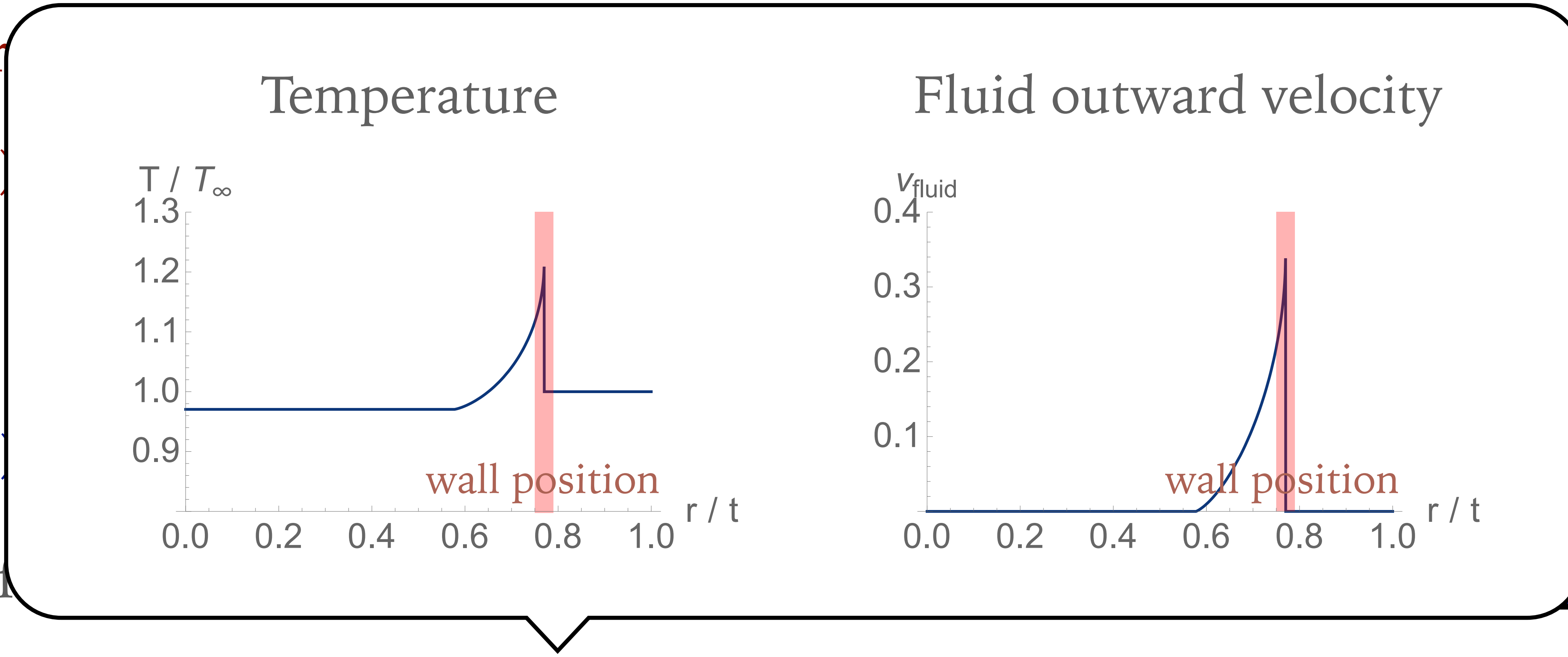

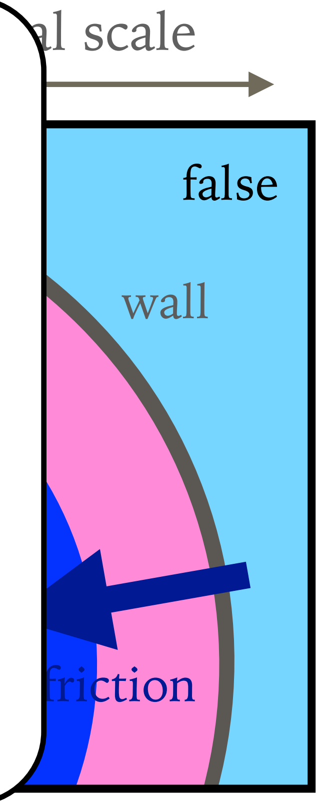

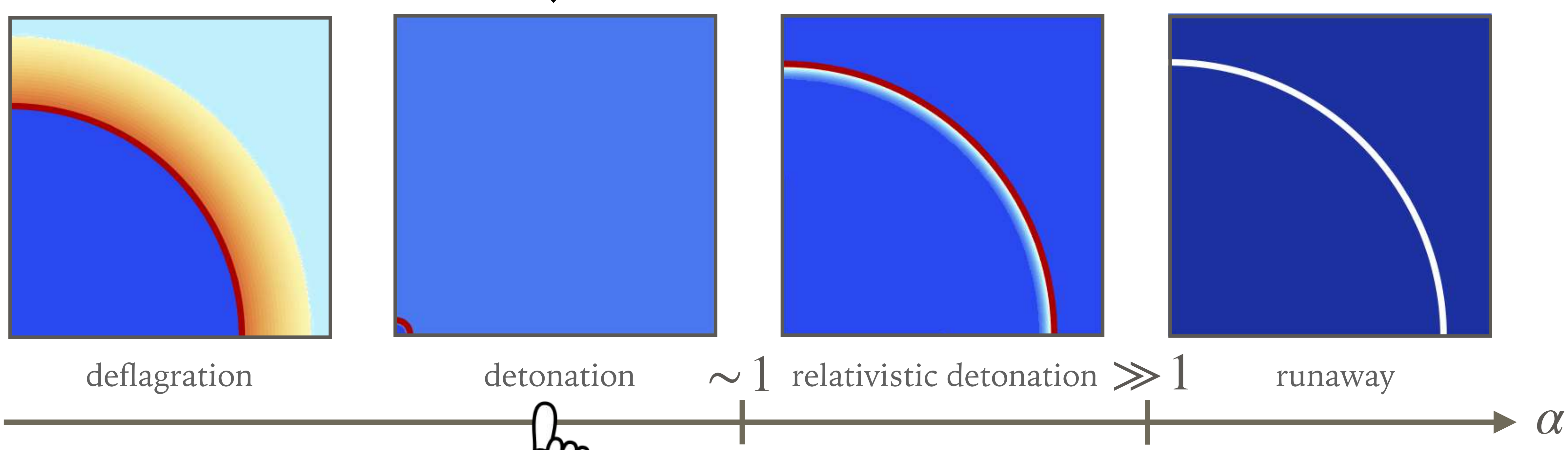

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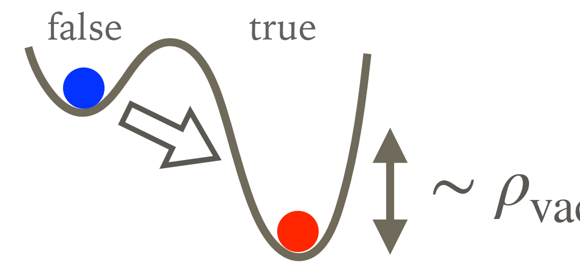

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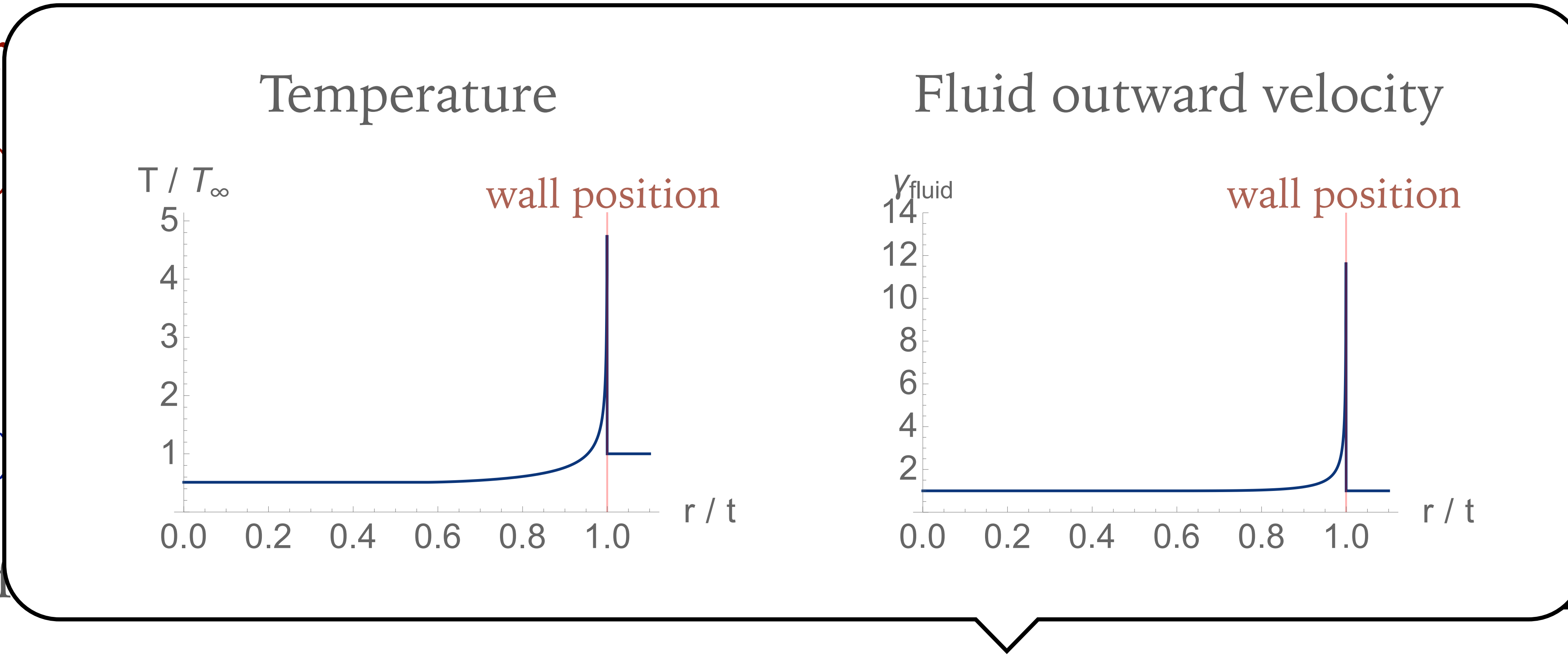

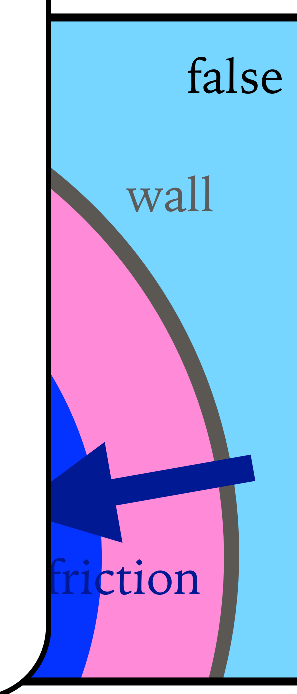

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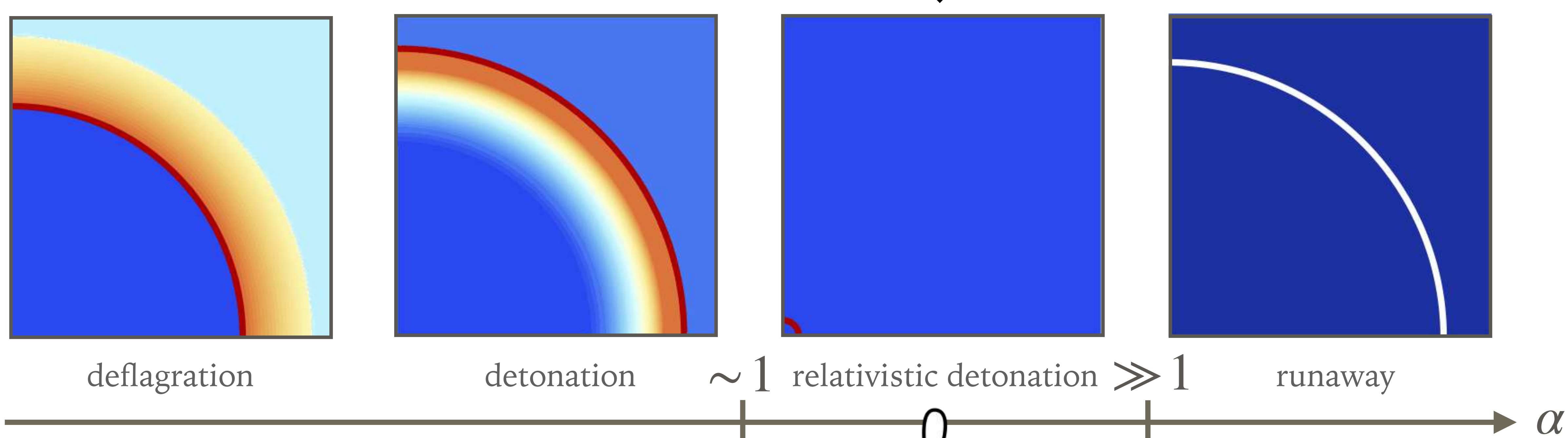

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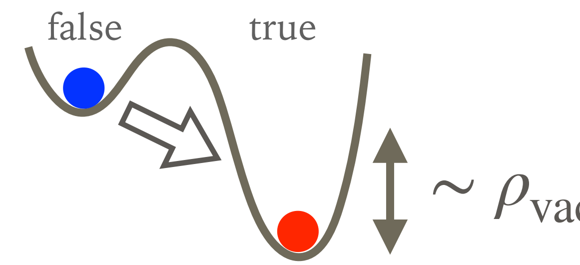

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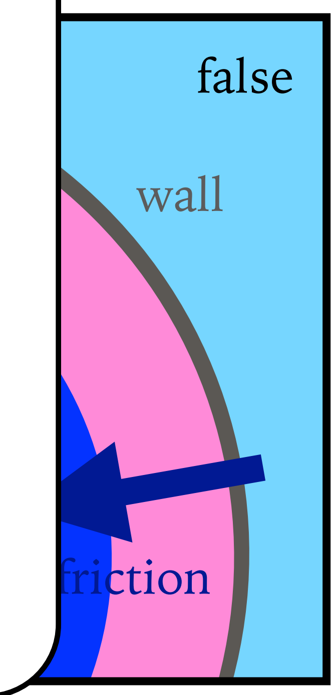
(1)

(2)

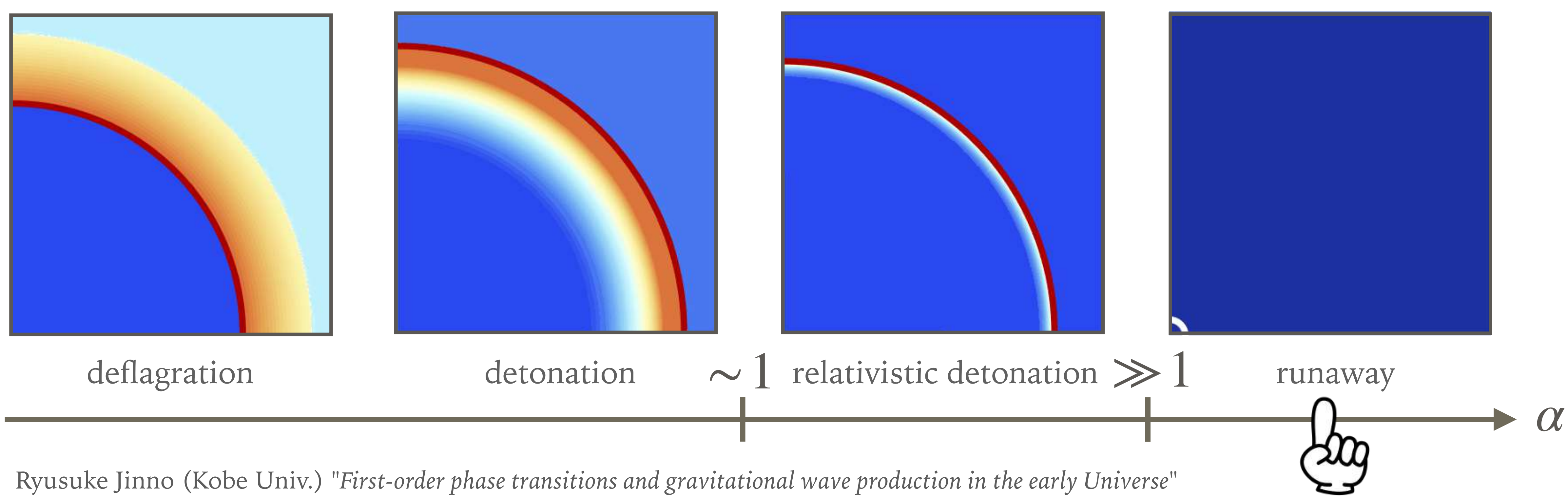


➤ Di

Plasma particles cannot stop the acceleration of the walls:

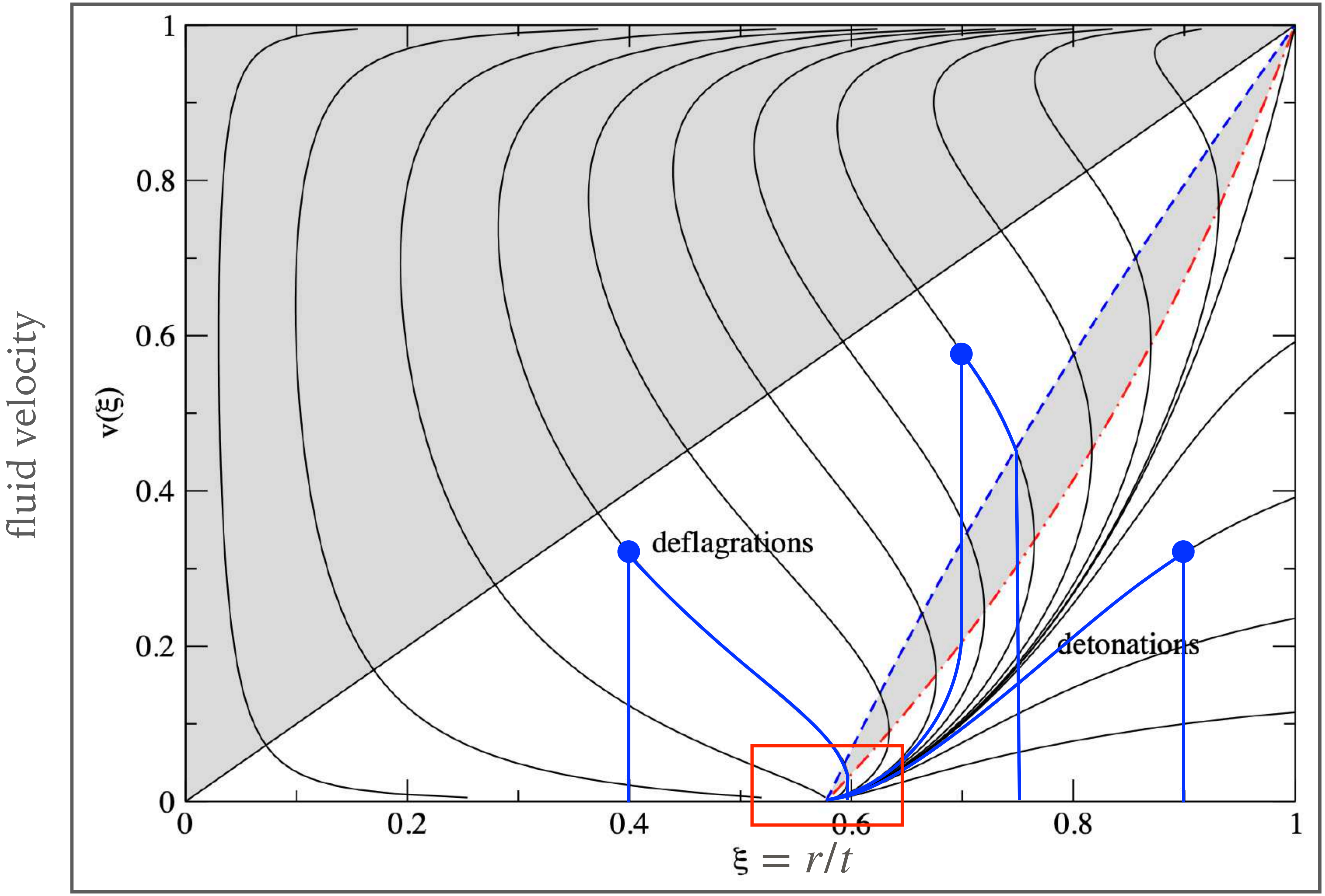
walls continue to accelerate until they collide with others

[Bodeker & Moore '09, '17]




Youtube "Explosions: 100 ton test detonation"

ASIDE: HOW THE PROFILE BEHAVES AROUND THE SOUND SPEED



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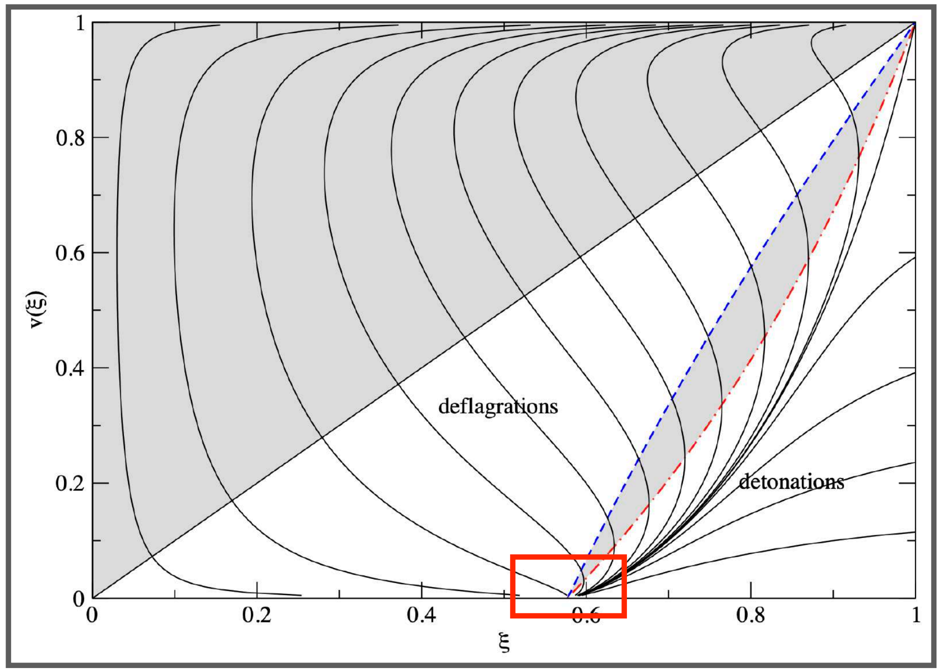


$$\frac{dv}{d\xi} = \frac{2v(1-v^2)}{\xi(1-\xi v)} \Big/ \left[\frac{(\xi - v)^2}{(1-\xi v)^2} \frac{1}{c_s^2} - 1 \right]$$

$$\downarrow$$

$$\frac{dv}{d\xi'} \sim \frac{v}{\xi' + (c_s^2 - 1)v}, \qquad \xi' \equiv \xi - c_s$$

$$\downarrow$$

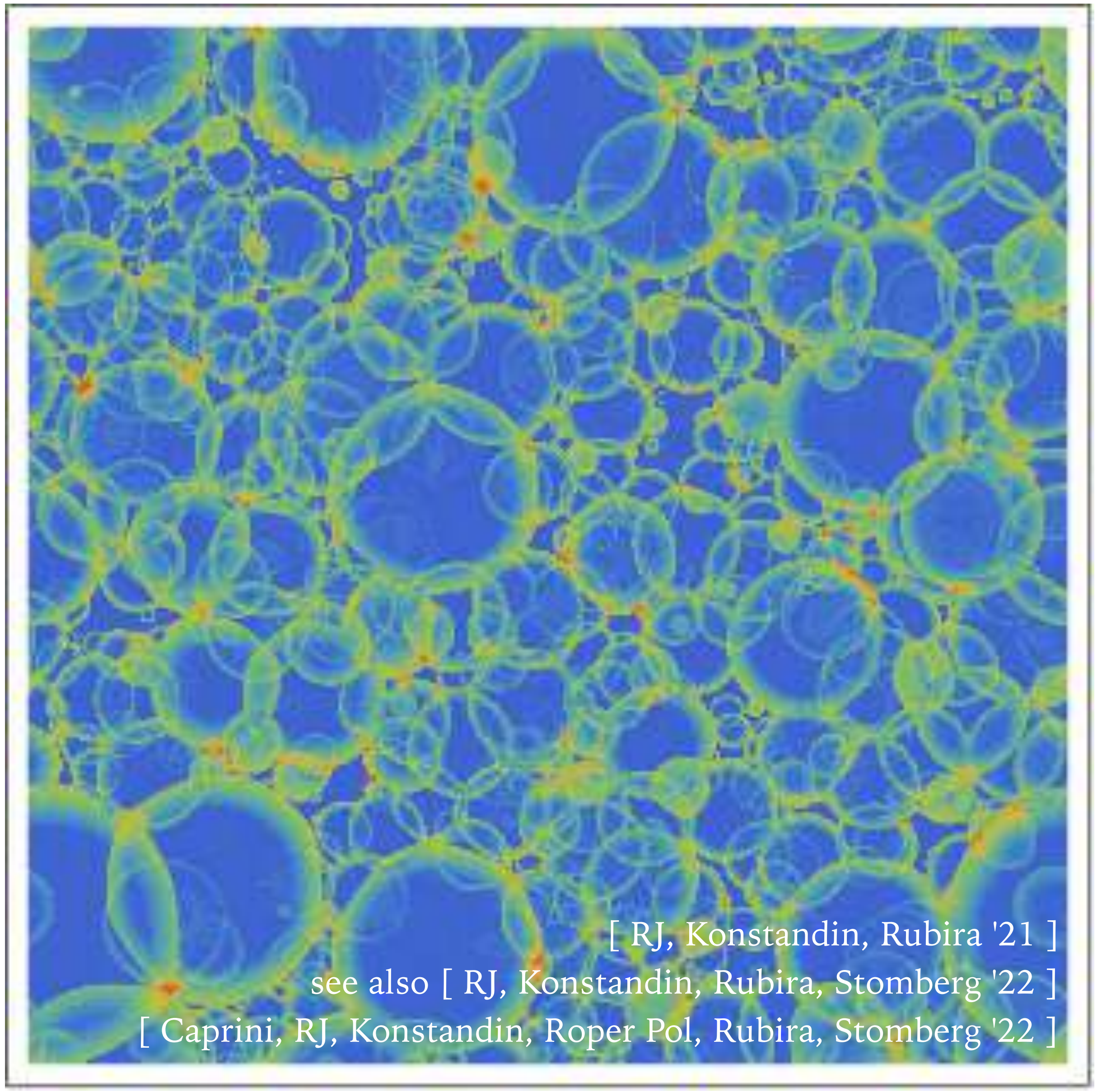
$$v \sim -\frac{\xi'}{\mathrm{ProductLog}(-\mathrm{const.} \times \xi')}$$

$$\downarrow$$

$$\left.\frac{dv}{d\xi'}\right|_{\xi'=0} = 0, \qquad \left.\frac{d^n v}{d\xi'^n}\right|_{\xi'=0} = \infty \quad (n \geq 2)$$

BUBBLE COLLISION & FLUID DYNAMICS

- Bubbles collide, and fluid dynamics sets in (example for )



TRANSITION (≒ THERMODYNAMIC) PARAMETERS

- Remind the spirit of thermodynamics

  - Only a few parameters determine macroscopic properties

TRANSITION (≒ THERMODYNAMIC) PARAMETERS

- Remind the spirit of thermodynamics

  - Only a few parameters determine macroscopic properties

- What are parameters that describe the present macroscopic system?

Particle physics → Transition parameters → Prediction on GWs

Lagrangian $\mathscr{L}$

α : transition strength

β : nucleation increase rate

v_w : wall velocity

T_* : transition temperature

GW spectrum $\Omega_{\rm GW}$

GW non-Gaussianity ...

TRANSITION (≒ THERMODYNAMIC) PARAMETERS

see e.g. [Caprini et al. '16]
[Caprini et al. '20]

➤ Transition strength $\alpha \equiv \rho_{\rm vac}/\rho_{\rm plasma}$

- How much energy (= latent heat) is released, compared to the plasma energy

- The numerator $\rho_{\rm vac} = \rho_{\rm vac,false} - \rho_{\rm vac,true}$ is calculated from the Helmholtz free energy, through the relation $U = F + TS = F - T\left(\frac{\partial F}{\partial T}\right)_V$ as

$$\rho_{\rm vac,true} = V_{\rm eff}(\phi_{\rm true}, T) - T\left(\frac{\partial V_{\rm eff}(\phi_{\rm true}, T)}{\partial T}\right)$$

$$\rho_{\rm vac,false} = V_{\rm eff}(\phi_{\rm false}, T) - T\left(\frac{\partial V_{\rm eff}(\phi_{\rm false}, T)}{\partial T}\right)$$

TRANSITION (≒ THERMODYNAMIC) PARAMETERS

see e.g. [Caprini et al. '16]
[Caprini et al. '20]

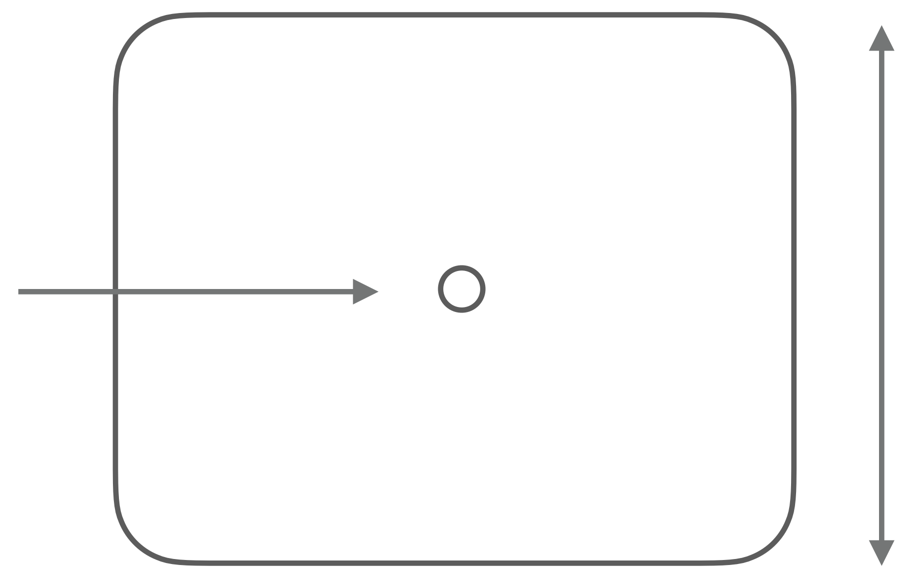
➤ Nucleation rate increase β : $\Gamma(t) \propto e^{\beta(t-t_*)+\cdots}$

- Calculate $\Gamma(T)$ as a function of temperature, using thermal field theory

- Translate $\Gamma(T)$ into $\Gamma(t)$ using (cosmological temperature) ⇔ (cosmological time)

- Taylor-expand the exponent around the typical transition time $t = t_*$

The first bubble
in the Hubble patch



Hubble radius

$$H^{-1} = \left(\frac{\dot{a}}{a}\right)^{-1}$$

TRANSITION (≒ THERMODYNAMIC) PARAMETERS

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Thermal field theory

$$\Gamma(T) \sim T^4 e^{-S_3/T}$$

=

Cosmology

$$H^4 \sim (T^2/M_P)^4$$

TRANSITION (≒ THERMODYNAMIC) PARAMETERS

see e.g. [Caprini et al. '16]
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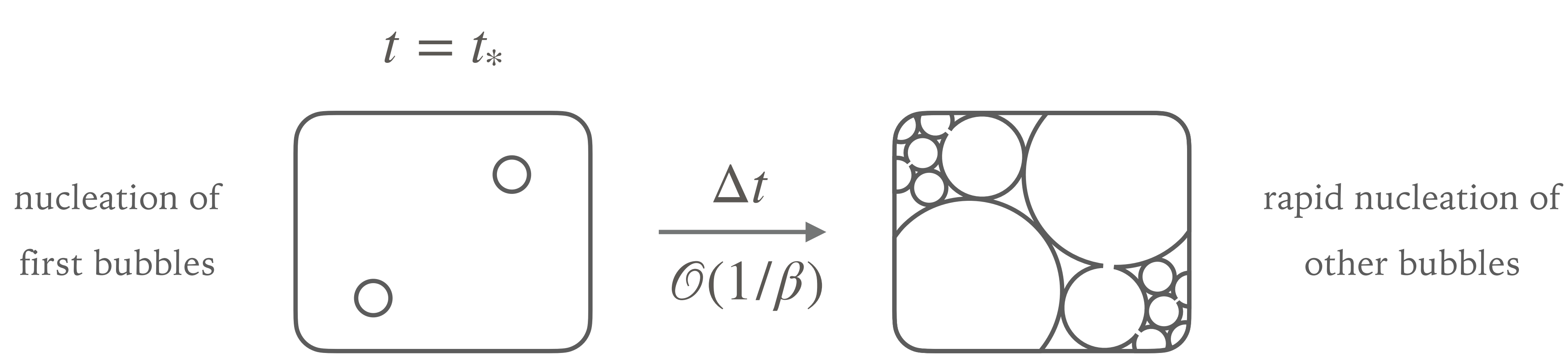
- Taylor-expand the exponent around the typical transition time $t = t_*$

- Interesting property: v_w/β gives the typical bubble size at the time of collision



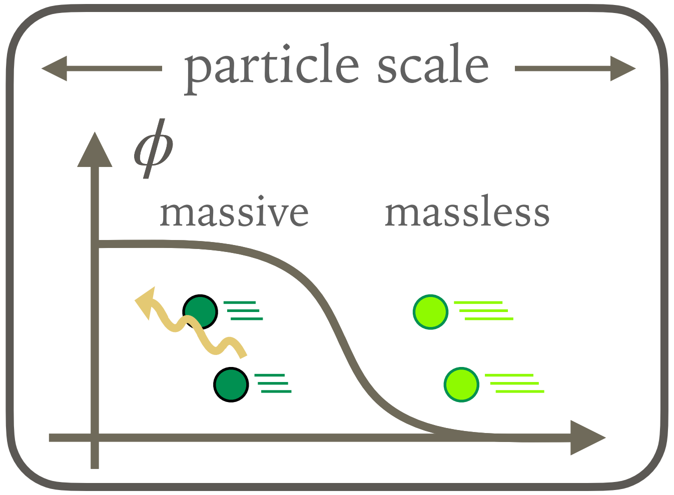
TRANSITION (≒ THERMODYNAMIC) PARAMETERS

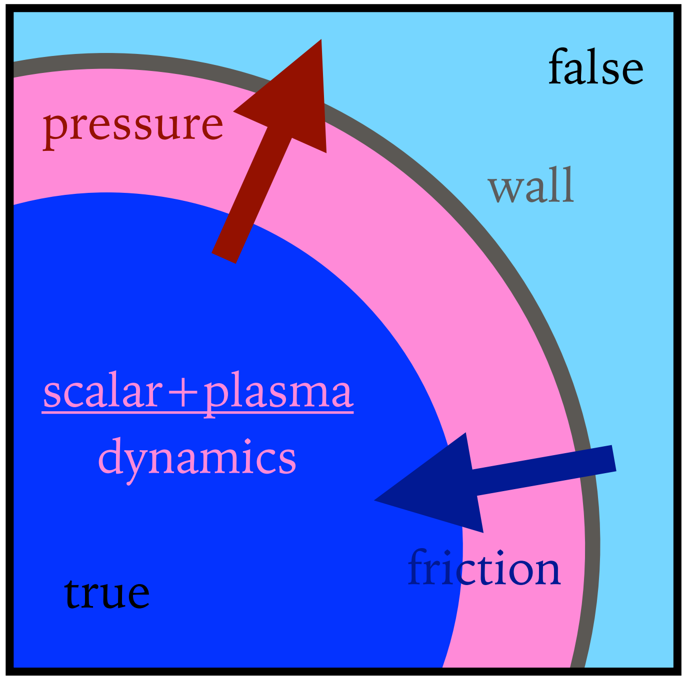
see e.g. [Caprini et al. '16]
[Caprini et al. '20]

➤ Wall velocity v_w

- Determined from "pressure vs. friction"

- In principle one should solve Boltzmann eq.,
  but people often put by hand
  (regarded as trade-off btwn. coupling ⇔ velocity)





➤ Transition temperature T_*

- Determined from your microphysical theory

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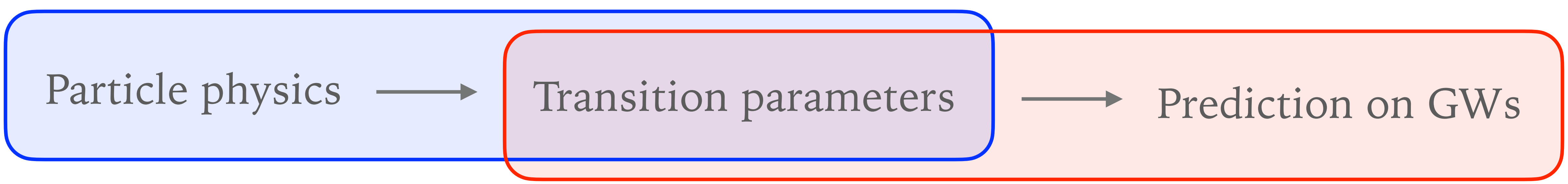

Lagrangian $\mathscr{L}$

α : transition strength

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GW spectrum $\Omega_{\rm GW}$

GRAVITATIONAL WAVES: A NEW PROBE TO THE UNIVERSE

- Einstein equation:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$

"Spacetime tells matter how to move. Matter tells spacetime how to curve."

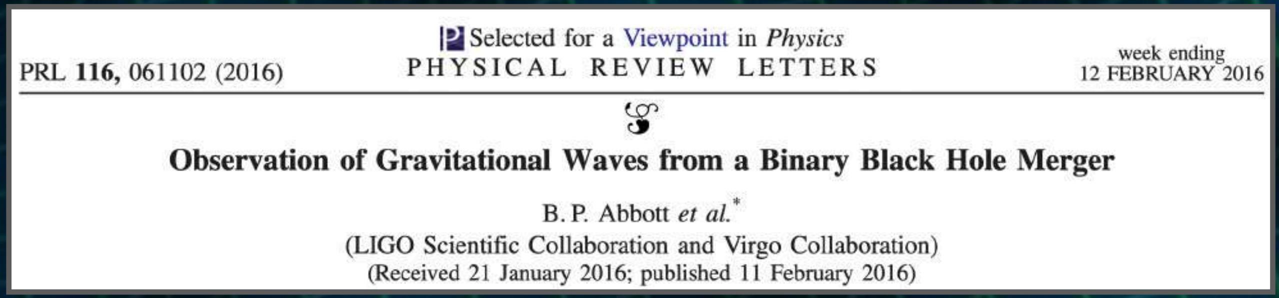
- Gravitational waves: transverse-traceless part of the metric

$$ds^2 = -dt^2 + a^2(\delta_{ij} + h_{ij})dx^i dx^j \qquad \partial_i h_{ij} = h_{ii} = 0$$

- After expanding the Einstein equation, GWs obey a wave equation sourced by the energy-momentum tensor of the system

$$\Box h_{ij} = 16\pi G \Lambda_{ij,kl} T_{kl}$$

- LIGO/Virgo detected GWs from binary black holes for the first time in 2015


PRL 116, 061102 (2016) Selected for a Viewpoint in Physics PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*[*]

(LIGO Scientific Collaboration and Virgo Collaboration)

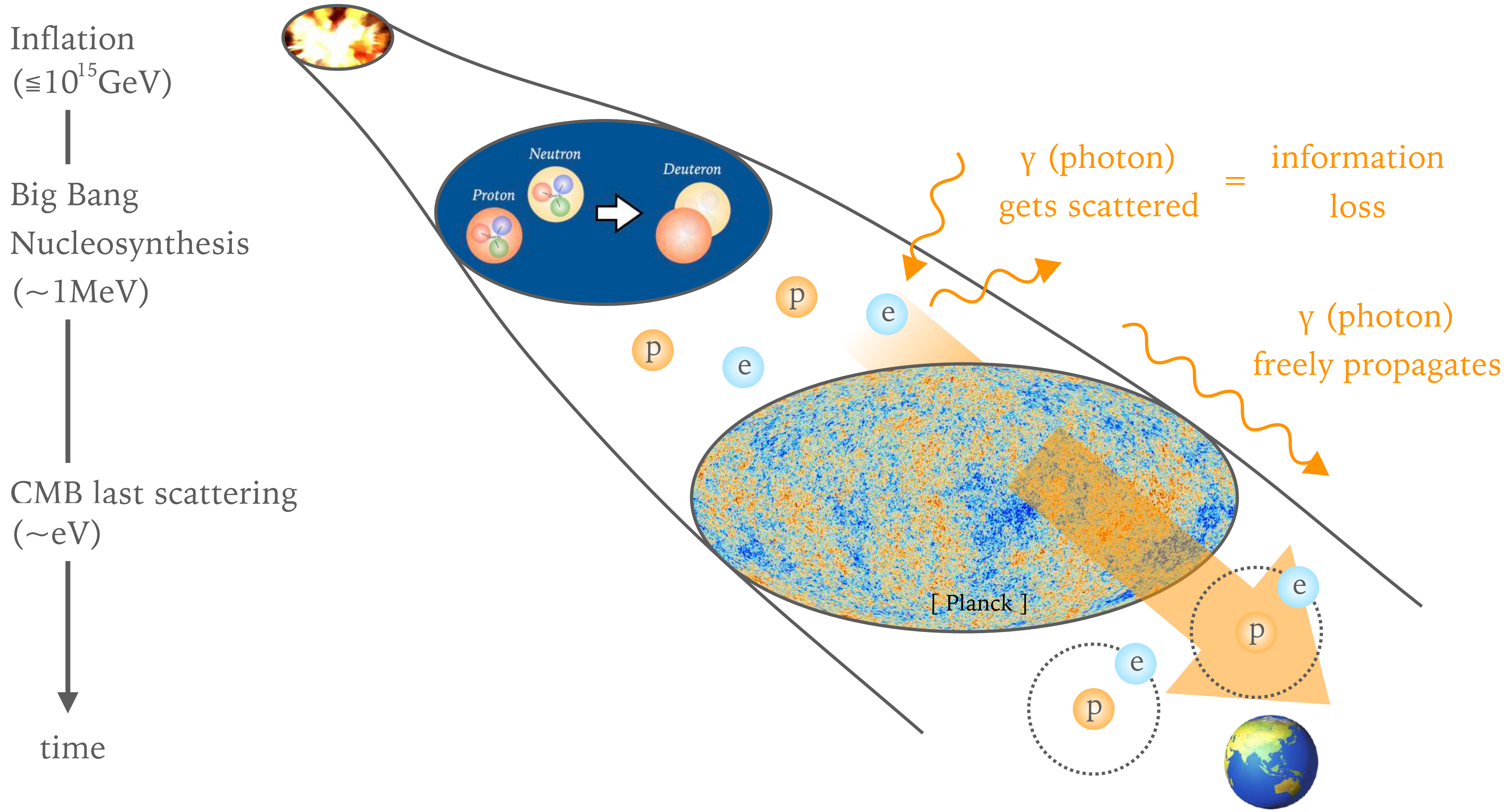
(Received 21 January 2016; published 11 February 2016)

$$36M_\odot + 29M_\odot \rightarrow 62M_\odot + 3M_\odot \text{ (GWs)}$$

[R. Hurt/Caltech-JPL]

GWS AS A PROBE OF THE EARLY UNIVERSE

➤ CMB (Cosmic Microwave Background) as a probe of the early Universe



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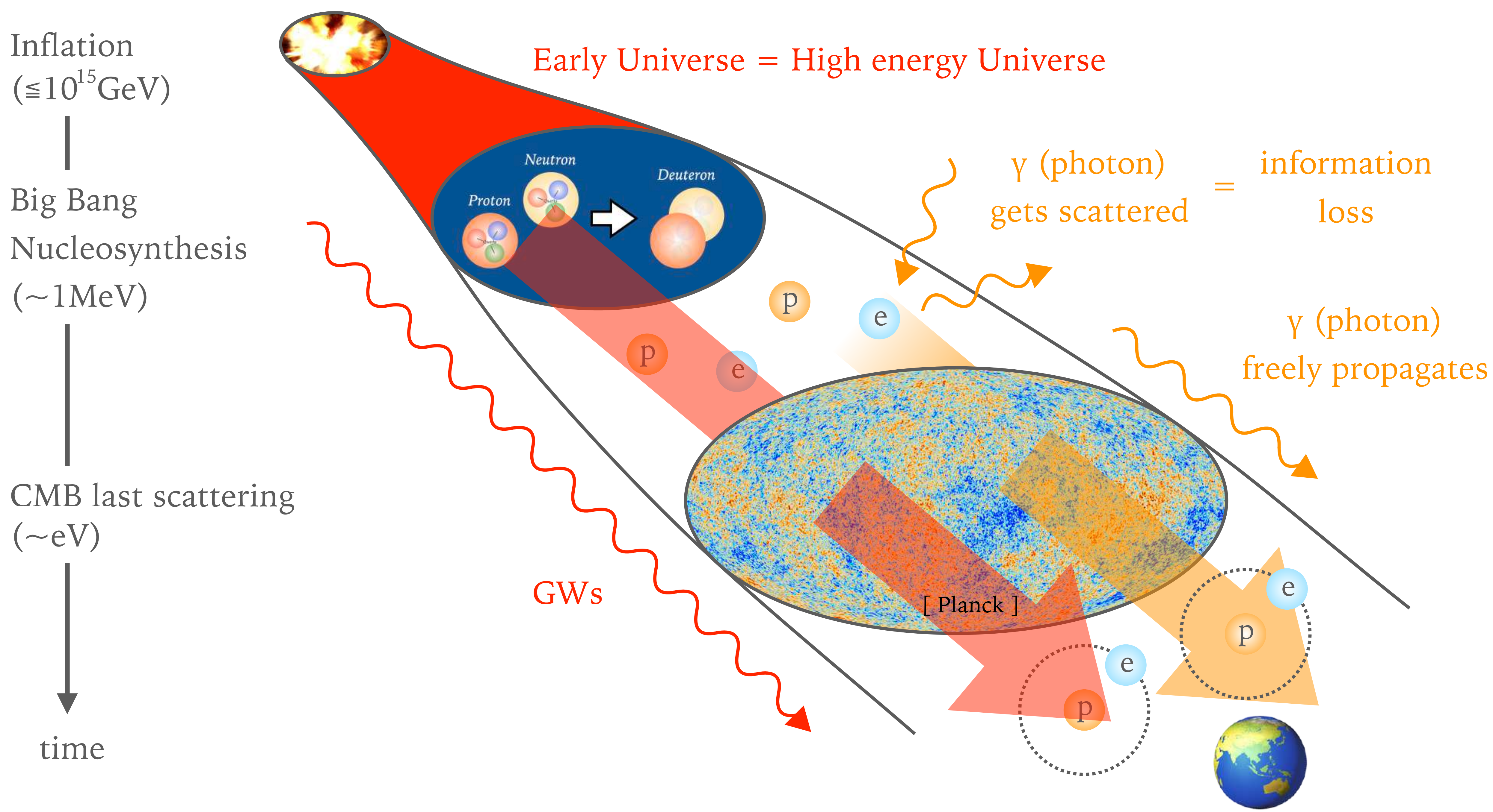
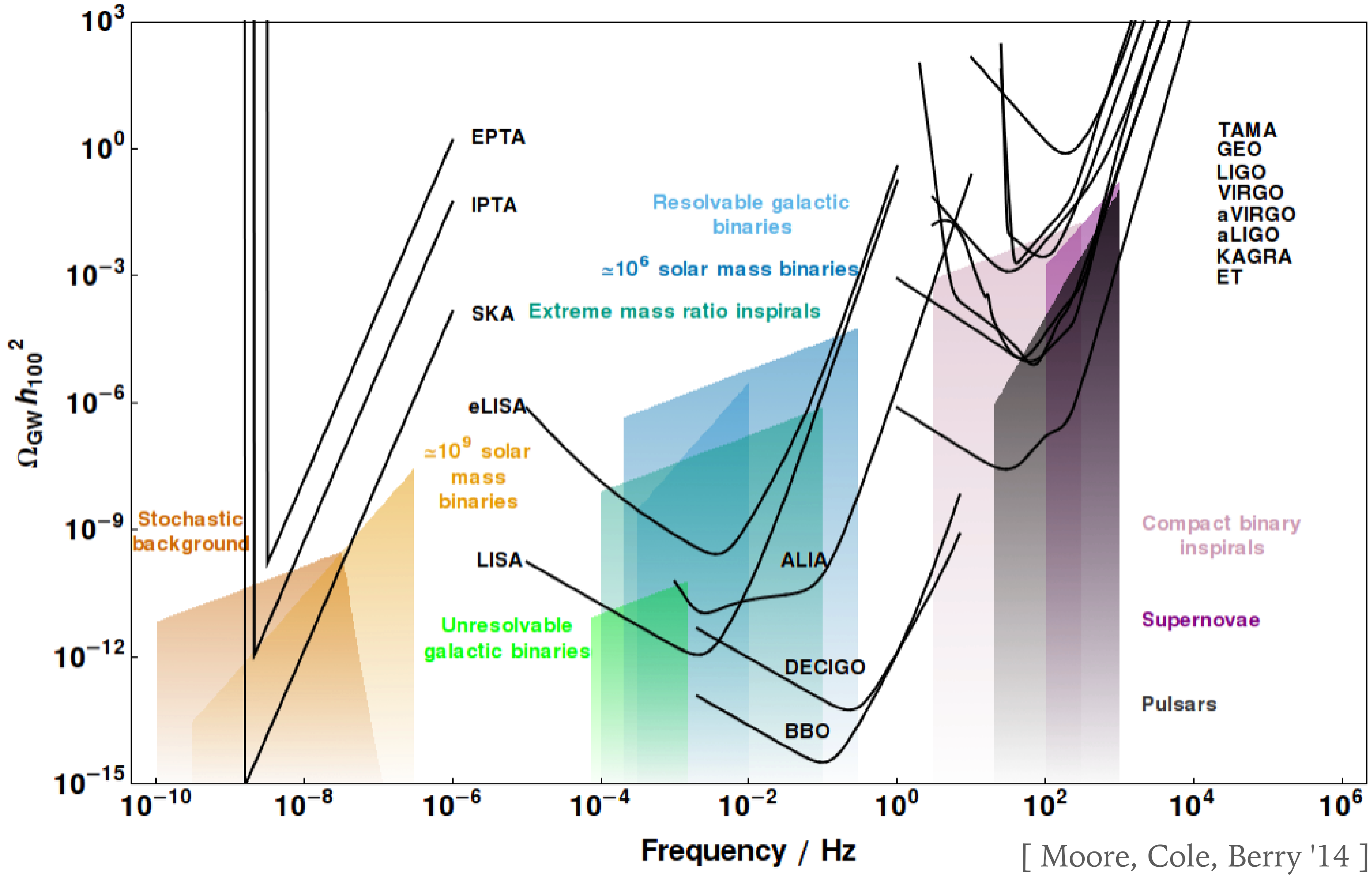


PRESENT & FUTURE OBSERVATIONS

Pulsar timing arrays	Space-borne interferometers	Ground-based interferometers
$\sim 10^{-8}$ Hz	$\sim$ mHz – Hz	~ 100 Hz

GW energy density per unit log freq. / total energy density of the Universe

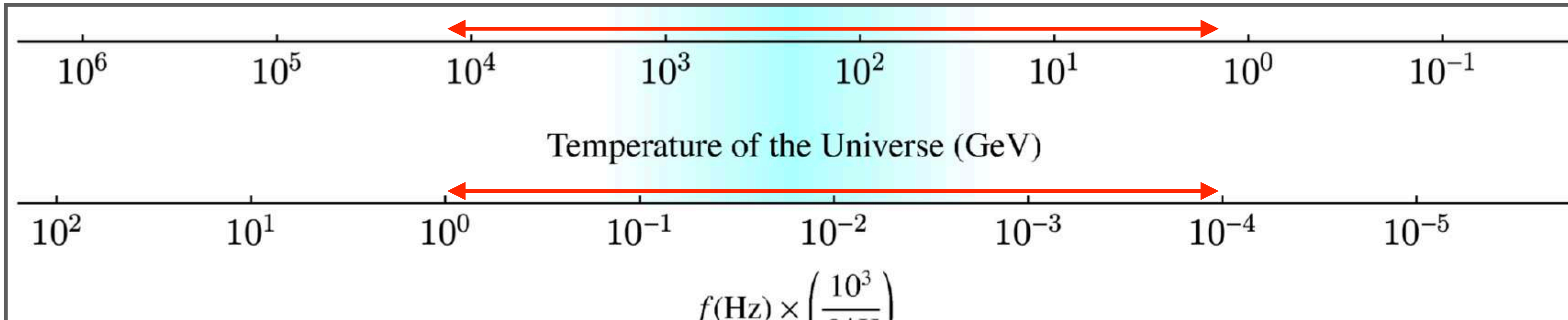


[Moore, Cole, Berry '14]

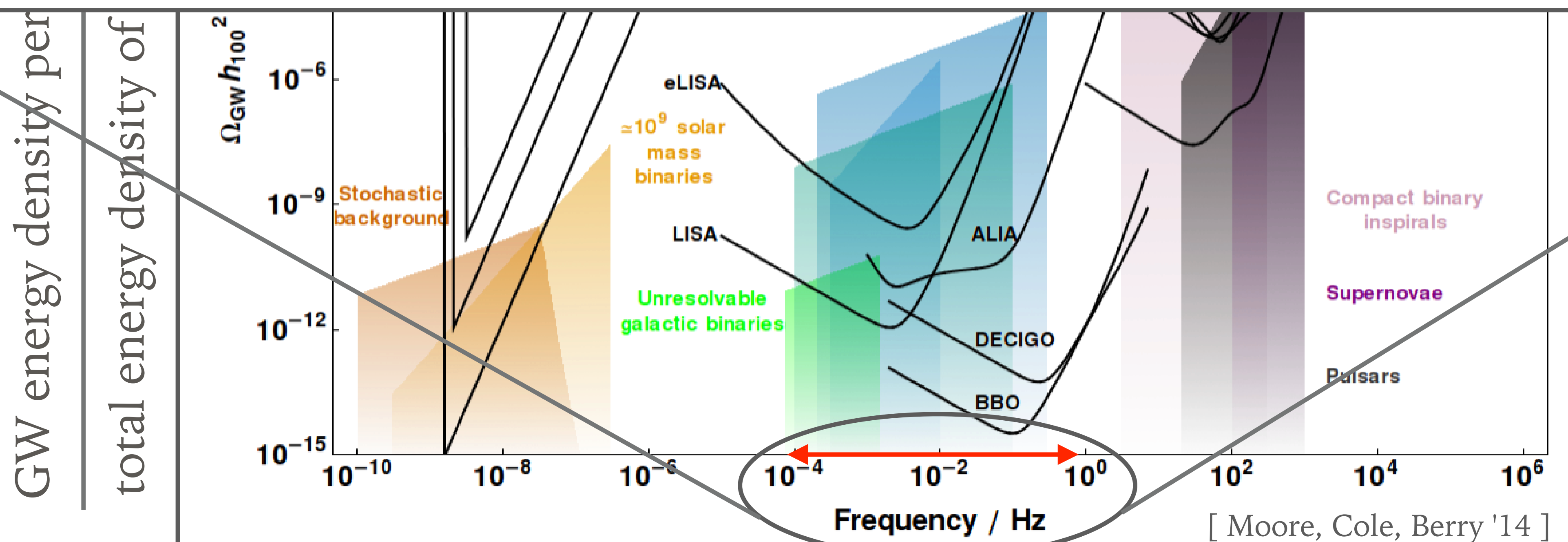
Present frequency of cosmological GWs

∝ Energy scale (temperature) at the time of production

TeV scale physics



[Auclair et al. '22]



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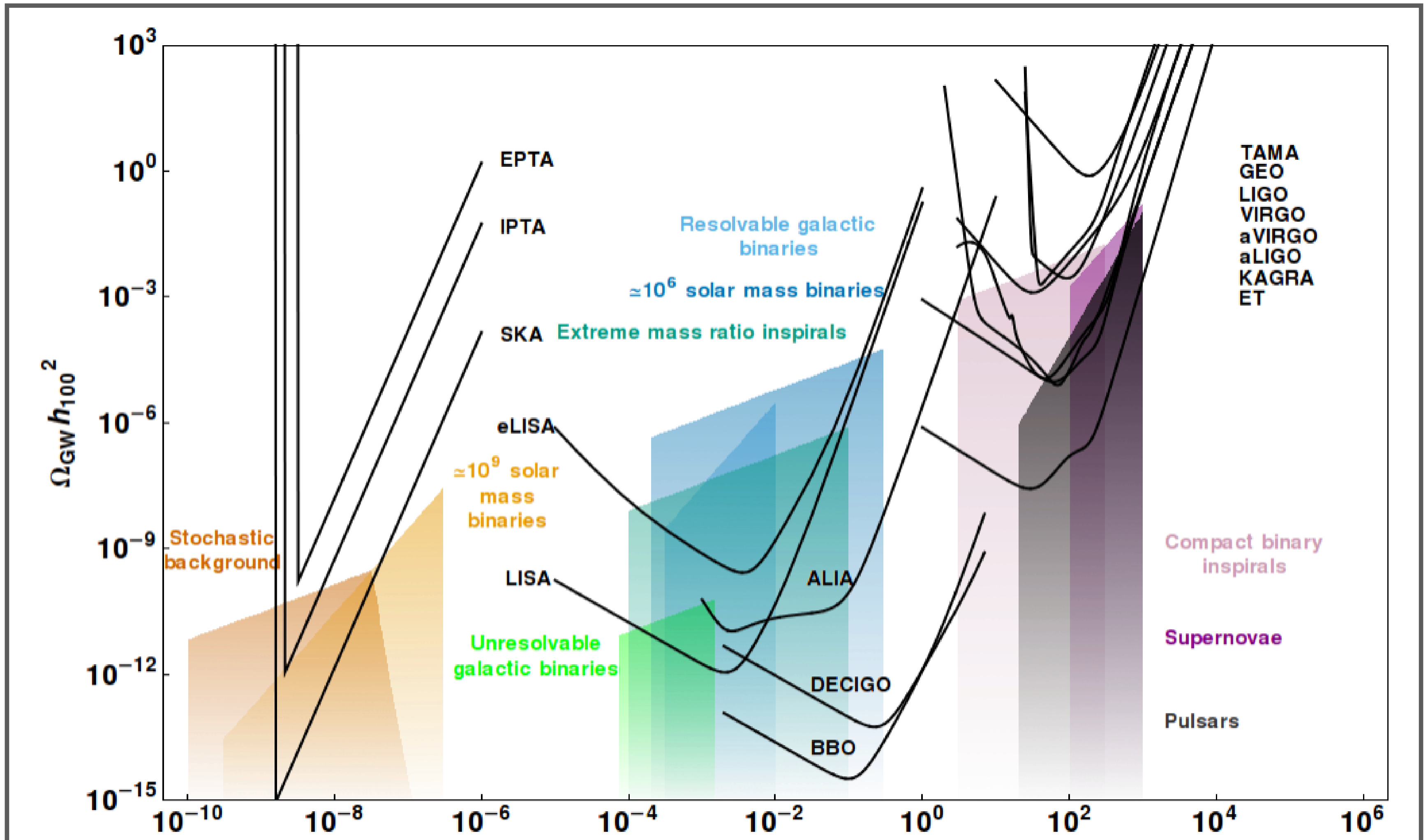
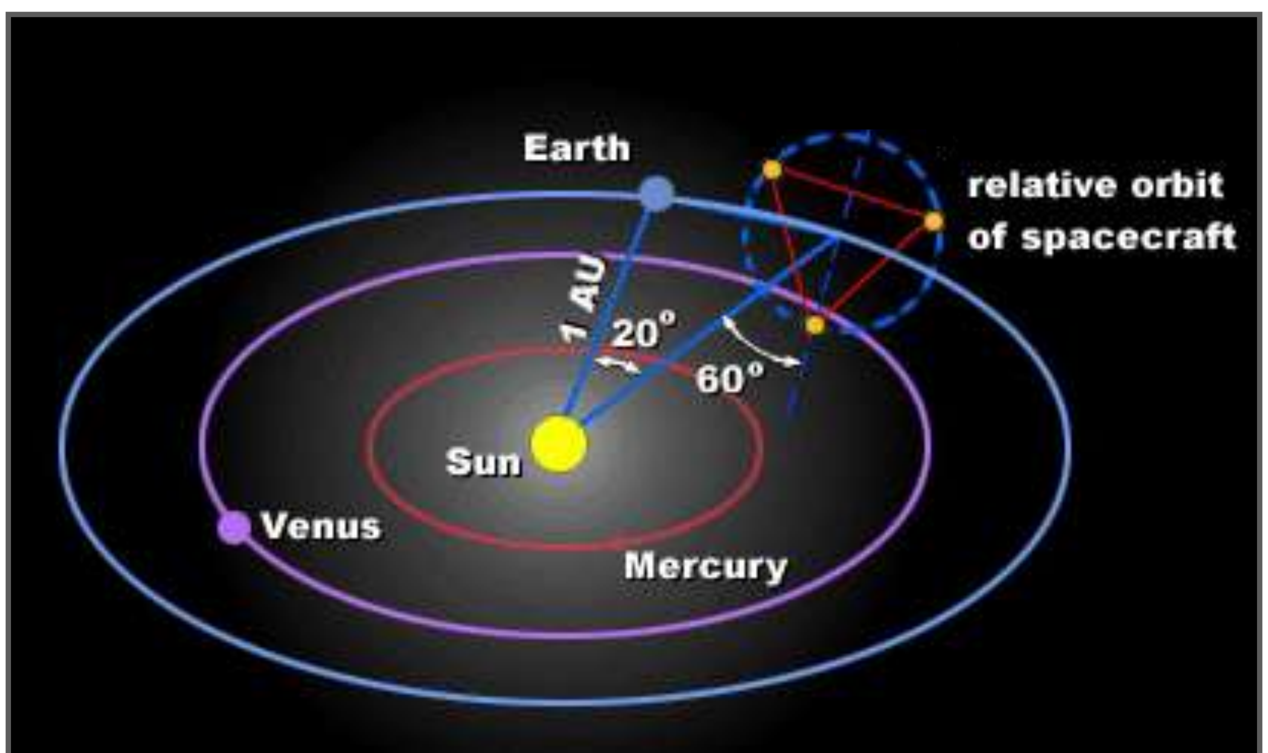


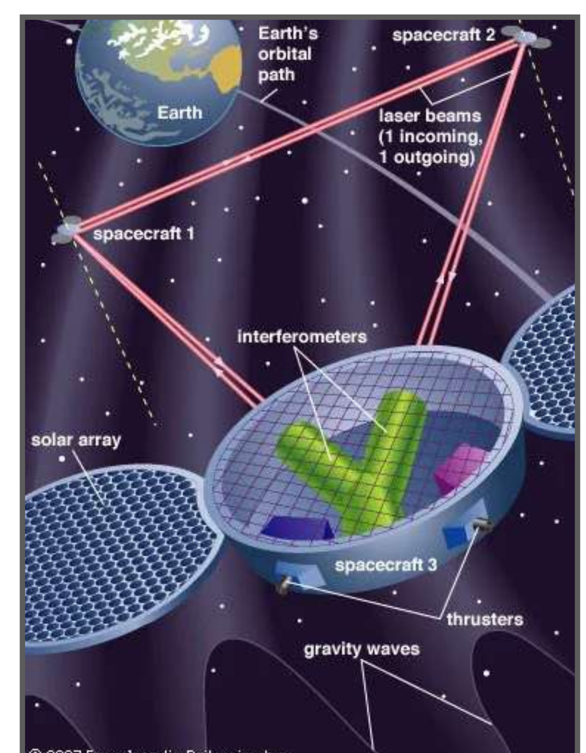
[Moore, Cole, Berry '14]

LISA (LASER INTERFEROMETER SPACE ANTENNA)

[LISA Mission L3 Proposal, https://www.elisascience.org/files/publications/LISA_L3_20170120.pdf] [Auclair et al. '22]

- Mission led by ***ESA*** (European Space Agency), together with ***NASA***
- Selected as L3 mission in 2017, planned to be ***launched in 2034***
- ***3 satellites*** forming an equilateral triangle in an Earth-trailing orbit
- Distance between satellites = $\mathbf{2.5 \times 10^6\ km}$
- Nominal mission of ***6 years***, with a ***duty cycle of around 75%***





[https://sci.esa.int/web/lisa/-/31704-schematic-of-lisa-orbit] [https://www.britannica.com/science/physics-science/The-study-of-gravitation]

LISA (LASER INTERFEROMETER SPACE ANTENNA)

s/LISA_L3_20170120.pdf] [Auclair et al. '22]

- with *NASA*
- *in* ~~*2034*~~ ***2037***
- railing orbit
-
- *75%*

→ THE EUROPEAN SPACE AGENCY

LISA factsheet

2689 VIEWS 6 LIKES

ESA / Science & Exploration / Space Science

Overview of the LISA mission.

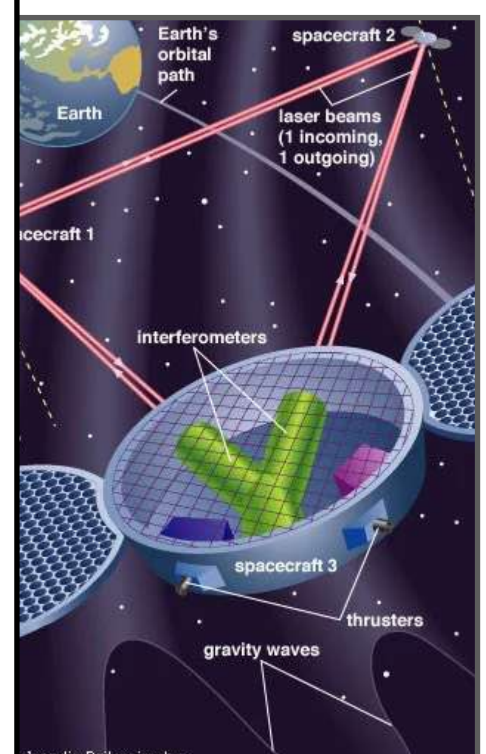
Name: The Laser Interferometer Space Antenna (LISA)

Planned launch: 2037

Mission theme: The gravitational Universe

Status: On 20 June 2017, LISA was selected as the third large-class mission, L3, under ESA's Cosmic Vision 2015-2025. LISA is currently in a more detailed phase of study and will be proposed for 'adoption' around 2024, after which construction can begin

[https://www.esa.int/Science_Exploration/Space_Science/LISA_factsheet]



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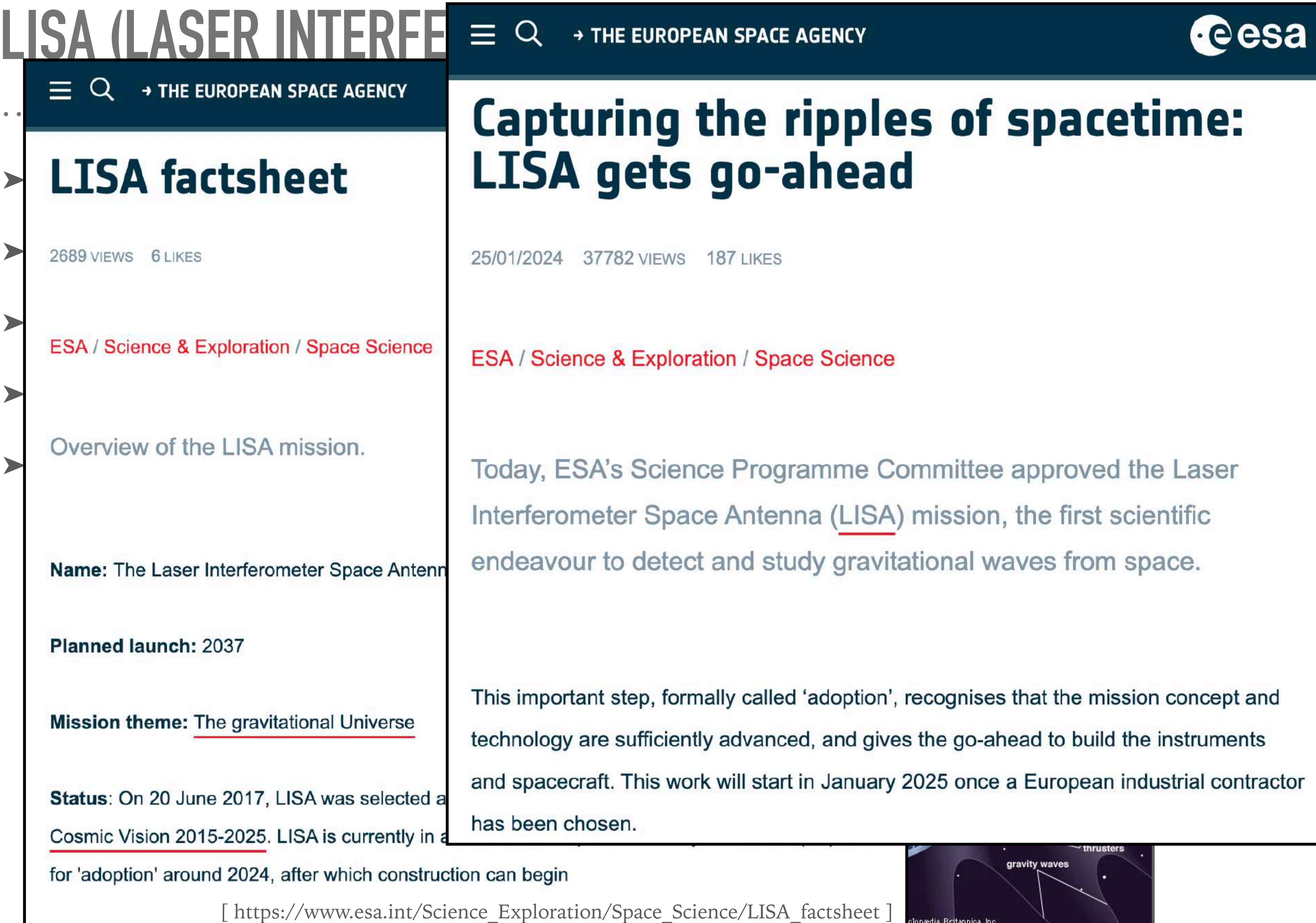
LISA (LASER INTERFE



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LISA factsheet

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esa

Capturing the ripples of spacetime: LISA gets go-ahead

25/01/2024 37782 VIEWS 187 LIKES

ESA / Science & Exploration / Space Science

Today, ESA's Science Programme Committee approved the Laser Interferometer Space Antenna (LISA) mission, the first scientific endeavour to detect and study gravitational waves from space.

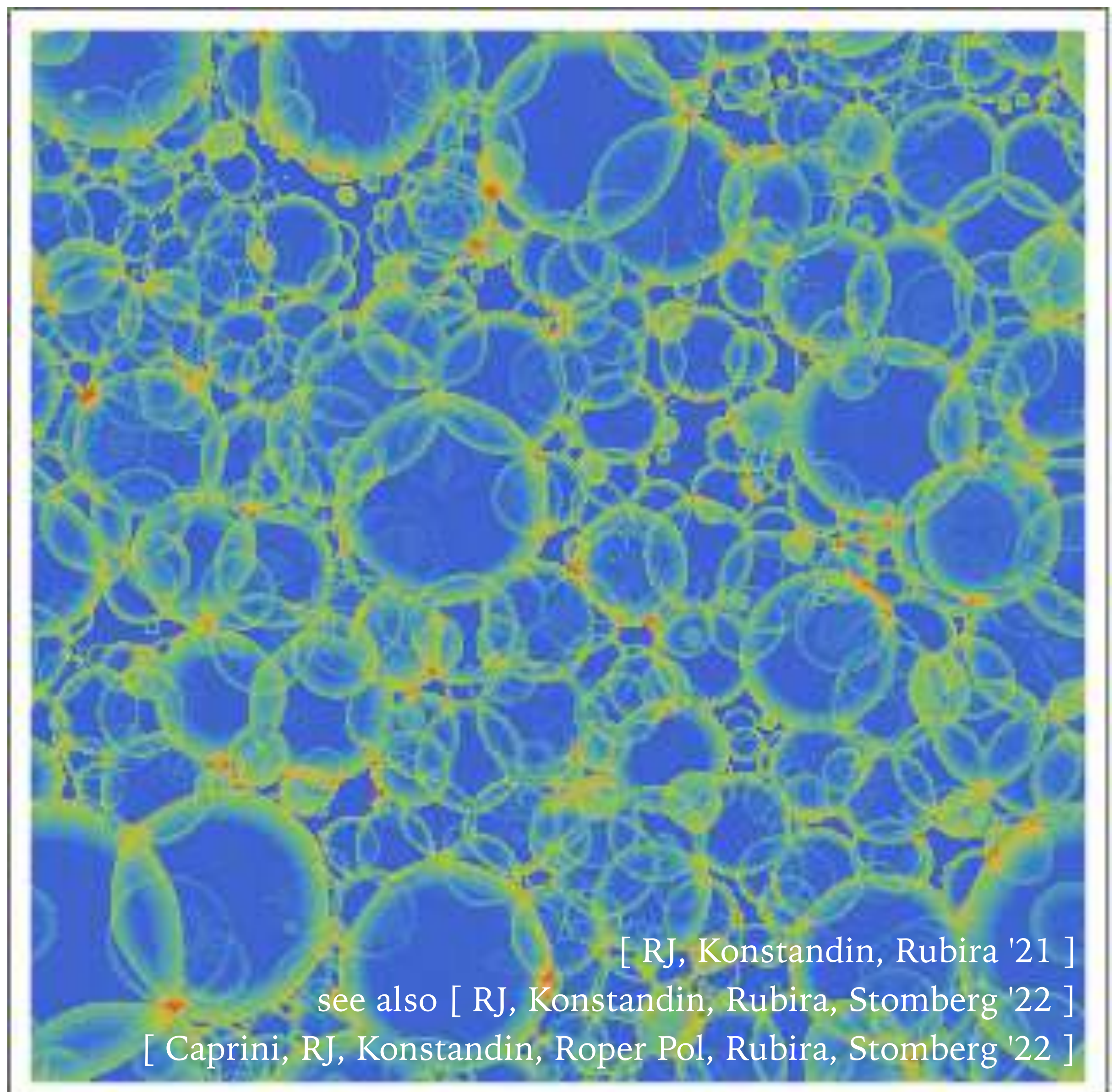
This important step, formally called 'adoption', recognises that the mission concept and technology are sufficiently advanced, and gives the go-ahead to build the instruments and spacecraft. This work will start in January 2025 once a European industrial contractor has been chosen.

[https://sci.esa.int/web/lisa/-/31704-schematic-of-lisa-orbit] [https://www.britannica.com/science/physics-science/The-study-of-gravitation]

BUBBLE COLLISION & FLUID DYNAMICS

➤ Bubbles collide, and fluid dynamics sets in (example for 

)

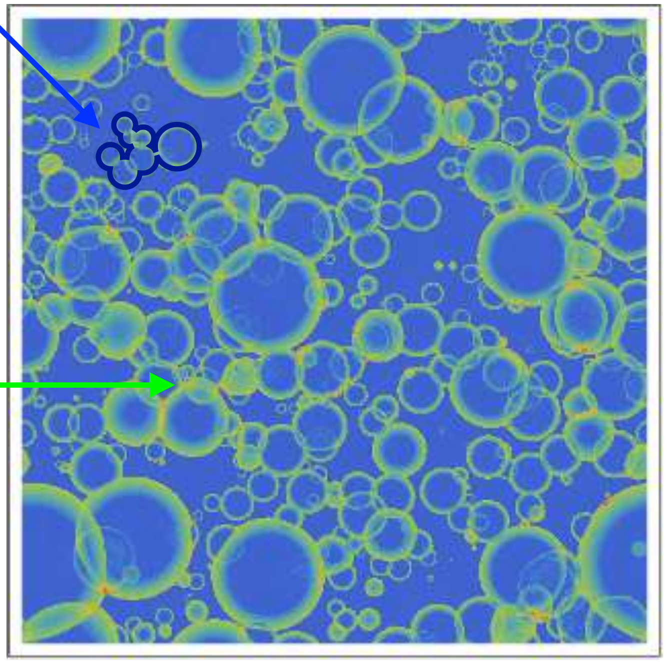


[RJ, Konstandin, Rubira '21]
see also [RJ, Konstandin, Rubira, Stomberg '22]
[Caprini, RJ, Konstandin, Roper Pol, Rubira, Stomberg '22]

GRAVITATIONAL WAVE SOURCES

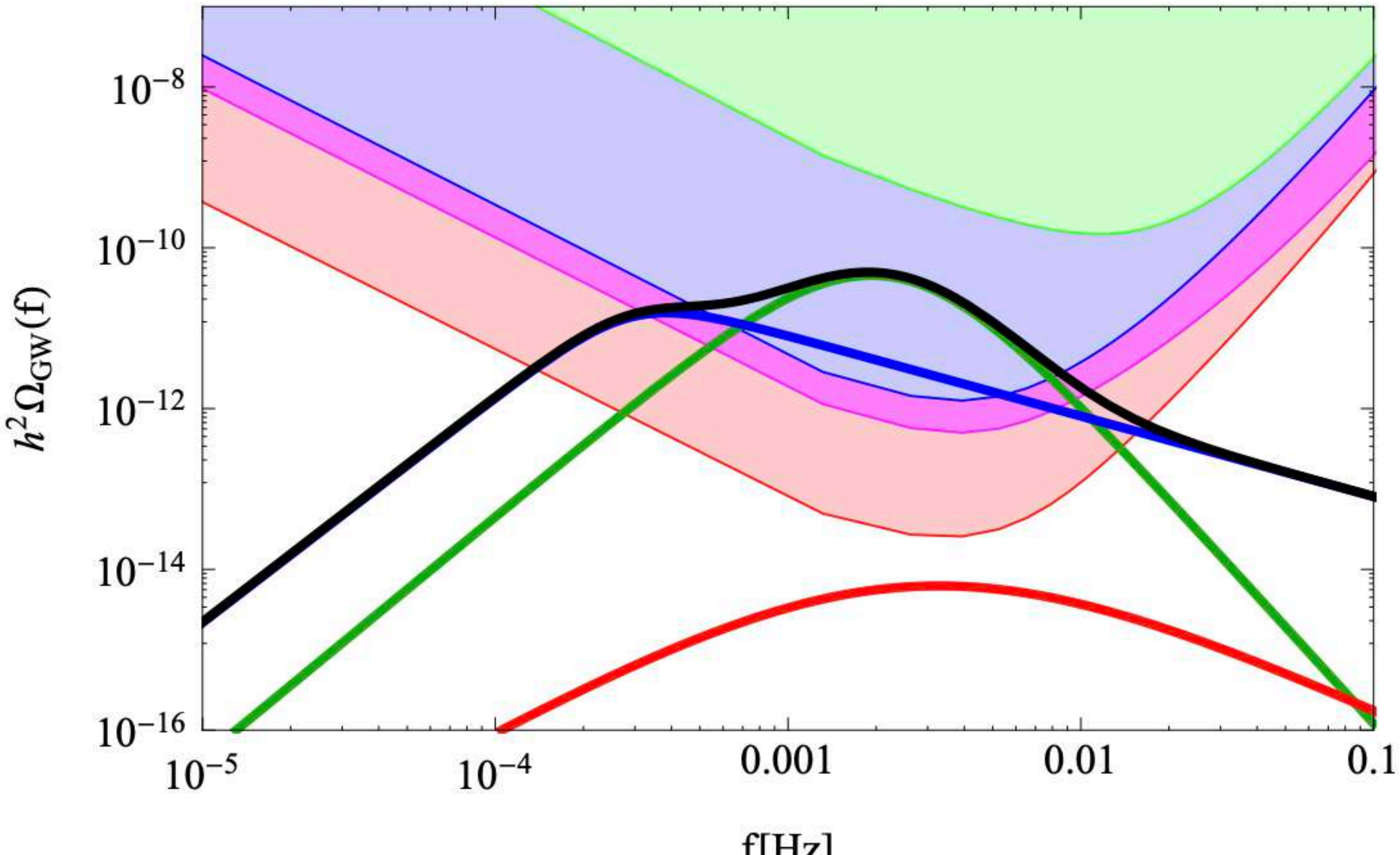
[Kosowsky, Turner, Watkins '92]
[Kosowsky, Turner '92]
[Kamionkowski, Kosowsky, Turner '93]
and e.g. [Caprini et al. '16] [Caprini et al. '20]

- **Bubble collision**
  - Kinetic & gradient energy of the scalar field (= order parameter field)
  - Dominant when the transition is extremely strong and the walls runaway
- **Sound waves**
  - Compression mode of the fluid motion
  - Dominant unless the transition is extremely strong
- **Turbulence**
  - Turbulent motion caused by fluid nonlinearity
  - Expected to develop at a later stage



important at later stage

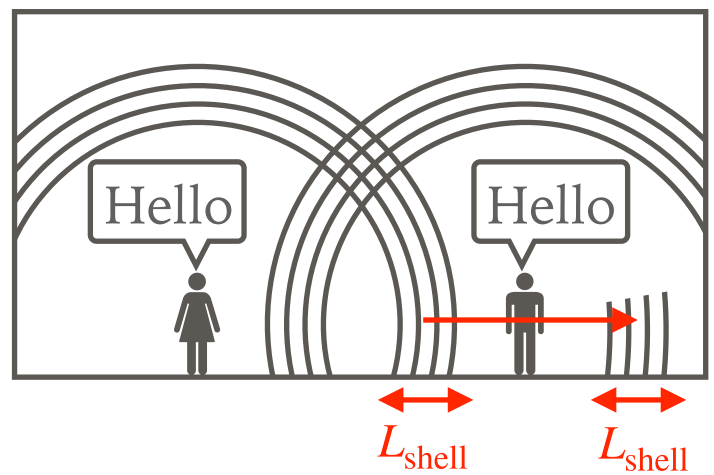
GRAVITATIONAL WAVE SPECTRUM



[Caprini et al. '16]

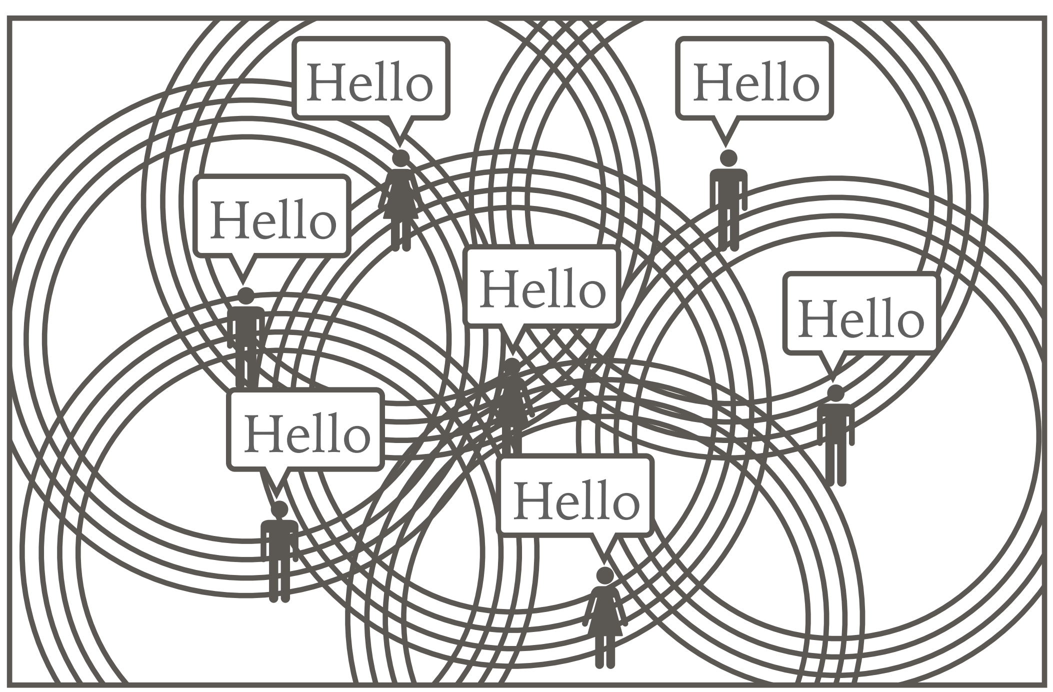
GRAVITATIONAL WAVES FROM SOUND WAVES

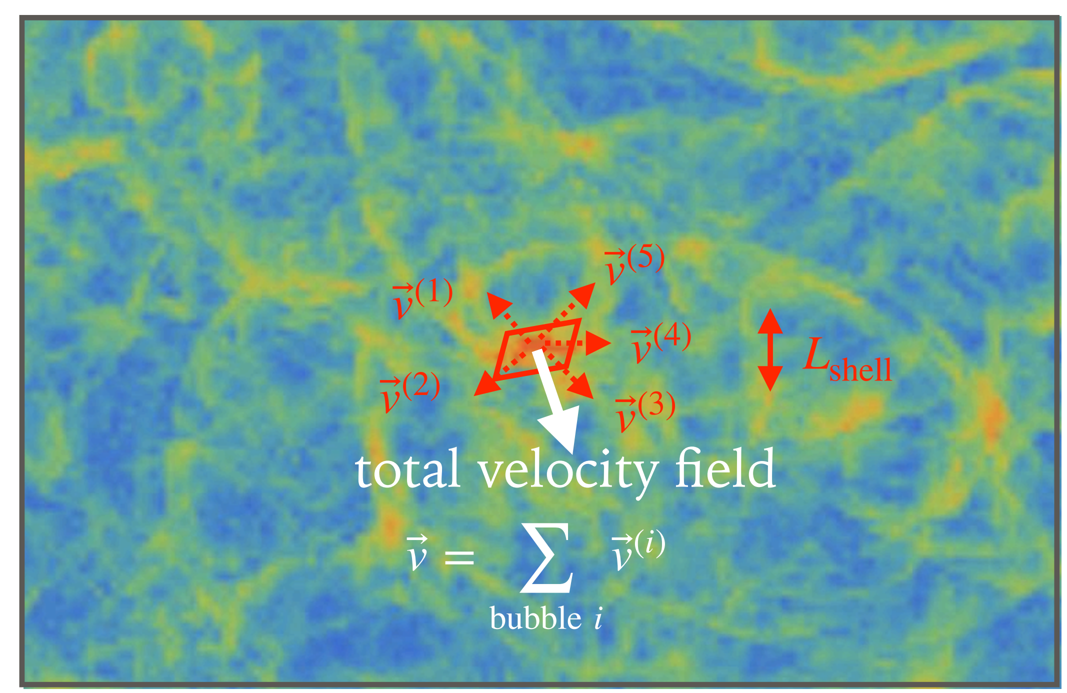
- Sound shells continue to propagate inside other bubbles



- Shell overlap creates random velocity fields everywhere, sourcing GWs

[Hindmarsh, Huber, Rummukainen, Weir '14, '15, '17] [Hindmarsh '15, +Hijazi '19]





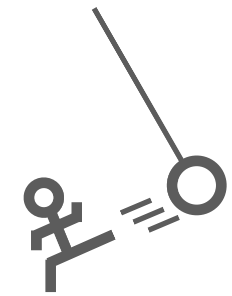
ROUGH ESTIMATE ON GW PRODUCTION

➤ BIG & RELATIVISTIC objects radiate more GWs

- Integrate the GW equation of motion over the coherence time Δt of the source

$$\Box h_{ij} \sim G T_{ij} \xrightarrow[\text{coherence time } \Delta t]{\text{integration over}} \dot{h}_{ij} \sim G T_{ij} \Delta t$$

oscillator



kicked oscillator

- GW energy density $\rho_{\rm GW} \sim G^{-1} \dot{h}_{ij}^2 \propto T_{ij}^2 \Delta t^2$ Note but: GWs from sound waves behave differently

1. Relativistic objects have larger $T_{ij} \propto \alpha$

2. Big bubbles typically have longer coherence time $\Delta t \propto \beta^{-1}$

1. Overview
2. First-order phase transitions
3. Dynamics of bubbles
4. Gravitational waves
5. **Recent topics**

SOME RECENT TOPICS

- Large-scale simulations: the Higgsless scheme

  [RJ, Konstandin, Rubira '21] [RJ, Konstandin, Rubira, Stomberg '22] [Caprini, RJ, Konstandin, Roper Pol, Rubira, Stomberg '24]

- GW signal from (almost) scale-invariant models

  [Konstandin, Servant '10] and many others, including [RJ, Takimoto '16]

- Seeded transitions from density perturbations/topological defects

  [RJ, Konstandin, Rubira, Van de Vis '21] [Blasi, Mariotti '22] [Blasi, RJ, Konstandin, Rubira, Stomberg '23]

- Particle splitting and next-leading-order (NLO) friction

  [Bodeker, Moore '17] [Azatov, Vanvlasselaer '21] [Gouttenoire, RJ, Sala '21] [Azatov, Barni, Petrossian-Byrne, Barni, Vanvlasselaer '24]

- Effect of gravity

  [Giombi, Hindmarsh '24] [RJ, Kume '24]

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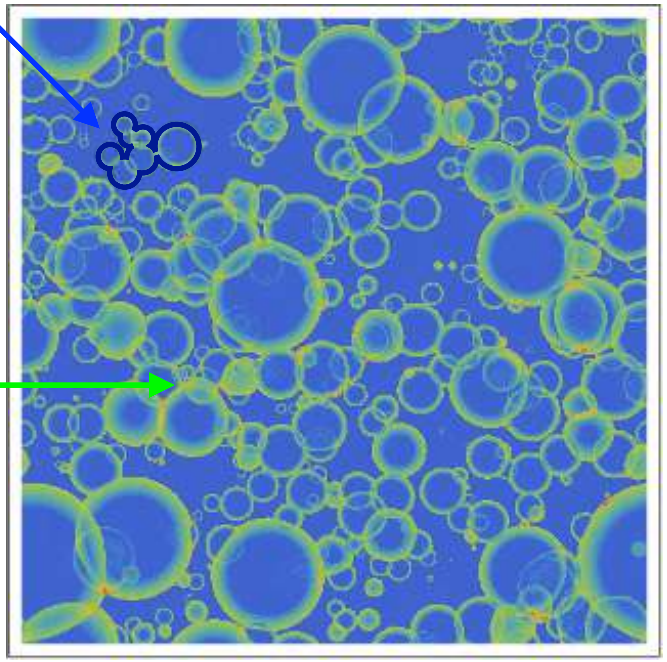


important at later stage

SOUND WAVE SIMULATIONS

➤ Fluid 3d simulation is harder than you might imagine:

- Shock waves
- Numerical viscosity → currently only 2 groups working on sound wave simulations
- Computational resources

➤ Our proposal: the Higgsless scheme

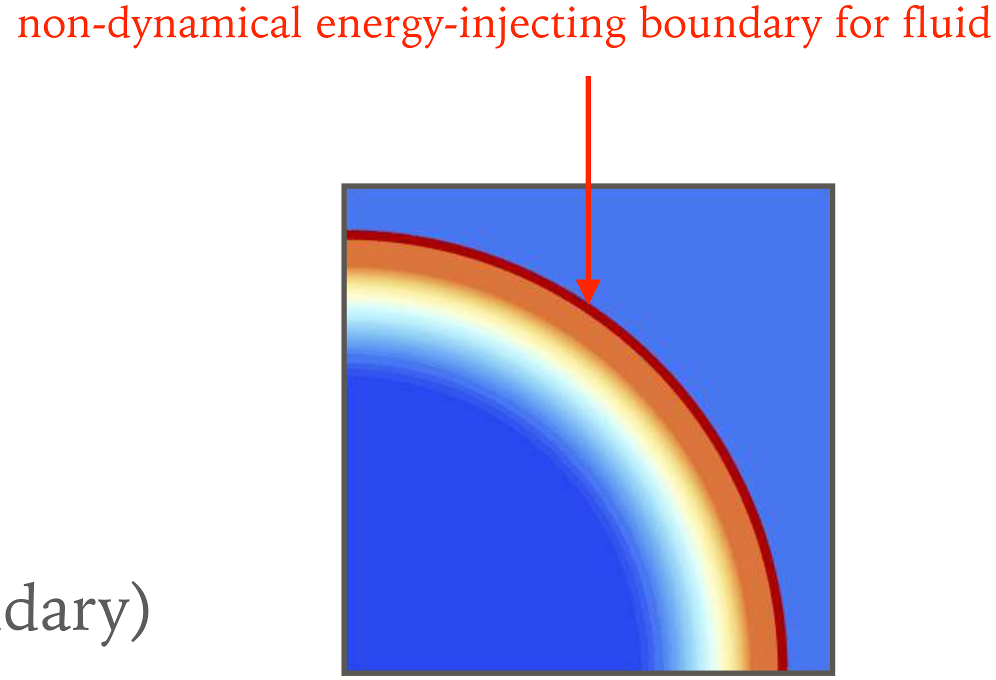
[RJ, Konstandin, Rubira '21] [RJ, Konstandin, Rubira, Stomberg '22]

[Caprini, RJ, Konstandin, Roper Pol, Rubira, Stomberg '24]

- We do ***not*** solve both the scalar field and fluid

  but rather "integrate out" the scalar field

  (= treat the scalar field as non-dynamical boundary)



HOW TO INTEGRATE THE HIGGS OUT

- The fluid evolution is determined from

  ① Energy-momentum conservation of the fluid $\partial_\mu T^{\mu\nu} = 0$

  ② Energy injection at the wall, parametrized by $\epsilon_{\rm vac} = \begin{cases} \epsilon_f & \text{(false vac.)} \\ \epsilon_t & \text{(true vac.)} \end{cases}$

- How can we implement these in simulations?

  ① Assume relativistic perfect fluid (for simplicity), $T^{\mu\nu} = w u^\mu u^\nu - g^{\mu\nu} p$

  ② Define $K^\mu \equiv T^{\mu 0}$, then $\partial_\mu T^{\mu\nu} = 0$ reduces to $\begin{cases} \partial_0 K^0 + \partial_i K^i = 0 \\ \partial_0 K^i + \partial_j T^{ij}(K^0, K^i) = 0 \end{cases}$

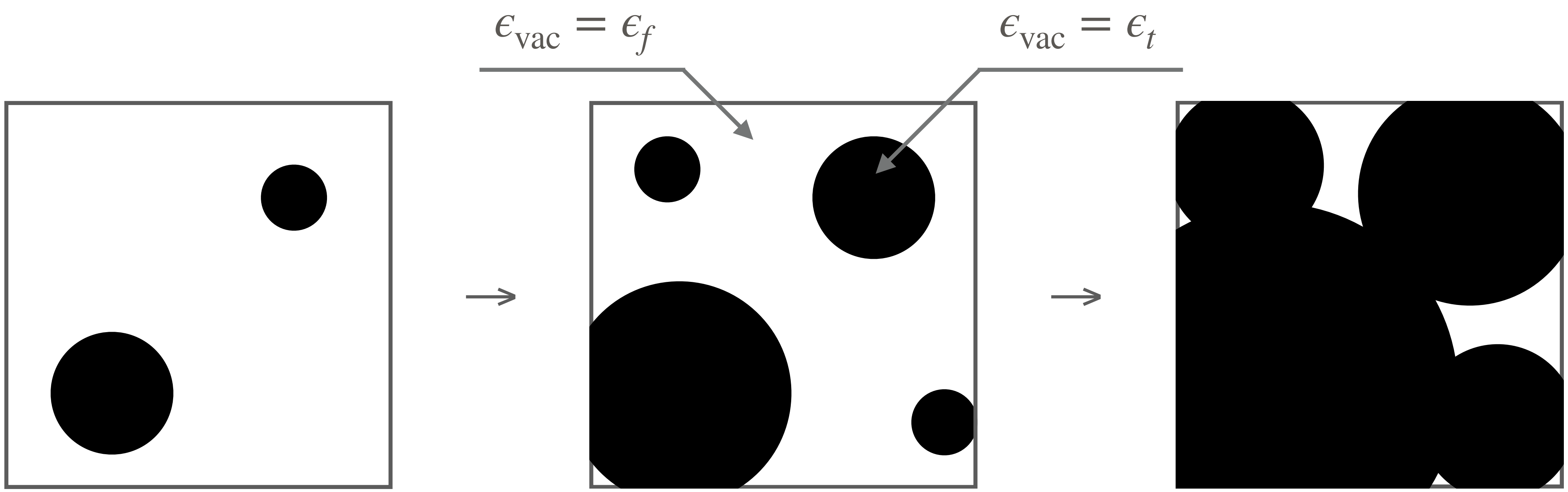
  ③ Where does the energy injection enter? Answer: in $T^{ij}(K^0, K^i)$

$$T^{ij}(K^0, K^i) = \frac{3}{2} \frac{K^i K^j}{(K^0 - \epsilon_{\rm vac}) + \sqrt{(K^0 - \epsilon_{\rm vac})^2 - \frac{3}{4} K^i K^i}}$$

RECIPE FOR THE HIGGSLESS SIMULATION

➤ We first numerically generate nucleation points,

and determine the false-true boundary of the bubbles



➤ We then evolve the fluid in this box according to $\begin{cases} \partial_0 K^0 + \partial_i K^i = 0 \\ \partial_0 K^i + \partial_j T^{ij}(K^0, K^i) = 0 \end{cases}$

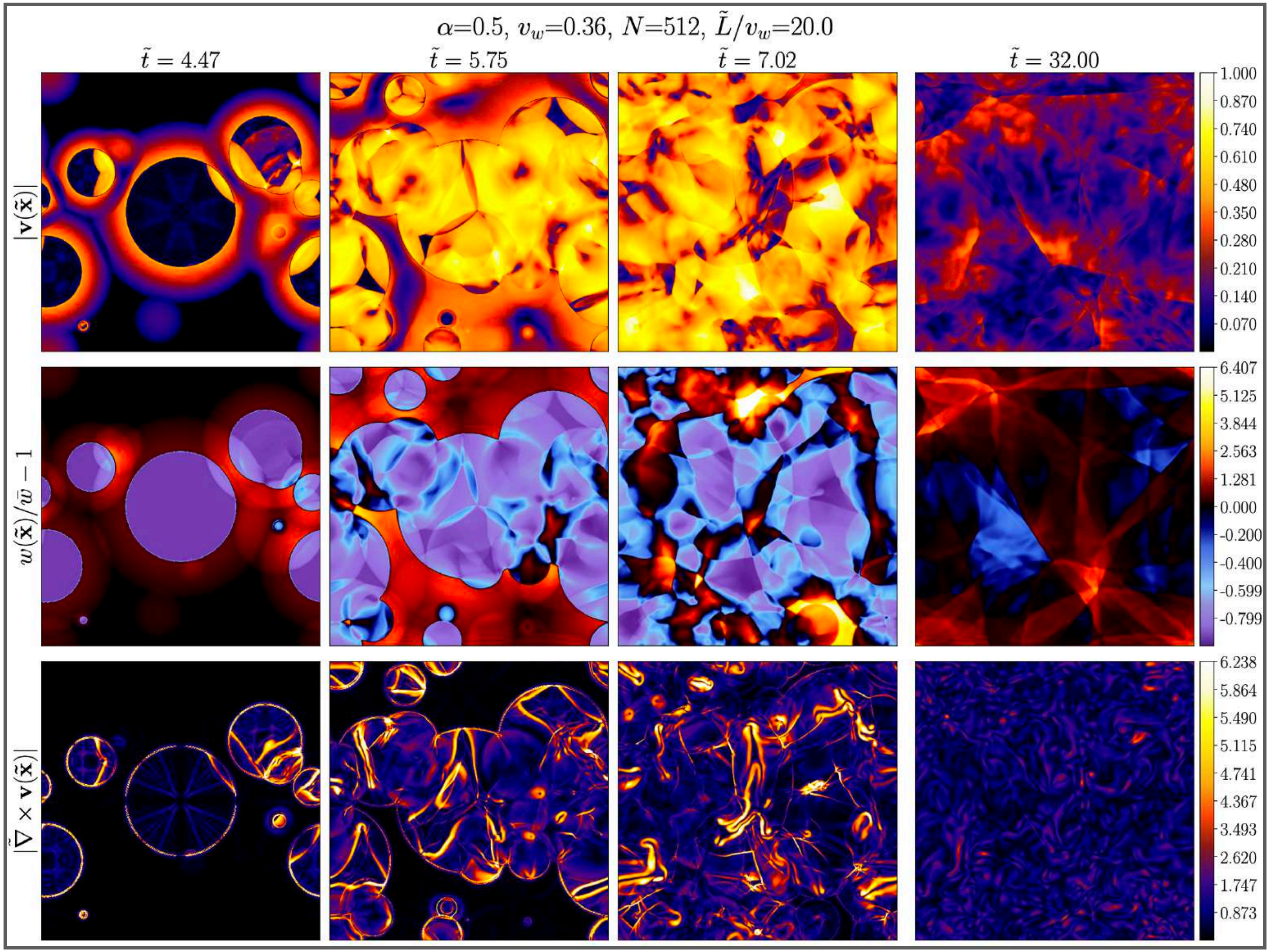
→ Fluid automatically develops profiles

RECIPE FOR THE HIGGSLESS SIMULATION

fluid velocity

enthalpy

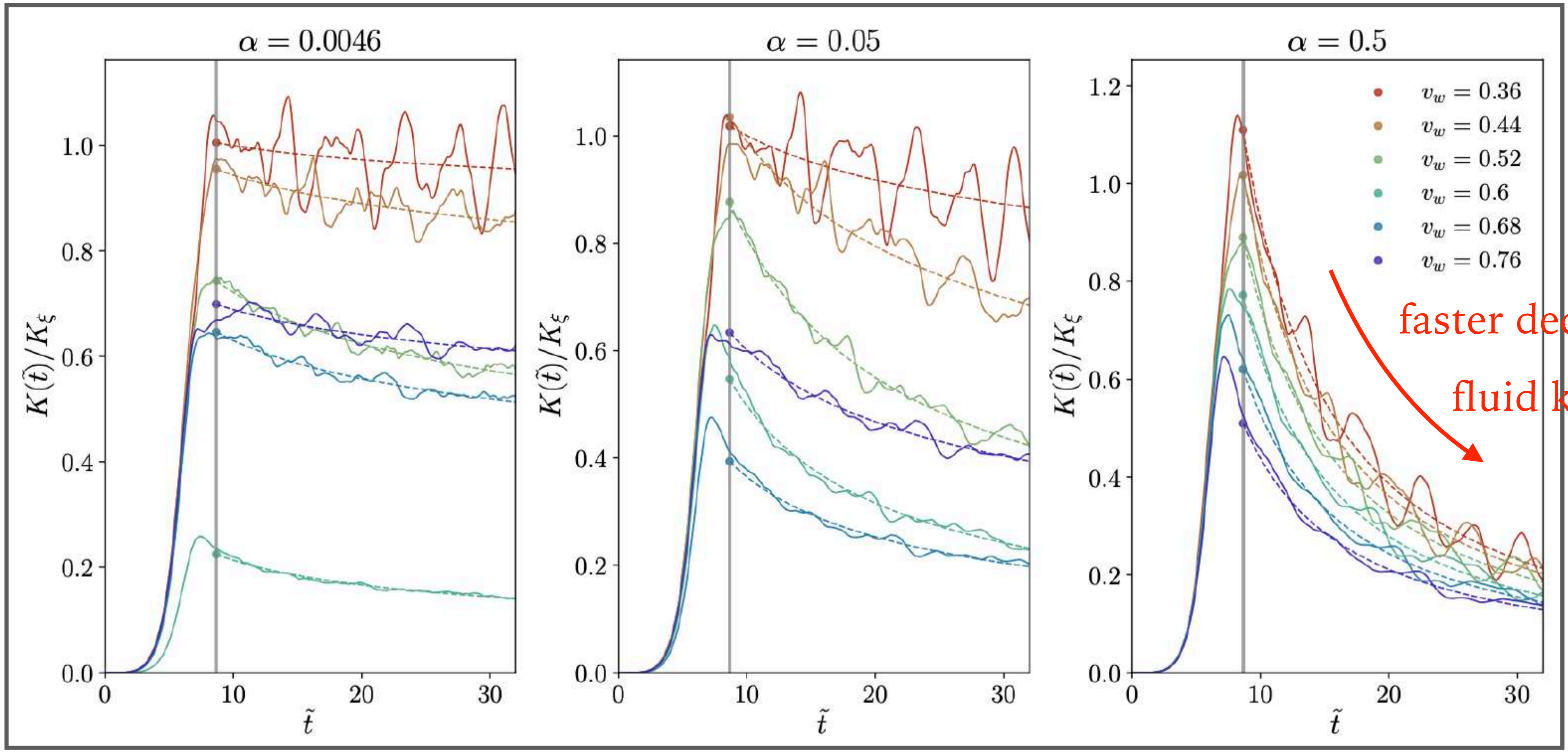
vorticity



RECIPE FOR THE HIGGSLESS SIMULATION

fluid kinetic energy

weak intermediate strong



time

MESSAGES

- The Higgsless simulation is now one of the largest simulations

  (spatial resolution: $N^3 = 256^3$ or 512^3 grids; simulation time: $T = 32/\beta$)

- We are now able to simulate the strong transition regime $\alpha \sim 1$,

  which was previously difficult due to shocks and numerical viscosities

- We are now also able to see sound waves developing into turbulence

SOME RECENT TOPICS

- Large-scale simulations: the Higgsless scheme

  [RJ, Konstandin, Rubira '21] [RJ, Konstandin, Rubira, Stomberg '22] [Caprini, RJ, Konstandin, Roper Pol, Rubira, Stomberg '24]

- GW signal from (almost) scale-invariant models

  [Konstandin, Servant '10] and many others, including [RJ, Takimoto '16]

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- Particle splitting and next-leading-order (NLO) friction

  [Bodeker, Moore '17] [Azatov, Vanvlasselaer '21] [Gouttenoire, RJ, Sala '21] [Azatov, Barni, Petrossian-Byrne, Barni, Vanvlasselaer '24]

- Effect of gravity

  [Giombi, Hindmarsh '24] [RJ, Kume '24]

SCALE-INVARIANT MODELS

[Iso, Okada, Orikasa '09]
[RJ, Takimoto '16]

➤ One example: Classically conformal B-L model

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$
q_L^i	**3**	**2**	+1/6	+1/3
u_R^i	**3**	**1**	+2/3	+1/3
d_R^i	**3**	**1**	-1/3	+1/3
l_L^i	**1**	**2**	+1/6	-1
e_R^i	**1**	**1**	-1	-1
ν_R^i	**1**	**1**	0	-1
H	**1**	**2**	-1/2	0
Φ	**1**	**1**	0	+2

TABLE I: Matter contents of the classically conformal $B-L$ model. In addition to the standard model matters, three generations of right-handed neutrinos ν_R^i and a $B-L$ charged complex scalar field Φ are introduced.

SCALE-INVARIANT MODELS

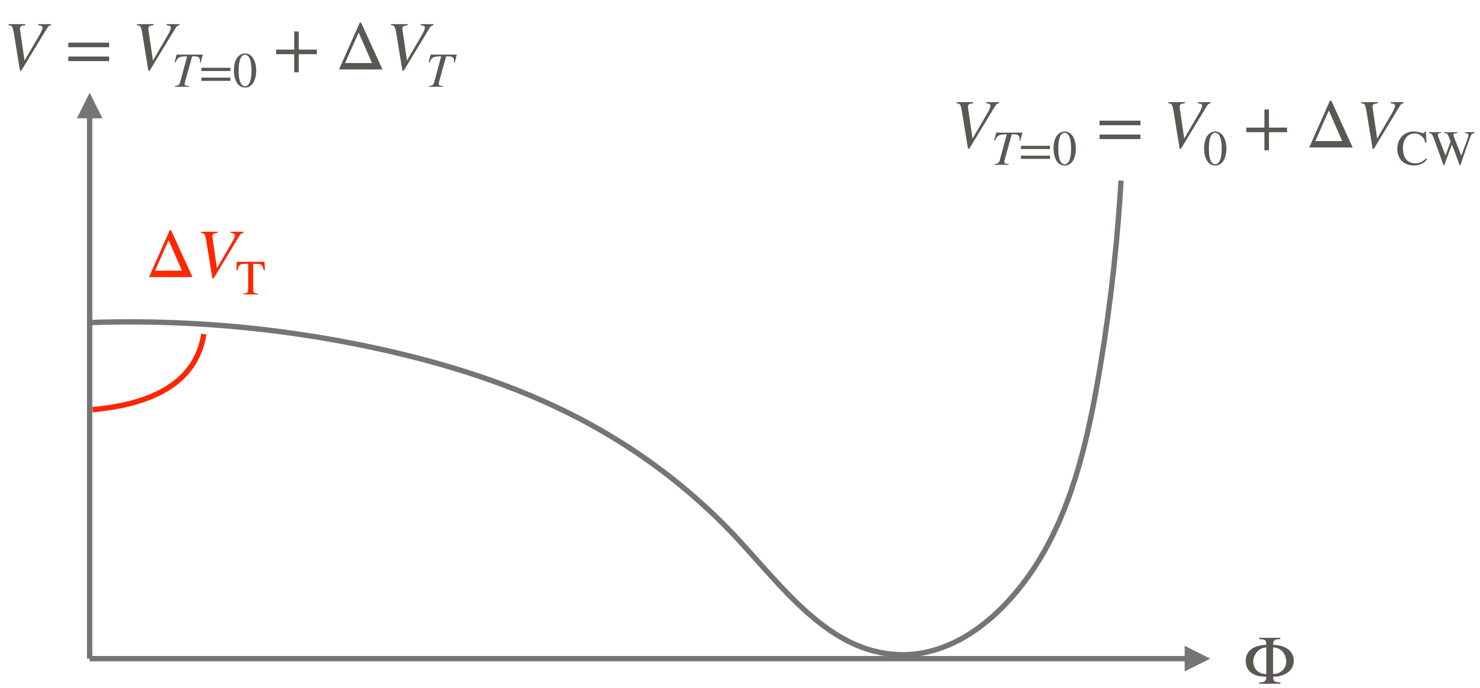
[Iso, Okada, Orikasa '09]
[RJ, Takimoto '16]
[Iso, Serpico, Shimada '17]

➤ Assumption: absence of mass scales

$$V_0 = \lambda_H |H|^4 + \lambda |\Phi|^4 - \lambda' |\Phi|^2 |H|^2$$

- Only quartic terms in the potential: no parameters with a finite mass dimension
- Scale dependence enters only through running of couplings

➤ Phase transition in Φ direction can be extremely strong



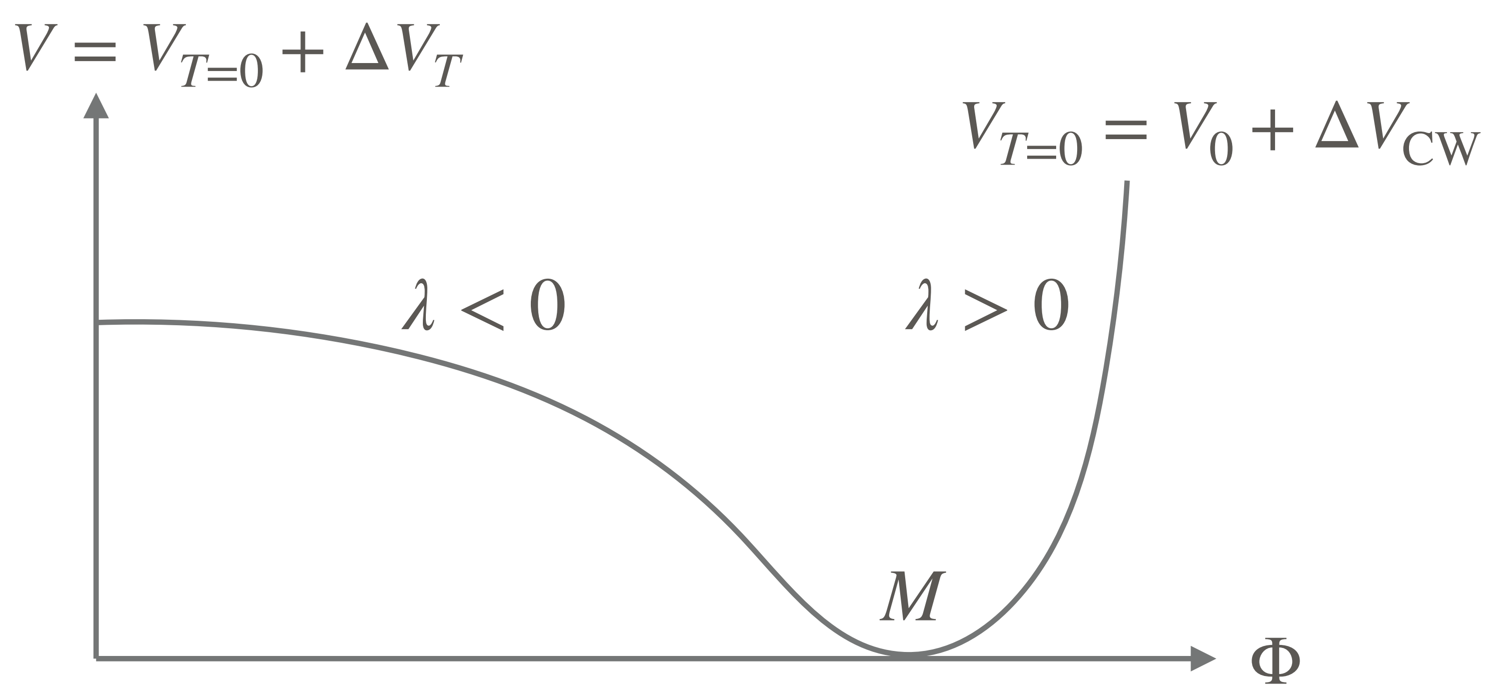
HOW SCALE-INVARIANT MODELS INDUCE STRONG FOPT

➤ Zero-temperature behavior

- The zero-temperature potential including loop corrections behaves as $\phi^4 \times$ (running coupling constant)

$$V(\phi) \sim \lambda(\phi)\phi^4$$

- The B-L scale M is generated a la Coleman-Weinberg

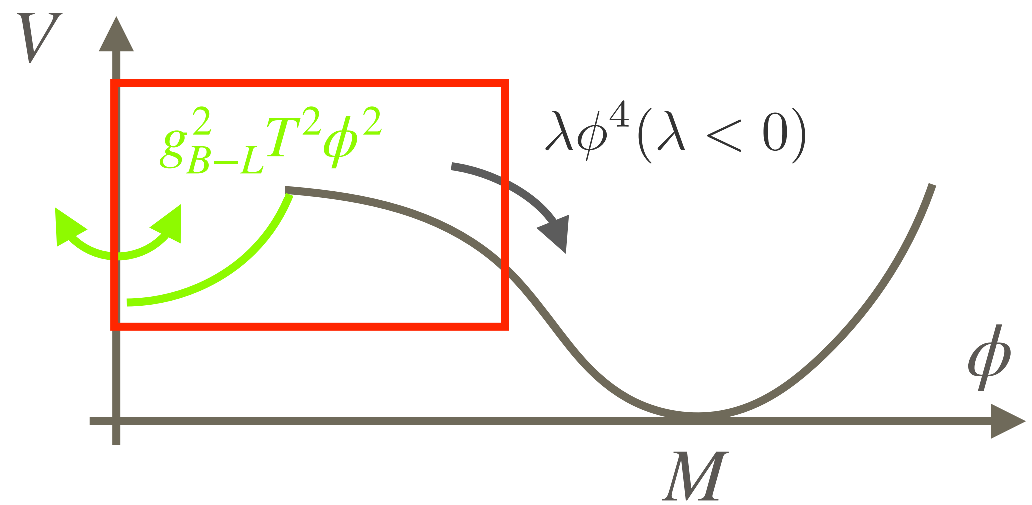


HOW SCALE-INVARIANT MODELS INDUCE STRONG FOPT

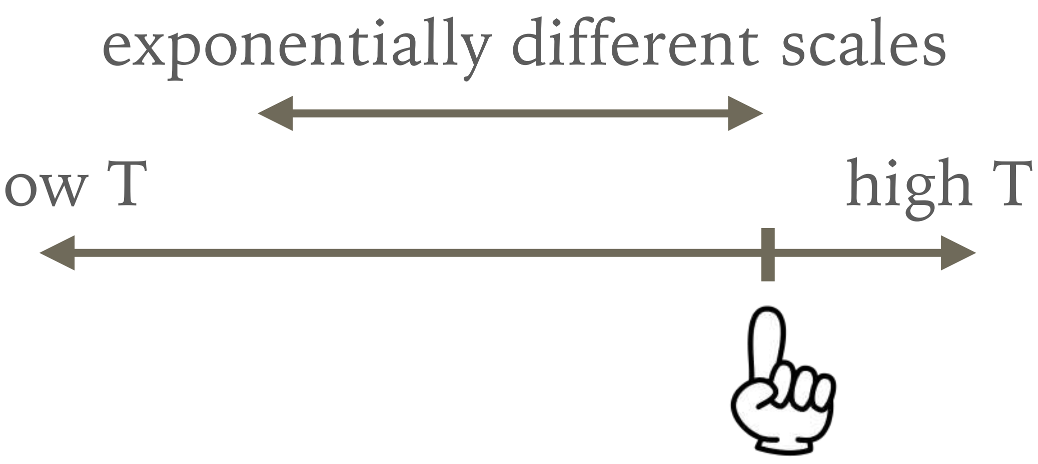
➤ Finite-temperature behavior

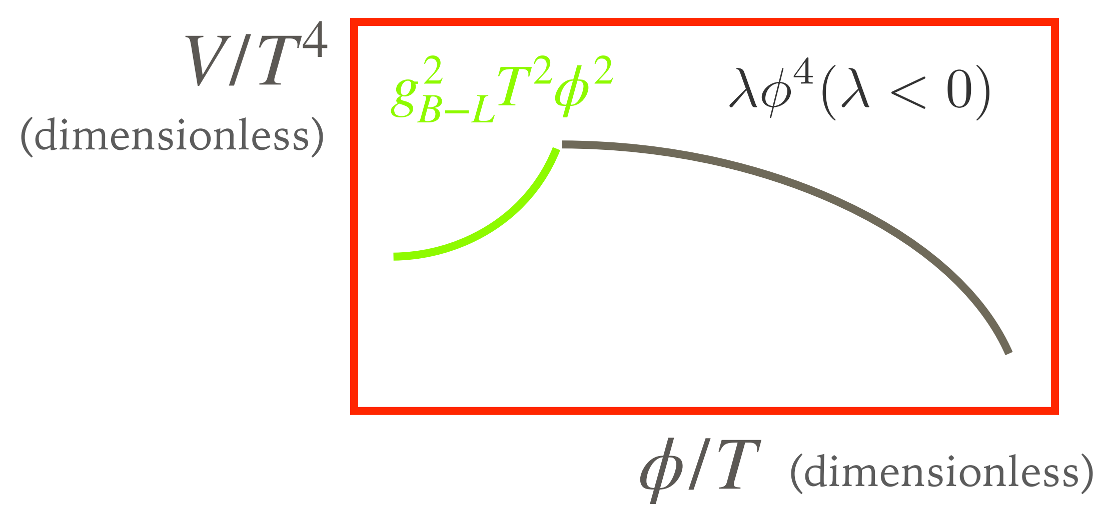
- Thermal corrections create a quadratic trap, which persists down to small T

$$V \sim g_{B-L}^2 T^2 \phi^2 + \lambda(\max(T,\phi))\phi^4$$



- Behavior of the potential as the temperature decreases





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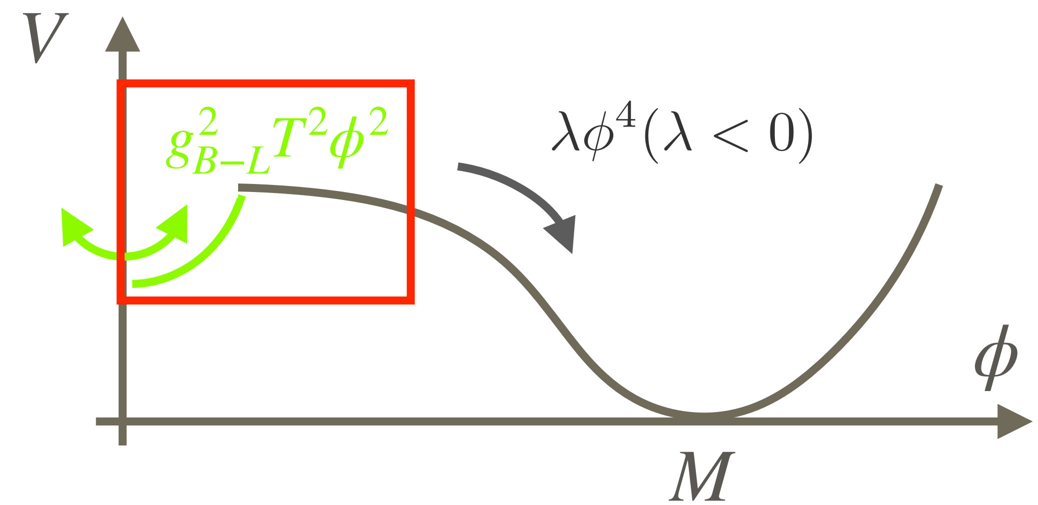

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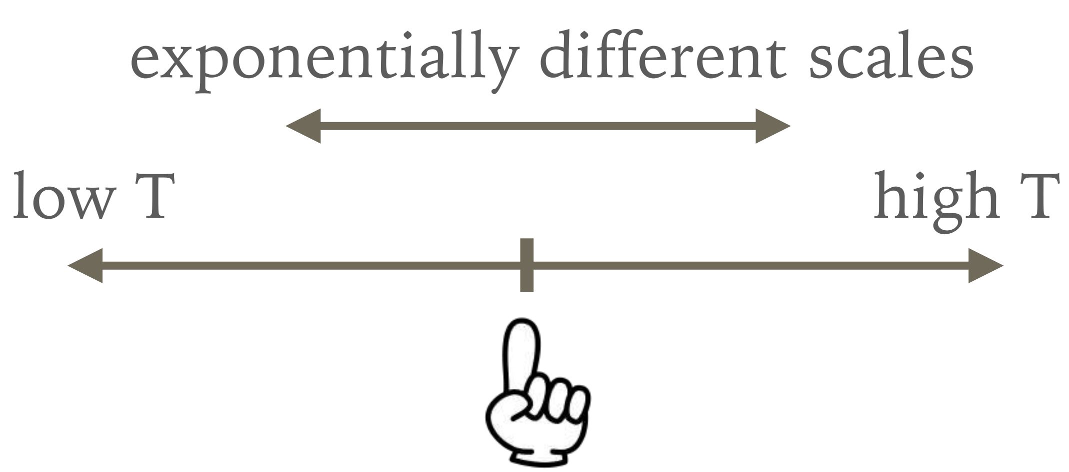

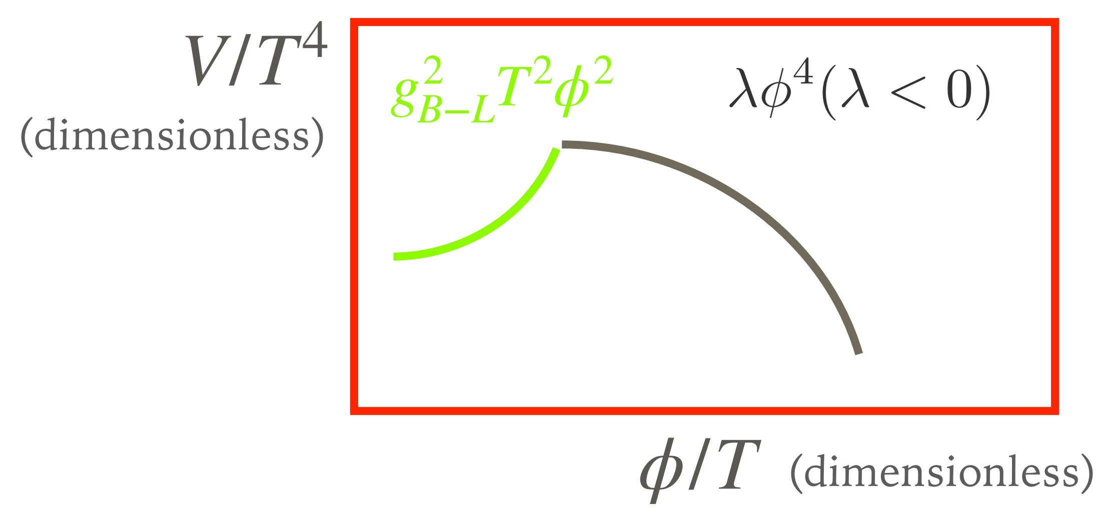

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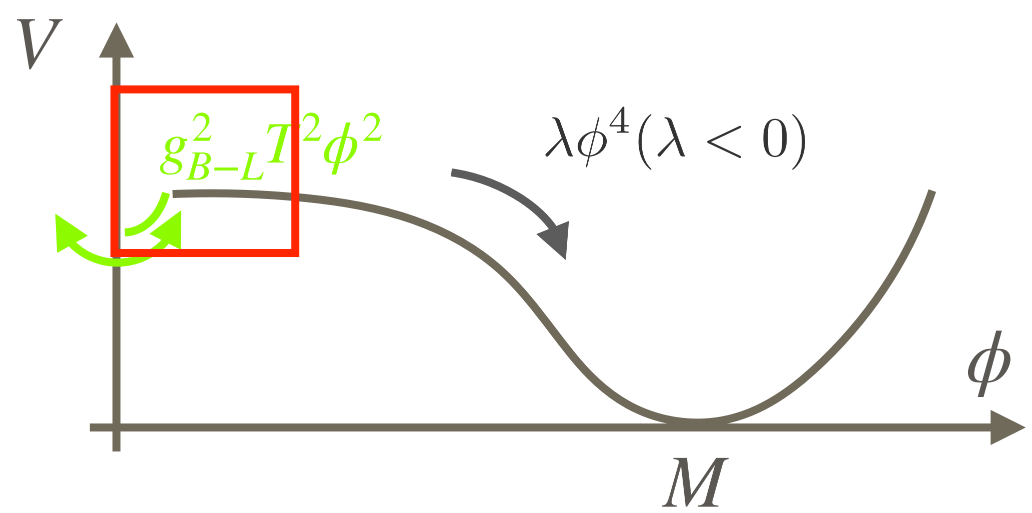

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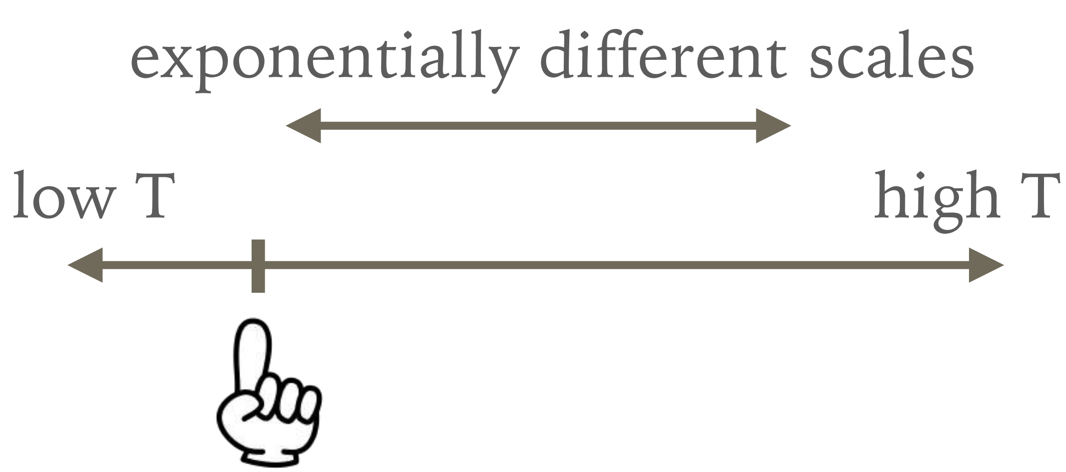
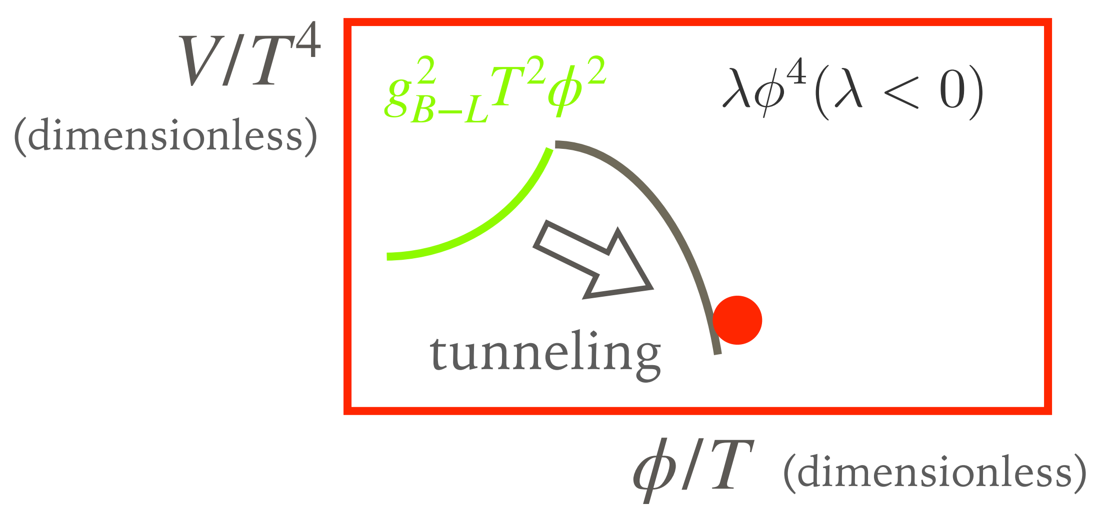
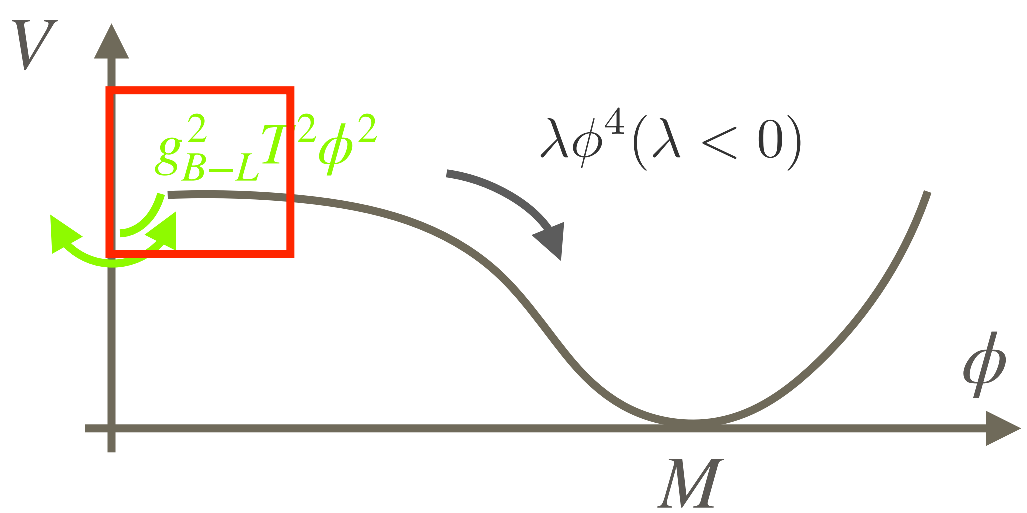




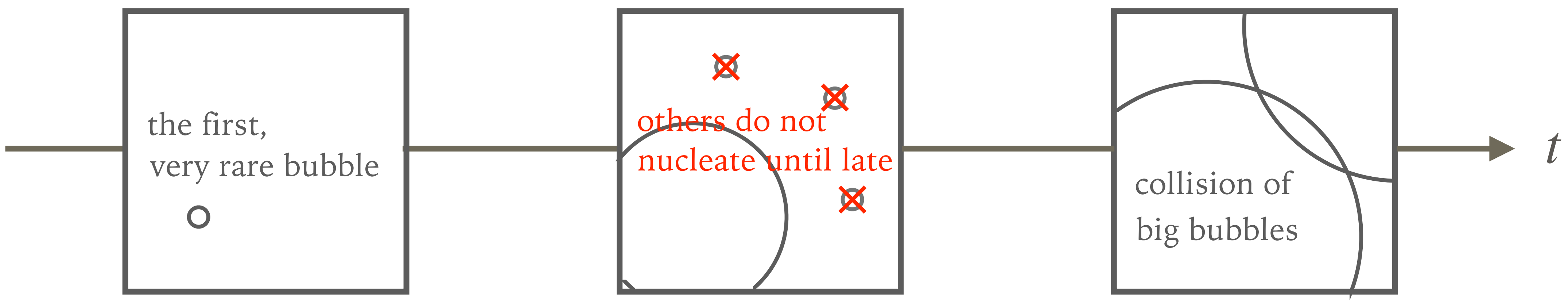
HOW SCALE-INVARIANT MODELS INDUCE STRONG FOPT

- When the B-L field tunnels, the energy release is huge



$\rightarrow$ large $\alpha \equiv \frac{\rho_{\rm vac}}{\rho_{\rm rad}}$

- The system changes only logarithmically, so bubble nucleation is slow

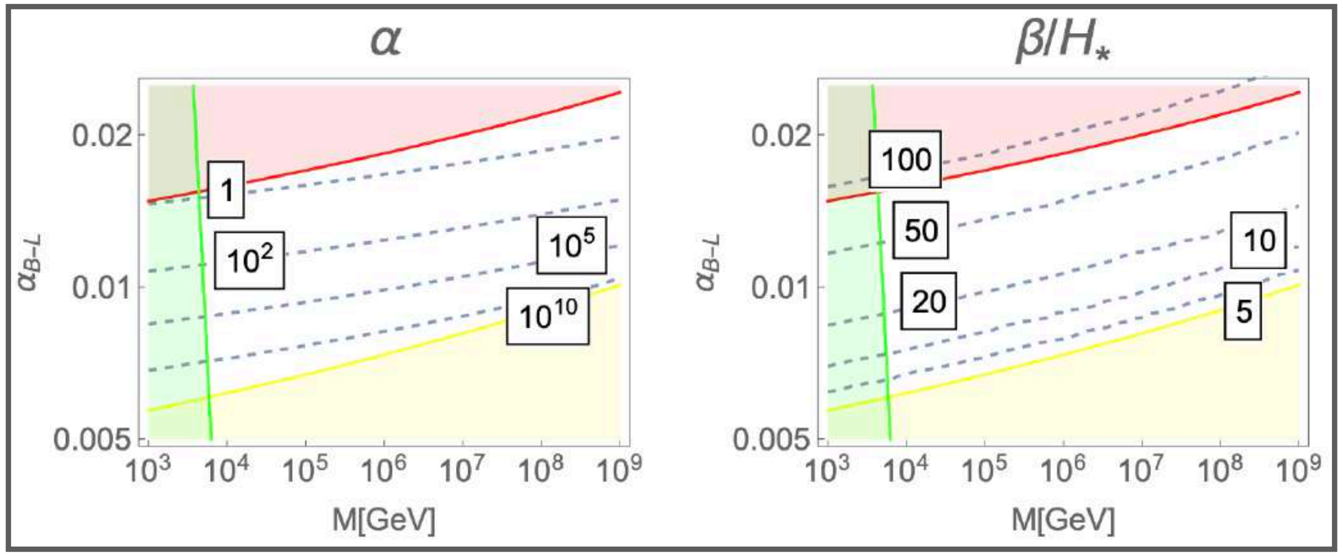


$\rightarrow$ collision of big bubbles (small β/H_*)

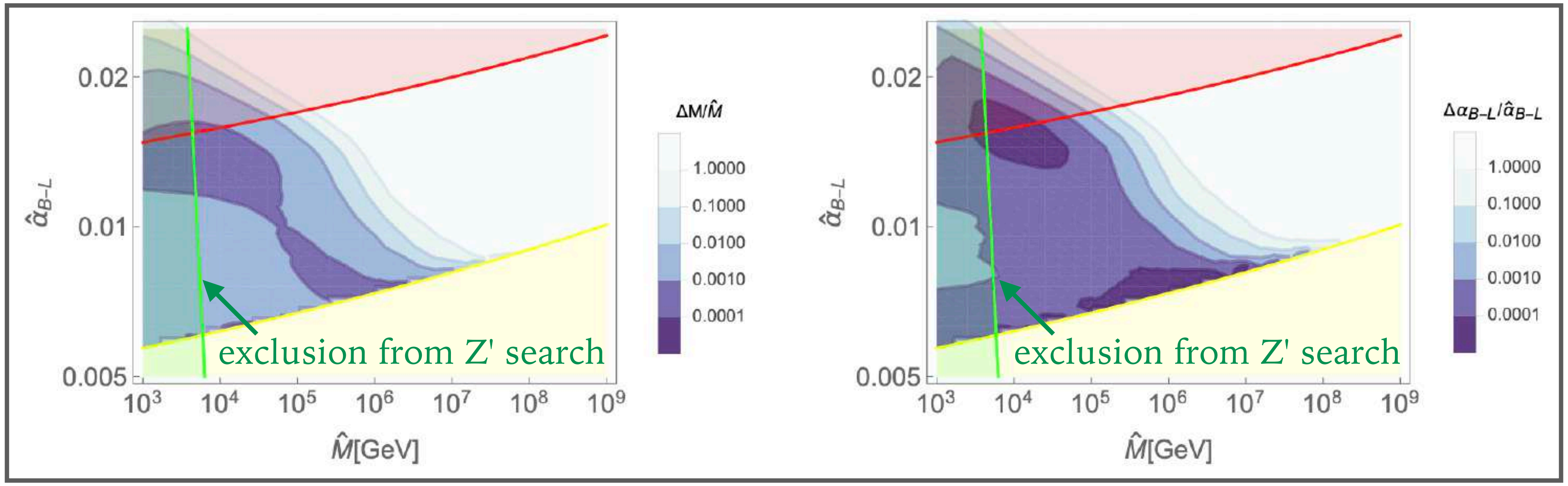
GW PREDICTIONS

[RJ, Takimoto '16]
[Hashino, RJ, Kanemura, Kakizaki, Takahashi, Takimoto '16]

- Transition parameters



- Synergy between collider and GW experiments



MESSAGES

- (Almost) scale-invariant models induce very strong FOPTs

SOME RECENT TOPICS

- Large-scale simulations: the Higgsless scheme

  [RJ, Konstandin, Rubira '21] [RJ, Konstandin, Rubira, Stomberg '22] [Caprini, RJ, Konstandin, Roper Pol, Rubira, Stomberg '24]

- GW signal from (almost) scale-invariant models

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- Seeded transitions from density perturbations/topological defects

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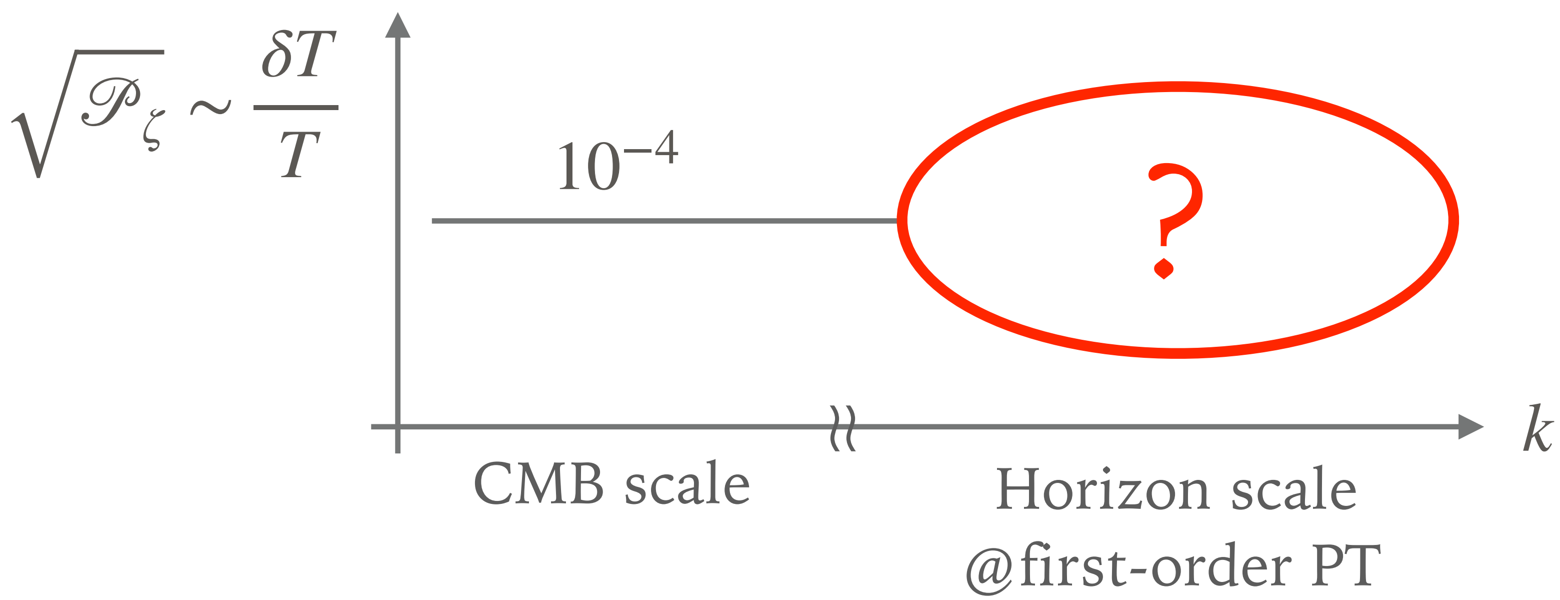
- Effect of gravity

  [Giombi, Hindmarsh '24] [RJ, Kume '24]

EFFECT OF DENSITY PERTURBATIONS

- Density (i.e. curvature) perturbations
  - Exist for sure (as long as we assume inflation) $\rightarrow$ Effects need to be studied
  - Constrained to $\zeta \sim \dfrac{\delta T}{T} \sim 10^{-4}$ at CMB scales, but unconstrained at larger k

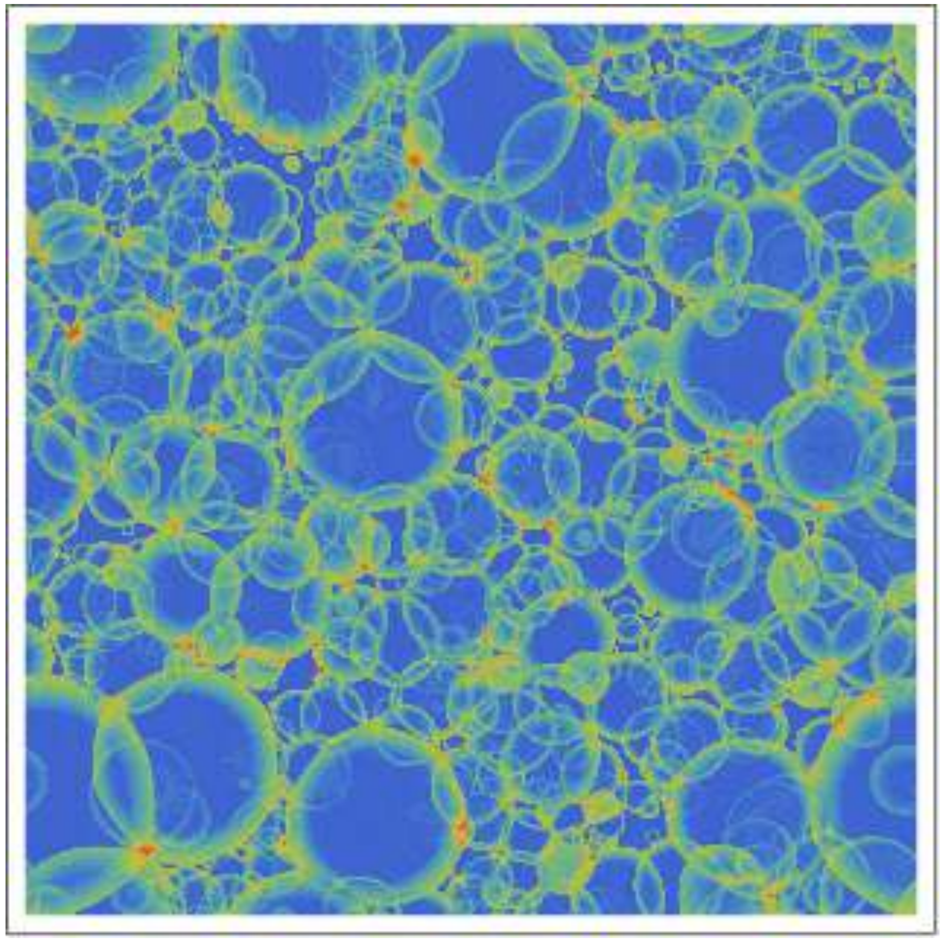


- The idea: biased nucleation time/position from density perturbations

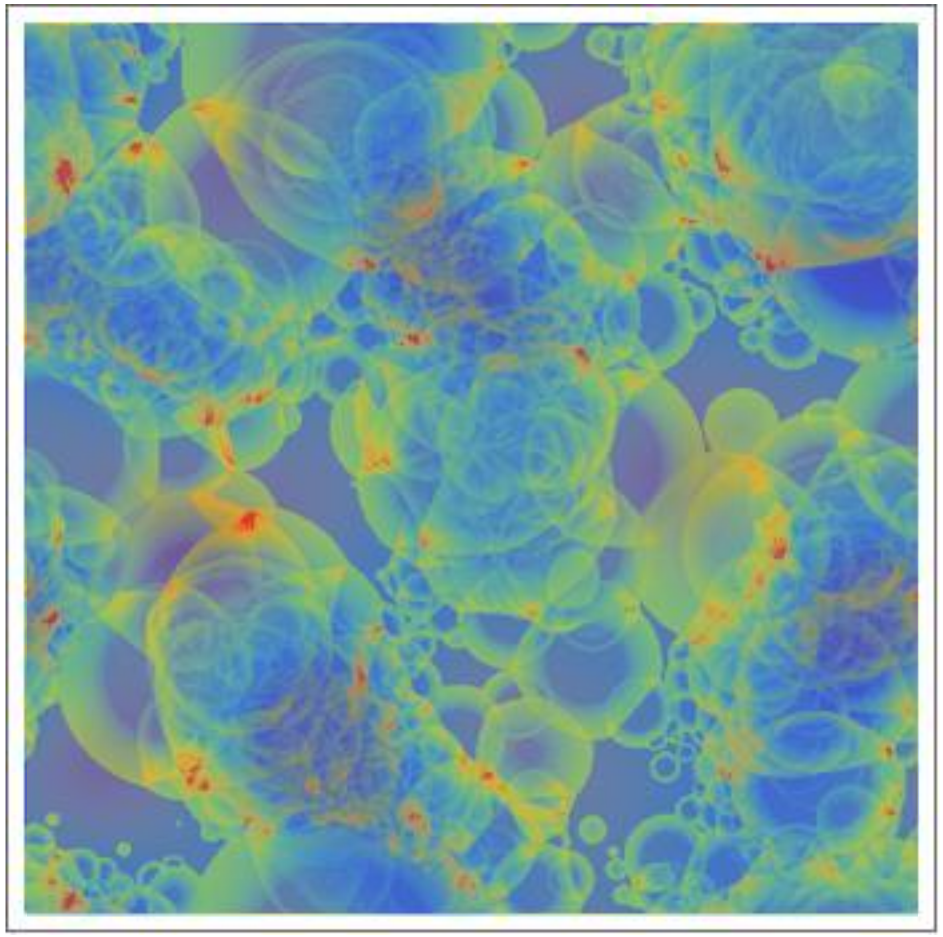
Summary:
- Density perturbations work as "effective big bubbles"
- To have interesting effects, the amplitude only needs to be $\dfrac{\delta T}{T} \sim \dfrac{H_*}{\beta} \ll 1$

CENTRAL IDEA

Without density perturbations



With density perturbations



CENTRAL IDEA

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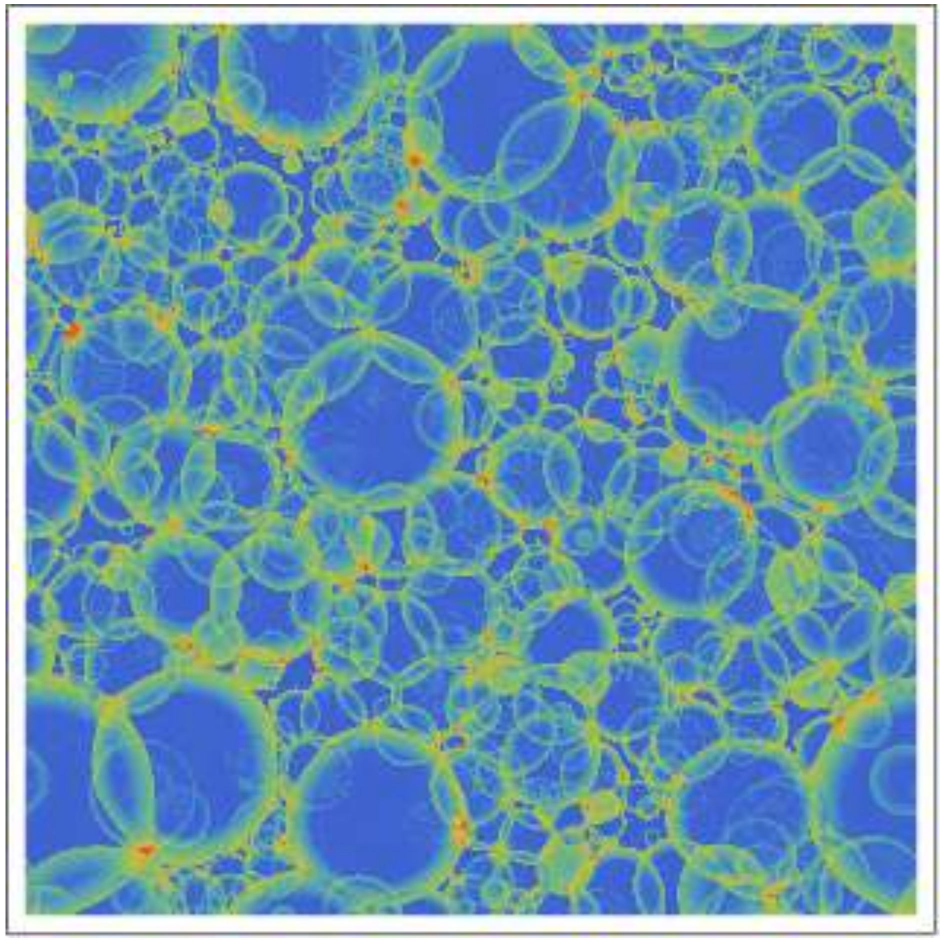
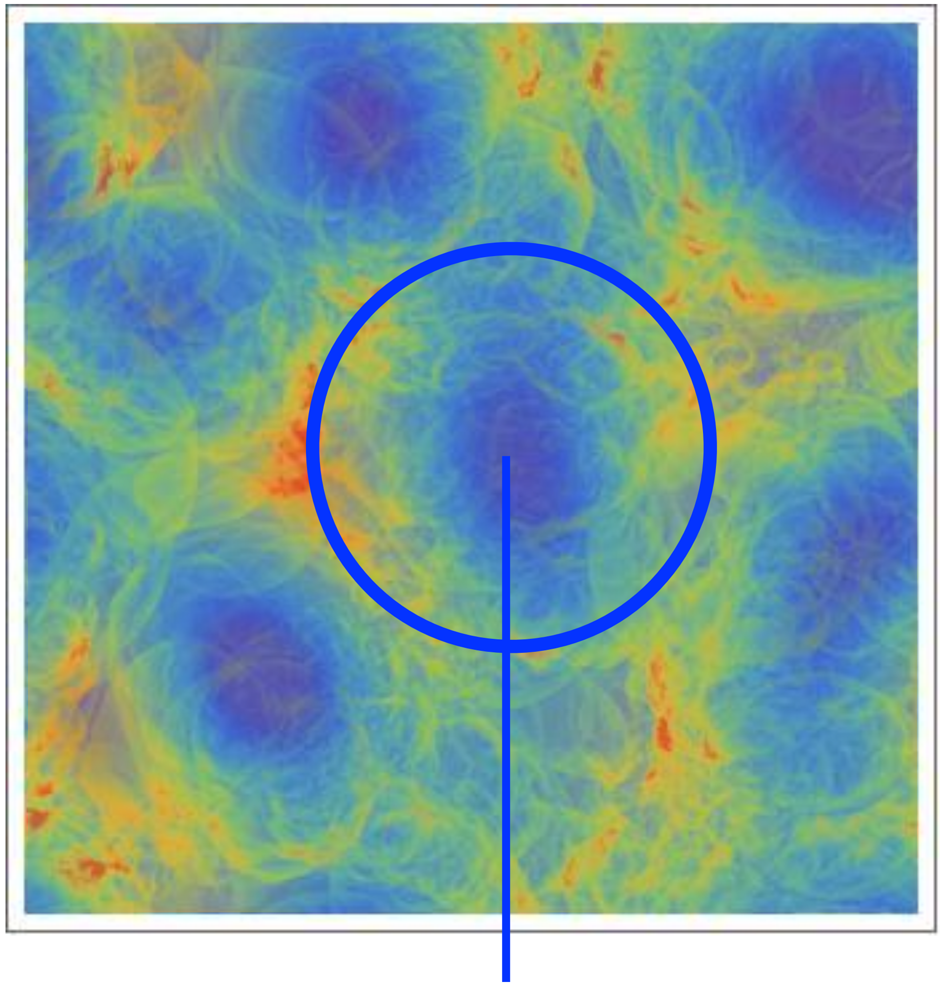


With density perturbations



formation of "effective big bubbles"
around the cold spots

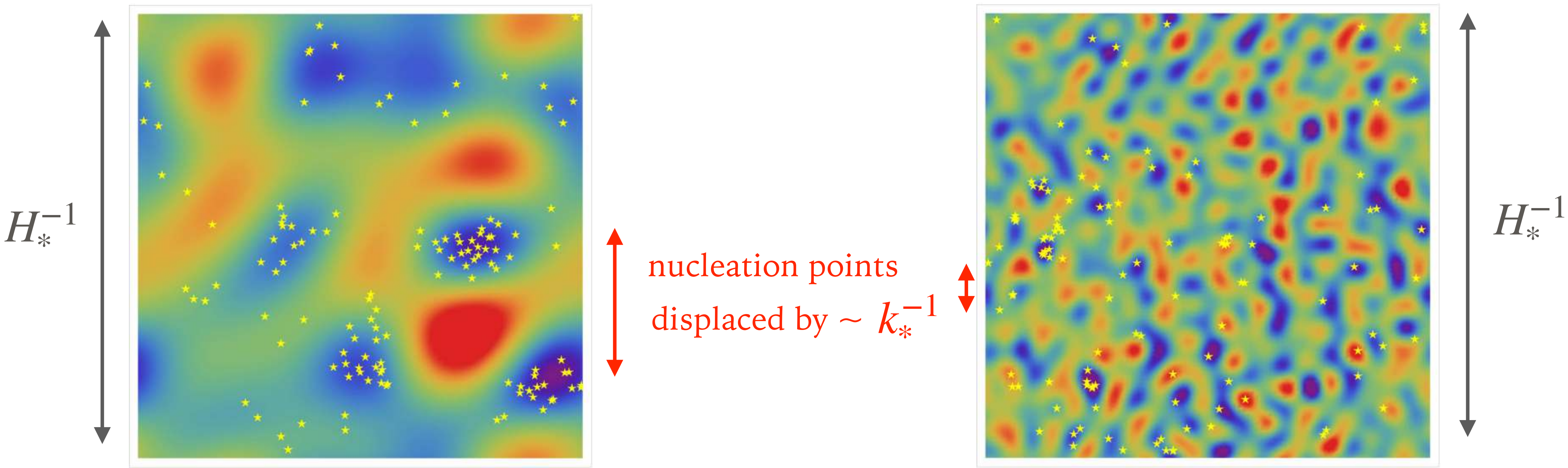
EFFECT OF DENSITY PERTURBATIONS

- Density perturbations are parameterized by two quantities

$$\begin{cases} \text{typical wavenumber } k_* & \rightarrow \text{ see below} \\ \text{typical normalized amplitude } \sigma \sim \dfrac{\delta T}{T} \Big/ \dfrac{H_*}{\beta} & \rightarrow \text{ effects set in once } > 1 \end{cases}$$

- Dependence of the nucleation points (★) on k_*

small k_* (= IR) large k_* (= UV)



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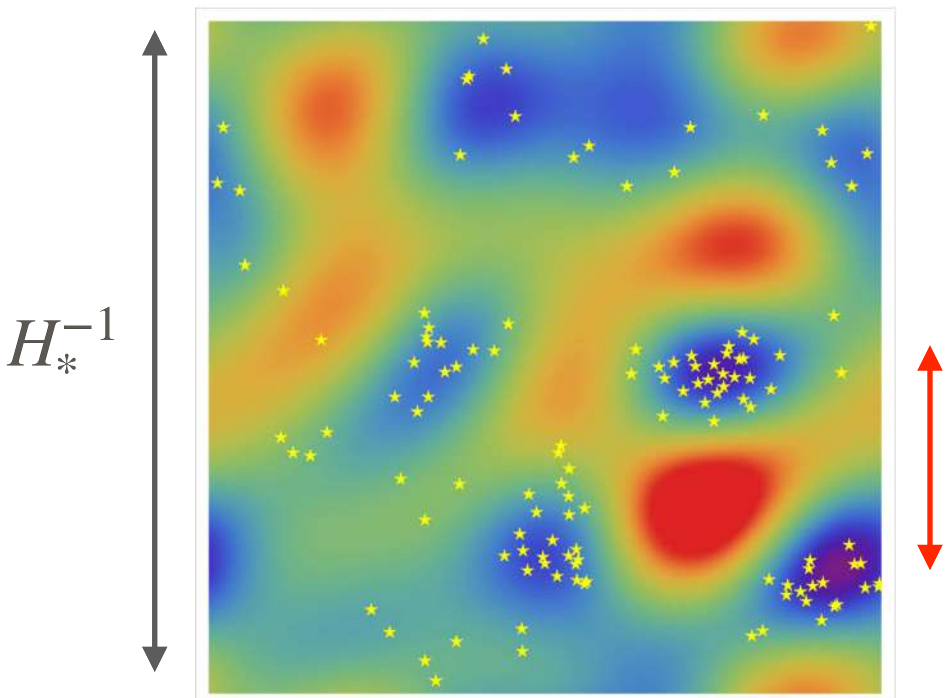
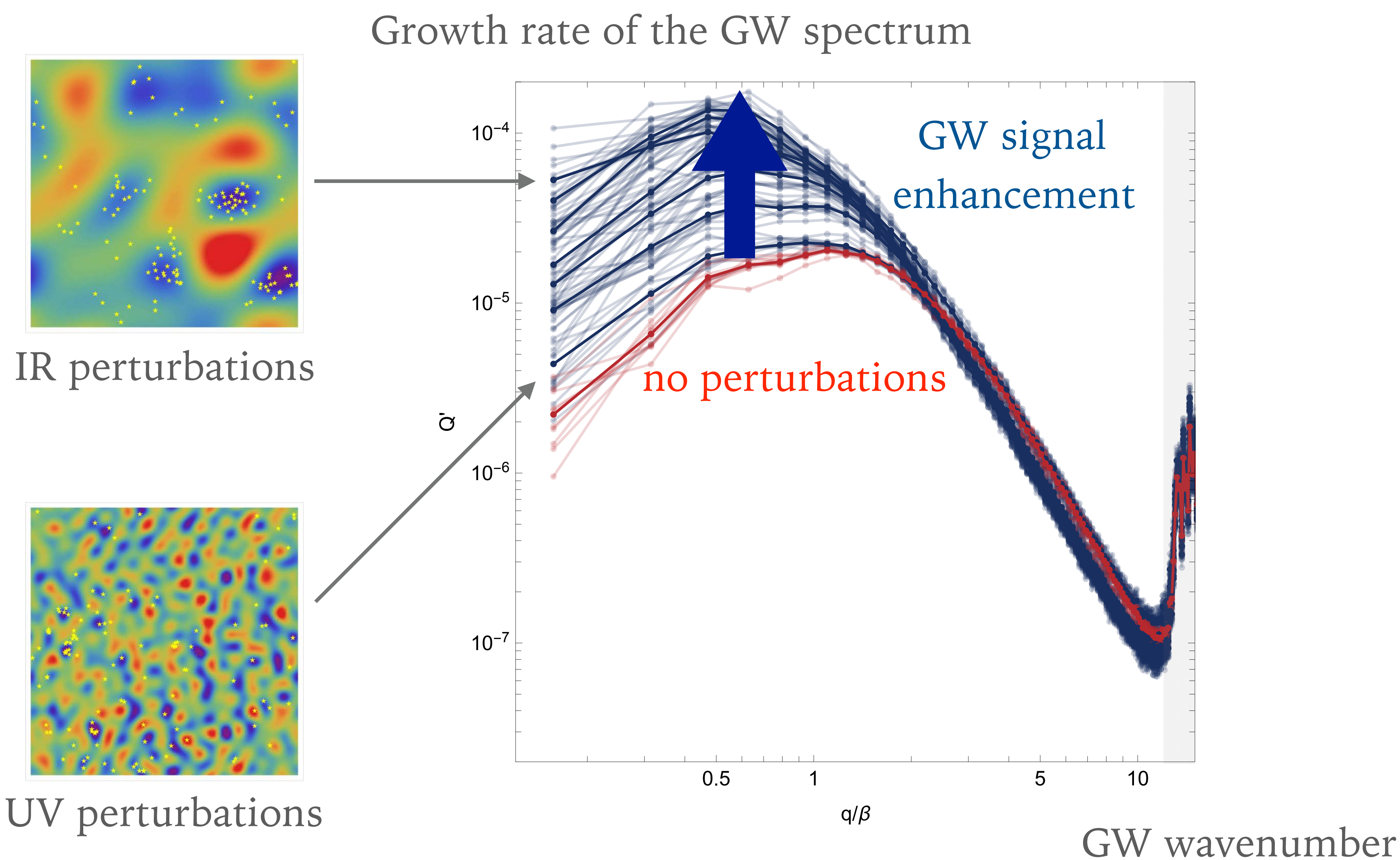


nucleation points displaced by $\sim k_*^{-1}$

Density perturbations work as "effective big bubbles"

GW ENHANCEMENT FROM DENSITY PERTURBATIONS

- Density perturbations with $H_* < k_* < \beta$ enhance the GW signal



MESSAGES

- Density perturbations can work as the source of "effective big bubbles"

- Similar ideas also apply to induced FOPTs from topological defects

SOME RECENT TOPICS

- Large-scale simulations: the Higgsless scheme

  [RJ, Konstandin, Rubira '21] [RJ, Konstandin, Rubira, Stomberg '22] [Caprini, RJ, Konstandin, Roper Pol, Rubira, Stomberg '24]

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- Effect of gravity

  [Giombi, Hindmarsh '24] [RJ, Kume '24]

CENTRAL QUESTION

- What happens in the extremely limit $\alpha \equiv \rho_{\rm vac}/\rho_{\rm rad} \ggg 1$?

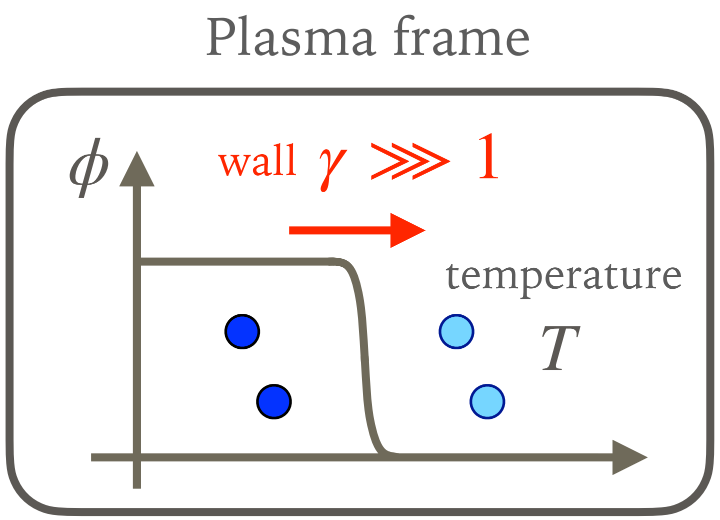
  - Bubble walls will move very fast

  - GW signals will be larger, at least naively

  - Particle splittings are be important

LEADING ORDER FRICTION

- Leading order friction (= particles getting mass m across the wall)

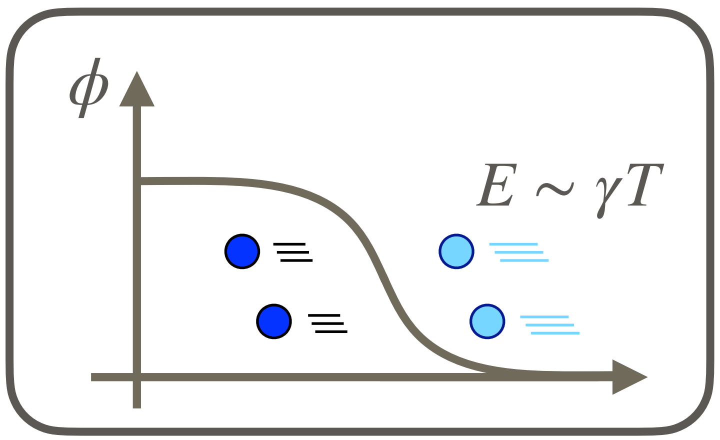


- Consider a wall moving with Lorentz factor $\gamma \gg 1$ in the plasma frame

LEADING ORDER FRICTION

- Leading order friction (= particles getting mass m across the wall)

Rest frame of the wall



- Consider a wall moving with Lorentz factor $\gamma \gg 1$ in the plasma frame

- In the wall frame, each particle has huge energy ($\gg T, \langle \phi \rangle$)

- Upon impinging, each particle gives momentum $\Delta p_z = E - \sqrt{E - m^2} \simeq \dfrac{m^2}{2E}$ to the wall

- After integrating over the phase space, the friction (= force/area = dim.-4 quantity) is

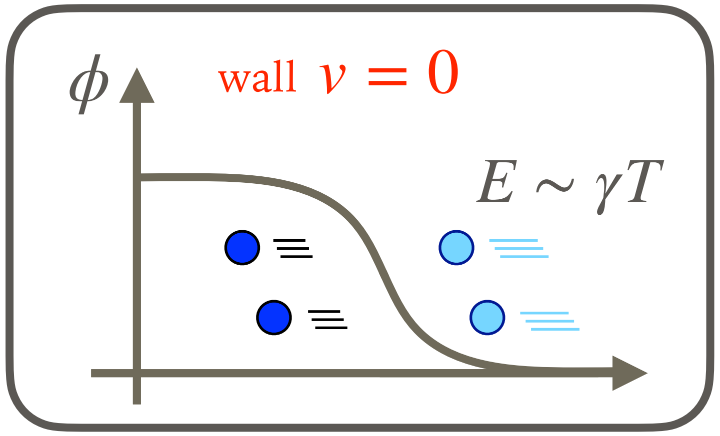
$$\mathscr{P} = \int \frac{d^3p}{(2\pi)^3} f_{\rm wall}(p) \frac{m^2}{2E} \ \ \text{(wall frame)} \ = \int \frac{d^3p}{(2\pi)^3} f_{\rm thermal}(p) \frac{m^2}{2E} \ \ \text{(plasma frame)} \ \sim m^2 T^2$$

NEXT-LEADING ORDER FRICTION

- Next-leading order friction (= particle splitting, transition splitting)

[Bodeker & Moore '17]

Rest frame of the wall



- Occurs when the impinging particles are gauge charged. The process is

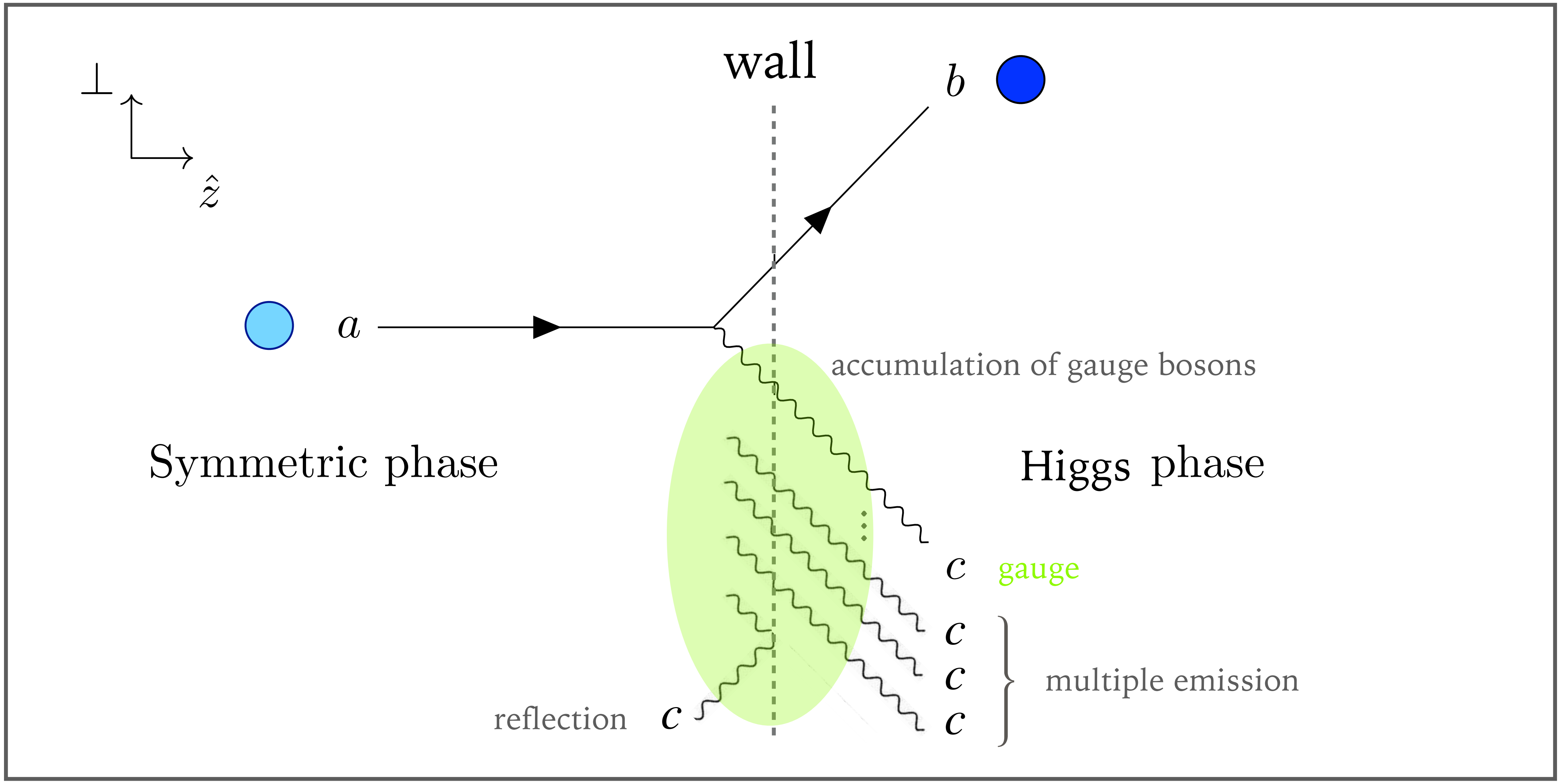
$$a \to bc \qquad\qquad c \text{ : gauge boson}$$

- The splitting probability involves the gauge coupling g , thus the name "next-leading"

- Following the original article, we consider c particle getting a mass

- [B&M '17] showed that NLO friction dominates LO when the wall is moving fast

ULTIMATE GOAL

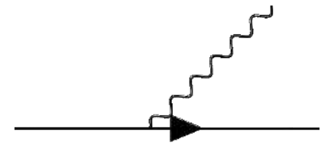


1) Momentum transfer from a particle to the wall, as a result of c emission: Δp

2) Averaged momentum transfer per each a particle impinging: $\langle \Delta p \rangle$

3) Friction pressure to the wall from a particles: $\mathscr{P} = \int \frac{d^3 p_a}{(2\pi)^3} f(p_a) \langle \Delta p \rangle \sim \gamma T_{\rm nuc}^3 \langle \Delta p \rangle$

SINGLE SPLITTING

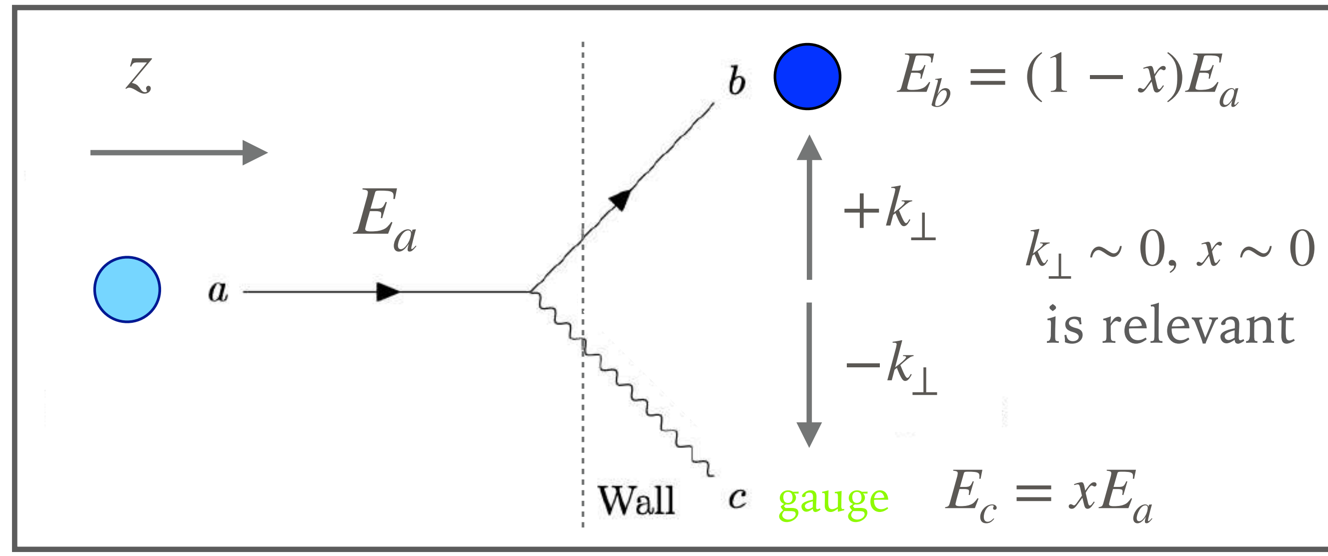


- First consider single splitting $a \to bc$, not multiple splitting $a \to bccc\cdots$

- Parametrization of kinematics

$$\begin{cases} x \;\; : \text{fraction of } a \text{ energy taken by } c \\ k_\perp : \text{perpendicular momenta of } b \;\&\; c \end{cases}$$

We are interested in $k_\perp \sim 0,\ x \sim 0$

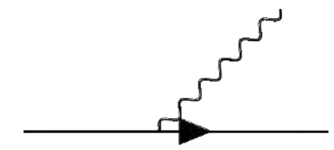


- Infinitesimal splitting probability $dP_{a\to bc}$ in terms of the matrix element $\mathscr{M}$

$$dP_{a\to bc} = \frac{d^3p_b}{(2\pi)^3 2E_b}\frac{d^3p_c}{(2\pi)^3 2E_c}\left\langle \phi_a \middle| \mathscr{T} \middle| p_b, p_c \right\rangle \left\langle p_b, p_c \middle| \mathscr{T} \middle| \phi_a \right\rangle = \cdots = \frac{d^2k_\perp}{(2\pi)^2}\frac{dE_c}{2\pi}\frac{1}{2p_{a,z}}\frac{1}{2p_{b,z}}\frac{1}{2p_{c,z}}|\mathscr{M}|^2$$

with
$$\begin{cases} \left|\phi_a\right\rangle = \displaystyle\int \frac{d^3p_a}{(2\pi)^3 2E_a}\phi_a(\vec{p}_a)\left|p_a\right\rangle & : \text{state localized around } \vec{p}_a \quad \text{with} \quad \left\langle \phi_a | \phi_a \right\rangle = 1 \\ \left\langle p_b, p_c \middle| \mathscr{T} \middle| p_a \right\rangle = \delta(\Sigma E)\,\delta^{(2)}(\Sigma \vec{p}_\perp)\mathscr{M} & : \text{conservation of energy and xy-momenta} \end{cases}$$

MATRIX ELEMENT FOR SINGLE SPLITTING



- Matrix element $\mathscr{M} \sim$ (vertex) $\times$ (mode functions)

  - Consider scalar QED for example

Lagrangian: $\mathscr{L} \supset |D_\mu \phi|^2 \sim ig(A^\mu \phi \partial_\mu \phi^* - A^\mu \phi^* \partial_\mu \phi) \sim g(p_{a\mu}(z) + p_{b\mu}(z))\epsilon^\mu(z) \times \chi_a(z)\chi_b^*(z)\chi_c^*(z)$

vertex function V

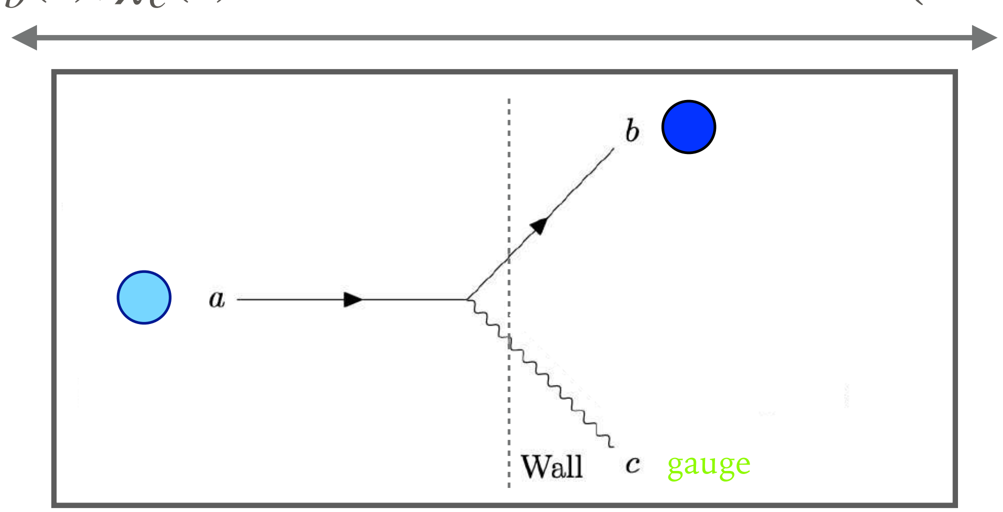
mode functions of a,b,c particles

Matrix element: $\mathscr{M} = \int dz \, V(z) \times \chi_a(z)\, \chi_b^*(z)\, \chi_c^*(z)$

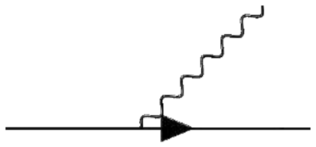
We consider sudden z-dependence $\begin{cases} V(z) = V_s \quad (z<0), & V(z) = V_h \quad (z>0) \\ \chi_i(z) = \chi_{i,s}(z) \quad (z<0), & \chi_i(z) = \chi_{i,h}(z) \quad (z>0) \end{cases}$

  - Note that the mode function is defined over all z :

$\chi_a(z), \chi_b(z), \chi_c(z)$ are all defined over $z \in (-\infty, \infty)$



MATRIX ELEMENT FOR SINGLE SPLITTING



- Comparison with the "usual" calculation of the matrix element

  - In "usual" QFT calculations (i.e. Lorentz-conserving bkg), z-integration returns δ :

$$\int dz\ \chi_a(z)\ \chi_b^*(z)\ \chi_c^*(z) = \int dz\ e^{ip_{az}z} e^{-ip_{bz}z} e^{-ip_{cz}z} \sim \delta(\Sigma p_z)$$

  - In the present case, breaking of z-translation by the wall gives rise to splitting.

  Defining the phase factor A as $\chi_a(z)\chi_b^*(z)\chi_c^*(z) \equiv e^{\frac{i}{2E_a}\int_0^z dz'\ A(z')}$, the matrix element becomes

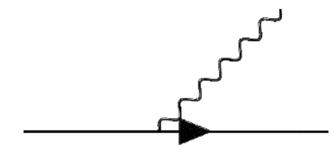
$$\mathscr{M} = V_s \int_{-\infty}^{0} dz\ \exp\left(i\left(\frac{A_s}{2E_a} - i0\right)z\right) + V_h \int_0^{\infty} dz\ \exp\left(i\left(\frac{A_h}{2E_a} + i0\right)z\right) = 2iE_a\left(\frac{V_h}{A_h} - \frac{V_s}{A_s}\right)$$

physical interpretation: "mismatch" across the two phases gives rise to splitting

  - We assume that the breaking of z-translation enters through the mass of c particle:

$$m_c = \begin{cases} m_{c,s} & (\text{symmetric phase}\ \ z < 0\,) \\ m_{c,h} & (\text{broken phase}\ \ z > 0\,) \end{cases}$$

CALCULATION OF VERTEX V AND PHASE A



- Calculation of vertex$V$

  ※ B&M '17 suggests that longitudinal is suppressed. But this still needs to be confirmed.

  - We consider only transverse modes ± for c particle

$$V \sim g(p_a^\mu + p_b^\mu)\,\epsilon_\mu(p_c) \quad \text{with} \quad \epsilon_\mu^\pm \simeq \frac{1}{\sqrt{2}}\left(0, 1, \pm i, -\frac{k_\perp}{xE_a}\right)$$

  Replacement $\Sigma_{\lambda=\pm}\epsilon_\mu^\lambda \epsilon_\nu^{\lambda *} = -g_{\mu\nu} + \cdots$ cannot be used when calculating $|V|^2$ [Altarelli, Parisi]

- Calculation of phase A in $\chi_a(z)\chi_b^*(z)\chi_c^*(z) \sim e^{\frac{i}{2E_a}\int^z dz'\, A(z')}$

  - Each mode function behaves as

$$\begin{cases} \chi_a \sim e^{iE_a z} \\ \chi_b \sim e^{i\sqrt{(1-x)^2E_a^2 - k_\perp^2}\,z} \sim e^{i(1-x)E_a\left(1 - \frac{k_\perp^2}{2(1-x)E_a}\right)z} \\ \chi_c \sim e^{i\int_0^z dz'\,\sqrt{(xE_a)^2 - m_c(z')^2 - k_\perp^2}} \sim e^{i\left(xE_a - \frac{m_c^2 + k_\perp^2}{2xE_a}\right)z} \end{cases}$$

  - For small x, the phase from χ_c dominates:

$$A \simeq \frac{m_c^2(z) + k_\perp^2}{x} \quad \to \quad A_s \simeq \frac{m_{c,s}^2 + k_\perp^2}{x}\ (z < 0) \qquad A_h \simeq \frac{m_{c,h}^2 + k_\perp^2}{x}\ (z > 0)$$

SOFT-COLLINEAR DIVERGENCE

- Finally, we get the matrix element

$$|\mathcal{M}|^2 = 16 g^2 C_{abc} E_a^2 \frac{k_\perp^2 (m_{c,h}^2 - m_{c,s}^2)^2}{(k_\perp^2 + m_{c,s}^2)^2 (k_\perp^2 + m_{c,h}^2)^2}$$

- Quick check: does $\mathcal{M}$ vanish in the transitionless limit? $\rightarrow$ yes :)

Transitionless limit $\rightleftarrows$ ϕ wall does not affect masses $\rightleftarrows$ $m_{c,s} = m_{c,h}$

- In some literature, $\mathcal{M}$ does not seem to vanish in this limit ($\rightarrow$ later)

- One problem: the single-splitting probability has soft-collinear divergence

collinear soft

$$dP_{a\to bc} \simeq \frac{g^2}{4\pi^2} \frac{dk_\perp^2}{k_\perp^2} \frac{dx}{x} \left(\frac{k_\perp^2}{k_\perp^2 + m_{c,s}^2} \right)^2 \left(\frac{m_{c,h}^2 - m_{c,s}^2}{k_\perp^2 + m_{c,h}^2} \right)^2$$

- The integration range is $(E_c^2 =)\,(xE_a)^2 > k_\perp^2 + m_{c,s}^2$ from consideration on energy

- In the next slide we consider possible candidates to tame the divergence

SOFT-COLLINEAR DIVERGENCE

- In order to avoid the divergence, we need either of a) b)

collinear soft

$$dP_{a\to bc} \simeq \frac{g^2}{4\pi^2}\frac{dk_\perp^2}{k_\perp^2}\frac{dx}{x}\left(\frac{k_\perp^2}{k_\perp^2+m_{c,s}^2}\right)^2\left(\frac{m_{c,h}^2-m_{c,s}^2}{k_\perp^2+m_{c,h}^2}\right)^2$$

a) lower limit for $k_\perp$ integration > 0

b) $m_{c,s} > 0$

Candidates for nonzero $\mu \equiv \max[m_{c,s}, \text{lower limit of } k_\perp]$ that tames the divergence

(1) Thermal mass :

Particles are thermalized in the symmetric phase, so at least $(m_{\rm th} \equiv)\, m_{c,s} \sim gT$

(2) Phase space saturation (for non-Abelian):

Accumulated c particles suppress further emission of c particles

$$m_{c,s}^2 \sim g^2 \int \frac{d^3p_c}{2E_c} f_c(p_c) \sim g^4 \int \frac{dk_\perp^2}{k_\perp^2} \int_{\frac{\sqrt{k_\perp^2+m_{c,s}^2}}{E_a}} \frac{dx}{x}\frac{\gamma T_{\rm nuc}^3}{2xE_a}\left(\frac{k_\perp^2}{k_\perp^2+m_{c,s}^2}\right)^2\left(\frac{m_{c,h}^2-m_{c,s}^2}{k_\perp^2+m_{c,h}^2}\right)^2$$

Screening on c particle from other c particles

The screening c particles comes from emission from a particles

$\rightarrow \quad (m_{\rm sat}^3 \equiv)\, m_{c,s}^3 \sim g^4 \gamma T_{\rm nuc}^3$: γ-enhanced screening

SOFT-COLLINEAR DIVERGENCE

- After all, we cut $k_\perp$ integration at

$$\mu = \begin{cases} m_{\text{thermal}}\,(\sim gT) & : \text{Abelian} \\ \text{Max}[m_{\text{thermal}}, m_{\text{sat}}] & : \text{non-Abelian} \end{cases}$$

- Subtleties

 - What happens for Abelian gauge boson emission in $T = 0$?

 Ultimately, if we consider multiple emissions, single a particle cannot emit infinitely many energetic c bosons. Taking into account this backreaction from the emitted c particles, we numerically confirmed that divergence is tamed (→ backup).

 - For non-abelian, for small momenta, previous derivation does not hold:

$$\mathscr{L} \sim \partial A \partial A + gAA\partial A + g^2 AAAA \sim p_c^2 A^2 + g p_c A^3 + g^2 A^4$$

leading...? perturbation...? leading...? perturbation...?

A : gauge field (= c boson field)

 This issue still needs to be addressed.

UNITARITY BREAKDOWN

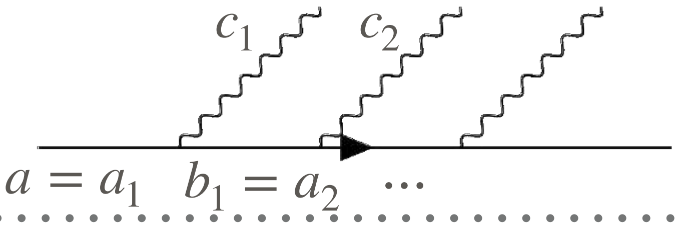
- Numerical estimate of the single-splitting probability

$$P_{a\to bc} = \int dP_{a\to bc}$$

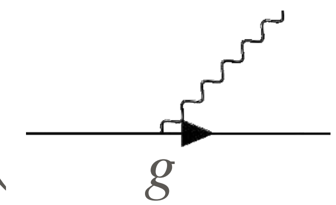
Emission probability P_E at LO in α		$\frac{T_\mathrm{nuc}}{T_\mathrm{start}} = 0.1$	$\frac{T_\mathrm{nuc}}{T_\mathrm{start}} = 10^{-3}$	$\frac{T_\mathrm{nuc}}{T_\mathrm{start}} = 10^{-6}$
$\mu \simeq \alpha^{1/2} T_\mathrm{nuc}$	$\alpha = 0.03$	0.6	$2.8 \gtrsim 1$	$4.1 \gg 1$
(1) thermal mass	$\alpha = 0.3$	$3.2 \gtrsim 1$	$24.5 \gg 1$	$38.3 \gg 1$
$\mu = m_\mathrm{sat}$	$\alpha = 0.03$	0.2	0.5	$2.0 \gtrsim 1$
(2) phase space	$\alpha = 0.3$	1.7	$5.5 \gg 1$	$17.3 \gg 1$

- What does this "probability $\gg$ 1" suggest?

MULTIPLE SPLITTING



- Single-splitting probability $\gg 1$ suggests the breakdown of perturbativity in g

$P\Big($  $\Big) \gg 1$

- Fortunately, it is known that resummation of leading-log diagrams at all order in g gives the Poisson distribution, recovering unitarity:

$$dP_{a\to bccc\cdots} = \sum_{n=1}^{\infty} \frac{1}{n!} \exp\left[-\underbrace{\int dP_{a\to bc}}_{\propto g^2}\right] \prod_{i=1}^{n} \underbrace{dP_{a_i\to b_i c_i}}_{\text{each } \propto g^2}$$

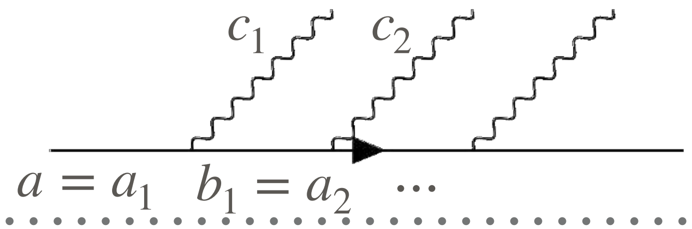
cf. Poisson distribution $P(n) = \frac{1}{n!}\lambda^n e^{-\lambda}$

So, the mean momentum exchange is $\langle \Delta p \rangle = \sum_{n=1}^{\infty} \frac{1}{n!} \exp\left[-\int dP_{a\to bc}\right] \prod_{i=1}^{n} \int dP_{a_i\to b_i c_i} \Delta p$

- In other words, our previous single-emission probability exceeds 1 because we were looking only at the leading order in g

$dP_{a\to bccc\cdots} \to dP_{a\to bc}$ leading order in g

MULTIPLE SPLITTING



- Evaluation of the mean momentum $\langle \Delta p \rangle$ taking multiple c into account

- We define $\left\{ \begin{array}{ll} E_{c,i} = x_i E_a & : i\text{-th } c \text{ particle energy} \\ k_{\perp,i} & : i\text{-th } c \text{ particle momentum} \end{array} \right\}$ as before.

- For each c particle with $(x_i, k_{\perp,i})$, we determine if it's transmitted or reflected according to

$$\left\{ \begin{array}{ll} x_i^2 E_a^2 - m_{c,h}^2 - k_{\perp,i}^2 > 0 & : z \text{ momentum large enough} \to \text{transmitted} \\ x_i^2 E_a^2 - m_{c,h}^2 - k_{\perp,i}^2 < 0 & : z \text{ momentum too small} \to \text{reflected} \end{array} \right.$$

c momentum after transmitted or reflected is estimated as

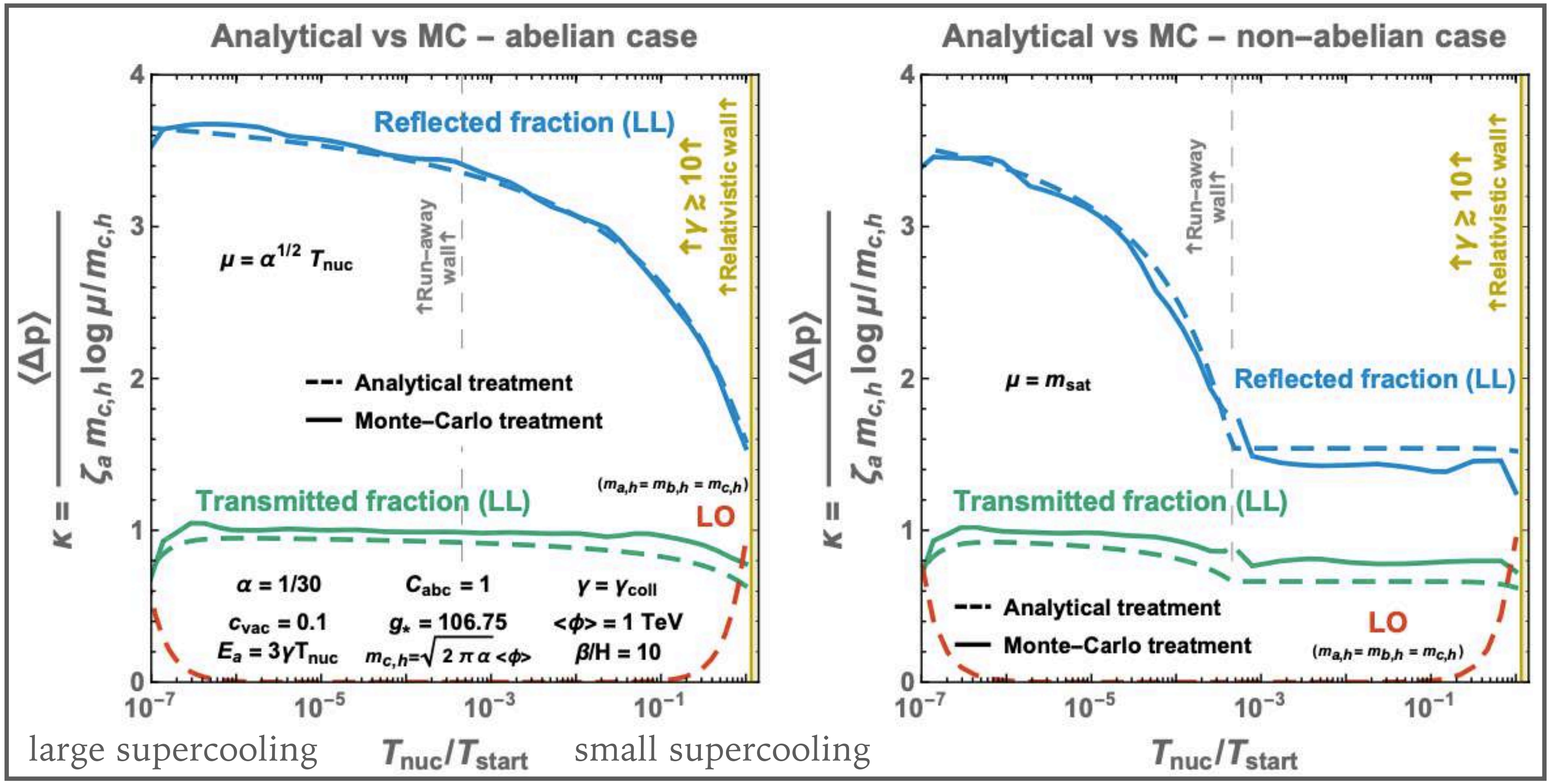
$$p_{cz}^i = \sqrt{x_i^2 E_a^2 - m_{c,h}^2 - k_{\perp,i}^2}\,\Theta(p_{c,h,i}^2) - \sqrt{x_i^2 E_a^2 - m_{c,s}^2 - k_{\perp,i}^2}\,\Theta(-p_{c,h,i}^2)$$

- As a result, momentum transfer from this multiple emission is estimated as

$$\Delta p = E_a - \sqrt{(1-X)^2 E_a^2 - K_\perp^2} - \sum_{i=1}^{n} p_{c,i,z} \quad \text{with} \quad X = \sum_{i=1}^{n} x_i \quad K_\perp = \sum_{i=1}^{n} k_{\perp,i}$$

a momentum b momentum c momenta

AVERAGE MOMENTUM TRANSFER



(large γ) (small γ)

- The vertical axis is essentially $\langle \Delta p \rangle$. If it's constant, $\mathscr{P} = \int \frac{d^3 p_a}{(2\pi)^3} f_a(p_a) \langle \Delta p \rangle \sim \gamma T_{\rm nuc}^3 \langle \Delta p \rangle \propto \gamma T_{\rm nuc}^3$

- Leading-log friction (LL = splitting) dominates leading-order friction (LO = mass)

- We also performed a Monte-Carlo simulation, which is shown as the solid lines.

BUBBLE LORENTZ FACTOR AT THE COLLISION TIME

- From $\mathscr{P} \sim \gamma T_{\rm nuc}^3 \langle \Delta p \rangle \propto g^2 \gamma m_{c,h} T_{\rm nuc}^3$ we obtained, we can determine if the bubbles reach a terminal velocity or run away at the collision time:

1) If the bubble expands infinitely, the wall reaches a terminal velocity

$$\mathscr{P} \sim g^2 \gamma_{\rm terminal} m_{c,h} T_{\rm nuc}^3 = \Delta V \qquad \rightarrow \qquad \gamma_{\rm terminal} \sim \frac{\Delta V}{g^2 m_{c,h} T_{\rm nuc}^3}$$

2) The question is whether $\gamma_{\rm terminal}$ is reached before or after the time of collision.

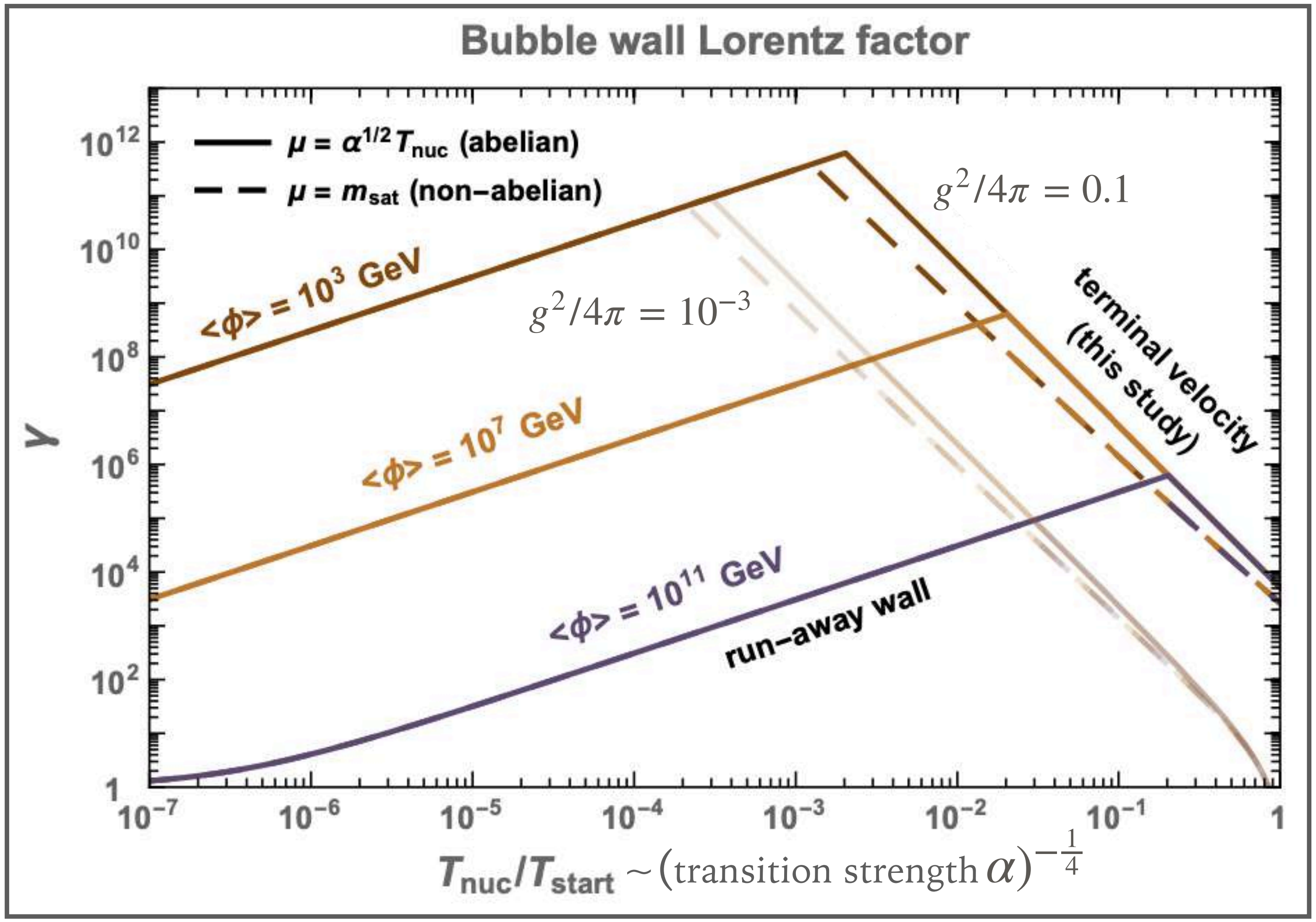
To see this, introduce the typical bubble size at collision time β^{-1} (~O(%) of 1/Hubble) and estimate the γ factor when the bubble expands to this size:

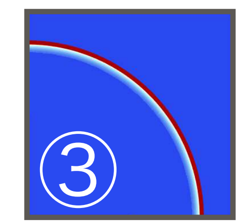
$$\gamma_{\rm run} \sim \frac{\beta^{-1}}{T_{\rm nuc}^{-1}}$$

3) The true γ factor is the smaller of the two:

$$\gamma = {\rm Min}\,[\,\gamma_{\rm terminal}, \gamma_{\rm run}\,]$$

BUBBLE LORENTZ FACTOR AT THE COLLISION TIME

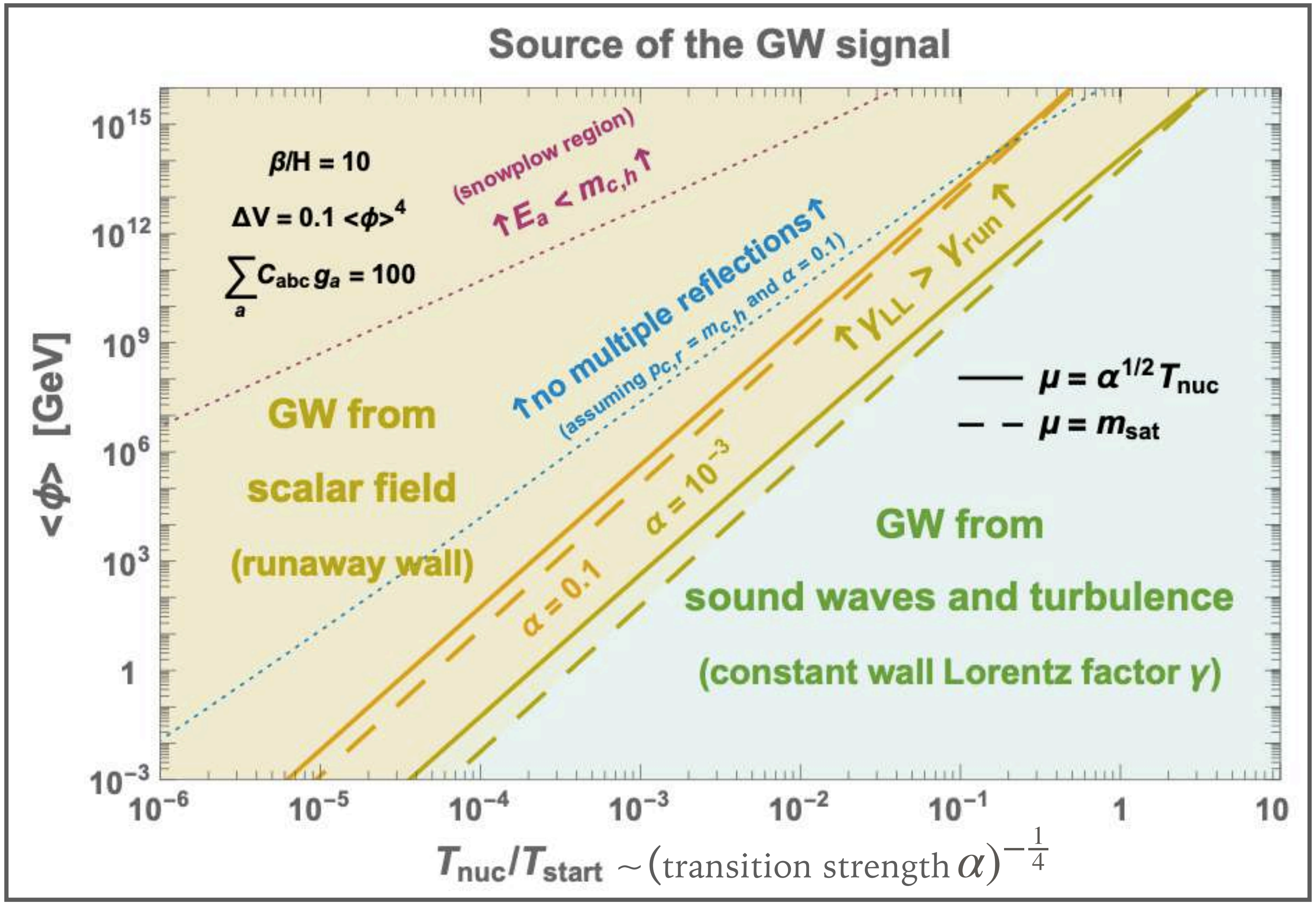




- Bubble walls reach a terminal velocity down to $T_{\rm nuc}/T_{\rm start} \sim 10^{-3}$, but there seems still some room for runaway transitions

GW SOURCE

➤ Whether the wall reaches a terminal velocity or runs away determines whether GWs are mainly sourced from fluid or scalar walls



SUMMARY

- First-order phase transitions and gravitational wave production are interesting phenomena that require understanding physics across scales

- In the 2030s, we enter the most exciting era of GW observations

- Don't miss the chance to contribute in this interesting era