Monopole-vortex continuity of $\mathcal{N}=1$ SYM on $\mathbb{R}^2\times S^1\times S^1$ with ‘t Hooft twist

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Based on:
[arXiv:2410.21392 [hep-th]] with Tatsuhiro Misumi (Kindai U.) and Yuya Tanizaki (YITP)
(special thanks to Mithat Ünsal(NCSU))
+ PRL **133**, 171902 (2024) [arXiv:2405.12402 [hep-th]] with Yuya Tanizaki (YITP)

Semiclassical approaches to confinement

Deformation (preserving confinement)

SU(N) Yang-Mills theory, etc.
(strongly coupled, hard problem)

→

Deformed theory
(weakly coupled, easy problem)

↑

Solve this theory (semiclassically)
& study confinement/vacuum structure

Two semiclassical approaches (for pure Yang-Mills)

Motto: deforming SU(N) YM to **weakly-coupled** theory with **keeping confinement**.

compactification

center-stabilizing deformation
(to avoid deconfinement transition)

3d monopole semiclassics

[Ünsal '07, Ünsal-Yaffe '08,…]

SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S^1}$ with "center-stabilizing deformation"

⇒ **confinement by 3d monopole gas**

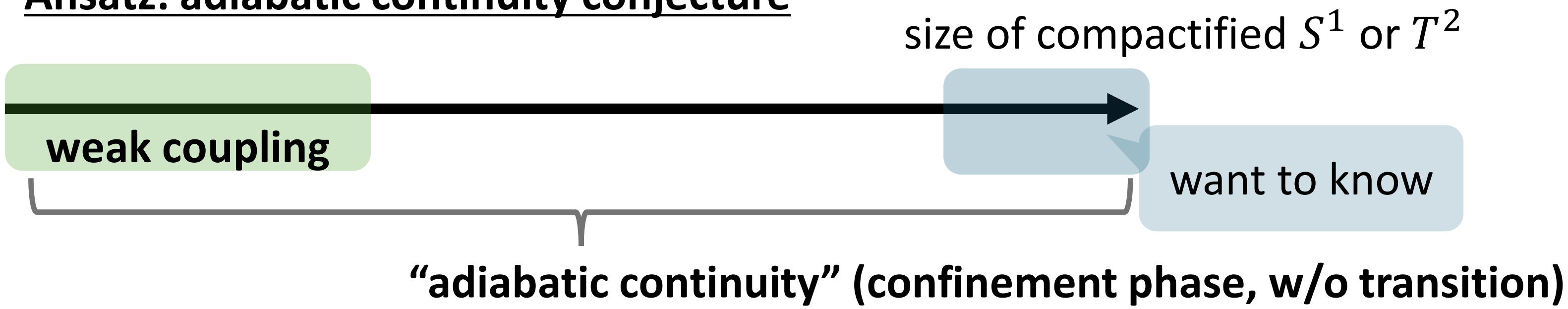
2d center-vortex semiclassics

[Tanizaki-Ünsal '22, …]

SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T^2}$ with 't Hooft flux ($\mathbb{Z}_N^{[1]}$ background)

⇒ **confinement by 2d center-vortex gas**

Ansatz: adiabatic continuity conjecture



Bridging two approaches [YH-Tanizaki ’24]

3d monopole semiclassics

[Ünsal ’07, Ünsal-Yaffe ’08,…]

SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S^1}$ with “center-stabilizing deformation”

⇒ **confinement by 3d monopole gas**

2d center-vortex semiclassics

[Tanizaki-Ünsal ’22, …]

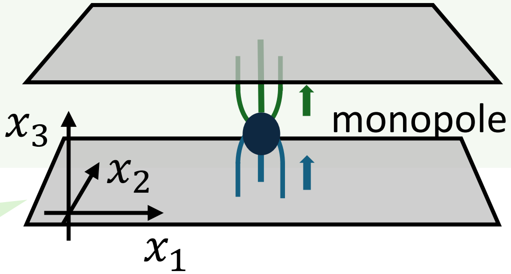
SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T^2}$ with ‘t Hooft flux

⇒ **confinement by 2d center-vortex gas**

Consider an interpolating setup: SU(N) Yang-Mills on $\mathbb{R}^2 \times \overbrace{(\boldsymbol{S^1})_3 \times (\boldsymbol{S^1})_4}^{\text{'t Hooft flux}}$

center-stabilizing deformation

Monopole in $\mathbb{R}^2 \times \boldsymbol{S^1}$



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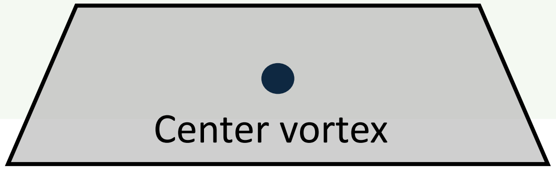
Center vortex in 2d



“monopole as junction of center vortices”

3d/2d continuity for SYM

[YH, Misumi, Tanizaki '24]

$\mathcal{N} = 1$ super-Yang-Mills theory

$\mathcal{N} = 1$ SU(N) SYM $=$ **One-flavor massless adjoint QCD**

Field contents: SU(N) gluon a_μ+ adjoint Weyl fermion λ (gluino)

- **Why $\mathcal{N} = 1$ SYM ?:**

Some quantities (e.g., Witten index/partition function) are exactly kept under the spatial compactification (without deformation):

adiabatic continuity is "exact"

⇒ nice playground for examining semiclassics!

spatial compactification:
periodic boundary condition for fermion

IR scenario

$U(1)_{\text{chiral}} : \lambda \to e^{i\,\alpha}\,\lambda, \qquad \bar{\lambda} \to e^{-i\,\alpha}\bar{\lambda}$

Only $(\mathbb{Z}_{2N})_{\text{chiral}}$ is non-anomalous.

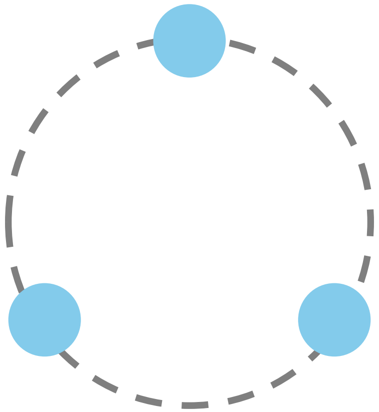
- **Symmetry:** $(\mathbb{Z}_{2N})_{\text{chiral}} \times \mathbb{Z}_N^{[1]}$

There is mixed anomaly.

- assume confinement ($\mathbb{Z}_N^{[1]}$ is unbroken)
  $\Rightarrow$ **(Natural) IR scenario:** $(\mathbb{Z}_{2N})_{\text{chiral}} \to \mathbb{Z}_2$ **SSB**

N vacua $\{\ |k\rangle\ \}_{k=0,\cdots,N-1}$ with chiral condensate $\langle k|\text{tr}(\lambda\lambda)|k\rangle \sim e^{2\pi i k/N}$

consistent with Witten index [Witten'82]



Outline of this talk

- Two semiclassics:
  - SYM on $\mathbb{R}^3 \times \boldsymbol{S^1}$ with periodic BC [Davies-Hollowood-Khoze-Mattis '99,…][Ünsal '07]

    At small S^1, the vacuum structure can be understood through monopole/bion gas
  - SYM on $\mathbb{R}^2 \times \boldsymbol{T^2}$ with 't Hooft flux [Tanizaki-Unsal '22]

    The vacuum structure is partially understood through center-vortex gas, but **some puzzling issues remain**.
- Aim of this talk: **understand/resolve (one of) these issues by the 3d/2d continuity.**

SYM on $\mathbb{R}^3 \times \boldsymbol{S^1}$ (with periodic BC)
⇒ 3d $U(1)^{N-1}$ gauge theory + monopoles

⟹

SYM on $\mathbb{R}^2 \times \boldsymbol{T^2}$ with 't Hooft flux

SU(N) SYM on $\mathbb{R}^2 \times (S^1)_3 \times (S^1)_4$ with 't Hooft flux (small L_4, varying L_3)
⇒ 3d EFT on $\mathbb{R}^2 \times (S^1)_3$ with $\left(\mathbb{Z}_N^{[0]}\right)_{3d}$-twisted boundary condition.

3d monopole-bion semiclassics

[Davies-Hollowood-Khoze-Mattis ’99,… for SYM] [Ünsal ’07, … for QCD(adj)]

- SYM on $\mathbb{R}^3 \times S^1$ (with periodic BC)
  3d EFT: two compact scalars (holonomy $\vec{\phi}$, dual photon $\vec{\sigma}$) & Cartan fermion $\vec{\lambda}$

SYM on $\mathbb{R}^3 \times S^1$ $\Rightarrow$ 3d gauge field + fermion + holonomy (adjoint scalar)
(adjoint higgsing: $SU(N) \to U(1)^{N-1}$ & 3d abelian duality)
$\Rightarrow$ (holonomy $\vec{\phi}$, dual photon $\vec{\sigma}$) + Cartan fermion $\vec{\lambda}$ **(+ BPS/KK monopole-instantons)**

Monopole amplitude

$$[\mathcal{M}_i] \sim e^{-\frac{8\pi^2}{Ng^2}} \, \mathrm{e}^{i\vec{\alpha}_i\cdot\vec{z}+i\frac{\theta}{N}} \left(\vec{\alpha}_i \cdot \vec{\lambda}\right)^2$$

- **BPS/KK Monopole carries two fermionic zeromodes**

$\Rightarrow$ leading to gluino condensate $\langle \mathrm{tr}(\lambda\lambda)\rangle \neq 0$; but no bosonic potential

- **Bion (monopole-antimonopole pair) induces bosonic potential (for $(\vec{\phi}, \vec{\sigma})$)**

In terms of superpotential, the bion-induced potential is automatically included

2d center-vortex semiclassics/Questions

- One can feel "N vacua" in 2d semiclassics (by considering $\mathbb{R} \times S^1 \times T^2$ setup [Tanizaki-Unsal '22])
- **A few unsatisfactory points:**
  - Everything becomes heavy due to 't Hooft-twisted compactification, so the 2d dilute-gas effective theory is not straightforward.

  Since the center-vortex carries two zeromodes, we'd like to write $[\mathcal{V}] \sim e^{-\frac{8\pi^2}{Ng^2}+i\frac{\theta}{N}}(\lambda\lambda)$ But λ is heavy…

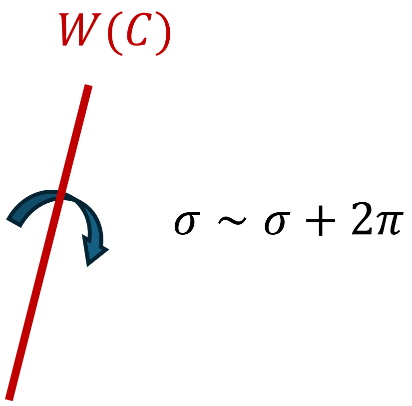
  - **Perimeter law of 2d Wilson loop? deconfinement? (counter-intuitive…)**
    [(From mixed anomaly,) Wilson loop should behave as a domain wall of discrete chiral symmetry in the 2d semiclassics.]

this talk

  - Role of magnetic bion in 2d? (magnetic bion causes confinement in 3d)

  Let's observe the reduction from 3d to 2d!

Wilson loop in 3d semiclassics



- Let us consider SU(2) $\mathcal{N} = 1$ SYM, for simplicity:

The dual photon is a compact boson $\sigma \sim \sigma + 2\pi$

- **Wilson loop: a defect operator with nontrivial monodoromy $\boldsymbol{\sigma \sim \sigma + 2\pi}$**

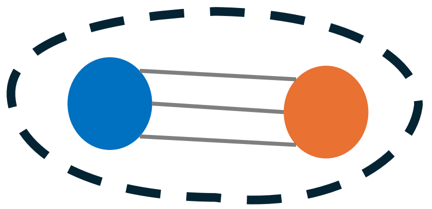
($\because$ Electromagnetic duality: 3d gauge field $\leftrightarrow$ compact scalar $\sigma \sim \sigma + 2\pi$)

- Bions give the bosonic potential:

Magnetic bion induces a potential $\sim \cos(2\sigma)$: **two minima**

Magnetic bion carries **no fermionic zeromodes**

but has **magnetic charge 2**

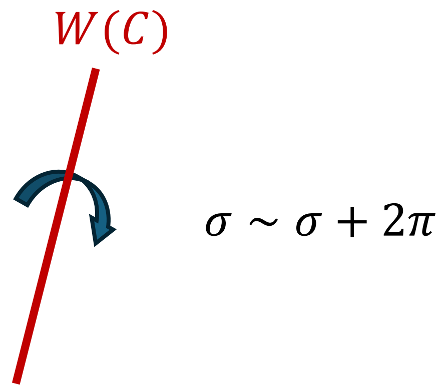


Magnetic bion
(molecule of BPS-$\overline{\mathrm{KK}}$ monopole)

- **Double string picture**:

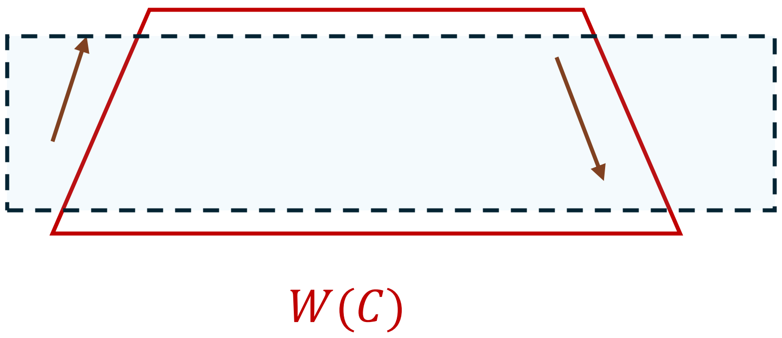
The Wilson loop (defect) emits two kinks ($\Delta\sigma = \pi$) [Anber-Poppitz-Sulejmanpasic '15]

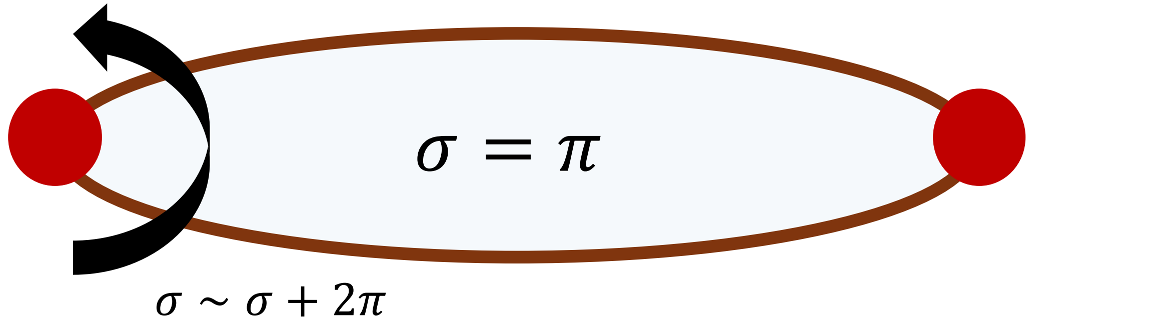
Wilson loop in 3d semiclassics



- Let us consider SU(2) $\mathcal{N}=1$ SYM, for simplicity.
- **Wilson loop: a defect operator with nontrivial monodoromy $\boldsymbol{\sigma\sim\sigma+2\pi}$**
- **Magnetic bion potential:** $\sim\cos(2\sigma)$
- **Double string picture**:

The Wilson loop emits two kinks [Anber-Poppitz-Sulejmanpasic '15]



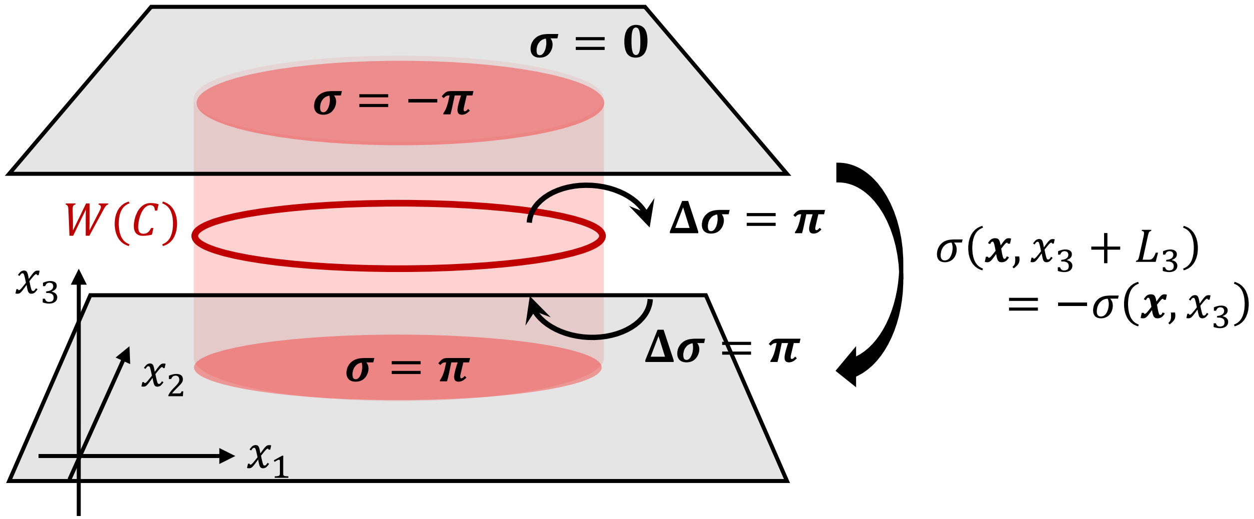


confining string = pair of two kinks

Wilson loop transmutes to domain wall

- Now, we look at the reduction from 3d semiclassics to 2d semiclassics:
  SU(2) $\mathcal{N}=1$ SYM on $\mathbb{R}^2 \times (\boldsymbol{S}^1)_3 \times (\boldsymbol{S}^1)_4$ (with small L_4, large-but-finite L_3).
  $\Rightarrow$ 3d EFT on $\mathbb{R}^2 \times (\boldsymbol{S}^1)_3$ with $\left(\mathbb{Z}_N^{[0]}\right)_{3\mathrm{d}}$-twisted boundary condition
  (= "charge-conjugation-twisted" BC for $N=2$)
- Consider a large Wilson loop $|C| \gg L_3$:

This is domain wall of $(\mathbb{Z}_{2N})_{\mathbf{chiral}}$!



(Area) = L_3 × (Perimeter)

Confining string extends along the compactified direction

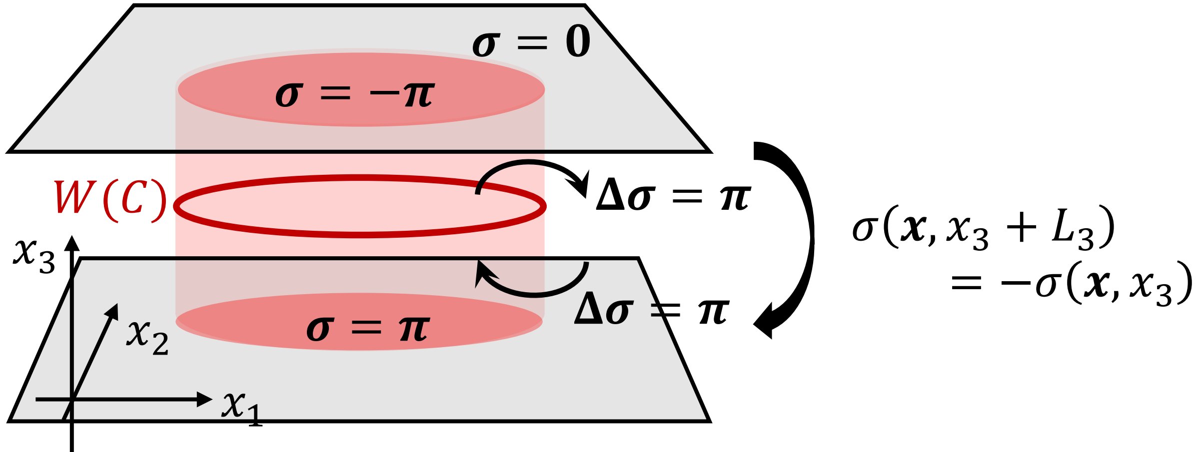
(Technical) Summary

- One of unclear points of 2d center-vortex semiclassics in $\mathcal{N}=1$ SYM:

2d Wilson loop follows the perimeter law. deconfinement? What happens?

- The 3d-2d continuity gives an explanation:
  The 3d double-string picture explains that **the Wilson loop becomes $(\mathbb{Z}_{2N})_{\mathbf{chiral}}$ domain wall** in the 2d perspective.

3d area law / 2d perimeter law
(Area) = L_3 × (Perimeter)



Generalization to SU(N)/ QCD(adj) is easy

Summary

Nice S^1/T^2 compactifications give **tractable confining theory**

- Spatial S^1 compactification: confinement by monopoles,
- ‘t Hooft-twisted T^2 compactification: confinement by center vortices,

and they are continuously connected.

This work: interplay between 3d/2d semiclassics in N=1 SYM (+QCD(adj)).
In this talk, we have focused on why 2d Wilson loop follows the perimeter law ("deconfinement"), whereas 2d semiclassics is continuously connected to 3d semiclassics.