The large charge sector of the NJL model

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Kyushu IAS-iTHEMS conference: *Non-perturbative methods in QFT*

Based on: [O. Antipin, JB, P. Panopoulos (2208.05839) and work in progress with S. Hellerman, D. Orlando, S. Reffert,]

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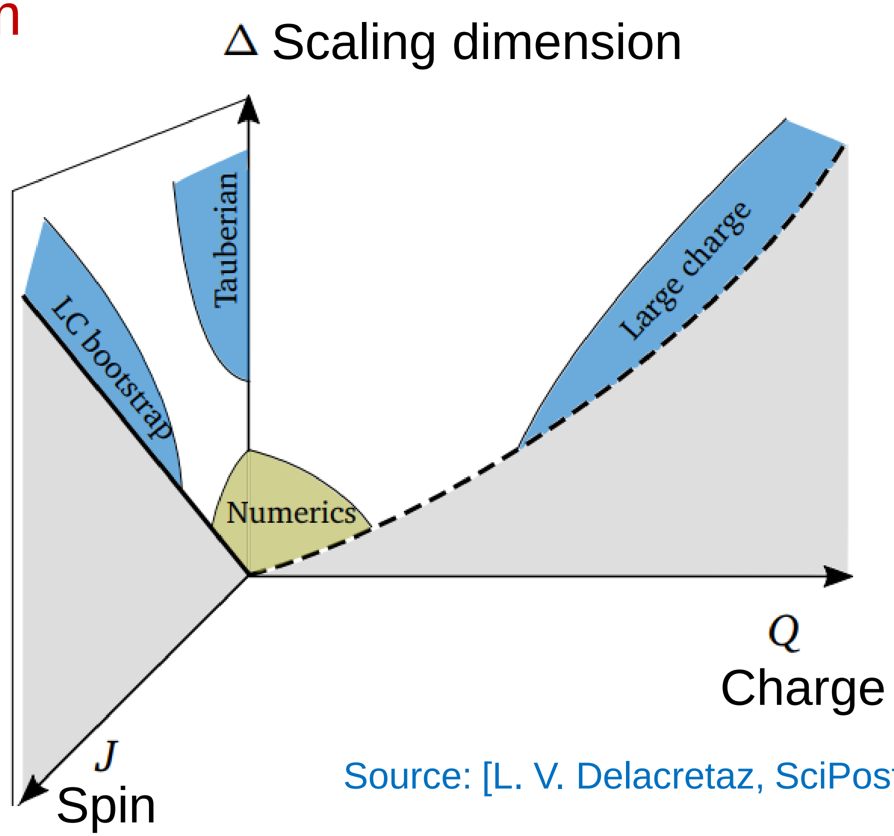
Solving non-SUSY CFTs in d>2

CFT Spectrum



Source: [L. V. Delacretaz, SciPost Phys. 9 (2020) 3, 034]

Many *non-perturbative methods* to access the various sector of the theory

Solving non-SUSY CFTs in d>2

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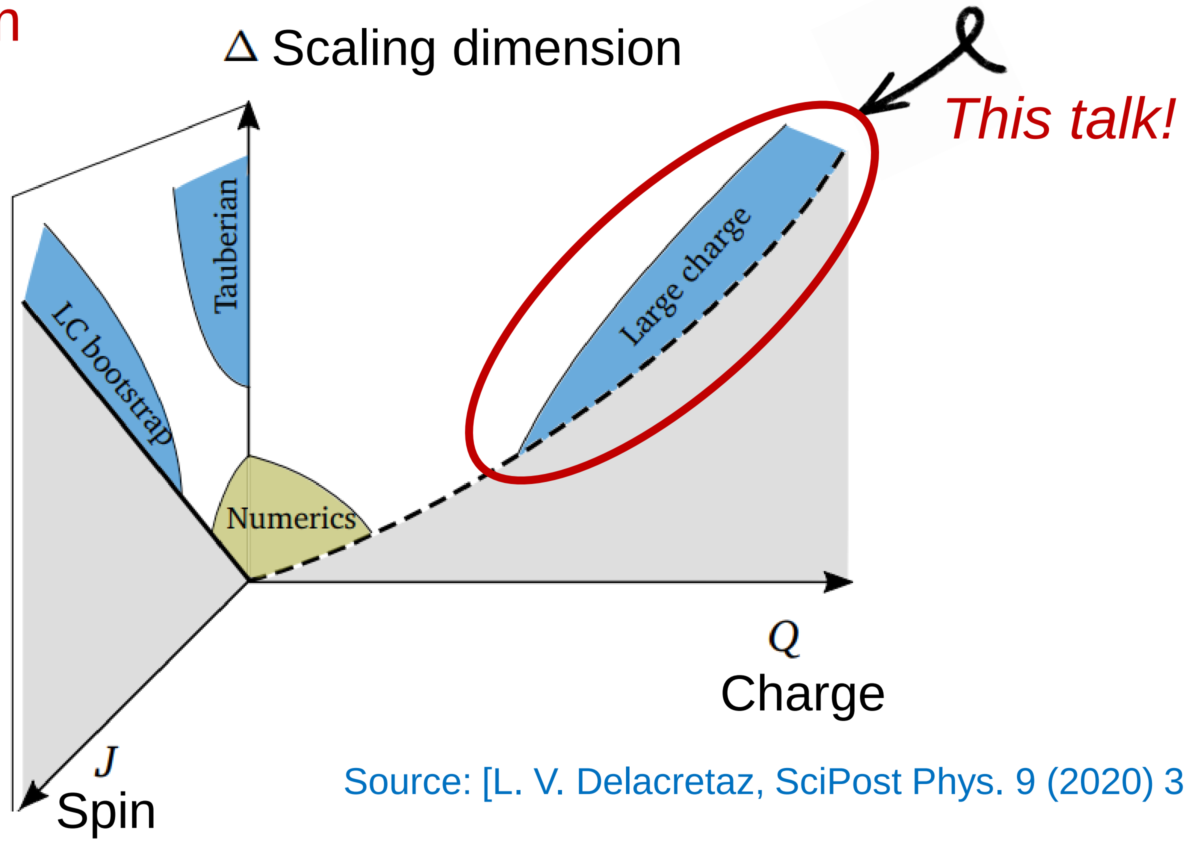


Source: [L. V. Delacretaz, SciPost Phys. 9 (2020) 3, 034]

The large-charge expansion

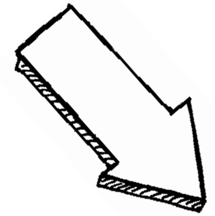
CFTs (QFTs) simplify in certain limits where a parameter is small/large.

Our large parameter(s): conserved charge(s) of the internal symmetry group of the CFT:

LARGE-CHARGE EXPANSION

[S. Hellerman, D. Orlando, S. Reffert, M. Watanabe (2015)]

This is a semiclassical expansion



Bohr's correspondence principle:

"*Quantum physics "classicalizes" in the presence of large quantum numbers.*"

Superfluid EFT

Consider a U(1) bosonic CFT with no moduli space : when the U(1) charge Q is the dominant large parameter the physics can be described via an EFT for Goldstone boson χ arising from the SSB induced by fixing the charge.

$$\mathcal{S} = -c_1 \int d^d x \sqrt{g} (-\partial_\mu \chi \; g^{\mu\nu} \partial_\nu \chi)^{d/2} + \dots$$

Derivative expansion ⟺ 1/Q expansion

The symmetry breaking defines a conformal superfluid phase of matter.

$$SO(d+1,1) \otimes U(1) \to SO(d) \otimes D' \qquad D' = D + \mu Q$$

D = dilations.
μ = chemical potential

A typical prediction is the scaling dimension of the lowest-lying operator with charge Q:

$$\Delta_Q = Q^{\frac{d}{d-1}} \left[\alpha_1 + \alpha_2 Q^{\frac{-2}{d-1}} + \alpha_3 Q^{\frac{-4}{d-1}} + \dots \right] + Q^0 \left[\beta_0 + \beta_1 Q^{\frac{-2}{d-1}} + \dots \right] + \mathcal{O}\left(Q^{-\frac{d}{d-1}} \right)$$

According to the state-operator correspondence Δ_Q is the ground state energy of the theory on the cylinder $\mathbb{R} \times S^{d-1}$.

Free fermions

[Z. Komargodski, M. Mezei, S. Pal, A. Raviv-Moshe (2021)]

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu + \mu\gamma_0)\psi$$

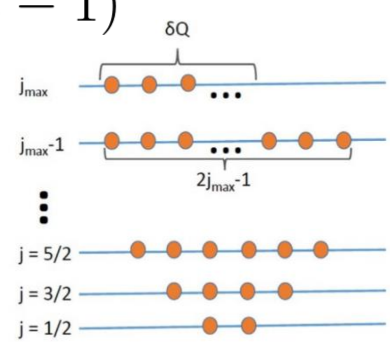
We have again: $\Delta_Q \underset{Q\to\infty}{\sim} A Q^{\frac{d}{d-1}}$

However, the large charge sector of the theory is not a superfluid but a Fermi surface. No SSB, Goldstones, different subleading corrections.

One particle energies: $\omega_\pm = p \pm \mu \Longrightarrow$ Fermi momentum: $p_F = \mu$

We can compute the ground state energy on $\mathbb{R}\times S^{d-1}$ by summing over the energy levels up to the Fermi surface:

$$\Delta_Q = \sum_{\ell=1}^{\ell_{\max}} n_\ell(\omega_+ + \omega_-) \qquad Q = \sum_{\ell=1}^{\ell_{\max}} n_\ell \Longrightarrow \Delta_Q \sim \frac{2^{\frac{1-[d/2]}{d-1}}}{d\Gamma(d-1)}(\Gamma(d)Q)^{\frac{d}{d-1}}$$



n_l is the degeneracy of the energy levels where l labels the eigenvalues λ_l of the Dirac operator ($p \to \lambda_\ell$).

Goal

“investigate the large charge sector of interacting fermionic CFTs”

We focus on a specific theory: the NJL (Nambu-Jona-Lasinio) model.

[Y. Nambu, G. Jona-Lasinio (1961)]

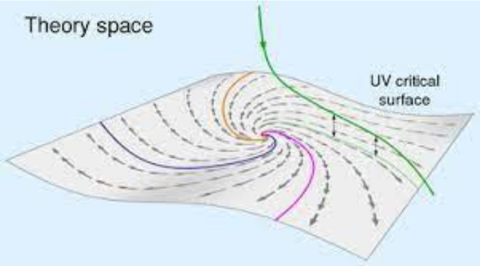
$$\mathcal{L} = \bar{\Psi}_i \Gamma^\mu \partial_\mu \Psi_i - \frac{g}{N}\left[\left(\bar{\Psi}_i \Psi_i\right)^2 - \left(\bar{\Psi}_i \Gamma_5 \Psi_i\right)^2\right] \qquad i = 1, \ldots N$$

In d > 2 the four-fermion interaction is non-renormalizable.

For 2<d<4, at large N the model flows to an interacting fixed point in the UV.

Non-perturbative renormalizability

“Asymptotic safety”



The “NJL models”

It is believed that in 2<d<4 dimensions there are three different description of the same CFT:

[J. Zinn-Justin (1991), L. Fei, S. Giombi, I. R. Klebanov, G. Tarnopolsky]

NJLY U(N) model. Weakly coupled IR fixed point in d=4-ε

$$\mathcal{L} = \partial_\mu \bar{\phi} \partial^\mu \phi + \bar{\psi}_j \gamma_\mu \partial^\mu \psi^j + g\, \bar{\psi}_{Rj} \bar{\phi} \psi_L^j + g\, \bar{\psi}_{Lj} \phi \psi_R^j + \frac{(4\pi)^2 \lambda}{24} \left(\bar{\phi}\phi\right)^2$$

NJL U(N) model. UV fixed point studied via the 1/N expansion

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Thirring U(2N) model. Weakly coupled UV fixed point in d=2+ε

$$\mathcal{L} = \bar{\psi}_j \gamma^\mu \partial_\mu \psi^j - g \left(\bar{\psi}_j \gamma_\mu T^a \psi^j \right)^2$$

Charging the NJLY model

[O. Antipin, JB, P. Panopoulos (2022)]

$$\mathcal{L} = \partial_\mu \bar{\phi} \partial^\mu \phi + \bar{\psi}_j \gamma_\mu \partial^\mu \psi^j + g\, \bar{\psi}_{Rj} \bar{\phi} \psi^j_L + g\, \bar{\psi}_{Lj} \phi \psi^j_R + \frac{(4\pi)^2 \lambda}{24} \left(\bar{\phi}\phi\right)^2$$

It has a U(N) flavor symmetry and a $U(1)_A$ chiral symmetry acting as

$$\phi \to e^{-2i\alpha}\phi, \qquad \psi_{Lj} \to e^{-i\alpha}\psi_{Lj}, \quad \psi_{Rj} \to e^{i\alpha}\psi_{Rj}$$

Wilson-Fisher fixed point in d=4-ε:

$$\frac{g^{2*}}{(4\pi)^2} \sim \frac{\epsilon}{4(1+N)} \qquad \lambda^* \sim \frac{3\left(\sqrt{N\,(N+38)+1} - N + 1\right)}{20\,(N+1)}\epsilon$$

We compute the scaling dimension of the lowest operator with $U(1)_A$ charge Q (Φ^Q) i.e. the ground state energy on $\mathbb{R}\times S^{d-1}$ in the double scaling limit

$$Q \to \infty,\ \epsilon \to 0,\ Q\epsilon = \text{fixed.} \qquad \Longrightarrow \qquad \Delta_Q = Q \sum_{j=0}^{\infty} \frac{\Delta_j(Q\epsilon)}{Q^j}$$

Semiclassics

$$\Delta_Q = Q \sum_{j=0}^{\infty} \frac{\Delta_j(Q\epsilon)}{Q^j}$$

This is a semiclassical expansion. In fact, the path integral for the ground state energy on the cylinder reads

$$\phi = \frac{\rho}{\sqrt{2}} e^{i\chi}$$

$$\langle Q | e^{-HT} | Q \rangle = \int D\psi D\bar{\psi} D\rho D\chi e^{-Q S_{\text{eff}}}$$

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Evolution operator

Arbitrary state with charge Q

$$\langle Q| e^{-HT} |Q\rangle \underset{T\to\infty}{\propto} e^{-\Delta_Q T}$$

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For large Q the path integral is dominated by the extrema of S_{eff}.

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Conformal coupling

NJLY action with couplings $\begin{cases} g^2 \to Qg^2 \\ \lambda \to Q\lambda \end{cases}$

Stems from $|Q\rangle$*, "charge fixing".*

For large Q the path integral is dominated by the extrema of S_{eff}.

Leading order: Δ_0

$$\phi = \frac{\rho}{\sqrt{2}} e^{i\chi}$$

The ground state is spatially homogeneous: $\langle \rho \rangle \neq 0 \quad \langle \chi \rangle = -i\mu t$

μ is the chemical potential.

The ground state breaks a linear combination of time translations on the cylinder and $U(1)_A$. This is the SSB pattern of the conformal superfluid phase.

No Fermi surface. One particle energies of the Fermions are positive:

$$\omega_\pm(p) = \sqrt{\frac{3g^2\left(\mu^2 - m^2\right)}{8\pi^2\lambda} + \left(\frac{\mu}{2} \pm p\right)^2}$$

Intuitively, due to Yukawa interactions the charge leaks from the fermions into the scalars. By plugging the classical solution into S_{eff} we have

$$\frac{4\Delta_0}{\lambda Q} = \frac{3^{\frac{2}{3}} x^{\frac{1}{3}}}{3^{\frac{1}{3}} + x^{\frac{2}{3}}} + \frac{3^{\frac{1}{3}}\left(3^{\frac{1}{3}} + x^{\frac{2}{3}}\right)}{x^{\frac{1}{3}}} \qquad x \equiv 6\lambda Q + \sqrt{36(\lambda Q)^2 - 3}$$

This classical result resums an infinite. series of Feynman diagrams One can also easily compute the NLO Δ_1 which is given by a sum over the one particle energies (functional determinant).

Qε<<1: ε-expansion

By expanding the Δ_i's in the limit of small Qε, we obtain the conventional perturbative expansion

$$\Delta_Q = Q + \left[\frac{Q\left(-\sqrt{N(N+38)+1}+N-11\right)}{20(N+1)} + \frac{Q^2\left(\sqrt{N(N+38)+1}-N+1\right)}{20(N+1)}\right]\epsilon$$
$$+\left[\left(\frac{64N^4+3748N^3+10557N^2+5581N-50}{2000(N+1)^3\sqrt{N(N+38)+1}} - \frac{32N^3+391N^2+1797N+25}{1000(N+1)^3}\right)Q\right.$$
$$+\left(\frac{-84N^4-4318N^3-3327N^2-4251N+80}{2000(N+1)^3\sqrt{N(N+38)+1}} + \frac{84N^3+1722N^2+2729N+80}{2000(N+1)^3}\right)Q^2$$
$$\left.-\left(\frac{(1-N)\sqrt{N(N+38)+1}+N^2+18N+1}{100(N+1)^2}\right)Q^3\right]\epsilon^2 + \left[a_4Q^4+a_3Q^3+a_2Q^2+a_1Q\right]\epsilon^3+\dots.$$

Red terms: Δ_0

Blue terms: Δ_1

Complete 2-loop (ε^2) scaling dimension obtained by combining our results with the known perturbative results for Q= 1.

Infinite number of checks for future diagrammatic computations.

Qε>>1: EFT matching

In the large Qε limit we obtain the form predicted by the EFT approach

$$\Delta_Q = Q^{\frac{d}{d-1}}\left[\alpha_1 + \alpha_2 Q^{\frac{-2}{d-1}} + \alpha_3 Q^{\frac{-4}{d-1}} + \dots\right] + Q^0\left[\beta_0 + \beta_1 Q^{\frac{-2}{d-1}} + \dots\right] + \mathcal{O}\left(Q^{-\frac{d}{d-1}}\right)$$

Microscopic calculation of α_1, α_2, … i.e of the Wilson coefficients of the large charge EFT.

We take $\epsilon = 1$

	α_1		α_2	
	LO	NLO	LO	NLO
$N = 1/2$	0.655	0.545	0.572	0.217
$N = 1$	0.644	0.596	0.582	0.244
$N = 2$	0.608	0.595	0.617	0.312
$N = 3$	0.578	0.567	0.649	0.377
$N = 4$	0.553	0.536	0.679	0.435
$N = 5$	0.532	0.507	0.705	0.488

SUSY limit: Wess-Zumino model

For N=1/2 the IR fixed point of the NJLY model feature emergent supersymmetry and reduces to the critical Wess-Zumino model

The chiral charge becomes the R charge $$R = \frac{2}{3} Q$$

Supersymmetry protects the dimension of Φ and $Φ^2$ becomes a descendant of Φ. Then

$$\Delta_{Q=2} = \Delta_{Q=1} + 1 \qquad \Delta_{Q=1} = \frac{D-1}{2} R = \frac{D-1}{3}$$

One can check the above by looking at our results

$$\Delta_Q = Q + \frac{1}{6}(Q-3)Q\epsilon - \frac{1}{18}(Q-2)(Q-1)Q\epsilon^2 + \frac{1}{108}(Q-2)(Q-1)Q(4Q-11+12\zeta(3))\epsilon^3 + \dots$$

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The NJL model in 2<d<4

$$\mathcal{L} = \bar{\Psi}_i \Gamma^\mu \partial_\mu \Psi_i - \frac{g}{N}\left[\left(\bar{\Psi}_i \Psi_i\right)^2 - \left(\bar{\Psi}_i \Gamma_5 \Psi_i\right)^2\right] \qquad i = 1, \ldots N$$

To study the model at large N one uses the Hubbard–Stratonovich transformation and introduces an auxiliary complex scalar Φ.

$$\mathcal{L} = \bar{\Psi}\left(\Gamma_\mu \partial^\mu + \Phi\frac{1+\Gamma_5}{2} + \bar{\Phi}\frac{1-\Gamma_5}{2}\right)\Psi + \frac{N}{4g}\bar{\Phi}\Phi$$

- Integrating out Φ one recovers the original Lagrangian.
- Integrating out the fermions one generates the 1/N expansion.
- $U(1)_A$ acts as $\Psi_i \to e^{i\alpha\Gamma_5}\Psi_i \qquad \Phi \to e^{-2i\alpha}\Phi$

We compute Δ_Q in 2<d<4 to the leading order in 1/N. This allows us to test the equivalence to the NJLY and Thirring models.

Charging the NJL model in 2<d<4

We introduce a chemical potential μ for the $U(1)_A$ charge. Then the ground state energy is the Legendre transform of the grand potential Ω(μ):

$$\Delta_Q = \Omega(\mu) + \mu Q\big|_{\mu=\mu(Q)} \qquad Q = -\frac{\partial \Omega}{\partial \mu} \qquad \frac{\partial \Omega}{\partial \Phi_0} = 0$$

Ω(μ) is the functional determinant generated by integrating out the fermions and is given by a sum over the one-particle energies.

$$\Omega(\mu) = \sum_{\ell=1}^{\infty} n_\ell (\omega_+ + \omega_-) \qquad \omega_\pm^2 = \Phi_0^2 + (\lambda_\ell \pm \mu)^2$$

λ_l are the eigenvalues of the Dirac operator on the (d-1)-sphere and have degeneracy n_l.

$$\lambda_\ell = \ell + \frac{d}{2} \qquad n_\ell = \frac{4\,\Gamma(\ell + d)}{\Gamma(d)\Gamma(\ell + 1)}$$

The one particle energies are always positive: no Fermi surface. SSB of the conformal superfluid phase.

Results

$$\frac{\Delta_Q}{2N} = \frac{Q}{4N} + \delta_2 \left(\frac{Q}{N}\right)^2 + \delta_3 \left(\frac{Q}{N}\right)^3 + \mathcal{O}\left(\left(\frac{Q}{N}\right)^4\right)$$

$Q/N \ll 1$

$$\begin{aligned}\delta_2 = -\Bigg[&\frac{1}{\pi^{7/2} 2^{d+2} (d-2)^4 \csc^3\left(\frac{\pi d}{2}\right) \Gamma\left(\frac{d+4}{2}\right)^2} \left(d^2-4\right) \Gamma(d-1) \Gamma\left(\frac{d+2}{2}\right) \Bigg(d(d+2)\Big(\pi(d-2)\cot\left(\frac{\pi d}{2}\right) \\ &+2\Big) \csc\left(\frac{\pi d}{2}\right) \Gamma\left(\frac{1-d}{2}\right) \Gamma(d) + \sqrt{\pi} 2^{d+2} \left(\pi(d-2)\cot\left(\frac{\pi d}{2}\right) - 2\right) \csc(\pi d) \Gamma\left(\frac{d}{2}+2\right)\Bigg) \\ &\times \left(2^{2-d}\left(\pi(d-2)\cot\left(\frac{\pi d}{2}\right)+2\right)\Gamma\left(\frac{3}{2}-\frac{d}{2}\right) + \frac{\pi^{3/2}(d-2)\left(\pi(d-2)\cot\left(\frac{\pi d}{2}\right)-2\right)\csc(\pi d)}{\Gamma\left(2-\frac{d}{2}\right)\Gamma(d-1)}\right)^2\Bigg]^{-1}\end{aligned}$$

- d=4-ε $\quad \delta_2 = \frac{\epsilon}{8} - \frac{\epsilon^2}{16} + \mathcal{O}\left(\epsilon^3\right)$ Matches the NJLY model.
- d=2+ε $\quad \delta_2 = \frac{1}{16}\epsilon - \frac{\zeta(3)}{64}\epsilon^4 + \mathcal{O}\left(\epsilon^5\right)$ Should match the Thirring model. No $O(\varepsilon^2)$, $O(\varepsilon^3)$ terms!
- d=3 $\quad \frac{\Delta_Q}{2N} = \frac{1}{2}\left(\frac{Q}{2N}\right) + \frac{2}{\pi^2}\left(\frac{Q}{2N}\right)^2 + \dots$

[N. Dondi, S. Hellerman, I. Kalogerakis, R. Moser, D. Orlando, S. Reffert (2022)]

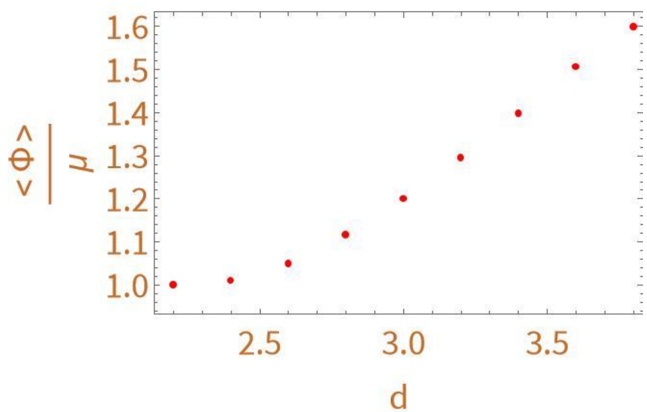
Results

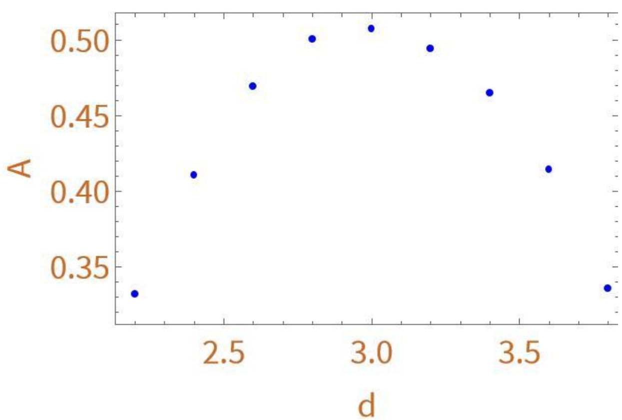
$$\Delta_Q \sim A\left(\frac{Q}{2N}\right)^{\frac{d}{d-1}} \qquad A = \left(f_{Q,1}\right)^{-\frac{d}{d-1}}\left(f_{Q,1}+f_{\Delta,1}\right)$$

$Q/N \gg 1$

In the Q/N>>1 limit, the results can be expressed in terms of integrals that one has to compute numerically.

$$f_{Q,1} = \frac{1}{\Gamma(d-1)}\int_0^\infty dk\ k^{d-2}\left(\frac{1-k}{\sqrt{(k-2)k+k_0^2}}+\frac{k+1}{\sqrt{k(k+2)+k_0^2}}\right) \qquad f_{\Delta,1} = \frac{1}{\Gamma(d-1)}\int_0^\infty d\ k\ k^{d-3}\left(2k^2 - k\left(\sqrt{(k-2)k+k_0^2}+\sqrt{k(k+2)+k_0^2}\right)+k_0^2-1\right)$$





The gap is a monotonic growing function of d. The coefficient of the leading term reaches its maximum in the physical dimension d=3.

Fermi surface in the NJL model

The large $U(1)_A$ charge sector of the NJL model realizes the conformal superfluid phase.

However, the NJL model has also a $U(1)_B$ ”baryon” symmetry:

One particle energies: $\omega_\pm = p \pm \mu$ $\Longrightarrow$ Fermi momentum: $p_F = \mu$

The ground state is a Fermi surface (scalar field not charged under $U(1)_B$).

[N. Dondi, S. Hellerman, I. Kalogerakis, R. Moser, D. Orlando, S. Reffert (2022)]

We can compute the ground state energy by summing over the energy levels up to the Fermi surface:

$$\Delta_Q = \sum_{\ell=1}^{\ell_{\max}} n_\ell(\omega_+ + \omega_-) \Longrightarrow \Delta_Q \sim \frac{2^{\frac{1-[d/2]}{d-1}}}{d\Gamma(d-1)} \left(\frac{\Gamma(d)Q}{2N} \right)^{\frac{d}{d-1}}$$

Same result obtained for the free fermion upon replacing $Q \to \frac{Q}{2N}$

However, the Fermi surface may be non-perturbatively unstable (BCS).

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Conclusions

"The large charge expansion represents an important non-perturbative methods in QFT"

- We studied the large $U(1)_A$ charge sector of the NJL model.
- The ground state is a conformal superfluid which can be described via the large charge EFT.
- We determined the EFT Wilson coefficients to the leading order in 1/N in $2<d<4$ and to the next-to-leading order in the ε-expansion.
- We provided a nontrivial test of the equivalence between different formulations of the CFT.