No-go theorems for higher-spin charges in AdS_2

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w/ A. Antunes & N. Levine

Non-perturbative methods in QFT 12/03/2025 Kyushu

One sentence summary:

It is impossible to non-trivially* deform

a) A free field in AdS

b) CFTs in AdS*

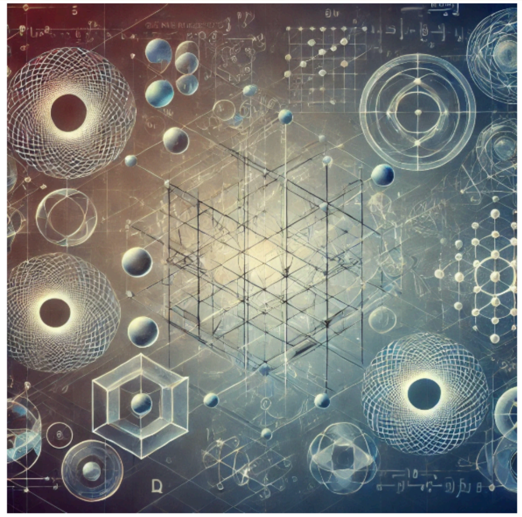
while preserving higher-spin charges

* trivial deformation = change the mass

Outline:

1. Motivation and basics
2. Higher-spin currents and charges in AdS
3. Example: free massive boson
4. No-go theorems
5. Outlook

1. Motivation and basics



" You can talk about whatever you like "

Yuya Kusuki

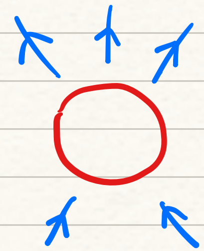
- Integrability is one of the most powerful Non-perturbative methods in QFT
  it allows to solve interacting QFTs by leveraging Symmetry,

- Anti de-Sitter space is a great playground: (Shota Komatsu's talk)
  - × massive QFT in $AdS_d \rightarrow$ conformal theory in $\mathbb{R}^{d-1}$
    (critical points of long range models)
  - × most symmetric box: solving QFTs using a method called the *conformal bootstrap*. ?

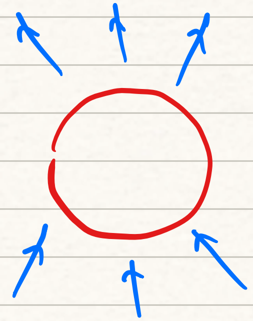
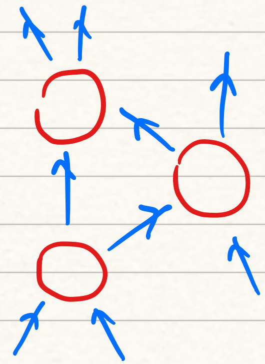
[Soichiro Shimamori's talk]

- Integrability in 2d flat space

  - No particle production

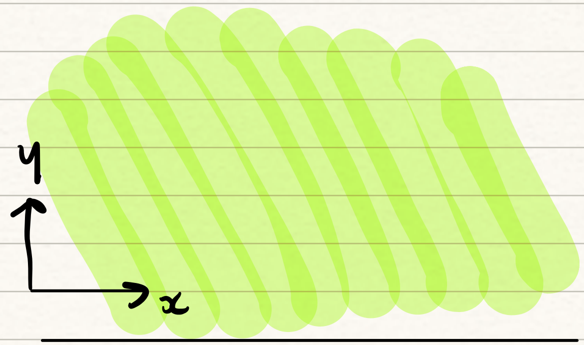
 $= 0$

  - Factorized scattering (Yang-Baxter)

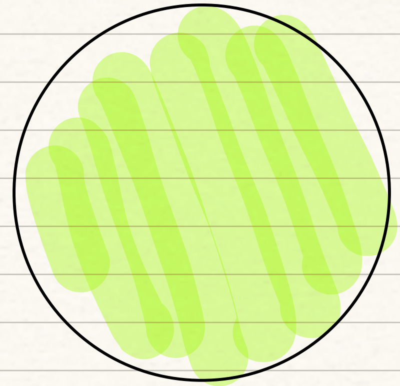
 $=$ 

- Wealth of exact results: S-matrix, (finite size) spectra, form factors
- Examples:
  - Sine-Gordon: $S = \int (\partial\phi)^2 + \cos(\beta\phi)$
  - Deformed Minimal Models $S = S_{CFT} + \int \phi_{(1,3)}$ [Yu Nakayama's talk]

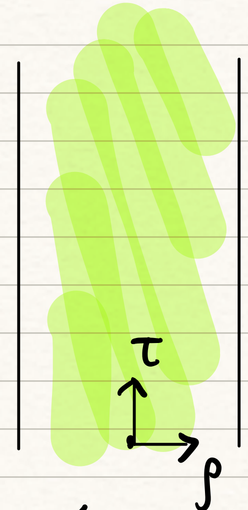
- (Euclidean) Anti de-Sitter space



$$ds^2 = \frac{1}{y^2}\left(dx^2 + dy^2\right)$$



$$ds^2 = \frac{4}{(1-r^2)^2}\left(dr^2 + r^2 d\theta^2\right)$$



$$ds^2 = \frac{1}{(\cos\rho)^2}\left(d\tau^2 + d\rho^2\right)$$

- Maximally symmetric space : can rotate around any point
  can translate in any direction

- Killing vectors $\left(\xi^A \quad A = 0, 1, 2\right)$ form $SL(2, \mathbb{R}) \simeq SO(2,1)$

translations: $\left[p = \partial_x\right]$ dilatations: $\left[d = x\partial_x + y\partial_y\right]$ sp. conformal: $\left[K = (x^2 - y^2)\partial_x + 2xy\partial_y\right]$

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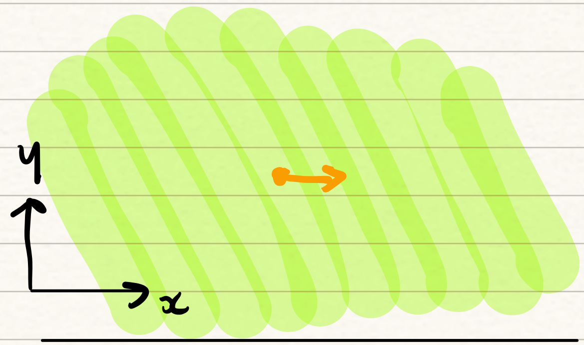

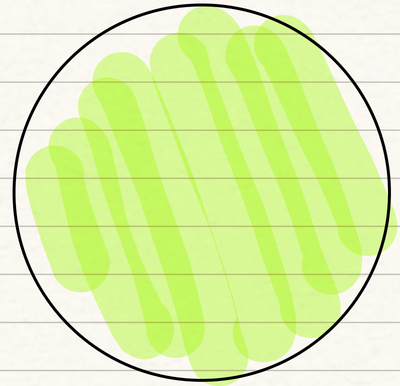

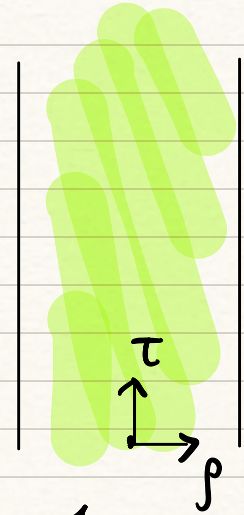

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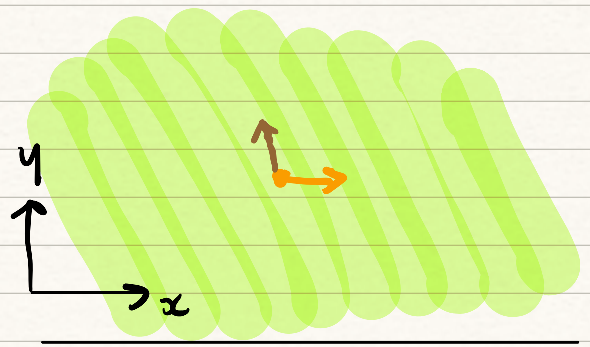

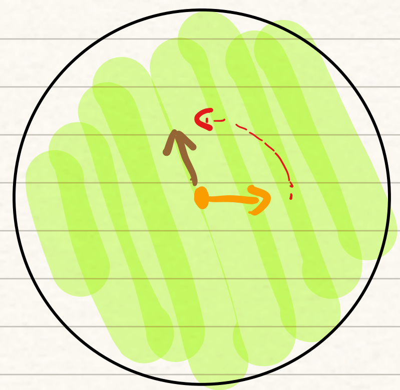

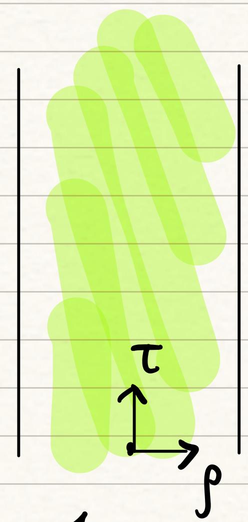

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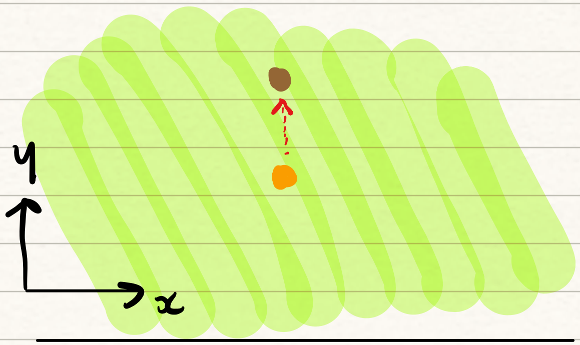

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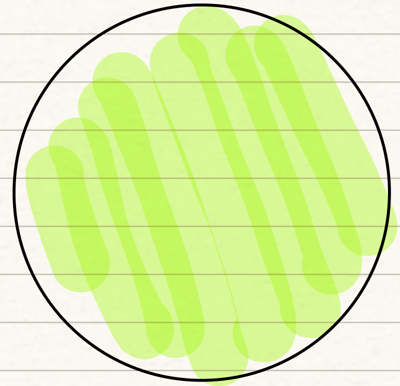

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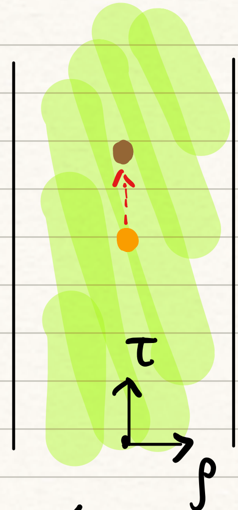

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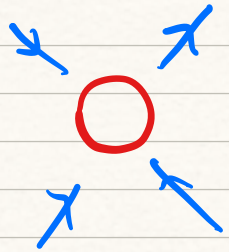
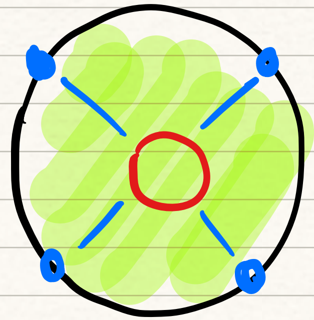
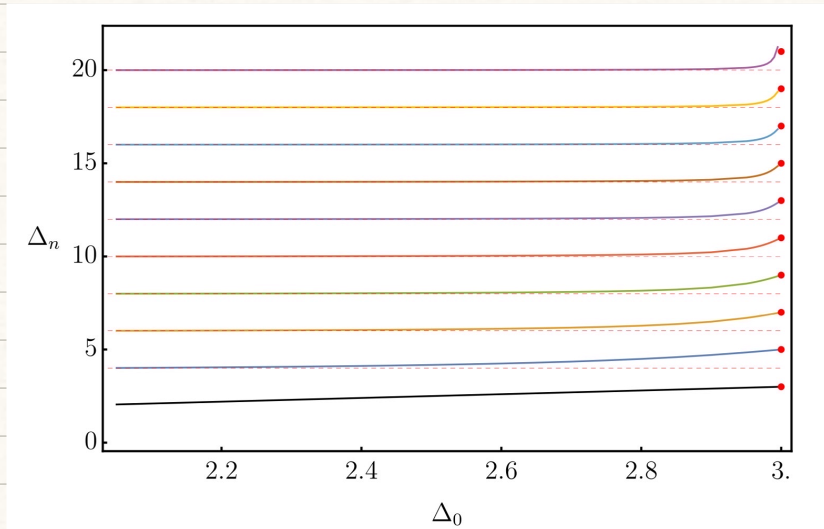
Integrability in AdS_2 ?

- S-matrix elements $\rightsquigarrow$ correlators of boundary ops.





" $C_{\phi\phi\phi^n} = 0 \quad n > 2$ " ?



[Paulos, Zan, 2020]

Integrability from higher-spin symmetry

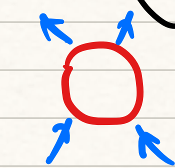
- Need ∞ many conserved charges

$\boxed{\text{higher-spin currents}}$ Non-local

Flat space: $\boxed{\partial_\mu T^{\mu}{}_{\underbrace{z\cdots z}_{s}} = 0} \Rightarrow Q_s$

$$Q_s |p\rangle = p^s |p\rangle \Rightarrow \sum_i p_i^s = 0 \Rightarrow \text{factorized scattering}$$

but non trivial



$\boxed{\text{What are the consequences of HS charges in AdS?}}$

2. Higher-spin currents and charges

charge

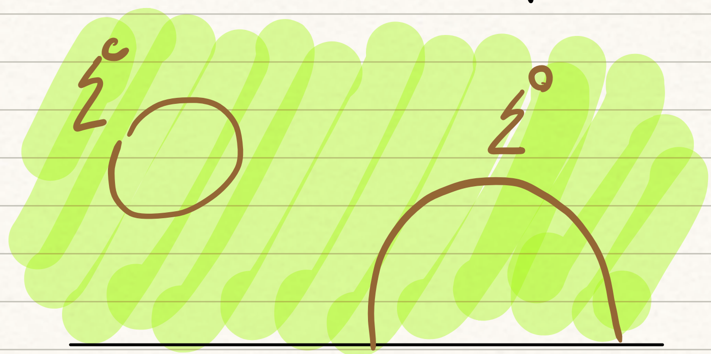
$$Q_\zeta^{(j)}(\Sigma) = \int_\Sigma d\Sigma^\mu \, \zeta^{\mu_2 \cdots \mu_j} \, T_{\mu \mu_2 \cdots \mu_j}$$

rank-j tensor operator

Killing tensor

$$\nabla_{(\mu} \zeta_{\nu_1 \cdots \nu_{j-1})} = 0$$

codimension-1 surface:

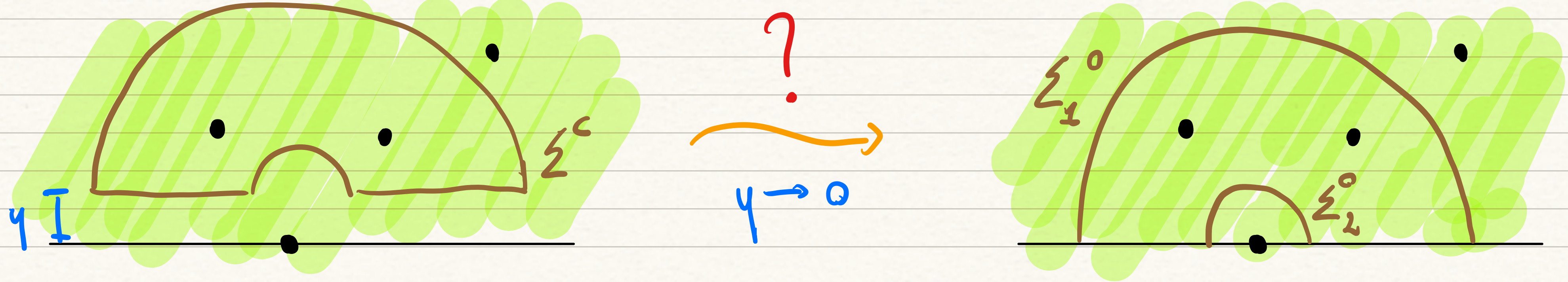


Minimal requirement:

* $\nabla^\mu \left(\zeta^{\mu_2 \cdots \mu_j} T_{\mu \mu_2 \cdots \mu_j} \right) = 0$

* guarantees $Q_\zeta(\Sigma^c)$ is topological, but what about $Q_\zeta(\Sigma^o)$?

[Ingo Runkel's talk]



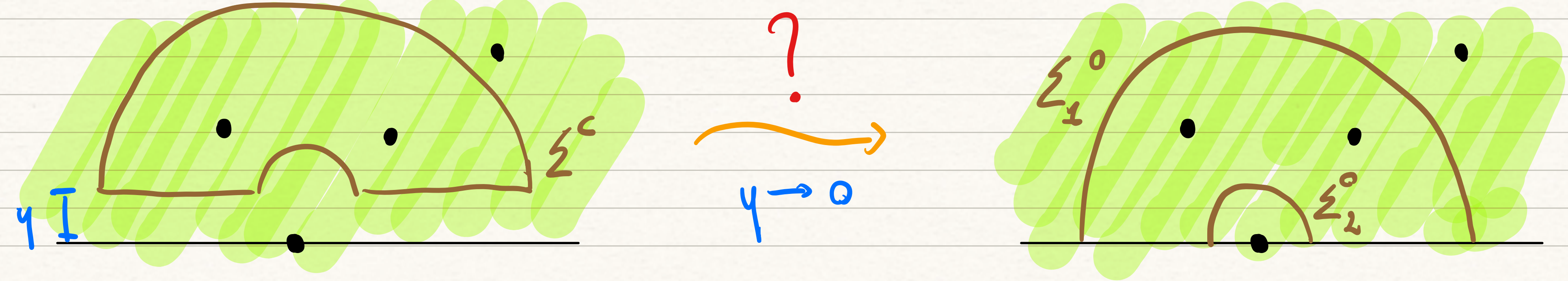
✓ Then, $Q_\xi(\Sigma_i^0)$ is topological. $\Leftarrow$ $\left[\xi^{\mu_2 \cdots \mu_J} T_{y \mu_2 \cdots \mu_J} \xrightarrow[y \to 0]{} 0 \right]$

- Condition on Boundary OPE of the current :

× $T_{\mu \cdots \mu_J} \sim \sum_\Delta \mathcal{O}_\Delta^\pm$ ← parity ← scaling dimensions

$$\Delta_+ > J-1 \qquad \Delta_- > J \qquad (J \text{ even})$$

$$\Delta_+ > J \qquad \Delta_- > J-1 \qquad (J \text{ odd})$$



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generically violated!

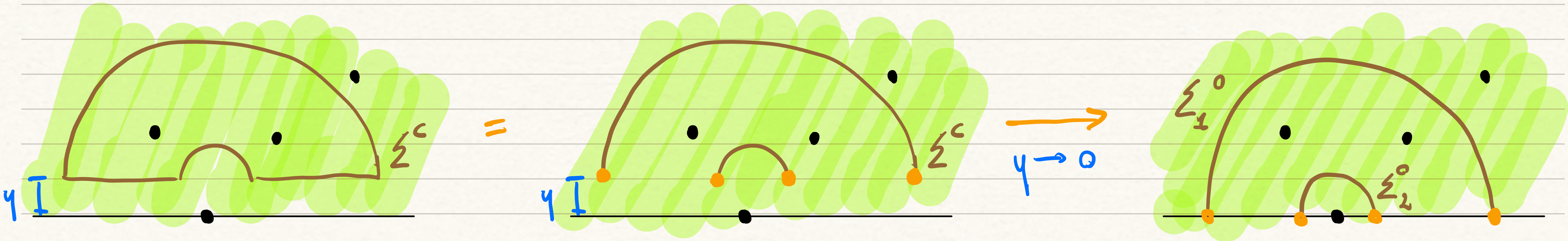
- Condition is sufficient but not necessary:

j even: $$\left[\zeta^{\mu_2\ldots\mu_j}\, T_{j\,\mu_2\ldots\mu_j}\Big|_{\mathcal{O}^+_\Delta \text{ family}}(x,y) = \partial_x\left(\mathcal{F}_\Delta(\zeta, y, \partial_x)\,\mathcal{O}^+_\Delta(x)\right)\right]$$

total derivative for ANY ζ!

(j odd: same for $\mathcal{O}^-_\Delta$)

$\Rightarrow$ if $\exists\ \Delta_+ < j-1$, they contribute a boundary term



• = counterterms which keep charge finite and conserved as $y \to 0$

[Klebanov, Witten 1999]

✓ This generalizes an argument by Klebanov & Witten to any QFT and any spin.

- The other condition is instead necessary and sufficient:

$$\Delta_- > j \quad (j \text{ even}) \qquad\qquad \Delta_+ > j \quad (j \text{ odd})$$

- In fact, conservation implies

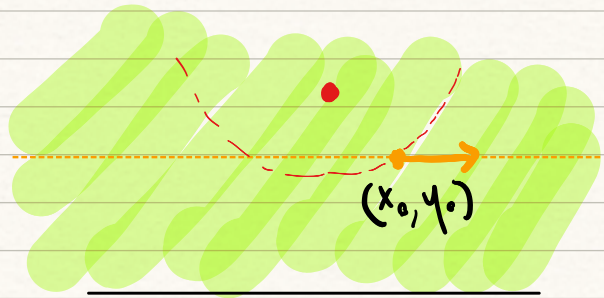
$$\Delta_- = j \quad (j \text{ even}) \qquad\qquad \Delta_+ = j \quad (j \text{ odd})$$

Hence:

$$\boxed{\begin{array}{l}\text{A conserved (higher-spin) current } T_{\mu_1\cdots\mu_j} \text{ generates}\\ \text{conserved charges acting on boundary operators}\\ \text{iff} \qquad T_{\mu_1\cdots\mu_j} \underset{\text{boundary}}{\sim} \cancel{C^-_\Delta} \quad (j \text{ even})\\ \qquad\qquad\qquad\qquad\qquad\;\; \cancel{C^+_\Delta} \quad (j \text{ odd})\end{array}}$$

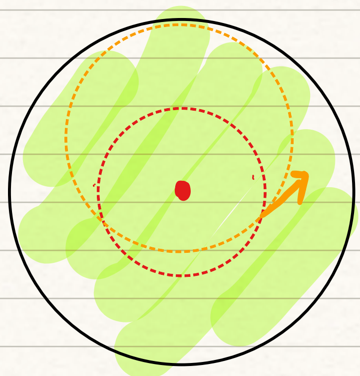
- (Generalized) Cardy condition ~ boundary conditions preserve HS symmetry

How many components must we conserve?

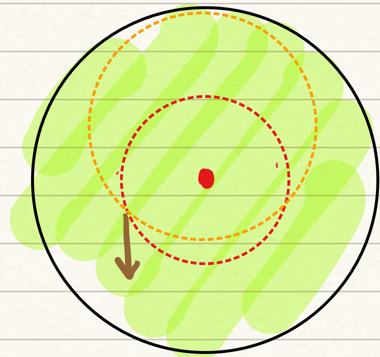


$$\boxed{\nabla_\mu T^{\mu x}(x_0, y_0) = 0}$$

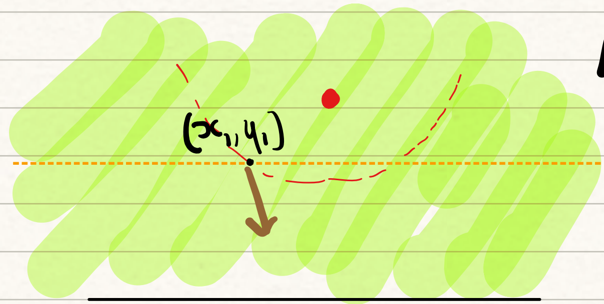
Map to the disk



and rotate

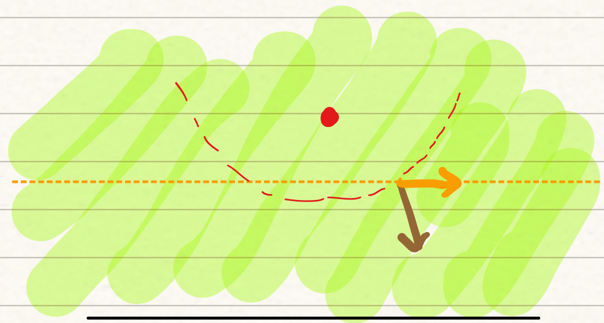


Map back



$$\xi_\nu \nabla_\mu T^{\mu\nu}(x_1, y_1) = 0$$

Translate



$$\boxed{\xi_\nu \nabla_\mu T^{\mu\nu}(x_0, y_0) = 0}$$

How many components must we conserve?

In general:
$$\left[\nabla^{\mu_1}\left(\zeta^{\mu_2\cdots\mu_J} T_{\mu_1\cdots\mu_J}\right) = 0 \implies \nabla^{\mu} T_{\mu\mu_2\cdots\mu_J} = 0\right]$$

for 1 fixed ζ

Mechanism:
$$\zeta^{\mu_2\cdots\mu_J}\nabla^{\mu_1} T_{\mu_1\cdots\mu_J} = 0 \xrightarrow[\text{under isometries}]{\text{invariance}} (\mathcal{L}_\xi \zeta)^{\mu_2\cdots\mu_J}\nabla^{\mu_1} T_{\mu_1\cdots\mu_J} = 0$$

Lie derivative
ξ = Killing vector

Space of AdS_2 Killing tensors:

- $\zeta_{\mu_1\cdots\mu_J} = \alpha_{A_1\cdots A_J}\,\xi^{A_1}\cdots\xi^{A_J}$ homogeneous poly. of ξ^A [Thompson 1986]
- Form spin J IRREPS of $SL(2,\mathbb{R})$ under isometries $\mathcal{L}_\xi \zeta$ (+ traces)

$$p^J \underset{K}{\overset{P}{\rightleftarrows}} p^{J-1}d \underset{K}{\overset{P}{\rightleftarrows}} \cdots \underset{K}{\overset{P}{\rightleftarrows}} K^J$$

How many components must we conserve?

$$(\mathcal{L}_\xi \zeta)^{\mu_2 \cdots \mu_j} \nabla^{\mu_1} T_{\mu_1 \cdots \mu_j} = 0 \quad \rightsquigarrow \quad \zeta_w^{\mu_2 \cdots \mu_j} \nabla^{\mu_1} T_{\mu_1 \cdots \mu_j} = 0$$

$w = -(j-1) \ldots j-1$ full $sl(2,\mathbb{R})$ irrep

- Spin $j-1$ Killing tensors form (overcomplete) basis of rank-$(j-1)$ symmetric tensors

$$\left[\nabla^{\mu_1}\left(\zeta^{\mu_2 \cdots \mu_j} T_{\mu_1 \cdots \mu_j}\right) = 0 \quad \Longrightarrow \quad \nabla^{\mu} T_{\mu \mu_2 \cdots \mu_j} = 0 \right]$$

for a fixed ζ

Consequences:

- If Cardy conditions are satisfied, we get full multiplet of charges
- If $\mathrm{Traces}\,(T^{\mu_1 \cdots \mu_j}) \neq 0$ we also get all lower spin charges

3. Example: free massive boson

$$S = \int_{AdS_2} \frac{1}{2}(\partial\Phi)^2 + m^2\Phi^2$$

dual to GFF for φ:

$$m^2 = \Delta_\varphi(\Delta_\varphi - 1)$$

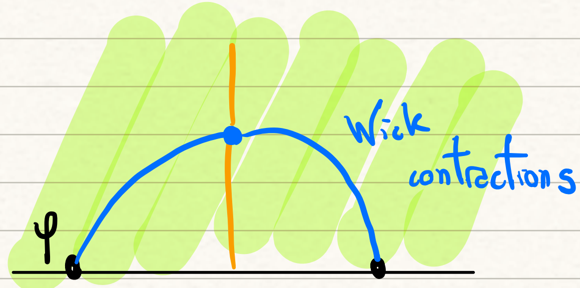
- Higher-spin currents for all even J, e.g.

$$T_{\mu\nu\rho\sigma} = \Phi\overleftrightarrow{\nabla}_{(\mu}\overleftrightarrow{\nabla}_{\nu}\overleftrightarrow{\nabla}_{\rho}\overleftrightarrow{\nabla}_{\sigma)}\Phi + 14\, g_{(\mu\nu}\Phi\overleftrightarrow{\nabla}_{\rho}\overleftrightarrow{\nabla}_{\sigma)}\Phi + 9\, g_{(\mu\nu}g_{\rho\sigma)}\Phi^2$$

[Bekaert, Meunier, 2010]

- We can build spin 3 charges $Q^{(3)}_{-3}, Q^{(3)}_{-2}, \dots Q^{(3)}_{3}$

$$Q^{(3)}_{-3} = \int_0^\infty dy\, T_{xxxx}$$



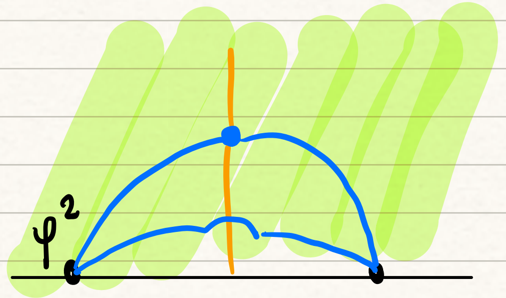
$$\boxed{[Q^{(3)}_{-3}, \varphi] = 16\,\partial_x^3\varphi}$$

- Does $Q^{(3)}_{-3}$ just produce conformal descendants? No!

$$\left[Q^{(3)}_{-3}, \varphi^2\right] = \# \, \varphi \, \partial_x^3 \varphi = \alpha \, \partial_x^3 \varphi^2 + \beta \, \partial_x (\varphi \partial_x^2 \varphi + \ldots)$$

acts constituent-wise

New primary!



- All states @ fixed particle number live in a single higher spin multiplet
- Higher-spin symmetry implies integer spaced primary spectrum:

$$\Delta = n \Delta_\varphi + m$$

- Constituent-wise action reminiscent of action on multi-particle states in flat space:

$$Q_J |p_1, \ldots p_n\rangle = \sum_{i=1}^{n} (p_i)^{J-1} |p_1, \ldots p_n\rangle$$

used to prove factorization

[Alkalaev, Bekaert, 2020]

easy to prove using constituent-wise action

[Zamolodchikov^2, 1978]

[Dorey, 1996]

Structure of the algebra and more on free rep

- Commuting w/ isometries produces the multiplet:

$$[K, Q^{(3)}_{-3}] \propto Q^{(3)}_{-2} \quad \text{etc.}$$

follows from Heisenberg equation, or integrated version of an argument above.

$(J, w) \rightsquigarrow$ unique $\zeta^{\mu_1 \cdots \mu_J} = \alpha_{A_1 \cdots A_J} \xi^{A_1} \cdots \xi^{A_J}$

$$Q^{(3)}_{-2}\, \varphi(0)|0\rangle \propto (P^2 D + PDP + DP^2)\, \varphi(0)|0\rangle$$

universal enveloping algebra*

- Commutators can exceed available weights at spin J

$\Rightarrow$ generate new IRREPS:

$SL(2,\mathbb{R})$ spin, follows from Jacobi id.

$$[Q^{(3)}_{-2}, Q^{(3)}_{-3}] = Q^{(5)}_{-5}$$

weight is simply additive

Consistent to set $Q^{(5)}_{w} = 0$, but in GFF they exist $\longleftrightarrow$ rank-6 current, etc.

$$hs(\lambda)$$

[Vasiliev, 1991]
[Alkalaev, Bekaert, 2020]

4. No-go theorems

Theorem: [Continuous interacting deformations of a free field cannot preserve any higher-spin charge]

- We focus on a free scaler, but free fermion works as well
- Similar Result for CFTs
- Theorem constrains both bulk and boundary interactions
  $\rightsquigarrow$ relevant for long-range models

[Benedetti, Lauria, Mazac, Van Vliet, 2024]

Proof.

- Step 1 : One h.s. charge $\overset{\text{isometries}}{\rightsquigarrow}$ one full $SL(2,\mathbb{R})$ multiplet

- Step 2 : One multiplet $\overset{\text{commute}}{\rightsquigarrow}$ higher-spin multiplets

  One multiplet $\underset{\text{commute or trace}}{\rightsquigarrow}$ lower-spin multiplets

  This step uses the continuity assumption : $[Q, Q'] \neq 0$

- Step 3 : constraints of h.s. charges on correlators.

  Let's delve into this

- Step 3.1 : action of charges on boundary operator.

perturbing parameter (dimensionless) $\lambda = \lambda_0 R$

$$\mathcal{O}_0(\lambda) = \varphi + \mathcal{O}(\lambda)$$

$$\left[Q^{(3)}_{-3}, \mathcal{O}_0\right] = \partial_x^3 \mathcal{O}_0 + \omega(\lambda)\,\partial_x^2 \mathcal{O}_1 + \alpha(\lambda)\,\partial_x \mathcal{O}_2 + \gamma(\lambda)\,\mathcal{O}_3$$

but $\Delta_n = \Delta_0 + n$ primaries do not exist as $\lambda \to 0$, @ generic Δ_φ

By continuity:

$$\boxed{\left[Q^{(J)}_{-J}, \mathcal{O}_0\right] = \partial_x^J \mathcal{O}_0} \qquad \lambda \in \text{h.s. symmetric range}$$

$$\Rightarrow \quad \left(\partial^J_{x_1} + \dots + \partial^J_{x_n}\right) \langle \mathcal{O}_0(x_1) \dots \mathcal{O}_0(x_n) \rangle = 0$$

• Step 3.2 : solving the constraint

[Maldacena, Zhiboedov, 2010]
[Zamolodchikov^2, 1978]

× 4-point function:

$$\left(\partial_{x_1}^3 + \ldots \partial_{x_n}^3\right) \langle O_0(x_1) O_0(x_2) O_0(x_3) O_0(x_n) \rangle = 0$$

$$\Rightarrow \quad \langle O_0 O_0 O_0 O_0 \rangle = t \, \text{GF Boson} + (1-t) \, \text{GF Fermion}$$

continuity : only 1 boson in UV $\rightsquigarrow$ $t = 1$

× n-point function: $\left(\sum_i \partial_{x_i}^J\right) \langle O_0(x_1) \ldots O_0(x_n) \rangle = 0$ J odd

momentum space + famous argument for S-matrix factorization

$$\Rightarrow \quad \langle O_0(p_1) \ldots O_0(p_{2n}) \rangle = \underbrace{\delta(p_1+p_2)\,\delta(p_3+p_4) \ldots \delta(p_{2n-1}+p_{2n}) \, f(p_1, \ldots p_{2n}) + }_{\text{permutations}}$$

(2n+1 point-function = 0)

- conformal invariance:

$$\langle \mathcal{O}_0(p_1) \cdots \mathcal{O}_0(p_{2n}) \rangle = \sum_{\text{perm}} \frac{C_{(12)(34)\cdots(2n-1\,2n)}}{x_{12}^{2\Delta_0}\, x_{34}^{2\Delta_0} \cdots x_{2n-1,2n}^{2\Delta_0}}$$

- Permutation symmetry

$$\boxed{\langle \mathcal{O}_0(p_1) \cdots \mathcal{O}_0(p_{2n}) \rangle = \text{Generalized free boson}}$$

∎

Comments:

- @ "special values" of Δ_φ result unchanged @ least at leading order in perturbation theory.
- Interesting leeway for a tower of fields w/ fine tuned masses: " $\Delta\varphi_i = \Delta\varphi_0 + i$ "

Two extensions:

1. Continuous deformations of a CFT:

- A bit harder: in UV there are always integer spaced quasi primaries:

$$[Q^{(3)}_{-3}, D] = \partial_x^3 D + \# \, \partial D^2$$

displacement $= T|_{\partial AdS_2}$

exists! $T^2|_{\partial AdS}$

⇒ weaker statement based on $\gamma_D \neq \gamma_{D^2}$ when deforming by a primary (could be extended to any $Q^{(j)}$ working out the action from Virasoro)

[Antunes, Milam, Lauria, Van Rees '23 '24]

2. Boundary localized interactions:

- A bit easier, can relax continuity because bulk always has free field

⇒ stronger statement valid at IR of boundary RG.

- E.g. long-range Ising, must have $\Delta = 1$ ops. to violate Cardy conditions

5. Summary

- Free fields and CFTs have higher spin charges which cannot be preserved under deformations
- AdS killing tensors are demanding: require full conservation
- Charge conservation can be broken at the boundary
  $\Rightarrow$ protected boundary operators
- Higher spin charges imply sum rules for BOE data
  (see paper)

& Outlook

- Weakly broken charges & flat space limit

- Integrability in AdS via different mechanisms
  (e.g. non-local charges)

- Explicit examples : $\frac{1}{2}$ BPS Wilson loop in $\mathcal{N}=4$ SYM.

Thank you

3841 m

3776 m

3403 m