Non-Supersymmetric Heterotic Branes

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Non-perturbative methods in QFT

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Non-Supersymmetric Heterotic Strings

- **We all are familiar with the $(E_8 \times E_8) \rtimes \mathbb{Z}_2$ and $\mathrm{Spin}(32)/\mathbb{Z}_2$ heterotic string theories.**

- **But there are known to be 7 other heterotic strings:**

$$\mathfrak{e}_8\ , \quad \mathfrak{u}(16)\ , \quad (\mathfrak{e}_7 \times \mathfrak{su}(2))^2\ , \quad \mathfrak{o}(8) \times \mathfrak{o}(24)\ ,$$
$$\mathfrak{o}(16) \times \mathfrak{e}_8\ , \quad \mathfrak{o}(32)\ , \quad \mathfrak{o}(16) \times \mathfrak{o}(16)$$

- **These were originally found in** [Kawai, Lewellen, Tye '86; Dixon, Harvey '86]**.**

  – Until very recently, it was unknown if these were all of the possibilities, or if there could be others. The completeness of this list was proven in [Boyle Smith, Lin, Tachikawa, Zheng '23] (see also [Rayhaun '23; Höhn, Möller '23]), where the set of all $c = 16$ spin-CFTs was classified.

- **All of these theories are non-supersymmetric. All but the last one has a closed string tachyon.**

Non-Supersymmetric Heterotic Strings

- **The closed string tachyon is worrying, but not fatal. It simply indicates that we are expanding around the wrong vacuum.**

- **We can *condense* the tachyon to obtain tachyon-free vacua in lower dimensions** [Helleman, Swanson '07; **JK** '20]

$$\begin{aligned}
\mathbb{R}^{1,9}\times\mathfrak{e}_8 \quad &\longrightarrow \quad \mathbb{R}^{1,7}\times\mathbb{R}_{\mathrm{lin.\,dil.}}\times\mathfrak{e}_8\\
\mathbb{R}^{1,9}\times\mathfrak{u}(16) \quad &\longrightarrow \quad \mathbb{R}^{1,6}\times\mathbb{R}_{\mathrm{lin.\,dil.}}\times\mathfrak{su}(16)\\
\mathbb{R}^{1,9}\times(\mathfrak{e}_7\times\mathfrak{su}(2))^2 \quad &\longrightarrow \quad \mathbb{R}^{1,4}\times\mathbb{R}_{\mathrm{lin.\,dil.}}\times\mathfrak{e}_7\times\mathfrak{e}_7\\
\mathbb{R}^{1,9}\times\mathfrak{o}(8)\times\mathfrak{o}(24) \quad &\longrightarrow \quad \mathbb{R}\times\mathbb{R}_{\mathrm{lin.\,dil.}}\times\mathfrak{o}(24)\\
\mathbb{R}^{1,9}\times\mathfrak{o}(16)\times\mathfrak{e}_8 \quad &\longrightarrow \quad \mathbb{R}\times\mathbb{R}_{\mathrm{lin.\,dil.}}\times\mathfrak{o}(8)\times\mathfrak{e}_8\\
\mathbb{R}^{1,9}\times\mathfrak{o}(32) \quad &\longrightarrow \quad \mathbb{R}\times\mathbb{R}_{\mathrm{lin.\,dil.}}\times\mathfrak{o}(24)
\end{aligned}$$

- **All of the resulting vacua are (perturbatively) stable, but have a linear dilaton. Note that we obtain vacua in** 9d, 8d, 6d, **and** 2d**; the latter are equivalent to the 2d heterotic strings in** [Davis, Larsen, Seiberg '05]**.**

Non-Supersymmetric Heterotic Strings

- **How should we interpret these vacua?**
- **A hint: another place in string theory where a linear dilaton arises is in the near horizon region of an NS5 brane.**
- **So maybe these non-supersymmetric strings exist in order to describe the near-horizon regions of some *new branes*?**
- **The $d = 9, 8, 6, 2$ vacua would be appropriate to describe the near-horizons of 7-,6-,4-,0-branes.**
- **The branes would be in the supersymmetric $(E_8 \times E_8) \rtimes \mathbb{Z}_2$ or $\mathrm{Spin}(32)/\mathbb{Z}_2$ heterotic strings, and would break all supersymmetries.**
- **Of course, no such branes were known. . .**

Non-Supersymmetric Heterotic Branes

- **A separate question: the old quantum gravity lore states that (see** [McNamara, Vafa '19] **for a modern version):**

> ANY CONSISTENT THEORY OF QUANTUM GRAVITY MUST CONTAIN OBJECTS CARRYING ALL POSSIBLE CHARGES.

- **Basically any topological invariant should count as a charge. Note that:**

$$\pi_0\left((E_8 \times E_8) \rtimes \mathbb{Z}_2\right) \simeq \mathbb{Z}_2 \qquad \pi_1(\mathrm{Spin}(32)/\mathbb{Z}_2) \simeq \mathbb{Z}_2$$

$$\pi_3\left((E_8 \times E_8) \rtimes \mathbb{Z}_2\right) \simeq \mathbb{Z} \times \mathbb{Z} \qquad \pi_7(\mathrm{Spin}(32)/\mathbb{Z}_2) \simeq \mathbb{Z}$$

- **What carries the charges?**

- **They would capture non-trivial configurations on S^1, S^2, S^4, S^8, which is just what is needed to surround a 7-,6-,4-,0-brane!**

- **Are the two related???**

The NS5-brane

- Recall another context in which a linear dilaton arises: the NS5-brane.
- In supergravity, we have the following extremal solution:

$$\begin{aligned} ds^2_{\mathsf{NS5}} &= dx_{\|}^2 + e^{2\phi} dx_{\perp}^2 \\ e^{2\phi} &= e^{2\phi(\infty)} + \frac{r_0^2}{r^2} \end{aligned} \qquad H_{mnp} = -\epsilon_{mnp}{}^{q}\, \partial_q \phi \qquad A_m = -2\rho^2\, \overline{\Sigma_{nm}} \frac{x^n}{r^2(r^2+\rho^2)}$$

where r is the transverse radial direction. [Callan, Harvey, Strominger '91]

- In the near-horizon limit $r \to 0$:

$$ds^2_{\mathsf{NS5}} = dx_{\|}^2 + \frac{r_0^2}{r^2} dr^2 + r_0^2 d\Omega_3^2 \qquad e^{2\phi} = \frac{r_0^2}{r^2}$$

- Or defining $y := \log \frac{r}{r_0}$,

$$ds^2_{\mathsf{NS5}} = dx_{\|}^2 + dy^2 + r_0^2 d\Omega_3^2 \qquad \phi = -y$$

The NS5-brane

- **So the extremal, near-horizon solution is:**

$$ds^2_{\mathsf{NS5}} = dx_{\|}^2 + dy^2 + r_0^2\, d\Omega_3^2 \qquad\qquad \phi = -y$$

with one unit of H flux through S^3 (in general r_0 is small though!)

- **There is an exact worldsheet description for this near-horizon solution** [Callan, Harvey, Strominger '91]

$$\mathbb{R}^{1,5} \times \mathbb{R}_{\text{linear dilaton}} \times \mathfrak{su}(2)_\bullet \times \mathfrak{g}$$

- **Intuitively: think of this as a 7d vacuum with a linear dilaton and $\mathfrak{su}(2) \times \mathfrak{g}$ gauge group.**

- **This is expected to be the holographic dual to the 6d LST living on the NS5 brane.** [Aharony, Berkooz, Kutasov, Seiberg '98]

The 6-brane

- **We now try an analogous thing for the 6-brane. Our considerations before suggest that the near-horizon limit is described by**

$$\mathbb{R}^{1,6} \times \mathbb{R}_{\text{linear dilaton}} \times \mathfrak{su}(16)_\bullet$$

- **To reproduce this, we note that there exists a black 6-brane solution for the $\mathfrak{so}(32)$ heterotic string** [Horowitz, Strominger '91]

$$\begin{aligned} ds^2 &= -\frac{(1-\frac{r_+}{r})}{(1-\frac{r_-}{r})}dt^2 + d\vec{x}^2 + \frac{dr^2}{(1-\frac{r_+}{r})(1-\frac{r_-}{r})} + r^2 d\Omega_2^2 \\ e^{-2\phi} &= e^{-2\phi(\infty)}\left(1-\frac{r_-}{r}\right) \qquad\qquad 8r_+r_- = \alpha' \sum_{i=1}^{16} q_i^2 \\ \frac{F_{\mathfrak{so}(32)}}{2\pi} &= \bigoplus_{i=1}^{16} \begin{pmatrix} 0 & q_i \\ -q_i & 0 \end{pmatrix} \frac{\text{vol}(S^2)}{4\pi} \end{aligned}$$

The 6-brane

- **Taking the extremal, near-horizon limit gives:**

$$ds^2 = dx_{\|}^2 + dy^2 + r_0^2\, d\Omega_2^2 \qquad\qquad \phi = -y$$

$$\frac{F_{\mathfrak{so}(32)}}{2\pi} = \bigoplus_{i=1}^{16} \begin{pmatrix} 0 & q_i \\ -q_i & 0 \end{pmatrix} \frac{\mathrm{vol}(S^2)}{4\pi}$$

so we have an infinite throat with S^2 of constant size $r_0 := \sqrt{\frac{\alpha'}{8}\sum_{i=1}^{16} {q_i}^2}$.

- **An important fact about the $\mathfrak{so}(32)$ heterotic string is that the global form of the gauge group is $\mathrm{Spin}(32)/\mathbb{Z}_2$. This makes it possible to choose $q_i = 1/2$.**

- **If we choose $q_i = 1/2$ for all i, then we preserve $\mathfrak{u}(16) \subset \mathfrak{so}(32)$.**

- **We expect this to give the six-brane, but note that $r_0 = (\alpha'/2)^{1/2}$, so that supergravity is not reliable.**

The 6-brane

- **At this point we transition to a worldsheet analysis. Worldsheet version of near-horizon limit would be (ignoring flux):**

$$\mathbb{R}^{1,6} \times \mathbb{R}_{\text{linear dilaton}} \times (\mathcal{N}=(0,1)\ \ S^2) \times \mathfrak{so}(32)_1$$

- **It turns out that**

$$\mathfrak{so}(32)_1 = [\mathfrak{su}(16)_1 \times \mathfrak{so}(2)_1]/(-1)^{\mathsf{F_L}}$$

- **With the flux, we can reorganize the worldsheet theory as**

$$\mathbb{R}^{1,6} \times \mathbb{R}_{\text{linear dilaton}} \times [(\mathcal{N}=(1,1)\ \ S^2) \times \mathfrak{su}(16)_1]/(-1)^{\mathsf{F_L}}$$

(because vector bundle of $\mathfrak{so}(2)_1$ = tangent bundle of S^2)

- **This is not a solution to the string equations of motion since it is not conformal. But it flows in the IR to the following theory:**

$$\mathbb{R}^{1,6} \times \mathbb{R}_{\text{linear dilaton}} \times \mathfrak{su}(16)_1$$

The 6-brane

- **We end up with an 8d spacetime with linear dilaton and $\mathfrak{su}(16)$ gauge group. This is precisely the 8d vacuum obtained via closed string tachyon condensation in the non-SUSY $\mathfrak{u}(16)$ string!**

- **So indeed, the near-horizon limit of the 6-brane is described by this vacuum.**

- **What charge does the 6-brane carry?**
  - Recall that it sources the following flux: $\int_{S^2} \frac{F_{\mathfrak{so}(32)}}{2\pi} = \frac{1}{2} \bigoplus_{i=1}^{16} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
  - This is incompatible with the vector representation, but is compatible with the adjoint and one of the spinor representations.
  - Given a $\mathrm{Spin}(32)/\mathbb{Z}_2$ bundle, the obstruction to it being a $SO(32)$ bundle is captured by a class $\widetilde{w}_2$ (c.f. $\pi_1(\mathrm{Spin}(32)/\mathbb{Z}_2) = \mathbb{Z}_2$).
  - This is the charge carried by the brane.

Other branes

- **The key tool in the above analysis was the identity**

$$\mathfrak{so}(32)_1 = [\mathfrak{su}(16)_1 \times \mathfrak{so}(2)_1]/(-1)^{\mathsf{F_L}}$$

- **There are three (actually 5) similar identities:**

$$\begin{aligned} \mathfrak{so}(32)_1 &= [\mathfrak{so}(24)_1 \times \mathfrak{so}(8)_1]/(-1)^{\mathsf{F_L}} \\ (\mathfrak{e}_8 \times \mathfrak{e}_8)_1 &= [(\mathfrak{e}_7 \times \mathfrak{e}_7)_1 \times \mathfrak{so}(4)_1]/(-1)^{\mathsf{F_L}} \\ (\mathfrak{e}_8 \times \mathfrak{e}_8)_1 &= [(\mathfrak{e}_8)_2 \times \mathfrak{so}(1)_1]/(-1)^{\mathsf{F_L}} \end{aligned}$$

- **These can be used to give exact worldsheet descriptions for 0-, 4-, and 7-branes, respectively.**

0-, 4-, and 6-branes

- **For $p = 0, 4, 6$, the idea is to use:**

$$
\begin{array}{ccl}
\mathbb{R}^{1,p} \times \mathbb{R}_{\text{lin. dil.}} & \times & [\mathcal{N} = (0,1)\ S^{8-p}] \times \mathfrak{g}_1 \\
& \Big\downarrow & \text{add flux and use } \mathfrak{g}_1 = [\mathfrak{h}_k \times \mathfrak{so}(8-p)_1]/(-1)^{\mathsf{F_L}} \\
\mathbb{R}^{1,p} \times \mathbb{R}_{\text{lin. dil.}} & \times & [\mathfrak{h}_k \times \left(\mathcal{N} = (1,1)\ S^{8-p}\right)]/(-1)^{\mathsf{F_L}} \\
& \Big\downarrow & \text{RG flow} \\
\mathbb{R}^{1,p} \times \mathbb{R}_{\text{lin. dil.}} & \times & \mathfrak{h}_k
\end{array}
$$

- **The results match with those coming from closed string tachyon condensation in non-SUSY strings.**

- **SUGRA solutions for the $p = 0, 4$ were given in** [Fukuda, Kobayashi, Watanabe, Yonekura '24]

The 7-brane

- The 7-brane is a bit more subtle. In that case we end up with an $\mathcal{N}=(1,1)$ S^1 sigma model, so there is seemingly no RG flow.

- However, say that we take the S^1 to have holonomy for the $\mathbb{Z}_2$ interchanging the two copies of E_8, and also give it anti-periodic spin structure.

- Now consider the operator

$$\psi_L \psi_R e^{i\widehat{\phi}}\ , \qquad \widehat{\phi} = \text{dual of } \phi$$

Because $\mathbb{Z}_2^{\text{perm}}$ is the quantum dual to $(-1)^{\mathsf{F_L}}$, the operator $e^{iw\widehat{\phi}}$ has charge $(-1)^{\mathsf{F_L}} = (-1)^w$. Then the above operator survives the gauging of $(-1)^{\mathsf{F_L}}$, and it also survives the GSO projection.

- So the above operator exists in the theory, but since

$$h_L = h_R = \frac{1}{2} + \frac{1}{4}R^2$$

when R is small it gives a tachyon. Condensing it gives $\mathbb{R}^{1,p} \times \mathbb{R}_{\text{lin. dil.}} \times \mathfrak{h}_k$ as before.

Summary of branes

- **We have now identified the following four branes:**
  - A 0-brane in the $\mathrm{Spin}(32)/\mathbb{Z}_2$ heterotic string. This brane serves as an endpoint for the heterotic string [Polchinski '05].
  - A 4-brane in the $(E_8 \times E_8) \rtimes \mathbb{Z}_2$ heterotic string. This brane can be interpreted as an $M5$ stretched between two $M9$s in Horava-Witten theory [Bergshoeff, Gibbons, Townsend '06].
  - A 6-brane in the $\mathrm{Spin}(32)/\mathbb{Z}_2$ heterotic string. It sources gauge configurations without vector structure.
  - A 7-brane in the $(E_8 \times E_8) \rtimes \mathbb{Z}_2$ heterotic string. This brane has monodromy for the $\mathbb{Z}_2$ factor exchanging the two copies of E_8.

SQFTs and TMF

- **Let's now make some more math-oriented comments.**
- **Begin by defining**

$$\mathrm{SQFT}_n := \{\mathcal{N}=(0,1) \text{ SQFTs with grav. anomaly } n\} \,/\, \sim$$

- **The equivalence relation is roughly "identification by continuous deformations" (including flowing up and down RG flows).**
  - More concretely: say $Y_{n+1} \subset \mathrm{SQFT}_{n+1}$ has a *boundary* if it has a non-compact direction in which $Y_{n+1} \to \mathbb{R}_{>0} \times X_n$, and denote $X_n = \partial Y_{n+1}$.
  - The equivalence relation is then defined similarly to bordism
- **There is a conjecture due to** [Stolz, Teichner '02; '11]**:**

$$\mathrm{SQFT}_n = \mathrm{TMF}_n$$

SQFTs and TMF

- **The groups TMF_n have been computed thanks to monumental efforts by mathematicians. For example, we have:**

$$\mathrm{TMF}_{-31} = \mathbb{Z}_2 + \mathbb{Z}_2 \qquad \mathrm{TMF}_{-26} = 0$$
$$\mathrm{TMF}_{-30} = \mathbb{Z}_2 + \mathbb{Z}_2 \qquad \mathrm{TMF}_{-25} = 0$$
$$\mathrm{TMF}_{-29} = 0 \qquad \mathrm{TMF}_{-24} = \mathbb{Z} + \mathbb{Z}$$
$$\mathrm{TMF}_{-28} = \mathbb{Z} + \mathbb{Z}_2 \qquad \mathrm{TMF}_{-23} = \mathbb{Z}_2$$
$$\mathrm{TMF}_{-27} = 0 \qquad \mathrm{TMF}_{-22} = \mathbb{Z}_2$$

SQFTs and TMF

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- **What are the invariants that label a TMF class?**
  - Given any SQFT, one can compute its ordinary/mod-2 elliptic genus $\mathrm{EG} : \mathrm{TMF}_\bullet \to \mathrm{KO}_\bullet((q))$. This gives one invariant.
  - However, EG is not enough to completely specify the TMF class. In particular, the classes in blue above live in $\ker(\mathrm{EG})$.

TMF and branes

- **Returning to our branes, recall that we had**

$$[\mathcal{N}=(0,1)\ S^{8-p}]\times\mathfrak{g}_1 \xrightarrow{\text{continuous def.}} \mathfrak{h}_k$$

so that $[(\mathcal{N}=(0,1)\ S^{8-p})\times\mathfrak{g}_1]=[\mathfrak{h}_k]$ **(for appropriate fluxes).**

- **Note that** $\mathfrak{g}_1$ **has** $(c_L,c_R)=(16,0)$ **and** S^{8-p} **has** $8-p$ **left- and right-moving bosons and** $8-p$ **right-moving fermions, so that** $(c_L,c_R)=(8-p,\frac{3}{2}(8-p))$**. In total then, the above classes have** $n=2(c_R-c_L)=-p-24$**, so that**

$$\begin{array}{ll}
\text{7-brane}: & [(\mathfrak{e}_8)_2]\in \mathrm{TMF}_{-31}\\
\text{6-brane}: & [(\mathfrak{su}(16))_1]\in \mathrm{TMF}_{-30}\\
\text{4-brane}: & [(\mathfrak{e}_7\times\mathfrak{e}_7)_1]\in \mathrm{TMF}_{-28}\\
\text{0-brane}: & [(\mathfrak{so}(24))_1]\in \mathrm{TMF}_{-24}
\end{array}$$

TMF and branes

- Going back to our previous list,

$$\mathrm{TMF}_{-31} = \mathbb{Z}_2 + \mathbb{Z}_2 \qquad \mathrm{TMF}_{-26} = 0$$

$$\mathrm{TMF}_{-30} = \mathbb{Z}_2 + \mathbb{Z}_2 \qquad \mathrm{TMF}_{-25} = 0$$

$$\mathrm{TMF}_{-29} = 0 \qquad \mathrm{TMF}_{-24} = \mathbb{Z} + \mathbb{Z}$$

$$\mathrm{TMF}_{-28} = \mathbb{Z} + \mathbb{Z}_2 \qquad \mathrm{TMF}_{-23} = \mathbb{Z}_2$$

$$\mathrm{TMF}_{-27} = 0 \qquad \mathrm{TMF}_{-22} = \mathbb{Z}_2$$

- For the 0-brane, it's easy to check that $\mathrm{EG}(\mathfrak{so}(24)_1)$ is non-trivial.

- For the other three cases, one can check that $\mathrm{EG}(\mathfrak{h}_k) = 0$, and that the angular theories give the generators of the blue elements!

TMF class = brane charge?

- **It seems like TMF classes give a "worldsheet version" of brane charges in the heterotic string. But not quite:**
  - **TMF can be "too small":** The angular part of the NS5 brane $[S^3_{H=1} \times \mathfrak{g}_1]$ is trivial in TMF, because it is trivial in Ω^{SUGRA} (may be non-trivial in equivariant TMF though).
  - **TMF can be "too big":** We usually measure charges by considering a sphere at infinity that surrounds the object. For a generic TMF class though, the "sphere at infinity" might not be a sphere (above we could always write $[\mathfrak{h}_k] = [(\mathcal{N} = (0,1)\ S^{8-p}) \times \mathfrak{g}_1]$, but we can't in general)
- **Nevertheless, we could still try to use TMF identities to motivate dualities.**

TMF and dualities

- **Example: it can be shown that**

$$[(\mathfrak{e}_8)_2] \times [S^1_p] = [\mathfrak{su}(16)_1]$$

which suggests the following duality

$$\text{HE with 7-brane on } S^1_p \;\; = \;\; \text{HO with 6-brane}$$

(precise duality would require appropriate Wilson lines)

- **By similar means, one might guess that**

$$\begin{aligned} \text{HE with 7-brane on } S^1_p \times S^3_{H=1} \quad &= \quad \text{HO with 6-brane on } S^3_{H=1} \\ &= \quad \text{HE with NS5 on } S^1_p \times S^1_a \\ &= \quad \text{HO with NS5 on } S^2_v \end{aligned}$$

($S^1_p = S^1$ with periodic spin structure; $S^1_a = S^1$ with anti-periodic spin structure and permutation twist)

Summary

- **We have found 0- and 6-branes in the $\mathrm{Spin}(32)/\mathbb{Z}_2$ heterotic string, as well as 4- and 7-branes in the $(E_8 \times E_8) \rtimes \mathbb{Z}_2$ heterotic string.**

- **The near-horizon limits of these branes were shown to be described by the stable, lower-dimensional vacua of the non-SUSY heterotic strings.**

- **By the holographic dictionary, the latter should provide holographic descriptions of the worldvolume theories of the branes. So the reason that the non-SUSY heterotic strings exist is to describe the worldvolume theories of the non-SUSY branes!**

- **Intriguing connections to TMF, which remain to be fully understood.**

- **Other non-SUSY branes are in the process of being uncovered!** [Dierigl, Heckman, Montero, Torres '22; . . .]

The End (for now)

Thank you!