Universal Aspects of Intersecting Defects

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iTHEMS

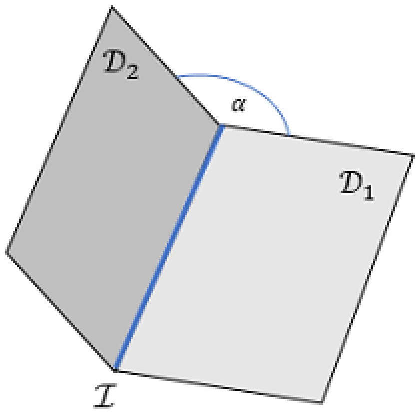
Overview

- 2411.14543. “On Intersecting Conformal Defects”, *T.S.*, *JHEP* (to appear).

- Short Introduction
- Two defects – RG on Intersections and wedges
- Three Defects – Corner Anomalous Dimension
- Summary & Conclusions

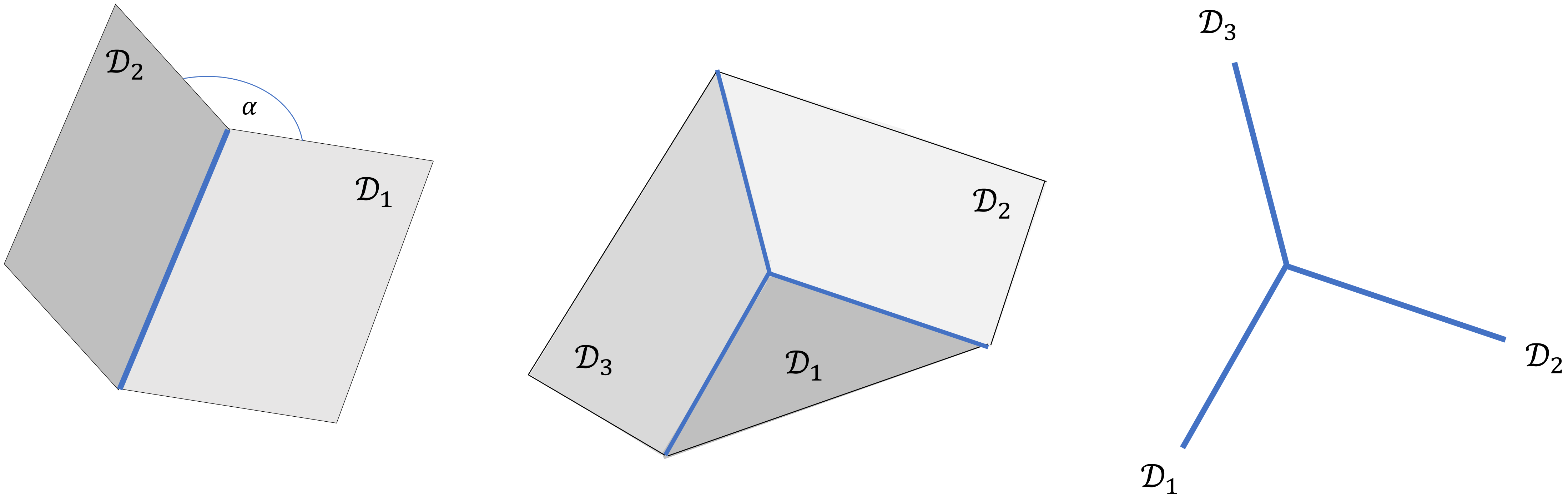
Short Introduction

- Defects are extended objects that are very common in QFT, coming with many exciting properties
  - Wilson-lines, domain walls…
  - Unique phase-diagrams, unique symmetries and anomalies…
- **Motivating question** – When defects meet, a new defect forms at the intersection. How do the properties of the original defects shape this new one?



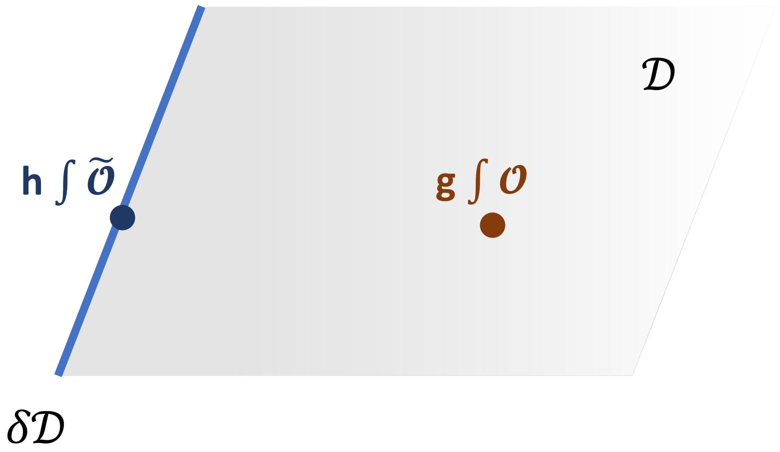
Short Introduction

- There are many interesting objects of this kind, incorporating defects of varying number and dimensionality



RG on the Intersection

- To begin to understand this large problem, we deploy conformal perturbation theory
  - Construct defects using local deformations from the bulk CFT
  - **Semi infinite plane -** Simplest object to begin with



$$S = S_{\text{CFT}} + g \int_{\mathcal{D}} \mathcal{O} + h \int_{\delta\mathcal{D}} \tilde{\mathcal{O}}$$

RG on the Intersection

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  - Construct defects using local deformations of the bulk CFT
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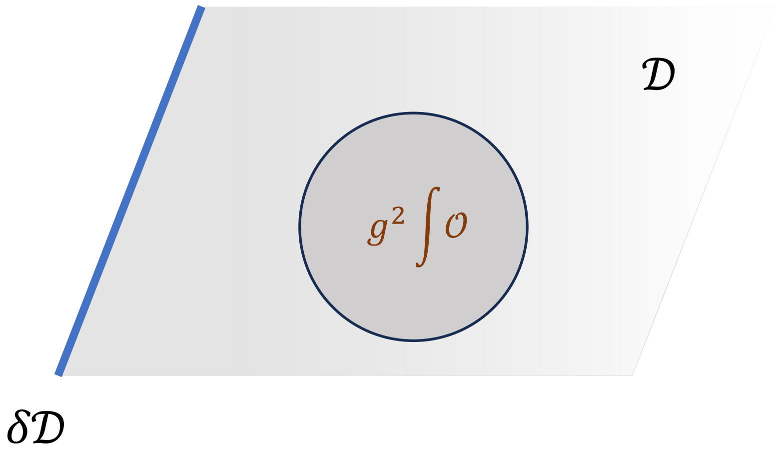


$$S = S_{\rm CFT} + g \int_{\mathcal{D}} \mathcal{O} + h \int_{\delta\mathcal{D}} \tilde{\mathcal{O}}$$

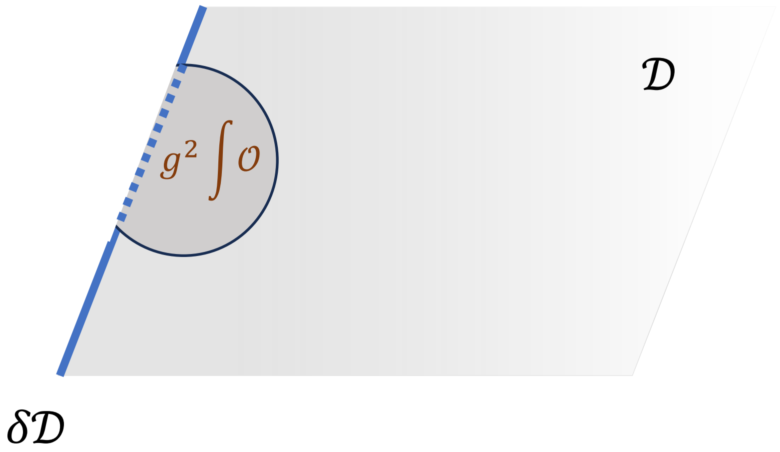
(slightly relevant)

$$\mu \frac{\partial g}{\partial \mu} = -\epsilon g + a g^2$$

OPE: $\mathcal{O}(x)\,\mathcal{O}(y) \sim \frac{\mathcal{O}(x)}{|x-y|^{\Delta}} + \dots$

RG on the Intersection

- **Semi infinite plane -** Simplest object to begin with
- From the $\mathcal{O} \times \mathcal{O} : \tilde{\mathcal{O}}$ OPE, we will have an additional boundary contribution for the edge coupling



$$S = S_{\mathrm{CFT}} + g \int_{\mathcal{D}} \mathcal{O} + h \int_{\delta\mathcal{D}} \tilde{\mathcal{O}}$$

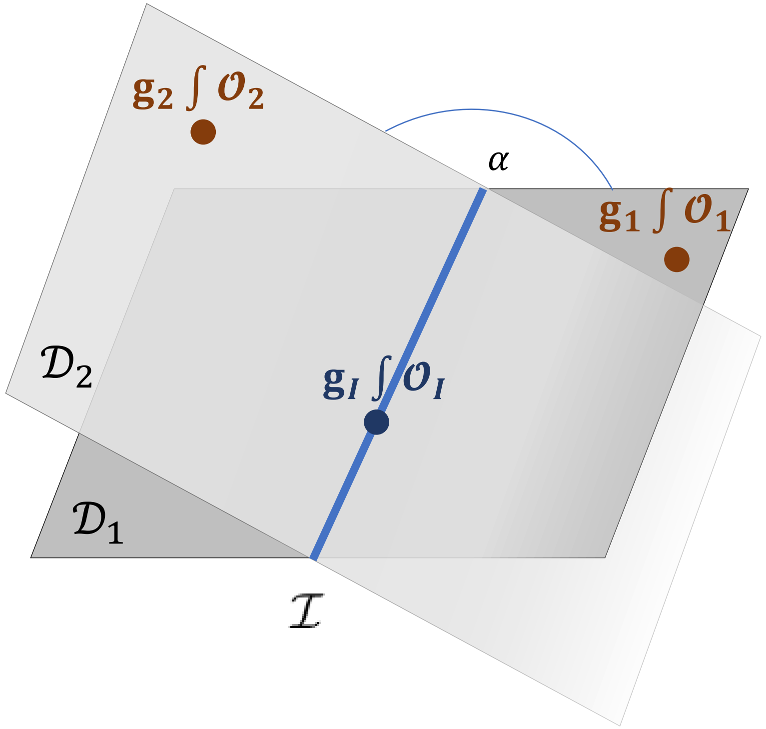
$$\mu \frac{\partial h}{\partial \mu} = -\epsilon h + a h^2 + b g h + \boldsymbol{c g^2}$$

OPE: $\mathcal{O}(x)\,\mathcal{O}(y) \sim \frac{\tilde{\mathcal{O}}(x)}{|x-y|^{\tilde{\Delta}}} + \dots$

RG on the Intersection

- **Intersecting planes**

$$S = S_{\mathrm{CFT}} + g_1 \int_{\mathcal{D}_1} \mathcal{O}_1 + g_2 \int_{\mathcal{D}_2} \mathcal{O}_2 + h \int_{\mathcal{I}} \mathcal{O}_{\mathcal{I}}$$

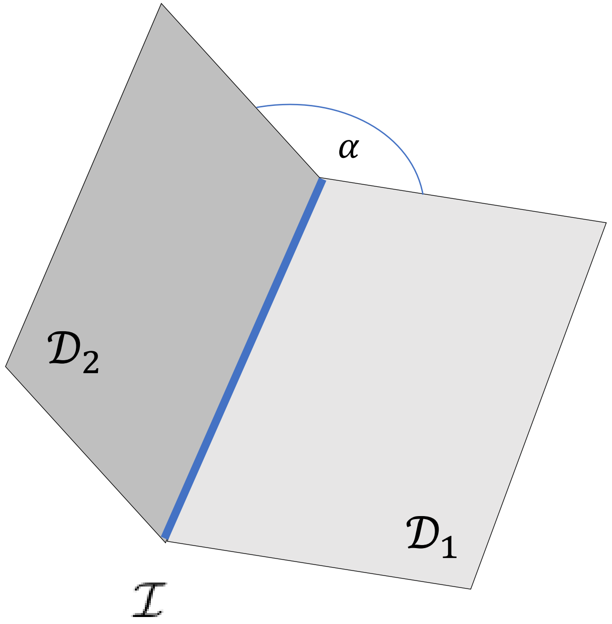


$$\mu \frac{\partial h}{\partial \mu} = -\epsilon h + a h^2 + b_1 g_1 h + b_2 g_2 h + \frac{\boldsymbol{c}}{\boldsymbol{\sin\alpha}} \boldsymbol{g_1}\, \boldsymbol{g_2}$$

RG on the Intersection

- **The wedge -** A combination of the two
  - Intersection is also a common edge for the two defects

$$S = S_{\mathrm{CFT}} + g_1 \int_{\mathcal{D}_1} \mathcal{O}_1 + g_2 \int_{\mathcal{D}_2} \mathcal{O}_2 + h \int_{\mathcal{I}} \mathcal{O}_{\mathcal{I}}$$



$$\mu \frac{\partial h}{\partial \mu} = -\epsilon h + a h^2 + b_1 g_1 h + b_2 g_2 h + \boldsymbol{c}\left(\frac{\boldsymbol{\pi - \alpha}}{\boldsymbol{\sin \alpha}}\, \boldsymbol{g_1\, g_2} - \frac{\boldsymbol{g_1^2 + g_2^2}}{\boldsymbol{2}}\right)$$

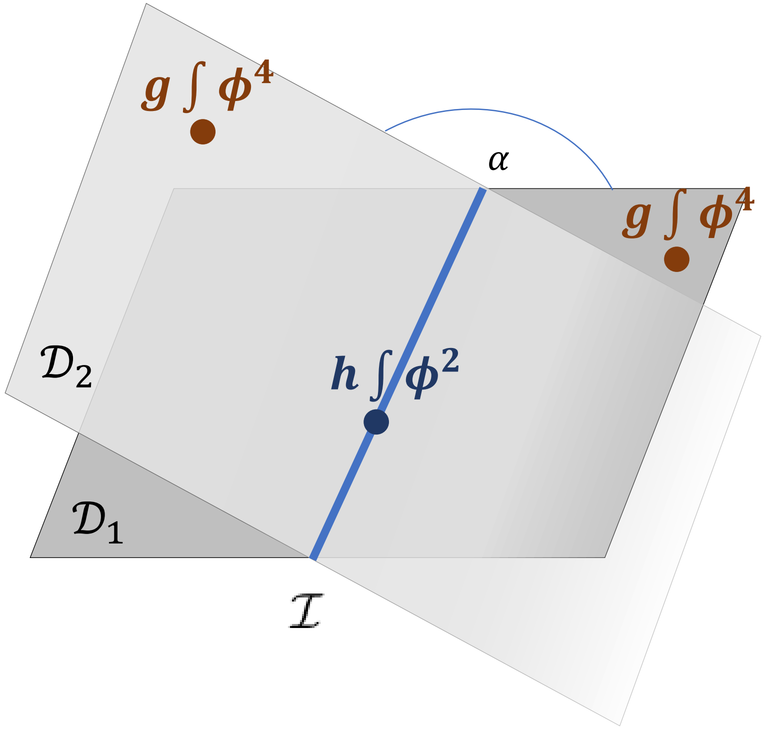
- $\alpha = \pi$
  - $g_1 \neq g_2$ - interface
  - $g_1 = g_2$ - composite defect *[Shimamori, Ge, Nishioka 24]*

RG on the Intersection

- **Example:** Tricritical bulk in $d = 3 - \epsilon$

$$S_{\text{bulk}} = \int d^{3-\epsilon}x \left(\frac{1}{2} (\partial\phi)^2 + \lambda\phi^6 \right)$$

$p_{\mathcal{D}} = 2 - \epsilon$ or 2.
$p_{\mathcal{I}} = 1 - \epsilon$ or 1.



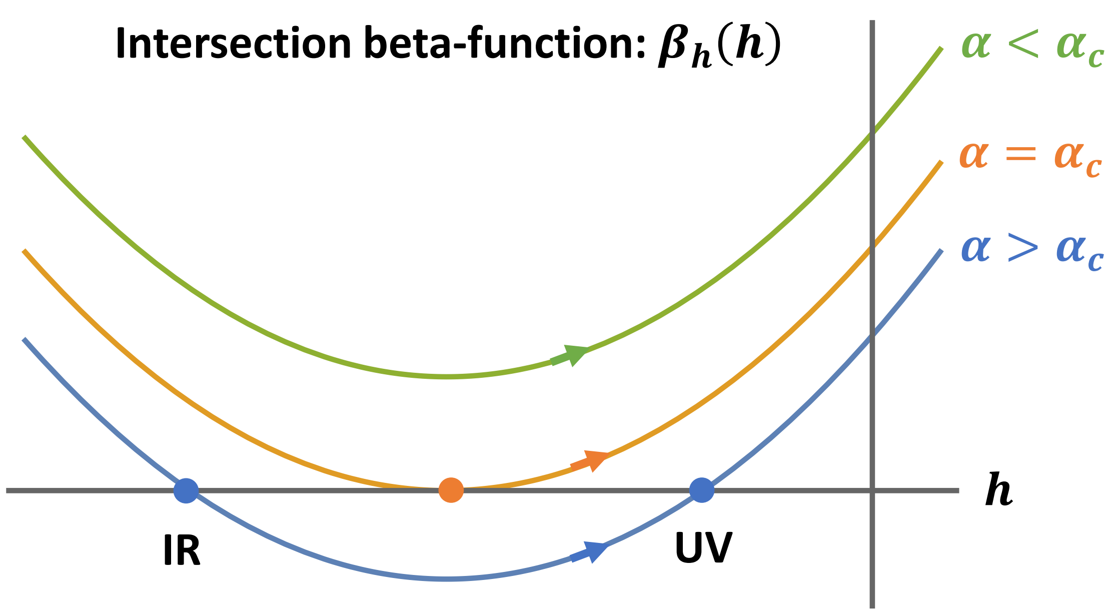
- Non-interacting f.p. ($\lambda = 0$)
  - $\Delta_{\phi^4} = 2 - 2\epsilon, \qquad \Delta_{\phi^2} = 1 - \epsilon$

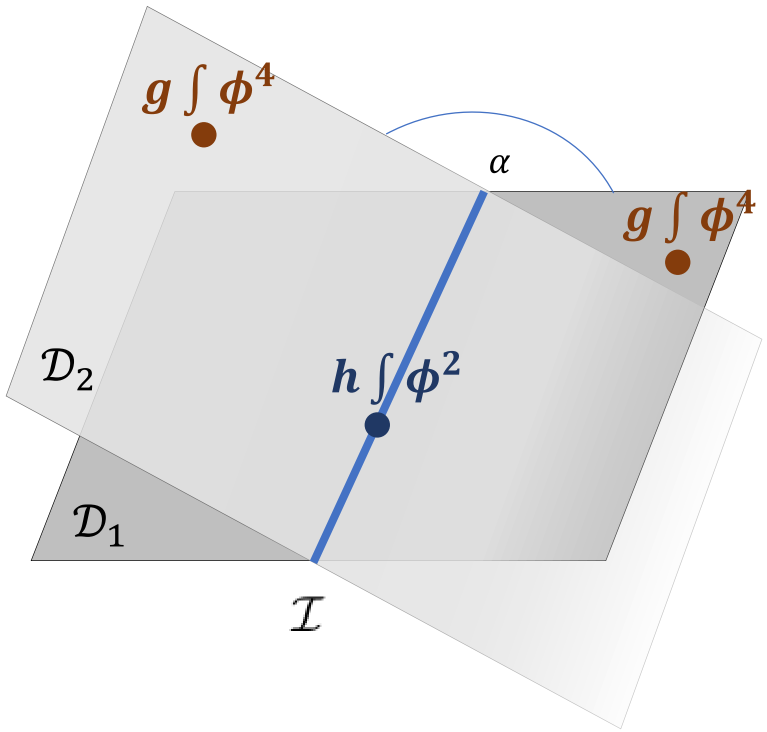
- Interacting f.p. ($\lambda^* = \frac{2\pi^2\epsilon}{3(3N+22)}$) *[Pisarski 82]*
  - $\gamma_{\phi^4} = -\frac{4(N+4)}{3N+22}\epsilon, \qquad \gamma_{\phi^2} = -\frac{5(N+2)(N+4)}{4(3N+22)^2}\epsilon^2$

RG on the Intersection

- **Example:** Tricritical bulk in $d = 3 - \epsilon$
- Look for fixed-points when bulk+defects are critical

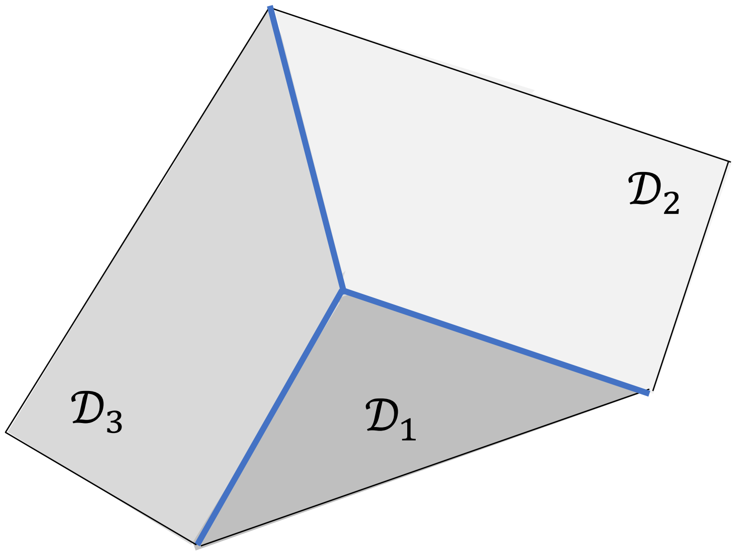
$$S_{\text{bulk}} = \int d^{3-\epsilon}x \left(\frac{1}{2}(\partial\phi)^2 + \lambda\phi^6\right)$$





Corners

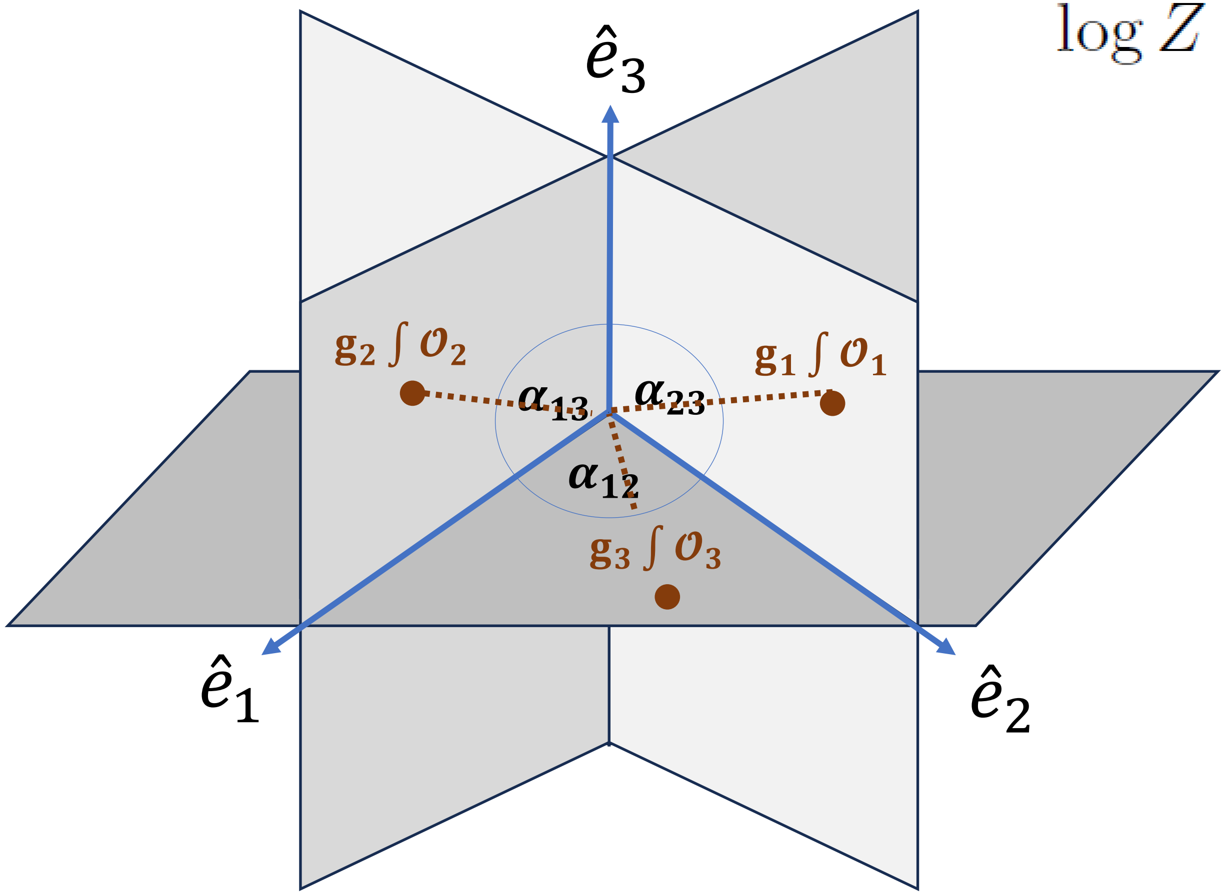
- We can consider mutual intersection of more than two defects
- For 3 defect for example, $\beta \subset g_1 g_2 g_3$ (cubic term)
  - Falls beyond the scope of conformal perturbation theory
- Remarkably, there are some critical 3-defect quantities that we can still compute – *corner anomalous dimension*.



$$\log Z = -\Gamma_{\text{corner}} \log\left(\frac{L}{a}\right) + \dots$$

Corners

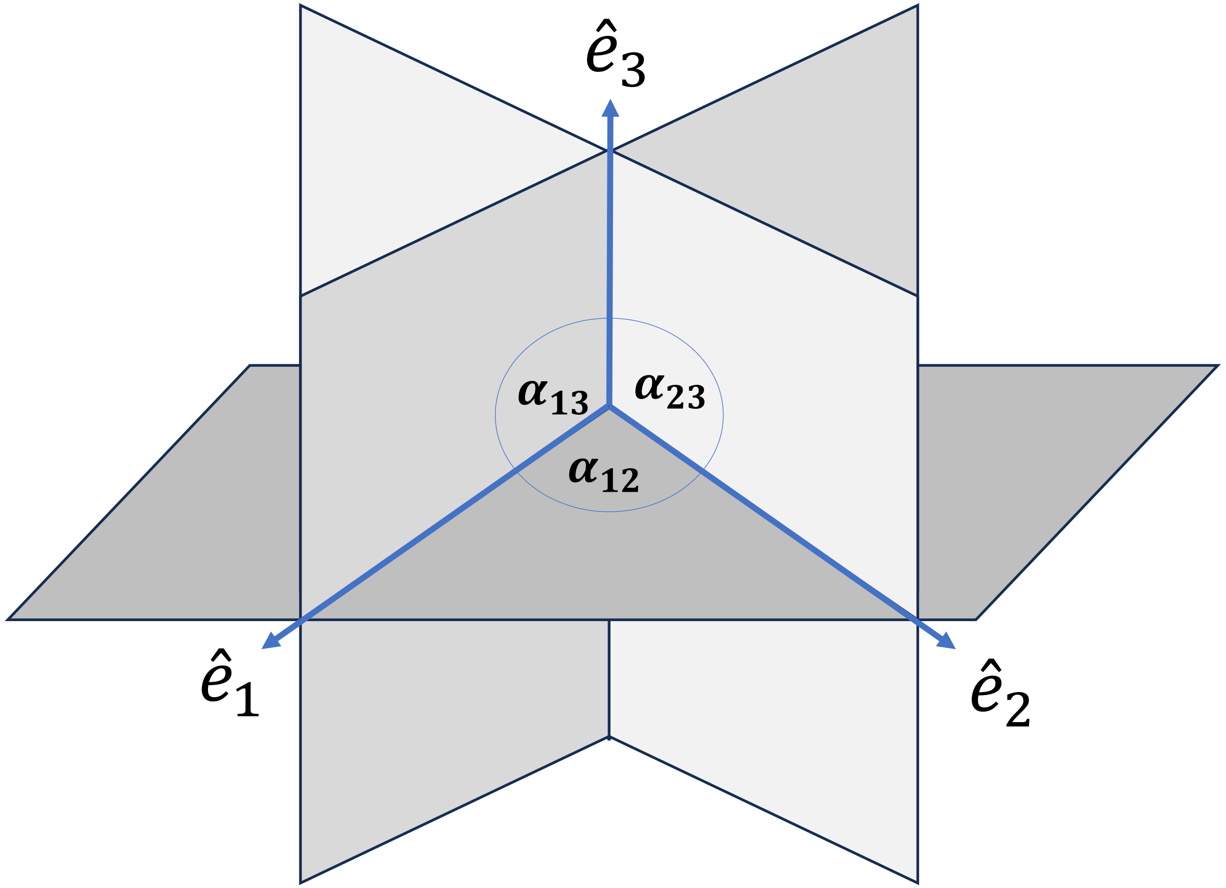
- Extended Trihedral Corners (Parametrized by $\hat{e}_a \cdot \hat{e}_b = \alpha_{ab}$)



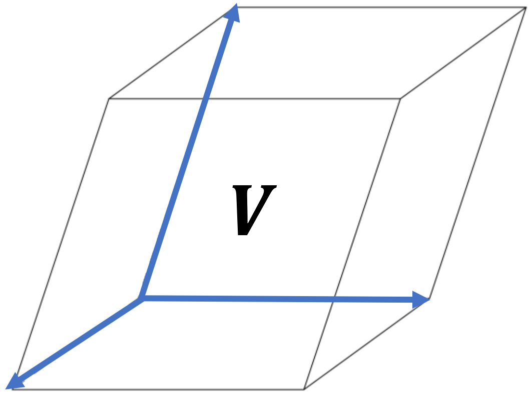
$$\log Z = -\Gamma_{\text{corner}} \log\left(\frac{L}{a}\right) + \ldots$$
$$= -g_1\, g_2\, g_3 \int_{\mathcal{D}_1} \int_{\mathcal{D}_2} \int_{\mathcal{D}_3} \langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \rangle + \ldots$$

Corners

- Extended Trihedral Corners

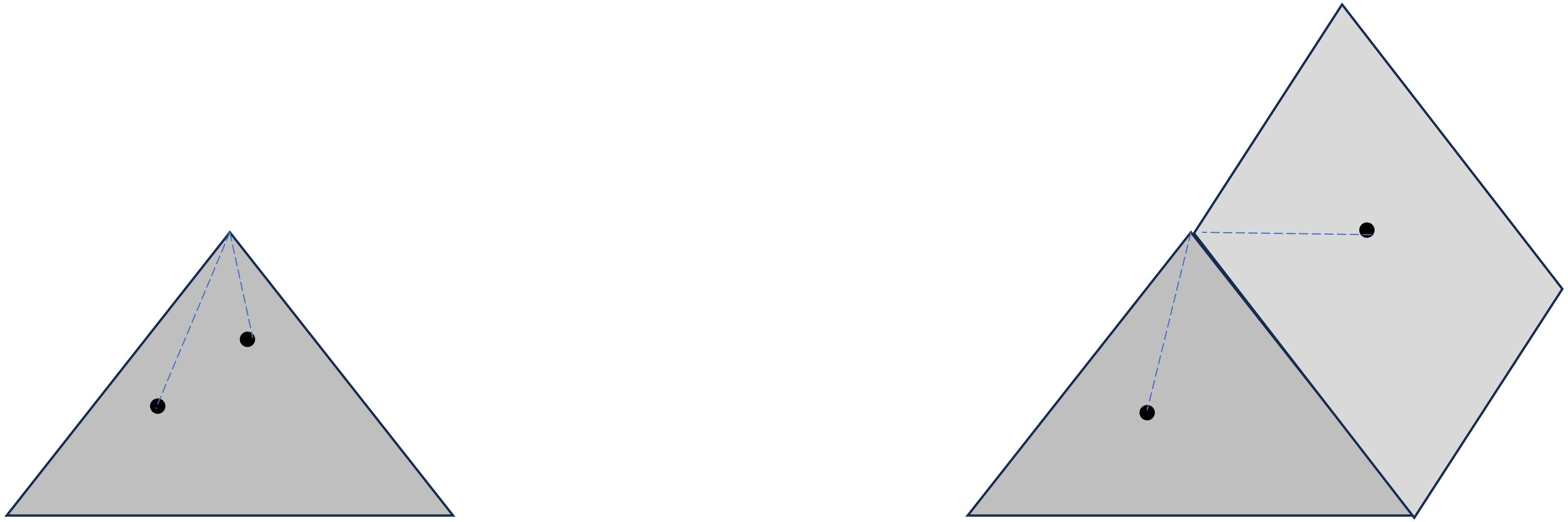


$$\Gamma_{\text{corner}} \propto \frac{\sin(\alpha_{12})\sin(\alpha_{23})\sin(\alpha_{13})}{V^2(\alpha_{12},\alpha_{23},\alpha_{13})} g_1\, g_2\, g_3$$



(unit parallelepiped)

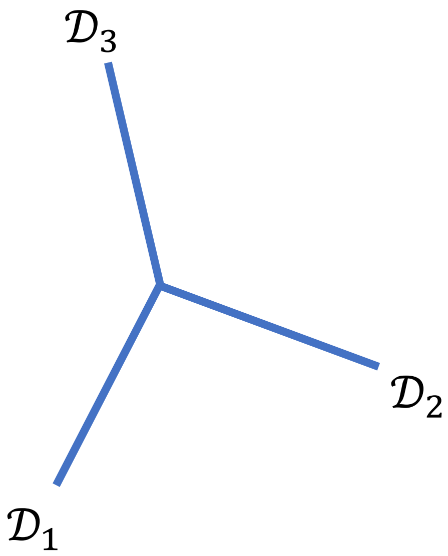
- Non-extended corner is more involved.
- There are more “primitive” structures that appear beforehand, at the level of the *2pt-function*



Corners

- 3-line corners

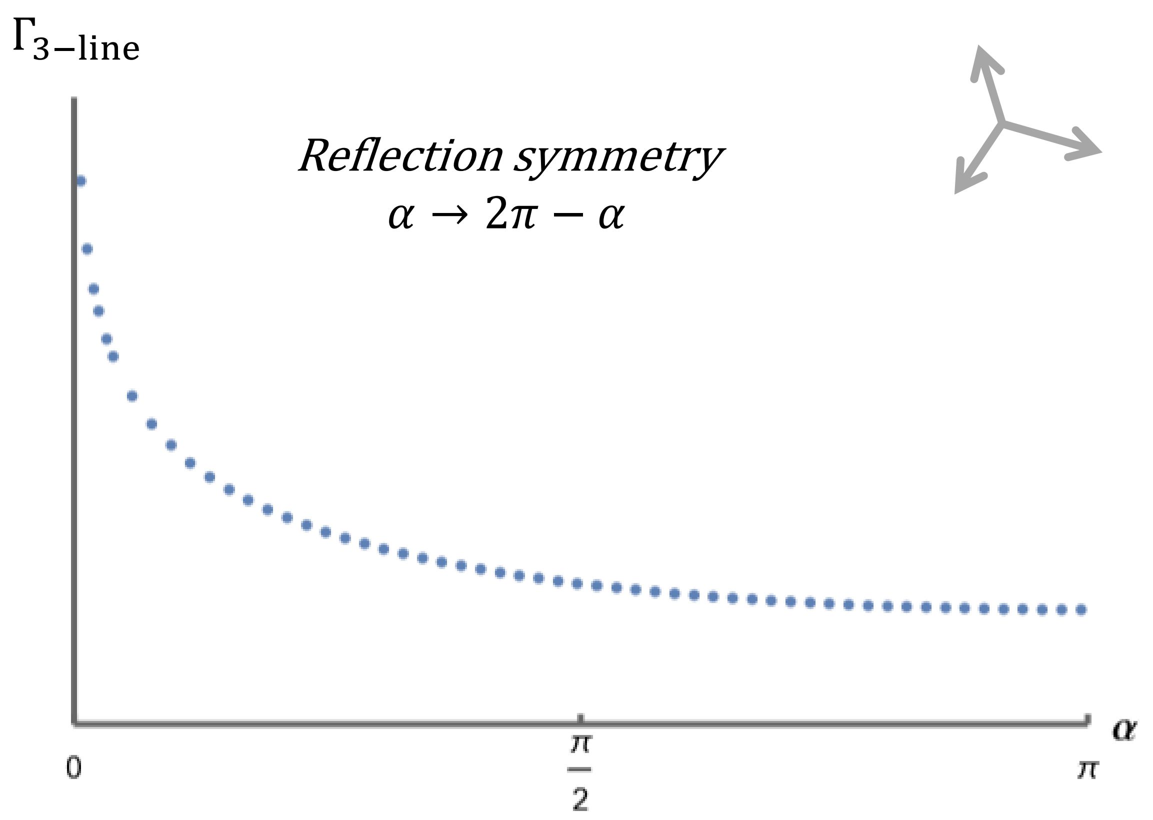
$$\Gamma = \underbrace{\sum_{a<b}^{3} \Gamma_{\text{cusp}}(\alpha_{ab})}_{\substack{\sim g^2 \\ \text{2-body potential}}} + \underbrace{\Gamma_{\text{3-line}}(\alpha_{12}, \alpha_{23}, \alpha_{13})}_{\substack{\sim g^3 \text{ (subleading)} \\ \text{3-body potential}}}$$

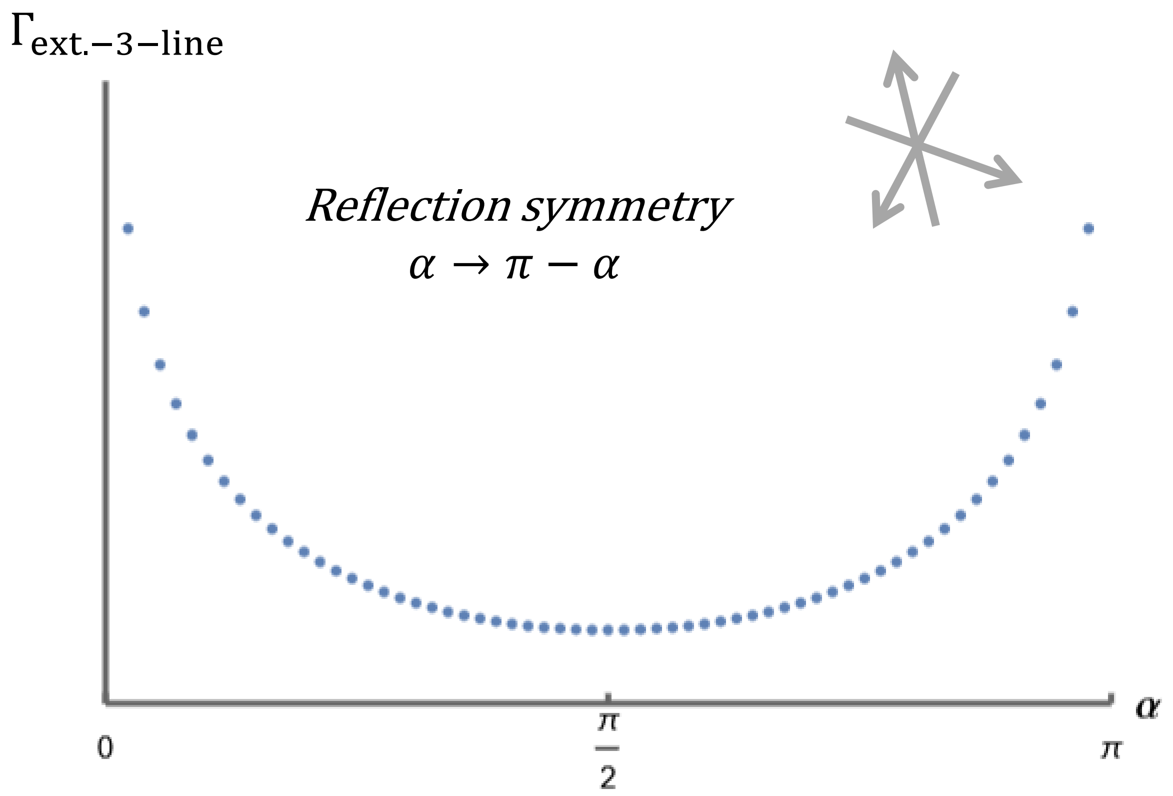


- Conformal mapping to $\mathbb{R} \times S^{d-1}$ - worldlines of point-like impurities *[Henkel, Patkos, Schlottmann 89]*
- $\Gamma_{3-\text{line}}$ can be computed analytically

Corners

- For simplification, let's consider two angles equal to $\frac{\pi}{2}$





Discussion

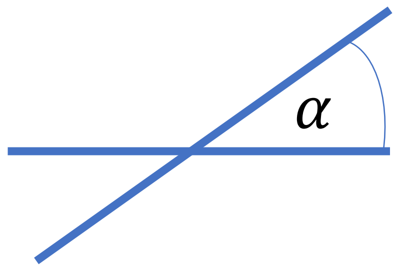
- **Summary**
  - When defects meet, new degrees of freedom arises
  - The RG flow is also governed by *how* the defects intersect
  - Conformal perturbation theory allowed us to study 2 and 3 defect configurations

- **Looking Forward**
  - DCFT + localized bulk deformation
  - Non-zero spin ($\partial_n \phi$)
  - Curved intersections ($y_1 = x^2$ & $y_2 = -x^2$)
  - Bootstrapping

- **Observation**

- Known example: cusp anomalous dimension *[Komargodski, He, Cuomo 24]*

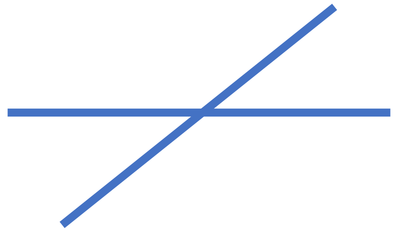
$$\left\langle e^{-g_1 \int_{\mathcal{D}_1} dx\, \mathcal{O}_1 - g_2 \int_{\mathcal{D}_2} dx\, \mathcal{O}_2} \right\rangle_{\mathrm{CFT}} = -\Gamma\left(\alpha\right) \log\left(\tfrac{L}{a}\right) + \ldots$$



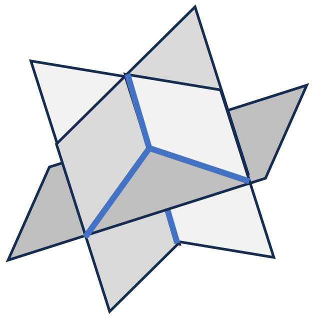
$$\Gamma\left(\alpha\right) = -\frac{2\pi}{\sin\alpha} g_1 g_2 + \ldots$$

- Observation: Inverse volume of parallelotopes

$$\Gamma(\alpha) = -\frac{2\pi}{\sin\alpha} g_1 g_2 + \ldots$$



$$\Gamma_{\text{ext.-trihedral}}(\alpha_{12}, \alpha_{23}, \alpha_{13}) = \frac{\sin(\alpha_{12})\sin(\alpha_{23})\sin(\alpha_{13})}{V^2(\alpha_{12}, \alpha_{23}, \alpha_{13})} 4\pi^4 C^{ijk} g_i^{(1)} g_j^{(2)} g_k^{(3)}$$



Thank You!