QFT in AdS

Shota Komatsu

Based on arXiv:1810.04185 with Carmi, Di Pietro
arXiv: 2007.13745 with Paulos, van Rees, Zhao
arXiv: 2312.09277 with Copetti, Di Pietro, Ji
WIP with Carmi, Ciccone, De Cesare
WIP with Carmi, Sharma

Theme

Quantum Field Theory on rigid Anti-de-Sitter space

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Quantum Field Theory on rigid Anti-de-Sitter space

- No gravity in AdS
- Boundary CFT with no stress tensor (boundary "CT")
- Still many similarities with AdS/CFT (e.g. boundary OPE)
- Encompasses the study of boundary CFT & defect CFT but extends them to massive QFT

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Quantum Field Theory on rigid Anti-de-Sitter space

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Goal of today's talk

Explain why studying QFT in AdS is interesting and exciting

Plan

1. Generalities on QFT in AdS
2. Non-perturbative S-matrix from flat-space limit
3. Asymptotic freedom in AdS
4. Boundary and defect CFT

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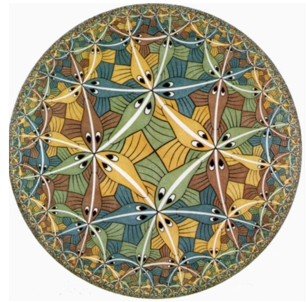
Why QFT in AdS?

[Callan, Wilczek]



Flat space (R^d)	Torus (T^d)	Sphere (S^d)	AdS
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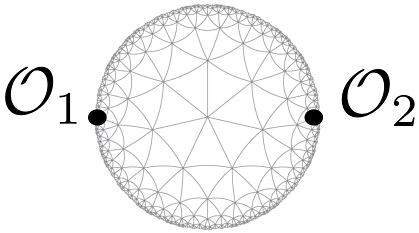
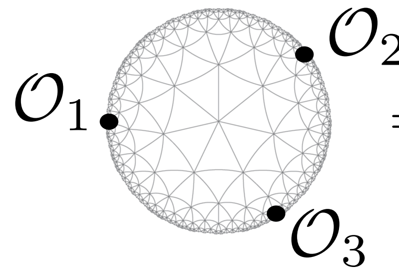
- AdS regulates strong IR effects while keeping virtues of flat space.
- Allows us to interpolate between weakly-coupled, small AdS regime ($L_{\mathrm{AdS}}\Lambda \ll 1$) and strongly-coupled, large AdS regime ($L_{\mathrm{AdS}}\Lambda \gg 1$)

Why QFT in AdS?

- Conformal correlators on the boundary (even for massive QFT).

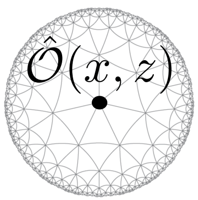


$$= \frac{1}{|x_1 - x_2|^{2\Delta}}$$

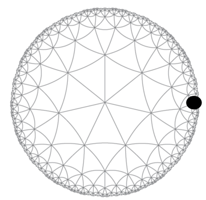


$$= \frac{C_{123}(L_{\text{AdS}}\Lambda)}{|x_1 - x_2|^{\Delta_1+\Delta_2-\Delta_3} \ldots}$$

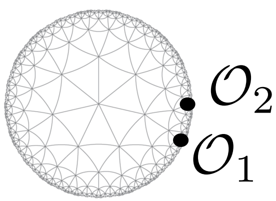
- Convergent OPE expansion (even for massive QFT).



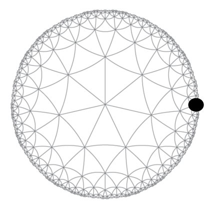
$$= \sum_{\mathcal{O}} f(x,z,\partial)$$



$\mathcal{O}(0)$



$$= \sum_{\mathcal{O}} g(x,\partial)$$



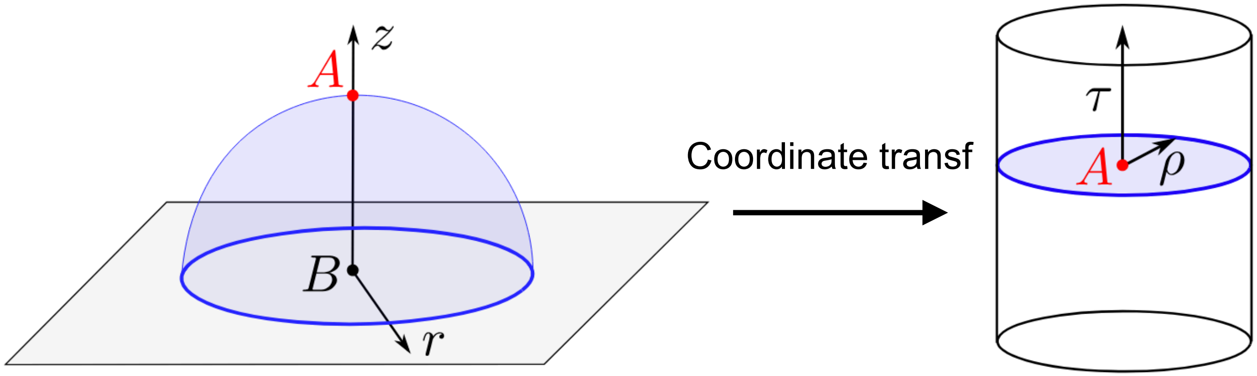
$\mathcal{O}(0)$

- No such expansions known in massive QFT in flat space
  (The lore says OPE is only asymptotic in massive theory)

- Extends similar relations in BCFT (& dCFT) to massive QFT.

Why QFT in AdS?

- State-operator correspondence



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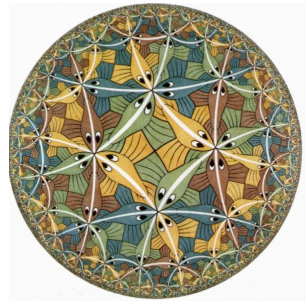
2. Non-perturbative S-matrix from flat-space limit
3. Asymptotic freedom in AdS
4. Boundary and defect CFT



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Non-perturbative S-matrix from flat-space limit

- Lots of recent progress on S-matrix bootstrap. cf. Andrea's talk!
- Analytic properties of S-matrices (in complex Mandelstam variable plane) is one of the most important inputs.
- The state of art is from 60's (!): axiomatic QFT, defines S-matrix by LSZ, uses very technical mathematical result (edge of wedge theorem etc)
- Even 2 to 2 amplitude not fully understood. Much less for higher-point.
- We have more analytic control over CFT correlators.
- Can we take the flat-space limit of AdS to say something about analyticity of the S-matrix?

S-matrix from CFT correlator

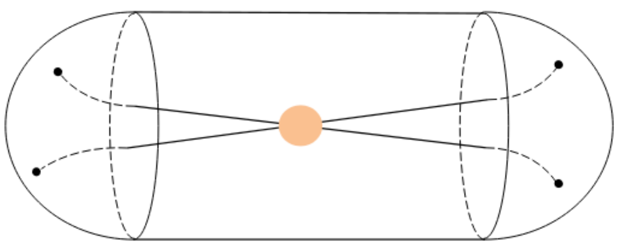
[Giddings, Polchinski, Penedones,….]

- Usual "AdS/CFT" dictionary:

$$m^2 L_{\mathrm{AdS}}^2 = \Delta(\Delta - d)$$

  - $\Delta \to \infty$ in the flat space limit

- Physical picture:



  - (Compton wavelength) $\ll$ AdS scale
  - Particles become classical

- The saddle-point equation of 4 geodesics meeting at a point gives

$$s = 4\left(\frac{1-\sqrt{\rho\bar{\rho}}}{1+\sqrt{\rho\bar{\rho}}}\right)^2 \qquad t = 4\left(\frac{\sqrt{\rho}+\sqrt{\bar{\rho}}}{1+\sqrt{\rho\bar{\rho}}}\right)^2 \quad \Bigg| \quad \left(\rho = \frac{1-\sqrt{1-z}}{1+\sqrt{1-z}}\right)$$

[SK, Paulos, van Rees, Zhao]

Position space formula for S-matrix from CFT

- More precise conjecture: [SK, Paulos, van Rees, Zhao]

$$G(s,t,u) = G_{\mathrm{GFF}}(s,t,u) + G_{\mathrm{contact}}(s,t,u)T(s,t,u)$$

$$\lim_{L_{\mathrm{AdS}}\to\infty} T(s,t,u) = T_{\mathrm{flat}}(s,t,u)$$

  - Under the assumption $\Delta_{\mathrm{next}} > \sqrt{2}\Delta_{\mathrm{lightest}}$

- Can be checked perturbatively.

- Assuming that the limit is finite in a subregion $s, t, u \leq 2m^2$, one can show that the analyticity of $T(s,t,u)$ on the first sheet of s, t, u -plane.

  [van Rees, Zhao]

- One can also derive the so-called extended unitarity with the same assumption.

- Both are beyond the results in 60's!

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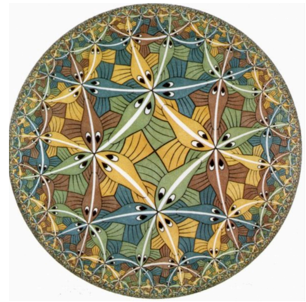

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Asymptotically Free Theory in AdS

- AdS allows us to study asymptotically free theory from perturbation, bootstrap, ….

- UV ($L_{\mathrm{AdS}}\Lambda \ll 1$): Weakly interacting massless particles

- IR ($L_{\mathrm{AdS}}\Lambda \gg 1$): strongly coupled, mass gap

- Depending on the boundary condition, the two regimes are either smoothly connected, or separated by a phase transition.

4d Yang-Mills in AdS

[Aharony, Berkooz, Tong, Yankielowicz]

- Yang-Mills in AdS admits two bc.

- Neumann (magnetic) bc: $F_{zj} = 0$

    - Boundary CFT contains gauge fields
    - Physical states are singlets of gauge group even at weak coupling.
    - Interpolation is expected to be smooth.
    - A great setup for "adiabatic continuity": large symmetry, bootstrap….

[Argyres, Dunne, Unsal ….]

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- Dirichlet (electric) bc: $F_{ij} = 0$
  - At UV, boundary CFT contains current J_μ dual to gluon.
  - Gluons (color charged states) should disappear in flat-space limit.
  - There should be a confinement transition.

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Q: How does the Dirichlet bc disappear?

Scenarios for phase transitions

[Aharony, Berkooz, Tong, Yankielowicz]

- **Higgs**: bulk charged scalar condenses. In CFT terms, this corresponds to a multiplet recombination by adjoint, marginal scalar.

$$\partial_\mu J^\mu = \mathcal{O}_{\mathrm{adjoint}}$$

- **Decoupling**: colored states (gluons) become "null states"

$$\langle J_\mu J_\nu \rangle \propto C_J \to 0$$

- **Tachyon**: some states become tachyonic (below BF bound). In BCFT, this means $\Delta_{\mathcal{O}} = 3/2$

Scenarios for phase transitions

[SK, Copetti, Di Pietro, Ji]

- **Singlet marginality**: a singlet scalar operator $\mathcal{O}$ becomes marginal at some point and triggers (boundary) RG flow.

$$\int_{\mathrm{AdS}} \frac{1}{2g^2} \mathrm{tr}\,(F_{\mu\nu})^2 + \int_{\partial\mathrm{AdS}} y\mathcal{O}$$

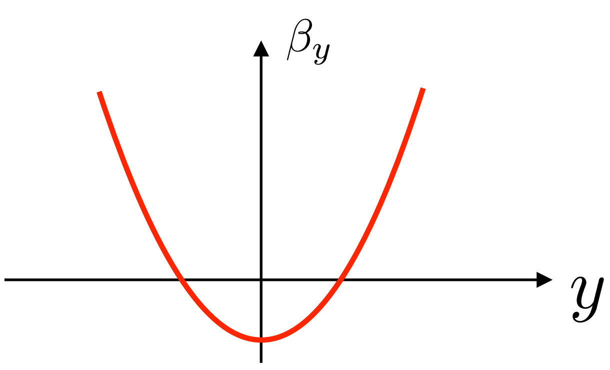
$$\beta_y = \# \left(\frac{1}{g_*^2} - \frac{1}{g^2} \right) + \# y^2$$

[Lauria, Milan, van Rees]

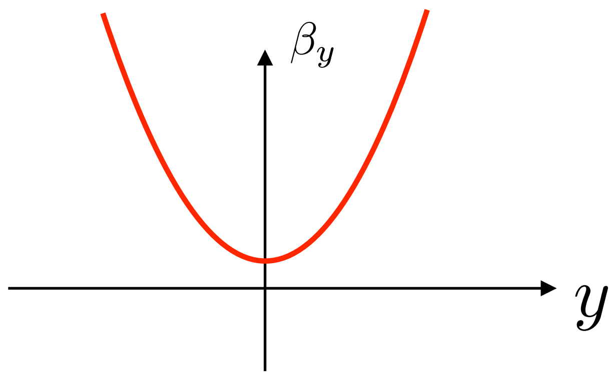
cf. [Fredenhagen, Gaberdiel, Keller]
"bulk-induced boundary perturbation"

- Zeros of beta function exists only for $g < g_*$.
- Fixed points merge and annihilate.

[Gorbenko, Rychkov, Zan]



$g < g_*$



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Scenarios for phase transitions

[SK, Copetti, Di Pietro, Ji]

- **Singlet marginality**: a singlet scalar operator $\mathcal{O}$ becomes marginal at some point and triggers (boundary) RG flow.

Q: Can we see explicitly which one is realized?

- Asymptotically free 2d theory: O(N) model at large N [SK, Copetti, Di Pietro, Ji]
- Perturbative analysis in Yang-Mills in AdS [Ciccone, De Cesare, Di Pietro, Serone]

O(N) model at large N

- O(N) nonlinear sigma model in 2d is a famous tractable example of asymptotically free theory.
- Target space: $\vec{n} \cdot \vec{n} = 1$
- UV Lagrangian has $n-1$ massless ("Goldstone") bosons.
- In flat space, the IR is gapped with n massive bosons.
- In AdS, there exists a SSB breaking boundary condition with $n-1$ massless bosons.

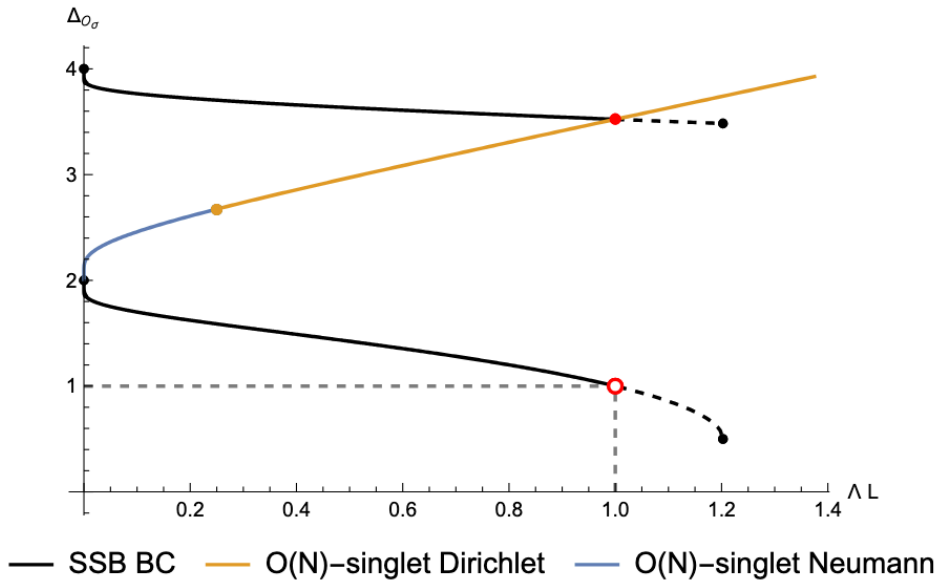
$$\vec{n}|_{\partial \mathrm{AdS}} = \vec{n}_0$$

- They correspond to exactly marginal scalars parametrizing conformal manifold.

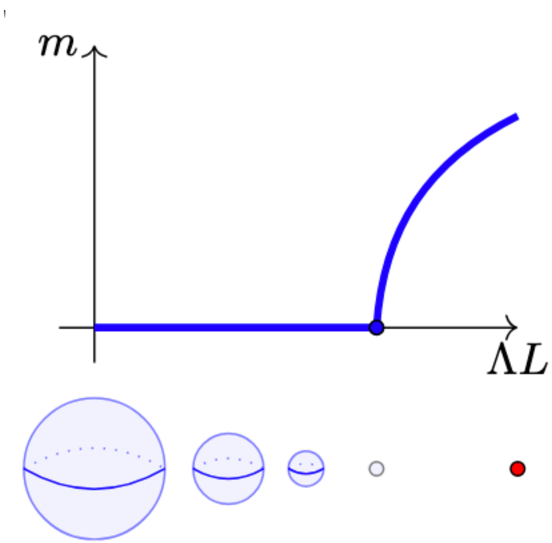
O(N) model at large N

[Inami, Ooguri]

- Applying the large N techniques, we find [SK, Copetti, Di Pietro, Ji]



$\vec{n}|_{\partial \mathrm{AdS}} = \vec{n}_0$



- The gapless phase disappears precisely when we have a singlet marginal operator.
- Supports the singlet marginality scenario.

Yang-Mills in AdS with Dirichlet bc

[Ciccone, De Cesare, Di Pietro, Serone]

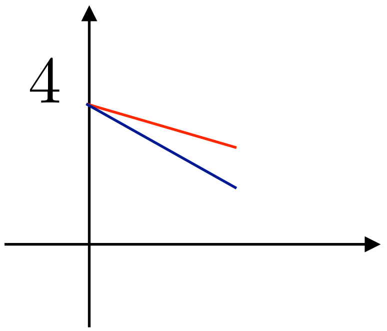
- One can compute the anomalous dimension of various operators perturbatively.

- Lightest adjoint scalar (relevant for **Higgs**): $J^\mu J_\mu$

- Lightest singlet scalar (relevant for **Marginality**): $\mathrm{tr}\left(J^\mu J_\mu\right)$

- Result:

$$\gamma_{\text{adjoint}} = -\frac{11 N_c}{3}\frac{g_{\text{YM}}^2}{16\pi^2} \qquad \gamma_{\text{singlet}} = -\frac{22 N_c}{3}\frac{g_{\text{YM}}^2}{16\pi^2}$$



- Supports singlet marginality.

- Ongoing work on numerical bootstrap.

[Di Pietro, Kousvos, Meineri, Piazza, Serone, Vichi]

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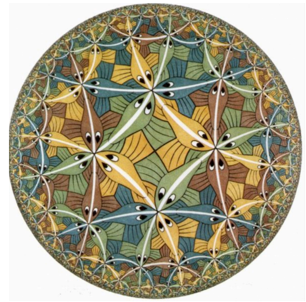

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QFT in AdS and defect CFT

- QFT in AdS encompasses the study of boundary or defect CFT.

- CFT in AdS_d : Boundary CFT

- CFT in $\mathrm{AdS}_{p+1} \times S^{d-p}$: p-dimensional defect CFT

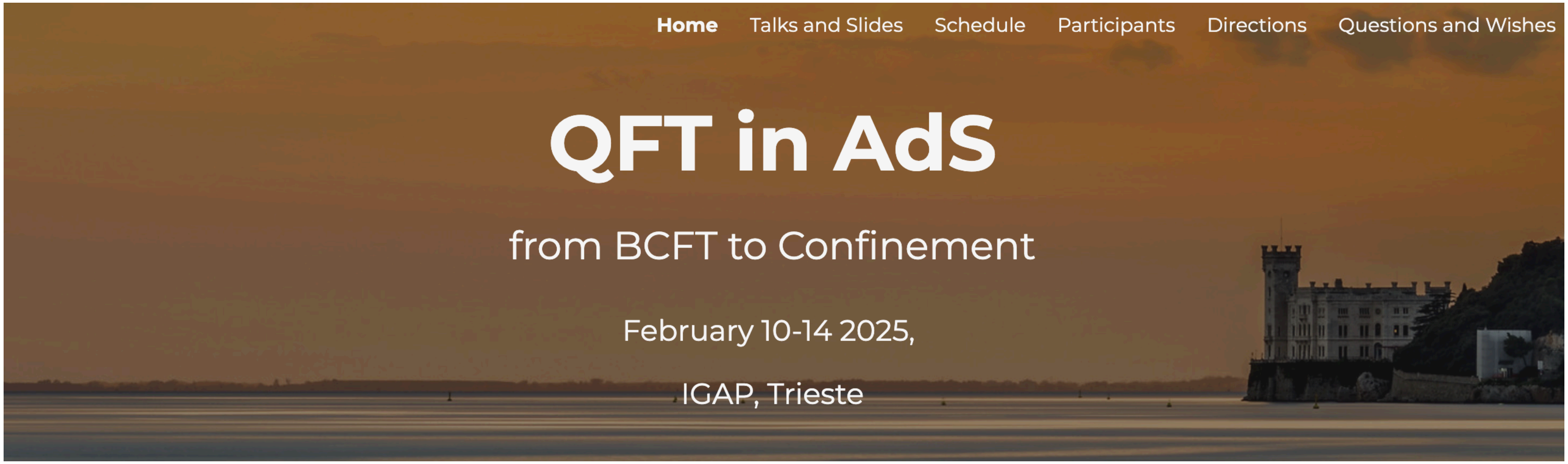
- By tuning the bulk QFT coupling so that the bulk becomes critical, one can extract BCFT, dCFT data. [Carmi, Di Pietro, Komatsu] [Giombi, Khanchandani] [Cuomo, Komargodski, Mezei]

- One can also "interpolate" between defects in different CFTs.
    - e.g. defects in Wilson-Fisher fixed point and Gaussian fixed point.

Conclusion

- QFT in AdS allows us to address many interesting questions.
    - Non-perturbative analyticity of S-matrix.
    - Mass gap and confinement.
    - Boundary CFT and defect CFT.

- It can be addressed by a variety of modern techniques.
    - Large N
    - Perturbation
    - Bootstrap (analytical, numerical)
    - Adiabatic continuity — Not much so far
    - Semiclassics, resurgence (IR renomalons?) — Not much so far
    - Supersymmetry, localization — Davide Bason's poster!



https://sites.google.com/view/qftinadsworkshop/home

QFT in AdS: Questions and Wish lists

- What are interesting questions to ask fc
Expectation: since the effective tempera
direction of AdS, there will be a plasma
region near the boundary is confined. C
or bootstrap? (See Riccardo's slides an
- QCD string in AdS: Does the Migdal-Ma
role? What is its interpretation in terms
- QFT in AdS always admits a converger
hand, the convergence of (bulk) OPE in general massive QFT is not
understood (the lore says that it is an asymptotic series but it's not even

- Flat space limit: In CFT, one can write an operator respor
wave with non-integer spin (i.e. lightray operators) while
such a clear understanding (i.e. physical object responsit
spin partial wave) in S-matrix in flat space. Can we gain i
taking the flat-space limit of AdS?
- Resurgence and QFT in AdS: Using the Hopf algebraic re
approach by Connes-Kreimer, one can sometimes write a
differential equation for anomalous dimensions in flat spa
the large order behavior and/or non-perturbative correctic
https://arxiv.org/abs/2005.04265). Can one do something
- Are there instantons in YM in AdS?
- Are there IR-renormalons in AdS? (Perhaps one can che
O(N) model)

- There was a recent work which studied Adjoint QCD (in 4d) by (softly) breaking supersymmetry of N=2 SYM https://arxiv.org/abs/2412.20547 . The effective description there was the Abelian Higgs model. This description makes some of the interesting questions accessible while others (like confining string in large N t Hooft limit) seem still difficult. On the other hand, YM in AdS allows us to study the theory keeping "full non-Abelian" degrees of freedom and seems promising for studying confining string. What happens if we (softly) break SUSY in N=2 SYM in AdS? Can one interpolate between the "Abelian" description (discussed in the paper above) and the "non-Abelian" description (i.e. bosonic YM in AdS at small coupling)?
- Can we see the transition from the conformal window to the spontaneous symmetry breaking phase as we vary NF in gauge theories in AdS (for instance we can study QED3 as a simpler example)? Probably some symmetric boundary condition stop to exist when we enter in the SSB phase
- Flow equation and supersymmetry: In superconformal field theory, there are often subsectors of supersymmetric operators in which the OPE is closed (e.g. chiral algebra for N=2 SCFT https://arxiv.org/abs/1312.5344). Can there be a similar subsector of ODE for supersymmetric QFT in AdS? For instance, a class of observables in the Maldacena Wilson loop in N=4